DELAY AT SIGNALIZED INTERSECTIONS CONSIDERING NON-STATIONARY TRAFFIC FLOW

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Word count: 4373 words text + 5 figures x 250 words (each) = 5623 words

Submission Date: November 11, 2016

ABSTRACT

Capacity Manuals use vehicle delay to assess the level of service for signalized intersections. Average delay is typically derived from capacity and traffic volume during an analysis period using adapted approaches on the basis of queueing theory. Variability of traffic volume during the analysis period can influence the resulting delay and needs to be taken into account. The impact of non-stationary arrival flow rates is not yet explained completely by queueing theory, therefore a simulation study is performed. The analysis of measurement data shows that traffic volume during the peak hour can be distributed in very different forms. A set of abstract flow patterns is derived, representing simplified flow profiles of the peak hour, split into 15-min-intervals. Microscopic traffic flow simulations are performed for different signal control programs and volume-to-capacity ratios to determine the delay caused by the different flow patterns. It is shown that in most cases delay is higher for non-stationary flow than for stationary flow. Finally, a correction factor is developed for the delay computation method used in the German Highway Capacity Manual (HBS) to better reflect the volume distribution within the design hour.

Keywords: Delay, Non-Stationarity, Signalized Intersection, Simulation, Level of Service

INTRODUCTION

Commonly, the quality or level of service (LOS) of an intersection regardless of its type is determined by average vehicle delay and is mainly influenced by traffic flow. The underlying computation methods mostly assume a certain uniformity of the traffic flow within the analysis period, but in reality vehicle volumes show different forms of variability during the peak hour. Therefore it is important to determine the influence of this variability and to adapt the procedures accordingly.

The U.S. Highway Capacity Manual (1) provides the peak hour factor (PHF) for any type of intersection to adjust the hourly flow rates of the peak hour if necessary. The German Highway Capacity Manual (HBS) (2) contains a “non-stationarity factor” within the computation method for the LOS of signalized intersections.

The task of investigating the degree of non-stationary traffic flow and its influence on the delay, as well as revising the HBS calculation methods is part of a research project. Its main topic is the assessment of traffic flow at intersections and it was carried out on behalf of the German Federal Highway Research Institute (BASt). The content of this paper is confined to signalized intersections and contains results of the research part on non-stationary traffic flow. First it focuses on the theoretical background of the calculation methods and the relevance of non-stationarity in a more general context, then it presents the practical investigation using empirical data and simulation, ending by some conclusions.

THEORETICAL BACKGROUND

To identify the possible effects of non-stationary traffic flow on delay it is crucial to understand how delay, or waiting time, is determined. Furthermore, non-stationary conditions may occur not only in the context of traffic engineering, so a short glance is taken at other research fields.

Evolution of calculation methods for average delay

In most guidelines, especially the U.S. HCM and the German HBS, the average delay for signalized intersections consists of two main parts. First, the course of the signal program cycles causes the so-called uniform delay (according to the HCM). The HBS describes it as the ‘basic waiting time due to the periodic change of red and green time’. Hence, the underlying computation method follows a deterministic approach, derived by Webster (3).

Second, the ‘waiting time due to the remaining queue at the end of the green time’ in the HBS corresponds to the incremental delay in the HCM. Additionally, the HCM takes into account the possible existence of a queue at the beginning of the analysis period.

That second part of the average delay, derived from the length of the queue, has a stochastic character. The principal idea is based on queueing theory, but not in its exact analytical form. Due to the random arrival process of vehicles during green time, steady-state queueing models were adapted first to compute the stochastic delay component. These models are subject to rather strong constraints, so that exact results are practically not possible to derive in the context of traffic flow. Hence, according to Rouphail et al. (4), already early modeling approaches, such as Webster’s, used approximation methods. But even in that way, steady-state queueing models are limited under realistic conditions, especially when demand reaches or actually exceeds capacity.

Therefore, time-dependent delay models evolved. The probably best-known procedure for intersections in general is the coordinate transformation technique by Kimber and Hollis (5). The main concept is to combine the stochastic steady-state model with a deterministic time-dependent queuing function. The steady-state model yields quite high delay values the closer the saturation gets to capacity. If capacity was reached, the resulting delay would be infinite, representing an asymptote at a volume-to-capacity ratio of 1 for the steady-state curve. The coordinate transformation technique then 'leans' that curve to the straight line of the deterministic formulation, thus using it as a new asymptote for the adapted steady-state-curve.

As explained by Rouphail et al., the Kimber and Hollis model shows some shortcomings when applied to signalized intersections. To obtain an improved formula for this type, Akçelik (6) utilized the same technique with different models representing the steady-state and the deterministic queue. He developed the model that is the basis of the computation methods for average delay in both guidelines considered here, the HCM and the HBS.

Non-stationarity and queueing theory

At first, the definition of non-stationarity is given. In general, it can be understood as the opposite or the absence of stationarity. Stationarity though is a property of a stochastic process and an essential principle in time series analysis. A time series is called (strictly) stationary if any sequence of its elements follows the same statistical distribution, at every point in time. If only the expectation value and variance are identical for two sequences, the time series is called weak stationary. (7)

Traffic flow, in a more practical context, is stationary, if it does not depend on time with a volume-to-capacity ratio smaller than one. Conversely, non-stationary traffic flow varies over time and may exceed capacity as well. (8)

As it is the basis of current delay calculation methods, stochastic queueing theory is considered regarding the existence and influence of non-stationarity on waiting time.

Queueing theory itself is a special case of Markov chains. Since the latter feature considerably more degrees of freedom for modeling, newer approaches, as those of Viti and van Zuylen (9) or Brilon and Wu (10), apply this concept to determine average delay at intersections.

In the field of stochastics, non-stationarity can be modeled by a Poisson process with an intensity function that itself is a random process, leading to varying momentary expectation values for the arrival rate. The impact of this changing arrival process on the average delay can in general not be derived analytically, though. It is expected that the more stationary the input process of a single-server queueing model with infinite capacity is, the smaller the average delay gets. This was formulated already in 1978 as Ross's conjecture (11).

Since then, this conjecture could not be proven in general, but only for special cases with certain restrictions, as by Bäuerle and Rolski (12). They use a Markov-modulated Poisson process as the arrival process, so the input rates change according to a transition matrix. It is shown that rarer changes of the arrival rate lead to higher average delays than more frequent changes.

Transferred to a traffic engineering context, this implies that a traffic flow that changes its flow rate every minute will yield shorter average delays than a traffic flow that changes between the same flow rates, but only every say 15 minutes. In both cases, hourly traffic volumes as kind of long-term average are the same.

Another technology sector being particularly sensitive to queueing and delay is telecommunications. Already as soon as in 1973, Heffes (13) points out the relevance of overflowing of a trunk group. Numerical methods are used to compare the implications of different degrees of peakedness on experienced delay representing time congestion. Identifying the assessed queueing system's performance as extremely sensitive to the peakedness of the input process forms the key statement of the study. Holtzman and Jagerman (14) differentiate between peakedness and non-stationarity, though stating the significance of considering the latter within the analysis of arrival processes.

Even though no ‘easy way’ could be found to include non-stationarity in calculation procedures in different fields of research, its relevance with regard to the impact on delay in general is evident.

Practical consideration of non-stationarity within the assessment of LOS

In order to take into account the traffic flow variability within the peak hour, the peak 15 minutes traffic volume is used by the HCM to calculate the peak flow rate as the main input parameter for the assessment of queue lengths and delay to obtain the level of service for an intersection. The HCM peak hour factor (PHF) is applied if appropriate data is missing. It serves to adjust the traffic volume of the peak hour to that of the unknown peak 15-minutes-interval. The PHF is applied to a complete intersection’s traffic flow rates. The resulting design hourly flow rates are then used for all further examinations and procedures.

Within the assessment of LOS for a signalized intersection following the German HBS, the so-called non-stationarity factor (NF) is separately calculated for every lane and used only within the calculation of the queue length, see equations (1) and (2). If no data is available concerning the 15-min peak interval, the NF is set to 1.1 for all lanes.

$$N_{GE,j} = \max \begin{cases} \dfrac{0.58 \cdot T \cdot C_{0,i,j}}{4} \cdot \left[\left(f_{in,j} \cdot x_j - 1\right) + \sqrt{\left(f_{in,j} \cdot x_j - 1\right)^2 + \dfrac{4 \cdot f_{in,j} \cdot x_j}{0.58 \cdot T \cdot C_{0,i,j}}} \right] \\ \dfrac{T \cdot C_{0,i,j}}{4} \cdot \left[\left(x_j - 1\right) + \sqrt{\left(x_j - 1\right)^2 + \dfrac{4 \cdot x_j}{T \cdot C_{0,i,j}}} \right] \end{cases} \quad (1)$$

$N_{GE,j}$	average queue at the end of the green time for lane j [veh]
T	analysis period (usually T=1h) [h]
$C_{0,i,j}$	capacity for movement i on lane j [veh/h]
$f_{in,j}$	non-stationarity factor for lane j [-]
x_j	volume-to-capacity ratio of lane j [-]

$$f_{in,j} = 1 + \frac{\left(\frac{q_{15,j}}{q_j} - 1\right)}{1.5} \quad (2)$$

$f_{in,j}$	non-stationarity factor for lane j [-]
$q_{15,j}$	arrival flow rate within the 15-min peak interval on lane j [veh/h]
q_j	arrival flow rate within the peak hour on lane j [veh/h]

Equation (1) for the calculation of the average queue length contains two subformulas: the second one complies exactly with the formula developed by Akçelik (6) and also used by the HCM; the first part contains the NF and additionally a factor of 0.58. Derived by Wu (8), these factors together represent the consideration of non-stationary traffic flow within the peak hour. The analysis period is virtually compressed to 0.58 of its length, while the arrival flow rate is raised by the NF that has a value of at least 1.0. The origin of the 0.58 value is not described explicitly in the HBS. Within the development process of Wu's formula a table is given containing several possible traffic flow profiles for the peak hour to be considered in the calculation. The profile in the form of a parabolic curve then is assigned a value of 0.582 as a correction factor for the analysis period. Thus, non-stationarity is considered when assessing the LOS according to the HBS, however implying a parabolic arrival flow profile implicitly.

PRACTICAL INVESTIGATION – MEASUREMENT AND SIMULATION

Within the following sections the investigation of the occurrence of non-stationary traffic flow and the assessment of its impact on vehicle delay are presented. It was assumed that arrival flow rates that do not show a parabolic or at least a symmetrical profile will lead to different average delay than symmetrical or even stationary profiles. To determine these differences, microscopic traffic flow simulation was used. Finally, a more faceted new factor is derived that could be integrated into the existing HBS calculation procedures.

Empirical data and flow patterns

As a basis for the identification of peak flow profiles and the design of flow patterns as input profiles for the simulation studies, empirical data was analyzed. Peak flow volumes were derived from vehicle counts of different permanent counting units within the city of Dresden for one day. Exemplified, the diagrams in Figure 1 show quite clearly for two of the counting points that peak arrival flow profiles may differ considerably from a parabolic or any generally symmetrical profile. Motivated by the variety of profile types, a set of simplified input profiles (flow patterns) was drawn up. As the minimum level of detail for traffic volume as input parameter requested by the HBS procedures is 15 minutes and this degree of complexity is still relatively easy to manage, 15-min-intervals were chosen to design the input flow patterns. It was intended to cover as much permutations as possible with reasonable effort, thus a set of 18 patterns of 6 types was selected.

Figure 2 contains a overview of this flow patterns. Each row represents a different type, as "increase", "decrease" or "boat form", while the columns show the levels of span between the intervals with the maximum and the minimum flow rate within the pattern.

Simulation setup

Using VISSIM 7, the microscopic simulation model was built as simple as possible: a link, sufficiently long in order to obtain a realistic driving behavior, with a signal. For every simulation setup, a fixed-time control program was chosen with a green time proportion of at least 50%. Also, the vehicle input (100% passenger cars) volume was varied, so that the resulting volume-to-capacity ratios are determined by the combination of control program and traffic volume, covering a range between 0.91 and 1.01, effectively.

Even though the flow patterns are defined by 15-min-intervals, the input and output intervals were set to 5 minutes in order to obtain more precise results. The simulation period of 1 hour for each sample was followed by a cooling down period of 15 min to ensure considering all arising delays actually. The simulation's main output, the average delay per vehicle, must not be compared directly with the average delay calculated using the HBS procedure as not necessarily the same delay components are included. Therefore, the simulation model was calibrated so that the VISSIM delays for a stationary input traffic flow equal the average delays derived by the HBS method including the saturation flow rate from simulation. The volume-to-capacity ratio considered for the calibration ranged between 0.92 and 0.96.

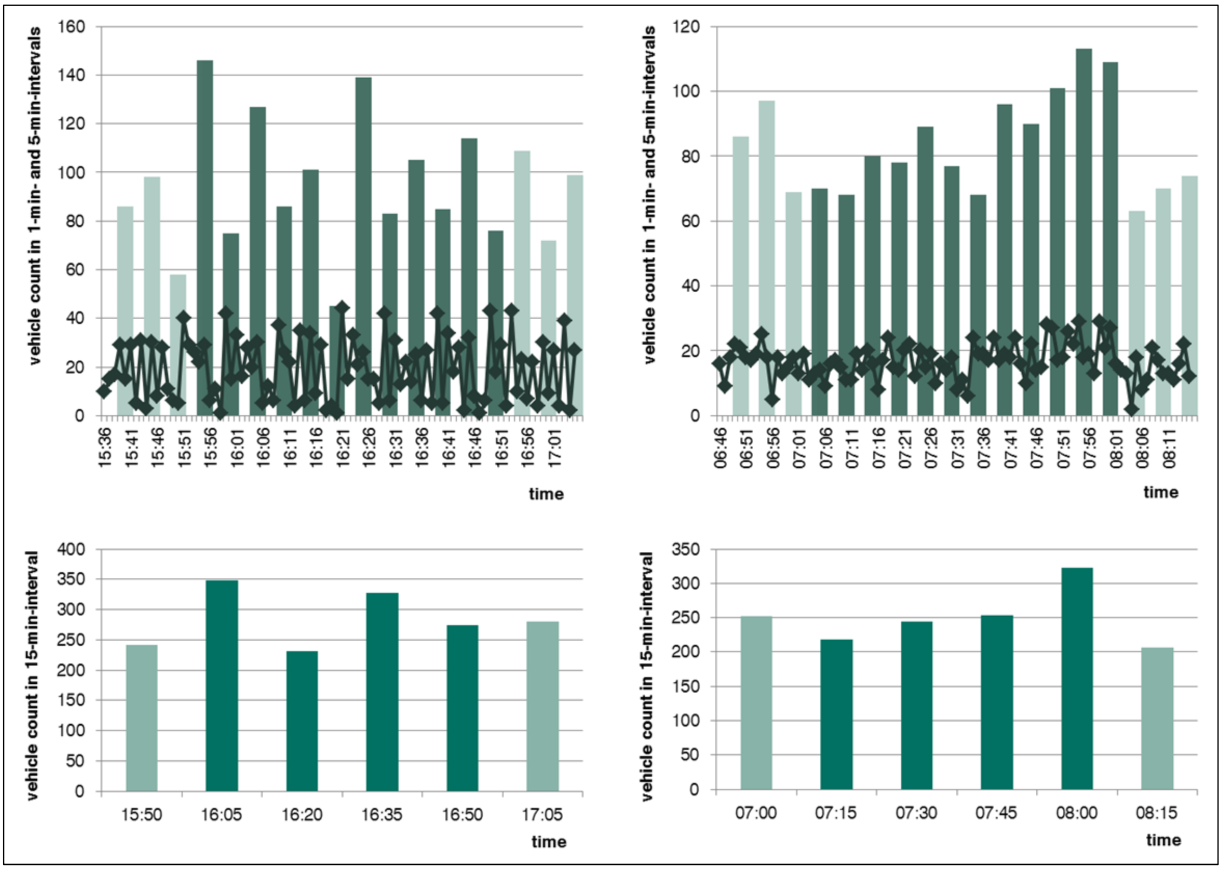

FIGURE 1 Vehicle counts for two counting points, aggregated to different levels of detail

FIGURE 2 Laboratory example traffic flow profiles (compressed view)

Simulation study results

Predictably, the simulation results differ considerably. Figure 3 contains a compressed view hereon represented by the total sum of square differences between the delays derived by simulation and those calculated using the HBS procedure, for all different variations of average input volume and signal program. The patterns are ordered by their deviation levels. Apart from just a few cases, the differences show the same direction: the simulation delays are generally smaller than the values determined by the HBS method.

As expected, the constant input profile yields the smallest deviation. Calibrated only for a subset of the possible volume-to-capacity ratios, this is a hint for the relevance of not only the arrival flow pattern but also of the volume-to-capacity ratio. Further, all variations of the parabolic profile (that is assumed for the calculation of the HBS NF) unsurprisingly show lower levels of deviation. The strongest differences are caused by the profiles with increasing traffic volumes. Within these the amplitude between the lowest and highest 15 min flow rate shows an effect, too.

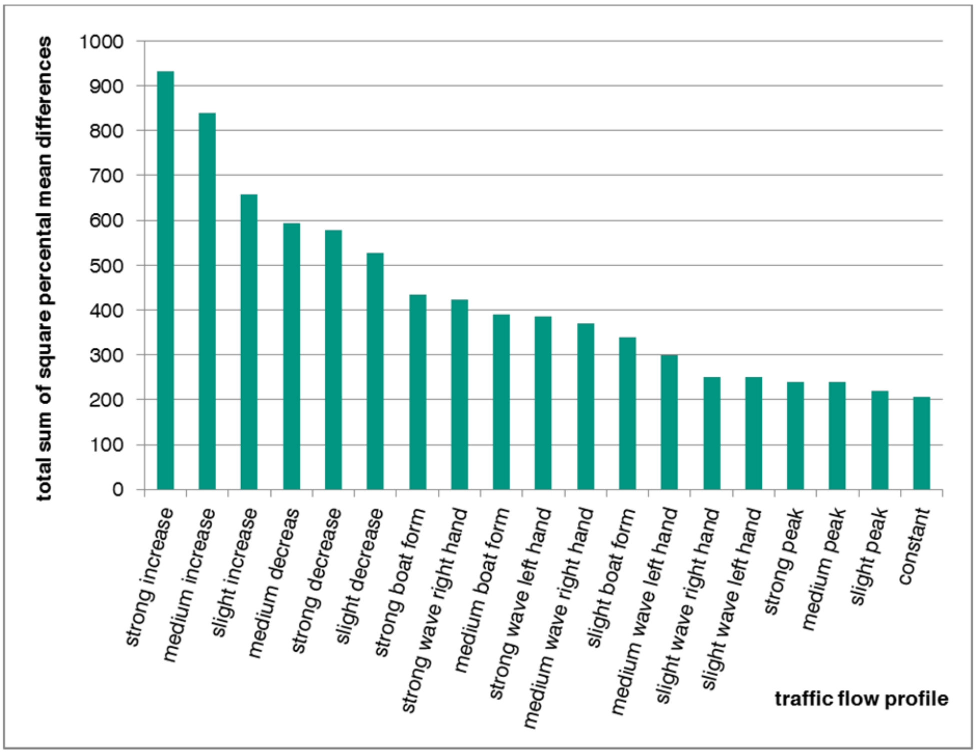

FIGURE 3 Deviation levels of non-stationary flow patterns

For a more detailed view, Figure 4 shows the resulting differences in percent for a subset of the patterns, grouped by the input volume and signal program variations. The "more parabolic" or "more stationary" a pattern is, or the less the maximum 15 min flow rate differs from the minimum 15 min flow rate, respectively, the better the HBS results fit the simulation outcomes. Also, the absolute length of the green time and the volume-to-capacity ratio seem to have an effect.

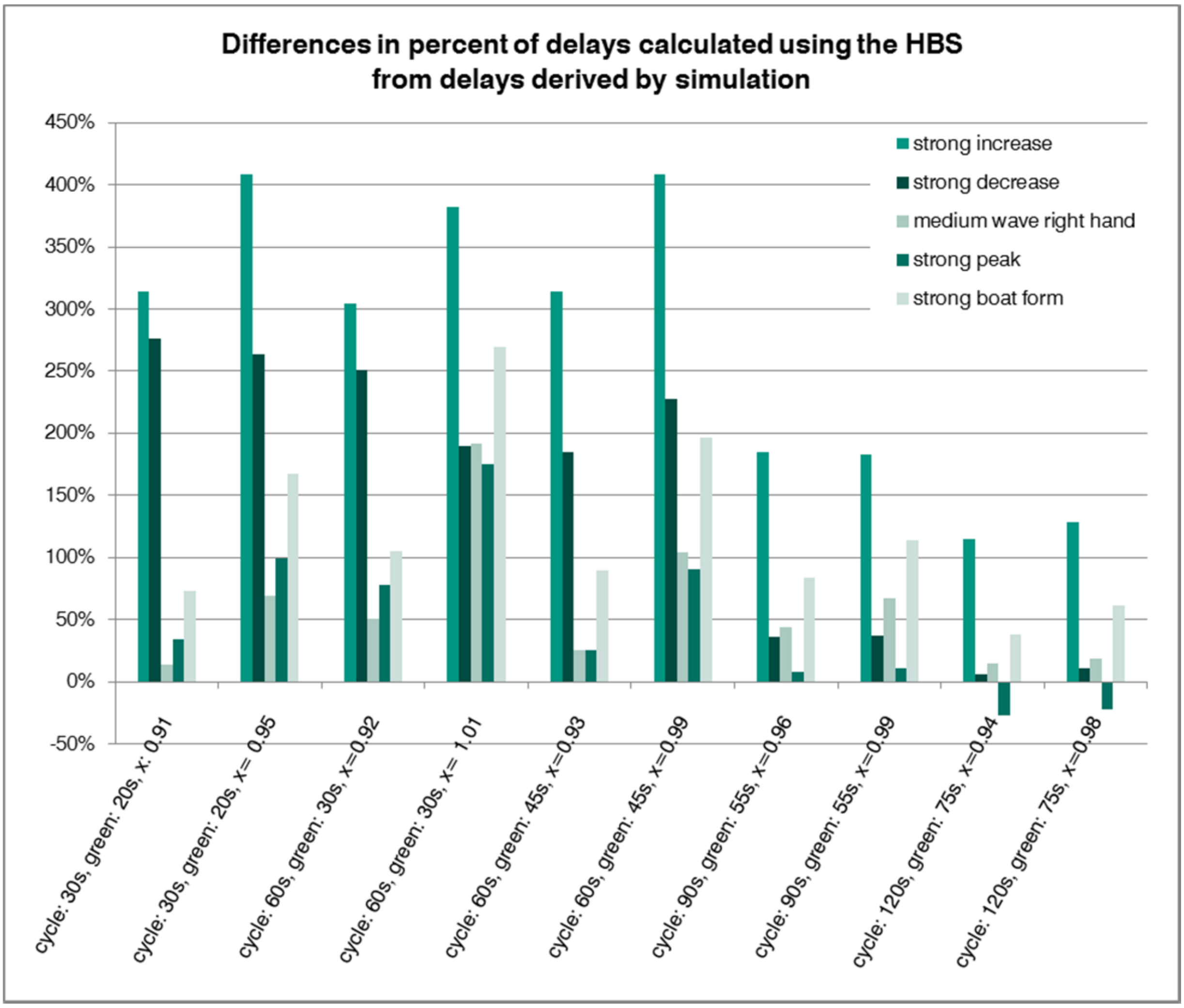

FIGURE 4 Differences of simulation to HBS calculated delay for selected flow patterns

Development of a new non-stationarity factor

By analysing the simulation results for the different patterns respecting deviations between them and regarding the delay calculated by the HBS procedure, some potential influencing values could be identified. The following characteristics of the traffic flow profiles were chosen to investigate whether and to what extent an effect exists:

- difference between the maximum 15 min flow rate and the average peak hour flow rate
- difference between the maximum to the minimum 15 min flow rate
- position of the peak 15 min interval within the peak hour
- position of the highest half-hour volume (first or second half-hour) of the peak hour
- volume-to-capacity ratio

Within the analysis process, an "ideal non-stationarity factor" was developed first. Therefore, a modified version of equation (1) was used, equally the original formula by Akçelik (6), with the term [$f_{in,ideal} \cdot x$] instead of just [x]. As the factor 0.58 in the first part of equation (1) corresponds to the assumption of a parabolic profile, it was dropped for the investigation of influences caused by differently shaped profiles. Thus, regarding every signal program variant and traffic flow profile, using the "ideal non-stationarity factor" leads to a minimum deviation of the calculated average delay from the average delay derived by simulation.

On the basis of this "ideal non-stationarity factor", different combinations of the potential cause variables were tested and examined within different possible formulations, as additive or multi-plicative compositions. Therefore the method of weighted least squares was employed. The total sum of square differences between the delay calculated using the formulation being evaluated and using the "ideal non-stationarity factor" was determined in order to be minimized. The result of this process was a subset of potentially best fitting formulations.

To identify the best version of this subset, these factors were applied to the peak hour arrival flow rates of the empirical data from the Dresden permanent counting units. Therefore, simulations were carried out with the same model as for the laboratory examples, with equal signal programs and volume-to-capacity ratios for the peak hour. To obtain the latter in order to harmonize the empirical samples, these arrival flow rate profiles were scaled to the matching peak hour volumes.

The total sum of square differences between the delay calculated with the new formulation and those derived by simulation was calculated for each combination of signal program and peak flow rate, using all potentially best fitting versions of the new non-stationarity factor. The aim was to identify the one not only fitting best, but also being rather robust than sensitive. The version matching both demands also turned out to yield the smallest deviations in cases where the traffic volume exceeds capacity.

Thus, the new formulation to determine the average queue length for calculating vehicle delay at intersections, signalized by fixed-time signal programs, considering non-stationary traffic flow with different characteristics of the flow rate profile is proposed by:

$$f_{IN,NEW,j} = 1 + 0.25 \cdot \left(\frac{q_{15,max,j} - q_{60,j}}{q_{60,j}} \right) - 0.01 \cdot n_{q,\frac{1}{2},j} - 0.03 \cdot x_j \tag{3}$$

$f_{IN,NEW}$ new non-stationarity factor

$q_{15,max}$ peak 15 min traffic flow rate within the peak hour [veh/h]

q_{60} peak hour traffic flow rate [veh/h]

$n_{q,\frac{1}{2}}$ parameter representing the larger flow rate of whether the first or second half-hour, whereas

$n_{q,\frac{1}{2}} = 1$ for $q_{30,1} > q_{30,2}$ (first half-hour flow rate is larger)

$n_{q,\frac{1}{2}} = 2$ for $q_{30,1} < q_{30,2}$

$n_{q,\frac{1}{2}} = 1{,}5$ if symmetrical and $n_{q,\frac{1}{2}} = 0$ if stationary

x_j volume-to-capacity ratio of lane j [-] to be used in:

$$N_{GE,j} = \frac{T \cdot C_{0,i,j}}{4} \cdot \left[\left(f_{IN,NEW,j} \cdot x_j - 1\right) + \sqrt{\left(f_{IN,NEW,j} \cdot x_j - 1\right)^2 + \frac{4 \cdot f_{IN,NEW,j} \cdot x_j}{T \cdot C_{0,i,j}}} \right] \tag{4}$$

$N_{GE,j}$	average queue at the end of the green time for lane j [veh]
T	analysis period (usually T=1h) [h]
$C_{0,i,j}$	capacity for movement i on lane j [veh/h]
$f_{IN,NEW,j}$	new non-stationarity factor for lane j [-]
x_j	volume-to-capacity ratio of lane j [-]

The total sum of square differences for all real-life flow rate profiles using the new factor is 88% lower than computing the differences by the HBS method, both in comparison to the simulation results.

Figure 5 summarizes the deviation comparison for all calculation variants considered from the simulation results for a subset of the flow patterns. Even if the new factor seems to underestimate the calculated delay compared to the simulation results, the differences concerning the amount of deviation is quite obvious.

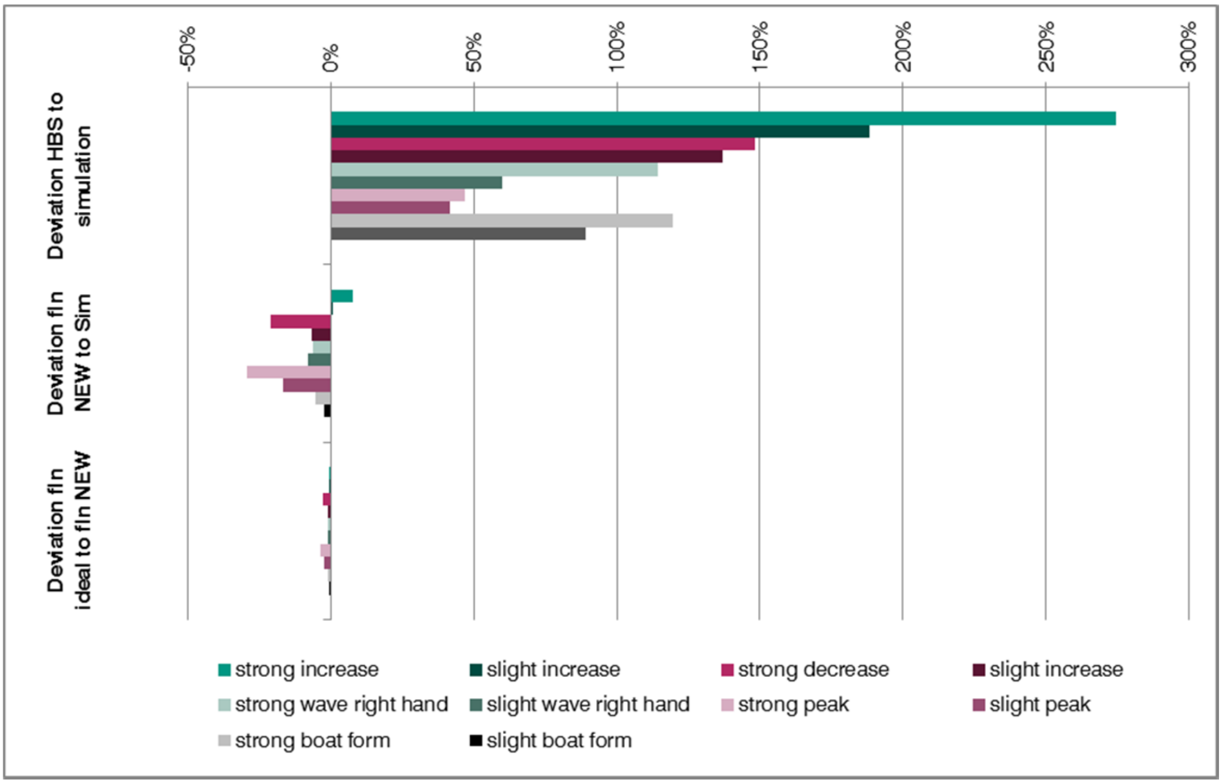

FIGURE 5 Deviations over all combinations of signal program and flow rate in percent

Related research on the PHF

Not only the research project in which this new non-stationarity factor was developed identified the relevance of non-stationary traffic flow to the LOS assessment of intersections, but also current research concerning the HCM peak hour factor investigates traffic flow variability during the peak hour.

Yi et al. (15) examined changes of the length of the interval the PHF is based on concerning stability and sensitivity on variations of the traffic conditions. Interval lengths from 3 to 20 minutes are analyzed using empirical data to find that longer intervals lead to an increase of the factor while intervals in a range from 10 to 15 minutes yield stable results for intersections.

Several studies (16, 17, 18) criticize the use of a single default PHF value in case of missing data. Tarko and Perez-Cartagena (16) as well as Lan and Abia (17) refer to the examination of intersections. Both studies point out the significance of traffic variability influencing the PHF and use empirical data to derive models for a better prediction of the PHF values. The model developed by Tarko and Perez-Cartagena contains hourly traffic volume, community population and time of day as influencing parameters. Lan and Abia derive another model, finding no transferability of Tarko and Perez-Cartagena’s model to their data, including the functional classification of roadways and the volume-to-capacity ratio as cause variables.

Although Dehman and Drakopoulos (18) analyze freeways rather than intersections, the key finding that LOS has an impact on the PHF is worth considering.

CONCLUSION

The analysis of the consideration of non-stationarity in a general context and of the impact on the assessment of traffic flow quality reveals a clear significance in both cases. Since there is no analytical way to include the influence of non-stationary traffic flow in the computation methods, the approximate solution by calculating the queue length using an additional factor adjusting the volume-to-capacity ratio was examined and modified. The new non-stationarity factor could be included in the HBS procedure to yield more accurate results by considering more characteristics of the arrival flow rates during the peak hour. Especially, the influence of the volume-to-capacity ratio should be emphasized, being supported by PHF research and being a constraint in classic queueing models. Nevertheless, more research, based on larger amounts of empirical data, is recommended.

ACKNOWLEDGEMENTS

The paper is based on research sponsored by the German Federal Ministry for Transport, Building and Urban Development, represented by the Federal Highway Research Institute, under project no. 03.0424/2007/DGB. The contents of this paper solely reflect the views of the authors.

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