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Zooming X-rays with a single rotation in X-ray prism zoom lenses (XPZL)

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Abstract: Prism arrays arranged to form a slightly open alligator mouth were found to focus incident X-rays, as with increasing distance from the object symmetry axis these rays hit an increasing number of refracting prism tips. Such an object is then formally a refractive lens. Due to the strong energy dependence of the refractive index of material for X-rays a refractive X-ray lens is chromatically focusing. The attractive feature of the alligator lens is the inherent zoomability possible as the mouth can easily be opened or closed. However, the required tolerances for the jaw rotations and the jaw positioning are so stringent, that the routine use of such systems has not been reported yet. This study will show that the related technical problems can be overcome by proper object fabrication. In fact the here presented objects can already be aligned in the production stage. Then the assembly can be made with simple tools. And the zooming is achieved by just a simple rotation. The transmission through the devices was found to be as expected, and thus performance-wise these objects can directly compete with other refractive X-ray focusing systems.

1. Introduction

Roentgen's failure to observe X-ray refraction in material [1] led to a long period, in which focusing of X-rays by use of refractive lenses was not tried. The refraction angle was undetectable in his experiments due to the fact, that the refractive index n of materials is only insignificantly different from unity for X-rays. In fact it is even slightly smaller than unity [2] and it is thus more conveniently written as $n = 1-\delta$ with

$$\delta = \frac{N_A}{2\pi} r_0 \lambda^2 \rho \frac{Z}{A}, \tag{1}$$

where N_A is Avogadro's number, r_0 is the classical electron radius, ρ is the density, λ is the photon wavelength, and Z and A are the atomic number and the atomic mass, respectively.

The relevant parameter for the design of refractive lens systems is the lens maker's equation [3], which in the thin lens approximation is given for a symmetric lens by

$$f_{\text{lens}} = \frac{R}{2(n-1)}, \tag{2}$$

where R is the on-axis radius of curvature and it is assumed, that for the environment, i.e. for air, one can use $n_{\text{air}} = 1$. The radius of curvature is positive in convex lens surfaces and it is negative in concave lens surfaces. Positive focal lengths will produce real images downstream of the lens. Then for X-rays with $\delta>0$ Eq. (2) can be written as

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$$f_{\text{lens}} = \frac{R}{2\delta}, \tag{3}$$

and thus for their focusing one will have to employ concave lens surfaces. Kirkpatrick and Baez [4] did not consider it to be practical to stack hundreds of lenses with R = 10 mm in order to achieve for refractive indices of the order of $\delta \approx 10^{-6}$ focal length of the order of 100 m. Then it was ignored, that immediately afterwards in 1949 Kirkpatrick [5,6] succeeded to focus X-rays by use of a curved single refracting surface. His sample was an extreme off-axis sector of a concave lens with very small on-axis radius of curvature [6,7]. The topic of focusing X-rays by refraction was then only revived in 1994, when Tomie [8,9] came up with the idea to drill a significant number of holes with small diameter on a line into material. Such stack of holes would focus in only one dimension. Tomie [9] thus proposes for the two-dimensional focusing to use two crossed stacks, or even radially symmetric cavities. The concept of drilled holes was then successfully tested by Snigirev et al [10] in the so-called compound refractive lenses (CRL). Ever since significant progress was made. In a first step real concave lenses could be made by punching rotationally symmetric surfaces into thin platelets. As the stamp could be formed more freely, the effect of the residual spherical aberrations inherent in spherical surfaces could be reduced significantly. Now the surfaces have the required parabolic profile [11]. Such profile can also be transferred into platelets of beryllium [12], the most robust of the highly transparent materials for use in combination with X-rays. Presently the ease of use especially of the latter objects, but also of different variants of lithographically produced arrays [13,14], have made the CRL the workhorse for focusing X-rays at high power synchrotron radiation sources with minimum spot size and spatial resolution in the sub-micrometer range rather competitive to other optics like diffractive zone plates or mirrors [15].

It is inconvenient that Eq. (1) shows a very significantly varying dispersion $\delta \propto \lambda^2$. According to Eq. (3) this makes refractive X-ray lenses chromatically focusing optics. A given lens stack will only focus a particular photon energy at a given position. And the focus position will move rapidly along the beam optical axis, when the photon energy is varied. This is not compatible with most experimental techniques, in which the sample needs to be scanned with high precision and stability through the small focal spot. Consequently in order to keep the focus position stationary, the so-called transfocators were developed, in which single lens platelets can be added/removed to/from the lens stack during operation [16–19]. The first of these systems were rather bulky [16]. In the meantime more compact solutions were identified, which permit an almost continuous zooming [17], though the zoomable system remains a rather complex device.

An alternative rather elegant solution for continuous zooming of X-rays was reported already in 2000 by Cederstroem et al [20]. It is depicted in Fig. 1(A). The refraction of X-rays by the properly varying amount of material perpendicular to the beam trajectory was achieved when two facing inclined saw-tooth arrays in the form of an ‘alligator mouth’ were hit at angles of grazing incidence. The zooming is readily achieved by opening/closing the ‘alligator mouth’. However, it turns out that the proper operation of such ‘alligator mouth’ is practically very difficult. Consequently the inventors pursued their feasibility studies almost exclusively on single prism arrays, avoiding in this way the need to align the facing array relatively very precisely [21]. The latter facilitation is especially important, when two arrays are to be crossed for two-dimensional focusing [21,22]. However, such system will then provide only one quarter of the possible aperture for the incident beam, which is a rather inconvenient drawback. The first test by the inventors proved the functioning of the concept [20], and even competitive focusing to sub-micrometer dimensions was achieved [21]. Cederström et al [20] showed, that the material distribution in the alligator lens perpendicular to the incident beam is approximately parabolic, as it is required in the ‘thin lens approximation’ and provided by CRL’s [11]. Then the alligator lens and the corresponding

CRL will essentially have equal transmission functions, when made for one-dimensional focusing with the same focal length and in the same material. Despite of these interesting transmission and zooming properties obtainable in this unconventional approach for X-ray focusing no related progress has been reported anymore.

This study will now show, that the operation of these prism arrays can be facilitated very much especially for two-dimensional focusing with full and thus competitive aperture, and that the alignment can be mostly eliminated. An easy to operate solution is discussed in chapter 2. And the functioning of the device in comparison to the expectations is verified with experimental data in chapter 3.

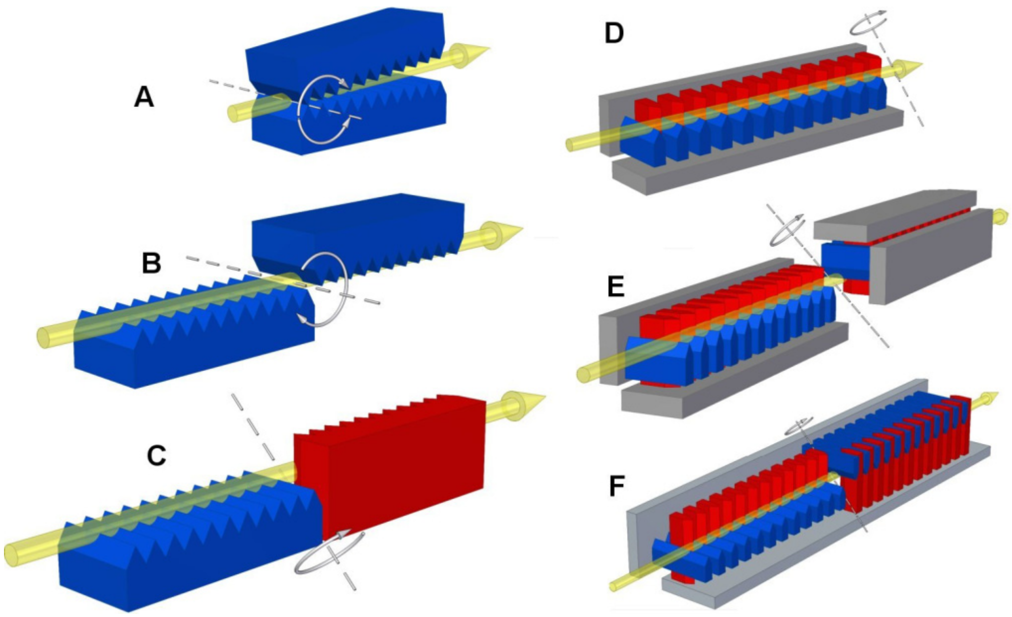

Fig. 1. The here described evolution of the saw-tooth refractive lens from an array, which is symmetrically oriented around the lenses optical axis (in A) and zooming in one direction, to an object, which can be zoomed for two-dimensional focusing by a simple rotation (in F). The dashed rotation axes are positioned such that the focus position remains stationary during zooming. Arrows indicate the required rotations for decreasing the inclinations of all arrays with respect to the incident beam equally. When looking along the rotation axes from bottom right to top left the rotations are clockwise and they keep the focal length constant for increasing photon energy. Blue prism arrays focus in the vertical direction, while red arrays focus horizontally. (A) Saw-tooth refractive lens as proposed by Cederström et al [20] for one dimensional focusing of X-rays. In the object in (B), looking like a blender blade, the zooming in one direction is achieved by a simple change of the array inclination. In the object in (C) instead the zooming is achieved in two orthogonal directions by a simple change of both array inclinations to be introduced by a rotation around an inclined axis at 45°. (D) The presented interdigitated array is obtained by removing every second prism in both prism arrays in (C) and by successively interdigitating them properly. (E) When two of the arrays as presented in (D) are put in series and in reversed orientation the final object can provide four-fold larger full aperture and zooming for two-dimensional focusing still employing only a single rotation. (F) The structure from (E) can also be prepared onto only two wafers, avoiding then the need for the relative alignment, which is included in the production process. The objects in the left and in the right half of the figure are independently drawn to scale. The length of the lenses in (C) and (D) are equal.

2. Optimization of a practical solution for full aperture prism arrays for two-dimensional focusing

Cederström et al [20] were presenting their refractive lens mostly as a symmetric orientation of two identical prism arrays as shown in Fig. 1(A), where a radiation beam exits from a slightly opened alligator mouth. The operation of such structure faces several technical challenges. First of all, in order to provide proper focusing the first two prisms in the inclined arrays have to almost touch each other at the end of the array. And secondly the rotations for changing the inclinations will have to be made around an axis in the array center by equal increments, however, in opposite directions. A significant simplification can be achieved, when the two arrays will be separated in longitudinal direction as shown in Fig. 1(B). Then the object will look from the side like a dented blender blade, and the zooming is achieved with just a single rotation, which varies properly simultaneously the inclination angle of both arrays. The relative alignment of both arrays, which requires all prism tips to lie in the same plane, could already be integrated into the production process, e.g. by lithography. Then neither the relative positioning of the rotation axis, nor the positioning of such one-dimensionally focusing device in the radiation beam are particularly demanding. When the prisms are now made significantly wider than needed, then one can even consider to incline the rotation axis with respect to the plane connecting the prism tips. In particular the 45° inclination with respect to the latter opens up the possibility for a very desirable feature. Then the second array could be operated in the orthogonal orientation with respect to the first one. As shown in Fig. 1(C) such crossed pair will then focus two-dimensionally and the simultaneous zooming can still be achieved with only a simple rotation around a properly inclined rotation axis positioned between both arrays. The same zooming effect can also be achieved, when the two orthogonal arrays will be interdigitated as shown in Fig. 1(D). This interdigitation will require to "remove" every second prism in the single arrays, and thus the number of prisms for focusing in each direction remains unaltered, when the total length of the object in Fig. 1(D) is identical with the length covered by both arrays in the object in Fig. 1(C). The focus remains stationary when the rotation axis is positioned at the end of the array closer to the focus. Undesirably at this point crossed "half-lenses" cover only one quadrant out of the 4 possible overlapping quadrants of a lens with full aperture. Obviously one can easily add to the lens in Fig. 1(D) another interdigitated array in series and in opposite orientation in order to form the object in Fig. 1(E). This tandem configuration can now provide maximum aperture, but it brings back alignment problems for the relative positioning and for the required simultaneous rotations around two independent rotation axes. The latter complication can be avoided and a single rotation can be used for zooming, when two properly (lithographically) produced blender blades from Fig. 1(B) with missing prisms will be interdigitated to form the structure shown in Fig. 1(F).

2.1 Optical properties of sawtooth prism arrays for focusing X-rays

The optimization of a sawtooth prism array for focusing X-rays can be made applying only a few simple equations, which were discussed already by Cederstroem et al [20]. The related parameters are explained in Fig. 2. This Fig. shows that each single prism array is just half of a plane-concave lens.

The amount of material $M(y)$ to be traversed by a beam being incident parallel to the x-axis and assuming a straight beam trajectory within the array, increases stepwise parabolically [20]

$$M\left(y\right)=y^{2}\left(\frac{N}{g}\right)\tan\frac{\theta}{2}. \tag{4}$$

This is a parabola with an on-axis radius of curvature given by

$$R=-\left(\frac{g}{N}\right)\frac{1}{2\tan\frac{\theta}{2}}. \tag{5}$$

By use of Eq. (3) applied to a plane-concave lens one then obtains

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$$f = -\frac{R}{\delta} = \left(\frac{g}{N}\right)\frac{1}{2\delta\tan\frac{\theta}{2}}, \tag{6}$$

and the transmission function becomes [11]

$$T(y) = \exp\left(-\frac{y^2}{2f\delta l}\right), \tag{7}$$

when l is the attenuation length of the array material.

The off-center distance Y for which the transmission has decayed to 1/e is then given as

$$Y = \sqrt{2f\delta l}. \tag{8}$$

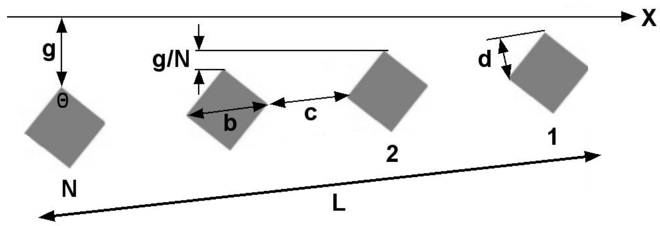

Fig. 2. Sketch of the mask, as it was used here for the production of the prisms realized as columns of properly rotated squares. The related dimensions as well as the operation parameters are indicated. The array contains N prisms. The index m for the prism number as well as the focal length are counted from the prism closest to the optical axis. θ is the prism tip angle, L the overall array length, b the diagonal of the squares, c the square separation and d the corresponding prism height. g is the distance of the last prism tip from the array optical axis, and g/N is thus the pitch between adjacent prisms.

Here Y will be considered to describe the "optimum" geometrical aperture for the prism array. The corresponding outermost ray traverses a distance $L' = l$ in material. Within the aperture Y 84% of the transmittable rays are collected, and the related average transmission is 75%. Consequently the condition $g = d = Y$ presents a rather appropriate operation condition for the array. For further analysis the important parameter for a refractive transmission optics is the "effective" aperture as it was introduced by Lengeler et al [11]. The latter "effective" aperture is the size of an ideally transparent aperture A, which collects the same amount of photons, which the single prism array can transmit within a chosen geometrical aperture. Obviously $A<Y$.

Without any geometrical aperture limitation the effective aperture for any concave lens with parabolic material distribution is given as

$$A_\infty = \int_0^{y=\infty} T(y)dy = \int_0^{y=\infty} \exp\left(-\frac{y^2}{2f\delta l}\right)dy = \frac{\sqrt{\pi}}{2}\sqrt{2f\delta l}. \tag{9}$$

When one installs an aperture with the opening Y according to Eq. (8) and with $Y<d$ and $Y<g$ upstream of the array the related effective aperture becomes

$$A_Y = \int_0^{y=Y} T(y)dy = 0.75\sqrt{2f\delta l} = 0.75Y. \tag{10}$$

Instead for $g<Y$, i.e. in an array with limited number of prisms N and of limited length L, the effective aperture becomes $>0.75g$. For $T(y)>0.4$ one can approximate

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$$\int_0^{y=g<Y} T(y)dy \approx \exp\left(-0.3\left(\frac{g}{Y}\right)^2\right) \approx \left(1-0.3\left(\frac{g}{Y}\right)^2\right)g$$

and thus

$$A_{g<Y} \approx \left(1-0.3\left(\frac{g}{Y}\right)^2\right)g. \tag{11}$$

At this point it is interesting to investigate the lens thickness in the direction of the trajectory of the radiation beam. This will be done for the earlier discussed condition $g = d = Y$, which is a good compromise between lens aperture and lens length. A stack of just touching concave lenses, whose diameter is chosen to be limited to Y according to Eq. (8), will then have a thickness L in beam direction of at least $L_{CRL} = l$. The related outermost ray passing in a single prism array made of the same material and inclined for the same focal length will progress half in material and half in empty spaces between the prisms. Consequently the alligator lens (AL) for one-dimensional focusing (Fig. 1(A)) has a thickness of at least $L_{AL} = 2l$, i.e. it is twice as thick as an equivalent CRL. The blender blade (BB) version in Fig. 1(B) is then twice longer than the alligator lens with $L_{BB} = 4l$, as is the simple series of half lenses for two-dimensional focusing in Fig. 1(C). And also the interdigitated half lenses in Fig. 1(D) have the same thickness. Consequently the series of two half lenses in Fig. 1(E) and 1(F) providing the full aperture (FA), will again be twice longer putting the total lens length to at least $L_{FA} = 8l$.

3. Discussion of experiment

3.1 Parameters of the prism array of optimized shape

In order to test the feasibility of the assembly and the operation of the here proposed interdigitated prism array concept for simple two-dimensional focusing and zooming a series of identical single prism arrays were produced by deep X-ray lithography. The prisms had square cross section (155 µm x 155 µm) as shown in Fig. 2, and they stood as perpendicular columns of height 400 µm on a 525 µm thick silicon substrate. The parameters of the prism array prepared for the interdigitation are then $d \approx 110$ µm for rectangular prisms with $\theta = 90°$. Each array is composed of $N = 36$ prisms with $b \approx 220$ µm and $c = 280$ µm filling a total length of $L \approx 18$ mm. The gap c was chosen larger than the width b in order to make interdigitation possible. The maximum permitted inclination of the array is achieved when $g = d$, in which case $g/N = 3.056$ µm. In this condition the most deflected ray will pass through $N = 36$ prism tips and correspondingly $L' = 3.96$ mm of material. For the proposed test of the device a maximum focal length of $f = 1$ m was considered to be the practical limitation. The refractive index for the chosen resist was derived to depend on the photon energy E according to $\delta = 2.714\ 10^{-4}*(keV/E)^2$ [17]. By use of Eq. (8) this leads to $f = 1$ m for $g = d$ and for 13 keV photon energy (corresponding wavelength is $\lambda = 0.095$ nm), which was then chosen for the experiment. At the latter photon energy one finds $l = 5.8$ mm$>L'$, which then leads to $Y>d$ and $Y>g$; in other words the prism depth and lens thickness are not the optimum for the given conditions for all chosen focal length $f<1$ m. In this case the effective aperture is always given by Eq. (11) and one finds $A_g>0.75g$. In the experiment the array inclination ϕ was varied in the range $0.1° < \phi < 0.35°$ leading to 0.29 m $< f < 1$ m. Interestingly according to Eq. (6) in the present array of limited length the maximum beam deflection angle given by g/f is constant for a given photon energy and independent on the focal length. This angle corresponds to the numerical aperture NA of the single array, and it is here for 13 keV photon energy $g/f = NA = 0.11$ mrad. When assembled to cover the full aperture $2g$ the present arrays could then have achieved in the indicated focal length range a diffraction limited spot size with dimensions of the order of $\lambda/(2NA) \approx 0.4$µm. The inclination of the assembled prism arrays with respect to the incident beam is very small during operation and thus the expected shadow images, as depicted in Fig. 3, will not allow one to appreciate the lens thickness, which in this case was about 37 mm.

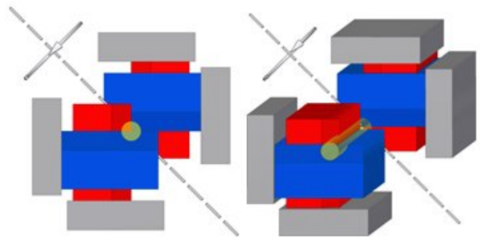

Fig. 3. The zoomable prism lens arrays from Fig. 1(E) as they would appear in a shadow image, when operated with very small inclination with respect to the incident beam.

3.2 Discussion of experimental data

In the experiment two properly interdigitated array structures, with the prism tips refracting in the vertical and in the horizontal direction, were oriented as shown in Fig. 1(D) by use of two independent hexapods in the MiQA experimental station [23] for imaging experiments at the IMAGE beamline [23] at the synchrotron radiation source at Karlsruhe Institute of Technology (KIT). The vertical source size is roughly Gaussian shaped with a full width at half maximum (FWHM) size of 0.059 mm, while the excessively large horizontal size of the X-ray beam was reduced by use of a pair of slits. The apparent source was then emitting intensity almost uniformly over a width of 1 mm. The assemblies were mounted at a source distance of about p = 39 m. By use of the hexapods the rotation around freely positionable rotation axes is possible. The rotation around a virtual rotation axes at the proposed positions and inclined by 45° with respect to the laboratory frame could thus easily be realized. In any case the two independent arrays were always carefully positioned with respect to each other for coinciding foci. The beam was registered downstream of the prism arrays by use of a CCD camera (PCO edge) behind a fluorescence screen (50 µm LuAG:Ce scintillator) with the magnification set to an equivalent pixel size of 1.625 µm in the scintillator plane (4x objective in combination with 6.5 µm CCD pixel size). This detector could be moved easily along the beam direction to the different focus positions.

The CCD camera presented a full width at half maximum of three CCD pixels for the point spread function, which corresponds to about 4.9 µm. This did not affect the determination of the horizontal focus size, while it made it impossible to precisely measure the size of the image in the vertical direction. In fact in this direction the expected focus size should always have been smaller than 2 µm, i.e. the image should mostly have filled only a single pixel. The present study can then not answer the question, whether the present prism arrays could have provided eventually even sub-micrometer foci. In fact for a pitch of g/N = 0.87 µm (f = 0.29 m) the expected image size was 0.6 µm and would thus have been obtainable with the array diffraction limit of the order of 0.4 µm. It is to note, however, that the prism array is a "thick" lens. Then the outermost rays, which are subject to the largest deflection, are the most aberrated rays, which will cross the optical axis either upstream or downstream of the nominal focal plane depending on whether they hit the prism array closer to the source or closer to the detector. In the ideally aligned lens the most aberrated rays will be off-set from the center ray in the nominal image plane in the same direction by $NA*L$, which would have been here $NA*L$ = 2 µm. This would have blurred the focus size. Then on the one hand sub-micrometer foci would have been difficult to achieve with the presented array, on the other hand the related aberrations could not be analyzed further due to the limited resolution of the detector in the present configuration.

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With this limitation the goal of this study was thus to show the feasibility of simple two-dimensional focusing and zooming by use of the proposed concept with the full available aperture of the system. For this purpose the comparison between the predicted and the observed spectral intensity enhancement is the most important parameter. The spectral intensity enhancement K is the ratio of radiation powers through a chosen reference area of 3.2 µm x 3.2 µm at the location of smallest beam dimensions with and without the X-ray optical system. It is simply given for $f \ll p$ by the ratio between the effective aperture A of the array and the FWHM of the image size i as $K = A/i$.

As the two prism arrays could here still be moved independently of each other the spectral intensity enhancement could be studied rather systematically for different prism array combinations. K_v and K_h can be determined independently for the vertical and the horizontal focusing arrays in the zones, which remained outside of the overlap region of the single interdigitated prism arrays. Simultaneously the spectral intensity enhancement K_{inter} can be measured in the single interdigitated array in the overlap area, where it is expected to be the product of the individual spectral intensity enhancements $K_{inter} = K_v * K_h$. And finally after proper positioning of the second array K_{full} can be measured in the common focus, where it is expected to increase fourfold to $K_{full} = 4K_{inter}$. For the proper comparison between expectation and experiment the minimum detector resolution of three pixels or 4.9 µm image size was considered in the simulation. This will limit the spectral intensity enhancement in the vertical direction. The enhancement is also limited in the horizontal direction, for which it can be shown for the chosen configuration of limited array length, that the single array geometrical aperture is always only 3.2-fold larger than the expected image size, leading thus to $K_h < 3$.

In Fig. 4 is presented a characteristic example of CCD images, in which the two point focus arrays are first aligned independently and then repositioned to overlap properly. The inclination in all prism arrays was set to $\phi = 0.2°$ providing a focal length of $f = 0.58$ m and a geometrical aperture in each single prism array of g = 63 µm (g/N = 1.75 µm), i.e. $g < d \approx 110$µm. The ideally expected spot size without aberrations would have been of the order of 1 µm.

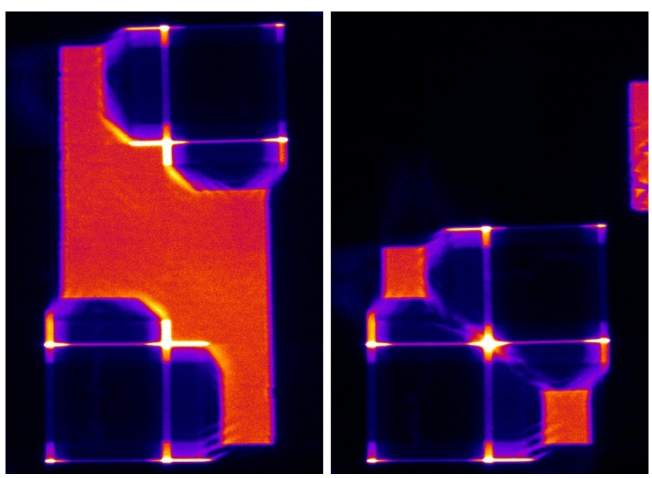

Fig. 4. At left is shown the intensity distribution registered in the image plane of two vertically separated interdigitated prism arrays adjusted for f = 0.58 m. At right the two independently movable arrays were positioned properly overlapping. In all cases, whether focused only in one direction or in the two orthogonal directions, the focus in the vertical direction measures 4.9 µm, while it measures 19.2 µm in the horizontal direction. In the chosen color coding the focused intensity is presented in white, i.e. in saturation, which is starting at a spectral intensity enhancement level compared to the incident beam (orange) of 2, while the maximum spectral intensity enhancement in the final focus is found to be 85. The black borders are produced by the shadow of the wafers and the respective holders.

In the vicinity of the two bright focal spots in the left picture in Fig. 4 one can identify brighter horizontal and vertical lines, which are caused by focusing in only one dimension. The correspondingly observed spectral intensity enhancements were derived to be in the wider vertical line $K_{h,exp} = 2.8$ and in the narrower horizontal line $K_{v,exp} = 9$. Both numbers as well as the horizontal image size are in agreement with the expectations. The spectral intensity enhancement of $K_{inter,exp} = 25$ measured in the overlap region corresponds to the product of the individual gains for both directions as expected. In this overlap region the sizes of the image remained unaltered compared to the focusing in only one direction. Consequently the additional interdigitated prisms, which do not contribute to the focusing in the orthogonal direction, do not lead to any focus size deterioration within the available spatial resolution of the detector. Now in this case the spectral intensity enhancement in the focus of two combined arrays was found to be 3.4 times larger ($K_{full,exp} = 85$) than the spectral intensity enhancement in the focus of a single interdigitated array. This is only about 15% less than the expected factor 4. This discrepancy might be due to some residual misalignment in the parallelism of the prism tips in corresponding prism arrays, which could have produced losses due to rays passing inappropriately both of them. The related misalignment is easier to correct for shorter detector distances than for longer. In fact for a shorter focal length of $f = 0.29$ m almost perfect performance was observed for the full aperture lens, while the discrepancy between the expected and the measured spectral intensity enhancements in the combined lens compared to the two independently focusing arrays increased for longer focal length. In this case the single arrays still focused according to the expectation, while the relative enhancement in the combination levelled off at 2.5 for $f = 1$ m.

4. Conclusion

From the here presented results one can conclude, that the interdigitated X-ray prism refractive structure performs as expected as far as the focusing capability and the structure transmission are concerned. The performance of this new arrangement of prisms as an X-ray zoom lens is thus competitive with CRLs, when used in the same conditions. This is a very promising and encouraging result, especially when one considers the easy zoomability of the prism arrays by a simple rotation. Obviously at a source with smaller horizontal source size higher spectral intensity enhancements would have been obtained. More spectral intensity enhancement could have been achieved by matching the presently small geometrical aperture g better with Y, which applies particularly for the more interesting shorter focal lengths. For a given lens material this would require using lens arrays longer than the 36 mm as they were chosen here. It was already pointed out, that the array thickness in beam direction cannot be ignored for the focusing as it will introduce aberrations into the focused image. This would then limit the obtainable spectral intensity enhancement and the related image size if the aberrations cannot be removed. Evans-Lutterodt et al [24] investigated the problem of the lens thickness and he showed that for a single CRL the parabolic profile is not anymore the proper lens shape for aberrations-free focusing. This argument is then discussed more systematically and in detail by Sanchez del Rio and Alianelli [25]. As far as the prism arrays are concerned Jark [26] reported the first related optimization, and Antimonov et al [27] discussed more possible alternatives. In fact in the prism arrays the aberrations removal can be achieved rather easily by varying the angle in the tips of the prisms along the array [26] or by varying the separation between the prism tips along the beam [27]. Both concepts are compatible with the interdigitation concept, however, they require different corrections for the interdigitated array closer to the source compared to the array closer to the focus. In both solutions the aberrations can be minimized in an image plane at fixed distance from the array, when the photon energy is zoomed. Then for a given correction the focal length is not a freely adjustable parameter anymore.

Acknowledgments

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