Next-to-leading order mixed QCD-electroweak corrections to Higgs boson production in gluon fusion

Zur Erlangung des akademischen Grades eines
DOKTORS DER NATURWISSENSCHAFTEN (Dr. rer. nat.)

von der KIT-Fakultät für Physik des
Karlsruher Instituts für Technologie (KIT)
angenommene

DISSERTATION

von

M.Sc. Marco Bonetti

Tag der mündlichen Prüfung: 28.06.2019

Referent: Prof. Dr. Kirill Melnikov

Korreferent: Prof. Dr. Matthias Steinhauser

NLO QCD-EW $gg \to H$

Karlsruher Institut für Technologie

Institut für Theoretische Teilchenpysik

M.Sc. Marco Bonetti

July 29, 2019

Abstract

In this thesis next-to-leading order mixed QCD-electroweak corrections to the Higgs boson gluon-fusion cross section are evaluated. Leading order and virtual next-to-leading order amplitudes are expressed in terms of two- and three-loop master integrals. These integrals are evaluated using differential equations augmented by the choice of a basis of uniformly transcendental functions. The integration constants are fixed in a systematic way by numerically matching the solutions to their large-mass expansion. The analytic results for the master integrals are expressed in terms of uniformly transcendental combinations of Goncharov polylogarithms. The real emission contributions are implemented using the soft-gluon approximation. The evaluation of the cross section shows that the ratio between QCD and mixed QCD-electroweak contributions to the cross section remains nearly unchanged from leading order to next-to-leading order. This result removes one of the important uncertainties if the Higgs boson gluon fusion cross section.

Contents

Overview

The fact that the Standard Model is a robust theory of Nature, culminated in the discovery of the Higgs boson in 2012. However, strong evidences from astrophysics and cosmology points towards the need to extend it, even if neither new fundamental particles nor interactions have been directly observed in collider experiments. Since effects of New Physics are searched for by looking for deviations from Standard Model expectations, precise theoretical predictions within the SM represent a key ingredient for the success of this effort.

The Higgs boson is a natural candidate for precision studies, both because it is a new particle and because of its peculiar role in the Standard Model. At present, the Higgs boson is studied at the LHC and its main production channel is gluon fusion. Given the importance of this production mode, it is desirable to decrease its theoretical uncertainty. One of the important sources of theoretical uncertainties is given by the missing three-loop computation of *mixed QCD-electroweak corrections to Higgs boson gluon fusion at next-to-leading order*.

This thesis provides the analytic calculation of these corrections to the Higgs boson gluon-fusion cross section for arbitrary values of electroweak bosons and Higgs boson masses of the leading order and virtual next-to-leading order amplitudes, while the real next-to-leading order corrections are evaluated in the soft-gluon approximation.

In Chapter 1 theoretical predictions for Higgs boson gluon fusion are reviewed. Mixed QCD-electroweak corrections are introduced and their current status is discussed.

The Feynman diagrams that contribute to QCD-electroweak corrections at leading and next-to-leading order is presented and discussed in Chapter 2. The form factors are extracted and renormalized, and IR singularities are made explicit.

Chapter 3 introduces the Feynman integrals required for the calculation of mixed QCD-electroweak corrections and describes their reduction to master integrals via integration-by-parts identities. Differential equations are used to calculate the master integrals and the notion of uniformly transcendental functions is reviewed and applied to simplify the solution of the differential equations. The large-mass expansion method is applied to systematically evaluate boundary conditions for master integrals both at leading and next-to-leading order.

In Chapter 4 the two-and three-loop master integrals appearing in leading order and next-to-leading order form factors, respectively, are calculated. At first, the equations are reduced to a canonical Fuchsian form, then the system is integrated in terms of Goncharov's polylogarithms and numerically matched to its large-mass expansion obtaining solutions in terms of uniformly transcendental functions. The analytic expressions for the form factors are constructed.

The next-to-leading order cross section for Higgs boson gluon fusion is computed in

Chapter 5, using a soft-gluon approximation to include the next-to-leading order real emission contributions. Conventions used to compute Feynman diagrams are listed in Appendix A. Leading order and virtual next-to-leading order Feynman diagrams, master integrals and uniformly transcendental bases at two and three loops are presented in Appendix B. Appendix C contains the analytic expressions for the leading order and next-to-leading order form factors.

The results presented in this thesis have been published in the papers [1–4]. Master integrals, form factors and differential equations are provided in a computer-readable format at `https://www.ttp.kit.edu/_media/progdata/2017/ttp17-047/anc.tar.gz`. This doctoral thesis has been typed using LaTeX. Feynman diagrams presented in this thesis have been realized using `JaxoDraw` [5].

Chapter 1.

Precision physics for the Higgs boson

In 2012, ATLAS and CMS collaborations announced the observation of a scalar elementary particle with the mass of 125 GeV. The properties of this particle were quite compatible with the expected properties of the Standard Model (SM) Higgs boson and the significance of the observation was almost 6 standard deviations [6, 7]. All subsequent tests performed so far point towards a close proximity of this new particle to the SM Higgs boson [8–16]. This discovery completes the SM and fixes its last undetermined parameter, the Higgs boson mass.

Although subject to unprecedented scrutiny, the SM has withstood all experimental checks up to this point. At the same time, its apparent inability to explain such experimental facts as neutrino masses and mixings, dark matter, and baryogenesis indicates that a “full” theory of Nature is yet to be found. Physics beyond the Standard Model (BSM) appears to be needed to explain phenomena in cosmology and astrophysics, but no new fundamental particles or violations of established laws of physics have been detected in collider and other terrestrial experiments until now.

As provocatively outlined in [17], the present situation shares many similarities with the situation of physics at the beginning of the 20th century. Indeed, Newtonian mechanics and Maxwell’s electrodynamics were able to adequately describe the majority of natural phenomena, but more and more deviations from their predictions were observed. A radical change in the theory of physics was needed to reconcile all experimental evidences. Special and general relativity together with quantum mechanics expanded the classical picture, providing explanations for the existing observations and successfully predicting new ones.

Many of the discrepancies that led to modern physics were discovered by comparing experimental results with precise theoretical predictions [18, 19]. The extensive and detailed work conducted within theories that existed at that time highlighted their weak points and directed the development of a new framework. In a similar way the SM can be used to identify issues with the existing understanding of Nature and guide the formulation of a new theory of fundamental interactions. For this reason, a continuous improvement of the understanding of the SM of particle physics at a very practical level remains as important as ever.

Currently, fundamental particle physics at the energy frontier is studied at the Large Hadron Collider (LHC) at CERN. Since all attempts to directly observe BSM particles

Table 1.1.: SM Higgs boson production cross section at the LHC (in pb) for $m_H = 125\,\text{GeV}$ and center-of-mass energy $\sqrt{s} = 13\,\text{TeV}$ [22].

ggH	VVH	WH	ZH	$t\bar{t}H$	**Total**
$44.1^{+11\%}_{-11\%}$	$3.78^{+2\%}_{-2\%}$	$1.37^{+2\%}_{-2\%}$	$0.88^{+5\%}_{-5\%}$	$0.51^{+9\%}_{-13\%}$	**50.6**

have been unsuccessful so far, a precise description of collider processes within the SM assumes a key role in unveiling New Physics. It is obvious that different SM processes have different sensitivities to BSM physics. The question of where to look to maximize chances to find it is, therefore, an important one. Both theoretical and experimental aspects of this question have to be considered to formulate a meaningful strategy.

On the theory side, the SM can be divided into three parts: matter, gauge interactions, and the Higgs sector. The matter part is described by Dirac fermions and does not possess self-interactions. The gauge sector is more complex: it is built upon the paradigm of gauge invariance [20, 21], which completely determines both self-interactions of gauge bosons and their couplings to matter fields once the gauge group representations are chosen. The Higgs sector is unusual in the sense that it does many different things at once, although –*a priori*– they are not related to each other. Indeed, the Higgs field breaks electroweak symmetry through self-interactions, couples and gives masses to electroweak bosons, restores the SM unitarity at high energies and gives masses to fermions. In addition, the Higgs sector is clearly the least explored part of the SM. It is, for example, far from clear that the Higgs boson does not couple to new, so far unknown particles. Such interactions may alter the Higgs couplings to SM particles and modify the Higgs boson production rates compared to SM predictions. The spontaneous symmetry breaking mechanism, the central part of the SM, is implemented in a rather simplistic way. Moreover, additional Higgs-like fields may hide at higher energies. The large number of options is clearly reflected in the diverse models of physics beyond the SM that have been explicitly constructed. The central role of the Higgs boson in the Standard Model and the fact that little is known about this particle makes it an ideal candidate for precision studies.

1.1. Higgs boson physics at the LHC

Higgs bosons are produced in collisions of high-energy protons at the Large Hadron Collider (LHC). The LHC is a circular proton-proton collider with a center-of-mass energy of $\sqrt{s} = 13\,\text{TeV}$. Higgs physics is mainly studied at two general-purpose detectors ATLAS and CMS.

The major Higgs boson production channels at the LHC are, in order of importance: gluon fusion (ggH), weak-boson fusion (VVH), associated Higgs boson production with an electroweak gauge boson (Higgs-*strahung*: WH or ZH), and associated $t\bar{t}$ production ($t\bar{t}H$). The corresponding cross sections are shown in Table 1.1.

The Higgs boson is an unstable particle. For this reason, its properties are inferred

Table 1.2.: SM Higgs boson branching ratios for $m_H = 125\,\text{GeV}$ and related uncertainties [22].

$b\bar{b}$	W^+W^-	$\tau^+\tau^-$	ZZ	$\gamma\gamma$	$Z\gamma$	$\mu^+\mu^-$
0.584	0.214	0.0627	0.0262	2.27×10^{-3}	1.53×10^{-3}	2.18×10^{-4}
+3.2%	+4.3%	+5.7%	+4.3%	+5.0%	+9.0%	+6.0%
−3.3%	−4.2%	−5.7%	−4.1%	−4.9%	−8.9%	−5.9%

from studies of its decay products. The Higgs boson mass of 125 GeV implies that the Higgs boson can decay in a variety of ways, many of which can be studied experimentally. Table 1.2 summarizes the main Higgs decay modes in order of their importance.

Higgs boson gluon fusion

As already mentioned, the LHC is a proton-proton collider. Protons are composite objects held together by the strong force, but their internal structure cannot be adequately described using perturbative quantum field theory. Nevertheless, for some hard scattering processes, for example Higgs boson production in gluon fusion, it was proven that the hadronic cross section can be computed according to the following *factorization theorem*

$$\sigma_{PP\to X} = \sum_{i,j} \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2\, f_{i/P}(x_1) f_{j/P}(x_2) \sigma_{ij\to X}(p_i, p_j) \left[1 + O\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)\right]. \quad (1.1)$$

In Eq. (1.1), Q indicates the transferred momentum, $\sigma_{PP\to X}$ is the *hadronic cross section* for the production of the final state X in proton collision, $\sigma_{ij\to X}$ is the *partonic cross section* calculated within perturbative QCD, and $f_{i/P}(x_1)$ ($f_{j/P}(x_2)$) is a *parton distribution function* (PDF), that describes the probability to find the parton i (j) with momentum $p_i^\mu = x_1 p_{1,h}^\mu$ ($p_j^\mu = x_2 p_{2,h}^\mu$) inside a proton P carrying momentum $p_{1,h}$ ($p_{2,h}$). PDFs are extracted from experimental data using Eq. (1.1) for a variety of final states X and then used to describe processes for other final states. PDFs for different partons in the proton are shown in Fig. 1.1. It follows from Eq. (1.1) that the factorization theorem is valid up to terms proportional to the ratio between the non-perturbative QCD scale $\Lambda_{\text{QCD}} \sim 0.3\,\text{GeV}$ and the hard momentum Q. Taking $Q \sim 30\,\text{GeV}$, Λ_{QCD}/Q is expected to provide a percent correction to the hard scattering cross section.

To understand which partonic processes are important for Higgs boson production, note that at leading order the partonic center-of-mass energy is fixed to $\sqrt{\hat{s}} = m_H$, since the only particle in the final state is the Higgs boson. This constrains the values of the Bjorken x since

$$\hat{s} = x_1 x_2 s = m_H^2 \qquad \xRightarrow[x_1 \sim x_2 = x]{} \qquad x \sim \sqrt{\frac{m_H^2}{s}} \sim 10^{-2}. \quad (1.2)$$

If follows from Fig. 1.1 that for such values of x, the gluon PDF is by far the largest; this is one of the reasons that makes gluon fusion the dominant production channels of

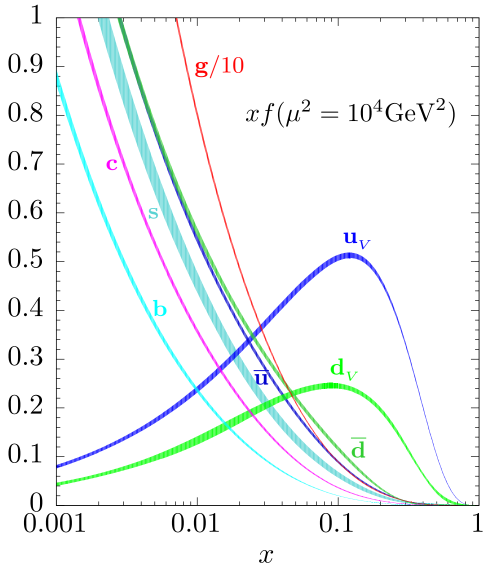


Figure 1.1.: PDF4LHC15 NNLO at typical LHC energy scale with a one-sigma uncertainty band from [23].

the Higgs boson at the LHC.

Numerically, the Higgs boson gluon-fusion production cross section at the LHC for $\sqrt{s} = 13\,\mathrm{TeV}$ and $m_H = 125\,\mathrm{GeV}$ is quite large [24]

$$\sigma = 48.58\,\mathrm{pb}^{+2.04\,\mathrm{pb}\ \ (+4.19\%)}_{-3.09\,\mathrm{pb}\ \ (-6.35\%)}\ (\mathrm{theory})\ \pm\ 1.56\,\mathrm{pb}\ (3.20\%)\ (\mathrm{PDF}{+}\alpha_S). \tag{1.3}$$

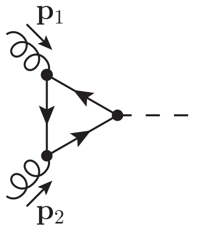
The cross section of Eq. (1.3) is assembled from different contributions shown in Table 1.3.

The largest contribution to the Higgs production cross section in gluon fusion comes from QCD interactions. In this case, the Higgs boson couples to gluons through quark

Table 1.3.: Breakdown of Higgs boson gluon fusion cross section at the LHC, $m_H = 125\,\mathrm{GeV}$, $\sqrt{s} = 13\,\mathrm{TeV}$, $\mu_F = \mu_R = m_H/2$ [24, 25].

Source	$\boldsymbol{\sigma}$ [pb]	**Relative contribution** [%]
LO, rEFT	16.00	32.9
NLO, rEFT	20.84	42.9
(t, b, c), exact NLO	-2.05	-4.2
NNLO, rEFT	9.56	19.7
NNLO, $1/m_t$	0.34	0.2
EW, QCD-EW	2.40	4.9
$\mathrm{N^3LO}$, rEFT	1.49	3.1
Total	48.58	100.0

loops. In practice, contributions of heavy quarks, especially of the top quark, dominate since [26–28]



$$\sim \begin{cases} \text{const.}, & \text{for } m_q \to +\infty, \\ \dfrac{m_q^2}{m_H^2} \log^2 \dfrac{m_H^2}{m_q^2}, & \text{for } m_q \to 0. \end{cases} \tag{1.4}$$

Because $m_H < 2m_t$, a useful approximation for computing the QCD contribution consists of considering the top quark infinitely heavy, "shrinking" its loop into an effective ggH vertex. This induced ggH interaction vertex follows from the Lagrangian

$$\mathcal{L}_{\text{HEFT}} \supset -\alpha_S \frac{C}{4v^2} G^a_{\mu\nu} G^{\mu\nu}_a \Phi^\dagger \Phi, \tag{1.5}$$

where Φ is the Higgs doublet and the Wilson coefficient C includes all corrections to the infinite top-mass effective vertex ggH that come from energy scales comparable to the top quark mass m_t. The description of Higgs boson interactions with gluons using the Lagrangian of Eq. (1.5) can be interpreted in the spirit of effective field theories, where the top quark is integrated out. Within this effective theory, N^{3}LO QCD corrections to the Higgs boson production cross section were recently computed in [29, 30] (NLO in [31–33] and NNLO in [34–36]). The validity of these calculations can be extended by rescaling the resulting cross sections by the ratio

$$R_{\text{LO}} = \frac{\sigma^{\text{LO}}_{\text{ex},t}}{\sigma^{\text{LO}}_{\text{EFT}}}, \tag{1.6}$$

where $\sigma^{\text{LO}}_{\text{ex},t}$ is the leading order cross section with full top mass dependence. To obtain the contributions showed in Table 1.3 and labelled as LO, NLO, NNLO, N^{3}LO rEFT (rescaled Effective Field Theory) the rescaled partonic cross sections are convoluted with NNLO PDFs.

To refine the result obtained in the $m_t \to +\infty$ limit, corrections due to the finite mass of the top quark are considered. These corrections are known exactly up to NLO, while for NNLO an expansion in the inverse top quark mass $1/m_t$ is available [37–41]. A further refinement is obtained by including bottom and charm quarks contributions; they have been evaluated through NLO [28, 32, 33, 42–47]. The impact of these corrections on the Higgs boson production cross section is shown in Table 1.3; they are labelled as "(t, b, c), exact NLO" (corrections up to NLO for exact values of m_t, m_b, and m_c) and as "NNLO, $1/m_t$" (corrections at NNLO expanded in $1/m_t$). The last contribution presented in Table 1.3 is due to electroweak corrections, known exactly at leading order ($\propto \alpha_S$) and only in various approximations at NLO ($\propto \alpha_S^2$). This contribution changes the Higgs boson cross section by about 5 %.

Table 1.4.: Breakdown of the theoretical uncertainty on Higgs boson gluon fusion [24, 25, 30].

δ(scale)	δ(PDF-TH)	δ(EW)	$\delta(t,b,c)$	$\delta(1/m_t)$	δ(PDF)	$\delta(\alpha_S)$
$^{+0.10\,\mathrm{pb}}_{-1.15\,\mathrm{pb}}$	$\pm 0.56\,\mathrm{pb}$	$\pm 0.49\,\mathrm{pb}$	$\pm 0.40\,\mathrm{pb}$	$\pm 0.49\,\mathrm{pb}$	$\pm 0.90\,\mathrm{pb}$	$^{+1.27\,\mathrm{pb}}_{-1.25\,\mathrm{pb}}$
$^{+0.21\%}_{-2.37\%}$	$\pm 1.16\%$	$\pm 1\%$	$\pm 0.83\%$	$\pm 1\%$	$\pm 1.86\%$	$^{+2.61\%}_{-2.58\%}$

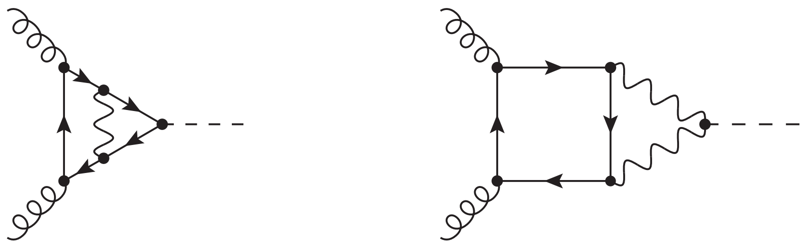
Each contribution to the $gg \to H$ cross section has an uncertainty associated with it; they are listed in Table 1.4. At the level of partonic cross sections, the first source of uncertainty is given by the residual scale dependence, a consequence of the fact that predictions are calculated through finite order in perturbative QCD. For Higgs boson production in gluon fusion it is customary to set the factorization and renormalization scales equal to each other $\mu_R = \mu_F = \mu$, choose $\mu = m_H/2$, and vary μ in the interval $[m_H/4, m_H]$ to determine the uncertainty δ(scale). Other sources of uncertainties include the expansion in $1/m_t$ at NNLO and the lack of knowledge of the bottom and charm mass effects beyond NLO ($\delta(1/m_t)$ and $\delta(t,b,c)$, respectively). The uncertainty in electroweak contributions δ(EW) is caused by the missing NLO computation, which includes exact dependencies on the Higgs, $W^{\pm}$, and Z masses.

PDF-related uncertainties have both theoretical and experimental origins. Theoretical PDF uncertainties (δ(PDF-TH)) are caused by the absence of N^{3}LO PDFs that are formally needed for a consistent N^{3}LO prediction. Experimental PDF (δ(PDF)) and α_S ($\delta(\alpha_S)$) uncertainties are evaluated according to PDF4LHC recommendations [48] and reflect the current knowledge of the parton distribution functions and the strong coupling constant. All theoretical uncertainties are added up linearly giving the theory error in Eq. (1.3), while the experimental uncertainties are added in quadratures to obtain the PDF + α_S error.

It is important to understand if the current knowledge of the Higgs boson production cross section is sufficient. A way to check this is to estimate a typical effect of BSM physics on the production cross section in gluon fusion. Consider a heavy fermion Q whose mass is not generated by the Higgs mechanism that couples to the Higgs boson via a Yukawa vertex $\overline{Q}HQ\, m_Q/v$. In this case the $\overline{Q}QH$ coupling is suppressed in the large m_Q limit, scaling as $C_Q m_H^2/m_Q$ (while the top quark coupling $\bar{t}tH$ is constant in the limit, being $C_t m_H^2/v$). The interference of Q and t contributions generates a variation in the $gg \to H$ cross section of the form

$$\sigma_{gg\to H} \propto C_t^2 \frac{m_H^4}{v^2}\left(1 + 2\frac{C_Q}{C_t}\frac{v}{m_Q} + \ldots\right). \tag{1.7}$$

By taking $C_t \sim C_Q$ and $m_Q \sim 5\,\mathrm{TeV}$, a a variation in the ggH coupling of $\delta g_{ggH}/g_{ggH} \sim 5\,\%$ is expected. Explicit calculations in specific models, such as supersymmetry [49], result in $O(1\text{–}5\,\%)$ variations of the ggH coupling. These variations can be detected if the total uncertainty of the theoretical predictions does not exceed a few percent for Higgs boson production cross section in gluon fusion. To decrease δ(scale), the N^{4}LO



(a) $q\bar{q}H$ coupling $\propto \alpha_S \alpha Y_q$ (b) VVH coumpling $\propto \alpha_S \alpha^2$

Figure 1.2.: The two classes of QCD-electroweak contributions to ggH at leading order.

EFT calculation is required, but it is almost impossible to perform such calculation with the current technology. All other contributions to the cross section have an $O(1\,\%)$ associated uncertainty. Among those, the calculation of three-loop QCD-electroweak corrections appears to be feasible although highly non-trivial. The focus of this thesis is the calculation of these corrections.

1.2. Mixed QCD-electroweak contributions

Mixed QCD-electroweak contributions can be divided into two categories, according to the Higgs coupling involved. The first category is related to the Higgs boson coupling to quarks ($q\bar{q}H$ coupling, Fig. 1.2a), the second to weak vector bosons (VVH coupling, Fig. 1.2b). For the $q\bar{q}H$ coupling the electroweak corrections consist of an electroweak boson line attached to the internal quarks, while for VVH diagrams a quark box acts as a connector between the gluons and the electroweak bosons. Both contributions appear for the first time at two loops.

Which of the two contributions is more important, is different for light ($m_q \ll v$) and heavy ($m_q \sim v$) quarks. Indeed, thanks to different Yukawa couplings, the top quark contribution is enhanced in $q\bar{q}H$ diagrams, while light quarks contributions are suppressed, cf. Eq. (1.4). On the contrary, VVH contributions are not suppressed by the light quark Yukawa couplings and, instead, are even somewhat enhanced by the total number of light quarks. The top contribution, on the other hand, is only a few percent with respect to the massless quarks contribution.

At present the two-loop leading order contributions are known for both classes of diagrams: corrections involving the top quark, both for $q\bar{q}H$ and VVH, have been evaluated as an expansion in $m_H^2/(4m_V^2)$ up to the fourth power [50]. This approximation is equivalent to an expansion of the $gg \to H$ amplitude in external momenta and, therefore, is evaluated in terms of vacuum bubble integrals that depend on masses m_t and m_V. In addition, a fully analytic expression for the amplitude for massless quarks in the VVH case is available [47, 51, 52]. These computations showed that the top quark contributions are only about $2\,\%$ of the light quark contributions and, therefore, are negligible.

An attempt to estimate the magnitude of the light-quark contributions at next-to-leading order in QCD was undertaken in [53, 54] using different ways to combine QCD and electroweak corrections. The results were not conclusive – depending on how the corrections were combined, the cross section was changed by either $+(1–2\,\%)$ or $+(5–6\,\%)$. This range is clearly unsatisfactory and an explicit calculation of the mixed corrections is called for.

A first direct calculation of next-to-leading order mixed QCD-electroweak contributions was performed in [55], where the limit $m_W, m_Z \gg m_H$ was employed.[1] In this limit, the calculation of the three-loop Feynman diagrams appearing in the amplitude is reduced to the evaluation of vacuum bubble integrals that are well-known. The contribution of next-to-leading order mixed QCD-electroweak corrections to the Higgs boson production cross section in this approximation amounts to $O\,(5\,\%)$.

To address the uncertainty related to the approximate knowledge of the QCD-electroweak contributions different approaches are used [25, 55]: difference between partial factorization and complete factorization, scale variation around $\mu_R = \mu_F = m_H/2$, variation of effective coupling $ggH_{\mathrm{QCD+EW}}$, etc. Most of these approaches lead to $O(1\,\%)$ variations of the $gg \to H$ cross section; for this reason, $1\,\%$ is conservatively chosen as the theoretical uncertainty.

To conclude, the imprecise knowledge of the next-to-leading order mixed QCD-electroweak corrections to $gg \to H$ is one of the important sources of theoretical uncertainties in the Higgs boson production cross section in gluon fusion. To eliminate it the exact computation of this corrections is needed. We will address this computation in what follows.

[1] Despite the unphysical nature of this limit, the fact that pure QCD gluon fusion contributions have been evaluated considering $m_t \gg m_H$ and that such result remains close to the exact one even for m_H up to $1\,\mathrm{TeV}$ is a good indication of its reliability.

Chapter 2.

Mixed QCD-electroweak $gg \to H$

2.1. Feynman diagrams

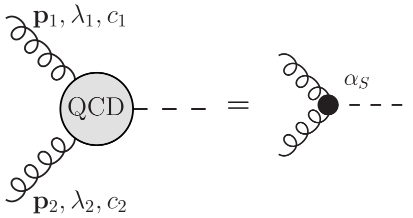
The mixed QCD-electroweak corrections to Higgs boson gluon fusion are obtained by considering the QCD terms obtained from the effective Lagrangian of Eq. (1.5) and the electroweak contributions presented in Fig. 1.2b, both described in Chapter 1. Both contributions are considered up to order α_S^2. At next-to-leading order we must account for virtual contributions, where an extra QCD loop is added, and real emissions, where a QCD parton is emitted into the final state. In this chapter, leading order and virtual next-to-leading order contributions are discussed, while the computation of the real emission contribution is addressed in Chapter 5.

Before discussing the calculation of the $gg \to H$ amplitudes in detail, we summarize the notation. We consider two incoming gluons g_1 and g_2 with on-shell momenta p_1 and p_2, color indices c_1 and c_2, and polarization labels λ_1 and λ_2, respectively. The momentum of the outgoing Higgs boson is $p_3^\mu = p_1^\mu + p_2^\mu$, with $(p_1 + p_2)^2 = p_3^2 = s = m_H^2$.

QCD contributions

QCD contributions are evaluated in the limit of an infinitely heavy top quark, see Eq. (1.5) of Section 1.1. Results are available in the literature [31, 33].

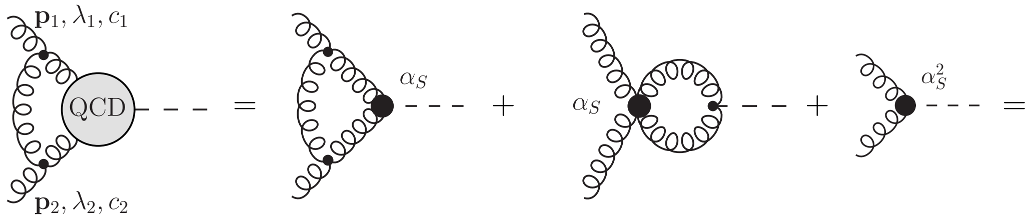
Only one diagram contributes to the amplitude at leading order. It gives



$$= \delta^{c_1 c_2} \epsilon_{\lambda_1}(\mathbf{p}_1) \cdot \epsilon_{\lambda_2}(\mathbf{p}_2) \mathrm{i} \frac{\alpha_S m_H^2}{6\pi v}. \qquad (2.1)$$

The contributions to the next-to-leading order amplitude arise both from loop effects and from higher order contributions to the Wilson coefficient of the effective ggH vertex.

The NLO contributions read [31, 33]



$$
= \delta^{c_1 c_2} \epsilon_{\lambda_1}(\mathbf{p}_1) \cdot \epsilon_{\lambda_2}(\mathbf{p}_2) \left(-\mathrm{i} \frac{\alpha_S^2 m_H^2}{6\pi v} \right) (4\pi)^{1+\varepsilon} \Gamma(1+\varepsilon) \times
$$

$$
\times \left(-\frac{\mu^2}{m_H^2} \right)^{2\varepsilon} \left[\frac{2N_C}{\varepsilon^2} - \frac{16\pi^4 N_C + 33}{48\pi^2} + O(\varepsilon) \right], \qquad (2.2)
$$

where μ is the renormalization scale.

Electroweak contributions

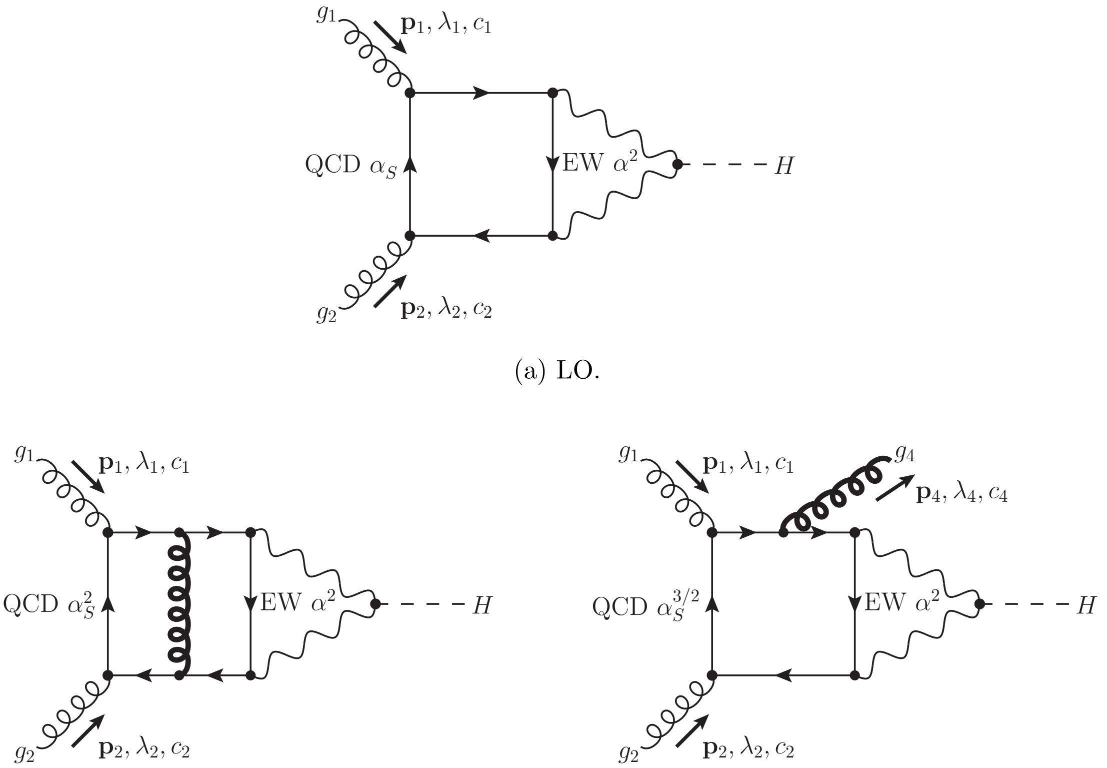
Mixed QCD-electroweak corrections to Higgs boson production in gluon fusion are sketched in Fig. 2.1. They appear for the first time at two loops. The two incoming gluons g_1 and g_2 annihilate to electroweak vector bosons V through a quark loop. The vector bosons subsequently fuse into a Higgs boson. The diagrams contributing at leading order and next-to-leading order have been generated using the computer program `QGRAF` [56] and the manipulation of Feynman diagrams has been performed using the computer language `FORM` [57]. The complete list of two-and three-loop Feynman diagrams at leading and next-to-leading order, respectively, is provided in Appendix B.

Both $W^\pm$ and Z bosons appear in the diagrams that contribute to mixed QCD-electroweak corrections. However, since electroweak interactions are considered at $O(\alpha^2)$, non-vanishing contributions contain diagrams either with $W^\pm$ or Z bosons. In the case of $W^\pm$, the first two generations of quarks are accounted for; when considering Z bosons, all quarks but the top quark are considered. Quarks are always considered to be massless and, therefore, the CKM matrix is taken to be equal to the identity matrix.[1] The conventions used for Feynman diagrams are listed in Appendix A.

Leading order

Three two-loop Feynman diagrams contribute to the leading order amplitude, they are shown in Fig. 2.2. The presence of electroweak vertices of the form $\gamma_\mu(C_v^{fV} + C_a^{fV}\gamma_5)$ between weak vector bosons and quarks introduces axial terms in the Feynman diagrams.

[1] All the quarks considered have a mass which is less than one twentieth of the mass of an electroweak boson. The largest quark-to-boson mass ratios are $m_c/m_W \lesssim 2\%$ for $W^\pm$ diagrams and $m_b/m_Z \lesssim 5\%$ for Z diagrams.



(a) LO.

(b) Virtual NLO.

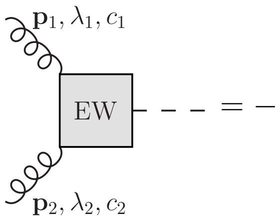
(c) Real NLO.

Figure 2.1.: Examples of mixed QCD-electroweak corrections for $gg \to H(g)$.

These terms are generated by Dirac structures of the form

$$(C_v^{fV^2} + C_a^{fV^2}) \operatorname{tr} \Gamma + 2 C_v^{fV} C_a^{fV} \operatorname{tr}[\gamma_5 \Gamma], \tag{2.3}$$

where Γ is a string of Dirac gamma matrices. Once the sum of all diagrams is computed, the part proportional to $\operatorname{tr}[\gamma_5\Gamma]$ vanishes. Moreover, for each diagram that contains $W^\pm$ bosons there exists another one containing Z bosons with exactly the same general structure, and vice versa. It is therefore possible to write the leading order amplitude as



$$= -\mathrm{i} \frac{\alpha^2 \alpha_S v}{64\pi \sin^4\theta_W} \left[C^W \mathcal{E}^{(0),c_1c_2}_{\lambda_1\lambda_2}\left(\frac{m_W^2}{s}\right) + C^Z \mathcal{E}^{(0),c_1c_2}_{\lambda_1\lambda_2}\left(\frac{m_Z^2}{s}\right)\right], \tag{2.4}$$

where $\mathcal{E}^{(0)}$ is a function proportional to the sum of the leading-order Feynman diagrams containing generic vector-boson lines and depending on the ratio m_V/s, v is the Higgs

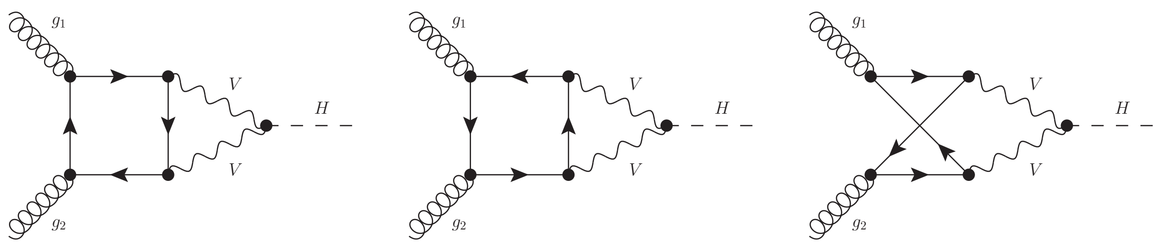


Figure 2.2.: LO QCD-electroweak Feynman diagrams.

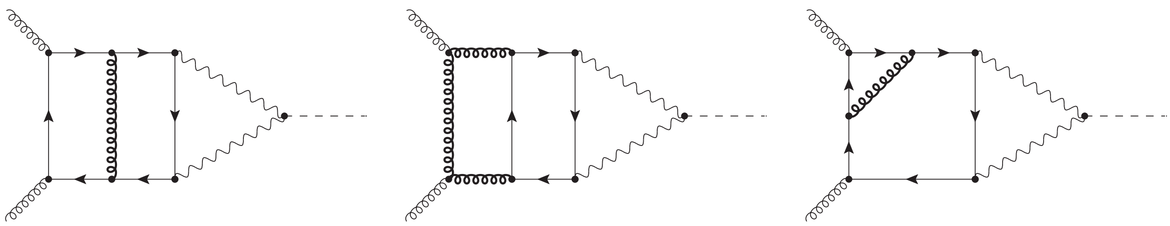


Figure 2.3.: Examples of virtual NLO mixed QCD-electroweak Feynman diagrams. NLO QCD corrections are shown with bold lines.

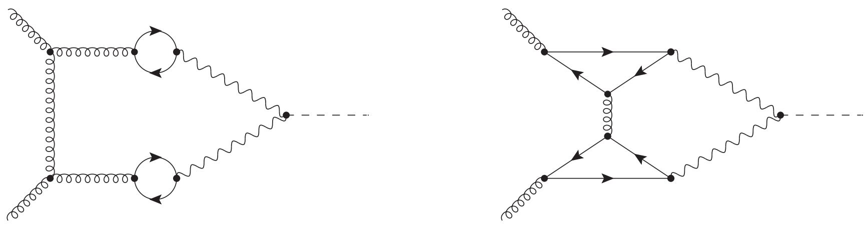
vacuum expectation value, θ_W is the Weinberg angle, and[2]

$$\begin{aligned} C^W &= 4, \\ C^Z &= \frac{2}{\cos^4\theta_W}\left(\frac{5}{4} - \frac{7}{3}\sin^2\theta_W + \frac{22}{9}\sin^4\theta_W\right). \end{aligned} \tag{2.5}$$

Next-to-leading order

Virtual NLO QCD-electroweak corrections are described by three-loop Feynman diagrams. Examples are shown in Fig. 2.3. The majority of the diagrams are similar for both Z and $W^\pm$ contributions. However, there are few exceptions that receive contributions only from the Z boson, see Fig. 2.4. However, all diagrams that distinguish Z and $W^\pm$ contributions vanish either because of color algebra (Fig. 2.4a), or because of Furry's theorem for vector current contribution or because complete generations of massless quarks enforce cancellation of the axial current contributions, Fig. 2.4b [58–61]. In the case of the axial current, the contribution of the bottom quark is not well defined without the top quark because of the axial anomaly. In this thesis this issue is ignored and all diagrams of the type shown in Fig. 2.4b are discarded. The bottom quark is nevertheless still included in all other three-loop QCD-electroweak diagrams. After these simplifications, there remain forty-seven non-vanishing three-loop Feynman diagrams for both $W^\pm$ and Z bosons. Similar to the leading-order case, no axial contribution is present at the level of the amplitude, thanks to charge-parity conservation and

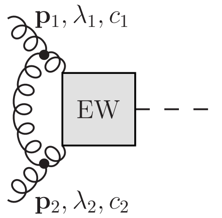
[2]By checking the Feynman diagrams, it is clear that the LO amplitude has mass dimension +1. All diagrams contain v as a coupling and s and m_V^2 as kinematic parameters. Once v is extracted the sum of Feynman diagrams $\mathcal{E}$ can only depend on the ratio m_V^2/s.



(a) Quark loop connection. (b) Two separate quark loops.

Figure 2.4.: Examples of diagrams that may provide Z-contributions but not $W^{\pm}$-contributions to the virtual NLO amplitude.

sum over complete generations of massless quarks [58–61]. Hence, the next-to-leading order mixed QCD-electroweak amplitude can be written in the same way as the leading order one



$$= -\mathrm{i}\frac{\alpha^2\alpha_S^2 v}{128\pi^2 \sin^4\theta_W}\left[C^W \mathcal{E}^{(1),c_1c_2}_{\lambda_1\lambda_2}\left(\frac{m_W^2}{s}\right) + C^Z \mathcal{E}^{(1),c_1c_2}_{\lambda_1\lambda_2}\left(\frac{m_Z^2}{s}\right)\right], \quad (2.6)$$

where $\mathcal{E}^{(1)}$ has the same role as $\mathcal{E}^{(0)}$ but is proportional to the virtual next-to-leading order Feynman diagrams, and C^W and C^Z are defined in Eq. (2.5).

2.2. Form factors

The amplitude can be decomposed into Lorentz and color structures, determined by the quantum numbers of the external particles and the symmetries of the theory. Such a decomposition is independent of the perturbative expansion. It reads

$$\mathcal{M}^{c_1c_2}_{\lambda_1\lambda_2}(\mathbf{p}_1, \mathbf{p}_2, \{m^2\}) = \sum_i \mathcal{F}_i(s, \{m^2\})\mathbb{T}^{c_1c_2}_{i,\lambda_1\lambda_2}(\mathbf{p}_1, \mathbf{p}_2, \epsilon_{\lambda_1}, \epsilon_{\lambda_2}), \quad (2.7)$$

where $\mathcal{F}_i$ are called *form factors* and $\mathbb{T}_i$ are the relevant Lorentz structures. The form factors are scalar functions of kinematic variables and internal masses, while the Lorentz structures contain all the information about quantum numbers of the external particles, for example their polarizations and color.

Tensorial decomposition

In the case of Higgs production in gluon fusion, the color structure is given by $\delta^{c_1c_2}$, since this is the only invariant structure that can be constructed from two color indices.

The Lorentz indices of the amplitude are made explicit by writing

$$\mathcal{M}^{c_1 c_2}_{\lambda_1 \lambda_2}(\mathbf{p}_1, \mathbf{p}_2, \{m^2\}) = \delta^{c_1 c_2} \epsilon^{\mu}_{\lambda_1}(\mathbf{p}_1) \epsilon^{\nu}_{\lambda_2}(\mathbf{p}_2) M_{\mu\nu}(\mathbf{p}_1, \mathbf{p}_2, \{m^2\}). \tag{2.8}$$

$M_{\mu\nu}$ is a linear combination of rank-2 Lorentz tensors constructed out of $p_{1\mu}$, $p_{2\mu}$, and the metric tensor $g_{\mu\nu}$.[3] All the possible tensorial structures but one disappear for physical external gluons.

The most general form of the tensor $M_{\mu\nu}$ is

$$M_{\mu\nu} = g_{\mu\nu}\, \mathcal{F}_0 + p_{1\mu} p_{1\nu}\, \mathcal{F}_1 + p_{2\mu} p_{2\nu}\, \mathcal{F}_2 + p_{1\mu} p_{2\nu}\, \mathcal{F}_3 + p_{2\mu} p_{1\nu}\, \mathcal{F}_4. \tag{2.9}$$

Consider now the Higgs boson rest frame, and take the $\hat{z}$ axis along $\mathbf{p}_1 = -\mathbf{p}_2$. The physical polarizations of the gluons belong to the $\langle \hat{x}, \hat{y} \rangle$ plane, therefore when $M_{\mu\nu}$ is contracted with $\epsilon^{\mu}_{\lambda_1} \epsilon^{\nu}_{\lambda_2}$ only the term $g_{\mu\nu}\mathcal{F}_0$ survives. The amplitude for $gg \to H$ is therefore written in terms of a single form factor, and reads

$$\mathcal{M}^{c_1 c_2}_{\lambda_1 \lambda_2}(\mathbf{p}_1, \mathbf{p}_2, \{m^2\}) = \delta^{c_1 c_2} \epsilon_{\lambda_1}(\mathbf{p}_1) \cdot \epsilon_{\lambda_2}(\mathbf{p}_2) \mathcal{F}(s, \{m^2\}). \tag{2.10}$$

The choice of physical polarizations is equivalent to impose the conditions

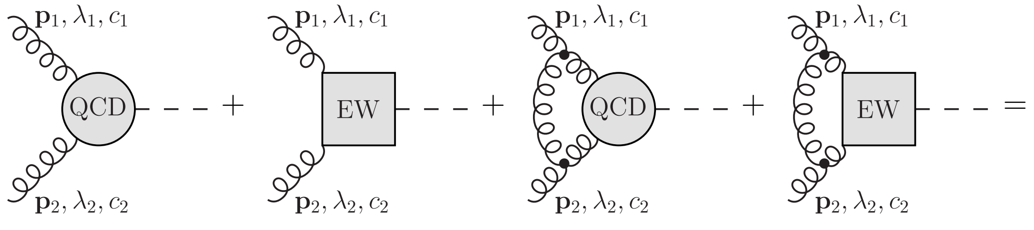
$$\begin{aligned} \epsilon_{\lambda_1}(\mathbf{p}_1) \cdot p_1 = \epsilon_{\lambda_1}(\mathbf{p}_1) \cdot p_2 = 0, \\ \epsilon_{\lambda_2}(\mathbf{p}_2) \cdot p_1 = \epsilon_{\lambda_2}(\mathbf{p}_2) \cdot p_2 = 0, \end{aligned} \tag{2.11}$$

and the sum over polarizations reads

$$\sum_{\lambda_i} \epsilon^{*\mu}_{\lambda_i}(\mathbf{p}_i) \epsilon^{\lambda_i, \nu}(\mathbf{p}_i) = -g^{\mu\nu} + \frac{p_1^{\mu} p_2^{\nu} + p_2^{\mu} p_1^{\nu}}{p_1 \cdot p_2}. \tag{2.12}$$

Extraction of the form factor

The form factor must be extracted from the amplitude written in terms of Feynman diagrams



$$= \delta^{c_1 c_2} \epsilon_{\lambda_1}(\mathbf{p}_1) \cdot \epsilon_{\lambda_2}(\mathbf{p}_2) \mathcal{F}(s, \{m^2\}). \tag{2.13}$$

To do so, it is necessary to build a *projector* $\mathbb{P}$, i.e. a tensor that selects the coefficients of the tensorial structure written in Eq. (2.10).

[3] Terms containing the Levi-Civita tensor do not appear since axial contributions have been ruled out in Section 2.1 [60, 61].

It is not difficult to build such a projector for the case under study. To select the color-singlet part of the Feynman diagrams it is sufficient to contract the amplitude with $\delta_{c_1 c_2}/(N_C^2-1)$ (the factor $1/(N_C^2-1)$ is introduced for normalization). For the Lorentz part, we use Eq. (2.12) and find

$$\begin{aligned}
&\sum_{\lambda_1,\lambda_2}\left[\epsilon^{*\lambda_1}(\mathbf{p}_1)\cdot\epsilon^{*\lambda_2}(\mathbf{p}_2)\right]\left[\epsilon_{\lambda_1}(\mathbf{p}_1)\cdot\epsilon_{\lambda_2}(\mathbf{p}_2)\right]= \\
&=\left[-g^{\mu\nu}+\frac{p_1^\mu p_2^\nu+p_2^\mu p_1^\nu}{p_1\cdot p_2}\right]\left[-g_{\mu\nu}+\frac{p_{1,\mu}p_{2,\nu}+p_{2,\mu}p_{1,\nu}}{p_1\cdot p_2}\right]=D-2,
\end{aligned} \tag{2.14}$$

therefore a factor $1/(D-2)$ must be included. Putting these pieces together, the projector reads

$$\mathbb{P}^{\lambda_1\lambda_2}_{c_1c_2}=\frac{\epsilon^{*\lambda_1}(\mathbf{p}_1)\cdot\epsilon^{*\lambda_2}(\mathbf{p}_2)}{D-2}\frac{\delta_{c_1c_2}}{N_C^2-1}. \tag{2.15}$$

The amplitude is written in terms of Feynman diagrams

$$\begin{aligned}
\mathcal{M}^{c_1c_2}_{\lambda_1\lambda_2}=-\mathrm{i}\frac{\alpha^2\alpha_S v}{64\pi\sin^4\theta_W}\Big[\, & C^W\left(\mathcal{E}^{(0),c_1c_2}_{\lambda_1\lambda_2}(m_W^2/s)+\frac{\alpha_S}{2\pi}\,\mathcal{E}^{(1),c_1c_2}_{\lambda_1\lambda_2}(m_W^2/s)\right) \\
&+C^Z\left(\mathcal{E}^{(0),c_1c_2}_{\lambda_1\lambda_2}(m_Z^2/s)+\frac{\alpha_S}{2\pi}\,\mathcal{E}^{(1),c_1c_2}_{\lambda_1\lambda_2}(m_Z^2/s)\right)\Big],
\end{aligned} \tag{2.16}$$

and the projector $\mathbb{P}^{\lambda_1\lambda_2}_{c_1c_2}$ is applied separately to each diagram in the sums $\mathcal{E}^{(i),c_1c_2}_{\lambda_1\lambda_2}$. The polarization vectors of each diagram are extracted and contracted with the projector, giving

$$\begin{aligned}
\mathbb{P}^{\lambda_1\lambda_2}_{c_1c_2}\mathcal{E}^{(i),c_1c_2}_{\lambda_1\lambda_2}&=\frac{\epsilon^{*\lambda_1}(\mathbf{p}_1)\cdot\epsilon^{*\lambda_2}(\mathbf{p}_2)}{D-2}\frac{\delta_{c_1c_2}}{N_C^2-1}\;\epsilon^\mu_{\lambda_1}(\mathbf{p}_1)\epsilon^\nu_{\lambda_2}(\mathbf{p}_2)E^{(i),c_1c_2}_{\mu\nu} \\
&=\frac{\delta_{c_1c_2}}{N_C^2-1}\frac{1}{D-2}\left[g^{\mu\nu}-\frac{p_1^\mu p_2^\nu+p_2^\mu p_1^\nu}{p_1\cdot p_2}\right]E^{(i),c_1c_2}_{\mu\nu}=F^{(i)},
\end{aligned} \tag{2.17}$$

where $E^{(i),c_1c_2}_{\mu\nu}$ are Feynman diagrams with free color and Lorentz indices. The projection operation reduces each Feynman diagram to a scalar function $F^{(i)}$. These functions are then added together to construct the form factor.

2.3. UV renormalization and IR subtraction

While the leading order part of the form factor is finite in $D\to 4$ dimensions, the virtual NLO contributions are divergent with poles starting at ε^{-2}, as explicitly shown for the QCD contributions in Eq. (2.2). These divergencies are both of ultraviolet (UV) and infrared (IR) origin.

To remove ultraviolet divergences, the strong coupling constant α_S must be renormalized. In the $\overline{\text{MS}}$ scheme the bare coupling constant $\alpha_S^{(0)}$ becomes

$$\alpha_S^{(0)} = \frac{\mathrm{e}^{\varepsilon\gamma_E}}{(4\pi)^\varepsilon}\alpha_S(\mu^2)\mu^{2\varepsilon}\left(1 - \frac{\beta_0}{\varepsilon}\frac{\alpha_S(\mu^2)}{2\pi}\right) + O\left(\alpha_S^3\right), \tag{2.18}$$

where γ_E is the Euler–Mascheroni constant and β_0 is the first coefficient of the QCD β-function. It reads

$$\beta_0 = \frac{11}{6}N_C - \frac{N_f}{3}, \tag{2.19}$$

where $N_C = 3$ is the number of colors and $N_f = 5$ is the number of massless fermions that contribute to the renormalization of the QCD coupling constant. It is not necessary to renormalize α since both axial and vector currents are conserved when massless quarks are considered.

Combining Eqs. (2.4), (2.6), and (2.10) together with the UV renormalization of Eq. (2.18), the complete form factor $\mathcal{F}$ of Eq. (2.13) can be written as

$$\begin{aligned} \mathcal{F} \; = \; & \mathrm{i}\frac{\alpha_S m_H^2}{6\pi v}\mathcal{A}_{\text{QCD}}\left(\frac{\mu^2}{s}\right) + \\ & - \mathrm{i}\frac{\alpha^2\alpha_S(\mu^2)v}{64\pi\sin^4\theta_W}\left[C^W\mathcal{A}_{\text{QCD-EW}}\left(\frac{m_W^2}{s},\frac{\mu^2}{s}\right) + C^Z\mathcal{A}_{\text{QCD-EW}}\left(\frac{m_Z^2}{s},\frac{\mu^2}{s}\right)\right], \end{aligned} \tag{2.20}$$

with C_i defined in Eq. (2.5). $\mathcal{A}_{\text{QCD}}$ includes QCD contributions, while $\mathcal{A}_{\text{QCD-EW}}$ contains mixed QCD-electroweak ones. Both of these functions can be written as an expansion in α_S

$$\mathcal{A} = A^{(0)} + \frac{\alpha_S(\mu^2)}{2\pi}A^{(1)} + O\left(\alpha_S^2\right). \tag{2.21}$$

The ultraviolet renormalization removes some of the $1/\varepsilon$ poles but infrared singularities remain. The general structure of these singularities in QCD is described by Catani's formula [62]. It reads

$$A^{(1)} = \mathbf{I}^{(1)}\left(\frac{\mu^2}{s}\right)A^{(0)} + A^{(1,\text{fin})}, \tag{2.22}$$

where

$$\mathbf{I}^{(1)}\left(\frac{\mu^2}{s}\right) = \left(\frac{-s - i0}{\mu^2}\right)^{-\varepsilon}\frac{\mathrm{e}^{\varepsilon\gamma_E}}{\Gamma(1-\varepsilon)}\left[-\frac{N_C}{\varepsilon^2} - \frac{\beta_0}{\varepsilon}\right]. \tag{2.23}$$

Note that the leading order amplitude in Eq.(2.22) is needed through order $O\left(\varepsilon^2\right)$. This pole structure is the same for both QCD and QCD-electroweak diagrams, since in both cases one order higher in α_S is considered to build the next-to-leading order amplitude. Eqs. (2.21) and (2.22) are combined into Eq. (2.20) to obtain the form factor. Its final expression reads

$$\begin{aligned}\mathcal{F} &= \mathrm{i}\frac{\alpha_S m_H^2}{6\pi v}\left[A^{(0)}_{\mathrm{QCD}} + \frac{\alpha_S(\mu^2)}{2\pi}\left(\mathbf{I}^{(1)}\left(\frac{\mu^2}{s}\right)A^{(0)}_{\mathrm{QCD}} + A^{(1,\mathrm{fin})}_{\mathrm{QCD}}\left(\frac{\mu^2}{s}\right)\right)\right] + \\ &\quad - \mathrm{i}\frac{\alpha^2\alpha_S(\mu^2)v}{64\pi\sin^4\theta_W}\sum_{V=W,Z} C^V\left[A^{(0)}_{\text{QCD-EW}}\left(\frac{m_V^2}{s}\right) + \right. \\ &\qquad \left. + \frac{\alpha_S(\mu^2)}{2\pi}\left(\mathbf{I}^{(1)}\left(\frac{\mu^2}{s}\right)A^{(0)}_{\text{QCD-EW}}\left(\frac{m_V^2}{s}\right) + A^{(1,\mathrm{fin})}_{\text{QCD-EW}}\left(\frac{m_V^2}{s}, \frac{\mu^2}{s}\right)\right)\right]. \qquad (2.24)\end{aligned}$$

Once the IR divergencies are made explicit by the Catani's operator, the form factor is written in terms of finite quantities. The calculation of the function $A^{(0)}_{\text{QCD-EW}}$ has required a non-negligible amount of work. The evaluation of $A^{(1,\mathrm{fin})}_{\text{QCD-EW}}$ (or, equivalently, of $A^{(1)}_{\text{QCD-EW}}$) is a highly non-trivial task, and the main subject of this thesis, which will be discussed in Chapters 3 and 4.

Chapter 3.

Evaluation of master integrals

3.1. From Feynman integrals to master integrals

Feynman integrals

The functions $A^{(0)}_{\text{QCD-EW}}$ and $A^{(1)}_{\text{QCD-EW}}$ introduced in Eq. (2.24) are linear combinations of scalar multi-dimensional *Feynman integrals* (FIs). A L-loop Feynman integral can be written as

$$I(\{s\},\{M^2\},\varepsilon) = \int \frac{\mathrm{d}^{4-2\varepsilon}k_1 \dots \mathrm{d}^{4-2\varepsilon}k_L}{\left[\mathrm{i}\pi^{2-\varepsilon}\Gamma(1+\varepsilon)\right]^L} \prod_{[j]\in\mathcal{J}} \frac{1}{[j]^{a_j}}, \tag{3.1}$$

where $\{s\}$ and $\{M^2\}$ indicate external kinematic variables and squares of internal masses, respectively, $\varepsilon = (4-D)/2$ is the dimensional regularization parameter, a_j is an integer number, and $[j]$ is an inverse propagator of the form

$$[j] = \mathcal{K}_j^2 - M_j^2. \tag{3.2}$$

In Eq. (3.2) $\mathcal{K}_j$ indicates a linear combination of external momenta p_i and loop momenta k_l. The set $\mathcal{J}$ of inverse propagators $[j]$ is called the *integral family* of the Feynman integral I.

The subset of inverse propagators $\tilde{\mathcal{J}}$ with positive a_j defines the *topology* of the Feynman integral, i.e. a graphical representation in terms of oriented edges and vertices. Different sets of denominators that belong to the same topology are related to each other by shifts of the loop momenta. When drawing a Feynman integral in this thesis, wavy lines are used to indicate massless propagators and external legs, and solid lines are used when the mass is non-zero. Lines of different thickness represent particles with different masses. A dot on a line indicates that the corresponding propagator is raised to the second power. Each extra dot on the same propagator increases the exponent by one.

In an actual calculation, scalar products that involve loop momenta may appear in the numerator. These scalar products can be written as linear combinations of inverse propagators, to express all Feynman integrals in the notation of Eq. (3.1). It is then easy to see that a L-loop Feynman integral with $E+1$ external legs (E independent external momenta) possesses $L(L+1)/2 + LE$ independent scalar products. At one loop it is always possible to map all the scalar products of a given Feynman integral into its inverse propagators ($\mathcal{J} = \tilde{\mathcal{J}}$), while from two loops on the number of inverse propagators is almost always smaller than the number of possible scalar products. The so-called *irreducible scalar products* require the introduction of additional inverse propagators. These inverse propagators never appear with positive powers a_j and, as long as they are independent, their choice is arbitrary. A rule of thumb is to select a collection of inverse propagators that makes the integral family symmetric under the exchange of loop and external momenta.

To calculate the leading order form factor from $A^{(0)}_{\text{QCD-EW}}$, $O(1000)$ two-loop Feynman integrals must be computed, while the calculation of $A^{(1)}_{\text{QCD-EW}}$ requires the evaluation of $O(10\,000)$ different three-loop Feynman integrals. To proceed, different integrals are grouped together into *integral families*. Inside the same integral family, the Feynman integrals can be further classified according to their *parent topology*. Parent topologies are Feynman integrals with a large number of inverse propagators, and their graphical representation usually resembles one or more Feynman diagrams of the relative amplitude. A Feynman integral belongs to a parent topology if it can be obtained by removing some inverse propagators from the parent one. The two-loop Feynman integrals are expressed in terms of two integral families (planar PP and non-planar NP), each containing one parent topology. The three-loop Feynman integrals require three integral families (one planar, VPP, and two non-planar, VNA and VNB) and a total of eleven parent topologies (five in VPP, four in VNA and two in VNB). Integral families and parent topologies are given in Appendix B.

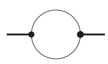
Reduction to master integrals

The computation of a large number of Feynman integrals, typical for multi-loop problems, is facilitated by the existence of linear algebraic relations among them. It is easy to derive these equations using the fact that integrals of total derivatives vanish in dimensional regularization. It follows that [63, 64]

$$\int \mathrm{d}^{4-2\varepsilon}k_1 \ldots \mathrm{d}^{4-2\varepsilon}k_L \, \frac{\partial}{\partial k_l^\mu} \left(q^\mu \prod_{j\in\mathcal{J}} \frac{1}{[j]^{a_j}} \right) = 0, \qquad q \in \{p, k\},\ l = 1, \ldots, L. \tag{3.3}$$

Eq. (3.3) produces $L(L+E)$ identities among different Feynman integrals, called *integration-by-parts identities* (IBPs).[1] Other useful information is provided by Lorentz identities (it has been proven in [66] that they can be constructed from the IBP identities) and symmetry relations between integrals.

As an example of how IBP identities are generated, consider the integral



$$= \int \frac{\left[\mathrm{i}\pi^{2-\varepsilon}\Gamma(1+\varepsilon)\right]^{-1} \mathrm{d}^{4-2\varepsilon}k}{[k^2 - M^2]\,[(k-p)^2 - M^2]}. \tag{3.4}$$

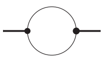
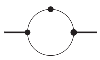
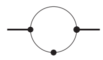
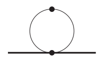
[1]One can understand IBPs as a consequence of the invariance of Feynman integrals under shifts of the loop momenta, as explained in [65].

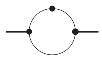
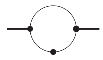
Two identities can be written

$$\int \mathrm{d}^{4-2\varepsilon}k\,\frac{\partial}{\partial k^\mu}\frac{k^\mu}{[k^2-M^2]\,[(k-p)^2-M^2]}=0, \tag{3.5}$$

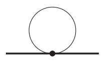
$$\int \mathrm{d}^{4-2\varepsilon}k\,\frac{\partial}{\partial k^\mu}\frac{p^\mu}{[k^2-M^2]\,[(k-p)^2-M^2]}=0, \tag{3.6}$$

and they lead to

$0 = (1-2\varepsilon)$  $- 2M^2$  $+ (p^2-2M^2)$  $-$  , (3.7)

$0 = p^2\Big($  $-$  $\Big)$. (3.8)

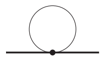
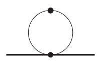
One IBP relation can be derived for the subtopology

 $= \displaystyle\int \frac{\left[\mathrm{i}\pi^{2-\varepsilon}\Gamma(1+\varepsilon)\right]^{-1}\,\mathrm{d}^{4-2\varepsilon}k}{[k^2-M^2]}$, (3.9)

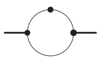
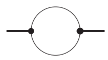
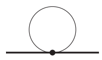
starting from

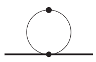
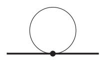
$$\int \mathrm{d}^{4-2\varepsilon}k\,\frac{\partial}{\partial k^\mu}\frac{k^\mu}{[k^2-M^2]}=0, \tag{3.10}$$

to obtain

$(1-\varepsilon)$  $- M^2$  $= 0.$ (3.11)

Eqs. (3.7), (3.8), and (3.11) are combined to give

 $= \dfrac{1-2\varepsilon}{4M^2-p^2}$  $+ \dfrac{-1+\varepsilon}{M^2\,(4M^2-p^2)}$  ,

 $= \dfrac{1-\varepsilon}{M^2}$  . (3.12)

Hence the two Feynman integrals with a dot are expressed in terms of two simpler integrals in an algebraic way.

By using IBP identities, Feynman integrals can be expressed through a small set of integrals. Elements of this set are called *master integrals* (MIs). Given the master integrals for a set of parent topologies, all other Feynman integrals are computed as linear combinations of the master integrals.

Finding master integrals requires a constructive algorithm. Ideally, this algorithm should be optimized to select simple candidates for the master integrals in an efficient way, since it becomes increasingly difficult to generate and solve IBPs beyond one loop. The algorithm described in [67] is currently widely used in many computer programs that generate and solve IBP identities.

To compute mixed QCD-electroweak corrections to Higgs production, we perform the reduction to master integrals at two and three loops using the computer programs

Reduze2 [68] and KIRA [69]. We find that $A^{(0)}_{\text{QCD-EW}}$ and $A^{(1)}_{\text{QCD-EW}}$ can be expressed as linear combinations of twelve two-loop master integrals and 95 three-loop master integrals, respectively. The master integrals can be found in Appendix B.

3.2. Differential equations for master integrals

The evaluation of master integrals is a highly non-trivial problem. Indeed, a direct integration over Feynman parameters is often prohibitively difficult, if not altogether impossible. A different approach to the problem consists of considering Feynman integrals as functions of kinematic variables and internal masses, that satisfy particular differential equations with respect to such variables [70–75]. By solving these equations and imposing suitable boundary conditions, Feynman integrals can be fully determined. The method of *differential equations for master integrals* has proven to be powerful and versatile. Generation of differential equations for a given set of Feynman integrals is implemented in many computer programs for reduction to master integrals, for example in Reduze2 [68].

There are many different ways to employ differential equations (DEs) to compute Feynman integrals. For example, the basis of master integrals $\mathbf{I}(\{s\},\{M^2\},\varepsilon)$ for the amplitude is differentiated with respect to kinematic variables $\{s\}$ and internal masses $\{M^2\}$, and IBP identities are used to express the result in terms of $\mathbf{I}$, leading to a *closed system of partial differential equations*

$$\frac{\partial \mathbf{I}(\mathbf{x},\varepsilon)}{\partial \mathbf{x}_i} = A_i(\mathbf{x},\varepsilon)\mathbf{I}(\mathbf{x},\varepsilon), \tag{3.13}$$

where $\mathbf{x} = (\{s\},\{M^2\})$.

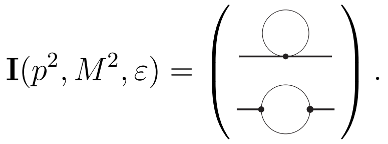
An additional relation for master integrals comes from the fact that they are functions with mass scaling dimension E, therefore they satisfy *Euler's scaling equation*

$$\sum_i \left[\mathbf{x}_i\right] \mathbf{x}_i \frac{\partial}{\partial \mathbf{x}_i} \mathbf{I}_k(\mathbf{x},\varepsilon) = E_k \mathbf{I}_k(\mathbf{x},\varepsilon), \tag{3.14}$$

$$E_k = 2\left[L(2-\varepsilon) - \sum_{j\in\mathcal{J}_k} \alpha_j\right], \tag{3.15}$$

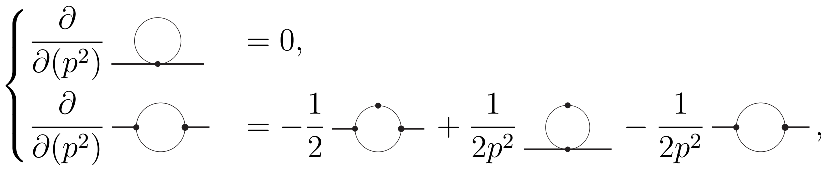
where $\mathbf{I}$ is the vector of master integrals, $[\mathbf{x}_i]$ is the mass dimension of the variable $\mathbf{x}_i$, and L is the number of loops. Practically speaking, the Euler's scaling equation is useful to check the consistency of the differential equations. A mismatch in this relation might indicate either that the system of IBPs is not complete or that some master integrals are not really independent.

To give an explicit example, consider the basis of master integrals

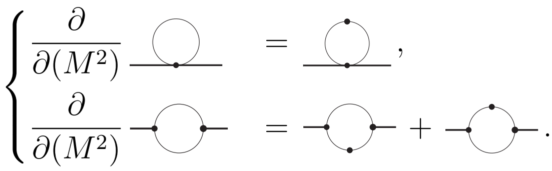


(3.16)

The master integrals are differentiated either with respect to p^2 or M^2, leading to

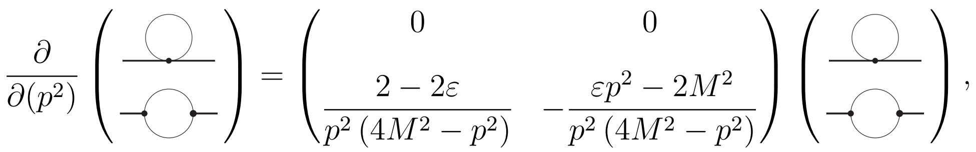


(3.17)

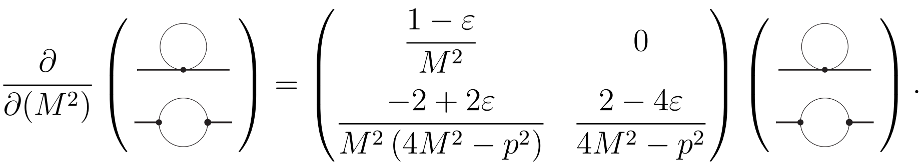


(3.18)

The IBP relations derived in Eq. (3.12) are applied. Two closed systems of partial differential equations, one with respect to p^2 and the other with respect to M^2, are obtained

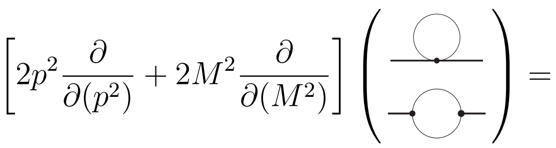


(3.19)

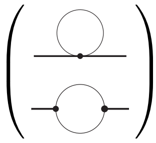


(3.20)

The differential equations are consistent with the Euler's scaling equation

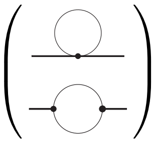


$$= \left[\begin{pmatrix} 0 & 0 \\ \dfrac{4-4\varepsilon}{4M^2-p^2} & \dfrac{2\varepsilon p^2 - 4M^2}{4M^2-p^2} \end{pmatrix} + \begin{pmatrix} 2-2\varepsilon & 0 \\ \dfrac{-4+4\varepsilon}{4M^2-p^2} & \dfrac{(4-8\varepsilon)M^2}{4M^2-p^2} \end{pmatrix} \right]$$



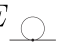
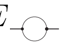
$$=$$

$$= \begin{pmatrix} 2-2\varepsilon & 0 \\ 0 & -2\varepsilon \end{pmatrix}$$

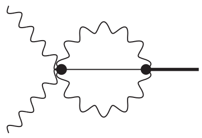


$$, \tag{3.21}$$

since (cf. Eq. (3.15))

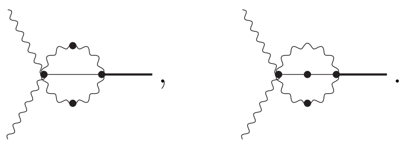
E  $= 2 - 2\varepsilon,$ $\qquad$ E  $= -2\varepsilon.$ $\qquad$ (3.22)

Master integrals possess a natural hierarchy given by their topologies, which translates into constraints on the system of differential equations. As can be guessed from the previous example, differentiating a master integral results in a linear combination of integrals whose topologies are either the original one or subtopologies of it. This implies that master integrals that contain fewer inverse propagators satisfy simpler differential equations, and that Feynman integrals belonging to unrelated topologies do not interact with each other. This rather simple picture is complicated by the fact that a single topology may require more than one master integral to be fully reduced. The issue appears already in simple topologies, for example in



$$, \tag{3.23}$$

that requires two master integrals. A possible choice of these integrals is



$$. \tag{3.24}$$

The number of master integrals for a given topology usually increases with the number of internal lines, going up from two for integrals with 3–4 internal lines to four for integrals with 6–9 internal lines.

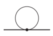
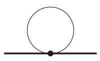
The presence of more than one master integral in a topology increases the complexity of the differential equations, since integrals that appear in the course of differentiation are reduced to linear combinations of all the master integrals of the topology. As a consequence, the general structure of the matrix of coefficients of the equations is (after rearranging the order of the single terms) *lower block-triangular* and relatively sparse. The lower block-triangular structure allows one to use a *bottom-up* approach to solving the differential equations: first the master integrals with smaller number of internal lines are determined, followed by the master integrals with a larger number of internal lines, where simpler integrals appear as inhomogeneous terms.

3.3. Uniformly transcendental functions

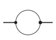
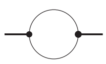
Definition

Except for the requirement that master integrals form a basis, their choice is to a large extent arbitrary. A natural choice [67, 68] is to use Feynman integrals with $a_j \in \{-1, 0, 1, 2\}$, giving a preference to simpler topologies with no inverse propagators in numerators. However, a system of equations for this choice of master integrals rarely shows a simple structure, despite the fact that the master integrals themselves are simple. In general, it is difficult to integrate a system of differential equations for Feynman integrals, therefore obtaining a simple system of differential equations is an important goal in practice.

To give an explicit example, consider once more the pair of one-loop master integrals given by the massive tadpole and the massive bubble. They can be evaluated by a direct integration over Feynman parameters

M  $=$ 

$$
\begin{aligned}
= \; & \frac{M^2}{\varepsilon} \\
& - M^2 \left[\log\frac{M^2}{\mu^2} - 1\right] \\
& + \varepsilon\,\frac{M^2}{2}\left[\log^2\frac{M^2}{\mu^2} - 2\log\frac{M^2}{\mu^2} + 2\right] \\
& + \varepsilon^2\,\frac{M^2}{6}\left[\log^3\frac{M^2}{\mu^2} - 3\log^2\frac{M^2}{\mu^2} + 6\log\frac{M^2}{\mu^2} - 6\right] \\
& + O\left(\varepsilon^3\right),
\end{aligned}
\tag{3.25}
$$

M  $=$ 

$$
\begin{aligned}
= \; & \frac{1}{\varepsilon} - \log M^2 - R\log\frac{R+1}{R} + R\log\frac{R-1}{R} + 2 \\
+ \; & \frac{\varepsilon}{4}\Bigg[4\log M^2\log(\rho+4) - 4R\log M^2\log\frac{R-1}{R} + 4\log M^2\log\frac{R-1}{R} \\
& +4R\log M^2\log\frac{R+1}{R} + 4\log M^2\log\frac{R+1}{R} + 2\log^2 M^2 - 8\log 2\log M^2 \\
& -8\log M^2 - 4R\,\mathrm{Li}_2\left(\frac{R+1}{2R}\right) + 4R\,\mathrm{Li}_2\left(\frac{R-1}{2R}\right) + 2\log^2(\rho+4) - 8\log 2\log(\rho+4) \\
& -8\log(\rho+4) - 4R\log(\rho+4)\log\frac{R-1}{R} + 4\log(\rho+4)\log\frac{R-1}{R} \\
& +4R\log(\rho+4)\log\frac{R+1}{R} + 4\log(\rho+4)\log\frac{R+1}{R} - 2R\log^2\frac{R-1}{R}
\end{aligned}
$$

$$
\begin{aligned}
&+2\log^2\frac{R-1}{R}+2R\log^2\frac{R+1}{R}+2\log^2\frac{R+1}{R}+4R\log 2\log\frac{R-1}{R}\\
&-8\log 2\log\frac{R-1}{R}-4R\log 2\log\frac{R+1}{R}-8\log 2\log\frac{R+1}{R}+8R\log\frac{R-1}{R}\\
&-8\log\frac{R-1}{R}-8R\log\frac{R+1}{R}+4\log\frac{R-1}{R}\log\frac{R+1}{R}-8\log\frac{R+1}{R}\\
&+16+8\log^2 2+16\log 2\quad\Bigg]+O\left(\varepsilon^2\right),
\end{aligned}
\tag{3.26}
$$

where $R=\sqrt{(\rho+4)/\rho}$, with $\rho=(-p^2)/M^2$ and p^2 is the incoming momentum squared, and μ^2 fixes the mass dimension of the integral. The corresponding differential equations are shown in Eqs. (3.19) and (3.20).

It is possible to derive much simpler differential equations and series expansions that contains the same amount of information. Indeed, consider the rescaled master integrals

$$
\mathbf{F} = \varepsilon\left(\frac{M^2}{\mu^2}\right)^{\varepsilon} = 1,
\tag{3.27}
$$

$$
\begin{aligned}
\mathbf{F} &= \varepsilon M^2\left(\frac{M^2}{\mu^2}\right)^{\varepsilon}\sqrt{\rho}\sqrt{\rho+4} =\\
&= -2\,\varepsilon\,\left[\log\left(\sqrt{\rho}+\sqrt{\rho+4}\right)-3\log 2\right]\\
&\quad+\varepsilon^2\,\left[-\mathrm{Li}_2\left(\frac{1}{2}+\frac{1}{2}\frac{\sqrt{\rho}}{\sqrt{\rho+4}}\right)+\mathrm{Li}_2\left(\frac{1}{2}-\frac{1}{2}\frac{\sqrt{\rho}}{\sqrt{\rho+4}}\right)\right.\\
&\quad\left.+2\log 2\ \log\sqrt{\rho+4}+2\log\left(\sqrt{\rho}+\sqrt{\rho+4}\right)\log\sqrt{\rho+4}\right]\\
&\quad+O\left(\varepsilon^3\right).
\end{aligned}
\tag{3.28}
$$

The differential equations in the new pair of variables (M^2,ρ) greatly simplify

$$
\frac{\partial}{\partial(M^2)}\begin{pmatrix}\mathbf{F}\\ \mathbf{F}\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}\begin{pmatrix}\mathbf{F}\\ \mathbf{F}\end{pmatrix},
\tag{3.29}
$$

$$
\frac{\partial}{\partial\rho}\begin{pmatrix}\mathbf{F}\\ \mathbf{F}\end{pmatrix}=\varepsilon\left[\frac{\partial}{\partial\rho}\begin{pmatrix}0&0\\-2\log\left(\sqrt{\rho}+\sqrt{\rho+4}\right)&-2\log\sqrt{\rho+4}\end{pmatrix}\right]\begin{pmatrix}\mathbf{F}\\ \mathbf{F}\end{pmatrix}.
\tag{3.30}
$$

The choice of this new functions $\mathbf{F}$ is less "natural" at first sight but it leads to extremely simple differential equations. Notice that

- $\mathbf{F}$ have no poles in ε;
- constants that appear both in M and $\mathbf{F}$ can be related to powers of logarithms, in the sense that

- π comes from $\log(-1)$, therefore it can be associated with the "logarithmic power" 1;
- $\zeta(2k)$, $k \in \mathbb{N}$, can be written as $q\pi^{2k}$, with $q \in \mathbb{Q}$, so that $\zeta(2k)$ can be associated with the "logarithmic power" $2k$;
- as a generalization of the above statements, one can assume that $\zeta(n)$ for $n \in \mathbb{N}$ has the "logarithmic power" n;
- and rational constants are assigned the "logarithmic power" 0;

- coefficients of ε^n terms have uniform "logarithmic power" n in $\mathbf{F}$;
- since
$$\log^n(x) \quad \Leftrightarrow \quad \int_{s_0}^{s} \frac{1}{\xi_n} \cdots \frac{1}{\xi_1} \, \mathrm{d}\xi_1 \ldots \mathrm{d}\xi_n, \tag{3.31}$$
the notion of "logarithmic power" can be associated with the *number of iterated integrations* (where the constant terms arise from the integration boundary s_0).

These properties find immediate confirmation in the structure of the differential equations, which possess:

- a $\mathrm{d}\log$-form with respect to the kinematic variables thanks to the fact that the coefficient of ε^n contains $\log^n s$ terms;
- an ε-homogeneous form, since the number of iterated integrations matches the ε power term by term in the expansion.

The above statements about G can be made more rigorous by introducing the concept of *uniformly transcendental functions* (UT functions) [76, 77]. These functions appear naturally in supersymmetric gauge theories, where *amplitudes* themselves are UT functions. Such results are extremely simple and compact and are expressed in terms of a very specific class of functions defined by iterated integrations on a small number of kernels, whose analytic properties are well known and allow to predict features of the final result such as discontinuities and integration constants. Despite the fact that SM amplitudes are not UT, the evaluation of master integrals simplifies in the UT framework. A rigorous discussion of the theory of UT functions is beyond the scope of this thesis; moreover, it is still under an active study by both mathematicians and physicists, see for example [78]. Below, a self-contained description of UT functions and their properties, useful for the present computation, is given.

The main concept behind UT functions is the concept of *weight*. A function of weight n can be written as n nested integrations of $\mathrm{d}\log R_i(\xi_i)$ forms, where $R_i(\xi_i)$ are rational functions

$$W(F) = n \quad \leftrightarrow \quad F(x) = \int_{x_0}^{x} \cdots \int_{x_0}^{\xi_{n-1}} \mathrm{d}\log R_n(\xi_n) \ldots \mathrm{d}\log R_1(\xi_1). \tag{3.32}$$

Constant values (such as π, $\zeta(k)$, $\log 2 \ldots$) are said to have weight n if they result from evaluating a weight n function at a rational point. Such assignment of weight has been shown to be unambiguous for a large number of cases. It is also important to note that a product of two quantities with weights n_1 and n_2 has weight n_1+n_2. This implies that

- $\log^n x$ has weight n;
- rational numbers have weight 0;
- π and $\log q$, $q \in \mathbb{Q}$, have weight 1;
- $\zeta(n)$, $n \in \mathbb{N}$, have weight n.

A function that admits a series expansion in the dimensional regularization parameter ε is said to be a *uniformly transcendental function* if and only if the coefficient of ε^k has weight k, for all $k \in \mathbb{Z}_{\geq 0}$.[2]

UT functions and canonical Fuchsian differential equations

UT functions are naturally related to differential equations written in the form presented in Eq. (3.30) which generalizes to [79]

$$\mathrm{d}\mathbf{F}(\mathbf{x},\varepsilon) = \varepsilon \sum_{k\in\mathcal{K}} \hat{A}_k \,\mathrm{d}\log R_k(\mathbf{x})\mathbf{F}(\mathbf{x},\varepsilon). \tag{3.33}$$

In Eq. (3.33) $\mathbf{F}(\mathbf{x},\varepsilon)$ is a vector of functions that depends on the set of variables $\mathbf{x}$, $\mathcal{K}$ is a finite set of points in the $\mathbf{x}$-space, $\hat{A}_k$ is a matrix with rational entries, and $R_k(\mathbf{x})$ are rational functions of $\mathbf{x}$. differential equations written in the form of in Eq. (3.33) are called *canonical Fuchsian differential equations.*

A system of differential equations

$$\frac{\partial}{\partial \mathbf{x}_i}\mathbf{F}(\mathbf{x},\varepsilon) = A_i(\mathbf{x},\varepsilon)\mathbf{F}(\mathbf{x},\varepsilon), \tag{3.34}$$

is said to be *canonical* if the coefficient matrices are homogeneous in ε

$$A_i(\mathbf{x},\varepsilon) = \varepsilon B_i(\mathbf{x}), \qquad \forall i \in \{1,\dots,\dim \mathbf{x}\}, \tag{3.35}$$

and it is said to be *Fuchsian* if it contains only simple poles (or, equivalently, d log-forms with rational functions as arguments) in all variables $\mathbf{x}$

$$A_i(\mathbf{x},\varepsilon) = \sum_{k\in\mathcal{K}} P_{i,k}(\varepsilon)\frac{\partial \log R_k(\mathbf{x})}{\partial \mathbf{x}_i}, \qquad \forall i \in \{1,\dots,\dim \mathbf{x}\}, \tag{3.36}$$

where $P_{i,k}(\varepsilon)$ is a rational function only containing ε.

[2]Equivalently, if one assigns weight -1 to the parameter ε, a uniformly transcendent function is a function of weight zero with no poles in ε.

Canonical Fuchsian systems of differential equations can be solved using a path-ordered exponential integral or a Dyson series [80]. The canonical form of the differential equations implies that the number of iterated integrations is equal to the order in the ε expansion of the solution of the differential equation term by term. The Fuchsian nature of the system allows us to straightforwardly assign a weight to each group of functions resulting from iterated integrations over rational $\mathrm{d}\log$ forms. The solution takes the form

$$\begin{aligned}\mathbf{F}(\mathbf{x},\varepsilon) = \quad & \mathbf{F}_0^{(0)} + \\ & + \varepsilon \left[\int_0^{\mathbf{x}} \mathcal{B}(\xi_1)\, \mathrm{d}\xi_1 \mathbf{F}_0^{(0)} + \mathbf{F}_0^{(1)} \right] + \\ & + \varepsilon^2 \left[\int_0^{\mathbf{x}} \mathcal{B}(\xi_1) \int_0^{\xi_1} \mathcal{B}(\xi_2)\, \mathrm{d}\xi_2\, \mathrm{d}\xi_1 \mathbf{F}_0^{(0)} + \int_0^{\mathbf{x}} \mathcal{B}(\xi_1)\, \mathrm{d}\xi_1 \mathbf{F}_0^{(1)} + \mathbf{F}_0^{(2)} \right] + \\ & + O\left(\varepsilon^3\right),\end{aligned} \tag{3.37}$$

where $\mathbf{F}_0 = \sum_n \varepsilon^n \mathbf{F}_0^{(n)}$ is the vector of integration constants,

$$\mathcal{B}(\mathbf{x}) = \sum_{k \in \mathcal{K}} \hat{A}_k\, \mathrm{d}\log R_k(\mathbf{x}), \tag{3.38}$$

and the integral $\int_0^{\mathbf{x}}$ is computed along the path $\gamma(0, \mathbf{x})$ (in case of a single scale problem, $\mathbf{x} = x$ and a typical choice of $\gamma(0, x)$ is the real axis interval $[0, x]$).

Thanks to the Fuchsian form of the matrix $\mathcal{B}$, the nested integrals in Eq. (3.37) can be expressed in terms of multiple polylogarithms [81, 82], or Goncharov's polylogarithms (GPLs) [83–86].

GPLs are defined recursively through the equation

$$G(\mathbf{m}_w; x) := \begin{cases} \dfrac{1}{w!} \log^w x, & \text{if } \mathbf{m}_w = (0, \dots, 0), \\ \displaystyle\int_0^x \frac{1}{\xi - m_w} G(\mathbf{m}_{w-1}; \xi)\, \mathrm{d}\xi, & \text{if } \mathbf{m}_w \neq (0, \dots, 0), \end{cases} \tag{3.39}$$

where $\mathbf{m}_w = (m_w, \mathbf{m}_{w-1})$, and $G(; x) := 1$. The weight of a GPL is then the number of entries of $\mathbf{m}_w$. Each element of $\mathbf{m}_w$ is called a *letter*, while the set of all different letters is called *alphabet*.

GPLs possess useful properties that follow from their definition in terms of iterated integrals [87–89]. They include

- *scaling relation*

$$G(m_w, \dots, m_1; x) = G(a m_w, \dots, a m_1; a x) \qquad \text{if } m_1 \neq 0; \tag{3.40}$$

- *shuffle algebra*

$$G(\mathbf{a};x)G(\mathbf{b};x) = \sum_{\mathbf{c}\in\mathbf{a}\sqcup\!\sqcup\,\mathbf{b}} G(\mathbf{c};x), \tag{3.41}$$

  where $\mathbf{a} \sqcup\!\sqcup\, \mathbf{b}$ denotes the *shuffle product* of the lists $\mathbf{a}$ and $\mathbf{b}$. The shuffle product is defined as the set of all possible lists containing all the elements of $\mathbf{a}$ and $\mathbf{b}$, for which the relative order of the elements in $\mathbf{a}$ and in $\mathbf{b}$ is the same as in the original lists;

- $G(m_w, \ldots; x)$ diverges for $x \to m_w$. In particular, $G(0_w, \ldots, 0_1; x \to 0)$ is proportional to $\lim_{x\to 0} \log^w x \to \infty$. Exceptions are $G(1, 0, \ldots, 0; x \to 1)$, which is finite, and $G(0, m_{w-1}, \ldots, m_1; x \to 0)$, which is zero if at least one of the m_i is different from zero;

- A value of a GPL of weight n at a rational point is given by a linear combination of constants of weight n with rational coefficients.

UT functions and boundary conditions

To fully solve the differential equations, the integration constants $\mathbf{F}_0^{(n)}$ must be fixed. A unique solution for the Cauchy problem is usually achieved by using regularity conditions and by matching the solutions to *boundary conditions* at a specific point $\mathbf{x}_0$. In this thesis we rely only on the matching to boundary conditions to resolve the integration constants, thanks to the fact that for this particular calculation all the boundary conditions are constructed using a single and relatively simple technique (see Section 3.4) and the matching is performed in a systematic way.

For UT functions, the matching procedure greatly simplifies. By choosing a rational point $\mathbf{x}_0$ as a matching point, the solution of the differential equations $\mathbf{F}(\mathbf{x}, \varepsilon)$ is given by a series in ε where the k-th order coefficient is given by a linear combination of constants of weight k with rational coefficients. In parallel, also the boundary functions $\mathbf{L}(\varepsilon)$ (which correspond to $\mathbf{F}$ evaluated in $\mathbf{x} \to \mathbf{x}_0$ and are constructed independently from the differential equations) must be UT ε-series with constant coefficients. By performing the matching

$$\lim_{\mathbf{x}\to\mathbf{x}_0} [\mathbf{F}(\mathbf{x}, \varepsilon) - \mathbf{L}(\varepsilon)] = 0 \tag{3.42}$$

the integration constants $\mathbf{F}_0(\varepsilon)$ of Eq. (3.37) can be determined order by order in ε.

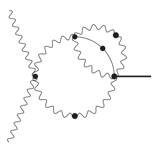
UT functions represent a useful tool for evaluating master integrals by integrating differential equations. First of all, they generate simple differential equations that are solved as Dyson series that contain only Goncharov polylogarithms, which are well-understood functions both analytically and numerically. The solution contains coefficients of weight k at order ε^k, so only a particular set of GPLs and constants can appear at each given order. The determination of the integration constants is also simplified by the weighted structure of both the boundary functions and the solutions to the differential equations.

3.4. Large-mass expansion

To determine the boundary conditions for the integrals that appear in the calculation of the $gg \to H$ amplitude we consider the case when the mass M^2 becomes infinitely large at fixed s. In this limit the master integrals and the UT functions can be easily computed using the so-called *large-mass expansion* procedure [90].

To construct the $1/M^2$ expansion of a Feynman integral, we proceed using the method of regions.[3] We note that in the case of mixed QCD-electroweak corrections to Higgs boson gluon fusion a typical integral depends on two different scales: the energy of the external legs $p_i \sim \sqrt{s}$, and the internal mass M. To provide a non-vanishing contribution in dimensional regularization, the loop momentum k that flows through any subset of internal lines has to scale either as $k \sim \sqrt{s}$ or as $k \sim M$. All possible combinations of scalings of loop momenta are allowed and have to be considered, provided that momentum conservation constraints are satisfied at each vertex. In particular, a *large-momentum conservation law* holds: large momentum $k \sim M$ cannot be created, destroyed or provided by external legs. For this reason it must flow through at least one closed loop composed of the internal lines of a given integral. Once a valid momentum scaling is identified for a particular Feynman integral, the *integrand* is Taylor-expanded in all small momenta (both the external and the loop ones) and then integrated. This procedure is performed for all possible momenta assignments and the results are summed up to obtain the large-mass expansion of the original Feynman integral.[4]

As an example, consider the master integral I_7 that appears in the NLO calculation (see Appendix B). It reads



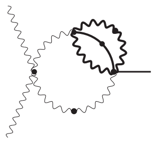
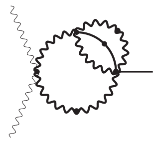
$$= \int \frac{\left[\mathrm{i}\pi^{2-\varepsilon}\Gamma(1+\varepsilon)\right]^{-3} \, \mathrm{d}^{4-2\varepsilon}k_1 \, \mathrm{d}^{4-2\varepsilon}k_2 \, \mathrm{d}^{4-2\varepsilon}k_3}{\left[(k_1)^2 - M^2\right]^2 \left[(k_1+k_2)^2\right]^2 \left[(k_2+k_3)^2\right] \left[(k_3)^2\right] \left[(k_3-p)^2\right]^2}, \tag{3.43}$$

where $p^2 = s$ is the external momentum. Among all the choices of large and small momenta that fulfill the large-momentum conservation law, only two configurations give non-vanishing contributions. Indeed, it is possible that the large momenta flow through the two-loop self-energy sub-graph and the small momentum flows in the "outer" loop,

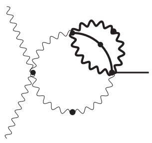
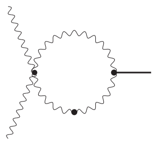
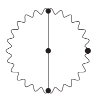
[3] An alternative approach, based on the decomposition in 1-PI large-mass subgraphs and co-subgraphs, is explained in [90, 91].

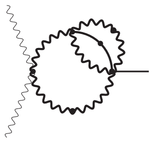
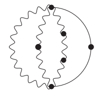
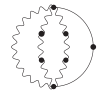
[4] We note that a single choice of loop momenta is often insufficient to account for all possible flowing routes of the large momentum. Multiple choices must be considered, and every new pattern of large-momentum flow must be included to get a consistent large-mass expansion.

or that all the three loop momenta are large (bold lines indicate large momenta)

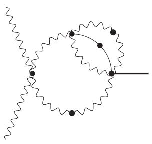
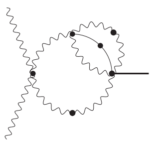
 ,  . (3.44)

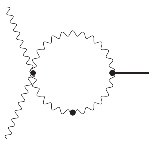
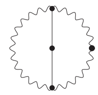
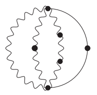
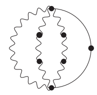
Lines with large momenta "decouple" from adjacent propagators: they become tadpoles that multiply a diagram obtained by shrinking the lines with large momenta to a point. For the two possible kinematic configurations described above, the large-mass expansion reads

 $\Rightarrow$  $\times$  , (3.45)

 $\Rightarrow$  $+\; s\,\dfrac{2(1+\epsilon)}{2-\epsilon}$  $+\; O\left(\dfrac{(-s)^2}{(M^2)^4}\right).$ (3.46)

In the first case the result is exact since, upon the expansion, the tadpole produces a term that cancels one of the propagators of the massless bubble. The large-mass expansion of the master integral I_7 is obtained by summing over the two possible momenta configurations described above. The final result reads

$\lim_{M^2\to+\infty}$  $= \lim_{M^2\gg s}$  $=$

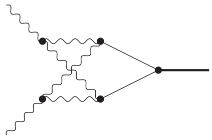
$= \Bigg[$  $\times$  $\Bigg] + \Bigg[$  $+\; s\,\dfrac{2(1+\epsilon)}{2-\epsilon}$  $\Bigg] + O\left(\dfrac{s^2}{M^8}\right).$

(3.47)

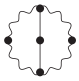
All integrals that appear on the right hand side of this equation can be easily evaluated.

Clearly, the number of different momentum configurations that need to be considered increases with the topological complexity of the integrals. Nevertheless, the large-mass expansion always leads to integrals that are straightforward to compute, even in cases with a large number of loops and many propagators. Indeed, upon expanding in s/M^2, the most complex integrals that arise are tadpoles and one-or two-loop massless triangles and bubbles. All these integrals are well-known, see e.g. [92, 93].

In many cases the large-mass expansion produces no contribution at the integral level. As an example, consider the two-loop master integral I_9 of the LO form factor (Appendix B)

 . (3.48)

Only the configuration where large momentum flows through all propagators may give non-vanishing contributions



$$+ O\left(\frac{s}{M^6}\right). \tag{3.49}$$

However, as can be immediately seen by dimensional analysis, such a contribution scales as $O\left(M^{-4}\right)$ and, therefore, vanishes for $M \to +\infty$. Note, however, that such statement may not hold true when considering UT functions obtained from master integrals, since they may be multiplied by powers of M^2. As a general rule, while Taylor expansion in small momenta can be safely performed master integral by master integral, the limit $s/M^2 \to 0$ must always be applied to the whole UT function.

Chapter 4.

Two- and three-loop $gg \to H$ results

In this Chapter the results and the techniques discussed in Chapter 3 are applied to compute the Feynman integrals that appear in the leading and next-to-leading order functions $A^{(0)}_{\text{QCD-EW}}$ and $A^{(1)}_{\text{QCD-EW}}$.

4.1. Preliminary analysis

With the help of the computer program `Reduze2` [68], integration-by-parts identities are generated and the reduction to master integrals is performed both for two-and three-loop Feynman integrals. Using dedicated routines available in `Reduze2` differential equations with respect to s and M^2 are derived for all the master integrals.

The first step in solving the system of differential equations is to redefine all but one variable to be dimensionless. We choose

$$\overline{s} := s, \qquad \omega := -\frac{M^2}{s}, \tag{4.1}$$

where the minus sign in the definition of ω is used to have a positive-definite variable in the Euclidean region $s \leq 0$ and $M^2 > 0$.

Since $\overline{s}$ is now the only dimensionful variable, the dependence of any master integral on it follows uniquely from their mass dimension. It is possible to factor out such dependence via the rescaling

$$\mathbf{I}_k(\overline{s}, \omega, \varepsilon) = (-\overline{s})^{-a_k - L\varepsilon}\, \mathcal{I}_k(\omega, \varepsilon). \tag{4.2}$$

In Eq. (4.2), L is the number of loops and a_k is an integer determined by the canonical mass dimension of the integral $\mathbf{I}_k$. The differential equations in s now reduce to

$$\partial_{\overline{s}} \mathcal{I}(\omega, \varepsilon) = 0. \tag{4.3}$$

The non-trivial information is contained in the dimensionless functions $\mathcal{I}_k(\omega, \varepsilon)$. The goal now is to find a transformation of $\mathcal{I}(\omega, \varepsilon)$ into a vector of UT functions $\mathbf{F}(\omega, \varepsilon)$. The new system of equations will be solved using the methods described in Chapter 3.

4.2. Reduction to UT basis

Given a set of master integrals, it is not always possible to map them into a set of UT functions. It was conjectured in [77] that in many physically relevant cases this mapping is possible. While this statement has not been rigorously proven, it is expected to be true at least for those cases in which the master integrals can be expressed in terms of Chen iterated integrals. General criteria to find candidate UT integrals and functions are given in [79, 94–96].

Even when a UT set of functions exists, its construction is a highly non-trivial procedure for which a universal algorithm is not yet known. For the calculations presented in this thesis, a UT basis exists and the general procedure to construct the sets of UT functions for two-and three-loop master integrals exploits the lower block-triangular nature of the differential equations, following the ideas described in [79]. The plan is to start constructing the UT bases from the simplest integrals and gradually move to more complex ones.

Master integrals with the smallest number of internal lines can often be evaluated by means of Feynman parameters or directly found in the literature, therefore their UT versions are easy to construct. Having determined the master integrals for lower topologies, the non-homogeneous terms in the differential equations of higher master integrals are fixed, and the corresponding UT functions are determined by requiring that the differential equations are canonical and Fuchsian. For all the cases considered here this procedure has proven to be successful.[1]

There are two main approaches to find UT expressions for master integrals. On the one hand, it is possible to work directly with the master integrals themselves, modifying the powers of the inverse propagators. This process is usually attempted first, in order to remove poles in ε and to bring the system as close as possible to an ε-linear form. On the other hand, the matrix of coefficients of the differential equation can be used to construct algebraic transformations to cast the system into a canonical $\mathrm{d}\log$ form. This is usually done as a second step, in order to remove non-Fuchsian poles from the differential equations and to bring the system into a canonical form.

Manipulating master integrals

The considerations described below allow one to choose master integrals that are close to UT ones. These considerations are not algorithmic, but they have proven to provide useful guidelines on many occasions.

First of all, candidate master integrals are found by adjusting powers of the inverse propagators. It is well understood how to do this to obtain a canonical form for simple integrals, such as bubbles and triangles where one of the guiding principles is to ensure that integrals are free from ultraviolet divergences. After that, the candidate integrals are multiplied by a polynomial in ε of the form $\varepsilon^{a_1}(c_1 + c_2\varepsilon)^{a_2}$, to bring the system as

[1] The canonical $\mathrm{d}\log$-form of the differential equations *alone* does not guarantee that the solution will be UT, since mixed weights might arise in the integration constants.

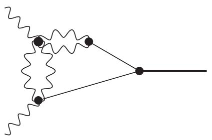
close as possible to a linear ε-form [97, 98]

$$\frac{\partial \mathcal{F}(\mathbf{x},\varepsilon)}{\partial \mathbf{x}_i} = [A_{i,0}(\mathbf{x}) + \varepsilon A_{i,1}(\mathbf{x})]\,\mathcal{F}(\mathbf{x},\varepsilon). \tag{4.4}$$

Building-block approach

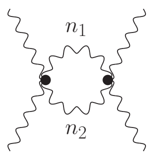
A graphical method that helps to select the right powers and prefactors for an integral is the *building-block approach*. A given master integral can be viewed as a collection of sub-integrals (such as bubbles, triangles, boxes...). A UT version of the sub-integrals is constructed. Typically, a UT version of the initial integral is found once all the building blocks are put back together. The effectiveness of the building-block approach decreases as the topology grows in complexity.

Consider for example the three-loop master integral I_{44}



$$= \frac{1}{[\mathrm{i}\pi^{2-\varepsilon}\Gamma(1+\varepsilon)]^3} \int \frac{\mathrm{d}^{4-2\varepsilon}k_1}{[(k_1)^2 - M^2]^{a_1}\,[(k_1-p_1-p_2)^2 - M^2]^{a_2}} \times$$
$$\times \left[\int \frac{\mathrm{d}^{4-2\varepsilon}k_2}{[(k_1+k_2)^2]^{a_3}\,[(k_2)^2]^{a_4}} \int \frac{\mathrm{d}^{4-2\varepsilon}k_2}{[(k_1+k_3-p_1)^2]^{a_5}\,[(k_3)^2]^{a_6}} \right]. \tag{4.5}$$

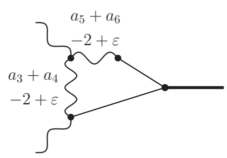
The integrals over k_2 and k_3 are one-loop massless bubbles with external momentum k_1. They read



$$= \frac{1}{[\mathrm{i}\pi^{2-\varepsilon}\Gamma(1+\varepsilon)]} \int \frac{\mathrm{d}^{4-2\varepsilon}k}{[(k)^2]^{n_1}\,[(k+k_1)^2]^{n_2}} =$$
$$= \left(-\frac{k_1^2}{\mu^2}\right)^{-\varepsilon} \left(k_1^2\right)^{2-n_1-n_2} \frac{\Gamma(n_1+n_2+\varepsilon-2)\Gamma(2-n_1-\varepsilon)\Gamma(2-n_2-\varepsilon)}{\Gamma(n_1)\Gamma(n_2)\Gamma(-n_1-n_2+4-2\varepsilon)}, \tag{4.6}$$

therefore Eq. (4.5) can be rewritten as

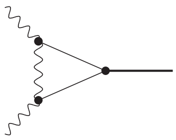
$$\int \frac{C_{a_3,a_4}(\varepsilon)C_{a_5,a_6}(\varepsilon)\,[\mathrm{i}\pi^{2-\varepsilon}\Gamma(1+\varepsilon)]\;\mathrm{d}^{4-2\varepsilon}k_1}{[(k_1)^2-M^2]^{a_1}\,[(k_1-p_1-p_2)^2-M^2]^{a_2}\,[(k_1)^2]^{a_3+a_4+\varepsilon}\,[(k_1-p_1)^2]^{a_5+a_6+\varepsilon}} =$$
$$= C_{a_3,a_4}(\varepsilon)C_{a_5,a_6}(\varepsilon) \left[\vphantom{\int} \right. \tag{4.7}$$



$$\left. \vphantom{\int} \right],$$

where C_{a_3,a_4} and C_{a_5,a_6} contain Euler gamma and rational functions of ε.

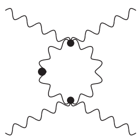
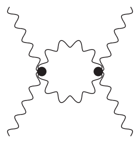
One possibility to obtain a UT structure is that both the functions C_{a_i,a_j} and the Feynman integral are UT functions. A UT form for the massive one-loop triangle is known to be

ε^2  $. \qquad (4.8)$

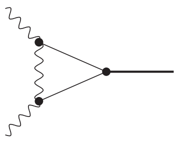
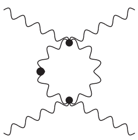
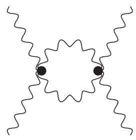
The values of the powers a_i are constrained by matching the Feynman integral of Eq. (4.7) to Eq. (4.8). ε is set to zero while matching the powers of the denominators. We find

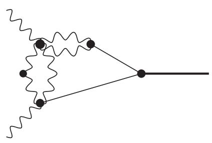
$$\begin{cases} a_3 + a_4 - 2 &= 1 \\ a_5 + a_6 - 2 &= 0 \end{cases} \quad \Rightarrow \quad \begin{cases} a_3 &= 2 \\ a_4 &= 1 \\ a_5 &= 1 \\ a_6 &= 1 \end{cases}, \tag{4.9}$$

in this way also the functions C_{a_3,a_4} and C_{a_5,a_6} are fixed. What remains to do to get a UT function now is to construct an ε-prefactor to make the two bubbles UT. By expanding Eq. (4.6) and requiring uniform weight we see that

ε  , $\qquad \varepsilon(1-2\varepsilon)$  , $\qquad (4.10)$

are UT functions. Upon combining the three pieces together, a UT function for I_{44} is found. It reads

ε^2  $\otimes\ \varepsilon$  $\otimes\ \varepsilon(1-2\varepsilon)$  $\Rightarrow$

$\Rightarrow \quad \varepsilon^4(1-2\varepsilon)$  $. \qquad (4.11)$

Unitarity cuts

Another useful procedure to select candidate integrals for the UT basis is to inspect the generalized unitarity cuts of the master integrals, as explained in [79, 99]. The idea is that by replacing a propagator by a Dirac delta function $1/(\mathcal{K}^2 - M^2) \to \delta(\mathcal{K}^2 - M^2)$, the differential equation that an integral satisfies does not change except that all integrals on the right hand side where this propagator is not present are set to zero. Thanks to this observation, by cutting a master integral in different ways it is possible to inspect different subsets of the differential equations that it satisfies. By replacing all propagators of a given integral with Dirac delta functions (*maximal cut*), differential equations which involve only master integrals of the highest topology are obtained, since all other integrals drop out [100].

Note that the analysis of the cuts can be simplified by using the so-called Baikov representation [101, 102]. Without going into details, we note that a UT integral must have a d log representation for all its cuts and *unit leading singularity*, i.e. only constant prefactors [79].

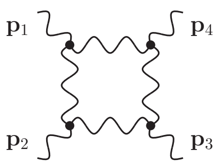
For the present calculation integrals with many propagators are required; they satisfy complicated differential equations with up to four coupled master integrals appearing in the same topology. The study of all the different cuts is therefore prohibitive and in most cases only maximal cuts can be computed. As explained above, the latter only allows us to inspect parts of differential equation that correspond to the highest topologies. What is usually done then is to look for a master integral I such that its maximal cut is of the d log-iterated form

$$I = C(\mathbf{x}) \int \cdots \int \mathrm{d}\log R_n(\mathbf{x}, \xi_1, \ldots, \xi_n) \ldots \mathrm{d}\log R_1(\mathbf{x}, \xi_1), \tag{4.12}$$

where $C(\mathbf{x})$ is a function of the kinematic invariants $\mathbf{x}$ and $R(\mathbf{x}, \xi_1, \ldots, \xi_k)$ are rational functions. If this requirements are fulfilled, the integral I is often a good candidate for the UT basis.

Once the d log-form is achieved for the maximal cut of all the coupled highest-level master integrals, the homogeneous part of the equations is ensured to be canonical. Of course, integrals of lower topologies that appear in the differential equations still do not have a a canonical form, but the study of the maximal cut has proven to be sufficient to provide a good starting point for the successful application of other methods.

Consider as an example the one-loop massless box with all incoming momenta

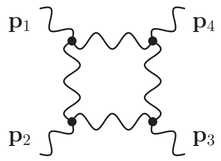


$$= \int \frac{\left[\mathrm{i}\pi^{2-\varepsilon}\Gamma(1+\varepsilon)\right]^{-1} \, \mathrm{d}^{4-2\varepsilon}k}{(k-p_1)^2(k)^2(k+p_2)^2(k+p_2+p_3)^2}. \tag{4.13}$$

First, we rewrite the integral in terms of Baikov variables using the `Mathematica` routines provided in [102]. The Baikov variables for this example are defined as

$$\begin{aligned} x_1 &= (k-p_1)^2, & x_2 &= (k)^2, \\ x_3 &= (k+p_2)^2, & x_4 &= (k+p_2+p_3)^2, \end{aligned} \tag{4.14}$$

and give



$$
= \int \frac{\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3\,\mathrm{d}x_4}{\Gamma(1+\varepsilon)} \frac{1}{x_1x_2x_3x_4} \times
$$

$$
\times \Big[4^{\varepsilon+1}[-st(s+t)]^{\varepsilon} \left[s^2\left(t^2-2t(x_2+x_4)+(x_2-x_4)^2\right)-2st(t(x_1+x_3)+ \right. \\
\left. -x_2x_3-x_4x_3+2x_2x_4-x_1(x_2-2x_3+x_4))+t^2(x_1-x_3)^2\right]^{-\varepsilon-\frac{1}{2}}\Big]. \tag{4.15}
$$

This integral is quite simple, therefore we can inspect all its cuts and explicitly construct a function showing the d log-form of Eq. (4.12) with unitlieading singularity $C(\mathbf{x}) = 1$. We include the a numerator of the form

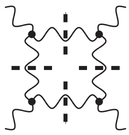
$$
\begin{aligned}
\mathcal{N} &= C_0 + C_1x_1 + C_2x_2 + C_3x_3 + C_4x_4 + C_{11}x_1^2 + C_{22}x_2^2 + C_{33}x_3^2 + C_{44}x_4^2 + \\
&\quad + C_{12}x_1x_2 + C_{13}x_1x_3 + C_{14}x_1x_4 + C_{23}x_2x_3 + C_{24}x_2x_4 + C_{34}x_3x_4,
\end{aligned} \tag{4.16}
$$

in order to consider also subtopologies in our analysis. We work in $D = 4$ ($\varepsilon = 0$) for simplicity.

The maximal cut is obtained by replacing all propagators with Dirac delta functions

$$
\frac{1}{x_1x_2x_3x_4} \Rightarrow \delta(x_1)\delta(x_2)\delta(x_3)\delta(x_4). \tag{4.17}
$$

We integrate in $D = 4$ and find

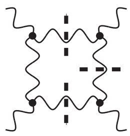


$$
\Big[\quad \mathcal{N} \Big] \propto \frac{C_0}{st}. \tag{4.18}
$$

To have unit leading singularity for the maximal cut we must impose

$$
C_0 = st \qquad \text{or} \qquad C_0 = 0. \tag{4.19}
$$

We cut now only three propagators, for example x_1, x_2, and x_3. We get



$$
\Big[\quad \mathcal{N} \Big] \propto \int \mathrm{d}x_4 \left[\frac{C_0}{stx_4} - \frac{(t^2C_{44}+tC_4+C_0)}{st(x_4-t)} - \frac{C_{44}}{s}\right]. \tag{4.20}
$$

We see immediately that C_{44} must be set to zero to eliminate the only non-logarithmic part of the expression. To have unit leading singularities in all logarithms we must impose

$$\begin{cases} C_{44} &= 0 \\ C_4 &= s \\ C_0 &= 0 \end{cases} \quad \text{or} \quad \begin{cases} C_{44} &= 0 \\ C_4 &= 0 \\ C_0 &= st \end{cases} \quad \text{or} \quad \begin{cases} C_{44} &= 0 \\ C_4 &= -s \\ C_0 &= st \end{cases} \quad \text{or} \quad \begin{cases} C_{44} &= 0 \\ C_4 &= 0\,. \\ C_0 &= 0 \end{cases} \tag{4.21}$$

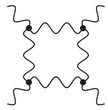
A similar result is found for other cuts involving only three propagators

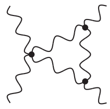
$$C_{11} = C_{22} = C_{33} = C_{44} = 0, \tag{4.22}$$
$$C_1 = \{0, \pm t\} = C_3, \tag{4.23}$$
$$C_2 = \{0, \pm s\} = C_4. \tag{4.24}$$

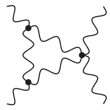
The same procedure is applied to integrals with one or two cut propagators. In this way we obtain the additional constraints

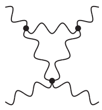
$$C_{12} = C_{13} = C_{14} = C_{23} = C_{24} = C_{34} = 0. \tag{4.25}$$

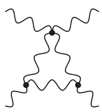
A set of UT candidates is therefore

$$C_1 = C_2 = C_3 = C_4 = 0 \quad \text{and} \quad C_0 = st \quad \Rightarrow \quad st$$  , (4.26)

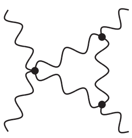
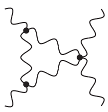
$$C_0 = C_1 = C_2 = C_3 = 0 \quad \text{and} \quad C_4 = s \quad \Rightarrow \quad s$$  , (4.27)

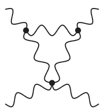
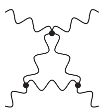
$$C_0 = C_1 = C_3 = C_4 = 0 \quad \text{and} \quad C_2 = s \quad \Rightarrow \quad s$$  , (4.28)

$$C_0 = C_1 = C_2 = C_4 = 0 \quad \text{and} \quad C_3 = t \quad \Rightarrow \quad t$$  , (4.29)

$$C_0 = C_2 = C_3 = C_4 = 0 \quad \text{and} \quad C_1 = t \quad \Rightarrow \quad t$$  . (4.30)

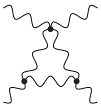
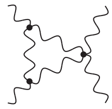
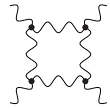
All the other candidates are linear combinations of these five. The triangle integrals are not independent, since

 $=$  , (4.31)

 $=$  . (4.32)

The basis of master integrals is constructed using both the triangles and the box, rescaled by ε^2.[2] By defining $z := t/s$ we find

$$\mathbf{I}(z) = \begin{pmatrix} \varepsilon^2(-s)^{1+\varepsilon} \; [\text{diagram 1}] \\ \varepsilon^2(-s)^{1+\varepsilon} z \; [\text{diagram 2}] \\ \varepsilon^2(-s)^{2+\varepsilon} z \; [\text{diagram 3}] \end{pmatrix}, \tag{4.34}$$

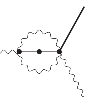




and the related differential equations read

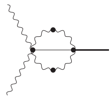
$$\frac{\mathrm{d}\mathbf{I}(z)}{\mathrm{d}z} = \varepsilon \left[\mathrm{d}\log z \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & -1 \end{pmatrix} + \mathrm{d}\log(1+z) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2 & -2 & 1 \end{pmatrix} \right] \mathbf{I}(z). \tag{4.35}$$

Consider the basis of master integrals for two-loop diagrams described in Section B.1. For master integrals like I_1, I_{10}, and I_{11} the power of the denominators and the form of the prefactors are fixed by checking the explicit expression of the integrals. For the other master integrals a combination of the building blocks method and direct inspection of the differential equations leads to a basis whose equations are linear in ε, cf. Eq. (4.4).

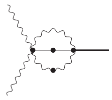
$$\mathcal{F}_1(\omega,\varepsilon) = \varepsilon^2(\varepsilon-1)(-s)^{2\varepsilon} \tag{4.36}$$

 ,

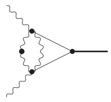
$$\mathcal{F}_2(\omega,\varepsilon) = -\varepsilon^2(-s)^{2\varepsilon+1}(\omega+1) \tag{4.37}$$

 ,

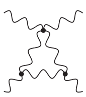
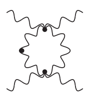
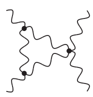
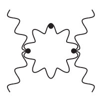
$$\mathcal{F}_3(\omega,\varepsilon) = -\varepsilon^2(-s)^{2\varepsilon+1}(\omega+1) \tag{4.38}$$

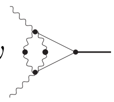
 ,

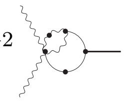
$$\mathcal{F}_4(\omega,\varepsilon) = \varepsilon^3(-s)^{2\varepsilon+1} \tag{4.39}$$

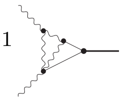
 ,

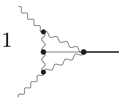
[2]This choice of master integrals is equivalent to the common one in terms of s-and t-bubbles, since

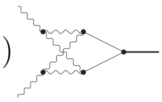
 $= -\frac{1}{\varepsilon}$  ,  $= -\frac{1}{\varepsilon}$  . (4.33)

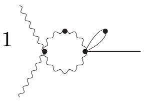
$\mathcal{F}_5(\omega,\varepsilon) \quad = \quad \varepsilon^2(-s)^{2\varepsilon+2}\omega$  , (4.40)

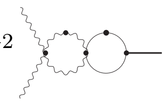
$\mathcal{F}_6(\omega,\varepsilon) \quad = \quad -\varepsilon^2(-s)^{2\varepsilon+2}$  , (4.41)

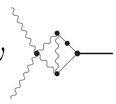
$\mathcal{F}_7(\omega,\varepsilon) \quad = \quad \varepsilon^4(-s)^{2\varepsilon+1}$  , (4.42)

$\mathcal{F}_8(\omega,\varepsilon) \quad = \quad \varepsilon^4(-s)^{2\varepsilon+1}$  , (4.43)

$\mathcal{F}_9(\omega,\varepsilon) \quad = \quad \varepsilon^4(-s)^{2\varepsilon+2}(4\omega+1)$  , (4.44)

$\mathcal{F}_{10}(\omega,\varepsilon) \quad = \quad \varepsilon^2(-s)^{2\varepsilon+1}$  , (4.45)

$\mathcal{F}_{11}(\omega,\varepsilon) \quad = \quad -\varepsilon^2(-s)^{2\varepsilon+2}$  , (4.46)

$\mathcal{F}_{12}(\omega,\varepsilon) \quad = \quad -\varepsilon^2(1-2\varepsilon)(-s)^{2\varepsilon+2}\omega$  . (4.47)

Manipulation of differential equations

A particular choice of master integrals often leads to a system of differential equations which is linear in ε or, at least, polynomial in ε. In many cases, such a system can be put into a canonical $\mathrm{d}\log$-form by an additional basis transformation that can be found by a direct inspection of the matrix of coefficients A.

Euler's variation of constants

Starting from an ε-linear system of differential equations of the form given in Eq. (4.4), the ε-independent part can often be removed. In a single-variable case, the ε-independent part of the system is removed by applying the matrix

$$\hat{S}_{A_0}(x) = P_x \mathrm{e}^{\int A_0(\xi)\,\mathrm{d}\xi} = \sum_{k=0}^{\infty} \int_{x_0}^{x} A_0(\xi_1) \cdots \int_{x_0}^{\xi_{k-1}} A_0(\xi_k)\,\mathrm{d}\xi_k \ldots \mathrm{d}\xi_1, \tag{4.48}$$

to the original set of functions $\mathcal{F}(x,\varepsilon)$, leading to

$$\mathbf{F}(x,\varepsilon) = \hat{S}_{A_0}^{-1}(x)\mathcal{F}(x,\varepsilon), \tag{4.49}$$

$$\frac{\partial \mathbf{F}(x,\varepsilon)}{\partial x} = \varepsilon \hat{S}_{A_0}^{-1}(x) A_1(x) \hat{S}_{A_0}(x) \mathbf{F}(x,\varepsilon). \tag{4.50}$$

A different way to construct the above transformation by means of algebraic operations is given in [97]. For the calculations presented in this thesis the basis change has always been obtained using Eq. (4.48) to take full advantage of the sparse nature of the matrices A_0, cf. [98].

In case of a n-scale problem the above construction is iterated over all the variables, one after the other

$$\hat{S}_{A_{0,j}}(\mathbf{x}) = P_{\mathbf{x}_j} \mathrm{e}^{\int A_{j,0}(\xi,\tilde{\mathbf{x}})\,\mathrm{d}\xi} = \sum_{k=0}^{+\infty} \int_{\mathbf{x}_0}^{\mathbf{x}} A_{j,0}(\xi_1, \tilde{\mathbf{x}}) \cdots \int_{\mathbf{x}_0}^{\tilde{\xi}_{k-1}} A_{j,0}(\xi_k, \tilde{\mathbf{x}})\,\mathrm{d}\xi_k \ldots \mathrm{d}\xi_1, \quad (4.51)$$

leading to

$$\mathbf{F}(\mathbf{x},\varepsilon) = \hat{S}^{-1}_{A_{n,0}}(\mathbf{x}) \ldots \hat{S}^{-1}_{A_{1,0}}(\mathbf{x}) \mathcal{F}(\mathbf{x},\varepsilon), \quad (4.52)$$

$$\frac{\partial \mathbf{F}(\mathbf{x},\varepsilon)}{\partial \mathbf{x}_i} = \varepsilon \hat{S}^{-1}_{A_{n,0}}(\mathbf{x}) \ldots \hat{S}^{-1}_{A_{1,0}}(\mathbf{x}) A_{i,1}(\mathbf{x}) \hat{S}_{A_{1,0}}(\mathbf{x}) \ldots \hat{S}_{A_{n,0}}(\mathbf{x}) \mathbf{F}(\mathbf{x},\varepsilon), \quad (4.53)$$

where

$$A_{j,0}(\xi, \tilde{\mathbf{x}}) = A_{j,0}(\mathbf{x}_1, \ldots, \mathbf{x}_{j-1}, \xi, \mathbf{x}_{j+1}, \ldots, \mathbf{x}_n), \quad (4.54)$$

$$\tilde{\xi}_k = (\mathbf{x}_1, \ldots, \mathbf{x}_{j-1}, \xi_k, \mathbf{x}_{j+1}, \ldots, \mathbf{x}_n). \quad (4.55)$$

It is not always possible to integrate out $A_0(\mathbf{x})$ in the presence of off-diagonal terms and even if the procedure works, the Fuchsian form of the system is not guaranteed to be preserved. Nevertheless, this procedure provides a practical way to achieve a canonical form for the majority of differential equations relevant for the present computations.

As an illustration, consider the coupled system of differential equations for integrals $\mathcal{F}_2$ and $\mathcal{F}_3$ (Eqs. (4.37) and (4.38)). The ε-independent part of the differential equations reads

$$\frac{\mathrm{d}}{\mathrm{d}\omega} \begin{pmatrix} \mathcal{F}_2 \\ \mathcal{F}_3 \end{pmatrix} = \frac{1}{\omega+1} \begin{pmatrix} 0 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathcal{F}_2 \\ \mathcal{F}_3 \end{pmatrix}. \quad (4.56)$$

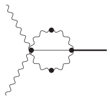
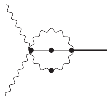
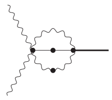
We integrate Eq. (4.56) and obtain

$$\begin{pmatrix} \mathcal{F}_2 \\ \mathcal{F}_3 \end{pmatrix} = \hat{S}_0 \begin{pmatrix} \mathcal{C}_2 \\ \mathcal{C}_3 \end{pmatrix}, \qquad \hat{S}_0 = \begin{pmatrix} -1 & -2\omega \\ 0 & \omega+1 \end{pmatrix}, \quad (4.57)$$

where $\mathcal{C}_2$ and $\mathcal{C}_3$ are the two integration constants. Since the above solution satisfies the system of differential equations for arbitrary $\mathcal{C}_2$ and $\mathcal{C}_2$, the matrix $\hat{S}_0$ satisfies the original differential equation

$$\frac{\mathrm{d}\hat{S}_0}{\mathrm{d}\omega} = \frac{1}{\omega+1} \begin{pmatrix} 0 & -2 \\ 0 & 1 \end{pmatrix} \hat{S}_0, \quad (4.58)$$

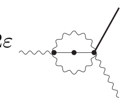
and can be considered to be a part of the (12×12) matrix $\hat{S}_{A_0}$. Finally, $\mathbf{F} = \hat{S}_0^{-1}\mathcal{F}$ reads

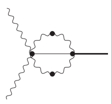
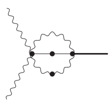
$$\mathbf{F}_2 = -\mathcal{F}_2 + \frac{2\omega}{1+\omega}\mathcal{F}_3 = -\varepsilon^2(-s)^{2\varepsilon+1}\left[(\omega+1)\right.$$  $$+\, 2\omega$$  $$\left.\right],$$
$$\mathbf{F}_3 = \frac{1}{1+\omega}\mathcal{F}_3 = \varepsilon^2(-s)^{2\varepsilon+1}$$  $$. \tag{4.59}$$

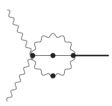
The system of differential equations for the functions $\mathbf{F}_2$ and $\mathbf{F}_3$ is guaranteed to have a canonical form. Moreover, the overall Fuchsian structure is not spoiled by the transformation

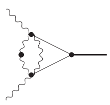
$$\frac{\mathrm{d}}{\mathrm{d}\omega}\begin{pmatrix}\mathbf{F}_2\\ \mathbf{F}_3\end{pmatrix} = \varepsilon\begin{pmatrix}-\dfrac{2}{\omega+1} & \dfrac{4}{\omega+1}\\ \dfrac{1}{\omega+1}-\dfrac{1}{\omega} & -\dfrac{2}{\omega+1}-\dfrac{1}{\omega}\end{pmatrix}\begin{pmatrix}\mathbf{F}_2\\ \mathbf{F}_3\end{pmatrix}. \tag{4.60}$$

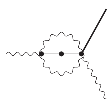
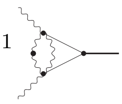
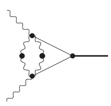
The procedure is applied block by block to $A_0 + \varepsilon A_1$ (cf. Eq. (4.4)) and the system of canonical Fuchsian differential equations is found. The canonical basis reads

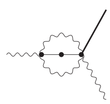
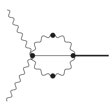
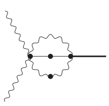
$$\mathbf{F}_1(\omega,\varepsilon) = \varepsilon^2(\varepsilon-1)(-s)^{2\varepsilon}$$  $$, \tag{4.61}$$

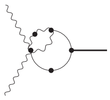
$$\mathbf{F}_2(\omega,\varepsilon) = -\varepsilon^2(-s)^{2\varepsilon+1}\left[(\omega+1)\right.$$  $$+\, 2\omega$$  $$\left.\right], \tag{4.62}$$

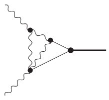
$$\mathbf{F}_3(\omega,\varepsilon) = \varepsilon^2(-s)^{2\varepsilon+1}$$  $$, \tag{4.63}$$

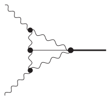
$$\mathbf{F}_4(\omega,\varepsilon) = \varepsilon^3(-s)^{2\varepsilon+1}$$  $$, \tag{4.64}$$

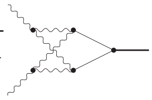
$$\mathbf{F}_5(\omega,\varepsilon) = \varepsilon^2\left[(1-\varepsilon)\frac{2}{\sqrt{1+4\omega}}(-s)^{2\varepsilon}\right.$$  $$+\,\varepsilon\frac{3(1+2\omega)}{\sqrt{1+4\omega}}(-s)^{2\varepsilon+1}$$  $$+$$
$$-(-s)^{2\varepsilon+2}\frac{\omega^2}{\sqrt{1+4\omega}}$$  $$\left.\right], \tag{4.65}$$

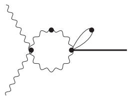
$$\mathbf{F}_6(\omega,\varepsilon) = \varepsilon^2\left[(1-\varepsilon)(-s)^{2\varepsilon}\frac{\sqrt{1+4\omega}}{2}\right.$$  $$+\,\frac{(\omega+1)\sqrt{1+4\omega}}{4}$$  $$+$$
$$+(-s)^{2\varepsilon+1} + (-s)^{2\varepsilon+1}\frac{(\omega+1)\sqrt{1+4\omega}}{2}$$  $$+$$

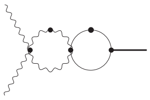
$+(-s)^{2\varepsilon+2}\omega\sqrt{1+4\omega}$  $\Bigg]$, (4.66)

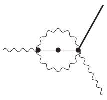
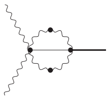
$\mathbf{F}_7(\omega,\varepsilon) \quad = \quad \varepsilon^4(-s)^{2\varepsilon+1}$  , (4.67)

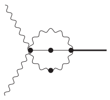
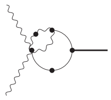
$\mathbf{F}_8(\omega,\varepsilon) \quad = \quad \varepsilon^4(-s)^{2\varepsilon+1}$  , (4.68)

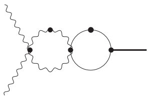
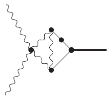
$\mathbf{F}_9(\omega,\varepsilon) \quad = \quad \varepsilon^4(-s)^{2\varepsilon+2}\sqrt{4\omega+1}$  , (4.69)

$\mathbf{F}_{10}(\omega,\varepsilon) \quad = \quad \varepsilon^2(-s)^{2\varepsilon+1}$  , (4.70)

$\mathbf{F}_{11}(\omega,\varepsilon) \quad = \quad -\varepsilon^2(-s)^{2\varepsilon+2}\sqrt{4\omega+1}$  , (4.71)

$\mathbf{F}_{12}(\omega,\varepsilon) \quad = \quad \varepsilon^2\Bigg[\frac{1-\varepsilon}{2}(-s)^{2\varepsilon}$  $+(-s)^{2\varepsilon+1}\frac{\omega+1}{4}$  $+$

$+(-s)^{2\varepsilon+1}\frac{\omega+1}{2}$  $+(-s)^{2\varepsilon+2}\omega$  $+$

$+(-s)^{2\varepsilon+2}$  $+(1-2\varepsilon)(-s)^{2\varepsilon+2}\omega$  $\Bigg]$. (4.72)

Algebraic transformations

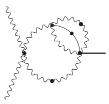
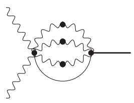
For larger sets of coupled master integrals or for master integrals with a larger number of denominators, the differential equations become too complex to be put into a canonical d log-form only by a clever choice of propagator powers and prefactors, or by integrating out non-homogeneous terms. When considering single scale integrals, like the two-and three-loop master integrals discussed in this thesis, the algebraic algorithm presented in [94] is applied. In practice, the implementation of this algorithm in the program `Fuchsia` [103] is used.[3] Similar techniques described in [95, 96] have also been used, together with their implementation in the `Mathematica` package `CANONICA` [105].

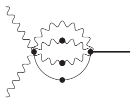
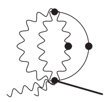
Despite the power and versatility of these algorithms, their blind application to the systems of differential equations results either in a failure of the procedure to reach

[3] Recently, the program `Libra` [104] became available. It has not been used in this thesis.

a canonical Fuchsian form or in an incorrect redefinition of the master integrals that have already been fixed. In the second case, the new system shows a canonical Fuchsian structure, but the integration constants $\mathbf{F}_0^{(n)}$ will be, in general, not of an uniform weight n. Again, by carefully selecting the master integrals and choosing their prefactors in such a way that at least no poles in ε are present, it is possible to overcome these problems.

As an example, consider the three-loop master integral I_7 from Eq. (3.43). Its UT version, obtained using `Fuchsia`, reads

$$(-s)^{3\varepsilon}\Big[-6s^2\omega(\varepsilon-1)\varepsilon^2$$  $$-\, s(\omega+1)\varepsilon^2(\varepsilon-\omega)$$  $$+$$

$$-3s(\omega-3)\varepsilon^2(\varepsilon-\omega)$$  $$-\,6(\varepsilon-1)\varepsilon^2(\varepsilon-\omega)$$  $$\Big]. \tag{4.73}$$

Notice the coefficient $(\varepsilon-\omega)$ that multiplies the last two master integrals: this prefactor cannot be obtained by means of the other techniques explained above, since it mixes ε with ω.

4.3. Canonical $\mathrm{d}\log$-form

The differential equations obtained from a UT basis of Master Integrals are homogeneous in ε

$$\mathrm{d}\mathbf{F}(\omega,\varepsilon) = \varepsilon\,\mathrm{d}\mathcal{B}(\omega)\mathbf{F}(\omega,\varepsilon), \tag{4.74}$$

For both the two-and three-loop cases the matrix $\mathrm{d}\mathcal{B}$ can be written in the $\mathrm{d}\log$-form

$$\begin{aligned} \mathrm{d}\mathcal{B}(\omega) = \quad & B_1 \quad \mathrm{d}\log\omega \\ + \quad & B_2 \quad \mathrm{d}\log(1+\omega) \\ + \quad & B_3 \quad \mathrm{d}\log(1+4\omega) \\ + \quad & B_4 \quad \mathrm{d}\log\frac{\sqrt{1+4\omega}-1}{\sqrt{1+4\omega}+1}, \end{aligned} \tag{4.75}$$

where B_i's are matrices whose entries are rational numbers.

It is convenient to remove the square roots to get Fuchsian expressions. The new variable

$$y := \frac{\sqrt{1+4\omega}-1}{\sqrt{1+4\omega}+1}, \tag{4.76}$$

is introduced. The differential equations become

$$\mathrm{d}\mathbf{F}(y,\varepsilon) = \varepsilon\,\mathrm{d}\mathcal{C}(y)\mathbf{F}(y,\varepsilon), \tag{4.77}$$

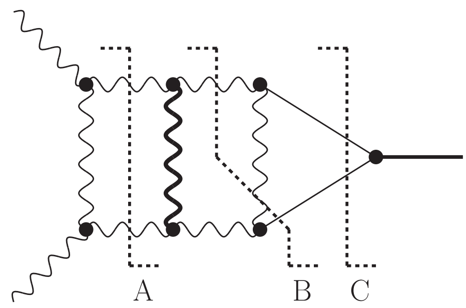


Figure 4.1.: Different cuts of the parent topologies. A bold line is used to indicate the extra propagator present in the three-loop diagrams.

$$
\begin{aligned}
\mathrm{d}\mathcal{C}(y) = \quad & C_1 \;\; \mathrm{d}\log y \\
+ \;\; & C_2 \;\; \mathrm{d}\log(1-y) \\
+ \;\; & C_3 \;\; \mathrm{d}\log(1+y) \\
+ \;\; & C_4 \;\; \mathrm{d}\log(1-y+y^2),
\end{aligned}
\tag{4.78}
$$

where, again, the matrices C_i contain rational numbers. The system can be integrated in terms of GPLs in y by defining the *integration kernels*

$$
f(0;y) = \frac{1}{y}, \quad f(1;y) = \frac{1}{y-1}, \quad f(-1;y) = \frac{1}{y+1}, \quad f(r;y) = \frac{2y-1}{y^2-y+1}. \tag{4.79}
$$

Then

$$
G(m_w, \mathbf{m}_{w-1}; y) = \int_0^y f(m_w;\tau) G(\mathbf{m}_{w-1};\tau)\,\mathrm{d}\tau, \tag{4.80}
$$

where $f(r;y)$ is a short-hand notation for

$$
f(r;y) = \frac{2y-1}{1-y+y^2} = \frac{1}{y-r_+} + \frac{1}{y-r_-} = f(r_+;y) + f(r_-;y), \qquad r_\pm = \mathrm{e}^{\pm \mathrm{i}\pi/3}. \tag{4.81}
$$

The functions $f(r_\pm;y)$ are necessary only for numerical evaluation, while the integration of the differential equations can be performed using the quadratic form indicated with the symbol r [106, 107]. Indeed, thanks to the linearity of both the differential equations and the GPLs, it is always possible to express generalized GPLs in terms of the canonical ones

$$
G(\ldots, r, \ldots; y) = G(\ldots, r_-, \ldots; y) + G(\ldots, r_+, \ldots; y)\,. \tag{4.82}
$$

One can relate the discontinuities of the logarithms in Eq. (4.79) to the discontinuities of the Feynman integrals present in $A^{(0)}_{\text{QCD-EW}}$ and $A^{(1)}_{\text{QCD-EW}}$. Indeed, all parent topologies both at two and three loops have the same cuts, as the graph in Fig. 4.1 shows. The branch points are:

Table 4.2.: Prescriptions for analytic continuation and different kinematic regions for s, ω, and y.

Definition	Prescr.	Euclidean r.	Minkowski region	
			Below thr.	Above thr.
s	$s+\mathrm{i}\,0^+$	$-\infty < s < 0$	$0 < s < 4M^2$	$4M^2 < s < +\infty$
$\omega = -\dfrac{M^2}{s}$	$\omega+\mathrm{i}\,0^+$	$0 < \omega < +\infty$	$-\infty < \omega < -\dfrac{1}{4}$	$-\dfrac{1}{4} < \omega < 0$
$y = \dfrac{\sqrt{1-4M^2/s}-1}{\sqrt{1-4M^2/s}+1}$	$y+\mathrm{i}\,0^+$	$0 < y < 1$	$\mathrm{e}^{\mathrm{i}\theta}$, $0 < \theta < \pi$	$-1 < y < 0$

- $y - 1 = 0 \quad \Rightarrow \quad s = 0$ from all on-shell massless lines (type A in Fig. 4.1);
- $y^2 - y + 1 = 0 \quad \Rightarrow \quad s = M^2$ from one on-shell massive line and all other on-shell lines being massless (type B);
- $y + 1 = 0 \quad \Rightarrow \quad s = 4M^4$ from two on-shell massive lines (type C);
- $y = 0 \quad \Rightarrow \quad s \to \infty$ since Feynman integrals are analytic functions in the complex plane.

Table 4.2 summarizes the changes of variables that have been used so far, prescriptions for their analytic continuation and Euclidean and Minkowski regions for each of them. Note that the two-and three-loop integrals that contribute to $gg \to H$ are strongly divergent in the $\varepsilon \to 0$ limit. The divergencies start at ϵ^{-4} and ε^{-6} for the two-and three-loop integrals, respectively. Moreover, in order to extract a finite remainder from the virtual contributions, the LO amplitude must be calculated up to order ε^2, according to Eq. (2.22). All UT functions are constructed to be free of ε-poles, therefore both the two-and three-loop master integrals must be evaluated up to order ε^6 (corresponding to weight 6). The time needed to produce numerical values with this precision is relatively small, never exceeding one hour for single UT function on a desktop machine, thanks to the fact that GPLs are evaluated at $y \to 1$.

4.4. Determination of the integration constants

Integration constants are determined by matching the large-mass expansion of the UT functions to the solutions of the differential equations in the limit $y \to 1$.[4] As it was

[4]This limit corresponds to $M^2 \to \infty$, or $\omega \to \infty$.

Table 4.3.: Constants appearing at each order in ε.

Weight	0	1	2	3	4	5	6
Values	1	—	π^2	$\zeta(3)$	π^4	$\pi^2\zeta(3)$ $\zeta(5)$	π^6 $\zeta^2(3)$

discussed earlier, both the boundary conditions and the solutions can be written as rational linear combinations of UT-weighted constants.

Analytic expressions for GPLs with complex letters $\mathrm{e}^{\pm \mathrm{i}\pi/3}$ evaluated at $y = 1$ are available [108]. Instead of evaluating one by one all the GPLs at $y = 1$ and then add the resulting constants, we evaluate the whole linear combinations of GPLs order by order in ε with very high precision, and match the result to a linear combination of weighted constants. This procedure is more efficient than the determination of constants for each integral separately, as it automatically accounts for possible cancellations of constants between different GPLs. The*Ansatz* for constants appearing at a given weight is constructed as follows:

$\boldsymbol{\varepsilon^0}$ rational numbers;

$\boldsymbol{\varepsilon^1}$ π, $\log 2$;

$\boldsymbol{\varepsilon^2}$ weight 2 products of the previous constants;

$\boldsymbol{\varepsilon^3}$ $\zeta(3)$, weight 3 products of the previous constants;

$\boldsymbol{\varepsilon^4}$ weight 4 products of the previous constants;

$\boldsymbol{\varepsilon^5}$ $\zeta(5)$, weight 5 products of the previous constants;

$\boldsymbol{\varepsilon^6}$ weight 6 products of the previous constants.

The UT solutions written in terms of GPLs are evaluated with high precision at $y = 1$ using an implementation of the program `GINAC` [86] in `Mathematica` [109] and their numerical values computed to at least 750 digits of precision are matched to rational linear combinations of elements of the *Ansatz* using the PSLQ algorithm. The integration constants are determined and are written as linear combinations of the UT constants listed in Table 4.3, with rational coefficients. At weight 1, all integration constants are zero.

At two loops, the large-mass expansion of any integral vanishes except for the following cases

$$
\begin{aligned}
\mathbf{L}_1(y,\varepsilon) = & -(1-y)^{4\varepsilon}\frac{\Gamma(1+2\varepsilon)\Gamma(1-\varepsilon)}{2\Gamma(1+\varepsilon)} = \\
= & -\frac{1}{2}+\varepsilon\left[-2G(1;y)\right]+\varepsilon^2\left[-4G^2(1;y)-\frac{\pi^2}{6}\right]+ \\
& +\varepsilon^3\left[\zeta(3)-\frac{2}{3}G(1;y)\left[8G^2(1;y)+\pi^2\right]\right]+ \\
& +\varepsilon^4\left[-\frac{4}{3}G(1;y)\left[4G^3(1;y)+\pi^2G(1;y)-3\zeta(3)\right]-\frac{\pi^4}{20}\right]+ \\
& +\varepsilon^5\left[\frac{1}{3}\zeta(3)\left[24G^2(1;y)+\pi^2\right]-\frac{1}{45}G(1;y)\times\right. \\
& \qquad\left.\times\left[192G^4(1;y)+80\pi^2G^2(1;y)+9\pi^4\right]+3\zeta(5)\right]+ \\
& +\varepsilon^6\left[\frac{4}{3}\zeta(3)\left[8G^2(1;y)+\pi^2\right]G(1;y)+12\zeta(5)G(1;y)-\frac{128}{45}G^6(1;y)+\right. \\
& \qquad\left.-\frac{16}{9}\pi^2G^4(1;y)-\frac{2}{5}\pi^4G^2(1;y)-\zeta^2(3)-\frac{61\pi^6}{3780}\right]+ \\
& +O\left(\varepsilon^7\right),
\end{aligned}
\tag{4.83}
$$

$$
\begin{aligned}
\mathbf{L}_2(y,\varepsilon) = & \ (1-y)^{4\varepsilon}\frac{\Gamma(1+2\varepsilon)\Gamma(1-\varepsilon)}{\Gamma(1+\varepsilon)} = \\
= & \ 1+\varepsilon\left[4G(1;y)\right]+\varepsilon^2\left[8G^2(1;y)+\frac{\pi^2}{3}\right]+ \\
& +\varepsilon^3\left[\frac{4}{3}G(1;y)\left[8G^2(1;y)+\pi^2\right]-2\zeta(3)\right]+ \\
& +\varepsilon^4\left[\frac{8}{3}G(1;y)\left[4G^3(1;y)+\pi^2G(1;y)-3\zeta(3)\right]+\frac{\pi^4}{10}\right]+ \\
& +\varepsilon^5\left[-\frac{2}{3}\zeta(3)\left[24G^2(1;y)+\pi^2\right]+\frac{2}{45}G(1;y)\left[192G^4(1;y)+\right.\right. \\
& \qquad\left.\left.+80\pi^2G^2(1;y)+9\pi^4\right]-6\zeta(5)\right]+ \\
& +\varepsilon^6\left[-\frac{8}{3}\zeta(3)\left[8G^2(1;y)+\pi^2\right]G(1;y)-24\zeta(5)G(1;y)+\right. \\
& \qquad+\frac{4}{45}\left[64G^4(1;y)+40\pi^2G^2(1;y)+9\pi^4\right]G^2(1;y)+ \\
& \qquad\left.+2\zeta^2(3)+\frac{61\pi^6}{1890}\right]+ \\
& +O\left(\varepsilon^7\right),
\end{aligned}
\tag{4.84}
$$

$$
\begin{aligned}
\mathbf{L}_{10}(y,\varepsilon) = &\ (1-y)^{4\varepsilon}\frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)} = \\
= &\ \frac{1}{2} + \varepsilon\left[2G(1;y)\right] + \varepsilon^2\left[4G^2(1;y) - \frac{\pi^2}{12}\right] + \\
&+ \varepsilon^3\frac{1}{3}\left[16G^3(1;y) - \pi^2 G(1;y) - 3\zeta(3)\right] + \\
&+ \varepsilon^4\left[-\frac{2}{3}G(1;y)\left[-8G^3(1;y) + \pi^2 G(1;y) + 6\zeta(3)\right] - \frac{\pi^4}{80}\right] + \\
&+ \varepsilon^5\Bigg[-8\zeta(3)G^2(1;y) + \frac{64}{15}G^5(1;y) - \frac{8}{9}\pi^2 G^3(1;y) - \frac{1}{20}\pi^4 G(1;y) + \\
&\qquad + \frac{\pi^2\zeta(3)}{6} - 3\zeta(5)\Bigg] + \\
&+ \varepsilon^6\Bigg[\frac{2}{3}\zeta(3)\left[\pi^2 - 16G^2(1;y)\right]G(1;y) - 12\zeta(5)G(1;y) + \\
&\qquad + \frac{128}{45}G^6(1;y) - \frac{8}{9}\pi^2 G^4(1;y) - \frac{1}{10}\pi^4 G^2(1;y) + \\
&\qquad + \zeta^2(3) - \frac{79\pi^6}{30240}\Bigg] + \\
&+ O\left(\varepsilon^7\right).
\end{aligned} \tag{4.85}
$$

The integration constants are fixed using the above expressions in the large-mass expansion. They read

$$
\begin{aligned}
\mathbf{F}_{0,1}^{(0)} &= -\frac{1}{2}, \\
\mathbf{F}_{0,1}^{(1)} &= 0, \\
\mathbf{F}_{0,1}^{(2)} &= -\frac{\pi^2}{6}, \\
\mathbf{F}_{0,1}^{(3)} &= \zeta(3), \\
\mathbf{F}_{0,1}^{(4)} &= -\frac{\pi^4}{20}, \\
\mathbf{F}_{0,1}^{(5)} &= \frac{1}{3}\left[\pi^2\zeta(3) + 9\zeta(5)\right], \\
\mathbf{F}_{0,1}^{(6)} &= -\zeta^2(3) - \frac{61\pi^6}{3780},
\end{aligned} \tag{4.86}
$$

$$
\begin{aligned}
\mathbf{F}_{0,2}^{(0)} &= 1, \\
\mathbf{F}_{0,2}^{(1)} &= 0, \\
\mathbf{F}_{0,2}^{(2)} &= -\frac{\pi^2}{3}, \\
\mathbf{F}_{0,2}^{(3)} &= -10\zeta(3), \\
\mathbf{F}_{0,2}^{(4)} &= -\frac{11\pi^4}{90}, \\
\mathbf{F}_{0,2}^{(5)} &= \frac{10\pi^2\zeta(3)}{3} - 54\zeta(5), \\
\mathbf{F}_{0,2}^{(6)} &= 50\zeta^2(3) - \frac{121\pi^6}{1890},
\end{aligned} \tag{4.87}
$$

$$
\begin{aligned}
\mathbf{F}_{0,3}^{(0)} &= 0, \\
\mathbf{F}_{0,3}^{(1)} &= 0, \\
\mathbf{F}_{0,3}^{(2)} &= \frac{\pi^2}{6}, \\
\mathbf{F}_{0,3}^{(3)} &= 8\zeta(3), \\
\mathbf{F}_{0,3}^{(4)} &= \frac{7\pi^4}{72}, \\
\mathbf{F}_{0,3}^{(5)} &= 48\zeta(5) - 3\pi^2\zeta(3), \\
\mathbf{F}_{0,3}^{(6)} &= \frac{127\pi^6}{2160} - 48\zeta^2(3),
\end{aligned}
\tag{4.88}
$$

$$
\begin{aligned}
\mathbf{F}_{0,4}^{(0)} &= 0, \\
\mathbf{F}_{0,4}^{(1)} &= 0, \\
\mathbf{F}_{0,4}^{(2)} &= 0, \\
\mathbf{F}_{0,4}^{(3)} &= -2\zeta(3), \\
\mathbf{F}_{0,4}^{(4)} &= -\frac{\pi^4}{180}, \\
\mathbf{F}_{0,4}^{(5)} &= -\frac{\pi^2\zeta(3)}{3} - 12\zeta(5), \\
\mathbf{F}_{0,4}^{(6)} &= 9\zeta^2(3) - \frac{37\pi^6}{3780},
\end{aligned}
\tag{4.89}
$$

$$
\begin{aligned}
\mathbf{F}_{0,5}^{(0)} &= 0, \\
\mathbf{F}_{0,5}^{(1)} &= 0, \\
\mathbf{F}_{0,5}^{(2)} &= -\frac{\pi^2}{3}, \\
\mathbf{F}_{0,5}^{(3)} &= -4\zeta(3), \\
\mathbf{F}_{0,5}^{(4)} &= -\frac{41\pi^4}{180}, \\
\mathbf{F}_{0,5}^{(5)} &= \pi^2\zeta(3) - 30\zeta(5), \\
\mathbf{F}_{0,5}^{(6)} &= 21\zeta^2(3) - \frac{97\pi^6}{756},
\end{aligned}
\tag{4.90}
$$

$$
\begin{aligned}
\mathbf{F}_{0,6}^{(0)} &= 0, \\
\mathbf{F}_{0,6}^{(1)} &= 0, \\
\mathbf{F}_{0,6}^{(2)} &= \frac{\pi^2}{4}, \\
\mathbf{F}_{0,6}^{(3)} &= 6\zeta(3), \\
\mathbf{F}_{0,6}^{(4)} &= \frac{5\pi^4}{48}, \\
\mathbf{F}_{0,6}^{(5)} &= 36\zeta(5) - \frac{5\pi^2\zeta(3)}{2}, \\
\mathbf{F}_{0,6}^{(6)} &= \frac{77\pi^6}{1440} - 36\zeta^2(3),
\end{aligned}
\tag{4.91}
$$

$$
\begin{aligned}
\mathbf{F}_{0,7}^{(0)} &= 0, \\
\mathbf{F}_{0,7}^{(1)} &= 0, \\
\mathbf{F}_{0,7}^{(2)} &= 0, \\
\mathbf{F}_{0,7}^{(3)} &= \zeta(3), \\
\mathbf{F}_{0,7}^{(4)} &= \frac{5\pi^4}{72}, \\
\mathbf{F}_{0,7}^{(5)} &= \frac{7\pi^2\zeta(3)}{6} + 6\zeta(5), \\
\mathbf{F}_{0,7}^{(6)} &= \frac{3\zeta^2(3)}{2} + \frac{13\pi^6}{270},
\end{aligned}
\tag{4.92}
$$

$$
\begin{aligned}
\mathbf{F}_{0,8}^{(0)} &= 0, \\
\mathbf{F}_{0,8}^{(1)} &= 0, \\
\mathbf{F}_{0,8}^{(2)} &= -\frac{\pi^2}{6}, \\
\mathbf{F}_{0,8}^{(3)} &= 4\zeta(3), \\
\mathbf{F}_{0,8}^{(4)} &= -\frac{\pi^4}{9}, \\
\mathbf{F}_{0,8}^{(5)} &= \frac{\pi^2\zeta(3)}{3} + 20\zeta(5), \\
\mathbf{F}_{0,8}^{(6)} &= -16\zeta^2(3) - \frac{173\pi^6}{3780},
\end{aligned}
\tag{4.93}
$$

$$
\begin{aligned}
\mathbf{F}_{0,9}^{(0)} &= 0, \\
\mathbf{F}_{0,9}^{(1)} &= 0, \\
\mathbf{F}_{0,9}^{(2)} &= -\frac{\pi^2}{3}, \\
\mathbf{F}_{0,9}^{(3)} &= -24\zeta(3), \\
\mathbf{F}_{0,9}^{(4)} &= \frac{13\pi^4}{45}, \\
\mathbf{F}_{0,9}^{(5)} &= \frac{46\pi^2\zeta(3)}{3} - 100\zeta(5), \\
\mathbf{F}_{0,9}^{(6)} &= 264\zeta^2(3) + \frac{397\pi^6}{945},
\end{aligned}
\tag{4.94}
$$

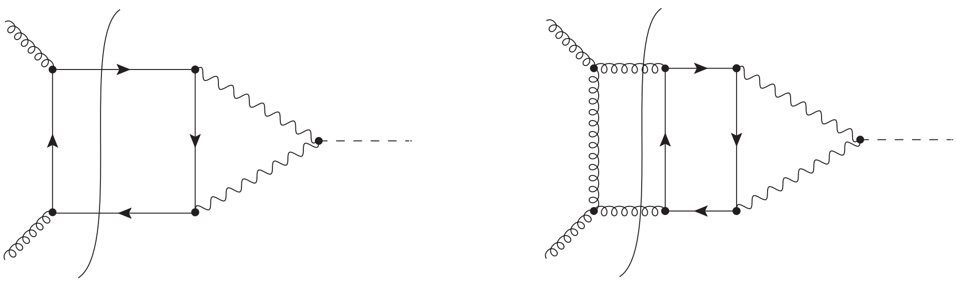
$$
\begin{aligned}
\mathbf{F}_{0,10}^{(0)} &= 1, \\
\mathbf{F}_{0,10}^{(1)} &= 0, \\
\mathbf{F}_{0,10}^{(2)} &= -\frac{\pi^2}{6}, \\
\mathbf{F}_{0,10}^{(3)} &= -2\zeta(3), \\
\mathbf{F}_{0,10}^{(4)} &= -\frac{\pi^4}{40}, \\
\mathbf{F}_{0,10}^{(5)} &= \frac{\pi^2\zeta(3)}{3} - 6\zeta(5), \\
\mathbf{F}_{0,10}^{(6)} &= 2\zeta^2(3) - \frac{79\pi^6}{15120},
\end{aligned}
\tag{4.95}
$$

$$
\begin{aligned}
\mathbf{F}_{0,11}^{(0)} &= 0, \\
\mathbf{F}_{0,11}^{(1)} &= 0, \\
\mathbf{F}_{0,11}^{(2)} &= \frac{\pi^2}{6}, \\
\mathbf{F}_{0,11}^{(3)} &= 2\zeta(3), \\
\mathbf{F}_{0,11}^{(4)} &= -\frac{\pi^4}{360}, \\
\mathbf{F}_{0,11}^{(5)} &= 6\zeta(5) - \pi^2\zeta(3), \\
\mathbf{F}_{0,11}^{(6)} &= -6\zeta^2(3) - \frac{47\pi^6}{15120},
\end{aligned}
\tag{4.96}
$$

$$
\begin{aligned}
\mathbf{F}_{0,12}^{(0)} &= 0, \\
\mathbf{F}_{0,12}^{(1)} &= 0, \\
\mathbf{F}_{0,12}^{(2)} &= \frac{\pi^2}{12}, \\
\mathbf{F}_{0,12}^{(3)} &= 4\zeta(3), \\
\mathbf{F}_{0,12}^{(4)} &= \frac{77\pi^4}{720}, \\
\mathbf{F}_{0,12}^{(5)} &= 30\zeta(5) - \frac{3\pi^2\zeta(3)}{2}, \\
\mathbf{F}_{0,12}^{(6)} &= \frac{1711\pi^6}{30240} - 30\zeta^2(3).
\end{aligned}
\tag{4.97}
$$

The basis of three-loop master integrals has been determined following the same steps as discussed here for the two-loop case. However, by adding one loop, the size and complexity of the expressions grow considerably, as well as the number of strategies employed to obtain and integrate the UT basis. The intermediate expressions are therefore not suitable for providing useful and self-contained examples, but the complete UT basis is available in Appendix B.

All the UT functions both at two and three loops have been checked for at least two different values of s and M^2 against numerical results obtained using the programs `SecDec` [110] and `pySecDec` [111]. In all cases agreement was found. Analytic expressions for both the two-loop and three-loop contributions are given in the ancillary files of [1, 2].



(a) Cut for $s > 0$ at LO.

(b) Non-zero contribution to the cut for $s > 0$ at NLO.

Figure 4.2.: Examples of cut diagrams contributing to the discontinuity for $0 < s < m_{W,Z}^2$.

4.5. Final result for $gg \to H$ amplitude

Despite the presence of poles up to order ε^{-4} in the individual integrals, the LO amplitude is finite, since all the poles cancel when all diagrams are added. At NLO, where ε^{-6} terms might appear, the virtual amplitude diverges as ε^{-2} so also here a cancellation of higher poles happens, as expected from Eq. (2.22).

Two-loop and three-loop results are combined as explained in Section 2.3. The analytic expressions for the mixed QCD-electroweak contributions of Eq. (2.24) from Section 2.3 are given in Appendix C.[5] Both $A^{(0)}_{\text{QCD-EW}}$ and $A^{(1,\text{fin})}_{\text{QCD-EW}}$ depend on y through either GPLs or rational expressions. The NLO part also depends on μ^2 through the logarithm $\log(s/\mu^2)$, on N_f, and on N_C. Taking $m_H = 125.09\,\text{GeV}$, $m_W = 80.385\,\text{GeV}$, $m_Z = 91.1876\,\text{GeV}$ (see [22]), $N_C = 3$, $N_f = 5$ and $\mu_R = \sqrt{s} = m_H$, the following numerical results are obtained

$$\begin{aligned}
\mathbf{A}_Z^{(0)} &= A^{(0)}_{\text{QCD-EW}}(m_Z^2/m_H^2, \mu^2/m_H^2 = 1) = -6.880846 - \mathrm{i}\, 0.5784119\,, \\
\mathbf{A}_W^{(0)} &= A^{(0)}_{\text{QCD-EW}}(m_W^2/m_H^2, \mu^2/m_H^2 = 1) = -10.71693 - \mathrm{i}\, 2.302953\,, \\
\mathbf{A}_Z^{(1,\text{fin})} &= A^{(1,\text{fin})}_{\text{QCD-EW}}(m_Z^2/m_H^2, \mu^2/m_H^2 = 1) = -2.975801 - \mathrm{i}\, 41.19509\,, \\
\mathbf{A}_W^{(1,\text{fin})} &= A^{(1,\text{fin})}_{\text{QCD-EW}}(m_W^2/m_H^2, \mu^2/m_H^2 = 1) = -11.31557 - \mathrm{i}\, 54.02989\,.
\end{aligned} \tag{4.98}$$

Considering the amplitude as a function of $s = m_H^2$ and the analysis of the cuts conducted in Section 4.3, one expects both $A^{(0)}$ and $A^{(1,\text{FIN})}$ to show a similar ratio between real and imaginary parts, since both receive imaginary contributions from discontinuities at $s = 0$ and $s = m_{W,Z}^2$ ($s = 4m_{W,Z}$ does not contribute since $m_{W,Z}^2 < m_H^2 < 4m_{W,Z}^2$).

[5] A `Mathematica`-readable form of the expressions is available in [2] or at `https://www.ttp.kit.edu/_media/progdata/2017/ttp17-047/anc.tar.gz`.

However, the difference in the relative importance of imaginary parts at NLO compared to LO is striking. The reason for this can be traced back to the fact that, contrary to expectations, the LO amplitude is actually *real* in the region $0 < s < m_{W,Z}^2$, and that the discontinuity at $s = 0$ contributes only to the imaginary part of the NLO amplitude [50].

To better understand this point, note that the imaginary part of the LO amplitude in the region $0 < s < m_{W,Z}^2$ is obtained by cutting through the fermion loop so that it corresponds to the sequence of physical processes $gg \to q\bar{q} \quad | \quad q\bar{q} \to H$, see Fig. 4.2a. Since the Higgs boson cannot couple to massless fermions, this contribution must vanish. It is important to stress that this is a feature of the *amplitude*, and it does not hold at the level of individual master integrals.

The same is not true at NLO, where more cuts contribute to the discontinuity at $s = 0$. In particular, a cut through a gluon loop provides a non-vanishing contribution to the imaginary part of the amplitude for $0 < s < m_{W,Z}^2$ from the re-scattering process $gg \to gg \quad | \quad gg \to H$, see Fig. 4.2b.

Chapter 5.

Soft-gluon approximation

5.1. Hadronic cross section

To compute the next-to-leading order cross section for Higgs boson gluon fusion both virtual and real NLO contributions have to be included to obtain a physical result. At next-to-leading order, the partonic process that we are interested in reads

$$g + g \to H + X, \tag{5.1}$$

where X describes the additional QCD radiation. The inclusive cross section up to next-to-leading order reads

$$\sigma^{\mathrm{NLO}}_{PP\to H}(p_{1,h}, p_{2,h}) = \sum_{i,j} \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2\, f_{i/P}(x_1) f_{j/P}(x_2) \left[\sigma^{\mathrm{LO}}_{ij\to H} + \sigma^{\mathrm{NLO}}_{ij\to H}\right], \tag{5.2}$$

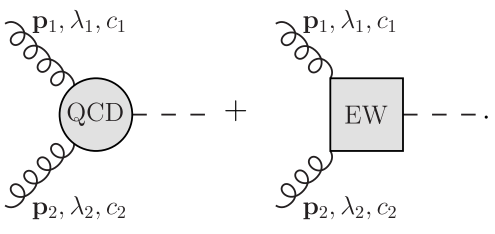
where $\sigma^{\mathrm{LO}}_{ij\to H}$ and $\sigma^{\mathrm{NLO}}_{ij\to H}$ are finite contributions to the $gg \to H$ cross section that depend on the momenta of the incoming gluons $p_1 = x_1 p_{1,h}$ and $p_2 = x_2 p_{2,h}$. $p_{1,h}$ and $p_{2,h}$ are the momenta of the two incoming protons.

As long as the Higgs boson is the only particle in the final state, the only possible pair of initial partons is a pair of gluons. The expression for the leading order differential cross section is

$$\mathrm{d}\sigma^{\mathrm{LO}}_{ij\to H} = \frac{\delta_{ig}\delta_{jg}}{2s} \frac{\mathcal{M}^{(0)}{}^{\lambda_1\lambda_2}_{c_1c_2} \mathcal{M}^{(0)\dagger}{}^{c_1c_2}_{\lambda_1\lambda_2}}{\left(N_C^2-1\right)^2 (D-2)^2} (2\pi)^D \delta^{(D)}(p_1+p_2-p_H) \frac{\mathrm{d}^{D-1}\mathbf{p}_H}{(2\pi)^3 2E_H}, \tag{5.3}$$

where $s = 2p_1 \cdot p_2 = 2x_1x_2 p_{1,h} \cdot p_{2,h}$ and the sum over polarization and color indices is understood. The leading order amplitude reads

$$\mathcal{M}^{(0)}{}^{\lambda_1\lambda_2}_{c_1c_2} =$$



(5.4)

The QCD and EW amplitudes have been defined in Section 2.1, Eqs. (2.1) and (2.4), respectively. The cross section is finite in $D = 4$ dimensions.

The next-to-leading part of the cross section contains different contributions, each one being divergent if taken alone. Renormalizing the strong coupling constant α_S in the $\overline{\mathrm{MS}}$ scheme, the cross section is written as

$$\sigma^{\mathrm{NLO}}_{ij\to H} = \sigma^{\mathrm{V}}_{ij\to H} + \sigma^{\mathrm{R}}_{ij\to H} + \frac{\alpha_S(\mu^2)}{2\pi\varepsilon}\left(\hat{P}^{(0)}_{i\to k}\otimes\sigma^{\mathrm{LO}}_{kj\to H} + \sigma^{\mathrm{LO}}_{ik\to H}\otimes\hat{P}^{(0)}_{j\to k}\right). \tag{5.5}$$

In Eq. (5.5), the functions $\hat{P}^{(0)}_{l\to k}$ are the Altarelli–Parisi splitting kernels and the symbol $\otimes$ indicates the convolution of two functions

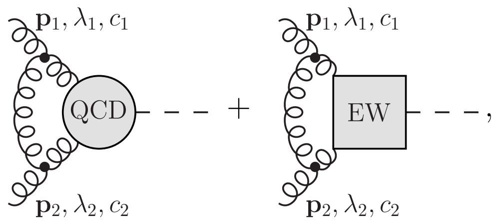
$$\left[g_1\otimes g_2\right](z) = \int_0^1 \mathrm{d}x\,\mathrm{d}y\, g_1(x)g_2(y)\delta(z-xy). \tag{5.6}$$

The virtual contributions are written as

$$\mathrm{d}\sigma^{\mathrm{V}}_{ij\to H} = \frac{\delta_{ig}\delta_{jg}}{2s}\frac{2\Re\left[\mathcal{M}^{(0)}{}^{\lambda_1\lambda_2}_{c_1c_2}\mathcal{M}^{(1)\dagger}{}^{c_1c_2}_{\lambda_1\lambda_2}\right]}{\left(N_C^2-1\right)^2\left(D-2\right)^2}(2\pi)^D\delta^{(D)}(p_1+p_2-p_H)\frac{\mathrm{d}^{D-1}\mathbf{p}_H}{(2\pi)^3 2E_H}, \tag{5.7}$$

with

$$\mathcal{M}^{(1)}{}^{\lambda_1\lambda_2}_{c_1c_2} = \tag{5.8}$$



where higher order terms are defined in Section 2.1, Eqs. (2.2) and (2.6). The virtual contributions contain both soft and collinear IR divergencies, according to Catani's formula, Eq. (2.22) and (2.23).

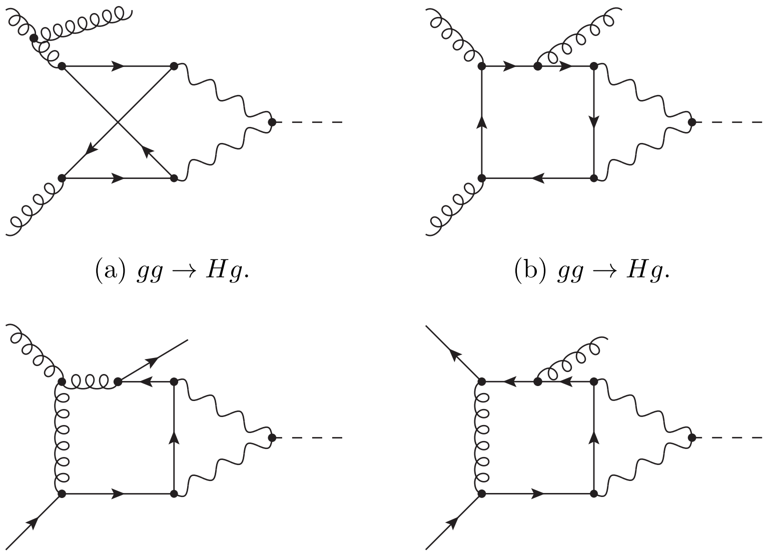
The real emission contributions may contain terms where initial states different from gluons play a role: the presence of an extra colored particle in the final state allows to select also quarks as incoming particles. In principle, all channels

$$gg\to Hg, \qquad qg\to Hq, \qquad \overline{q}g\to H\overline{q}, \qquad q\overline{q}\to Hg, \tag{5.9}$$

contribute to $\mathrm{d}\sigma^{\mathrm{R}}_{ij\to H}$ and should be considered. These contributions contain soft and collinear divergencies that are cancelled by similar poles in the virtual contributions, and an additional renormalization of parton distribution functions. This renormalization reads

$$f^{(0)}_{i/P}(x) = f_{i/P}(x,\mu^2) + \frac{\alpha_S(\mu^2)}{2\pi\varepsilon}\left[\hat{P}^{(0)}_{j\to i}\otimes f_{j/P}\right](x,\mu^2) + O\left(\alpha_S^2\right). \tag{5.10}$$

It produces the last term in Eq. (5.5) and removes the collinear divergencies still present in $\mathrm{d}\sigma^{\mathrm{V}}_{ij\to H} + \mathrm{d}\sigma^{\mathrm{R}}_{ij\to H}$. The full next-to-leading order cross section $\sigma^{\mathrm{NLO}}_{ij\to H}$ is therefore finite



(a) $gg \to Hg$. (b) $gg \to Hg$.

(c) $qg \to Hq$. (d) $q\overline{q} \to Hg$.

Figure 5.1.: Examples of Feynman diagrams for NLO real corrections to $gg \to H$.

in $D = 4$ dimensions.

Real emission corrections

The real emission corrections consist of 66 Feynman diagrams (up to crossings) that contribute to the amplitude at order $\alpha_S^{3/2}\alpha^2$ in the coupling constants (see Fig. 5.1). These diagrams contain two-loop four-point Feynman integrals, both planar and non-planar, that depend in a non-trivial way on four kinematic scales (s, t, u, and m_V^2). Their reduction to master integrals and subsequent evaluation is a challenging tasks [112]. A different approach to the calculation is presented in [113], where the NLO contributions are evaluated in the small mass limit for m_V^2.

It is possible in first approximation to neglect quark-initiated contributions (qg, $\overline{q}g$, and $q\overline{q}$) since they are suppressed by low luminosity, as explained in [114]. In the QCD case, when considering gluons in the initial state, the real emission contributions are relatively well described by the *soft-gluon approximation* [115–117], and it is reasonable to expect that such approximation will also provide a reliable estimate of the mixed QCD-electroweak contributions to Higgs boson gluon fusion. The soft gluon approximation accounts for the contributions of the real gluon emission by a universal formula that depends on the leading order cross section. In this way, the only non-universal piece at NLO that is required is the finite part of the virtual correction, computed in Chapter 4.

5.2. Evaluation of the cross section

Neglecting the quark contributions, we write the inclusive hadronic cross section given in Eq. (5.2) as

$$\sigma_{PP\to H} = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2\, f_{g/P}(x_1,\mu^2) f_{g/P}(x_2,\mu^2)\; z\; \sigma_0\; G(z,\mu^2,\alpha_S(\mu^2)). \tag{5.11}$$

In Eq. (5.11), $z = m_H^2/(s x_1 x_2)$, σ_0 is the Born cross section for $gg \to H$, m_H is the mass of the Higgs boson, s is the hadronic center-of-mass energy squared, $\alpha_s(\mu)$ is the strong coupling constant, and μ is the factorization scale which is set equal to the renormalization scale. G is called the *hard coefficient function*. It contains all the information about contributions beyond leading order. It admits a perturbative expansion in the renormalized QCD coupling $\alpha_S(\mu^2)$

$$G(z,\mu^2,\alpha_S(\mu^2)) = G^{(0)}(z) + \frac{\alpha_S(\mu^2)}{2\pi} G^{(1)}(z,\mu^2) + O\left(\left(\frac{\alpha_S(\mu^2)}{2\pi}\right)^2\right). \tag{5.12}$$

Leading order cross section

The leading order expression for the inclusive partonic cross section is obtained by integrating Eq. (5.3) over the phase space of the Higgs boson. The integration results in

$$\sigma^{\mathrm{LO}}_{gg\to H} = \frac{\pi}{m_H^4} \frac{\mathcal{M}^{(0)}{}^{\lambda_1\lambda_2}_{c_1c_2} \mathcal{M}^{(0)\dagger}{}^{c_1c_2}_{\lambda_1\lambda_2}}{(N_C^2-1)^2\,(D-2)^2} z\delta(1-z). \tag{5.13}$$

This expression is matched to Eq. (5.11) using the following definitions

$$\begin{aligned} \sigma_0 &= \frac{\pi}{m_H^4} \frac{\mathcal{M}^{(0)}{}^{\lambda_1\lambda_2}_{c_1c_2} \mathcal{M}^{(0)\dagger}{}^{c_1c_2}_{\lambda_1\lambda_2}}{(N_C^2-1)^2\,(D-2)^2}, \\ G^{(0)}(z) &= \delta(1-z). \end{aligned} \tag{5.14}$$

As mentioned above, σ_0 is the Born cross section. It reads

$$\sigma_0 = \frac{\alpha_S^2}{576\pi v^2} \mathcal{B}_0(m_H, m_W, m_Z), \tag{5.15}$$

where $\mathcal{B}_0$ contains both QCD and QCD-electroweak contributions at leading order. It is normalized in such a way that $\mathcal{B}_0 = 1$ if only pure QCD contributions are taken into account. Considering also QCD-electroweak corrections, the expression for $\mathcal{B}_0$ becomes

$$\mathcal{B}_0 = |A^{(0)}|^2, \tag{5.16}$$

where

$$A^{(0)} = \begin{cases} A^{(0)}_{\mathrm{QCD}} & = 1, \\ A^{(0)}_{\mathrm{QCD+EW}} & = 1 - \dfrac{3\alpha^2 v^2}{32 m_H^2 \sin^4\theta_W} \left(C_W \mathbf{A}^{(0)}_W + C_Z \mathbf{A}^{(0)}_Z \right), \end{cases} \tag{5.17}$$

with

$$\begin{aligned} C_W &= 4, \\ C_Z &= \frac{2}{\cos^4\theta_W} \left[\frac{5}{4} - \frac{7}{3}\sin^2\theta_W + \frac{22}{9}\sin^4\theta_W \right], \end{aligned} \tag{5.18}$$

and

$$\begin{aligned} \mathbf{A}^{(0)}_W &= -10.71693 - \mathrm{i}\, 2.302953, \\ \mathbf{A}^{(0)}_Z &= -6.880846 - \mathrm{i}\, 0.5784119, \end{aligned} \tag{5.19}$$

as written in Eq. (4.98), Section 4.5. Notice that, according to Eq. (5.16), also the square of the mixed QCD-electroweak corrections is included in the cross section. Numerically, this makes a tiny difference and we keep this contribution for the sake of convenience.

Numerical results

We choose the following numerical values for Standard Model parameters: $m_H = 125\,\mathrm{GeV}$, $m_W = 80.398\,\mathrm{GeV}$, $m_Z = 91.88\,\mathrm{GeV}$, $\alpha_{\mathrm{QED}}(m_Z) = 1/128.0$, $\sin^2\theta_W = 0.2233$, $G_F = 1.166\,39 \times 10^{-5}\,\mathrm{GeV}^{-2}$. The Higgs field vacuum expectation value is defined as $v = \left(G_F\sqrt{2}\right)^{-1/2}$. The numerical values for α_S and gluon PDFs are provided by the NNPDF30 set [118]. Specifically, $\mathrm{NNPDF30}_{\text{lo-as-0130}}$ and $\mathrm{NNPDF30}_{\text{nlo-as-0118}}$ are used for leading and next-to-leading order computations, respectively. The hadronic center-of-mass energy squared is taken to be $s = (13\,\mathrm{TeV})^2$. The renormalization and factorization scales are set equal to each other and their central values are taken to be $\mu_F = \mu_R = \mu = m_H/2$.[1] The evaluation of the cross section is performed using the computer language `FORTRAN`.

The cross section of Eq. (5.11) is evaluated for the two cases of pure QCD and QCD+EW cross section, using the appropriate definition of $A^{(0)}$ from Eq. (5.17). We find

$$\begin{aligned} \sigma^{\mathrm{LO}}_{\mathrm{QCD}} &= 20.6\,\mathrm{pb}, \\ \sigma^{\mathrm{LO}}_{\mathrm{QCD+EW}} &= 21.7\,\mathrm{pb}. \end{aligned} \tag{5.20}$$

It follows from Eq. (5.20) that the inclusion of electroweak corrections increases the gluon fusion cross section by

$$\sigma^{\mathrm{LO}}_{\mathrm{QCD+EW}} - \sigma^{\mathrm{LO}}_{\mathrm{QCD}} = +1.1\,\mathrm{pb} \qquad \Rightarrow \qquad \frac{\sigma^{\mathrm{LO}}_{\mathrm{QCD+EW}} - \sigma^{\mathrm{LO}}_{\mathrm{QCD}}}{\sigma^{\mathrm{LO}}_{\mathrm{QCD}}} = +5.3\,\%. \tag{5.21}$$

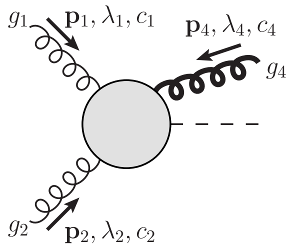
[1] It is noted, however, that the relative change in LO and NLO QCD cross sections due to mixed QCD-electroweak contributions is practically independent of the central scale.

Next-to-leading order cross section

The virtual next-to-leading order contributions to the partonic cross section have been evaluated in Chapter 4. The real corrections are implemented in the limit of a soft emitted gluon.

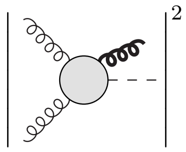
Eikonal approximation for real emissions

Consider the partonic process $gg \to Hg$

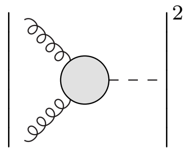


. (5.22)

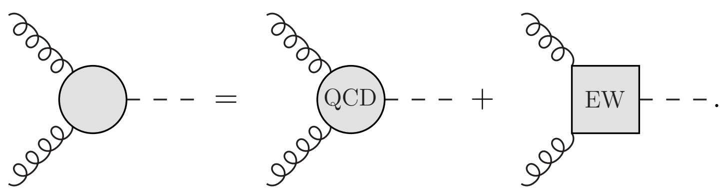
In the limit where the energy E_4 of the final state gluon g_4 approaches zero the amplitude *squared* factorizes into a universal *eikonal factor* and the LO amplitude squared



$$\xRightarrow[E_4 \to 0]{} \quad 4\pi\alpha_S N_C \frac{2p_1 \cdot p_2}{(p_1 \cdot p_4)(p_2 \cdot p_4)}$$



$$+ O(1)\,, \tag{5.23}$$

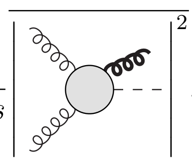


. (5.24)

The leading soft contribution is given by the emission of the soft gluon off the external legs g_1 and g_2 (cf. Fig. 5.1a). Emissions from internal lines are suppressed by at least *two powers* of the soft momentum p_4 [119–123].

In the limit $E_4 \to 0$ the phase space of the inclusive cross section can be factorized into the $gg \to H$ phase space and the phase space of a soft final state [116]. The partonic cross section for the real corrections $\sigma_{gg\to Hg}$ with a final state containing a Higgs boson and a gluon with momenta p_H and p_4, respectively, reads

$$\sigma^{\mathrm{R}} = \int [p_H] \int [p_4]\, (2\pi)^D \delta^{(D)}(p_1 + p_2 - p_H - p_4) \frac{1}{2s}$$



, (5.25)

where $[p] = \mathrm{d}^{D-1}\mathbf{p}/[(2\pi)^{D-1} 2E_p]$ and the line over the picture of the process indicates averaging over the initial states. The identity

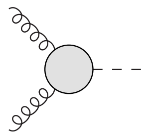
$$1 = \frac{1}{2\pi} \int [K] \int \mathrm{d}Q^2\, \delta^+(K^2 - Q^2)(2\pi)^D \delta^{(D)}(K - p_H), \tag{5.26}$$

(where δ^+ indicates that the positive solution is taken) is inserted in Eq. (5.25). In the soft limit

$$\delta^{(D)}(K - p_H)\delta^{(D)}(p_1 + p_2 - p_H - p_4) \underset{E_4 \to 0}{\Longrightarrow} \delta^{(D)}(p_1 + p_2 - p_H)\delta^{(D)}(p_1 + p_2 - K - p_4), \tag{5.27}$$

and a factorized expression for the cross section can be written as

$$\mathrm{d}\sigma^{\mathrm{R}} \underset{E_4 \to 0}{\Longrightarrow} \frac{\mathrm{d}Q^2}{2\pi} \int [p_H]\, \frac{1}{2s} \left| \right.$$



$$\left. \right|^2 (2\pi)^D \delta^{(D)}(p_1 + p_2 - p_H) \times$$

$$\times \int [K] \int (2\pi)^D\, [p_4]\, \delta^{(D)}(p_1 + p_2 - K - p_4) 4\pi\alpha_S N_C \frac{2 p_1 \cdot p_2}{(p_1 \cdot p_4)(p_2 \cdot p_4)} \times$$

$$\times\, \delta^+(K^2 - Q^2). \tag{5.28}$$

The first line of Eq. (5.28) is the leading order cross section, while the second line contains the soft part of the process.

The soft part of the cross section is integrated over the momentum p_4, giving

$$\frac{\mathrm{d}\sigma^{\mathrm{R}}}{\mathrm{d}Q^2} = -\frac{\sigma_0}{s} \frac{\alpha_S^{(0)}}{2\pi} \frac{(4\pi/s)^{\varepsilon}}{(1-z)^{1+2\varepsilon}} N_C \frac{4\Gamma(1-\varepsilon)}{\varepsilon\Gamma(1-2\varepsilon)}, \tag{5.29}$$

where $\alpha_S^{(0)}$ indicates the bare (unrenormalized) strong coupling constant. The term $1/(1-z)^{1+2\varepsilon}$ admits the following expansion in ε

$$\frac{1}{(1-z)^{1+2\varepsilon}} = -\frac{1}{2\varepsilon}\delta(1-z) + \mathcal{D}_0(z) - 2\varepsilon\mathcal{D}_1(z) + O\left(\varepsilon^2\right). \tag{5.30}$$

The functions $\mathcal{D}_0$ and $\mathcal{D}_1$ have the form

$$\mathcal{D}_0(z) = \left[\frac{1}{1-z}\right]_+, \qquad \mathcal{D}_1(z) = \left[\frac{\log(1-z)}{1-z}\right]_+, \tag{5.31}$$

where the *plus distributions* are defined as

$$\int_0^1 \mathrm{d}z\, \mathcal{D}_n(z) g(z) := \int_0^1 \mathrm{d}z \left[\frac{\log^n(1-z)}{1-z}\right] [g(z) - g(1)]. \tag{5.32}$$

Virtual contributions

The divergent part of the virtual contributions to the cross section is extracted using Catani's representation of the amplitude after the renormalization of α_S (as discussed in Section 2.3). This contribution reads

$$\frac{\mathrm{d}\sigma^{\mathrm{V}}}{\mathrm{d}Q^2} = \frac{\sigma_0}{s}\frac{\alpha_S}{2\pi}\delta(1-z)\left[-\left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon}\frac{\mathrm{e}^{\varepsilon\gamma_E}\Gamma(\varepsilon)}{(4\pi)^{\varepsilon}\Gamma(1-2\varepsilon)\Gamma(2\varepsilon)}\left(\frac{N_C}{\varepsilon^2}+\frac{\beta_0}{\varepsilon}\right)+\frac{\sigma^{(1,\mathrm{fin})}}{\sigma_0}\right], \tag{5.33}$$

where $\beta_0 = 11N_C/6 - N_f/3$ and $\sigma^{(1,\mathrm{fin})}$ is defined as

$$\frac{\sigma^{(1,\mathrm{fin})}}{\sigma_0} = \frac{\int [p_H]\, 2\Re\left[\mathcal{M}^{(0)}{}^{\lambda_1\lambda_2}_{c_1c_2}\mathcal{M}^{(1,\mathrm{fin})\dagger}{}^{c_1c_2}_{\lambda_1\lambda_2}\right]}{\int [p_H]\,\mathcal{M}^{(0)}{}^{\lambda_1\lambda_2}_{c_1c_2}\mathcal{M}^{(0)\dagger}{}^{c_1c_2}_{\lambda_1\lambda_2}}. \tag{5.34}$$

Notice that the term containing $\mathrm{e}^{\varepsilon\gamma_E}$ in Eq. (5.33) is obtained by taking twice the real part of the Catani's operator $\mathbf{I}^{(1)}$ of Eq. (2.23), Section 2.3, and rewriting $(-1)^{\varepsilon}$ in terms of Euler's gamma functions.

Soft-gluon cross section

The renormalization of the PDFs is the last ingredient needed to compute Eq. (5.5). It reads

$$\frac{\mathrm{d}\sigma^{\mathrm{PDF}}}{\mathrm{d}Q^2} = \frac{\sigma_0}{s}\frac{\alpha_S}{2\pi}\frac{2}{\varepsilon}\hat{P}^{(0)}_{g\to g}(z). \tag{5.35}$$

In the soft limit the Altarelli–Parisi splitting kernels become

$$\hat{P}^{(0)}_{g\to g}(z) = \beta_0\delta(1-z) + 2N_C\mathcal{D}_0(z). \tag{5.36}$$

Eqs. (5.29), (5.33), (5.35) are integrated in Q^2 and inserted in Eq. (5.5). The divergent parts in ε cancel in the sum. The result in the $\varepsilon\to 0$ limit reads

$$\begin{aligned}\sigma^{\mathrm{NLO}}_{gg\to H} &= \int \mathrm{d}Q^2\left[\frac{\mathrm{d}\sigma^{\mathrm{V}}}{\mathrm{d}Q^2}+\frac{\mathrm{d}\sigma^{\mathrm{R}}}{\mathrm{d}Q^2}+\frac{\mathrm{d}\sigma^{\mathrm{PDF}}}{\mathrm{d}Q^2}\right] = \\ &= \sigma_0\, z\,\frac{\alpha_S}{2\pi}\left[8N_C\left(\mathcal{D}_1(z)+\frac{\mathcal{D}_0(z)}{2}\log\frac{m_H^2}{\mu^2}\right)\right. \\ &\qquad \left. +\left(\frac{2\pi^2}{3}N_C+\frac{\sigma^{(1,\mathrm{fin})}}{\sigma_0}+2\beta_0\log\frac{m_H^2}{\mu^2}\right)\delta(1-z)\right],\end{aligned} \tag{5.37}$$

where we have used $Q^2/s = p_H^2/s = m_H^2/s = z$. As a last step, the dependence on $\log(s/\mu^2)$ in $\mathcal{M}^{(1,\mathrm{fin})}$ is made explicit

$$\mathcal{M}^{(1,\mathrm{fin})}{}^{\lambda_1\lambda_2}_{c_1c_2} = \mathcal{M}^{(1,\mathrm{fin})}_{\mu^2=s}{}^{\lambda_1\lambda_2}_{c_1c_2} - \beta_0\log\frac{s}{\mu^2}\mathcal{M}^{(0)}{}^{\lambda_1\lambda_2}_{c_1c_2}, \tag{5.38}$$

therefore

$$\frac{\sigma^{(1,\mathrm{fin})}}{\sigma_0} = \frac{\int [p_H]\, 2\Re\left[\mathcal{M}^{(0)}{}^{\lambda_1\lambda_2}_{c_1c_2}\mathcal{M}^{(1,\mathrm{fin})\dagger}{}^{c_1c_2}_{\mu^2=s\ \ \lambda_1\lambda_2}\right]}{\int [p_H]\, \mathcal{M}^{(0)}{}^{\lambda_1\lambda_2}_{c_1c_2}\mathcal{M}^{(0)\dagger}{}^{c_1c_2}_{\lambda_1\lambda_2}} - 2\beta_0 \log\frac{s}{\mu^2}. \tag{5.39}$$

The final result reads

$$\sigma^{\mathrm{NLO}}_{gg\to H} = z\ \sigma_0\ \frac{\alpha_S}{2\pi}\left[8N_C\left(\mathcal{D}_1(z) + \frac{\mathcal{D}_0(z)}{2}\log\frac{m_H^2}{\mu^2}\right) + \left(\frac{2\pi^2}{3}N_C + \frac{\sigma^{(1,\mathrm{fin})}_{\mu^2=s}}{\sigma_0}\right)\delta(1-z)\right]. \tag{5.40}$$

This expression is compared with Eq. (5.11) and the hard coefficient function $G^{(1)}$ is extracted. It reads

$$G^{(1)}(z,\mu^2) = 8N_C\left(\mathcal{D}_1(z) + \frac{\mathcal{D}_0(z)}{2}\log\frac{m_H^2}{\mu^2}\right) + \left(\frac{2\pi^2}{3}N_C + \mathcal{V}\right)\delta(1-z). \tag{5.41}$$

Where $\mathcal{V}$ stands for $\sigma^{(1,\mathrm{fin})}_{\mu^2=s}/\sigma_0$. $\mathcal{V}$ is the only *non-universal* contribution in the soft limit. It is determined using the results of Eq. (4.98) of Section 4.5 and next-to-leading order pure QCD corrections from the literature [31]. They are computed as

$$\mathcal{V} = \frac{2\Re\left[A^{(1,\mathrm{fin})}_{\mu^2=s}A^{(0)*}\right]}{|A^{(0)}|^2}, \tag{5.42}$$

where

$$A^{(1,\mathrm{fin})}_{\mu^2=s} = \begin{cases} A^{(1,\mathrm{fin})}_{\mathrm{QCD}} & = \dfrac{11}{2}, \\ A^{(1,\mathrm{fin})}_{\mathrm{QCD+EW}} & = \dfrac{11}{2} - \dfrac{3\alpha^2 v^2}{32 m_H^2 \sin^4\theta_W}\left(C_W \mathbf{A}^{(1,\mathrm{fin})}_W + C_Z \mathbf{A}^{(1,\mathrm{fin})}_Z\right), \end{cases} \tag{5.43}$$

with

$$\begin{aligned} \mathbf{A}^{(1,\mathrm{fin})}_W &= -11.315691\ -\mathrm{i}\,54.029527, \\ \mathbf{A}^{(1,\mathrm{fin})}_Z &= -2.975666\ \ -\mathrm{i}\,41.195540, \end{aligned} \tag{5.44}$$

as shown in Eq. (4.98), Section 4.5.

Numerical results

With the same specifications as for the leading order, the cross section of Eq. (5.11) at NLO evaluates to

$$\begin{aligned} \sigma^{\mathrm{NLO}}_{\mathrm{QCD}} &= 26.30\,\mathrm{pb}, \\ \sigma^{\mathrm{NLO}}_{\mathrm{QCD+EW}} &= 27.70\,\mathrm{pb}. \end{aligned} \tag{5.45}$$

The modification of the cross section is

$$\sigma^{\rm NLO}_{\rm QCD+EW} - \sigma^{\rm NLO}_{\rm QCD} = +1.40\,{\rm pb} \qquad \Rightarrow \qquad \frac{\sigma^{\rm NLO}_{\rm QCD+EW} - \sigma^{\rm NLO}_{\rm QCD}}{\sigma^{\rm NLO}_{\rm QCD}} = +5.32\,\%. \tag{5.46}$$

Improved soft-gluon approximation

It is known that in the case of Higgs production the soft-gluon approximation underestimates the exact NLO QCD corrections. An attempt to improve on this was made in [117], where sub-leading terms were taken into account. It was argued there, using analyticity considerations in Mellin space and information on universal sub-leading terms in the $z \to 1$ limit that arise from soft-gluon kinematics and, also, from the collinear splitting kernels, that a useful extension of the soft approximation is obtained by replacing the plus-distribution $\mathcal{D}_1(z)$, that appears in Eq.(5.41), with

$$\widetilde{\mathcal{D}}_1(z) = \mathcal{D}_1(z) + \delta\mathcal{D}_1(z), \tag{5.47}$$

where

$$\delta\mathcal{D}_1(z) = \frac{2-3z+2z^2}{1-z}\log\frac{1-z}{\sqrt{z}} - \frac{\log(1-z)}{1-z}. \tag{5.48}$$

Notice that $\delta\mathcal{D}_1(z)$ is an integrable function of z and not a plus-distribution.

Using this improved formula for the soft-gluon approximation the QCD and QCD+EW cross sections becomes

$$\begin{aligned} \sigma^{\rm NLO+}_{\rm QCD} &= 32.66\,{\rm pb}, \\ \sigma^{\rm NLO+}_{\rm QCD+EW} &= 34.41\,{\rm pb}. \end{aligned} \tag{5.49}$$

The modification of the cross section is

$$\sigma^{\rm NLO+}_{\rm QCD+EW} - \sigma^{\rm NLO+}_{\rm QCD} = +1.75\,{\rm pb} \qquad \Rightarrow \qquad \frac{\sigma^{\rm NLO+}_{\rm QCD+EW} - \sigma^{\rm NLO+}_{\rm QCD}}{\sigma^{\rm NLO+}_{\rm QCD}} = +5.35\,\%. \tag{5.50}$$

The ratio is in line with the previous soft-gluon approximation.

Full NLO contributions

The increase in the hadronic cross section from pure QCD to QCD-electroweak processes is stable around $+5.3\,\%$, and the full NLO QCD cross section for Higgs boson gluon fusion in all partonic channels can be automatically computed. It is therefore possible to give an approximate result for the full NLO QCD-electroweak hadronic cross section for $PP \to H$ by multiplying the full QCD NLO cross section by $+5.3\,\%$.

The full NLO QCD cross section is generated with the computer code `MCFM` [124–127]. For $\mu = m_H/2$ and including all partonic channels the QCD cross section reads

$$\sigma^{\rm NLO,full}_{\rm QCD} = 35.4\,{\rm pb}. \tag{5.51}$$

$$\sigma_{\mathrm{QCD}}^{\mathrm{NLO,full}} - \sigma_{\mathrm{QCD}}^{\mathrm{NLO+}} = +2.74\,\mathrm{pb} \qquad \Rightarrow \qquad \frac{\sigma_{\mathrm{QCD}}^{\mathrm{NLO,full}} - \sigma_{\mathrm{QCD}}^{\mathrm{NLO+}}}{\sigma_{\mathrm{QCD}}^{\mathrm{NLO+}}} = +8.4\,\%. \tag{5.52}$$

The estimate for the full NLO QCD-electroweak cross section is

$$\frac{\sigma_{\mathrm{QCD+EW}}^{\mathrm{NLO,full}} - \sigma_{\mathrm{QCD}}^{\mathrm{NLO,full}}}{\sigma_{\mathrm{QCD}}^{\mathrm{NLO,full}}} \stackrel{!}{=} +5.3\,\% \qquad \Rightarrow \qquad \sigma_{\mathrm{QCD+EW}}^{\mathrm{NLO,full}} = 37.28\,\mathrm{pb}. \tag{5.53}$$

The variation of the next-to-leading order cross section for mixed QCD-electroweak corrections given by the inclusion of quark channels is therefore expected to be

$$\sigma_{\mathrm{QCD+EW}}^{\mathrm{NLO,full}} - \sigma_{\mathrm{QCD+EW}}^{\mathrm{NLO+}} = +2.87\,\mathrm{pb}, \tag{5.54}$$

In particular the cross section can be divided into

$$\underbrace{37.28\,\mathrm{pb}}_{\sigma_{\mathrm{QCD+EW}}^{\mathrm{NLO,full}}} \;=\; \underbrace{34.41\,\mathrm{pb}}_{\sigma_{\mathrm{QCD+EW}}^{\mathrm{NLO+}}} + \underbrace{2.74\,\mathrm{pb}}_{\sigma_{\mathrm{QCD}}^{q}} + \underbrace{0.13\,\mathrm{pb}}_{\sigma_{\Delta\mathrm{EW}}^{q}}, \tag{5.55}$$

where $\sigma_{\mathrm{QCD}}^{q} = \sigma_{\mathrm{QCD}}^{\mathrm{NLO,full}} - \sigma_{\mathrm{QCD}}^{\mathrm{NLO+}}$ is the contribution of pure QCD quark-initiated processes and $\sigma_{\Delta\mathrm{EW}}^{q}$ is the estimate of the mixed QCD-EW contributions to quark channels. This last contribution is very small (+0.3 %), as expected from [114].

Analysis of the results

It follows from the different cases showed above that the QCD-electroweak contributions increase *both* the leading and next-to-leading order cross sections by $O(+5.3\,\%)$. This outcome is consistent with the estimate of the impact of mixed QCD-EW corrections obtained in [55].

The fact that the increase in the cross section remains essentially unchanged between the soft-gluon approximation of Eq. (5.46) and its refined version of Eq. (5.50) is a good indication of the robustness of the result, since $\delta\mathcal{D}_1$ represents our attempt to go beyond the soft-gluon approximation.

In general, the soft approximation for real gluon emission that has been employed here does not describe correctly the structure-dependent radiation that arises when gluons are emitted from the internal lines of the diagrams. As mentioned before, the contribution of the true structure-dependent radiation to the cross section is suppressed by *two powers* of the gluon energy relative to the soft gluon approximation [119–123]. For this reason, there is a good chance that the structure-dependent radiation plays a relatively minor role and that the soft gluon approximation employed here provides a sufficiently good description of real emissions. Further improvement on this result are then only possible with the exact computations of the real emission contributions.

Conclusions

In recent years, multi-loop computations have experienced a rapid advancement, thanks to a close interplay among phenomenology, formal scattering amplitudes, and mathematics. As a result, new techniques and automated tools have been developed and applied to the computation of higher and higher order corrections to many processes of interest, reducing their theoretical uncertainty to unprecedented small values. In this thesis, we computed the next-to-leading order mixed QCD-electroweak corrections to Higgs boson gluon fusion. These corrections, previously unknown, represent an important ingredient to refine the theoretical Standard Model predictions for the main production channel of Higgs bosons at the LHC.

We considered diagrams where gluons interact with light quarks; the quarks later couple either to $W^{\pm}$ or Z bosons and these last particles annihilate into the Higgs boson. We reduced the single form factor for Higgs boson gluon fusion at leading and virtual next-to-leading order to a linear combination of master integrals and evaluated them by solving their differential equations. We exploited the notion of uniformly transcendental functions and canonical Fuchsian equations to express the result in terms of weighted linear combinations of Goncharov polylogarithms. We fixed the integration constants in a systematic way by numerically matching the large-mass expansion of the UT functions valid in the limit $m_{W,Z} \gg m_H$ to the solution of the differential system with high precision. The analytic result for the amplitude has been obtained for an arbitrary relation between the Higgs boson and the electroweak gauge boson masses. The real next-to-leading order contributions is the last ingredient needed; it has been evaluated in the soft-gluon limit, giving a reliable result for the next-to-leading order Higgs boson gluon fusion cross section.

The mixed QCD-electroweak corrections increase the Standard Model theoretical prediction for the Higgs boson gluon fusion cross section by about five percent, both at leading and next-to-leading order, settling a debate about the magnitude of this class of contributions and removing one of the main theoretical uncertainties in the Standard Model prediction for $gg \to H$.

To further improve the result presented in this thesis, the exact analytic computation of the real emission contributions is required. Such computation, even if remarkably challenging, appears to be within reach of the current methods and techniques for multi-loop computations and is under active study.

Acknowledgements

This doctoral thesis has been produced at the *Institut für Theoretische Teilchenphysik*, in the *Karlsruher Institut für Technologie*. The author has been supported by a graduate fellowship from the *DFG* Research Training Group 1694/2 "*Elementarteilchenphysik bei höchster Energie und höchster Präzision*", from May 2016 until August 2019. The author acknowledges the collaboration and supervision of Prof. Dr. Kirill Melnikov and the collaboration of Dr. Lorenzo Tancredi, and is deeply in debt with both of them. The author wishes to thank his *Korreferent* Prof. Dr. Matthias Steinhauser.

The author and his collaborators wish to thank Dr. Robbert Rietkerk, Dr. Raoul Röntsch, and Dr. Christopher Wever for their help and support in many steps of this project; Dr. Erich Weihs for providing them with his `Mathematica` routines to numerically evaluate GPLs with arbitrary precision using `Ginac`; Prof. Dr. Roberto Bonciani for his assistance with comparing the results obtained in this doctoral thesis with [47]; Dr. Oleksandr Gituliar for his help with the program `Fuchsia`; Prof. Dr. Fabrizio Caola for useful comments; Prof. Dr. Charalampos Anastasiou for pointing out a mistake in the calculation of the next-to-leading order cross section for Higgs boson gluon fusion. The author wishes to thank Konstantin Asteriadis, Maximilian Delto, Dr. Thomas Deppisch, Florian Herren, Kirill Kudashkin, and David Wellmann for their help and support.

Appendix A.

Notations and conventions

A.1. Feynman rules

In this thesis, when referring to Feynman diagrams, the following conventions are understood:

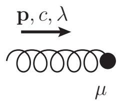
- bold momenta $\mathbf{p}$, $\mathbf{q}$... indicate on-shell particles;
- Greek superscripts and subscripts μ, ν... represent Lorentz indices, with the exception of λ, which is used to indicate polarization states of the external gluons;
- capital Latin subscripts A, B... are Dirac indices, while r, s... refer to spin states of the external quarks and anti-quarks;
- Latin superscripts and subscripts refer to color structure; in particular, a, b... span the adjoint representation (used for gluons) of the color group, while i, j... span the fundamental representation (used for quarks).

Quarks are considered massless, therefore only one spinor U is required to fully describe the states of both particle and antiparticle. As a consequence, there is no misalignment between flavour states and mass states, resulting in the CKM matrix being the identity matrix.

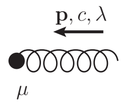
EW massive vector bosons are described by a single particle V, since W and Z boson never appear in the same diagram in mixed QCD-electroweak corrections. The Feynman rules used in this thesis and presented below follow the conventions of Ref. [128].

External legs

- Gluon external leg

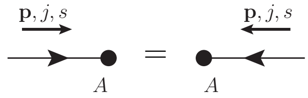


$$= \epsilon^{\lambda}_{\mu}(\mathbf{p}), \tag{A.1}$$

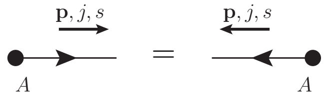


$$= \epsilon^{*\lambda}_{\mu}(\mathbf{p}). \tag{A.2}$$

- Quark external leg

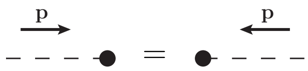


$$= U_{s,A}(\mathbf{p}), \tag{A.3}$$



$$= \overline{U}_{s,A}(\mathbf{p}). \tag{A.4}$$

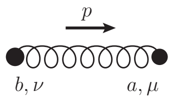
- Higgs boson external leg



$$= 1. \tag{A.5}$$

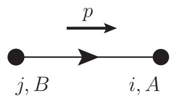
Propagators

- Gluon propagator



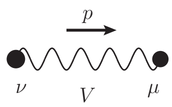
$$= -\mathrm{i}\delta_{ab}\left[\frac{g_{\mu\nu}}{p^2+\mathrm{i}0} - (1-\xi_G)\frac{p_\mu p_\nu}{(p^2)^2}\right]. \tag{A.6}$$

- Quark propagator



$$= \mathrm{i}\delta_{ij}\frac{\not{p}_{AB}}{p^2+\mathrm{i}0}. \tag{A.7}$$

- Massive vector boson propagator

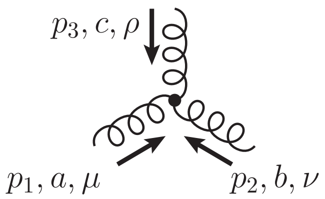


$$= \frac{-\mathrm{i}}{p^2 - m_V^2 + \mathrm{i}0}\left[g_{\mu\nu} - (1-\xi_V)\frac{p_\mu p_\nu}{p^2 - \xi_V m_V^2}\right], \tag{A.8}$$

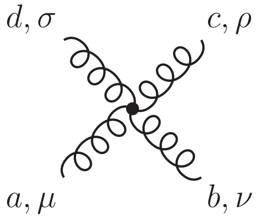
where $m_V = m_W = gv/2$ or $m_V = m_Z = gv/(2\cos\theta_W)$, $v = 246.22\,\mathrm{GeV}$ is the Higgs vacuum expectation value (VEV), $g = e/\sin\theta_W = \sqrt{4\pi\alpha}/\sin\theta_W$ is the weak charge (α is the fine-structure constant), and θ_W is the weak mixing angle.

Vertices

- three- and four-point gluon vertices



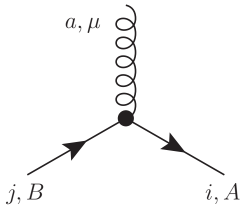
$$\begin{aligned} &= -g_S f^{abc}\left[g^{\mu\nu}(p_1-p_2)^\rho + \right. \\ &\qquad \left. + g^{\nu\rho}(p_2-p_3)^\mu + g^{\rho\mu}(p_3^\rho - p_1)^\nu\right], \end{aligned} \tag{A.9}$$



$$\begin{aligned} &= -\mathrm{i}g_S^2\left[f_{eab}f^e{}_{cd}\left(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right) + \right. \\ &\qquad + f_{eac}f^e{}_{db}\left(g_{\mu\sigma}g_{\rho\nu} - g_{\mu\nu}g_{\rho\sigma}\right) + \\ &\qquad \left. + f_{ead}f^e{}_{bc}\left(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma}\right)\right], \end{aligned} \tag{A.10}$$

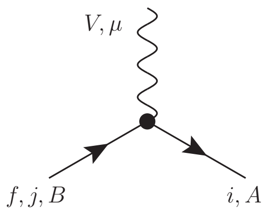
where $g_S = \sqrt{4\pi\alpha_S}$ is the strong charge (α_S is the strong coupling constant).

- Quark-gluon interaction



$$= -\mathrm{i}g_S\gamma^\mu_{AB}T^a_{ji}. \tag{A.11}$$

- Quark-massive vector boson interaction



$$= -\mathrm{i}\delta_{ij}\frac{g}{\cos\theta_W}\gamma^\mu_{AC}\left(C_v^{fV}\delta_{CB} - C_a^{fV}\gamma_{5CB}\right), \tag{A.12}$$

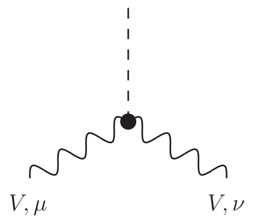
where, for $V = W$

$$C_v^{(u,d,s,c)W} = C_a^{(u,d,s,c)W} = 1, \tag{A.13}$$

while for $v = Z$

$$\begin{aligned} C_v^{(u,c)Z} &= +\frac{1}{4} - \frac{2}{3}\sin^2\theta_W, & C_a^{(u,c)Z} &= +\frac{1}{4}, \\ C_v^{(d,s,b)Z} &= -\frac{1}{4} + \frac{1}{3}\sin^2\theta_W, & C_a^{(d,s,b)Z} &= -\frac{1}{4}. \end{aligned} \tag{A.14}$$

- Higgs boson-massive vector boson interaction



$$= \mathrm{i}\frac{g^2 v}{2} C^{VVH} g_{\mu\nu} \tag{A.15}$$

where

$$C^{WWH} = 1, \qquad C^{ZZH} = \frac{1}{\cos^2\theta_W}. \tag{A.16}$$

A.2. Relevant identities for color factors

Considering a gauge group $SU(N_C)$, quarks (antiquarks) transform as elements of $\mathbf{N_C}$ ($\overline{\mathbf{N_C}}$), therefore they can be represented as vectors ψ_i (covectors $\overline{\psi}_i$), $i = 1, \ldots, N_C$. Gauge bosons transform as elements of $\mathbf{N_C^2} - \mathbf{1}$, and can be represented using $N_C \times N_C$ hermitian matrices T^a, $a = 1, \ldots, N_C^2 - 1$.[1] Conventions enlisted in [129] are followed.

For the gauge group $SU(N_C)$ the color factors of the fundamental (C_F) and adjoint (C_A) representation have the form

$$C_F = \frac{N_C^2 - 1}{2N_C}, \qquad C_A = N_C. \tag{A.17}$$

The Dynkin index is taken to be $T_F = 1/2$.

Structure constants are defined as

$$[T^a, T^b] = \mathrm{i} f^{abc} T_c, \qquad \{T^a, T^b\} = \frac{\delta^{ab}}{N_C}\mathbb{I}_{N_C} + d^{abc} T_c, \tag{A.18}$$

where $\mathbb{I}_{N_C}$ is the $N_C \times N_C$ identity matrix.

[1]When $N_C = 3$ Gell-Mann matrices for $SU(3)$ QCD can be used.

Relations among T matrices

$$T^aT^b = \frac{1}{2}\left[\frac{\delta^{ab}}{N_C}\mathbb{I}_{N_C} + (d^{abc} + \mathrm{i}f^{abc})T_c\right], \tag{A.19}$$

$$(T^a)_{ij}(T_a)_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{\delta_{ij}\delta_{kl}}{N_C}\right), \tag{A.20}$$

$$\operatorname{tr} T^a = 0, \tag{A.21}$$

$$\operatorname{tr}(T^aT^b) = \frac{\delta^{ab}}{2}, \tag{A.22}$$

$$\operatorname{tr}(T^aT^bT^c) = \frac{d^{abc} + \mathrm{i}f^{abc}}{4}, \tag{A.23}$$

$$\operatorname{tr}(T^aT^bT^aT^c) = -\frac{\delta^{bc}}{4N_C}. \tag{A.24}$$

Relations among structure constants

$$f^{abe}f^{cd}{}_e = \frac{2}{N_C}(\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc}) + (d^{ace}d^{bd}{}_e - d^{bce}d^{ad}{}_e), \tag{A.25}$$

$$f^{ab}{}_b = 0, \tag{A.26}$$

$$d^{ab}{}_b = 0, \tag{A.27}$$

$$f^{acd}f^b{}_{cd} = N_C\delta^{ab}, \tag{A.28}$$

$$f^{acd}d^b{}_{cd} = 0, \tag{A.29}$$

$$d^{acd}d^b{}_{cd} = \frac{N_C^2 - 4}{N_C}\delta^{ab}. \tag{A.30}$$

Appendix B.

Feynman diagrams and master integrals

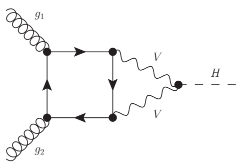
Feynman diagrams and Master Integrals are presented.

Feynman diagrams have been generated using `QGRAF` [56] and drawn using `QGRAF Diagram Drawer` [130]. Line coding follows Feynman rules presented in App. A. Wavy lines stand for a generic massive vector boson V.

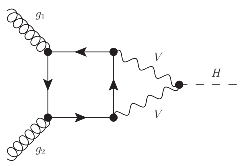
Master Integrals are drawn using `LiteRed` [131], a package for `Mathematica` [109]. Wavy lines indicate massless propagators, while internal straight lines indicate propagators carrying mass M.

B.1. LO amplitude $gg \to H$

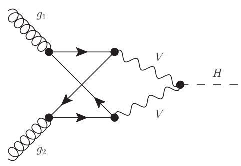
Feynman diagrams

(-1) 

D_1

(-1) 

D_2

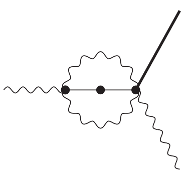
(-1) 

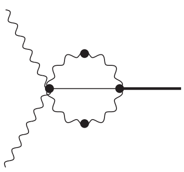
D_3

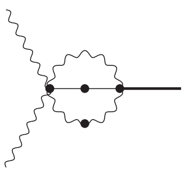
Master Integrals

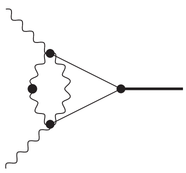
The integral families are listed in Table B.1. The parent topologies are depicted in Fig. B.1.

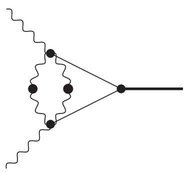
Planar PP integral family


I_1


I_2


I_3


I_4


I_5

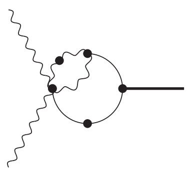

I_6

Table B.1.: Integral families for 2-loop FIs in $A^{(0)}_{\text{QCD-EW}}$ and $A^{(1,\text{fin})}_{\text{QCD-EW}}$.

Label	Families of denominators	
	PP	NP
1	$(k_1)^2$	$(k_1)^2$
2	$(k_1+p_1)^2$	$(k_1+p_1)^2$
3	$(k_2)^2$	$(k_2)^2$
4	$(k_1-p_2)^2$	$(k_2+p_2)^2$
5	$(k_1-k_2+p_1)^2 \; - \; M^2$	$(k_1-k_2+p_1)^2 \; - \; M^2$
6	$(k_2-k_1+p_2)^2 \; - \; M^2$	$(k_2-k_1+p_2)^2 \; - \; M^2$
7	$(k_1+k_2)^2$	$(k_1+k_2)^2$

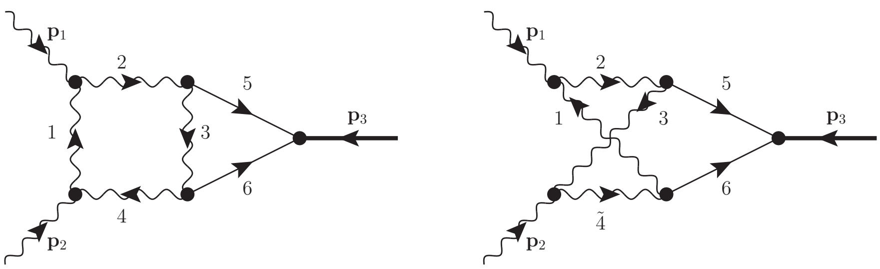
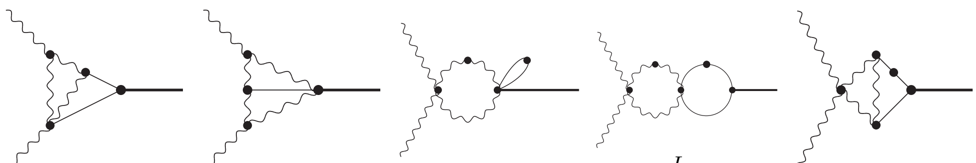
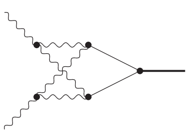


Figure B.1.: Planar (PP, left) and non-planar (NP, right) parent topologies for 2-loop FIs in $A^{(0)}_{\text{QCD-EW}}$ and $A^{(1,\text{fin})}_{\text{QCD-EW}}$. Cfr. Table B.1 for momenta assignments and propagator labels.



I_7 I_8 I_{10} I_{11} I_{12}

Non-planar NP integral family



I_9

UT basis

$$\mathbf{F}_1(y,\varepsilon) = \varepsilon^2(\varepsilon-1)(-s)^{2\varepsilon}I_1, \tag{B.1}$$

$$\mathbf{F}_2(y,\varepsilon) = -\varepsilon^2(-s)^{2\varepsilon+1}\left[\frac{y^2-y+1}{(1-y)^2}I_2+\frac{2y}{(1-y)^2}I_3\right], \tag{B.2}$$

$$\mathbf{F}_3(y,\varepsilon) = \varepsilon^2(-s)^{2\varepsilon+1}I_3, \tag{B.3}$$

$$\mathbf{F}_4(y,\varepsilon) = \varepsilon^3(-s)^{2\varepsilon+1}I_4, \tag{B.4}$$

$$\begin{aligned}\mathbf{F}_5(y,\varepsilon) &= \varepsilon^2\left[2(1-\varepsilon)(-s)^{2\varepsilon}I_1+\right.\\ &\left.-\varepsilon\frac{3(y^2+1)}{(y-1)(y+1)}(-s)^{2\varepsilon+1}I_4+\frac{y^2}{(y-1)^3(y+1)}(-s)^{2\varepsilon+2}I_5\right],\end{aligned} \tag{B.5}$$

$$\begin{aligned}\mathbf{F}_6(y,\varepsilon) &= \varepsilon^2\left[(1-\varepsilon)(-s)^{2\varepsilon}\frac{y+1}{2(1-y)}I_1+(-s)^{2\varepsilon+1}\frac{y^3+1}{4(1-y)^3}I_2+\right.\\ &\left.+(-s)^{2\varepsilon+1}\frac{y^3+1}{2(1-y)^3}I_3+(-s)^{2\varepsilon+2}\frac{y(y+1)}{(1-y)^3}I_6\right],\end{aligned} \tag{B.6}$$

$$\mathbf{F}_7(y,\varepsilon) = \varepsilon^4(-s)^{2\varepsilon+1}I_7, \tag{B.7}$$

$$\mathbf{F}_8(y,\varepsilon) = \varepsilon^4(-s)^{2\varepsilon+1}I_8, \tag{B.8}$$

$$\mathbf{F}_9(y,\varepsilon) = \varepsilon^4(-s)^{2\varepsilon+2}\frac{y+1}{1-y}I_9, \tag{B.9}$$

$$\mathbf{F}_{10}(y,\varepsilon) = \varepsilon^2(-s)^{2\varepsilon+1}I_{10}, \tag{B.10}$$

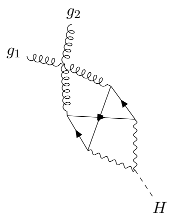
$$\mathbf{F}_{11}(y,\varepsilon) = -\varepsilon^2(-s)^{2\varepsilon+2}\frac{y+1}{1-y}I_{11}, \tag{B.11}$$

$$\begin{aligned}\mathbf{F}_{12}(y,\varepsilon) &= \varepsilon^2\left[\frac{1-\varepsilon}{2}(-s)^{2\varepsilon}I_1+(-s)^{2\varepsilon+1}\frac{y^2-y+1}{4(1-y)^2}I_2+\right.\\ &+(-s)^{2\varepsilon+1}\frac{y^2-y+1}{2(1-y)^2}I_3+(-s)^{2\varepsilon+2}\omega I_6+\\ &\left.+(-s)^{2\varepsilon+2}I_{11}+(1-2\varepsilon)(-s)^{2\varepsilon+2}\frac{y}{(1-y)^2}I_{12}\right].\end{aligned} \tag{B.12}$$

B.2. NLO virtual corrections $gg \to H$

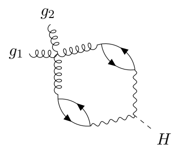
Feynman diagrams

$\left(-\frac{1}{2}\right)$



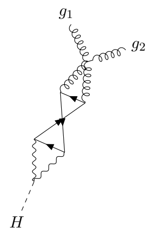
D_1

$\left(+\frac{1}{2}\right)$



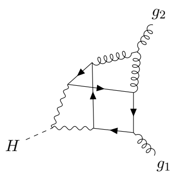
D_2

(-1)



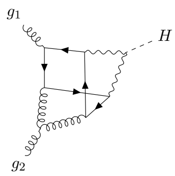
D_3

$(+1)$



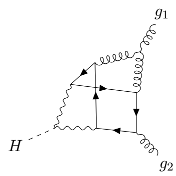
D_4

$(+1)$



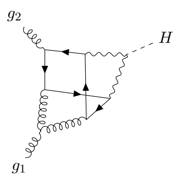
D_5

$(+1)$



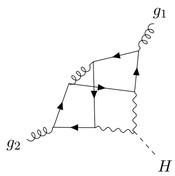
D_6

$(+1)$



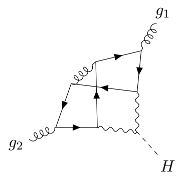
D_7

(-1)



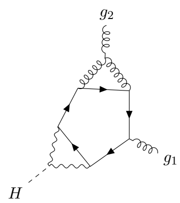
D_8

(-1)



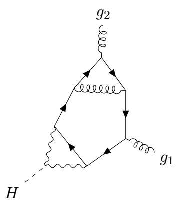
D_9

$(+1)$



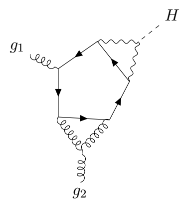
D_{10}

(-1)



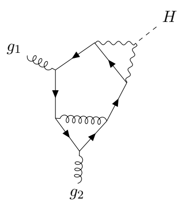
D_{11}

$(+1)$



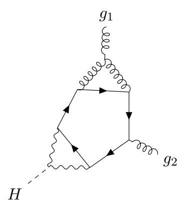
D_{12}

(-1)



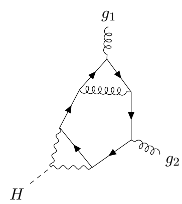
D_{13}

$(+1)$

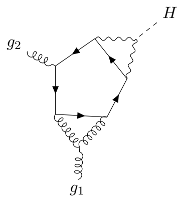


D_{14}

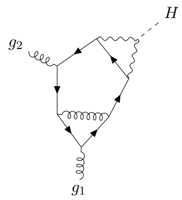
(-1)



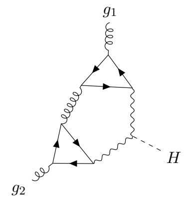
D_{15}

(+1) 

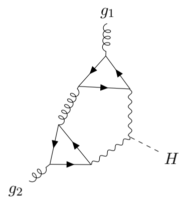
D_{16}

(−1) 

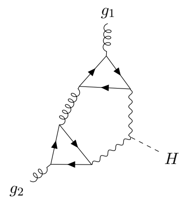
D_{17}

(+1) 

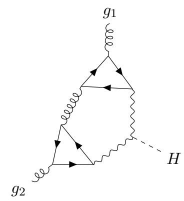
D_{18}

(+1) 

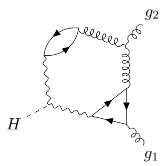
D_{19}

(+1) 

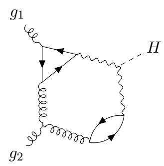
D_{20}

(+1) 

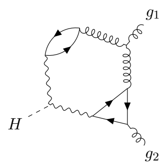
D_{21}

(−1) 

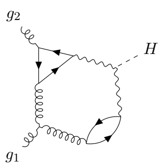
D_{22}

(−1) 

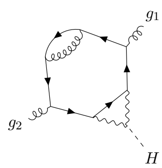
D_{23}

(−1) 

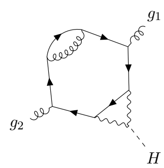
D_{24}

(−1) 

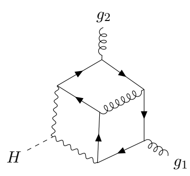
D_{25}

(−1) 

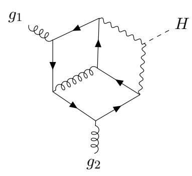
D_{26}

(−1) 

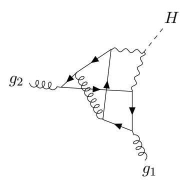
D_{27}

(−1) 

D_{28}

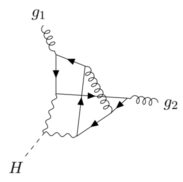
(−1) 

D_{29}

(−1) 

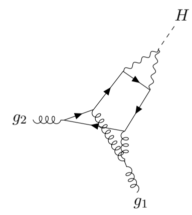
D_{30}

(-1)



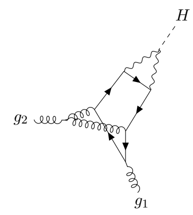
D_{31}

$(+1)$



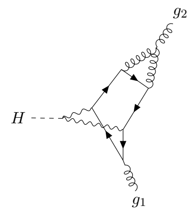
D_{32}

$(+1)$



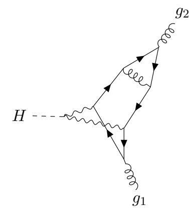
D_{33}

$(+1)$



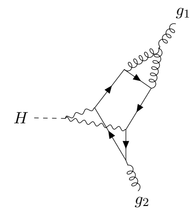
D_{34}

(-1)



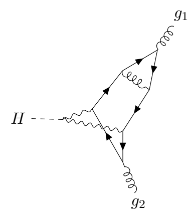
D_{35}

$(+1)$



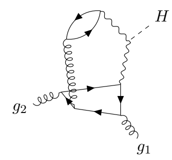
D_{36}

(-1)



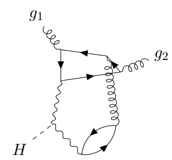
D_{37}

$(+1)$



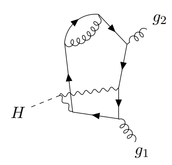
D_{38}

$(+1)$



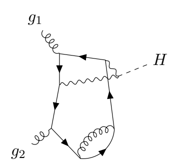
D_{39}

(-1)



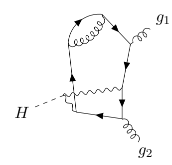
D_{40}

(-1)



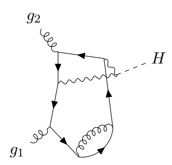
D_{41}

(-1)



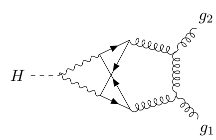
D_{42}

(-1)



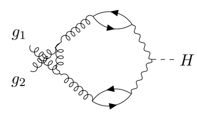
D_{43}

(-1)



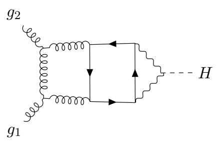
D_{44}

$(+1)$



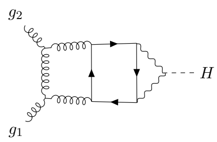
D_{45}

(−1)



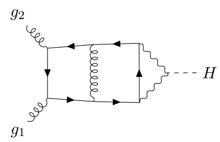
D_{46}

(−1)



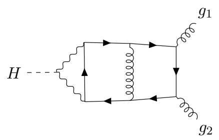
D_{47}

(−1)



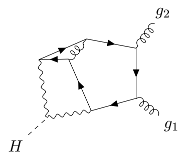
D_{48}

(−1)



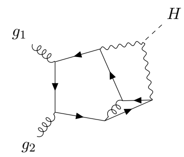
D_{49}

(−1)



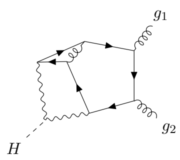
D_{50}

(−1)



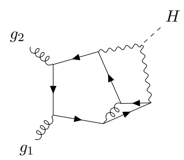
D_{51}

(−1)



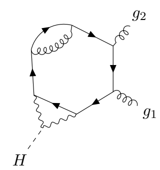
D_{52}

(−1)



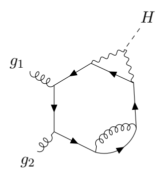
D_{53}

(−1)



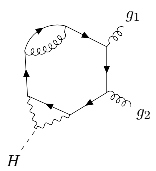
D_{54}

(−1)



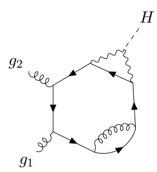
D_{55}

(−1)



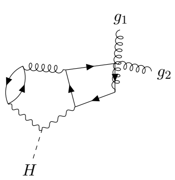
D_{56}

(−1)



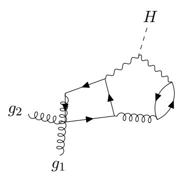
D_{57}

(+1)



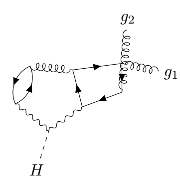
D_{58}

(+1)



D_{59}

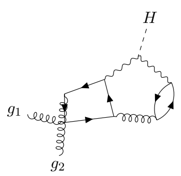
(+1)

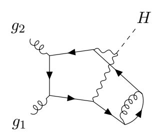


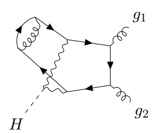
D_{60}

Table B.2.: Integral families for 3-loop FIs in $A^{(1,\text{fin})}_{\text{QCD-EW}}$.

Label	Families of denominators		
	VPP	VNA,	VNB
1	$(k_1)^2$	$(k_2)^2$	$(k_1)^2$
2	$(k_2)^2$	$(k_3)^2 \; - \; M^2$	$(k_2)^2$
3	$(k_3)^2 \; - \; M^2$	$(k_1+p_3)^2$	$(k_3)^2 \; - \; M^2$
4	$(k_1+p_3)^2$	$(k_3+p_3)^2 \; - \; M^2$	$(k_1+p_3)^2$
5	$(k_2+p_3)^2$	$(k_3-k_2)^2$	$(k_2+p_3)^2$
6	$(k_3+p_3)^2 \; - \; M^2$	$(k_2-k_1)^2$	$(k_3+p_3)^2 \; - \; M^2$
7	$(k_3-k_2)^2$	$(k_3-k_1)^2$	$(k_3-k_2)^2$
8	$(k_2-k_1)^2$	$(k_1-p_1)^2$	$(k_2-k_1)^2$
9	$(k_3-k_1)^2$	$(k_2-p_1)^2$	$(k_1-p_1)^2$
10	$(k_1-p_1)^2$	$(k_1-k_2-p_1)^2$	$(k_3-p_1)^2$
11	$(k_2-p_1)^2$	$(k_1-k_2+k_3+p_3)^2$	$(k_1-k_2+k_3+p_3)^2$
12	$(k_3-p_1)^2$	$(k_3-k_2-p_2)^2$	$(k_2-k_1-p_2)^2$

(+1) 

(−1) 

(−1) 

D_{61} D_{62} D_{63}

Master Integrals

The integral families are listed in Table B.2. The parent topologies are depicted in Fig. B.2, B.3, and B.4.

A circled number besides the graph indicates that the corresponding propagator appears in the numerator with power one.

The MIs with an asterisk ($*$) do not contribute to the NLO form factor, but do appear in the differential equations.

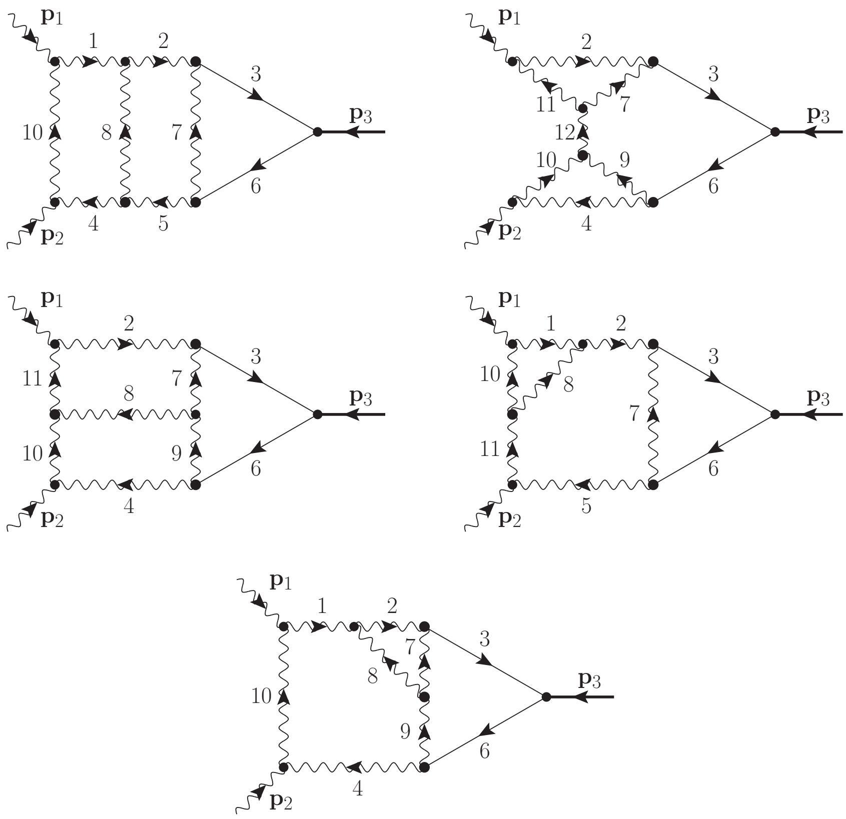


Figure B.2.: Planar (VPP) parent topologies for 3-loop FIs in $A^{(1,\text{fin})}_{\text{QCD-EW}}$. Cfr. Table B.2 for momenta assignments and propagator labels.

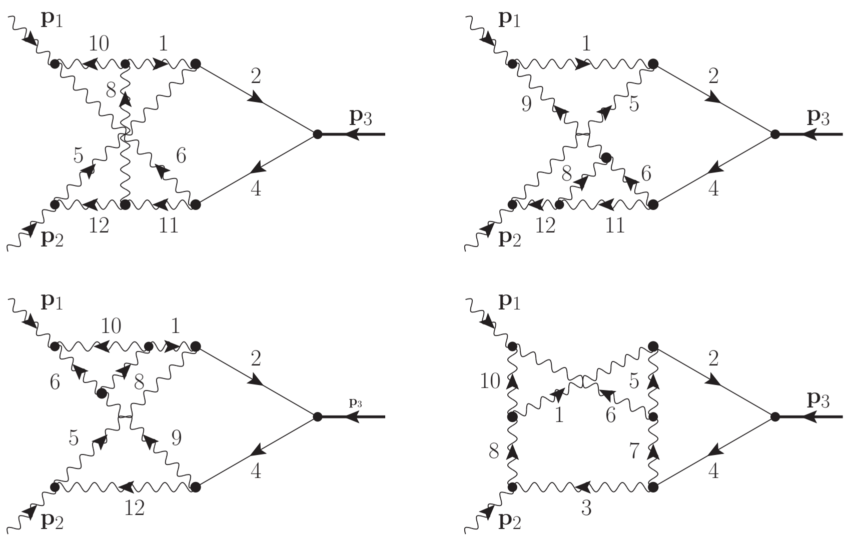


Figure B.3.: Non-planar type A (VNA) parent topologies for 3-loop FIs in $A^{(1,\text{fin})}_{\text{QCD-EW}}$. Cfr. Table B.2 for momenta assignments and propagator labels.

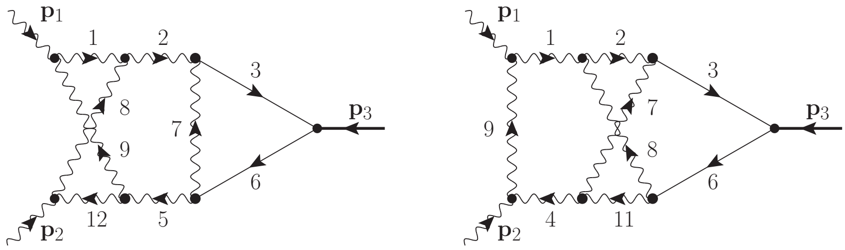
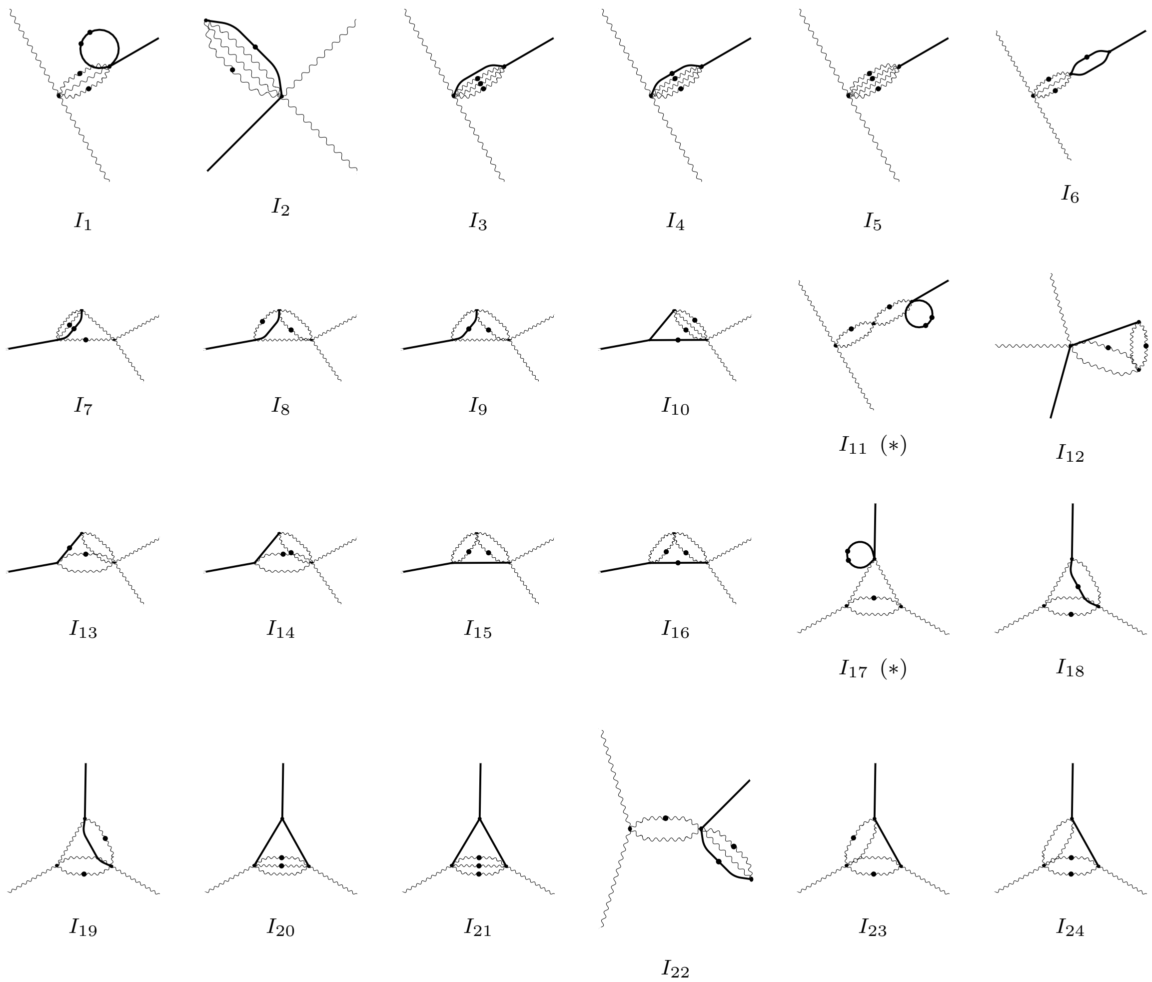
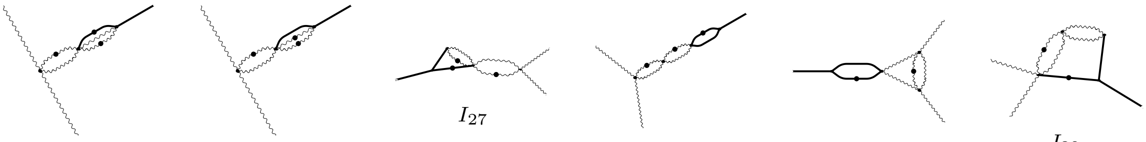


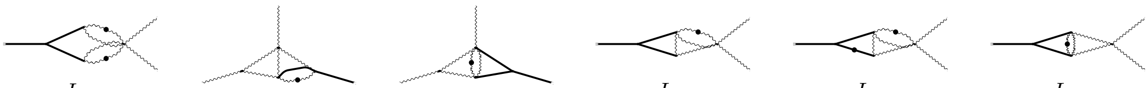
Figure B.4.: Non-planar type B (VNB) parent topologies for 3-loop FIs in $A^{(1,\mathrm{fin})}_{\text{QCD-EW}}$. Cfr. Table B.2 for momenta assignments and propagator labels.

Planar VPP integral family

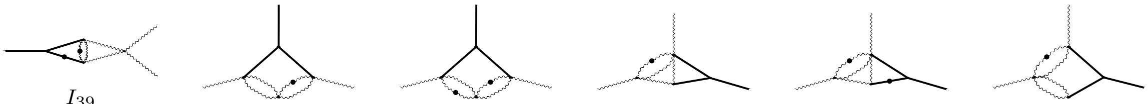




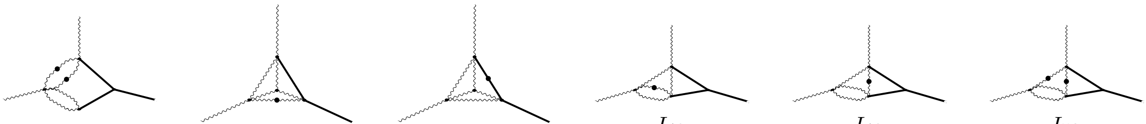
I_{25} I_{26} I_{27} I_{28} (*) I_{29} (*) I_{30}



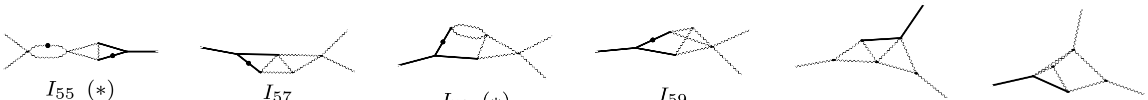
I_{31} I_{34} I_{35} I_{36} I_{37} I_{38}



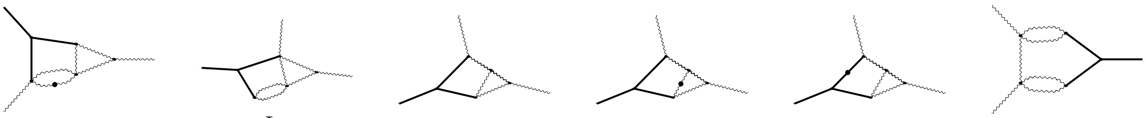
I_{39} I_{40} I_{41} I_{42} I_{43} I_{44}



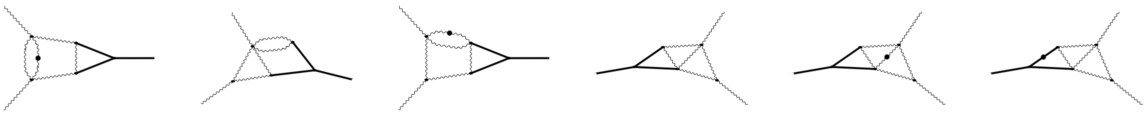
I_{45} I_{46} I_{47} I_{48} I_{49} I_{50}



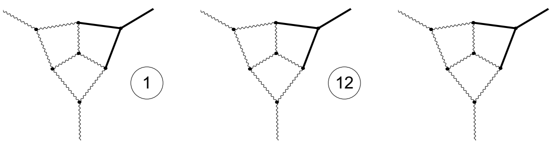
I_{55} (*) I_{57} I_{58} (*) I_{59} I_{61} I_{62}



I_{63} I_{64} I_{65} I_{66} I_{67} I_{68} (*)

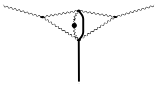


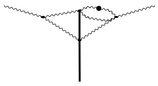
I_{69} (*) I_{70} I_{71} I_{72} I_{73} I_{74}

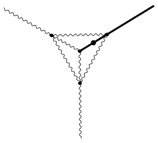


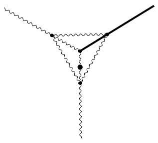
I_{97} I_{98} I_{99}

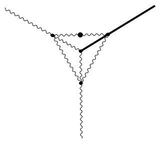
Non-planar type A VNA integral family

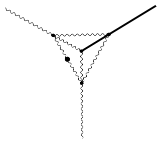

I_{32}

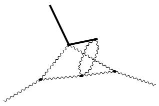

I_{33}

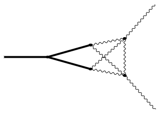

I_{51}

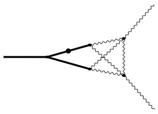

I_{52}

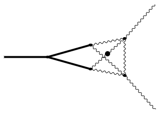

I_{53}

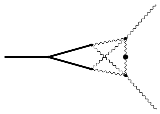

I_{54}

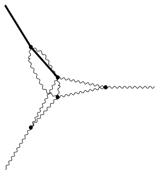

I_{60}

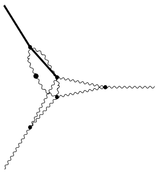

I_{75}

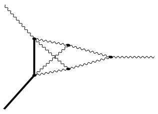

I_{76}

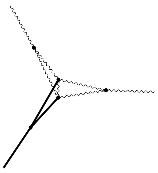

I_{77}

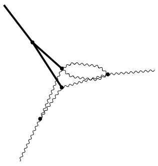

I_{78}

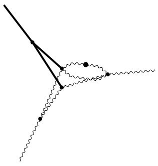

I_{79}

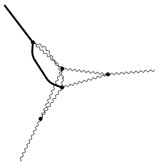

I_{80}

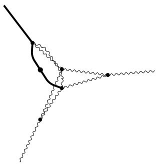

I_{81}

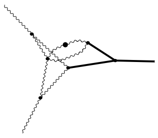

I_{83}

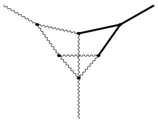

I_{84}

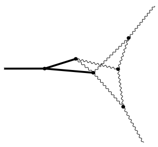

I_{85}

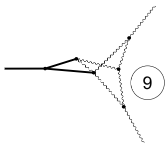

I_{86}


I_{87}

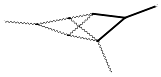

I_{88}

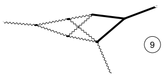

I_{89}


I_{92}

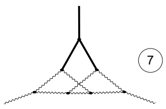


I_{93}

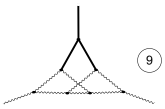

I_{94}



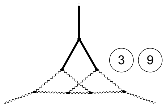
I_{95}



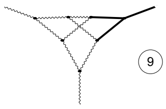
I_{100}



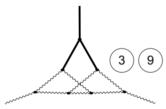
I_{101}



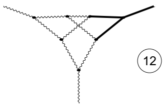
I_{102}



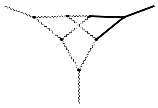
I_{103}



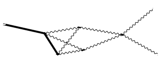
I_{104}

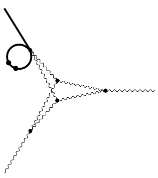


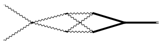
I_{105}

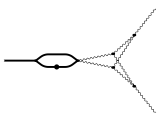

I_{106}

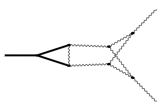
Non-planar type B VNB integral family


I_{56}


I_{82} $(*)$


I_{90}


I_{91} $(*)$


I_{96} $(*)$

UT basis

$$\mathbf{F}_1 = \frac{(-s)^{3\varepsilon}s^2y\varepsilon^2 I_1}{(y-1)^2}, \tag{B.13}$$

$$\mathbf{F}_2 = -(-s)^{3\varepsilon}(\varepsilon-1)\varepsilon^3 I_2, \tag{B.14}$$

$$\mathbf{F}_3 = -(-s)^{3\varepsilon}s\varepsilon^3 I_4, \tag{B.15}$$

$$\mathbf{F}_4 = -\frac{s\left(y^2-y+1\right)\varepsilon^3 I_3(-s)^{3\varepsilon}}{(y-1)^2} - \frac{3sy\varepsilon^3 I_4(-s)^{3\varepsilon}}{(y-1)^2}, \tag{B.16}$$

$$\mathbf{F}_5 = -(-s)^{3\varepsilon}s\varepsilon^3 I_5, \tag{B.17}$$

$$\mathbf{F}_6 = -\frac{(-s)^{3\varepsilon}s^2(y+1)\varepsilon^3 I_6}{y-1}, \tag{B.18}$$

$$\begin{aligned}\mathbf{F}_7 = &\frac{3s\left(3y^2-7y+3\right)\varepsilon^2\left(\varepsilon y^2-2\varepsilon y-y+\varepsilon\right)I_4(-s)^{3\varepsilon}}{(y-1)^4} - \frac{6(\varepsilon-1)\varepsilon^2\left(\varepsilon y^2-2\varepsilon y-y+\varepsilon\right)I_2(-s)^{3\varepsilon}}{(y-1)^2}+\\ &-\frac{6s^2y(\varepsilon-1)\varepsilon^2 I_7(-s)^{3\varepsilon}}{(y-1)^2} - \frac{s\left(y^2-y+1\right)\varepsilon^2\left(\varepsilon y^2-2\varepsilon y-y+\varepsilon\right)I_3(-s)^{3\varepsilon}}{(y-1)^4},\end{aligned} \tag{B.19}$$

$$\begin{aligned}\mathbf{F}_8 = &\frac{6(y+1)(\varepsilon-1)\varepsilon^3 I_2(-s)^{3\varepsilon}}{y-1} + \frac{s(y+1)\left(y^2-y+1\right)\varepsilon^3 I_3(-s)^{3\varepsilon}}{(y-1)^3}+\\ &+\frac{3s(y+1)\left(y^2-y+1\right)\varepsilon^3 I_4(-s)^{3\varepsilon}}{(y-1)^3} - \frac{6s^2y(y+1)\varepsilon^3 I_{10}(-s)^{3\varepsilon}}{(y-1)^3},\end{aligned} \tag{B.20}$$

$$\mathbf{F}_9 = -(-s)^{3\varepsilon}s\varepsilon^4 I_{20}, \tag{B.21}$$

$$\mathbf{F}_{10} = -\frac{6(y-1)(\varepsilon-1)\varepsilon^3 I_2(-s)^{3\varepsilon}}{y+1} - \frac{6s\left(y^2+1\right)\varepsilon^4 I_{20}(-s)^{3\varepsilon}}{(y-1)(y+1)} - \frac{s^2y^2\varepsilon^3 I_{21}(-s)^{3\varepsilon}}{(y-1)^3(y+1)}, \tag{B.22}$$

$$\begin{aligned}\mathbf{F}_{11} = &\frac{6(y-1)^2(\varepsilon-1)\varepsilon^3 I_2(-s)^{3\varepsilon}}{y^2-y+1} + \frac{s\varepsilon^3\left(5\varepsilon y^2-13\varepsilon y+2y+5\varepsilon\right)I_8(-s)^{3\varepsilon}}{y^2-y+1}+\\ &+\frac{s\varepsilon^3\left(5\varepsilon y^2+5y^2-13\varepsilon y-7y+5\varepsilon+5\right)I_9(-s)^{3\varepsilon}}{y^2-y+1} - \frac{2s^2y\varepsilon^2 I_1(-s)^{3\varepsilon}}{y^2-y+1},\end{aligned} \tag{B.23}$$

$$\begin{aligned}\mathbf{F}_{12} = &\frac{s^2y\varepsilon^2 I_1(-s)^{3\varepsilon}}{y^2-y+1} - \frac{3(y-1)^2(\varepsilon-1)\varepsilon^3 I_2(-s)^{3\varepsilon}}{y^2-y+1}+\\ &-\frac{s\varepsilon^3\left(3\varepsilon y^2-7\varepsilon y+y+3\varepsilon\right)I_8(-s)^{3\varepsilon}}{y^2-y+1} - \frac{s\varepsilon^3\left(3\varepsilon y^2+3y^2-7\varepsilon y-4y+3\varepsilon+3\right)I_9(-s)^{3\varepsilon}}{y^2-y+1},\end{aligned} \tag{B.24}$$

$$\mathbf{F}_{13} = -\frac{(-s)^{3\varepsilon}s^3y\varepsilon^2 I_{11}}{(y-1)^2}, \tag{B.25}$$

$$\mathbf{F}_{14} = \frac{(-s)^{3\varepsilon}sy(\varepsilon-1)\varepsilon^3 I_{12}}{(y-1)^2}, \tag{B.26}$$

$$\mathbf{F}_{15} = \frac{s\left(3y^2-5y+3\right)\varepsilon^3(2\varepsilon-1)I_{13}(-s)^{3\varepsilon}}{(y-1)^2} + \frac{s\varepsilon^3\left(3\varepsilon y^2-2\varepsilon y-y+3\varepsilon\right)I_{14}(-s)^{3\varepsilon}}{(y-1)^2}, \tag{B.27}$$

$$\mathbf{F}_{16} = \frac{s\left(y^2-y+1\right)\varepsilon^3(2\varepsilon-1)I_{13}(-s)^{3\varepsilon}}{(y-1)^2} + \frac{s\varepsilon^3\left(\varepsilon y^2+2\varepsilon y-y+\varepsilon\right)I_{14}(-s)^{3\varepsilon}}{(y-1)^2}, \tag{B.28}$$

$$\mathbf{F}_{17} = \frac{(-s)^{3\varepsilon}s^2\left(2y^4-5y^3+7y^2-5y+2\right)\varepsilon^3 I_{16}}{(y-1)^4} - \frac{(-s)^{3\varepsilon}s\varepsilon^3\left(10\varepsilon y^2+y^2-16\varepsilon y-2y+10\varepsilon+1\right)I_{15}}{(y-1)^2}, \tag{B.29}$$

$$\mathbf{F}_{18} = \frac{4(-s)^{3\varepsilon}s\varepsilon^3\left(13\varepsilon y^2+y^2-19\varepsilon y-2y+13\varepsilon+1\right)I_{15}}{(y-1)^2} - \frac{(-s)^{3\varepsilon}s^2\left(11y^4-26y^3+37y^2-26y+11\right)\varepsilon^3 I_{16}}{(y-1)^4}, \tag{B.30}$$

$$\mathbf{F}_{19} = \frac{(-s)^{3\varepsilon} s^2 y \varepsilon^3 I_{17}}{(y-1)^2}, \tag{B.31}$$

$$\mathbf{F}_{20} = -(-s)^{3\varepsilon} s \varepsilon^4 I_{18}, \tag{B.32}$$

$$\mathbf{F}_{21} = -(-s)^{3\varepsilon} s \varepsilon^4 I_{19}, \tag{B.33}$$

$$\mathbf{F}_{22} = -\frac{(-s)^{3\varepsilon} s^2 y(\varepsilon-1)\varepsilon^2 I_{22}}{(y-1)^2}, \tag{B.34}$$

$$\mathbf{F}_{23} = -(-s)^{3\varepsilon} s \varepsilon^4 I_{23}, \tag{B.35}$$

$$\mathbf{F}_{24} = \frac{2(-s)^{3\varepsilon} s \left(y^2-y+1\right) \varepsilon^4 I_{23}}{(y-1)^2} - \frac{(-s)^{3\varepsilon} s y \varepsilon^3 (2\varepsilon-1)^2 I_{24}}{(y-1)^2(\varepsilon+1)}, \tag{B.36}$$

$$\mathbf{F}_{25} = (-s)^{3\varepsilon} s^2 \varepsilon^3 I_{25}, \tag{B.37}$$

$$\mathbf{F}_{26} = \frac{2s^2 y \varepsilon^3 I_{25}(-s)^{3\varepsilon}}{(y-1)^2} + \frac{s^2 \left(y^2-y+1\right) \varepsilon^3 I_{26}(-s)^{3\varepsilon}}{(y-1)^2}, \tag{B.38}$$

$$\begin{aligned}\mathbf{F}_{27} = {} & \frac{s^2 y(y+1)(\varepsilon-1)\varepsilon^2 I_{22}(-s)^{3\varepsilon}}{(y-1)^3} + \frac{4s^3 y(y+1)\varepsilon^3 I_{27}(-s)^{3\varepsilon}}{(y-1)^3} + \\ & - \frac{2s^2(y+1)\left(y^2-y+1\right)\varepsilon^3 I_{25}(-s)^{3\varepsilon}}{(y-1)^3} - \frac{s^2(y+1)\left(y^2-y+1\right)\varepsilon^3 I_{26}(-s)^{3\varepsilon}}{(y-1)^3},\end{aligned} \tag{B.39}$$

$$\mathbf{F}_{28} = \frac{(-s)^{3\varepsilon} s^3 (y+1)\varepsilon^3 I_{28}}{y-1}, \tag{B.40}$$

$$\mathbf{F}_{29} = -\frac{(-s)^{3\varepsilon} s^2 (y+1)\varepsilon^4 I_{29}}{y-1}, \tag{B.41}$$

$$\begin{aligned}\mathbf{F}_{30} = {} & \frac{4s(y+1)\left(y^2-y+1\right)\varepsilon^4 I_{15}(-s)^{3\varepsilon}}{(y-1)^3} + \frac{2s^2 y(y+1)\varepsilon^3(2\varepsilon-1) I_{30}(-s)^{3\varepsilon}}{(y-1)^3} + \\ & - \frac{s y(y+1)(\varepsilon-1)\varepsilon^3 I_{12}(-s)^{3\varepsilon}}{(y-1)^3} - \frac{s^2(y+1)\left(y^2-y+1\right)^2 \varepsilon^3 I_{16}(-s)^{3\varepsilon}}{(y-1)^5},\end{aligned} \tag{B.42}$$

$$\mathbf{F}_{31} = \frac{s\left(y^2-y+1\right)\varepsilon^3(2\varepsilon-1) I_{13}(-s)^{3\varepsilon}}{(y-1)(y+1)} + \frac{s\varepsilon^3\left(\varepsilon y^2 - 4\varepsilon y + y + \varepsilon\right) I_{14}(-s)^{3\varepsilon}}{(y-1)(y+1)} \frac{s^2 y^2 \varepsilon^3 (2\varepsilon-1) I_{31}(-s)^{3\varepsilon}}{(y-1)^3(y+1)}, \tag{B.43}$$

$$\mathbf{F}_{32} = -(-s)^{3\varepsilon} s \varepsilon^5 I_{32}, \tag{B.44}$$

$$\mathbf{F}_{33} = -(-s)^{3\varepsilon} s \varepsilon^5 I_{33}, \tag{B.45}$$

$$\mathbf{F}_{34} = -(-s)^{3\varepsilon} s \varepsilon^5 I_{34}, \tag{B.46}$$

$$\mathbf{F}_{35} = -(-s)^{3\varepsilon} s \varepsilon^5 I_{35}, \tag{B.47}$$

$$\mathbf{F}_{36} = (-s)^{3\varepsilon} s \varepsilon^4 (2\varepsilon-1) I_{36}, \tag{B.48}$$

$$\begin{aligned}\mathbf{F}_{37} = {} & \frac{6(y-1)(y+1)(\varepsilon-1)\varepsilon^3 I_2(-s)^{3\varepsilon}}{y} + \frac{s(y+1)\left(y^2-y+1\right)\varepsilon^3 I_3(-s)^{3\varepsilon}}{(y-1)y} + \frac{3s(y+1)\varepsilon^3 I_4(-s)^{3\varepsilon}}{y-1} + \\ & + \frac{6s(y-1)(y+1)\varepsilon^4 I_8(-s)^{3\varepsilon}}{y} + \frac{6s(y-1)(y+1)\varepsilon^3(\varepsilon+1) I_9(-s)^{3\varepsilon}}{y} + \frac{6s^2(y+1)\varepsilon^3(2\varepsilon-1) I_{37}(-s)^{3\varepsilon}}{y-1} + \\ & - \frac{3s^2(y+1)\varepsilon^3 I_{10}(-s)^{3\varepsilon}}{y-1} - \frac{3s^2(y-1)(y+1)\varepsilon^3 I_6(-s)^{3\varepsilon}}{y} - \frac{6s(y-1)(y+1)\varepsilon^4(2\varepsilon-1) I_{36}(-s)^{3\varepsilon}}{y},\end{aligned} \tag{B.49}$$

$$\mathbf{F}_{38} = (-s)^{3\varepsilon} s \varepsilon^4 (2\varepsilon-1) I_{38}, \tag{B.50}$$

$$\begin{aligned}\mathbf{F}_{39} = {} & \frac{3s(y+1)\left(y^2-3y+1\right)\varepsilon^3 I_4(-s)^{3\varepsilon}}{(y-1)^3} + \frac{12s^2 y(y+1)\varepsilon^3(2\varepsilon-1) I_{39}(-s)^{3\varepsilon}}{(y-1)^3} + \\ & - \frac{6(y+1)(\varepsilon-1)\varepsilon^3 I_2(-s)^{3\varepsilon}}{y-1} - \frac{18s(y+1)\varepsilon^4(2\varepsilon-1) I_{38}(-s)^{3\varepsilon}}{y-1} - \frac{s(y+1)\left(y^2-y+1\right)\varepsilon^3 I_3(-s)^{3\varepsilon}}{(y-1)^3},\end{aligned} \tag{B.51}$$

$$\mathbf{F}_{40} = -\frac{sy(\varepsilon-1)\varepsilon^3 I_{12}(-s)^{3\varepsilon}}{(y-1)(y+1)} - \frac{2s\left(y^2+1\right)\varepsilon^4(2\varepsilon-1)I_{40}(-s)^{3\varepsilon}}{(y-1)(y+1)} - \frac{s^2y^2\varepsilon^3(2\varepsilon+1)I_{41}(-s)^{3\varepsilon}}{(y-1)^3(y+1)}, \tag{B.52}$$

$$\mathbf{F}_{41} = (-s)^{3\varepsilon}s\varepsilon^4(2\varepsilon-1)I_{40}, \tag{B.53}$$

$$\mathbf{F}_{42} = -(-s)^{3\varepsilon}s\varepsilon^5 I_{42}, \tag{B.54}$$

$$\begin{aligned}\mathbf{F}_{43} = &\frac{2s(y-1)(y+1)\varepsilon^4 I_{18}(-s)^{3\varepsilon}}{y^2+1} + \frac{s(y-1)(y+1)\varepsilon^4 I_{20}(-s)^{3\varepsilon}}{y^2+1} + \\ &+ \frac{10s(y-1)(y+1)\varepsilon^5 I_{42}(-s)^{3\varepsilon}}{y^2+1} - \frac{4s^2y(y+1)\varepsilon^4 I_{43}(-s)^{3\varepsilon}}{(y-1)\left(y^2+1\right)},\end{aligned} \tag{B.55}$$

$$\mathbf{F}_{44} = (-s)^{3\varepsilon}s\varepsilon^4(2\varepsilon-1)I_{44}, \tag{B.56}$$

$$\begin{aligned}\mathbf{F}_{45} = &\frac{2sy(\varepsilon-1)\varepsilon^3 I_{12}(-s)^{3\varepsilon}}{(y-1)(y+1)} + \frac{2s\left(y^2-y+1\right)\varepsilon^4 I_{23}(-s)^{3\varepsilon}}{(y-1)(y+1)} + \frac{sy\varepsilon^3(2\varepsilon-1)^2 I_{24}(-s)^{3\varepsilon}}{(y-1)(y+1)(\varepsilon+1)} + \\ &+ \frac{6s\left(y^2+1\right)\varepsilon^4(2\varepsilon-1)I_{44}(-s)^{3\varepsilon}}{(y-1)(y+1)} + \frac{2s^2y^2\varepsilon^3(2\varepsilon-1)I_{45}(-s)^{3\varepsilon}}{(y-1)^3(y+1)},\end{aligned} \tag{B.57}$$

$$\mathbf{F}_{46} = -(-s)^{3\varepsilon}s\varepsilon^5 I_{46}, \tag{B.58}$$

$$\mathbf{F}_{47} = -(-s)^{3\varepsilon}s\varepsilon^5 I_{47}, \tag{B.59}$$

$$\mathbf{F}_{48} = -(-s)^{3\varepsilon}s\varepsilon^5 I_{48}, \tag{B.60}$$

$$\mathbf{F}_{49} = (-s)^{3\varepsilon}s\varepsilon^4(2\varepsilon-1)I_{49}, \tag{B.61}$$

$$\begin{aligned}\mathbf{F}_{50} = &\frac{6(\varepsilon-1)\varepsilon^2\left(12\varepsilon y^4-48\varepsilon y^3-y^3+72\varepsilon y^2\right)I_2(-s)^{3\varepsilon}}{(y-1)^3(y+1)}\frac{6(\varepsilon-1)\varepsilon^2\left(+4y^2-48\varepsilon y-y+12\varepsilon\right)I_2(-s)^{3\varepsilon}}{(y-1)^3(y+1)} + \\ &+ \frac{6s^2y\left(y^2-4y+1\right)(\varepsilon-1)\varepsilon^2 I_7(-s)^{3\varepsilon}}{(y-1)^3(y+1)} + \frac{108s\left(y^2+1\right)\varepsilon^4(2\varepsilon-1)I_{49}(-s)^{3\varepsilon}}{(y-1)(y+1)} + \\ &+ \frac{48s^2y^2\varepsilon^3(2\varepsilon-1)I_{50}(-s)^{3\varepsilon}}{(y-1)^3(y+1)} - \frac{72s\left(y^2+1\right)\varepsilon^5 I_{48}(-s)^{3\varepsilon}}{(y-1)(y+1)} - \frac{6s^2y\left(y^2-4y+1\right)(\varepsilon-1)\varepsilon^2 I_{22}(-s)^{3\varepsilon}}{(y-1)^3(y+1)} + \\ &- \frac{s\left(y^2-y+1\right)\varepsilon^2\left(6\varepsilon y^4-12\varepsilon y^3+y^3+12\varepsilon y^2\right)I_3(-s)^{3\varepsilon}}{(y-1)^5(y+1)} + \\ &- \frac{s\left(y^2-y+1\right)\varepsilon^2\left(-4y^2-12\varepsilon y+y+6\varepsilon\right)I_3(-s)^{3\varepsilon}}{(y-1)^5(y+1)} + \\ &- \frac{3(-s)^{3\varepsilon}sy\varepsilon^2\left(6\varepsilon y^4-3y^4-12\varepsilon y^3+19y^3\right)I_4}{(y-1)^5(y+1)} - \frac{3(-s)^{3\varepsilon}sy\varepsilon^2\left(+12\varepsilon y^2-34y^2-12\varepsilon y+19y+6\varepsilon-3\right)I_4}{(y-1)^5(y+1)},\end{aligned} \tag{B.62}$$

$$\mathbf{F}_{51} = -2s\varepsilon^5 I_{54}(-s)^{3\varepsilon} - (-s)^{3\varepsilon}s\varepsilon^5 I_{53}, \tag{B.63}$$

$$\mathbf{F}_{52} = -s\varepsilon^5 I_{54}(-s)^{3\varepsilon} - (-s)^{3\varepsilon}s\varepsilon^5 I_{53}, \tag{B.64}$$

$$\begin{aligned}\mathbf{F}_{53} = &\,33s\varepsilon^3(2\varepsilon-1)I_{13}(-s)^{3\varepsilon} + \frac{3s\varepsilon^3\left(21\varepsilon y^2+3\varepsilon y-8y+21\varepsilon\right)I_{14}(-s)^{3\varepsilon}}{y^2-y+1} + \\ &+ \frac{8s\varepsilon^3\left(16\varepsilon y^2-31\varepsilon y+3y+16\varepsilon\right)I_{18}(-s)^{3\varepsilon}}{y^2-y+1} + \frac{12s(\varepsilon-1)\varepsilon^3\left(5\varepsilon y^2-11\varepsilon y+2y+5\varepsilon\right)I_{52}(-s)^{3\varepsilon}}{y^2-y+1} + \\ &+ \frac{3s\varepsilon^4\left(71\varepsilon y^2-111\varepsilon y+8y+71\varepsilon\right)I_{53}(-s)^{3\varepsilon}}{y^2-y+1} + \frac{6s\left(27y^2-19y+27\right)\varepsilon^5 I_{54}(-s)^{3\varepsilon}}{y^2-y+1} + \\ &- \frac{72(y-1)^2(\varepsilon-1)\varepsilon^3 I_2(-s)^{3\varepsilon}}{y^2-y+1} - \frac{12s(y-1)^2\varepsilon^3 I_5(-s)^{3\varepsilon}}{y^2-y+1} - \frac{6s\varepsilon^3\left(5\varepsilon y^2+24\varepsilon^2 y-25\varepsilon y+4y+5\varepsilon\right)I_{51}(-s)^{3\varepsilon}}{y^2-y+1},\end{aligned} \tag{B.65}$$

$$\begin{aligned}\mathbf{F}_{54} = &-15s\varepsilon^3(2\varepsilon-1)I_{13}(-s)^{3\varepsilon} + \frac{4s\varepsilon^3\left(16\varepsilon y^2-31\varepsilon y+3y+16\varepsilon\right)I_{18}(-s)^{3\varepsilon}}{y^2-y+1} - \frac{36(y-1)^2(\varepsilon-1)\varepsilon^3 I_2(-s)^{3\varepsilon}}{y^2-y+1} + \\ &- \frac{6s(y-1)^2\varepsilon^3 I_5(-s)^{3\varepsilon}}{y^2-y+1} - \frac{3s\varepsilon^3\left(35\varepsilon y^2-47\varepsilon y+4y+35\varepsilon\right)I_{14}(-s)^{3\varepsilon}}{y^2-y+1} + \\ &- \frac{6s\varepsilon^3\left(-15\varepsilon y^2+12\varepsilon^2 y+5\varepsilon y+2y-15\varepsilon\right)I_{51}(-s)^{3\varepsilon}}{y^2-y+1} - \frac{12s(\varepsilon-1)\varepsilon^3\left(15\varepsilon y^2-12\varepsilon y-y+15\varepsilon\right)I_{52}(-s)^{3\varepsilon}}{y^2-y+1} +\end{aligned}$$

$$- \frac{3s\varepsilon^4 \left(17\varepsilon y^2 + 3\varepsilon y - 4y + 17\varepsilon\right) I_{53}(-s)^{3\varepsilon}}{y^2 - y + 1} - \frac{6s \left(39y^2 - 43y + 39\right) \varepsilon^5 I_{54}(-s)^{3\varepsilon}}{y^2 - y + 1}, \tag{B.66}$$

$$\begin{aligned}\mathbf{F}_{55} =\ & \frac{2s^2 \left(y^2 - y + 1\right) \varepsilon^3 I_{25}(-s)^{3\varepsilon}}{(y-1)^3} + \frac{s^2 \left(y^2 - y + 1\right) \varepsilon^3 I_{26}(-s)^{3\varepsilon}}{(y-1)^3} - \frac{4s^3 \varepsilon^3 I_{28}(-s)^{3\varepsilon}}{y-1} + \\ & - \frac{2s^3 y\varepsilon^3 (2\varepsilon - 1) I_{55}(-s)^{3\varepsilon}}{(y-1)^2} - \frac{s^2 y(\varepsilon - 1)\varepsilon^2 I_{22}(-s)^{3\varepsilon}}{(y-1)^3} - \frac{4s^3 y\varepsilon^3 I_{27}(-s)^{3\varepsilon}}{(y-1)^3},\end{aligned} \tag{B.67}$$

$$\mathbf{F}_{56} = (-s)^{3\varepsilon} s\varepsilon^5 (2\varepsilon - 1) I_{56}, \tag{B.68}$$

$$\begin{aligned}\mathbf{F}_{57} =\ & \frac{6 \left(y^2 - y + 1\right) (\varepsilon - 1)\varepsilon^3 I_2(-s)^{3\varepsilon}}{(y-1)^2 y} + \frac{s \left(y^2 - y + 1\right)^2 \varepsilon^3 I_3(-s)^{3\varepsilon}}{(y-1)^4 y} + \frac{3s \left(4y^2 - 7y + 4\right) \varepsilon^3 I_4(-s)^{3\varepsilon}}{(y-1)^4} + \\ & + \frac{6s\varepsilon^4 I_8(-s)^{3\varepsilon}}{y} + \frac{6s\varepsilon^3(\varepsilon + 1) I_9(-s)^{3\varepsilon}}{y} + \frac{3s^2 y(\varepsilon - 1)\varepsilon^2 I_{22}(-s)^{3\varepsilon}}{(y-1)^4} + \frac{12s^3 y\varepsilon^3 I_{27}(-s)^{3\varepsilon}}{(y-1)^4} + \\ & + \frac{6s^2 \varepsilon^3 (2\varepsilon - 1) I_{37}(-s)^{3\varepsilon}}{(y-1)^2} + \frac{12s^2 y\varepsilon^3 (2\varepsilon - 1) I_{39}(-s)^{3\varepsilon}}{(y-1)^4} + \frac{6s^2 y\varepsilon^4 (2\varepsilon - 1) I_{57}(-s)^{3\varepsilon}}{(y-1)^2} + \\ & - \frac{18s\varepsilon^4 (2\varepsilon - 1) I_{38}(-s)^{3\varepsilon}}{(y-1)^2} - \frac{3s^2 (y+1)^2 \varepsilon^3 I_{10}(-s)^{3\varepsilon}}{(y-1)^4} - \frac{6s^2 \left(y^2 - y + 1\right) \varepsilon^3 I_{25}(-s)^{3\varepsilon}}{(y-1)^4} + \\ & - \frac{3s^2 \left(y^2 - y + 1\right) \varepsilon^3 I_{26}(-s)^{3\varepsilon}}{(y-1)^4} - \frac{3s^2 \varepsilon^3 I_6(-s)^{3\varepsilon}}{y} - \frac{6s\varepsilon^4 (2\varepsilon - 1) I_{36}(-s)^{3\varepsilon}}{y},\end{aligned} \tag{B.69}$$

$$\begin{aligned}\mathbf{F}_{58} =\ & \frac{2sy^2 (\varepsilon - 1)\varepsilon^3 I_{12}(-s)^{3\varepsilon}}{(y-1)^4} + \frac{2s^2 y \left(y^2 - y + 1\right)^2 \varepsilon^3 I_{16}(-s)^{3\varepsilon}}{(y-1)^6} + \frac{s^2 y(\varepsilon - 1)\varepsilon^2 I_{22}(-s)^{3\varepsilon}}{(y-1)^3} + \frac{4s^3 y\varepsilon^3 I_{27}(-s)^{3\varepsilon}}{(y-1)^3} + \\ & - \frac{2s^2 y\varepsilon^3 (2\varepsilon - 1)^2 I_{58}(-s)^{3\varepsilon}}{(y-1)^2} - \frac{2s^2 \left(y^2 - y + 1\right) \varepsilon^3 I_{25}(-s)^{3\varepsilon}}{(y-1)^3} - \frac{s^2 \left(y^2 - y + 1\right) \varepsilon^3 I_{26}(-s)^{3\varepsilon}}{(y-1)^3} + \\ & - \frac{8sy \left(y^2 - y + 1\right) \varepsilon^4 I_{15}(-s)^{3\varepsilon}}{(y-1)^4} - \frac{4s^2 y^2 \varepsilon^3 (2\varepsilon - 1) I_{30}(-s)^{3\varepsilon}}{(y-1)^4} - \frac{4s \left(y^2 - y + 1\right) \varepsilon^3 (2\varepsilon - 1) I_{13}(-s)^{3\varepsilon}}{(y-1)^2 (y+1)^2} + \\ & - \frac{4s\varepsilon^3 \left(\varepsilon y^2 - 4\varepsilon y + y + \varepsilon\right) I_{14}(-s)^{3\varepsilon}}{(y-1)^2 (y+1)^2} - \frac{4s^2 y^2 \varepsilon^3 (2\varepsilon - 1) I_{31}(-s)^{3\varepsilon}}{(y-1)^4 (y+1)^2},\end{aligned} \tag{B.70}$$

$$\begin{aligned}\mathbf{F}_{59} =\ & \frac{6 \left(y^2 - y + 1\right) (\varepsilon - 1)\varepsilon^3 I_2(-s)^{3\varepsilon}}{(y-1) y} + \frac{s \left(y^2 - y + 1\right)^2 \varepsilon^3 I_3(-s)^{3\varepsilon}}{(y-1)^3 y} + \frac{3s \left(2y^2 - 3y + 2\right) \varepsilon^3 I_4(-s)^{3\varepsilon}}{(y-1)^3} + \\ & + \frac{6s(y-1)\varepsilon^4 I_8(-s)^{3\varepsilon}}{y} + \frac{6s(y-1)\varepsilon^3(\varepsilon + 1) I_9(-s)^{3\varepsilon}}{y} + \frac{6s^2 \varepsilon^3 (2\varepsilon - 1) I_{37}(-s)^{3\varepsilon}}{y-1} + \\ & + \frac{3s^2 y\varepsilon^4 (2\varepsilon - 1) I_{59}(-s)^{3\varepsilon}}{(y-1)^2} - \frac{3s^2 \left(y^2 + 1\right) \varepsilon^3 I_{10}(-s)^{3\varepsilon}}{(y-1)^3} - \frac{3s^2 (y-1)\varepsilon^3 I_6(-s)^{3\varepsilon}}{y} + \\ & - \frac{6s(y-1)\varepsilon^4 (2\varepsilon - 1) I_{36}(-s)^{3\varepsilon}}{y} - \frac{6s \left(y^2 - y + 1\right) \varepsilon^3 (2\varepsilon - 1) I_{13}(-s)^{3\varepsilon}}{(y-1)(y+1)^2} + \\ & - \frac{6s\varepsilon^3 \left(\varepsilon y^2 - 4\varepsilon y + y + \varepsilon\right) I_{14}(-s)^{3\varepsilon}}{(y-1)(y+1)^2} - \frac{6s^2 y^2 \varepsilon^3 (2\varepsilon - 1) I_{31}(-s)^{3\varepsilon}}{(y-1)^3 (y+1)^2},\end{aligned} \tag{B.71}$$

$$\mathbf{F}_{60} = (-s)^{3\varepsilon} s\varepsilon^5 (2\varepsilon - 1) I_{60}, \tag{B.72}$$

$$\mathbf{F}_{61} = -(-s)^{3\varepsilon} s\varepsilon^6 I_{61}, \tag{B.73}$$

$$\mathbf{F}_{62} = -(-s)^{3\varepsilon} s\varepsilon^6 I_{62}, \tag{B.74}$$

$$\mathbf{F}_{63} = -\frac{(-s)^{3\varepsilon} s^2 y\varepsilon^5 I_{63}}{(y-1)^2}, \tag{B.75}$$

$$\mathbf{F}_{64} = (-s)^{3\varepsilon} s\varepsilon^5 (2\varepsilon - 1) I_{64}, \tag{B.76}$$

$$\mathbf{F}_{65} = -(-s)^{3\varepsilon} s\varepsilon^6 I_{65}, \tag{B.77}$$

$$\mathbf{F}_{66} = -\frac{(-s)^{3\varepsilon} s^2 y\varepsilon^4 (\varepsilon + 1) I_{66}}{(y-1)^2}, \tag{B.78}$$

$$\mathbf{F}_{67} = \frac{(-s)^{3\varepsilon} s^2 (y+1)\varepsilon^5 I_{67}}{y-1}, \tag{B.79}$$

$$\mathbf{F}_{68} = -(-s)^{3\varepsilon} s\varepsilon^4 (2\varepsilon-1)^2 I_{68}, \tag{B.80}$$

$$\mathbf{F}_{69} = (-s)^{3\varepsilon} s^2 \varepsilon^5 I_{69}, \tag{B.81}$$

$$\mathbf{F}_{70} = (-s)^{3\varepsilon} s\varepsilon^5 (2\varepsilon-1) I_{70}, \tag{B.82}$$

$$\mathbf{F}_{71} = (-s)^{3\varepsilon} s^2 \varepsilon^5 I_{71}, \tag{B.83}$$

$$\begin{aligned}\mathbf{F}_{72} = &\ \frac{s\varepsilon^4 I_8(-s)^{3\varepsilon}}{y^2+1} + \frac{s\varepsilon^3(\varepsilon+1) I_9(-s)^{3\varepsilon}}{y^2+1} + \frac{2s\varepsilon^4 I_{18}(-s)^{3\varepsilon}}{y^2+1} + \frac{s\varepsilon^4 I_{20}(-s)^{3\varepsilon}}{y^2+1} + \\ &+ \frac{s\varepsilon^5 I_{34}(-s)^{3\varepsilon}}{y^2+1} + \frac{2s\varepsilon^4(2\varepsilon-1) I_{36}(-s)^{3\varepsilon}}{y^2+1} + \frac{2s^2\varepsilon^5 I_{73}(-s)^{3\varepsilon}}{y^2+1} - \frac{2s\varepsilon^5 I_{35}(-s)^{3\varepsilon}}{y^2+1} + \\ &- \frac{4s\varepsilon^5 I_{42}(-s)^{3\varepsilon}}{y^2+1} - \frac{8s\varepsilon^6 I_{72}(-s)^{3\varepsilon}}{y^2+1} - \frac{2s^2 y(y+1)\varepsilon^5 I_{74}(-s)^{3\varepsilon}}{(y-1)\left(y^2+1\right)},\end{aligned} \tag{B.84}$$

$$\mathbf{F}_{73} = -(-s)^{3\varepsilon} s\varepsilon^6 I_{72}, \tag{B.85}$$

$$\mathbf{F}_{74} = (-s)^{3\varepsilon} s^2 \varepsilon^5 I_{73}, \tag{B.86}$$

$$\mathbf{F}_{75} = 2s\varepsilon^5(6\varepsilon-1) I_{75}(-s)^{3\varepsilon} + \frac{s^2(y+1)\varepsilon^5 I_{78}(-s)^{3\varepsilon}}{y-1}, \tag{B.87}$$

$$\begin{aligned}\mathbf{F}_{76} = &\ \frac{2s\varepsilon^4 I_{14}(-s)^{3\varepsilon}}{y-1} + \frac{4s\varepsilon^4 I_{18}(-s)^{3\varepsilon}}{y-1} + \frac{8s\varepsilon^5 I_{42}(-s)^{3\varepsilon}}{y-1} + \frac{4s(\varepsilon-1)\varepsilon^4 I_{52}(-s)^{3\varepsilon}}{y-1} + \\ &+ \frac{2s^2 y(y+1)\varepsilon^4(6\varepsilon-1) I_{76}(-s)^{3\varepsilon}}{(y-1)^3} + \frac{8s^2 y\varepsilon^5 I_{77}(-s)^{3\varepsilon}}{(y-1)^3} + \frac{s^2(y+1)\varepsilon^5 I_{78}(-s)^{3\varepsilon}}{y-1} + \\ &- \frac{2s\varepsilon^4 I_{51}(-s)^{3\varepsilon}}{y-1} - \frac{4s\varepsilon^5 I_{53}(-s)^{3\varepsilon}}{y-1} - \frac{4s\varepsilon^5(6\varepsilon-1) I_{75}(-s)^{3\varepsilon}}{y-1},\end{aligned} \tag{B.88}$$

$$\mathbf{F}_{77} = -\frac{(-s)^{3\varepsilon} s^2 y\varepsilon^5 I_{77}}{(y-1)^2}, \tag{B.89}$$

$$\mathbf{F}_{78} = \frac{(-s)^{3\varepsilon} s^2 (y+1)\varepsilon^5 I_{78}}{y-1}, \tag{B.90}$$

$$\mathbf{F}_{79} = 5(-s)^{3\varepsilon} s\varepsilon^5(2\varepsilon-1) I_{79} - \frac{(-s)^{3\varepsilon} s^2 \left(y^2-y+1\right)\varepsilon^5 I_{80}}{(y-1)^2}, \tag{B.91}$$

$$\mathbf{F}_{80} = 5s\varepsilon^5(2\varepsilon-1) I_{79}(-s)^{3\varepsilon} + \frac{s^2\left(y^2-y+1\right)\varepsilon^5 I_{80}(-s)^{3\varepsilon}}{(y-1)^2}, \tag{B.92}$$

$$\begin{aligned}\mathbf{F}_{81} = &\ \frac{6\left(y^2+1\right)(\varepsilon-1)\varepsilon^3 I_2(-s)^{3\varepsilon}}{y} + \frac{s\left(y^2+1\right)\left(y^2-y+1\right)\varepsilon^3 I_3(-s)^{3\varepsilon}}{(y-1)^2 y} + \frac{3s\left(y^2+1\right)\varepsilon^3 I_4(-s)^{3\varepsilon}}{(y-1)^2} + \\ &+ \frac{6s\left(y^2+1\right)\varepsilon^4 I_{19}(-s)^{3\varepsilon}}{y} + \frac{6s\left(y^2+1\right)\varepsilon^5 I_{46}(-s)^{3\varepsilon}}{y} - \frac{3s(y-1)^2(\varepsilon-1)\varepsilon^5 I_{81}(-s)^{3\varepsilon}}{y},\end{aligned} \tag{B.93}$$

$$\mathbf{F}_{82} = \frac{(-s)^{3\varepsilon} s^3 y\varepsilon^4 I_{82}}{(y-1)^2}, \tag{B.94}$$

$$\begin{aligned}\mathbf{F}_{83} = &\ \frac{6(\varepsilon-1)\varepsilon^3 I_2(-s)^{3\varepsilon}}{y+1} + \frac{s\left(y^2-y+1\right)\varepsilon^3 I_3(-s)^{3\varepsilon}}{(y-1)^2(y+1)} + \frac{6s\varepsilon^5 I_{32}(-s)^{3\varepsilon}}{y+1} + \\ &+ \frac{6s(y-1)\varepsilon^5(6\varepsilon-1) I_{83}(-s)^{3\varepsilon}}{y+1} - \frac{24s\varepsilon^5 I_{35}(-s)^{3\varepsilon}}{y+1} - \frac{3s\left(y^2-3y+1\right)\varepsilon^3 I_4(-s)^{3\varepsilon}}{(y-1)^2(y+1)},\end{aligned} \tag{B.95}$$

$$\mathbf{F}_{84} = (-s)^{3\varepsilon} s\varepsilon^5(2\varepsilon-1) I_{84}, \tag{B.96}$$

$$\mathbf{F}_{85} = \frac{(-s)^{3\varepsilon} s^2(y+1)\varepsilon^5 I_{85}}{y-1}, \tag{B.97}$$

$$\mathbf{F}_{86} = (-s)^{3\varepsilon} s\varepsilon^5(2\varepsilon-1) I_{86} - \frac{(-s)^{3\varepsilon} s^2 y\varepsilon^5 I_{87}}{(y-1)^2}, \tag{B.98}$$

$$\mathbf{F}_{87} = (-s)^{3\varepsilon} s^2 \varepsilon^5 I_{87}, \tag{B.99}$$

$$\mathbf{F}_{88} = \frac{(-s)^{3\varepsilon} s^3 y(y+1)\varepsilon^5 I_{88}}{(y-1)^3} - \frac{(-s)^{3\varepsilon} s^2 (y-2)\varepsilon^5 I_{80}}{(y-1)^2}, \tag{B.100}$$

$$\mathbf{F}_{89} = \frac{(-s)^{3\varepsilon} s^2 \left(y^2-y+1\right)\varepsilon^6 I_{89}}{(y-1)^2}, \tag{B.101}$$

$$\mathbf{F}_{90} = -\frac{(-s)^{3\varepsilon} s^2 (y+1)\varepsilon^5 (2\varepsilon-1) I_{90}}{y-1}, \tag{B.102}$$

$$\mathbf{F}_{91} = \frac{(-s)^{3\varepsilon} s^3 (y+1)\varepsilon^5 I_{91}}{y-1}, \tag{B.103}$$

$$\begin{aligned}
\mathbf{F}_{92} = {} & \frac{216(y-2)(\varepsilon-1)\varepsilon^3 I_2(-s)^{3\varepsilon}}{y+1} + \frac{s\left(5y^3-108y^2+108y-103\right)\varepsilon^3 I_3(-s)^{3\varepsilon}}{(y-1)^2(y+1)} + \frac{72s(8y-1)\varepsilon^5 I_{32}(-s)^{3\varepsilon}}{y+1} + \\
& + \frac{1152s^2 y(y+1)\varepsilon^4 I_{43}(-s)^{3\varepsilon}}{(y-1)\left(y^2+1\right)} + \frac{288s^2 y(y+1)\varepsilon^5 I_{74}(-s)^{3\varepsilon}}{(y-1)\left(y^2+1\right)} + \frac{144s^2 y\varepsilon^5(2\varepsilon+1) I_{92}(-s)^{3\varepsilon}}{(y-1)(y+1)} + \\
& + \frac{1152s\varepsilon^6 I_{93}(-s)^{3\varepsilon}}{y+1} - \frac{576s\varepsilon^5(6\varepsilon y-y-4\varepsilon+1) I_{83}(-s)^{3\varepsilon}}{y+1} - \frac{144s\varepsilon^5(2\varepsilon-1) I_{86}(-s)^{3\varepsilon}}{y+1} + \\
& - \frac{72s^2 y\varepsilon^2 I_1(-s)^{3\varepsilon}}{(y-1)^2(y+1)} - \frac{3s\left(93y^3-206y^2+226y-15\right)\varepsilon^3 I_4(-s)^{3\varepsilon}}{(y-1)^2(y+1)} + \\
& - \frac{144s^2\left(y^3-y^2-3y+2\right)\varepsilon^5 I_{87}(-s)^{3\varepsilon}}{(y-1)^2(y+1)} - \frac{144s\varepsilon^4 I_8(-s)^{3\varepsilon}}{y^2+1} - \frac{144s\varepsilon^3(\varepsilon+1) I_9(-s)^{3\varepsilon}}{y^2+1} + \\
& - \frac{288s\left(2y^2-1\right)\varepsilon^4 I_{18}(-s)^{3\varepsilon}}{y^2+1} - \frac{144s\left(2y^2-1\right)\varepsilon^4 I_{20}(-s)^{3\varepsilon}}{y^2+1} - \frac{144s\varepsilon^5 I_{34}(-s)^{3\varepsilon}}{y^2+1} + \\
& - \frac{288s\varepsilon^4(2\varepsilon-1) I_{36}(-s)^{3\varepsilon}}{y^2+1} - \frac{288s\left(12y^3+5y^2+11y+4\right)\varepsilon^5 I_{35}(-s)^{3\varepsilon}}{(y+1)\left(y^2+1\right)} + \\
& - \frac{576s\left(2y^3+y^2-9y-10\right)\varepsilon^5 I_{42}(-s)^{3\varepsilon}}{(y+1)\left(y^2+1\right)} - \frac{288s\left(y^3-y^2-3y-5\right)\varepsilon^6 I_{72}(-s)^{3\varepsilon}}{(y+1)\left(y^2+1\right)} - \frac{288s^2\left(y^4-3\right)\varepsilon^5 I_{73}(-s)^{3\varepsilon}}{(y-1)(y+1)\left(y^2+1\right)},
\end{aligned} \tag{B.104}$$

$$\begin{aligned}
\mathbf{F}_{93} = {} & \frac{24s^2 y\varepsilon^2 I_1(-s)^{3\varepsilon}}{(y-1)^2} + \frac{24(15y+23)(\varepsilon-1)\varepsilon^3 I_2(-s)^{3\varepsilon}}{y+1} + \frac{s(27y+59)\left(y^2-y+1\right)\varepsilon^3 I_3(-s)^{3\varepsilon}}{(y-1)^2(y+1)} + \\
& + \frac{192s(9y+5)\varepsilon^5 I_{35}(-s)^{3\varepsilon}}{y+1} - 1536s\varepsilon^5 I_{42}(-s)^{3\varepsilon} - 384s\varepsilon^6 I_{72}(-s)^{3\varepsilon} + 384s^2\varepsilon^5 I_{73}(-s)^{3\varepsilon} + \\
& + \frac{192s\varepsilon^5(8\varepsilon y-y-4\varepsilon+1) I_{83}(-s)^{3\varepsilon}}{y+1} + 48s\varepsilon^5(2\varepsilon-1) I_{86}(-s)^{3\varepsilon} + \frac{48s^2\left(3y^2-7y+3\right)\varepsilon^5 I_{87}(-s)^{3\varepsilon}}{(y-1)^2} + \\
& - 384s\varepsilon^6 I_{93}(-s)^{3\varepsilon} - \frac{24s(9y+1)\varepsilon^5 I_{32}(-s)^{3\varepsilon}}{y+1} - \frac{3s\left(9y^3-4y^2-132y+41\right)\varepsilon^3 I_4(-s)^{3\varepsilon}}{(y-1)^2(y+1)},
\end{aligned} \tag{B.105}$$

$$\begin{aligned}
\mathbf{F}_{94} = {} & \frac{s\left(y^2-y+1\right)\varepsilon^2\left(2\varepsilon y^2-y^2-4\varepsilon y+2\varepsilon\right) I_3(-s)^{3\varepsilon}}{(y-1)^4 y(y+1)} + \frac{3s\varepsilon^2\left(3y^3+2\varepsilon y^2-7y^2-4\varepsilon y+3y+2\varepsilon\right) I_4(-s)^{3\varepsilon}}{(y-1)^4(y+1)} + \\
& + \frac{6s^2 y(\varepsilon-1)\varepsilon^2 I_7(-s)^{3\varepsilon}}{(y-1)^2(y+1)} + \frac{24s\varepsilon^4 I_{23}(-s)^{3\varepsilon}}{y+1} + \frac{48s\varepsilon^5 I_{42}(-s)^{3\varepsilon}}{y+1} + \frac{12s\varepsilon^5 I_{46}(-s)^{3\varepsilon}}{y(y+1)} + \\
& + \frac{96s\varepsilon^5 I_{48}(-s)^{3\varepsilon}}{y+1} + \frac{36s\varepsilon^4(2\varepsilon-1) I_{49}(-s)^{3\varepsilon}}{y+1} + \frac{6s(y-1)(\varepsilon-1)\varepsilon^5 I_{81}(-s)^{3\varepsilon}}{y(y+1)} + \frac{6s^2 y\varepsilon^5(2\varepsilon+1) I_{94}(-s)^{3\varepsilon}}{(y-1)(y+1)} + \\
& + \frac{48s(y+2)\varepsilon^6 I_{95}(-s)^{3\varepsilon}}{y+1} - \frac{18s\varepsilon^4 I_{20}(-s)^{3\varepsilon}}{y+1} - \frac{96s\varepsilon^5 I_{35}(-s)^{3\varepsilon}}{y+1} - \frac{24s\varepsilon^4(2\varepsilon-1) I_{44}(-s)^{3\varepsilon}}{y+1} + \\
& - \frac{24s\varepsilon^6 I_{65}(-s)^{3\varepsilon}}{y+1} - \frac{6s^2 y(\varepsilon-1)\varepsilon^2 I_{22}(-s)^{3\varepsilon}}{(y-1)^2(y+1)} - \frac{12s^2 y\varepsilon^4(\varepsilon+1) I_{66}(-s)^{3\varepsilon}}{(y-1)^2(y+1)} + \\
& - \frac{12s(2y-1)\varepsilon^4 I_{19}(-s)^{3\varepsilon}}{y(y+1)} - \frac{6(\varepsilon-1)\varepsilon^2\left(6\varepsilon y^3-14\varepsilon y^2+y^2+10\varepsilon y-2\varepsilon\right) I_2(-s)^{3\varepsilon}}{(y-1)^2 y(y+1)},
\end{aligned} \tag{B.106}$$

$$\mathbf{F}_{95} = (-s)^{3\varepsilon} s\varepsilon^6 I_{95}, \tag{B.107}$$

$$\mathbf{F}_{96} = -12s\varepsilon^4 I_{18}(-s)^{3\varepsilon} - 4s\varepsilon^4 I_{19}(-s)^{3\varepsilon} + 6s^2\varepsilon^5 I_{73}(-s)^{3\varepsilon} + 6s^2\varepsilon^6 I_{92}(-s)^{3\varepsilon} - 3s^3\varepsilon^6 I_{96}(-s)^{3\varepsilon}, \tag{B.108}$$

$$\mathbf{F}_{97} = \frac{(-s)^{3\varepsilon} s^2 y\varepsilon^6 I_{97}}{(y-1)^2}, \tag{B.109}$$

$$\mathbf{F}_{98} = -13s^2\varepsilon^6 I_{98}(-s)^{3\varepsilon} - \frac{14s^2 y\varepsilon^6 I_{97}(-s)^{3\varepsilon}}{(y-1)^2}, \tag{B.110}$$

$$
\begin{aligned}
\mathbf{F}_{99} = {} & \frac{234s^2y\left(y^2-5y+3\right)\varepsilon^2I_1(-s)^{3\varepsilon}}{(y-1)^2\left(y^3+1\right)}+\frac{104s(4y-1)\varepsilon^3I_5(-s)^{3\varepsilon}}{y^2-1}+\frac{156s(3y+1)\varepsilon^4I_{18}(-s)^{3\varepsilon}}{y^2-1}+\\
& +\frac{702s\varepsilon^4I_{20}(-s)^{3\varepsilon}}{y+1}+\frac{78s^2y(3y-5)(\varepsilon-1)\varepsilon^2I_{22}(-s)^{3\varepsilon}}{(y-1)^3(y+1)}+\frac{156s\left(y^3-26y^2+32y-13\right)\varepsilon^4I_{23}(-s)^{3\varepsilon}}{(y-1)^3(y+1)}+\\
& +\frac{78sy(11y-5)\varepsilon^3(2\varepsilon-1)^2I_{24}(-s)^{3\varepsilon}}{(y-1)^3(y+1)(\varepsilon+1)}+\frac{1404s\varepsilon^5I_{35}(-s)^{3\varepsilon}}{y+1}+\frac{936s\varepsilon^5I_{42}(-s)^{3\varepsilon}}{y+1}+\\
& +\frac{468sy\varepsilon^5I_{46}(-s)^{3\varepsilon}}{y^2-1}+\frac{156s(3y+1)\varepsilon^5I_{47}(-s)^{3\varepsilon}}{y^2-1}+\frac{936s\varepsilon^6I_{61}(-s)^{3\varepsilon}}{y+1}+\frac{936s\varepsilon^6I_{62}(-s)^{3\varepsilon}}{y^2-1}+\\
& +\frac{936s^2y\varepsilon^5I_{63}(-s)^{3\varepsilon}}{(y-1)^2(y+1)}+\frac{468s^2\varepsilon^5I_{73}(-s)^{3\varepsilon}}{y^2-1}+\frac{234s^3y^2\varepsilon^5(4\varepsilon+1)I_{99}(-s)^{3\varepsilon}}{(y-1)^3(y+1)}+\\
& -\frac{468s\left(2y^2-2y+1\right)\varepsilon^3(2\varepsilon-1)I_{13}(-s)^{3\varepsilon}}{(y-1)^3}-\frac{936s\varepsilon^4(2\varepsilon-1)I_{44}(-s)^{3\varepsilon}}{y+1}-\frac{1872s\varepsilon^5I_{48}(-s)^{3\varepsilon}}{y+1}+\\
& -\frac{1872s\varepsilon^6I_{65}(-s)^{3\varepsilon}}{y+1}-\frac{312sy(\varepsilon-1)\varepsilon^3I_{12}(-s)^{3\varepsilon}}{(y-1)^2(y+1)}-\frac{468s^2y\varepsilon^4(\varepsilon+1)I_{66}(-s)^{3\varepsilon}}{(y-1)^2(y+1)}+\\
& -\frac{18s^2y(115y+167)\varepsilon^6I_{97}(-s)^{3\varepsilon}}{(y-1)^2(y+1)}-\frac{78s^2y(3y-5)(\varepsilon-1)\varepsilon^2I_7(-s)^{3\varepsilon}}{(y-1)^3(y+1)}+\\
& -\frac{468s\varepsilon^3\left(2\varepsilon y^3+9\varepsilon y^2-3y^2-4\varepsilon y+y+\varepsilon\right)I_{14}(-s)^{3\varepsilon}}{(y-1)^3(y+1)}+\\
& -\frac{13s\left(y^2-y+1\right)\varepsilon^2\left(3\varepsilon y^3-21\varepsilon y^2-3y^2+33\varepsilon y+5y-15\varepsilon\right)I_3(-s)^{3\varepsilon}}{(y-1)^5(y+1)}-\frac{468s(y-2)\varepsilon^4I_{19}(-s)^{3\varepsilon}}{y^2-1}+\\
& -\frac{234s(3y-1)\varepsilon^5I_{34}(-s)^{3\varepsilon}}{y^2-1}-\frac{936s^2\varepsilon^5I_{71}(-s)^{3\varepsilon}}{y^2-1}-\frac{468s^2\left(y^2-2\right)\varepsilon^6I_{98}(-s)^{3\varepsilon}}{y^2-1}+\\
& -\frac{234s\varepsilon^3\left(\varepsilon y^3-16\varepsilon y^2+2y^2+24\varepsilon y-4y-7\varepsilon\right)I_8(-s)^{3\varepsilon}}{y^4-y^3+y-1}+\\
& -\frac{234s\varepsilon^3\left(\varepsilon y^3+y^3-16\varepsilon y^2-10y^2+24\varepsilon y+12y-7\varepsilon-7\right)I_9(-s)^{3\varepsilon}}{y^4-y^3+y-1}+\\
& -\frac{78(-s)^{3\varepsilon}(\varepsilon-1)\varepsilon^2\left(3\varepsilon y^5-48\varepsilon y^4-3y^4+147\varepsilon y^3+8y^3-183\varepsilon y^2\right)I_2}{(y-1)^3\left(y^3+1\right)}+\\
& -\frac{78(-s)^{3\varepsilon}(\varepsilon-1)\varepsilon^2\left(-8y^2+102\varepsilon y+5y-21\varepsilon\right)I_2}{(y-1)^3\left(y^3+1\right)}+\\
& +\frac{39(-s)^{3\varepsilon}s\varepsilon^2\left(3\varepsilon y^5-26\varepsilon y^4-9y^4\right)I_4}{(y-1)^5(y+1)}+\frac{39(-s)^{3\varepsilon}s\varepsilon^2\left(+83\varepsilon y^3+36y^3-111\varepsilon y^2-44y^2+62\varepsilon y+15y-11\varepsilon\right)I_4}{(y-1)^5(y+1)},
\end{aligned}
\tag{B.111}
$$

$\mathbf{F}_{100} = [s(23y^6-12y^5-63y^4+149y^2-12y+63)\varepsilon^5I_{32}(-s)^{3\varepsilon}]/[7(y+1)^2(y^4+1)] + [s(7y^4+8y^3-8y+35)\varepsilon^5I_{48}(-s)^{3\varepsilon}]/[y^4+1] + [s\varepsilon^4(202\varepsilon y^4-101y^4-438\varepsilon+319)I_{49}(-s)^{3\varepsilon}]/[10(y^4+1)] + [2s(y-1)^2(63569y^4-34965y^3+34965y-6361)\varepsilon^5I_{54}(-s)^{3\varepsilon}]/[11655(y^2-y+1)(y^4+1)] + [2s\varepsilon^6(14\varepsilon y^4-7y^4+8\varepsilon y^3-4y^3-8\varepsilon y+4y+198\varepsilon-59)I_{65}(-s)^{3\varepsilon}]/[(y^4+1)(2\varepsilon-1)] + [s^2y\varepsilon^4(\varepsilon+1)(26\varepsilon y^4-13y^4+20\varepsilon y^3-10y^3-20\varepsilon y+10y+516\varepsilon-133)I_{66}(-s)^{3\varepsilon}]/[5(y-1)^2(y^4+1)(2\varepsilon-1)] + [s^2(y+1)\varepsilon^5I_{78}(-s)^{3\varepsilon}]/[y-1] + [s\varepsilon^5(146\varepsilon y^4-73y^4+446\varepsilon-173)I_{84}(-s)^{3\varepsilon}]/[5(y^4+1)] + [4s^2(y^2-y+1)\varepsilon^6(14\varepsilon y^4-7y^4+9\varepsilon-17)I_{89}(-s)^{3\varepsilon}]/[5(y-1)^2(y^4+1)(2\varepsilon-1)] + [40s\varepsilon^6I_{102}(-s)^{3\varepsilon}]/[y^4+1] - [838988s\varepsilon^4I_{19}(-s)^{3\varepsilon}]/[174825] - [3s^2(y+1)\varepsilon^5I_{67}(-s)^{3\varepsilon}]/[5(y-1)] - [2s^2y^2(y+1)\varepsilon^6I_{100}(-s)^{3\varepsilon}]/[(y-1)(y^4+1)] - [s^2(5y^6-3y^4+17y^2+9)\varepsilon^5I_{85}(-s)^{3\varepsilon}]/[(y-1)^2(y^4+1)] - [2s^2y(y^4+2y^3-2y+13)\varepsilon^6I_{101}(-s)^{3\varepsilon}]/[(y-1)^2(y^4+1)] - [s^2y(257y^6+643y^4+5657y^2+6043)\varepsilon^3I_{10}(-s)^{3\varepsilon}]/[450(y-1)^4(y^4+1)] - [4s(4y^6+3y^4+46y^2+45)\varepsilon^5(6\varepsilon-1)I_{83}(-s)^{3\varepsilon}]/[7(y+1)^2(y^4+1)] - [s^2y^2(18723y^6-19423y^4+14523y^2-23623)\varepsilon^3I_{21}(-s)^{3\varepsilon}]/[1050(y-1)^4(y+1)^2(y^4+1)] - [2s\varepsilon^5(272\varepsilon y^4-136y^4-300\varepsilon y^3+150y^3+300\varepsilon y-150y+147\varepsilon-136)I_{33}(-s)^{3\varepsilon}]/[25(y^4+1)(2\varepsilon-1)] - [2s\varepsilon^5(374\varepsilon y^4-187y^4+2624\varepsilon-687)I_{46}(-s)^{3\varepsilon}]/[25(y^4+1)(2\varepsilon-1)] - [4s\varepsilon^5(804\varepsilon^2y^4-536\varepsilon y^4+67y^4+2344\varepsilon^2-1096\varepsilon+137)I_{75}(-s)^{3\varepsilon}]/[35(y^4+1)(2\varepsilon-1)] - [s\varepsilon^5(626\varepsilon y^4-313y^4+600\varepsilon y^3-300y^3-600\varepsilon y+300y-6074\varepsilon+1787)I_{47}(-s)^{3\varepsilon}]/[75(y^4+1)(2\varepsilon-1)] - [2s^2y\varepsilon^5(662\varepsilon y^4-331y^4+1712\varepsilon-331)I_{77}(-s)^{3\varepsilon}]/[35(y-1)^2(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s\varepsilon^3(-1153346\varepsilon y^7+576673y^7+1864800\varepsilon^2y^6-405214\varepsilon y^6-263593y^6-1864800\varepsilon^2y^5-738946\varepsilon y^5+835673y^5-9324000\varepsilon^2y^4+9324000\varepsilon y^4+29836800\varepsilon^2y^3-34926946\varepsilon y^3+3788273y^3-39160800\varepsilon^2y^2+43106786\varepsilon y^2-3216193y^2+27972000\varepsilon^2y-34512546\varepsilon y+4047273y-9324000\varepsilon^2+9324000\varepsilon)I_5]/[466200y(y^2-y+1)(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s(\varepsilon-1)\varepsilon^3(16546\varepsilon y^6-8273y^6+3000\varepsilon^2y^5-60674\varepsilon y^5+29587y^5+3000\varepsilon^2y^4+500\varepsilon y^4-1000y^4-18000\varepsilon^2y^3+14500\varepsilon y^3+1000y^3+12000\varepsilon^2y^2-5454\varepsilon y^2-273y^2+15000\varepsilon^2y-82674\varepsilon y+37587y-15000\varepsilon^2+15000\varepsilon)I_{12}]/[750(y-1)^2(y+1)(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s\varepsilon^4(185908\varepsilon^2y^8-185908\varepsilon y^8+46477y^8-4200\varepsilon^2y^7+4200\varepsilon y^7-1050y^7-665784\varepsilon^2y^6+665784\varepsilon y^6-166446y^6+12600\varepsilon^2y^5-12600\varepsilon y^5+3150y^5+747684\varepsilon^2y^4-724934\varepsilon y^4+182546y^4-12600\varepsilon^2y^3+12600\varepsilon y^3-3150y^3-672784\varepsilon^2y^2+627284\varepsilon y^2-159446y^2+4200\varepsilon^2y-4200\varepsilon y+1050y+561776\varepsilon^2-539026\varepsilon+136069)I_{20}]/[350(y-1)^2(y+1)^2(y^4+1)(2\varepsilon-1)^2] + [(-s)^{3\varepsilon}s^2y(\varepsilon-1)\varepsilon^2(1646\varepsilon y^4-823y^4-1200\varepsilon y^3+600y^3+1200\varepsilon y-600y+9646\varepsilon-2323)I_{22}]/[300(y-1)^2(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s\varepsilon^3(2\varepsilon-1)(-81002\varepsilon y^6+40501y^6+60300\varepsilon^2y^5+46164\varepsilon y^5-38157y^5+60300\varepsilon^2y^4+36850\varepsilon y^4-33500y^4-361800\varepsilon^2y^3+264650\varepsilon y^3+33500y^3+241200\varepsilon^2y^2-486352\varepsilon y^2+107501y^2+301500\varepsilon^2y-359186\varepsilon y+28843y-301500\varepsilon^2+301500\varepsilon)I_{24}]/[10050(y-1)^2(y+1)(y^4+1)(\varepsilon+1)] + [2(-s)^{3\varepsilon}s^2y\varepsilon^3(-3218\varepsilon y^5+1609y^5+3150\varepsilon^2y^4-1575\varepsilon y^4+6300\varepsilon^2y^3-3150\varepsilon y^3-12600\varepsilon^2y^2+14175\varepsilon y^2-3218\varepsilon y+1609y+15750\varepsilon^2-15750\varepsilon)I_{31}]/[1575(y-1)^3(y+1)(y^4+1)] - [(-s)^{3\varepsilon}s\varepsilon^5(2206\varepsilon y^4-1103y^4+280\varepsilon y^3-140y^3-280\varepsilon y+140y+6126\varepsilon-2363)I_{42}]/[35(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s\varepsilon^4(954\varepsilon y^7-477y^7+100\varepsilon^2y^6+150\varepsilon y^6-100y^6+$

$$600\varepsilon^2y^5-2602\varepsilon y^5+1151y^5+500\varepsilon^2y^4-400\varepsilon y^4+200y^4-1400\varepsilon^2y^3+3554\varepsilon y^3-1677y^3+100\varepsilon^2y^2+150\varepsilon y^2-100y^2+2000\varepsilon^2y-5202\varepsilon y+2351y+500\varepsilon^2-500\varepsilon)I_{44}]/[25(y-1)y(y+1)(y^4+1)] + [(-s)^{3\varepsilon}s^2y\varepsilon^3(-674\varepsilon y^5+337y^5+300\varepsilon^2y^4-150\varepsilon y^4+600\varepsilon^2y^3-300\varepsilon y^3-1200\varepsilon^2y^2+1350\varepsilon y^2-674\varepsilon y+337y+1500\varepsilon^2-1500\varepsilon)I_{45}]/[75(y-1)^3(y+1)(y^4+1)] + [2(-s)^{3\varepsilon}s\varepsilon^3(30108\varepsilon^2y^6-15054\varepsilon y^6+311484\varepsilon^3y^5-398800\varepsilon^2y^5+173443\varepsilon y^5-25957y^5-139860\varepsilon^3y^4+169968\varepsilon^2y^4-73329\varepsilon y^4+11655y^4+139860\varepsilon^3y^2-74787\varepsilon^2y^2-3399\varepsilon y^2-11655y^2+31764\varepsilon^3y-154045\varepsilon^2y+103513\varepsilon y-2647y+65073\varepsilon^2-61674\varepsilon)I_{51}]/[11655(y^2-y+1)(y^4+1)(2\varepsilon-1)] + [2(-s)^{3\varepsilon}s\varepsilon^4(191600\varepsilon^2y^6-95800\varepsilon y^6-25270\varepsilon^2y^5-39279\varepsilon y^5+25957y^5+168290\varepsilon^2y^4-60835\varepsilon y^4-11655y^4+751040\varepsilon^2y^2-224005\varepsilon y^2+11655y^2-608020\varepsilon^2y+123891\varepsilon y+2647y+727730\varepsilon^2-189040\varepsilon)I_{53}]/[11655(y^2-y+1)(y^4+1)(2\varepsilon-1)] + [2(-s)^{3\varepsilon}s\varepsilon^6(-14\varepsilon y^5+7y^5+30\varepsilon^2y^4+25\varepsilon y^4-20y^4-180\varepsilon^2y^2+125\varepsilon y^2+20y^2+300\varepsilon^2y-274\varepsilon y-13y-150\varepsilon^2+150\varepsilon)I_{61}]/[5y(y^4+1)(2\varepsilon-1)] + [2(-s)^{3\varepsilon}s^2\varepsilon^5(-34\varepsilon y^5+17y^5+30\varepsilon^2y^4-35\varepsilon y^4+10y^4-180\varepsilon^2y^2+185\varepsilon y^2-10y^2+300\varepsilon^2y-334\varepsilon y+17y-150\varepsilon^2+150\varepsilon)I_{63}]/[5(y-1)^2(y^4+1)(2\varepsilon-1)] + [2(-s)^{3\varepsilon}s\varepsilon^5(-304\varepsilon y^5+152y^5+50\varepsilon^2y^4-25\varepsilon y^4-300\varepsilon^2y^2+275\varepsilon y^2+500\varepsilon^2y-754\varepsilon y+252y-250\varepsilon^2+250\varepsilon)I_{70}]/[25y(y^4+1)] - [(-s)^{3\varepsilon}s\varepsilon^3(7145480\varepsilon y^8-3572740y^8+3729600\varepsilon^2y^7-17291926\varepsilon y^7+7713563y^7-7806902\varepsilon y^6+3903451y^6-22377600\varepsilon^2y^5+23848168\varepsilon y^5-1667684y^5+41025600\varepsilon^2y^4-49730920\varepsilon y^4+7616060y^4-18648000\varepsilon^2y^3+3220874\varepsilon y^3+7713563y^3-22377600\varepsilon^2y^2+12705898\varepsilon y^2+3903451y^2+37296000\varepsilon^2y-51676232\varepsilon y+9521116y-18648000\varepsilon^2+18648000\varepsilon)I_{13}]/[466200(y-1)^2y(y+1)(y^4+1)] - [(-s)^{3\varepsilon}s^2y(\varepsilon-1)\varepsilon^2(1646\varepsilon y^4-823y^4-1200\varepsilon y^3+600y^3+1200\varepsilon y-600y+9646\varepsilon-2323)I_7]/[300(y-1)^2(y^4+1)(2\varepsilon-1)] - [8(-s)^{3\varepsilon}s\varepsilon^4(196\varepsilon^2y^6-196\varepsilon y^6+49y^6+20\varepsilon^3y^4-520\varepsilon^2y^4+505\varepsilon y^4-125y^4+40\varepsilon^3y^3-40\varepsilon^2y^3+10\varepsilon y^3-80\varepsilon^3y^2+296\varepsilon^2y^2-211\varepsilon y^2+54y^2+100\varepsilon^3-620\varepsilon^2+520\varepsilon-130)I_{40}]/[5(y-1)(y+1)(y^4+1)(2\varepsilon-1)] - [2(-s)^{3\varepsilon}s^2y\varepsilon^3(2\varepsilon+1)(-152\varepsilon y^5+76y^5+10\varepsilon^2y^4-5\varepsilon y^4+20\varepsilon^2y^3-10\varepsilon y^3-40\varepsilon^2y^2+45\varepsilon y^2-152\varepsilon y+76y+50\varepsilon^2-50\varepsilon)I_{41}]/[5(y-1)^3(y+1)(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s\varepsilon^4(449360\varepsilon y^8-224680y^8+20100\varepsilon^2y^7-513612\varepsilon y^7+251781y^7-309530\varepsilon y^6+154765y^6-120600\varepsilon^2y^5+482994\varepsilon y^5-186222y^5+221100\varepsilon^2y^4+982010\varepsilon y^4-345280y^4-100500\varepsilon^2y^3-1294162\varepsilon y^3+446081y^3-120600\varepsilon^2y^2-1090080\varepsilon y^2+349065y^2+201000\varepsilon^2y+915144\varepsilon y-306822y-100500\varepsilon^2+100500\varepsilon)I_{23}]/[5025(y-1)^2y(y+1)(y^4+1)(2\varepsilon-1)] - [2(-s)^{3\varepsilon}s\varepsilon^5(1446\varepsilon y^6-723y^6+2342\varepsilon y^5-1171y^5+3346\varepsilon y^4-1673y^4+13521\varepsilon y^2-4573y^2+11792\varepsilon y-1521y+15421\varepsilon-5523)I_{35}]/[175(y+1)^2(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s\varepsilon^2(2357826\varepsilon^2y^{10}-1178913\varepsilon y^{10}+1246478\varepsilon^2y^9+2072909\varepsilon y^9-1348074y^9-3293826\varepsilon^2y^8-1217403\varepsilon y^8+1432158y^8-10542348\varepsilon^2y^7-1263354\varepsilon y^7+3267264y^7+7098208\varepsilon^2y^6+1394380\varepsilon y^6-3154242y^6+21899148\varepsilon^2y^5+18466722\varepsilon y^5-5153148y^5+29592992\varepsilon^2y^4-25905412\varepsilon y^4+4871958y^4-143329548\varepsilon^2y^3+7035846\varepsilon y^3+10474464y^3+42960382\varepsilon^2y^2-23416307\varepsilon y^2+285558y^2+20652670\varepsilon^2y+16393813\varepsilon y-3805074y-5333182\varepsilon^2+1301591\varepsilon)I_4]/[327600(y-1)^4(y+1)^2(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s(y^2-y+1)\varepsilon^2(6429922\varepsilon^2y^8-3214961\varepsilon y^8-2246192\varepsilon^2y^7+224380\varepsilon y^7+449358y^7-15293236\varepsilon^2y^6+6504386\varepsilon y^6+571116y^6+7487792\varepsilon^2y^5-3332212\varepsilon y^5-205842y^5+34294036\varepsilon^2y^4-8957018\varepsilon y^4-7487792\varepsilon^2y^3-2833220\varepsilon y^3+1923558y^3-89986036\varepsilon^2y^2+17424386\varepsilon y^2+2864316y^2+2246192\varepsilon^2y-6389812\varepsilon y+1268358y+27864114\varepsilon^2-5742057\varepsilon)I_3]/[982800(y-1)^4(y+1)^2(y^4+1)(2\varepsilon-1)] - [2(-s)^{3\varepsilon}s(\varepsilon-1)\varepsilon^3(60216\varepsilon^2y^6-30108\varepsilon y^6-122718\varepsilon^2y^5+113273\varepsilon y^5-25957y^5+36906\varepsilon^2y^4-41763\varepsilon y^4+11655y^4+153456\varepsilon^2y^2-111693\varepsilon y^2-11655y^2-239268\varepsilon^2y+183203\varepsilon y-2647y+130146\varepsilon^2-123348\varepsilon)I_{52}]/[11655(y^2-y+1)(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s\varepsilon^3(2799766\varepsilon^2y^6-1399883\varepsilon y^6-7789666\varepsilon^2y^5+5452253\varepsilon y^5-778710y^5+3499066\varepsilon^2y^4-2448833\varepsilon y^4+349650y^4-12468284\varepsilon^2y^2+3145567\varepsilon y^2-349650y^2+8177684\varepsilon^2y-142147\varepsilon y-79410y-11768984\varepsilon^2+2096617\varepsilon)I_{18}]/[174825(y^2-y+1)(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s\varepsilon^3(9554120\varepsilon^2y^{10}-4777060\varepsilon y^{10}+3729600\varepsilon^3y^9-49099464\varepsilon^2y^9+31475458\varepsilon y^9-3929063y^9+18648000\varepsilon^3y^8+8686120\varepsilon^2y^8-14256452\varepsilon y^8+2625696y^8-18648000\varepsilon^3y^7-1523512\varepsilon^2y^7+16184188\varepsilon y^7-3049216y^7-70862400\varepsilon^3y^6+74690528\varepsilon^2y^6-16514050\varepsilon y^6-3888807y^6+164102400\varepsilon^3y^5-313751456\varepsilon^2y^5+143288654\varepsilon y^5-11388263y^5-74592000\varepsilon^3y^4+118709320\varepsilon^2y^4-42228452\varepsilon y^4+760896y^4-93240000\varepsilon^3y^3+89851688\varepsilon^2y^3-11787812\varepsilon y^3-4914016y^3+145454400\varepsilon^3y^2-283581192\varepsilon^2y^2+132785010\varepsilon y^2-11348007y^2-55944000\varepsilon^3y+84065608\varepsilon^2y-32708804\varepsilon y-18648000\varepsilon^3+18648000\varepsilon^2)I_{14}]/[466200(y-1)^2y(y+1)(y^2-y+1)(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}(\varepsilon-1)\varepsilon^2(146559756\varepsilon^2y^{10}-73279878\varepsilon y^{10}-588216596\varepsilon^2y^9+287713588\varepsilon y^9+3197355y^9+784401684\varepsilon^2y^8-393933552\varepsilon y^8+866355y^8-269291488\varepsilon^2y^7+139307744\varepsilon y^7-2331000y^7-500777240\varepsilon^2y^6+239331910\varepsilon y^6+5528355y^6+441785488\varepsilon^2y^5-264762164\varepsilon y^5+12222210y^5-151902760\varepsilon^2y^4+43138670\varepsilon y^4+6693855y^4+430008512\varepsilon^2y^3-219666256\varepsilon y^3+2331000y^3-1160156996\varepsilon^2y^2+537941788\varepsilon y^2+11355855y^2+1030002084\varepsilon^2y-552475752\varepsilon y+9024855y-423484444\varepsilon^2+211742222\varepsilon)I_2]/[1165500(y-1)^2(y+1)^2(y^2-y+1)(y^4+1)(2\varepsilon-1)] ,$$

$$\mathbf{F}_{101} = [1475351s\varepsilon^4I_{19}(-s)^{3\varepsilon}]/[699300] + [s^2y^2(19073y^6-19073y^4+14873y^2-23273)\varepsilon^3I_{21}(-s)^{3\varepsilon}]/[2100(y-1)^4(y+1)^2(y^4+1)] + [s\varepsilon^5(414\varepsilon y^4-207y^4-600\varepsilon y^3+300y^3+600\varepsilon y-300y+164\varepsilon-207)I_{33}(-s)^{3\varepsilon}]/[50(y^4+1)(2\varepsilon-1)] + [2s\varepsilon^5(66\varepsilon y^4-33y^4+14\varepsilon y^3-7y^3-14\varepsilon y+7y+262\varepsilon-96)I_{42}(-s)^{3\varepsilon}]/[7(y^4+1)(2\varepsilon-1)] + [s\varepsilon^5(958\varepsilon y^4-479y^4+9958\varepsilon-2479)I_{46}(-s)^{3\varepsilon}]/[100(y^4+1)(2\varepsilon-1)] + [3s^2(y+1)\varepsilon^5I_{67}(-s)^{3\varepsilon}]/[20(y-1)] + [4s\varepsilon^5(264\varepsilon^2y^4-176\varepsilon y^4+22y^4+1034\varepsilon^2-456\varepsilon+57)I_{75}(-s)^{3\varepsilon}]/[35(y^4+1)(2\varepsilon-1)] + [2s^2y\varepsilon^5(218\varepsilon y^4-109y^4+743\varepsilon-109)I_{77}(-s)^{3\varepsilon}]/[35(y-1)^2(y^4+1)(2\varepsilon-1)] + [s(y^6-y^4+85y^2+83)\varepsilon^5(6\varepsilon-1)I_{83}(-s)^{3\varepsilon}]/[7(y+1)^2(y^4+1)] + [2s^2(y^6-y^4+4y^2+2)\varepsilon^5I_{85}(-s)^{3\varepsilon}]/[(y-1)^2(y^4+1)] + [s^2y^2(y+1)\varepsilon^6I_{100}(-s)^{3\varepsilon}]/[(y-1)(y^4+1)] + [2s^2y(y+2)(y^2-2y+3)\varepsilon^6I_{101}(-s)^{3\varepsilon}]/[(y-1)^2(y^4+1)] - [s^2(y+1)\varepsilon^5I_{78}(-s)^{3\varepsilon}]/[2(y-1)] - [2s(y^4+2y^3-2y+8)\varepsilon^5I_{48}(-s)^{3\varepsilon}]/[y^4+1] - [20s\varepsilon^6I_{102}(-s)^{3\varepsilon}]/[y^4+1] - [s\varepsilon^4(26\varepsilon y^4-13y^4-134\varepsilon+92)I_{49}(-s)^{3\varepsilon}]/[5(y^4+1)] - [2s\varepsilon^5(16\varepsilon y^4-8y^4+91\varepsilon-33)I_{84}(-s)^{3\varepsilon}]/[5(y^4+1)] - [s^2y(193y^6-193y^4-5207y^2-5593)\varepsilon^3I_{10}(-s)^{3\varepsilon}]/[900(y-1)^4(y^4+1)] - [s(8y^6-6y^5-35y^4+71y^2-6y+28)\varepsilon^5I_{32}(-s)^{3\varepsilon}]/[7(y+1)^2(y^4+1)] - [s(y-1)^2(14113y^4-11655y^3+11655y-9197)\varepsilon^5I_{54}(-s)^{3\varepsilon}]/[3885(y^2-y+1)(y^4+1)] - [s\varepsilon^6(138\varepsilon y^4-69y^4+80\varepsilon y^3-40y^3-80\varepsilon y+40y+1978\varepsilon-589)I_{65}(-s)^{3\varepsilon}]/[10(y^4+1)(2\varepsilon-1)] - [s\varepsilon^5(6\varepsilon y^4-3y^4-300\varepsilon y^3+150y^3+300\varepsilon y-150y+3356\varepsilon-1053)I_{47}(-s)^{3\varepsilon}]/[75(y^4+1)(2\varepsilon-1)] - [2s^2(y^2-y+1)\varepsilon^6(6\varepsilon y^4-3y^4+\varepsilon-13)I_{89}(-s)^{3\varepsilon}]/[5(y-1)^2(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}(\varepsilon-1)\varepsilon^2(162830886\varepsilon^2y^{10}-81415443\varepsilon y^{10}-626549726\varepsilon^2y^9+308776033\varepsilon y^9+2249415y^9+814750554\varepsilon^2y^8-407212107\varepsilon y^8-81585y^8-267217228\varepsilon^2y^7+138270614\varepsilon y^7-2331000y^7-454157240\varepsilon^2y^6+217917790\varepsilon y^6+4580415y^6+365119228\varepsilon^2y^5-222637274\varepsilon y^5+10326330y^5-105282760\varepsilon^2y^4+21724550\varepsilon y^4+5745915y^4+432082772\varepsilon^2y^3-220703386\varepsilon y^3+2331000y^3-1129808126\varepsilon^2y^2+524663233\varepsilon y^2+10407915y^2+991668954\varepsilon^2y-531413307\varepsilon y+8076915y-407213314\varepsilon^2+203606657\varepsilon)I_2]/[2331000(y-1)^2(y+1)^2(y^2-y+1)(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s(y^2-y+1)\varepsilon^2(4672582\varepsilon^2y^8-2336291\varepsilon y^8-2246192\varepsilon^2y^7+490828\varepsilon y^7+316134y^7-9157756\varepsilon^2y^6+3969542\varepsilon y^6+304668y^6+7487792\varepsilon^2y^5-3065764\varepsilon y^5-339066y^5+30779356\varepsilon^2y^4-7199678\varepsilon y^4-7487792\varepsilon^2y^3-2566772\varepsilon y^3+1790334y^3-83850556\varepsilon^2y^2+14889542\varepsilon y^2+2597868y^2+2246192\varepsilon^2y-6123364\varepsilon y+1135134y+26106774\varepsilon^2-4863387\varepsilon)I_3]/[1965600(y-1)^4(y+1)^2(y^4+1)(2\varepsilon -$$

$$1)] + [(-s)^{3\varepsilon} s\varepsilon^2(2656102\varepsilon^2y^{10}-1328051\varepsilon y^{10}+202986\varepsilon^2y^9+1795311\varepsilon y^9-948402y^9-6212902\varepsilon^2y^8+508583\varepsilon y^8+1298934y^8-592964\varepsilon^2y^7-$$
$$4106462\varepsilon y^7+2201472y^7+4477408\varepsilon^2y^6+2971228\varepsilon y^6-3287466y^6+19812164\varepsilon^2y^5+17911526\varepsilon y^5-4353804y^5+26972192\varepsilon^2y^4-24328564\varepsilon y^4+$$
$$4738734y^4-133380164\varepsilon^2y^3+4192738\varepsilon y^3+9408672y^3+40041306\varepsilon^2y^2-21690321\varepsilon y^2+152334y^2+19609178\varepsilon^2y+16116215\varepsilon y-3405402y-$$
$$5034906\varepsilon^2+1152453\varepsilon)I_4]/[655200(y-1)^4(y+1)^2(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s^2y(\varepsilon-1)\varepsilon^2(1158\varepsilon y^4-579y^4-1200\varepsilon y^3+600y^3+1200\varepsilon y-600y+$$
$$9158\varepsilon-2079)I_7]/[600(y-1)^2(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s\varepsilon^3(5522438\varepsilon y^8-2761219y^8+3729600\varepsilon^2y^7-11752804\varepsilon y^7+4944002y^7-2267780\varepsilon y^6+$$
$$1133890y^6-22377600\varepsilon^2y^5+22225126\varepsilon y^5-856163y^5+41025600\varepsilon^2y^4-51353962\varepsilon y^4+8427581y^4-18648000\varepsilon^2y^3+8759996\varepsilon y^3+4944002y^3-$$
$$22377600\varepsilon^2y^2+18245020\varepsilon y^2+1133890y^2+37296000\varepsilon^2y-53299274\varepsilon y+10332637y-18648000\varepsilon^2+18648000\varepsilon)I_{13}]/[932400(y-1)^2y(y+1)(y^4+$$
$$1)] + [(-s)^{3\varepsilon}s\varepsilon^3(5884998\varepsilon^2y^{10}-2942499\varepsilon y^{10}+3729600\varepsilon^3y^9-31192100\varepsilon^2y^9+20304096\varepsilon y^9-2820223y^9+18648000\varepsilon^3y^8+10112198\varepsilon^2y^8-$$
$$16667891\varepsilon y^8+3474896y^8-18648000\varepsilon^3y^7-97434\varepsilon^2y^7+13772749\varepsilon y^7-2200016y^7-70862400\varepsilon^3y^6+88928770\varepsilon^2y^6-25850851\varepsilon y^6-2779967y^6+$$
$$164102400\varepsilon^3y^5-299513214\varepsilon^2y^5+133951853\varepsilon y^5-10279423y^5-74592000\varepsilon^3y^4+120135398\varepsilon^2y^4-44639891\varepsilon y^4+1610096y^4-93240000\varepsilon^3y^3+$$
$$91277766\varepsilon^2y^3-14199251\varepsilon y^3-4064816y^3+145454400\varepsilon^3y^2-265673828\varepsilon^2y^2+121613648\varepsilon y^2-10239167y^2-55944000\varepsilon^3y+80396486\varepsilon^2y-$$
$$30874243\varepsilon y-18648000\varepsilon^3+18648000\varepsilon^2)I_{14}]/[932400(y-1)^2y(y+1)(y^2-y+1)(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s\varepsilon^3(144146\varepsilon^2y^6-72073\varepsilon y^6-$$
$$324686\varepsilon^2y^5+254395\varepsilon y^5-46026y^5+214076\varepsilon^2y^4-176968\varepsilon y^4+34965y^4-1382659\varepsilon^2y^2+382472\varepsilon y^2-34965y^2+1272049\varepsilon^2y-305045\varepsilon y+$$
$$23904y-1312729\varepsilon^2+277577\varepsilon)I_{18}]/[34965(y^2-y+1)(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s\varepsilon^4(622090\varepsilon y^8-311045y^8+40200\varepsilon^2y^7-739740\varepsilon y^7+$$
$$359820y^7-331576\varepsilon y^6+165788y^6-241200\varepsilon^2y^5+689358\varepsilon y^5-234129y^5+442200\varepsilon^2y^4+1687390\varepsilon y^4-552245y^4-201000\varepsilon^2y^3-2300840\varepsilon y^3+$$
$$748420y^3-241200\varepsilon^2y^2-1892676\varepsilon y^2+554388y^2+402000\varepsilon^2y+1553658\varepsilon y-475329y-201000\varepsilon^2+201000\varepsilon)I_{23}]/[20100(y-1)^2y(y+1)(y^4+$$
$$1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s\varepsilon^5(302\varepsilon y^6-151y^6+2304\varepsilon y^5-1152y^5+4102\varepsilon y^4-2051y^4+24452\varepsilon y^2-7851y^2+21204\varepsilon y-1852y+28252\varepsilon-$$
$$9751)I_{35}]/[350(y+1)^2(y^4+1)(2\varepsilon-1)] + [2(-s)^{3\varepsilon}s\varepsilon^4(28\varepsilon^2y^6-28\varepsilon y^6+7y^6+40\varepsilon^3y^4-676\varepsilon^2y^4+646\varepsilon y^4-159y^4+80\varepsilon^3y^3-80\varepsilon^2y^3+20\varepsilon y^3-$$
$$160\varepsilon^3y^2+228\varepsilon^2y^2-58\varepsilon y^2+17y^2+200\varepsilon^3-876\varepsilon^2+676\varepsilon-169)I_{40}]/[5(y-1)(y+1)(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s^2y\varepsilon^3(2\varepsilon+1)(-152\varepsilon y^5+76y^5+$$
$$10\varepsilon^2y^4-5\varepsilon y^4+20\varepsilon^2y^3-10\varepsilon y^3-40\varepsilon^2y^2+45\varepsilon y^2-152\varepsilon y+76y+50\varepsilon^2-50\varepsilon)I_{41}]/[5(y-1)^3(y+1)(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s(\varepsilon-1)\varepsilon^3(9064\varepsilon^2y^6-$$
$$4532\varepsilon y^6-7876\varepsilon^2y^5+34622\varepsilon y^5-15342y^5-14246\varepsilon^2y^4-16187\varepsilon y^4+11655y^4+102304\varepsilon^2y^2-86117\varepsilon y^2-11655y^2-124426\varepsilon^2y+104552\varepsilon y+$$
$$7968y+78994\varepsilon^2-97772\varepsilon)I_{52}]/[11655(y^2-y+1)(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s\varepsilon^6(-14\varepsilon y^5+7y^5+30\varepsilon^2y^4+25\varepsilon y^4-20y^4-180\varepsilon^2y^2+125\varepsilon y^2+$$
$$20y^2+300\varepsilon^2y-274\varepsilon y-13y-150\varepsilon^2+150\varepsilon)I_{61}]/[5y(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s\varepsilon^5(-124\varepsilon y^5+62y^5+50\varepsilon^2y^4-25\varepsilon y^4-300\varepsilon^2y^2+275\varepsilon y^2+$$
$$500\varepsilon^2y-574\varepsilon y+162y-250\varepsilon^2+250\varepsilon)I_{70}]/[25y(y^4+1)] - [(-s)^{3\varepsilon}s^2y\varepsilon^3(-674\varepsilon y^5+337y^5+300\varepsilon^2y^4-150\varepsilon y^4+600\varepsilon^2y^3-300\varepsilon y^3-1200\varepsilon^2y^2+$$
$$1350\varepsilon y^2-674\varepsilon y+337y+1500\varepsilon^2-1500\varepsilon)I_{45}]/[150(y-1)^3(y+1)(y^4+1)] - [(-s)^{3\varepsilon}s^2y\varepsilon^3(-3218\varepsilon y^5+1609y^5+3150\varepsilon^2y^4-1575\varepsilon y^4+$$
$$6300\varepsilon^2y^3-3150\varepsilon y^3-12600\varepsilon^2y^2+14175\varepsilon y^2-3218\varepsilon y+1609y+15750\varepsilon^2-15750\varepsilon)I_{31}]/[1575(y-1)^3(y+1)(y^4+1)] - [(-s)^{3\varepsilon}s\varepsilon^4(114\varepsilon y^7-$$
$$57y^7+200\varepsilon^2y^6+300\varepsilon y^6-200y^6+1200\varepsilon^2y^5-3410\varepsilon y^5+1405y^5+1000\varepsilon^2y^4-800\varepsilon y^4+400y^4-2800\varepsilon^2y^3+5314\varepsilon y^3-2457y^3+200\varepsilon^2y^2+300\varepsilon y^2-$$
$$200y^2+4000\varepsilon^2y-8610\varepsilon y+3805y+1000\varepsilon^2-1000\varepsilon)I_{44}]/[100(y-1)y(y+1)(y^4+1)] - [(-s)^{3\varepsilon}s\varepsilon^3(2\varepsilon-1)(-151150\varepsilon y^6+75575y^6+120600\varepsilon^2y^5+$$
$$103182\varepsilon y^5-81741y^5+120600\varepsilon^2y^4+73700\varepsilon y^4-67000y^4-723600\varepsilon^2y^3+529300\varepsilon y^3+67000y^3+482400\varepsilon^2y^2-961850\varepsilon y^2+209575y^2+$$
$$603000\varepsilon^2y-707518\varepsilon y+52259y-603000\varepsilon^2+603000\varepsilon)I_{24}]/[40200(y-1)^2(y+1)(y^4+1)(\varepsilon+1)] - [(-s)^{3\varepsilon}s^2\varepsilon^5(-14\varepsilon y^5+7y^5+30\varepsilon^2y^4-35\varepsilon y^4+$$
$$10y^4-180\varepsilon^2y^2+185\varepsilon y^2-10y^2+300\varepsilon^2y-314\varepsilon y+7y-150\varepsilon^2+150\varepsilon)I_{63}]/[5(y-1)^2(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s^2y\varepsilon^4(\varepsilon+1)(54\varepsilon y^4-27y^4+$$
$$40\varepsilon y^3-20y^3-40\varepsilon y+20y+1034\varepsilon-267)I_{66}]/[20(y-1)^2(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s^2y(\varepsilon-1)\varepsilon^2(1158\varepsilon y^4-579y^4-1200\varepsilon y^3+600y^3+1200\varepsilon y-$$
$$600y+9158\varepsilon-2079)I_{22}]/[600(y-1)^2(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s(\varepsilon-1)\varepsilon^3(11018\varepsilon y^6-5509y^6+1500\varepsilon^2y^5-27592\varepsilon y^5+13421y^5+1500\varepsilon^2y^4+$$
$$250\varepsilon y^4-500y^4-9000\varepsilon^2y^3+7250\varepsilon y^3+500y^3+6000\varepsilon^2y^2+18\varepsilon y^2-1509y^2+7500\varepsilon^2y-38592\varepsilon y+17421y-7500\varepsilon^2+7500\varepsilon)I_{12}]/[750(y-$$
$$1)^2(y+1)(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s\varepsilon^3(4532\varepsilon^2y^6-2266\varepsilon y^6+184104\varepsilon^3y^5-203384\varepsilon^2y^5+86350\varepsilon y^5-15342y^5-139860\varepsilon^3y^4+$$
$$144392\varepsilon^2y^4-60541\varepsilon y^4+11655y^4+139860\varepsilon^3y^2-100363\varepsilon^2y^2+9389\varepsilon y^2-11655y^2-95616\varepsilon^3y+41371\varepsilon^2y+16420\varepsilon y+7968y+39497\varepsilon^2-$$
$$48886\varepsilon)I_{51}]/[11655(y^2-y+1)(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s\varepsilon^4(149348\varepsilon^2y^6-74674\varepsilon y^6-89168\varepsilon^2y^5+13900\varepsilon y^5+15342y^5+126038\varepsilon^2y^4-$$
$$39709\varepsilon y^4-11655y^4+708788\varepsilon^2y^2-202879\varepsilon y^2+11655y^2-671918\varepsilon^2y+177070\varepsilon y-7968y+685478\varepsilon^2-167914\varepsilon)I_{53}]/[11655(y^2-y+1)(y^4+$$
$$1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s\varepsilon^3(-715834\varepsilon y^7+357917y^7+1864800\varepsilon^2y^6+6474\varepsilon y^6-469437y^6-1864800\varepsilon^2y^5-301434\varepsilon y^5+616917y^5-9324000\varepsilon^2y^4+$$
$$9324000\varepsilon y^4+29836800\varepsilon^2y^3-34489434\varepsilon y^3+3569517y^3-39160800\varepsilon^2y^2+43518474\varepsilon y^2-3422037y^2+27972000\varepsilon^2y-34075034\varepsilon y+3828517y-$$
$$9324000\varepsilon^2+9324000\varepsilon)I_5]/[932400y(y^2-y+1)(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s\varepsilon^4(25828\varepsilon^2y^8-25828\varepsilon y^8+6457y^8-8400\varepsilon^2y^7+8400\varepsilon y^7-2100y^7-$$
$$661992\varepsilon^2y^6+661992\varepsilon y^6-165498y^6+25200\varepsilon^2y^5-25200\varepsilon y^5+6300y^5+803392\varepsilon^2y^4-757892\varepsilon y^4+192098y^4-25200\varepsilon^2y^3+25200\varepsilon y^3-6300y^3-$$
$$675992\varepsilon^2y^2+584992\varepsilon y^2-151498y^2+8400\varepsilon^2y-8400\varepsilon y+2100y+777564\varepsilon^2-732064\varepsilon+185641)I_{20}]/[1400(y-1)^2(y+1)^2(y^4+1)(2\varepsilon-1)^2] ,$$

$$\mathbf{F}_{102} = [s^2y(y+1)(\varepsilon-1)\varepsilon^2(80\varepsilon y^2-15y^2+24\varepsilon y-12y+80\varepsilon-15)I_7(-s)^{3\varepsilon}]/[3(y-1)(y^4+1)(2\varepsilon-1)] + [s^2y(y+1)(2893y^4+5400y^2+$$
$$2893)\varepsilon^3I_{10}(-s)^{3\varepsilon}]/[225(y-1)^3(y^4+1)] - [92]/[25]\ s\varepsilon^4I_{19}(-s)^{3\varepsilon} + [4s(y-1)(y+1)\varepsilon^5(28\varepsilon y^2-9y^2-4\varepsilon y+2y+28\varepsilon-9)I_{42}(-s)^{3\varepsilon}]/[(y^4+$$
$$1)(2\varepsilon-1)] + [20s(y-1)(y+1)(y^2+1)\varepsilon^5(9\varepsilon-2)I_{46}(-s)^{3\varepsilon}]/[(y^4+1)(2\varepsilon-1)] + [2s(y-1)(y+1)(y^2+1)\varepsilon^4(32\varepsilon-21)I_{49}(-s)^{3\varepsilon}]/[y^4+$$
$$1] + [4s(y-1)(y+1)(\varepsilon-1)\varepsilon^3(3\varepsilon^2y^2-4\varepsilon y^2-2\varepsilon^2y-\varepsilon y+y+3\varepsilon^2-4\varepsilon)I_{52}(-s)^{3\varepsilon}]/[(y^4+1)(2\varepsilon-1)] + [12s(y-1)^3(y+1)\varepsilon^5I_{54}(-s)^{3\varepsilon}]/[y^4+$$
$$1] + [8s(y-1)(y+1)(y^2+1)\varepsilon^5(22\varepsilon^2-8\varepsilon+1)I_{75}(-s)^{3\varepsilon}]/[(y^4+1)(2\varepsilon-1)] + [60s^2y(y+1)(y^2+1)\varepsilon^6I_{77}(-s)^{3\varepsilon}]/[(y-1)(y^4+1)(2\varepsilon-$$
$$1)] + [2s^2(y+1)\varepsilon^5I_{78}(-s)^{3\varepsilon}]/[y-1] + [4s(y-1)(41y^4+84y^2+41)\varepsilon^5(6\varepsilon-1)I_{83}(-s)^{3\varepsilon}]/[7(y+1)(y^4+1)] + [4s^2(y+1)(y^4+6y^2+$$
$$1)\varepsilon^5I_{85}(-s)^{3\varepsilon}]/[(y-1)(y^4+1)] + [4s^2(y+1)(y^2+1)(y^2-y+1)\varepsilon^6(\varepsilon+2)I_{89}(-s)^{3\varepsilon}]/[(y-1)(y^4+1)(2\varepsilon-1)] + [8s^2y(y+1)(3y^2-y+$$
$$3)\varepsilon^6I_{101}(-s)^{3\varepsilon}]/[(y-1)(y^4+1)] - [4s(y-1)(y+1)(7y^2-4y+7)\varepsilon^5I_{48}(-s)^{3\varepsilon}]/[y^4+1] - [20s(y-1)(y+1)(y^2+1)\varepsilon^5(3\varepsilon-1)I_{84}(-s)^{3\varepsilon}]/[y^4+$$
$$1] - [40s(y-1)(y+1)(y^2+1)\varepsilon^6I_{102}(-s)^{3\varepsilon}]/[y^4+1] - [4s^2y^2(y+1)\varepsilon^6I_{100}(-s)^{3\varepsilon}]/[(y-1)(y^4+1)] - [4s(13y^5-7y^4+63y^3-63y^2+13y-$$
$$7)\varepsilon^5I_{32}(-s)^{3\varepsilon}]/[7(y+1)(y^4+1)] - [s^2y^2(21173y^4+4200y^2+21173)\varepsilon^3I_{21}(-s)^{3\varepsilon}]/[525(y-1)^3(y+1)(y^4+1)] - [4s(y-1)(y+1)\varepsilon^4(23\varepsilon^2y^2-$$
$$4\varepsilon y^2-2\varepsilon^2y+3\varepsilon y-y+23\varepsilon^2-4\varepsilon)I_{53}(-s)^{3\varepsilon}]/[(y^4+1)(2\varepsilon-1)] - [8s(y-1)(y+1)\varepsilon^6(46\varepsilon y^2-13y^2-4\varepsilon y+2y+46\varepsilon-13)I_{65}(-s)^{3\varepsilon}]/[(y^4+1)(2\varepsilon-$$
$$1)] - [4s(y-1)(y+1)\varepsilon^5(67\varepsilon y^2-21y^2+12\varepsilon y-6y+67\varepsilon-21)I_{47}(-s)^{3\varepsilon}]/[3(y^4+1)(2\varepsilon-1)] - [2s\varepsilon^5(79\varepsilon y^4+23y^4-600\varepsilon y^3+300y^3+600\varepsilon y-$$
$$300y-171\varepsilon+23)I_{33}(-s)^{3\varepsilon}]/[25(y^4+1)(2\varepsilon-1)] - [2s^2y(y+1)\varepsilon^4(\varepsilon+1)(49\varepsilon y^2-12y^2-4\varepsilon y+2y+49\varepsilon-12)I_{66}(-s)^{3\varepsilon}]/[(y-1)(y^4+1)(2\varepsilon-$$
$$1)] - [s^2y(y+1)(\varepsilon-1)\varepsilon^2(80\varepsilon y^2-15y^2+24\varepsilon y-12y+80\varepsilon-15)I_{22}(-s)^{3\varepsilon}]/[3(y-1)(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s(y^2-y+1)\varepsilon^2(10259426\varepsilon^2y^6-$$
$$1034713\varepsilon y^6+2246192\varepsilon^2y^5-3307096\varepsilon y^5+409500y^5-26171634\varepsilon^2y^4+3967617\varepsilon y^4+1146600y^4-5241600\varepsilon^2y^3-3057600\varepsilon y^3+1474200y^3-$$

$$27086974\varepsilon^2y^2+4425287\varepsilon y^2+1146600y^2+2246192\varepsilon^2y-3307096\varepsilon y+409500y+11174766\varepsilon^2-1492383\varepsilon)I_3]/[491400(y-1)^3(y+1)(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s\varepsilon^3(932400\varepsilon^2y^6-932400\varepsilon y^6-1864800\varepsilon^2y^5+2463262\varepsilon y^5-376931y^5+1305360\varepsilon^2y^4-1015280\varepsilon y^4-51800y^4-1305360\varepsilon^2y^2+1015280\varepsilon y^2+51800y^2+1864800\varepsilon^2y-2343778\varepsilon y+317189y-932400\varepsilon^2+932400\varepsilon)I_5]/[46620y(y^4+1)(2\varepsilon-1)] + [2(-s)^{3\varepsilon}s(\varepsilon-1)\varepsilon^3(3750\varepsilon^2y^7-3750\varepsilon y^7-3750\varepsilon^2y^6+19293\varepsilon y^6-8709y^6-2250\varepsilon^2y^5-19317\varepsilon y^5+10221y^5+5250\varepsilon^2y^4-3500\varepsilon y^4-500y^4-5250\varepsilon^2y^3+3500\varepsilon y^3+500y^3+2250\varepsilon^2y^2+8293\varepsilon y^2-4709y^2+3750\varepsilon^2y-30317\varepsilon y+14221y-3750\varepsilon^2+3750\varepsilon)I_{12}]/[375(y-1)^2(y+1)(y^4+1)(2\varepsilon-1)] - [2(-s)^{3\varepsilon}s\varepsilon^3(3539\varepsilon^2y^4-957\varepsilon y^4-300\varepsilon^2y^3+450\varepsilon y^3-150y^3+300\varepsilon^2y-450\varepsilon y+150y-2711\varepsilon^2+543\varepsilon)I_{18}]/[75(y^4+1)(2\varepsilon-1)] + [(-s)^{3\varepsilon}s\varepsilon^3(2\varepsilon-1)(301500\varepsilon^2y^7-301500\varepsilon y^7-301500\varepsilon^2y^6+284350\varepsilon y^6+8575y^6-180900\varepsilon^2y^5+538682\varepsilon y^5-148741y^5+422100\varepsilon^2y^4-227800\varepsilon y^4-67000y^4-422100\varepsilon^2y^3+227800\varepsilon y^3+67000y^3+180900\varepsilon^2y^2-526350\varepsilon y^2+142575y^2+301500\varepsilon^2y-272018\varepsilon y-14741y-301500\varepsilon^2+301500\varepsilon)I_{24}]/[10050(y-1)^2(y+1)(y^4+1)(\varepsilon+1)] + [4(-s)^{3\varepsilon}s^2y\varepsilon^3(7875\varepsilon^2y^6-7875\varepsilon y^6-3218\varepsilon y^5+1609y^5-4725\varepsilon^2y^4+6300\varepsilon y^4+6300\varepsilon^2y^3-3150\varepsilon y^3-4725\varepsilon^2y^2+6300\varepsilon y^2-3218\varepsilon y+1609y+7875\varepsilon^2-7875\varepsilon)I_{31}]/[1575(y-1)^3(y+1)(y^4+1)] + [2(-s)^{3\varepsilon}s\varepsilon^5(14853\varepsilon y^5-5239y^5-3647\varepsilon y^4+4011y^4+14700\varepsilon y^3-7350y^3-14700\varepsilon y^2+7350y^2+5403\varepsilon y-4889y-13097\varepsilon+4361)I_{35}]/[175(y+1)(y^4+1)(2\varepsilon-1)] + [4(-s)^{3\varepsilon}s\varepsilon^4(125\varepsilon^2y^8-125\varepsilon y^8+500\varepsilon^2y^7-1125\varepsilon y^7+500y^7+50\varepsilon^2y^6+75\varepsilon y^6-50y^6-200\varepsilon^2y^5+301\varepsilon y^5-163y^5+250\varepsilon^2y^4-200\varepsilon y^4+100y^4-200\varepsilon^2y^3+175\varepsilon y^3-100y^3+50\varepsilon^2y^2+75\varepsilon y^2-50y^2+500\varepsilon^2y-999\varepsilon y+437y+125\varepsilon^2-125\varepsilon)I_{44}]/[25(y-1)y(y+1)(y^4+1)] + [2(-s)^{3\varepsilon}s^2y\varepsilon^3(750\varepsilon^2y^6-750\varepsilon y^6-674\varepsilon y^5+337y^5-450\varepsilon^2y^4+600\varepsilon y^4+600\varepsilon^2y^3-300\varepsilon y^3-450\varepsilon^2y^2+600\varepsilon y^2-674\varepsilon y+337y+750\varepsilon^2-750\varepsilon)I_{45}]/[75(y-1)^3(y+1)(y^4+1)] + [2(-s)^{3\varepsilon}s(y-1)(y+1)\varepsilon^3(24y\varepsilon^3-3y^2\varepsilon^2-24y\varepsilon^2-3\varepsilon^2+4y^2\varepsilon+10y\varepsilon+4\varepsilon-2y)I_{51}]/[(y^4+1)(2\varepsilon-1)] + [4(-s)^{3\varepsilon}s\varepsilon^6(75\varepsilon^2y^6-75\varepsilon y^6-150\varepsilon^2y^5+136\varepsilon y^5+7y^5+105\varepsilon^2y^4-50\varepsilon y^4-20y^4-105\varepsilon^2y^2+50\varepsilon y^2+20y^2+150\varepsilon^2y-124\varepsilon y-13y-75\varepsilon^2+75\varepsilon)I_{61}]/[5y(y^4+1)(2\varepsilon-1)] + [4(-s)^{3\varepsilon}s^2\varepsilon^5(75\varepsilon^2y^6-75\varepsilon y^6-150\varepsilon^2y^5+156\varepsilon y^5-3y^5+105\varepsilon^2y^4-110\varepsilon y^4+10y^4-105\varepsilon^2y^2+110\varepsilon y^2-10y^2+150\varepsilon^2y-144\varepsilon y-3y-75\varepsilon^2+75\varepsilon)I_{63}]/[5(y-1)^2(y^4+1)(2\varepsilon-1)] + [4(-s)^{3\varepsilon}s\varepsilon^5(125\varepsilon^2y^6-125\varepsilon y^6-250\varepsilon^2y^5+261\varepsilon y^5-68y^5+175\varepsilon^2y^4-150\varepsilon y^4-175\varepsilon^2y^2+150\varepsilon y^2+250\varepsilon^2y-189\varepsilon y+32y-125\varepsilon^2+125\varepsilon)I_{70}]/[25y(y^4+1)] - [(-s)^{3\varepsilon}s\varepsilon^3(9324000\varepsilon^2y^9-9324000\varepsilon y^9-18648000\varepsilon^2y^8+31576076\varepsilon y^8-7629538y^8+13053600\varepsilon^2y^7-16640962\varepsilon y^7+2726081y^7+9324000\varepsilon^2y^6-7155938\varepsilon y^6-1084031y^6-31701600\varepsilon^2y^5+38954764\varepsilon y^5-5724482y^5+31701600\varepsilon^2y^4-34624324\varepsilon y^4+3559262y^4-9324000\varepsilon^2y^3+3871838\varepsilon y^3+2726081y^3-13053600\varepsilon^2y^2+13356862\varepsilon y^2-1084031y^2+18648000\varepsilon^2y-27245636\varepsilon y+5464318y-9324000\varepsilon^2+9324000\varepsilon)I_{13}]/[233100(y-1)^2y(y+1)(y^4+1)] - [16(-s)^{3\varepsilon}s\varepsilon^4(50\varepsilon^3y^6-218\varepsilon^2y^6+168\varepsilon y^6-42y^6-30\varepsilon^3y^4-106\varepsilon^2y^4+141\varepsilon y^4-34y^4+40\varepsilon^3y^3-40\varepsilon^2y^3+10\varepsilon y^3-30\varepsilon^3y^2-118\varepsilon^2y^2+153\varepsilon y^2-37y^2+50\varepsilon^3-206\varepsilon^2+156\varepsilon-39)I_{40}]/[5(y-1)(y+1)(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}(\varepsilon-1)\varepsilon^2(209213286\varepsilon^2y^6-104606643\varepsilon y^6-524087240\varepsilon^2y^5+277583620\varepsilon y^5-2913750y^5+524000914\varepsilon^2y^4-226258457\varepsilon y^4-8158500y^4-186480000\varepsilon^2y^3+133644000\varepsilon y^3-10489500y^3+372383286\varepsilon^2y^2-150449643\varepsilon y^2-8158500y^2-524087240\varepsilon^2y+277583620\varepsilon y-2913750y+360830914\varepsilon^2-180415457\varepsilon)I_2]/[582750(y-1)(y+1)(y^4+1)(2\varepsilon-1)] - [4(-s)^{3\varepsilon}s^2y\varepsilon^3(2\varepsilon+1)(25\varepsilon^2y^6-25\varepsilon y^6-152\varepsilon y^5+76y^5-15\varepsilon^2y^4+20\varepsilon y^4+20\varepsilon^2y^3-10\varepsilon y^3-15\varepsilon^2y^2+20\varepsilon y^2-152\varepsilon y+76y+25\varepsilon^2-25\varepsilon)I_{41}]/[5(y-1)^3(y+1)(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s\varepsilon^2(4979102\varepsilon^2y^8-1807051\varepsilon y^8-11512622\varepsilon^2y^7-6255689\varepsilon y^7+1228500y^7-19281600\varepsilon^2y^6+9859200\varepsilon y^6+573300y^6+58500030\varepsilon^2y^5-12214815\varepsilon y^5-2375100y^5-30528992\varepsilon^2y^4+23509096\varepsilon y^4-3439800y^4+54880978\varepsilon^2y^3-10405289\varepsilon y^3-2375100y^3-19281600\varepsilon^2y^2+9859200\varepsilon y^2+573300y^2-7893570\varepsilon^2y-8065215\varepsilon y+1228500y+2711906\varepsilon^2-673453\varepsilon)I_4]/[163800(y-1)^3(y+1)(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s\varepsilon^4(100500\varepsilon^2y^9-100500\varepsilon y^9-201000\varepsilon^2y^8-351876\varepsilon y^8+25188y^8+140700\varepsilon^2y^7+468726\varepsilon y^7-43413y^7+100500\varepsilon^2y^6+876890\varepsilon y^6-237445y^6-341700\varepsilon^2y^5-385108\varepsilon y^5+102104y^5+341700\varepsilon^2y^4+612924\varepsilon y^4-216012y^4-100500\varepsilon^2y^3-1092374\varepsilon y^3+345187y^3-140700\varepsilon^2y^2-684210\varepsilon y^2+151155y^2+201000\varepsilon^2y+579692\varepsilon y-139096y-100500\varepsilon^2+100500\varepsilon)I_{23}]/[5025(y-1)^2y(y+1)(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s\varepsilon^3(9324000\varepsilon^3y^9-9324000\varepsilon^2y^9+37296000\varepsilon^3y^8-44414524\varepsilon^2y^8+12883262\varepsilon y^8-42890400\varepsilon^3y^7+79912580\varepsilon^2y^7-37175804\varepsilon y^7+3971057y^7-24242400\varepsilon^3y^6+85909540\varepsilon^2y^6-52375996\varepsilon y^6+7740913y^6+46620000\varepsilon^3y^5-51954236\varepsilon^2y^5+12923518\varepsilon y^5+1864800y^5-46620000\varepsilon^3y^4+56284676\varepsilon^2y^4-15088738\varepsilon y^4-1864800y^4+24242400\varepsilon^3y^3-86054620\varepsilon^2y^3+51402196\varepsilon y^3-7217743y^3+42890400\varepsilon^3y^2-80057660\varepsilon^2y^2+36202004\varepsilon y^2-3447887y^2-37296000\varepsilon^3y+48744964\varepsilon^2y-15048482\varepsilon y-9324000\varepsilon^3+9324000\varepsilon^2)I_{14}]/[233100(y-1)^2y(y+1)(y^4+1)(2\varepsilon-1)] - [(-s)^{3\varepsilon}s\varepsilon^4(189690\varepsilon^2y^6-178315\varepsilon y^6+45235y^6+4200\varepsilon^2y^5-4200\varepsilon y^5+1050y^5+182678\varepsilon^2y^4-194053\varepsilon y^4+47857y^4-8400\varepsilon^2y^3+8400\varepsilon y^3-2100y^3+186190\varepsilon^2y^2-197565\varepsilon y^2+48735y^2+4200\varepsilon^2y-4200\varepsilon y+1050y+186178\varepsilon^2-174803\varepsilon+44357)I_{20}]/[175(y-1)(y+1)(y^4+1)(2\varepsilon-1)^2] \quad ,$$

$$\mathbf{F}_{103} = [4s(63y^3+67y^2+151y+21)\varepsilon^4I_{19}(-s)^{3\varepsilon}]/[21y(y+1)] + [2s(4y^5-13y^4+22y^3-26y^2+23y-8)\varepsilon^4I_{23}(-s)^{3\varepsilon}]/[(y-1)^2(y+1)(y^2-y+1)] + [s^2y\varepsilon^3(2\varepsilon-1)(2\varepsilon y^4-10\varepsilon y^3+3y^3-6y^2-10\varepsilon y+3y+2\varepsilon)I_{31}(-s)^{3\varepsilon}]/[3(y-1)^3(y+1)(y^2-y+1)(4\varepsilon+1)] + [2s(y-1)\varepsilon^5I_{32}(-s)^{3\varepsilon}]/[y+1] + [2s\varepsilon^5(3\varepsilon y^4+17\varepsilon y^3+4y^3-13\varepsilon y-3y-3\varepsilon)I_{34}(-s)^{3\varepsilon}]/[y(y^2+1)(4\varepsilon+1)] + [s^2(y+1)\varepsilon^3(2\varepsilon-1)(60\varepsilon y^2+13y^2-56\varepsilon y-12y+60\varepsilon+13)I_{37}(-s)^{3\varepsilon}]/[(y-1)(y^2-y+1)(4\varepsilon+1)] + [8s^2(y+1)\varepsilon^4(\varepsilon y^2-82\varepsilon y-21y+\varepsilon)I_{43}(-s)^{3\varepsilon}]/[3(y-1)(y^2+1)(4\varepsilon+1)] + [s(12y^5-29y^4+16y^3-29y+4)\varepsilon^5I_{46}(-s)^{3\varepsilon}]/[y(y+1)(y^2-y+1)] + [4s(11y^3-17y^2+17y-3)\varepsilon^5I_{48}(-s)^{3\varepsilon}]/[(y+1)(y^2-y+1)] + [3s(y^3-7y^2+7y-9)\varepsilon^4(2\varepsilon-1)I_{49}(-s)^{3\varepsilon}]/[(y+1)(y^2-y+1)] + [8s(y+1)(y^2-y+1)\varepsilon^5I_{51}(-s)^{3\varepsilon}]/[3(y-1)y(4\varepsilon+1)] + [16s(y+1)(y^2-y+1)\varepsilon^6I_{53}(-s)^{3\varepsilon}]/[3(y-1)y(4\varepsilon+1)] + [s^2(y+1)\varepsilon^4(2\varepsilon-1)(2\varepsilon y^2+2\varepsilon y+y+2\varepsilon)I_{59}(-s)^{3\varepsilon}]/[(y-1)(y^2-y+1)(4\varepsilon+1)] + [6s(y^3+y^2-y+7)\varepsilon^6I_{65}(-s)^{3\varepsilon}]/[(y+1)(y^2-y+1)] + [2s^2y(2y-1)\varepsilon^5I_{71}(-s)^{3\varepsilon}]/[y^2-y+1] + [4s^2(y+1)\varepsilon^5(\varepsilon y^2+30\varepsilon y+7y+\varepsilon)I_{74}(-s)^{3\varepsilon}]/[(y-1)(y^2+1)(4\varepsilon+1)] - [4]/[7]\ s^2\varepsilon^5I_{87}(-s)^{3\varepsilon} + [2s^2(y+1)\varepsilon^6I_{89}(-s)^{3\varepsilon}]/[y-1] + [32sy\varepsilon^6I_{95}(-s)^{3\varepsilon}]/[y+1] - [2s^2(y+1)\varepsilon^5I_{85}(-s)^{3\varepsilon}]/[y-1] - [2s^2(y+1)\varepsilon^6I_{103}(-s)^{3\varepsilon}]/[y-1] - [4s(y-1)\varepsilon^5(6\varepsilon-1)I_{83}(-s)^{3\varepsilon}]/[y+1] - [4s^2y\varepsilon^5(2\varepsilon+1)I_{94}(-s)^{3\varepsilon}]/[(y-1)(y+1)] - [2s(y-1)(3y^2-1)(\varepsilon-1)\varepsilon^5I_{81}(-s)^{3\varepsilon}]/[y(y+1)] - [16s^2y\varepsilon^2I_1(-s)^{3\varepsilon}]/[y^2-y+1] - [2sy(2y-1)\varepsilon^6I_{62}(-s)^{3\varepsilon}]/[y^2-y+1] - [2s(y-1)(y+1)\varepsilon^5(2\varepsilon-1)I_{70}(-s)^{3\varepsilon}]/[y^2-y+1] - [4sy(2y-1)\varepsilon^5I_{47}(-s)^{3\varepsilon}]/[3(y^2-y+1)] - [2s(y^3-7y^2+7y-9)\varepsilon^4(2\varepsilon-1)I_{44}(-s)^{3\varepsilon}]/[(y+1)(y^2-y+1)] - [2s^2y(y^3-3y^2+3y-5)\varepsilon^4(\varepsilon+1)I_{66}(-s)^{3\varepsilon}]/[(y-1)^2(y+1)(y^2-y+1)] - [sy\varepsilon^3(2\varepsilon-1)^2I_{24}(-s)^{3\varepsilon}]/[(y-1)^2(\varepsilon+1)] - [4s^2(y+1)^3\varepsilon^5(6\varepsilon-1)I_{76}(-s)^{3\varepsilon}]/[3(y-1)^3(4\varepsilon+1)] - [4s^2(y+1)(11y^2-14y+11)\varepsilon^6I_{77}(-s)^{3\varepsilon}]/[3(y-1)^3(4\varepsilon+1)] - [4s(y-1)(y+1)\varepsilon^6I_{54}(-s)^{3\varepsilon}]/[y(4\varepsilon+1)] - [16s(y+1)(y^2-y+1)(\varepsilon-1)\varepsilon^5I_{52}(-s)^{3\varepsilon}]/[3(y-1)y(4\varepsilon+1)] - [8s(y+1)(y^2-4y+1)\varepsilon^6(6\varepsilon-1)I_{75}(-s)^{3\varepsilon}]/[3(y-1)y(4\varepsilon+1)] - [2s^2(y+1)^3\varepsilon^6I_{78}(-s)^{3\varepsilon}]/[3(y-1)y(4\varepsilon+1)] - [s^2y\varepsilon^3(\varepsilon y^2-106\varepsilon y-27y+\varepsilon)I_{21}(-s)^{3\varepsilon}]/[9(y-1)^3(y+1)(4\varepsilon+1)] - [s^2(y-1)(y+1)\varepsilon^3(60\varepsilon y^2+13y^2-56\varepsilon y-12y+60\varepsilon+13)I_6(-s)^{3\varepsilon}]/[2y(y^2-y+1)(4\varepsilon+1)] + [(-s)^{3\varepsilon}(\varepsilon-1)\varepsilon^2(1216\varepsilon^2y^7+300\varepsilon y^7-224\varepsilon^2y^6-72\varepsilon y^6-5376\varepsilon^2y^5-1324\varepsilon y^5+5y^5+6816\varepsilon^2y^4+1876\varepsilon y^4+40y^4+96\varepsilon^2y^3-148\varepsilon y^3-40y^3-3840\varepsilon^2y^2-684\varepsilon y^2+69y^2+1760\varepsilon^2y+456\varepsilon y-448\varepsilon^2-108\varepsilon)I_2]/[12(y-1)^2y(y+1)(y^2-y+1)(4\varepsilon+1)] + [(-s)^{3\varepsilon}s\varepsilon^2(1296\varepsilon^2y^7+$$

$$
300\varepsilon y^7 - 3672\varepsilon^2 y^6 - 894\varepsilon y^6 + 4272\varepsilon^2 y^5 + 1088\varepsilon y^5 + 5y^5 - 840\varepsilon^2 y^4 - 98\varepsilon y^4 + 40y^4 - 3240\varepsilon^2 y^3 - 922\varepsilon y^3 - 40y^3 + 3888\varepsilon^2 y^2 + 1248\varepsilon y^2 + 69y^2 - 1176\varepsilon^2 y - 318\varepsilon y - 528\varepsilon^2 - 108\varepsilon)I_3]/[72(y-1)^4 y(y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s\varepsilon^2(1880\varepsilon^2 y^7 + 470\varepsilon y^7 - 4936\varepsilon^2 y^6 - 1318\varepsilon y^6 - 15y^6 + 4336\varepsilon^2 y^5 + 744\varepsilon y^5 - 85y^5 + 1760\varepsilon^2 y^4 + 2004\varepsilon y^4 + 385y^4 - 7520\varepsilon^2 y^3 - 4332\varepsilon y^3 - 607y^3 + 7184\varepsilon^2 y^2 + 4208\varepsilon y^2 + 603y^2 - 2152\varepsilon^2 y - 1342\varepsilon y - 207y - 552\varepsilon^2 - 138\varepsilon)I_4]/[24(y-1)^4(y+1)(y^2-y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s^2(\varepsilon-1)\varepsilon^2(12\varepsilon y^5 - 20\varepsilon y^4 - 5y^4 - 172\varepsilon y^3 - 40y^3 + 172\varepsilon y^2 + 40y^2 - 276\varepsilon y - 69y - 12\varepsilon)I_7]/[12(y-1)^2(y+1)(y^2-y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s\varepsilon^3(60\varepsilon^2 y^6 + 13\varepsilon y^6 + 234\varepsilon^2 y^5 + 60\varepsilon y^5 - 486\varepsilon^2 y^4 - 59\varepsilon y^4 + 16y^4 + 520\varepsilon^2 y^3 + 130\varepsilon y^3 - 546\varepsilon^2 y^2 - 71\varepsilon y^2 + 16y^2 + 286\varepsilon^2 y + 70\varepsilon y - 60\varepsilon^2 - 13\varepsilon)I_8]/[y(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s\varepsilon^3(60\varepsilon^2 y^6 + 73\varepsilon y^6 + 13y^6 + 234\varepsilon^2 y^5 + 294\varepsilon y^5 + 60y^5 - 486\varepsilon^2 y^4 - 417\varepsilon y^4 - 75y^4 + 520\varepsilon^2 y^3 + 650\varepsilon y^3 + 130y^3 - 546\varepsilon^2 y^2 - 489\varepsilon y^2 - 87y^2 + 286\varepsilon^2 y + 356\varepsilon y + 70y - 60\varepsilon^2 - 73\varepsilon - 13)I_9]/[y(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; + \; [2(-s)^{3\varepsilon} s\varepsilon^3(2\varepsilon-1)(\varepsilon y^5 + 12\varepsilon y^4 + 6y^4 - \varepsilon y^3 - 6y^3 - 11\varepsilon y^2 + 3y^2 + 24\varepsilon y + 3y - \varepsilon)I_{13}]/[3(y-1)^2 y(y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s\varepsilon^4(5\varepsilon y^8 + 437\varepsilon y^7 + 111y^7 - 330\varepsilon y^6 - 81y^6 + 131\varepsilon y^5 + 33y^5 + 490\varepsilon y^4 + 126y^4 - 13\varepsilon y^3 - 3y^3 - 186\varepsilon y^2 - 45y^2 + 293\varepsilon y + 75y + 5\varepsilon)I_{20}]/[3(y-1)y(y+1)(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s^2\varepsilon^5(6\varepsilon y^6 + 106\varepsilon y^5 + 27y^5 - 114\varepsilon y^4 - 29y^4 + 120\varepsilon y^3 + 30y^3 - 6\varepsilon y^2 - y^2 + 14\varepsilon y + 3y - 6\varepsilon)I_{73}]/[y(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon} s^2(y+1)\varepsilon^3(58\varepsilon y^4 + 13y^4 - 98\varepsilon y^3 - 22y^3 + 160\varepsilon y^2 + 36y^2 - 98\varepsilon y - 22y + 58\varepsilon + 13)I_{10}]/[2(y-1)^3(y^2-y+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon} s^2(\varepsilon-1)\varepsilon^2(12\varepsilon y^5 - 20\varepsilon y^4 - 5y^4 - 172\varepsilon y^3 - 40y^3 + 172\varepsilon y^2 + 40y^2 - 276\varepsilon y - 69y - 12\varepsilon)I_{22}]/[12(y-1)^2(y+1)(y^2-y+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon} s\varepsilon^3(6\varepsilon^2 y^7 - 24\varepsilon^2 y^6 - 14\varepsilon y^6 - 84\varepsilon^2 y^5 + 51\varepsilon y^5 + 3y^5 + 42\varepsilon^2 y^4 - 67\varepsilon y^4 + 9y^4 - 114\varepsilon^2 y^3 + 49\varepsilon y^3 - 9y^3 + 36\varepsilon^2 y^2 - 15\varepsilon y^2 + 9y^2 - 48\varepsilon^2 y - 4\varepsilon y - 6\varepsilon^2)I_{14}]/[3(y-1)^2 y(y+1)(y^2-y+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon} s(y-1)(y+1)\varepsilon^4(2\varepsilon-1)(54\varepsilon y^4 + 13y^4 - 110\varepsilon y^3 - 26y^3 + 168\varepsilon y^2 + 40y^2 - 110\varepsilon y - 26y + 54\varepsilon + 13)I_{36}]/[y(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; - \; [2(-s)^{3\varepsilon} s(y-1)(y+1)\varepsilon^6(6\varepsilon y^4 + 110\varepsilon y^3 + 27y^3 - 108\varepsilon y^2 - 28y^2 + 110\varepsilon y + 27y + 6\varepsilon)I_{72}]/[y(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon} s\varepsilon^4(56\varepsilon y^7 - 11840\varepsilon y^6 - 2953y^6 + 23904\varepsilon y^5 + 5934y^5 - 28688\varepsilon y^4 - 7179y^4 + 21688\varepsilon y^3 + 5415y^3 - 9680\varepsilon y^2 - 2462y^2 + 4952\varepsilon y + 1245y + 56\varepsilon)I_{18}]/[21(y-1)y(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; - \; [2(-s)^{3\varepsilon} s\varepsilon^5(6\varepsilon y^7 + 122\varepsilon y^6 + 30y^6 - 164\varepsilon y^5 - 41y^5 + 232\varepsilon y^4 + 57y^4 - 200\varepsilon y^3 - 49y^3 + 164\varepsilon y^2 + 41y^2 - 90\varepsilon y - 22y - 6\varepsilon)I_{35}]/[y(y+1)(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; - \; [4(-s)^{3\varepsilon} s\varepsilon^5(19\varepsilon y^8 - 391\varepsilon y^7 - 99y^7 + 472\varepsilon y^6 + 120y^6 + 119\varepsilon y^5 + 27y^5 - 374\varepsilon y^4 - 96y^4 + 119\varepsilon y^3 + 27y^3 + 472\varepsilon y^2 + 120y^2 - 391\varepsilon y - 99y + 19\varepsilon)I_{42}]/[3(y-1)y(y+1)(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; ,
$$

$$
\mathbf{F}_{104} = \; [s^2 y\varepsilon^3(60\varepsilon y^2 + 15y^2 - 59\varepsilon y - 15y + \varepsilon)I_{21}(-s)^{3\varepsilon}]/[18(y-1)^4(y+1)(4\varepsilon+1)] \; + \; [s(15y^3 + 48y^2 - 56y + 55)\varepsilon^4 I_{23}(-s)^{3\varepsilon}]/[4(y+1)(y^2-y+1)] \; + \; [s\varepsilon^5(59\varepsilon y - 25y + 3\varepsilon - 81)I_{32}(-s)^{3\varepsilon}]/[56(y+1)(\varepsilon+1)] \; + \; [s(11y^3 + 72y^2 - 78y + 77)\varepsilon^4(2\varepsilon-1)I_{49}(-s)^{3\varepsilon}]/[4(y+1)(y^2-y+1)] \; + \; [s^2\varepsilon^4(2\varepsilon-1)(8\varepsilon y^3 + 2y^3 - 10\varepsilon y^2 - 2y^2 + 6\varepsilon y + y - 2\varepsilon)I_{59}(-s)^{3\varepsilon}]/[2(y-1)^2(y^2-y+1)(4\varepsilon+1)] \; + \; [s\varepsilon^5(2\varepsilon-1)I_{70}(-s)^{3\varepsilon}]/[y^2-y+1] \; + \; [s^2 y^2(y^2-2y+2)\varepsilon^5 I_{71}(-s)^{3\varepsilon}]/[(y-1)^2(y^2-y+1)] \; + \; [2s^2(y+1)\varepsilon^5(16\varepsilon y^2 + 4y^2 - 17\varepsilon y - 4y - \varepsilon)I_{74}(-s)^{3\varepsilon}]/[(y-1)^2(y^2+1)(4\varepsilon+1)] \; + \; [4s(y^2-4y+1)\varepsilon^6(6\varepsilon-1)I_{75}(-s)^{3\varepsilon}]/[3(y-1)^2 y(4\varepsilon+1)] \; + \; [2s^2(y+1)^2\varepsilon^5(6\varepsilon-1)I_{76}(-s)^{3\varepsilon}]/[3(y-1)^4(4\varepsilon+1)] \; + \; [2s^2(11y^2 - 14y + 11)\varepsilon^6 I_{77}(-s)^{3\varepsilon}]/[3(y-1)^4(4\varepsilon+1)] \; + \; [s^2(y+1)\varepsilon^5(12\varepsilon y^2 + 3y^2 - 10\varepsilon y - 3y + 2\varepsilon)I_{78}(-s)^{3\varepsilon}]/[6(y-1)^2 y(4\varepsilon+1)] \; + \; [s(y-1)(17y^2-5)(\varepsilon-1)\varepsilon^5 I_{81}(-s)^{3\varepsilon}]/[4y(y+1)] \; - \; [1]/[2] \; s\varepsilon^5(2\varepsilon-1)I_{84}(-s)^{3\varepsilon} - \; [53]/[28] \; s\varepsilon^5(2\varepsilon-1)I_{86}(-s)^{3\varepsilon} + \; [s^2(21y^2 + 11y + 21)\varepsilon^5 I_{87}(-s)^{3\varepsilon}]/[28(y-1)^2] \; + \; [s^2 y\varepsilon^6 I_{92}(-s)^{3\varepsilon}]/[(y-1)^2] \; - 2s\varepsilon^6 I_{93}(-s)^{3\varepsilon} + \; [3s^2 y\varepsilon^5(2\varepsilon+1)I_{94}(-s)^{3\varepsilon}]/[(y-1)(y+1)] \; + \; [s^2\varepsilon^6 I_{103}(-s)^{3\varepsilon}]/[(y-1)^2] \; + \; [s^2 y\varepsilon^6 I_{104}(-s)^{3\varepsilon}]/[(y-1)^2] \; + \; [s^3 y^2\varepsilon^6 I_{106}(-s)^{3\varepsilon}]/[(y-1)^4] \; - \; [s^2(y+1)\varepsilon^5 I_{85}(-s)^{3\varepsilon}]/[2(y-1)] \; - \; [s^2(y^2-y+2)\varepsilon^6 I_{89}(-s)^{3\varepsilon}]/[(y-1)^2] \; - \; [s\varepsilon^5(4\varepsilon y - y - 8\varepsilon + 1)I_{83}(-s)^{3\varepsilon}]/[y+1] \; - \; [24sy\varepsilon^6 I_{95}(-s)^{3\varepsilon}]/[y+1] \; - \; [s(y-1)y\varepsilon^6 I_{62}(-s)^{3\varepsilon}]/[y^2-y+1] \; - \; [s(25y^2 - 25y + 17)\varepsilon^5 I_{47}(-s)^{3\varepsilon}]/[12(y^2-y+1)] \; - \; [s(2y^3 + 12y^2 - 13y + 13)\varepsilon^4(2\varepsilon-1)I_{44}(-s)^{3\varepsilon}]/[(y+1)(y^2-y+1)] \; - \; [s(24y^3 + 12y^2 - 17y + 31)\varepsilon^6 I_{65}(-s)^{3\varepsilon}]/[(y+1)(y^2-y+1)] \; - \; [s(79y^3 - 96y^2 + 92y - 21)\varepsilon^5 I_{48}(-s)^{3\varepsilon}]/[2(y+1)(y^2-y+1)] \; - \; [s^2 y(2y^3 + 6y^2 - 7y + 7)\varepsilon^4(\varepsilon+1)I_{66}(-s)^{3\varepsilon}]/[(y-1)^2(y+1)(y^2-y+1)] \; - \; [s(34y^5 - 105y^4 + 34y^3 + 18y^2 - 73y + 10)\varepsilon^5 I_{46}(-s)^{3\varepsilon}]/[4y(y+1)(y^2-y+1)] \; - \; [3s\varepsilon^6 I_{33}(-s)^{3\varepsilon}]/[\varepsilon+1] \; - \; [s\varepsilon^4 I_5(-s)^{3\varepsilon}]/[2(\varepsilon+1)] \; - \; [sy(\varepsilon-1)\varepsilon^3(2\varepsilon+1)I_{12}(-s)^{3\varepsilon}]/[(y-1)^2(\varepsilon+1)] \; - \; [s^2 y\varepsilon^2(541\varepsilon y^2 + 513y^2 - 1061\varepsilon y - 1033y + 541\varepsilon + 513)I_1(-s)^{3\varepsilon}]/[56(y-1)^2(y^2-y+1)(\varepsilon+1)] \; - \; [4s^2(y+1)\varepsilon^4(48\varepsilon y^2 + 12y^2 - 47\varepsilon y - 12y + \varepsilon)I_{43}(-s)^{3\varepsilon}]/[3(y-1)^2(y^2+1)(4\varepsilon+1)] \; - \; [s^2\varepsilon^3(24\varepsilon y^4 + 6y^4 - 8\varepsilon y^3 - 2y^3 - 36\varepsilon y^2 - 7y^2 + 56\varepsilon y + 12y - 44\varepsilon - 9)I_6(-s)^{3\varepsilon}]/[4y(y^2-y+1)(4\varepsilon+1)] \; - \; [2s\varepsilon^5(8y\varepsilon^2 - \varepsilon^2 + 6y\varepsilon - \varepsilon + y)I_{54}(-s)^{3\varepsilon}]/[y(\varepsilon+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s\varepsilon^2(7844\varepsilon^3 y^9 + 9805\varepsilon^2 y^9 + 1961\varepsilon y^9 - 36516\varepsilon^3 y^8 - 42873\varepsilon^2 y^8 - 8352\varepsilon y^8 + 21y^8 + 69308\varepsilon^3 y^7 + 80559\varepsilon^2 y^7 + 21492\varepsilon y^7 + 1421y^7 - 48564\varepsilon^3 y^6 - 75097\varepsilon^2 y^6 - 47981\varepsilon y^6 - 8092y^6 - 48744\varepsilon^3 y^5 + 4198\varepsilon^2 y^5 + 78380\varepsilon y^5 + 18634y^5 + 152744\varepsilon^3 y^4 + 87442\varepsilon^2 y^4 - 82144\varepsilon y^4 - 23646y^4 - 172884\varepsilon^3 y^3 - 126561\varepsilon^2 y^3 + 50859\varepsilon y^3 + 17892y^3 + 106044\varepsilon^3 y^2 + 90079\varepsilon^2 y^2 - 14740\varepsilon y^2 - 7595y^2 - 32932\varepsilon^3 y - 32849\varepsilon^2 y - 568\varepsilon y + 1365y + 3700\varepsilon^3 + 4625\varepsilon^2 + 925\varepsilon)I_4]/[336(y-1)^6(y+1)(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s^2(\varepsilon-1)\varepsilon^2(4\varepsilon^2 y^6 + 5\varepsilon y^6 + y^6 + 316\varepsilon^2 y^5 + 350\varepsilon y^5 + 70y^5 - 868\varepsilon^2 y^4 - 1091\varepsilon y^4 - 223y^4 + 1176\varepsilon^2 y^3 + 1473\varepsilon y^3 + 297y^3 - 804\varepsilon^2 y^2 - 1050\varepsilon y^2 - 210y^2 + 284\varepsilon^2 y + 349\varepsilon y + 65y - 12\varepsilon^2 - 12\varepsilon)I_7]/[24(y-1)^4(y+1)(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s\varepsilon^3(336\varepsilon^3 y^8 + 420\varepsilon^2 y^8 + 84\varepsilon y^8 + 2212\varepsilon^3 y^7 + 2709\varepsilon^2 y^7 + 539\varepsilon y^7 - 12756\varepsilon^3 y^6 - 14597\varepsilon^2 y^6 - 1791\varepsilon y^6 + 260y^6 + 23720\varepsilon^3 y^5 + 27101\varepsilon^2 y^5 + 3155\varepsilon y^5 - 520y^5 - 27500\varepsilon^3 y^4 - 31616\varepsilon^2 y^4 - 4016\varepsilon y^4 + 520y^4 + 25092\varepsilon^3 y^3 + 28753\varepsilon^2 y^3 + 3435\varepsilon y^3 - 520y^3 - 15024\varepsilon^3 y^2 - 17341\varepsilon^2 y^2 - 2267\varepsilon y^2 + 260y^2 + 4536\varepsilon^3 y + 5537\varepsilon^2 y + 1043\varepsilon y - 616\varepsilon^3 - 742\varepsilon^2 - 126\varepsilon)I_8]/[28(y-1)^2 y(y^2+1)(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s\varepsilon^3(336\varepsilon^3 y^8 + 756\varepsilon^2 y^8 + 504\varepsilon y^8 + 84y^8 + 2212\varepsilon^3 y^7 + 4921\varepsilon^2 y^7 + 3248\varepsilon y^7 + 539y^7 - 12756\varepsilon^3 y^6 - 25049\varepsilon^2 y^6 - 14884\varepsilon y^6 - 2339y^6 + 23720\varepsilon^3 y^5 + 46437\varepsilon^2 y^5 + 27192\varepsilon y^5 + 4223y^5 - 27500\varepsilon^3 y^4 - 54508\varepsilon^2 y^4 - 32624\varepsilon y^4 - 5112y^4 + 25092\varepsilon^3 y^3 + 49461\varepsilon^2 y^3 + 29124\varepsilon y^3 + 4503y^3 - 15024\varepsilon^3 y^2 - 30061\varepsilon^2 y^2 - 18104\varepsilon y^2 - 2815y^2 + 4536\varepsilon^3 y + 10073\varepsilon^2 y + 6580\varepsilon y + 1043y - 616\varepsilon^3 - 1358\varepsilon^2 - 868\varepsilon - 126)I_9]/[28(y-1)^2 y(y^2+1)(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s\varepsilon^3(2\varepsilon-1)(84\varepsilon^2 y^5 + 81\varepsilon y^5 + 15y^5 - 62\varepsilon^2 y^4 - 101\varepsilon y^4 - 21y^4 + 10\varepsilon^2 y^3 + 7\varepsilon y^3 - 3y^3 - 60\varepsilon^2 y^2 - 93\varepsilon y^2 - 15y^2 + 94\varepsilon^2 y + 88\varepsilon y + 12y - 2\varepsilon^2 - 2\varepsilon)I_{13}]/[6(y-1)^2 y(y+1)^2(\varepsilon+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s\varepsilon^3(84\varepsilon^3 y^7 + 105\varepsilon^2 y^7 + 21\varepsilon y^7 - 162\varepsilon^3 y^6 - 312\varepsilon^2 y^6 - 33\varepsilon y^6 + 9y^6 + 426\varepsilon^3 y^5 + 319\varepsilon^2 y^5 - 14\varepsilon y^5 - 15y^5 - 450\varepsilon^3 y^4 - 428\varepsilon^2 y^4 + 43\varepsilon y^4 + 21y^4 + 444\varepsilon^3 y^3 + 312\varepsilon^2 y^3 - 33\varepsilon y^3 - 9y^3 - 210\varepsilon^3 y^2 - 332\varepsilon^2 y^2 - 8\varepsilon y^2 + 6y^2 + 102\varepsilon^3 y + 118\varepsilon^2 y + 16\varepsilon y + 6\varepsilon^3 + 6\varepsilon^2)I_{14}]/[6(y-1)^2 y(y+1)^2(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s\varepsilon^4(14852\varepsilon^2 y^7 + 18901\varepsilon y^7 + 3797y^7 - 45620\varepsilon^2 y^6 - 56717\varepsilon y^6 - 11349y^6 + 67428\varepsilon^2 y^5 + 84663\varepsilon y^5 + 16983y^5 - 67216\varepsilon^2 y^4 - 83474\varepsilon y^4 - 16762y^4 + 43180\varepsilon^2 y^3 + 54423\varepsilon y^3 + 10991y^3 - 21484\varepsilon^2 y^2 - 26645\varepsilon y^2 - 5413y^2 + 6844\varepsilon^2 y + 8933\varepsilon y + 1837y + 112\varepsilon^2 + 112\varepsilon)I_{18}]/[84(y-1)^2 y(y^2+1)(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s\varepsilon^4(84\varepsilon y^8 + 21y^8 - 610\varepsilon y^7 - 150y^7 + 1440\varepsilon y^6 + 354y^6 - 2112\varepsilon y^5 - 525y^5 + 3386\varepsilon y^4 + 843y^4 - 3994\varepsilon y^3 - 1002y^3 + 3180\varepsilon y^2 + 798y^2 - 1332\varepsilon y - 339y - 10\varepsilon)I_{20}]/[12(y-1)^2 y(y+1)(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s^2 y\varepsilon^3(2\varepsilon-1)(24\varepsilon y^5 + 6y^5 - 74\varepsilon y^4 - 18y^4 + 106\varepsilon y^3 + 21y^3 - 72\varepsilon y^2 - 12y^2 + 34\varepsilon y + 3y - 2\varepsilon)I_{31}]/[6(y-1)^4(y+1)^2(y^2-y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s\varepsilon^5(156\varepsilon^2 y^3 + 27\varepsilon y^3 - 3y^3 - 84\varepsilon^2 y^2 - 84\varepsilon y^2 - 320\varepsilon^2 y - 561\varepsilon y - 115y - 84\varepsilon^2 - 84\varepsilon)I_{34}]/[28y(y^2+1)(\varepsilon+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon} s\varepsilon^5(132\varepsilon y^6 + 33y^6 - 170\varepsilon y^5 - 44y^5 + 338\varepsilon y^4 + 84y^4 - 218\varepsilon y^3 - 56y^3 + 178\varepsilon y^2 + 43y^2 - 14\varepsilon y - 4y + 6\varepsilon)I_{35}]/[y(y+1)(y^2+
$$

$1)(y^2-y+1)(4\varepsilon+1)] + [(-s)^{3\varepsilon}s^2\varepsilon^3(2\varepsilon-1)(24\varepsilon y^4+6y^4-8\varepsilon y^3-2y^3-36\varepsilon y^2-7y^2+56\varepsilon y+12y-44\varepsilon-9)I_{37}]/[2(y-1)^2(y^2-y+1)(4\varepsilon+1)] + [(-s)^{3\varepsilon}s\varepsilon^5(1236\varepsilon y^8+309y^8-1820\varepsilon y^7-474y^7-896\varepsilon y^6-210y^6+4788\varepsilon y^5+1191y^5-6208\varepsilon y^4-1557y^4+3836\varepsilon y^3+954y^3-1056\varepsilon y^2-270y^2+172\varepsilon y+57y+76\varepsilon)I_{42}]/[6(y-1)^2y(y+1)(y^2+1)(y^2-y+1)(4\varepsilon+1)] + [(-s)^{3\varepsilon}s^2\varepsilon^5(148\varepsilon y^7+37y^7-454\varepsilon y^6-112y^6+702\varepsilon y^5+172y^5-738\varepsilon y^4-180y^4+524\varepsilon y^3+126y^3-290\varepsilon y^2-68y^2+106\varepsilon y+23y-6\varepsilon)I_{73}]/[2(y-1)^2y(y^2+1)(y^2-y+1)(4\varepsilon+1)] - [(-s)^{3\varepsilon}s\varepsilon^6(132\varepsilon y^5+33y^5-138\varepsilon y^4-33y^4+138\varepsilon y^3+35y^3-8\varepsilon y^2-y^2+6\varepsilon y+2y-6\varepsilon)I_{72}]/[y(y^2+1)(y^2-y+1)(4\varepsilon+1)] - [(-s)^{3\varepsilon}s\varepsilon^4(2\varepsilon-1)(24\varepsilon y^6+6y^6+80\varepsilon y^5+20y^5-94\varepsilon y^4-23y^4+286\varepsilon y^3+70y^3-224\varepsilon y^2-54y^2+206\varepsilon y+50y-38\varepsilon-- [(-s)^{3\varepsilon}s\varepsilon^4(119\varepsilon y^5+119y^5-766\varepsilon y^4-780y^4+829\varepsilon y^3+829y^3+241\varepsilon y^2+241y^2-430\varepsilon y-444y+35\varepsilon+35)I_{19}]/[14(y-1)^2y(y+1)(\varepsilon+1)] - [(-s)^{3\varepsilon}s^2\varepsilon^3(24\varepsilon y^6+6y^6-32\varepsilon y^5-8y^5+14\varepsilon y^4+5y^4+50\varepsilon y^3+10y^3-104\varepsilon y^2-22y^2+82\varepsilon y+18y-42\varepsilon-9)I_{10}]/[4(y-1)^4(y^2-y+1)(4\varepsilon+1)]\ \ 9)I_{36}]/[2y(y^2+1)(y^2-y+1)(4\varepsilon+1)] - [(-s)^{3\varepsilon}s\varepsilon^4(24\varepsilon y^3+6y^3+72\varepsilon^3y^2-58\varepsilon^2y^2-49\varepsilon y^2-9y^2-8\varepsilon^2y+16\varepsilon y+6y+8\varepsilon^2+8\varepsilon)I_{51}]/[6(y-1)^2y(\varepsilon+1)(4\varepsilon+1)] - [(-s)^{3\varepsilon}s(\varepsilon-1)\varepsilon^4(-48\varepsilon y^3-12y^3+68\varepsilon^2y^2+89\varepsilon y^2+21y^2+16\varepsilon^2y-32\varepsilon y-12y-16\varepsilon^2-16\varepsilon)I_{52}]/[6(y-1)^2y(\varepsilon+1)(4\varepsilon+1)] - [(-s)^{3\varepsilon}s\varepsilon^5(144\varepsilon^2y^3+132\varepsilon y^3+24y^3-260\varepsilon^2y^2-257\varepsilon y^2-51y^2+128\varepsilon^2y+116\varepsilon y+24y+16\varepsilon^2+16\varepsilon)I_{53}]/[6(y-1)^2y(\varepsilon+1)(4\varepsilon+1)] - [(-s)^{3\varepsilon}s\varepsilon^2(3696\varepsilon^3y^9+4620\varepsilon^2y^9+924\varepsilon y^9-30708\varepsilon^3y^8-39057\varepsilon^2y^8-7845\varepsilon y^8+90576\varepsilon^3y^7+117784\varepsilon^2y^7+23687\varepsilon y^7+7y^7-135864\varepsilon^3y^6-175094\varepsilon^2y^6-32944\varepsilon y^6+490y^6+103884\varepsilon^3y^5+126971\varepsilon^2y^5+18502\varepsilon y^5-1561y^5-11028\varepsilon^3y^4-1101\varepsilon^2y^4+8982\varepsilon y^4+2079y^4-57576\varepsilon^3y^3-85242\varepsilon^2y^3-23340\varepsilon y^3-1470y^3+59328\varepsilon^3y^2+80012\varepsilon^2y^2+17611\varepsilon y^2+455y^2-27684\varepsilon^3y-34773\varepsilon^2y-6585\varepsilon y+5376\varepsilon^3+6552\varepsilon^2+1176\varepsilon)I_3]/[1008(y-1)^6y(y+1)(\varepsilon+1)(4\varepsilon+1)] - [(-s)^{3\varepsilon}s^2(\varepsilon-1)\varepsilon^2(4\varepsilon^2y^6+5\varepsilon y^6+y^6+316\varepsilon^2y^5+350\varepsilon y^5+70y^5-868\varepsilon^2y^4-1091\varepsilon y^4-223y^4+1176\varepsilon^2y^3+1473\varepsilon y^3+297y^3-804\varepsilon^2y^2-1050\varepsilon y^2-210y^2+284\varepsilon^2y+349\varepsilon y+65y-12\varepsilon^2-12\varepsilon)I_{22}]/[24(y-1)^4(y+1)(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] - [(-s)^{3\varepsilon}(\varepsilon-1)\varepsilon^2(3696\varepsilon^3y^9+4620\varepsilon^2y^9+924\varepsilon y^9-54300\varepsilon^3y^8-67539\varepsilon^2y^8-13491\varepsilon y^8+223936\varepsilon^3y^7+278576\varepsilon^2y^7+55655\varepsilon y^7+7y^7-430792\varepsilon^3y^6-534234\varepsilon^2y^6-104716\varepsilon y^6+490y^6+421764\varepsilon^3y^5+520121\varepsilon^2y^5+97552\varepsilon y^5-1561y^5-165228\varepsilon^3y^4-199311\varepsilon^2y^4-31248\varepsilon y^4+2079y^4-53352\varepsilon^3y^3-70302\varepsilon^2y^3-20184\varepsilon y^3-1470y^3+81024\varepsilon^3y^2+101924\varepsilon^2y^2+22363\varepsilon y^2+455y^2-31564\varepsilon^3y-39175\varepsilon^2y-7863\varepsilon y+4816\varepsilon^3+5992\varepsilon^2+1176\varepsilon)I_2]/[168(y-1)^4y(y+1)(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)]$,

$\mathbf{F}_{105} = [s^2y\varepsilon^2(219\varepsilon y^2+191y^2-417\varepsilon y-389y+219\varepsilon+191)I_1(-s)^{3\varepsilon}]/[28(y-1)^2(y^2-y+1)(\varepsilon+1)] + [s\varepsilon^3(3\varepsilon y^2+y^2-4\varepsilon y-2y+3\varepsilon+1)I_5(-s)^{3\varepsilon}]/[2(y^2-y+1)(\varepsilon+1)] + [s^2\varepsilon^3(44\varepsilon y^4+11y^4-96\varepsilon y^2-20y^2+128\varepsilon y+28y-92\varepsilon-19)I_6(-s)^{3\varepsilon}]/[4y(y^2-y+1)(4\varepsilon+1)] + [2sy(\varepsilon-1)\varepsilon^3(2\varepsilon+1)I_{12}(-s)^{3\varepsilon}]/[(y-1)^2(\varepsilon+1)] + [sy\varepsilon^3(2\varepsilon-1)^2I_{24}(-s)^{3\varepsilon}]/[2(y-1)^2(\varepsilon+1)] + [6s\varepsilon^6I_{33}(-s)^{3\varepsilon}]/[\varepsilon+1] + [8s^2(y+1)\varepsilon^4(48\varepsilon y^2+12y^2-47\varepsilon y-12y+\varepsilon)I_{43}(-s)^{3\varepsilon}]/[3(y-1)^2(y^2+1)(4\varepsilon+1)] + [s(5y^3+23y^2-27y+27)\varepsilon^4(2\varepsilon-1)I_{44}(-s)^{3\varepsilon}]/[(y+1)(y^2-y+1)] + [s(15y^5-50y^4+17y^3+9y^2-34y+3)\varepsilon^5I_{46}(-s)^{3\varepsilon}]/[y(y+1)(y^2-y+1)] + [s(33y^2-37y+25)\varepsilon^5I_{47}(-s)^{3\varepsilon}]/[6(y^2-y+1)] + [s(81y^3-98y^2+90y-19)\varepsilon^5I_{48}(-s)^{3\varepsilon}]/[(y+1)(y^2-y+1)] + [s(4y^2-5y+2)\varepsilon^6I_{62}(-s)^{3\varepsilon}]/[y^2-y+1] + [s(53y^3+19y^2-39y+67)\varepsilon^6I_{65}(-s)^{3\varepsilon}]/[(y+1)(y^2-y+1)] + [s^2y(5y^3+11y^2-15y+15)\varepsilon^4(\varepsilon+1)I_{66}(-s)^{3\varepsilon}]/[(y-1)^2(y+1)(y^2-y+1)] + [s(y^2-2y-1)\varepsilon^5(2\varepsilon-1)I_{70}(-s)^{3\varepsilon}]/[y^2-y+1] + [4s\varepsilon^5(5\varepsilon y^2-y^2-8\varepsilon y+y-\varepsilon)I_{83}(-s)^{3\varepsilon}]/[(y+1)^2] +s\varepsilon^5(2\varepsilon-1)I_{84}(-s)^{3\varepsilon}+ [2s^2y\varepsilon^5I_{85}(-s)^{3\varepsilon}]/[y-1] + [53]/[14]\ s\varepsilon^5(2\varepsilon-1)I_{86}(-s)^{3\varepsilon}+ [2s^2(y^2-y+2)\varepsilon^6I_{89}(-s)^{3\varepsilon}]/[(y-1)^2] +4s\varepsilon^6I_{93}(-s)^{3\varepsilon}+ [s^2\varepsilon^5(\varepsilon y^2-12\varepsilon y-6y-\varepsilon)I_{94}(-s)^{3\varepsilon}]/[(y-1)(y+1)] + [48sy\varepsilon^6I_{95}(-s)^{3\varepsilon}]/[y+1] +s^2\varepsilon^6I_{105}(-s)^{3\varepsilon}- [2s^2y\varepsilon^6I_{92}(-s)^{3\varepsilon}]/[(y-1)^2] - [2s^2\varepsilon^6I_{103}(-s)^{3\varepsilon}]/[(y-1)^2] - [s^2(y^2+1)\varepsilon^6I_{104}(-s)^{3\varepsilon}]/[(y-1)^2] - [s^2(21y^2+11y+21)\varepsilon^5I_{87}(-s)^{3\varepsilon}]/[14(y-1)^2] - [s^3y(y^2+1)\varepsilon^6I_{106}(-s)^{3\varepsilon}]/[(y-1)^4] - [3s(y-1)(5y^2-1)(\varepsilon-1)\varepsilon^5I_{81}(-s)^{3\varepsilon}]/[2y(y+1)] - [s^2(4y^4-11y^3+14y^2-7y+2)\varepsilon^5I_{71}(-s)^{3\varepsilon}]/[(y-1)^2(y^2-y+1)] - [s(14y^3+69y^2-81y+80)\varepsilon^4(2\varepsilon-1)I_{49}(-s)^{3\varepsilon}]/[2(y+1)(y^2-y+1)] - [s(23y^5-125y^3+227y^2-184y+63)\varepsilon^4I_{23}(-s)^{3\varepsilon}]/[2(y-1)^2(y+1)(y^2-y+1)] - [s\varepsilon^5(87\varepsilon y^2+3y^2+6\varepsilon y-162y+31\varepsilon-53)I_{32}(-s)^{3\varepsilon}]/[28(y+1)^2(\varepsilon+1)] - [4s^2(y+1)^2\varepsilon^5(6\varepsilon-1)I_{76}(-s)^{3\varepsilon}]/[3(y-1)^4(4\varepsilon+1)] - [4s^2(11y^2-14y+11)\varepsilon^6I_{77}(-s)^{3\varepsilon}]/[3(y-1)^4(4\varepsilon+1)] - [8s(y^2-4y+1)\varepsilon^6(6\varepsilon-1)I_{75}(-s)^{3\varepsilon}]/[3(y-1)^2y(4\varepsilon+1)] - [s^2(y+1)\varepsilon^5(12\varepsilon y^2+3y^2-10\varepsilon y-3y+2\varepsilon)I_{78}(-s)^{3\varepsilon}]/[3(y-1)^2y(4\varepsilon+1)] - [s^2y\varepsilon^3(72\varepsilon y^3+18y^3-23\varepsilon y^2-6y^2-46\varepsilon y-12y+\varepsilon)I_{21}(-s)^{3\varepsilon}]/[9(y-1)^4(y+1)^2(4\varepsilon+1)] - [4s^2(y+1)\varepsilon^5(16\varepsilon y^2+4y^2-17\varepsilon y-4y-\varepsilon)I_{74}(-s)^{3\varepsilon}]/[(y-1)^2(y^2+1)(4\varepsilon+1)] - [s^2\varepsilon^3(2\varepsilon-1)(44\varepsilon y^4+11y^4-96\varepsilon y^2-20y^2+128\varepsilon y+28y-92\varepsilon-19)I_{37}(-s)^{3\varepsilon}]/[2(y-1)^2(y^2-y+1)(4\varepsilon+1)] - [s^2\varepsilon^4(2\varepsilon-1)(20\varepsilon y^3+5y^3-28\varepsilon y^2-6y^2+16\varepsilon y+3y-4\varepsilon)I_{59}(-s)^{3\varepsilon}]/[2(y-1)^2(y^2-y+1)(4\varepsilon+1)] + [(-s)^{3\varepsilon}(\varepsilon-1)\varepsilon^2(3192\varepsilon^3y^{10}+3990\varepsilon^2y^{10}+798\varepsilon y^{10}-36828\varepsilon^3y^9-45699\varepsilon^2y^9-9123\varepsilon y^9+122764\varepsilon^3y^8+154687\varepsilon^2y^8+33246\varepsilon y^8+567y^8-177288\varepsilon^3y^7-221050\varepsilon^2y^7-44609\varepsilon y^7-91y^7+86732\varepsilon^3y^6+103347\varepsilon^2y^6+13976\varepsilon y^6-1631y^6+73080\varepsilon^3y^5+96194\varepsilon^2y^5+26320\varepsilon y^5+1694y^5-122820\varepsilon^3y^4-152153\varepsilon^2y^4-30292\varepsilon y^4+49y^4+57240\varepsilon^3y^3+66230\varepsilon^2y^3+6631\varepsilon y^3-1603y^3+2588\varepsilon^3y^2+6399\varepsilon^2y^2+5582\varepsilon y^2+1015y^2-12972\varepsilon^3y-15963\varepsilon^2y-3243\varepsilon y+4312\varepsilon^3+5362\varepsilon^2+1050\varepsilon)I_2]/[84(y-1)^4y(y+1)^2(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] + [(-s)^{3\varepsilon}s\varepsilon^2(3192\varepsilon^3y^{10}+3990\varepsilon^2y^{10}+798\varepsilon y^{10}-38604\varepsilon^3y^9-48927\varepsilon^2y^9-9819\varepsilon y^9+99684\varepsilon^3y^8+130737\varepsilon^2y^8+28596\varepsilon y^8+567y^8-79560\varepsilon^3y^7-102502\varepsilon^2y^7-20765\varepsilon y^7-91y^7-55164\varepsilon^3y^6-79343\varepsilon^2y^6-23038\varepsilon y^6-1631y^6+152328\varepsilon^3y^5+204914\varepsilon^2y^5+48232\varepsilon y^5+1694y^5-91788\varepsilon^3y^4-117563\varepsilon^2y^4-22954\varepsilon y^4+49y^4-32520\varepsilon^3y^3-50422\varepsilon^2y^3-17237\varepsilon y^3-1603y^3+71460\varepsilon^3y^2+97249\varepsilon^2y^2+23780\varepsilon y^2+1015y^2-33900\varepsilon^3y-42711\varepsilon^2y-8307\varepsilon y+4872\varepsilon^3+5922\varepsilon^2+1050\varepsilon)I_3]/[504(y-1)^6y(y+1)^2(\varepsilon+1)(4\varepsilon+1)] + [(-s)^{3\varepsilon}s\varepsilon^2(4812\varepsilon^3y^{10}+6015\varepsilon^2y^{10}+1203\varepsilon y^{10}-50344\varepsilon^3y^9-72422\varepsilon^2y^9-21763\varepsilon y^9-1701y^9+135992\varepsilon^3y^8+196030\varepsilon^2y^8+57476\varepsilon y^8+4242y^8-138344\varepsilon^3y^7-168926\varepsilon^2y^7-23491\varepsilon y^7+2555y^7-3492\varepsilon^3y^6-77837\varepsilon^2y^6-84019\varepsilon y^6-16226y^6+127952\varepsilon^3y^5+244668\varepsilon^2y^5+119712\varepsilon y^5+16604y^5-80660\varepsilon^3y^4-109617\varepsilon^2y^4-22335\varepsilon y^4+70y^4-50760\varepsilon^3y^3-126982\varepsilon^2y^3-86099\varepsilon y^3-14413y^3+95672\varepsilon^3y^2+176486\varepsilon^2y^2+85924\varepsilon y^2+11914y^2-49784\varepsilon^3y-77266\varepsilon^2y-28511\varepsilon y-3045y+8956\varepsilon^3+11195\varepsilon^2+2239\varepsilon)I_4]/[168(y-1)^6(y+1)^2(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] + [(-s)^{3\varepsilon}s^2\varepsilon^3(44\varepsilon y^6+11y^6-32\varepsilon y^5-8y^5-48\varepsilon y^4-9y^4+196\varepsilon y^3+44y^3-284\varepsilon y^2-63y^2+196\varepsilon y+44y-88\varepsilon-19)I_{10}]/[4(y-1)^4(y^2-y+1)(4\varepsilon+1)] + [(-s)^{3\varepsilon}s\varepsilon^4(210\varepsilon y^5+210y^5-1049\varepsilon y^4-1077y^4+1203\varepsilon y^3+1203y^3+27\varepsilon y^2+27y^2-377\varepsilon y-405y+42\varepsilon+42)I_{19}]/[14(y-1)^2y(y+1)(\varepsilon+1)] + [(-s)^{3\varepsilon}s^2(\varepsilon-1)\varepsilon^2(324\varepsilon^2y^6+405\varepsilon y^6+81y^6-340\varepsilon^2y^5-470\varepsilon y^5-94y^5-532\varepsilon^2y^4-671\varepsilon y^4-139y^4+1512\varepsilon^2y^3+1893\varepsilon y^3+381y^3-1460\varepsilon^2y^2-1870\varepsilon y^2-374y^2+604\varepsilon^2y+749\varepsilon y+145y-12\varepsilon^2-12\varepsilon)I_{22}]/[12(y-1)^4(y+1)(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] + [(-s)^{3\varepsilon}s\varepsilon^4(2\varepsilon-1)(44\varepsilon y^6+11y^6+176\varepsilon y^5+44y^5-216\varepsilon y^4-53y^4+604\varepsilon y^3+148y^3-476\varepsilon y^2-115y^2+428\varepsilon y+104y-80\varepsilon-19)I_{36}]/[2y(y^2+1)(y^2-y+1)(4\varepsilon+1)] + [(-s)^{3\varepsilon}s\varepsilon^3(-24\varepsilon^3y^5-6\varepsilon^2y^5+144\varepsilon^4y^4+44\varepsilon^3y^4-28\varepsilon^2y^4+3\varepsilon y^4+3y^4-216\varepsilon^4y^3-106\varepsilon^3y^3+59\varepsilon^2y^3-3\varepsilon y^3-6y^3+144\varepsilon^4y^2+60\varepsilon^3y^2-12\varepsilon^2y^2+3\varepsilon y^2+3y^2-40\varepsilon^3y-22\varepsilon^2y+8\varepsilon^3+8\varepsilon^2)I_{51}]/[3(y-1)^2y(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] + [(-s)^{3\varepsilon}s(\varepsilon-1)\varepsilon^3(48\varepsilon^3y^5+12\varepsilon^2y^5-40\varepsilon^3y^4-10\varepsilon^2y^4-9\varepsilon y^4-3y^4+68\varepsilon^3y^3+5\varepsilon^2y^3+15\varepsilon y^3+6y^3-72\varepsilon^3y^2-42\varepsilon^2y^2-9\varepsilon y^2-3y^2+80\varepsilon^3y+44\varepsilon^2y-16\varepsilon^3-16\varepsilon^2)I_{52}]/[3(y-1)^2y(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] + [(-s)^{3\varepsilon}s\varepsilon^4(96\varepsilon^3y^5+72\varepsilon^2y^5+12\varepsilon y^5-200\varepsilon^3y^4-146\varepsilon^2y^4-39\varepsilon y^4-3y^4+220\varepsilon^3y^3+139\varepsilon^2y^3+51\varepsilon y^3+6y^3-168\varepsilon^3y^2-114\varepsilon^2y^2-39\varepsilon y^2-3y^2+64\varepsilon^3y+40\varepsilon^2y+12\varepsilon y+16\varepsilon^3+16\varepsilon^2)I_{53}]/[3(y-1)^2y(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] + [2(-s)^{3\varepsilon}s\varepsilon^5(20\varepsilon^2y^3+$

$$17\varepsilon y^3+3y^3-26\varepsilon^2y^2-24\varepsilon y^2-4y^2+22\varepsilon^2y+19\varepsilon y+3y-2\varepsilon^2-2\varepsilon)I_{54}]/[y(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon}s\varepsilon^6(260\varepsilon y^5+65y^5-$$
$$268\varepsilon y^4-64y^4+268\varepsilon y^3+68y^3-8\varepsilon y^2+8\varepsilon y+3y-12\varepsilon)I_{72}]/[y(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon}s^2y\varepsilon^3(2\varepsilon-1)(60\varepsilon y^5+15y^5-196\varepsilon y^4-$$
$$48y^4+284\varepsilon y^3+60y^3-192\varepsilon y^2-36y^2+80\varepsilon y+9y-4\varepsilon)I_{31}]/[6(y-1)^4(y+1)^2(y^2-y+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon}s^2\varepsilon^5(180\varepsilon y^7+45y^7-552\varepsilon y^6-$$
$$135y^6+808\varepsilon y^5+195y^5-764\varepsilon y^4-182y^4+452\varepsilon y^3+103y^3-224\varepsilon y^2-47y^2+96\varepsilon y+17y-12\varepsilon)I_{73}]/[2(y-1)^2y(y^2+1)(y^2-y+1)(4\varepsilon+$$
$$1)] \; - \; [(-s)^{3\varepsilon}s\varepsilon^5(1260\varepsilon y^8+315y^8-1892\varepsilon y^7-492y^7-824\varepsilon y^6-192y^6+4764\varepsilon y^5+1185y^5-6232\varepsilon y^4-1563y^4+3908\varepsilon y^3+972y^3-1128\varepsilon y^2-$$
$$288y^2+196\varepsilon y+63y+76\varepsilon)I_{42}]/[3(y-1)^2y(y+1)(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon}s\varepsilon^5(252\varepsilon y^7+63y^7-28\varepsilon y^6-10y^6+252\varepsilon y^5+59y^5+$$
$$336\varepsilon y^4+80y^4-164\varepsilon y^3-47y^3+376\varepsilon y^2+90y^2-28\varepsilon y-11y+12\varepsilon)I_{35}]/[y(y+1)^2(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon}s\varepsilon^4(108\varepsilon y^9+27y^9-$$
$$622\varepsilon y^8-153y^8+1118\varepsilon y^7+276y^7-1152\varepsilon y^6-291y^6+1802\varepsilon y^5+450y^5-1088\varepsilon y^4-279y^4-526\varepsilon y^3-132y^3+1752\varepsilon y^2+435y^2-1318\varepsilon y-333y-$$
$$10\varepsilon)I_{20}]/[6(y-1)^2y(y+1)^2(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon}s\varepsilon^3(2\varepsilon-1)(108\varepsilon^2y^5+111\varepsilon y^5+21y^5-74\varepsilon^2y^4-116\varepsilon y^4-24y^4+34\varepsilon^2y^3+$$
$$37\varepsilon y^3+3y^3-72\varepsilon^2y^2-108\varepsilon y^2-18y^2+118\varepsilon^2y+118\varepsilon y+18y-2\varepsilon^2-2\varepsilon)I_{13}]/[3(y-1)^2y(y+1)^2(\varepsilon+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon}s\varepsilon^5(92\varepsilon^2y^3+$$
$$31\varepsilon y^3+2y^3-42\varepsilon^2y^2-42\varepsilon y^2-146\varepsilon^2y-263\varepsilon y-54y-42\varepsilon^2-42\varepsilon)I_{34}]/[7y(y^2+1)(\varepsilon+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon}s^2(\varepsilon-1)\varepsilon^2(324\varepsilon^2y^6+405\varepsilon y^6+$$
$$81y^6-340\varepsilon^2y^5-470\varepsilon y^5-94y^5-532\varepsilon^2y^4-671\varepsilon y^4-139y^4+1512\varepsilon^2y^3+1893\varepsilon y^3+381y^3-1460\varepsilon^2y^2-1870\varepsilon y^2-374y^2+604\varepsilon^2y+749\varepsilon y+$$
$$145y-12\varepsilon^2-12\varepsilon)I_7]/[12(y-1)^4(y+1)(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon}s\varepsilon^3(168\varepsilon^3y^7+210\varepsilon^2y^7+42\varepsilon y^7-240\varepsilon^3y^6-555\varepsilon^2y^6-90\varepsilon y^6+$$
$$9y^6+1236\varepsilon^3y^5+1046\varepsilon^2y^5-22\varepsilon y^5-48y^5-1452\varepsilon^3y^4-1426\varepsilon^2y^4+98\varepsilon y^4+72y^4+1272\varepsilon^3y^3+1032\varepsilon^2y^3-60\varepsilon y^3-36y^3-336\varepsilon^3y^2-595\varepsilon^2y^2-$$
$$40\varepsilon y^2+3y^2+204\varepsilon^3y+236\varepsilon^2y+32\varepsilon y+12\varepsilon^3+12\varepsilon^2)I_{14}]/[6(y-1)^2y(y+1)^2(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon}s\varepsilon^3(308\varepsilon^3y^8+385\varepsilon^2y^8+$$
$$77\varepsilon y^8-168\varepsilon^3y^7-266\varepsilon^2y^7-56\varepsilon y^7-2984\varepsilon^3y^6-3026\varepsilon^2y^6-153\varepsilon y^6+99y^6+6556\varepsilon^3y^5+6934\varepsilon^2y^5+474\varepsilon y^5-198y^5-7900\varepsilon^3y^4-8404\varepsilon^2y^4-$$
$$726\varepsilon y^4+198y^4+7928\varepsilon^3y^3+8586\varepsilon^2y^3+754\varepsilon y^3-198y^3-5252\varepsilon^3y^2-5770\varepsilon^2y^2-629\varepsilon y^2+99y^2+2156\varepsilon^3y+2562\varepsilon^2y+448\varepsilon y-644\varepsilon^3-$$
$$777\varepsilon^2-133\varepsilon)I_8]/[14(y-1)^2y(y^2+1)(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon}s\varepsilon^3(308\varepsilon^3y^8+693\varepsilon^2y^8+462\varepsilon y^8+77y^8-168\varepsilon^3y^7-434\varepsilon^2y^7-$$
$$322\varepsilon y^7-56y^7-2984\varepsilon^3y^6-4994\varepsilon^2y^6-2641\varepsilon y^6-379y^6+6556\varepsilon^3y^5+11682\varepsilon^2y^5+6276\varepsilon y^5+898y^5-7900\varepsilon^3y^4-14272\varepsilon^2y^4-8054\varepsilon y^4-$$
$$1178y^4+7928\varepsilon^3y^3+14706\varepsilon^2y^3+8208\varepsilon y^3+1178y^3-5252\varepsilon^3y^2-10006\varepsilon^2y^2-5861\varepsilon y^2-855y^2+2156\varepsilon^3y+4718\varepsilon^2y+3010\varepsilon y+448y-644\varepsilon^3-$$
$$1421\varepsilon^2-910\varepsilon-133)I_9]/[14(y-1)^2y(y^2+1)(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon}s\varepsilon^3(1188\varepsilon^3y^7+1821\varepsilon^2y^7+381\varepsilon y^7-5356\varepsilon^3y^6-6219\varepsilon^2y^6-$$
$$1073\varepsilon y^6+42y^6+564\varepsilon^3y^5+747\varepsilon^2y^5-153\varepsilon y^5-84y^5+13312\varepsilon^3y^4+17522\varepsilon^2y^4+3790\varepsilon y^4+84y^4-23684\varepsilon^3y^3-29493\varepsilon^2y^3-6145\varepsilon y^3-84y^3+$$
$$18780\varepsilon^3y^2+23853\varepsilon^2y^2+4863\varepsilon y^2+42y^2-6820\varepsilon^3y-8147\varepsilon^2y-1579\varepsilon y+112\varepsilon^3+112\varepsilon^2)I_{18}]/[42(y-1)^2y(y^2+1)(y^2-y+1)(\varepsilon+1)(4\varepsilon+1)] \; ,$$

$$\mathbf{F}_{106} = \; [3s(y-1)^2\varepsilon^3I_5(-s)^{3\varepsilon}]/[4(y^2-y+1)] \; + \; [s^2y\varepsilon^3(200\varepsilon y^2+51y^2+292\varepsilon y+75y-4\varepsilon)I_{21}(-s)^{3\varepsilon}]/[72(y-1)^3(y+1)^2(4\varepsilon+1)] \; + \; [s(6y^5+$$
$$77y^4-250y^3+326y^2-235y+78)\varepsilon^4I_{23}(-s)^{3\varepsilon}]/[4(y-1)^2(y+1)(y^2-y+1)] \; + \; [s^2y\varepsilon^3(2\varepsilon-1)(2\varepsilon y^4-10\varepsilon y^3+3y^3-6y^2-10\varepsilon y+3y+$$
$$2\varepsilon)I_{31}(-s)^{3\varepsilon}]/[6(y-1)^3(y+1)^2(y^2-y+1)(4\varepsilon+1)] \; + \; [s(y-1)\varepsilon^5I_{32}(-s)^{3\varepsilon}]/[(y+1)^2] \; + \; [s^2\varepsilon^3(2\varepsilon-1)(68\varepsilon y^3+17y^3+8\varepsilon y^2-2y^2+4y+76\varepsilon+$$
$$15)I_{37}(-s)^{3\varepsilon}]/[4(y-1)(y^2-y+1)(4\varepsilon+1)] \; + \; [3s(y^3+21y^2-20y+20)\varepsilon^4(2\varepsilon-1)I_{49}(-s)^{3\varepsilon}]/[2(y+1)(y^2-y+1)] \; + \; [s^2\varepsilon^4(2\varepsilon-1)(2\varepsilon y^2+2\varepsilon y+$$
$$y+2\varepsilon)I_{59}(-s)^{3\varepsilon}]/[2(y-1)(y^2-y+1)(4\varepsilon+1)] \; + \; [s^2(y^2+3y-3)\varepsilon^5I_{71}(-s)^{3\varepsilon}]/[4(y^2-y+1)] \; + \; [s^2(y+1)\varepsilon^5(72\varepsilon y+17y+4\varepsilon)I_{74}(-s)^{3\varepsilon}]/[2(y-$$
$$1)(y^2+1)(4\varepsilon+1)] \; + \; [s^2(y+1)\varepsilon^5(32\varepsilon y+9y-4\varepsilon)I_{78}(-s)^{3\varepsilon}]/[12(y-1)y(4\varepsilon+1)] \; + \; [5s(y-1)(3y^2-1)(\varepsilon-1)\varepsilon^5I_{81}(-s)^{3\varepsilon}]/[2y(y+$$
$$1)] \; + \; [23]/[14] \;\; s^2\varepsilon^5I_{87}(-s)^{3\varepsilon} + \; [s^2\varepsilon^6I_{89}(-s)^{3\varepsilon}]/[y-1] \; + \; [5s^2y\varepsilon^5(2\varepsilon+1)I_{94}(-s)^{3\varepsilon}]/[(y-1)(y+1)] \; - \; [s^2\varepsilon^5I_{85}(-s)^{3\varepsilon}]/[y-$$
$$1] \; - \; [s^2\varepsilon^6I_{103}(-s)^{3\varepsilon}]/[y-1] \; - \; [s^3y\varepsilon^6I_{106}(-s)^{3\varepsilon}]/[(y-1)^2] \; - \; [2s(21y+1)\varepsilon^6I_{95}(-s)^{3\varepsilon}]/[y+1] \; - \; [5s(504y^3-1933y^2-1261y+$$
$$168)\varepsilon^4I_{19}(-s)^{3\varepsilon}]/[168y(y+1)] \; - \; [2s(y-1)\varepsilon^5(6\varepsilon-1)I_{83}(-s)^{3\varepsilon}]/[(y+1)^2] \; - \; [s(y-1)\varepsilon^5(2\varepsilon-1)I_{70}(-s)^{3\varepsilon}]/[y^2-y+1] \; - \; [s(y^2+3y-$$
$$3)\varepsilon^6I_{62}(-s)^{3\varepsilon}]/[4(y^2-y+1)] \; - \; [s(y^2+3y-3)\varepsilon^5I_{47}(-s)^{3\varepsilon}]/[6(y^2-y+1)] \; - \; [351s^2y\varepsilon^2I_1(-s)^{3\varepsilon}]/[28(y^2-y+1)] \; - \; [s(y^3+21y^2-$$
$$20y+20)\varepsilon^4(2\varepsilon-1)I_{44}(-s)^{3\varepsilon}]/[(y+1)(y^2-y+1)] \; - \; [2s(31y^3-39y^2+40y-10)\varepsilon^5I_{48}(-s)^{3\varepsilon}]/[(y+1)(y^2-y+1)] \; - \; [s(23y^3+25y^2-$$
$$20y+38)\varepsilon^6I_{65}(-s)^{3\varepsilon}]/[(y+1)(y^2-y+1)] \; - \; [s^2y(y^3+22y^2-20y+19)\varepsilon^4(\varepsilon+1)I_{66}(-s)^{3\varepsilon}]/[2(y-1)^2(y+1)(y^2-y+1)] \; - \; [s(120y^5-$$
$$331y^4+104y^3+40y^2-235y+40)\varepsilon^5I_{46}(-s)^{3\varepsilon}]/[8y(y+1)(y^2-y+1)] \; - \; [sy\varepsilon^3(2\varepsilon-1)^2I_{24}(-s)^{3\varepsilon}]/[8(y-1)^2(\varepsilon+1)] \; - \; [2s^2(y+1)^2\varepsilon^5(6\varepsilon-$$
$$1)I_{76}(-s)^{3\varepsilon}]/[3(y-1)^3(4\varepsilon+1)] \; - \; [2s^2(11y^2-14y+11)\varepsilon^6I_{77}(-s)^{3\varepsilon}]/[3(y-1)^3(4\varepsilon+1)] \; - \; [4s(y^2-4y+1)\varepsilon^6(6\varepsilon-1)I_{75}(-s)^{3\varepsilon}]/[3(y-$$
$$1)y(4\varepsilon+1)] \; - \; [s^2(y+1)\varepsilon^4(200\varepsilon y+51y-4\varepsilon)I_{43}(-s)^{3\varepsilon}]/[3(y-1)(y^2+1)(4\varepsilon+1)] \; - \; [s\varepsilon^5(48\varepsilon y^3+15y^3+8\varepsilon y^2+120\varepsilon y+32y+$$
$$12\varepsilon)I_{34}(-s)^{3\varepsilon}]/[4y(y^2+1)(4\varepsilon+1)] \; - \; [s\varepsilon^5(2\varepsilon y^3+8\varepsilon y^2+3y^2+4\varepsilon y-2\varepsilon)I_{54}(-s)^{3\varepsilon}]/[y(y^2-y+1)(4\varepsilon+1)] \; - \; [s^2(y-1)\varepsilon^3(68\varepsilon y^3+17y^3+$$
$$8\varepsilon y^2-2y^2+4y+76\varepsilon+15)I_6(-s)^{3\varepsilon}]/[8y(y^2-y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon}s\varepsilon^2(7048\varepsilon^2y^8+1762\varepsilon y^8-16736\varepsilon^2y^7-5636\varepsilon y^7-363y^7+9944\varepsilon^2y^6+$$
$$10470\varepsilon y^6+2020y^6+16768\varepsilon^2y^5-7824\varepsilon y^5-3004y^5-27136\varepsilon^2y^4-4728\varepsilon y^4+490y^4+1600\varepsilon^2y^3+8896\varepsilon y^3+2148y^3+27256\varepsilon^2y^2-4178\varepsilon y^2-$$
$$2748y^2-24672\varepsilon^2y-2148\varepsilon y+981y+5928\varepsilon^2+1482\varepsilon)I_4]/[192(y-1)^4(y+1)^2(y^2-y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon}s\varepsilon^3(476\varepsilon^2y^6+119\varepsilon y^6+3432\varepsilon^2y^5+$$
$$830\varepsilon y^5-9020\varepsilon^2y^4-802\varepsilon y^4+351y^4+7284\varepsilon^2y^3+1751\varepsilon y^3-9524\varepsilon^2y^2-907\varepsilon y^2+351y^2+3880\varepsilon^2y+921\varepsilon y-532\varepsilon^2-105\varepsilon)I_8]/[28y(y^2+1)(y^2-$$
$$y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon}s\varepsilon^3(476\varepsilon^2y^6+595\varepsilon y^6+119y^6+3432\varepsilon^2y^5+4262\varepsilon y^5+830y^5-9020\varepsilon^2y^4-7014\varepsilon y^4-1153y^4+7284\varepsilon^2y^3+9035\varepsilon y^3+$$
$$1751y^3-9524\varepsilon^2y^2-7623\varepsilon y^2-1258y^2+3880\varepsilon^2y+4801\varepsilon y+921y-532\varepsilon^2-637\varepsilon-105)I_9]/[28y(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon}s\varepsilon^3(2\varepsilon-$$
$$1)(92\varepsilon y^5+21y^5-24\varepsilon y^4+18y^4-80\varepsilon y^3-66y^3-16\varepsilon y^2+42y^2+132\varepsilon y+9y-8\varepsilon)I_{13}]/[24(y-1)^2y(y+1)^2(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon}s\varepsilon^3(252\varepsilon^2y^7+$$
$$69\varepsilon y^7-660\varepsilon^2y^6-13\varepsilon y^6+30y^6+60\varepsilon^2y^5-201\varepsilon y^5+6y^5+1104\varepsilon^2y^4+298\varepsilon y^4-108y^4+132\varepsilon^2y^3-301\varepsilon y^3+30y^3-852\varepsilon^2y^2+99\varepsilon y^2+18y^2+$$
$$324\varepsilon^2y+49\varepsilon y+24\varepsilon^2)I_{14}]/[24(y-1)^2y(y+1)^2(y^2-y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon}s\varepsilon^3(10032\varepsilon^2y^6+2522\varepsilon y^6-18944\varepsilon^2y^5-5009\varepsilon y^5-63y^5+$$
$$24692\varepsilon^2y^4+6488\varepsilon y^4+63y^4-20436\varepsilon^2y^3-5417\varepsilon y^3-63y^3+10320\varepsilon^2y^2+2895\varepsilon y^2+63y^2-5832\varepsilon^2y-1479\varepsilon y-56\varepsilon^2)I_{18}]/[42(y-1)y(y^2+$$
$$1)(y^2-y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon}s^2(\varepsilon-1)\varepsilon^2(436\varepsilon y^4+121y^4-2000\varepsilon y^3-512y^3+1920\varepsilon y^2+480y^2-1356\varepsilon y-327y+48\varepsilon)I_{22}]/[96(y-1)^2(y+$$
$$1)(y^2-y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon}s\varepsilon^5(916\varepsilon y^7+235y^7-396\varepsilon y^6-97y^6+1220\varepsilon y^5+305y^5+528\varepsilon y^4+136y^4-132\varepsilon y^3-37y^3+932\varepsilon y^2+233y^2-$$
$$148\varepsilon y-39y+24\varepsilon)I_{35}]/[4y(y+1)^2(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; + \; [2(-s)^{3\varepsilon}s\varepsilon^5(353\varepsilon y^7+93y^7-106\varepsilon y^6-30y^6-534\varepsilon y^5-132y^5+1123\varepsilon y^4+$$
$$282y^4-1097\varepsilon y^3-273y^3+426\varepsilon y^2+108y^2-178\varepsilon y-48y-19\varepsilon)I_{42}]/[3(y-1)y(y+1)(y^2+1)(y^2-y+1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon}s\varepsilon^3(-40\varepsilon^2y^4-$$
$$12\varepsilon y^4+216\varepsilon^3y^3-46\varepsilon^2y^3+15\varepsilon y^3+9y^3-216\varepsilon^3y^2+54\varepsilon^2y^2-15\varepsilon y^2-9y^2+32\varepsilon^2y+12\varepsilon y+8\varepsilon^2)I_{51}]/[6(y-1)y(y^2-y+1)(4\varepsilon+$$
$$1)] \; + \; [(-s)^{3\varepsilon}s(\varepsilon-1)\varepsilon^3(80\varepsilon^2y^4+24\varepsilon y^4-52\varepsilon^2y^3-57\varepsilon y^3-9y^3+36\varepsilon^2y^2+57\varepsilon y^2+9y^2-64\varepsilon^2y-24\varepsilon y-16\varepsilon^2)I_{52}]/[6(y-1)y(y^2-y+$$
$$1)(4\varepsilon+1)] \; + \; [(-s)^{3\varepsilon}s^2\varepsilon^5(836\varepsilon y^5+203y^5-876\varepsilon y^4-211y^4+1424\varepsilon y^3+346y^3-612\varepsilon y^2-143y^2+604\varepsilon y+143y-24\varepsilon)I_{73}]/[8y(y^2+1)(y^2-$$
$$y+1)(4\varepsilon+1)] \; - \; [(-s)^{3\varepsilon}s\varepsilon^2(4128\varepsilon^2y^8+1032\varepsilon y^8-15288\varepsilon^2y^7-3726\varepsilon y^7+11256\varepsilon^2y^6+2234\varepsilon y^6-121y^6+10296\varepsilon^2y^5+4138\varepsilon y^5+391y^5-$$
$$25296\varepsilon^2y^4-6004\varepsilon y^4+32y^4+17304\varepsilon^2y^3+3522\varepsilon y^3-153y^3+1560\varepsilon^2y^2+1698\varepsilon y^2+327y^2-7704\varepsilon^2y-1830\varepsilon y+3744\varepsilon^2+840\varepsilon)I_3]/[576(y-$$

$$
\begin{aligned}
&1)^4y(y+1)^2(4\varepsilon+1)] \;-\; [(-s)^{3\varepsilon}s^2\varepsilon^3(68\varepsilon y^5+17y^5-64\varepsilon y^4-19y^4+96\varepsilon y^3+29y^3+52\varepsilon y^2+5y^2-40\varepsilon y-5y+72\varepsilon+15)I_{10}]/[8(y-1)^3(y^2-y+\\
&1)(4\varepsilon+1)] \;-\; [(-s)^{3\varepsilon}s\varepsilon^4(152\varepsilon^2y^4+42\varepsilon y^4-484\varepsilon^2y^3-93\varepsilon y^3+9y^3+468\varepsilon^2y^2+93\varepsilon y^2-9y^2-136\varepsilon^2y-42\varepsilon y-16\varepsilon^2)I_{53}]/[6(y-1)y(y^2-y+\\
&1)(4\varepsilon+1)] \;-\; [(-s)^{3\varepsilon}s^2(\varepsilon-1)\varepsilon^2(436\varepsilon y^4+121y^4-2000\varepsilon y^3-512y^3+1920\varepsilon y^2+480y^2-1356\varepsilon y-327y+48\varepsilon)I_7]/[96(y-1)^2(y+1)(y^2-y+\\
&1)(4\varepsilon+1)] \;-\; [(-s)^{3\varepsilon}(\varepsilon-1)\varepsilon^2(28896\varepsilon^2y^8+7224\varepsilon y^8-249712\varepsilon^2y^7-62316\varepsilon y^7+435920\varepsilon^2y^6+106040\varepsilon y^6-847y^6+26160\varepsilon^2y^5+17488\varepsilon y^5+\\
&2737y^5-631488\varepsilon^2y^4-157312\varepsilon y^4+224y^4+411216\varepsilon^2y^3+98856\varepsilon y^3-1071y^3+45936\varepsilon^2y^2+20640\varepsilon y^2+2289y^2-90896\varepsilon^2y-23172\varepsilon y+23968\varepsilon^2+\\
&5880\varepsilon)I_2]/[672(y-1)^2y(y+1)^2(y^2-y+1)(4\varepsilon+1)] \;-\; [(-s)^{3\varepsilon}s\varepsilon^6(170\varepsilon y^5+41y^5-172\varepsilon y^4-42y^4+198\varepsilon y^3+49y^3-34\varepsilon y^2-8y^2+36\varepsilon y+8y-\\
&6\varepsilon)I_{72}]/[y(y^2+1)(y^2-y+1)(4\varepsilon+1)] \;-\; [(-s)^{3\varepsilon}s\varepsilon^4(2\varepsilon-1)(68\varepsilon y^6+17y^6+244\varepsilon y^5+60y^5-240\varepsilon y^4-56y^4+764\varepsilon y^3+184y^3-516\varepsilon y^2-122y^2+\\
&512\varepsilon y+124y-64\varepsilon-15)I_{36}]/[4y(y^2+1)(y^2-y+1)(4\varepsilon+1)] \;-\; [(-s)^{3\varepsilon}s\varepsilon^4(184\varepsilon y^8+51y^8+1372\varepsilon y^7+336y^7-816\varepsilon y^6-210y^6+976\varepsilon y^5+\\
&243y^5-1804\varepsilon y^4-465y^4-1700\varepsilon y^3-426y^3+1656\varepsilon y^2+408y^2-3752\varepsilon y-945y-20\varepsilon)I_{20}]/[24(y-1)y(y+1)^2(y^2+1)(y^2-y+1)(4\varepsilon+1)] \;.
\end{aligned}
$$

Appendix C.

QCD-electroweak form factor at NLO

The form factor for mixed QCD-electroweak corrections to Higgs boson gluon fusion from Eq. (2.24) reads

$$
\begin{aligned}
\mathcal{F} = \quad & \mathrm{i}\frac{\alpha_S m_H^2}{6\pi v}\left[A^{(0)}_{\mathrm{QCD}} + \frac{\alpha_S(\mu^2)}{2\pi}\left(\mathbf{I}^{(1)}\left(\frac{\mu^2}{s}\right)A^{(0)}_{\mathrm{QCD}} + A^{(1,\mathrm{fin})}_{\mathrm{QCD}}\left(\frac{\mu^2}{s}\right)\right)\right] + \\
& - \mathrm{i}\frac{\alpha^2\alpha_S(\mu^2)v}{64\pi\sin^4\theta_W}\sum_{V=W,Z} C^V\left[A^{(0)}_{\text{QCD-EW}}(y) + \right. \\
& \left. \qquad + \frac{\alpha_S(\mu^2)}{2\pi}\left(\mathbf{I}^{(1)}\left(\frac{\mu^2}{s}\right)A^{(0)}_{\text{QCD-EW}}(y) + A^{(1,\mathrm{fin})}_{\text{QCD-EW}}\left(y, \frac{\mu^2}{s}\right)\right)\right].
\end{aligned}
\tag{C.1}
$$

See also the ancillary file of [2] for `Mathematica`-readable expressions.

$A^{(0)}_{\text{QCD-EW}}$

$A^{(0)}_{\text{QCD-EW}}(y) = \frac{-4}{3(y-1)^3}[-2\pi^2G(1;y)y^3 - 12G^2(1;y)G(r;y)y^3 - 6G(r;y)y^3 - 24G(1;y)G(0,1;y)y^3 + 12G(1;y)G(0,r;y)y^3 + 24G(1;y)G(r,1;y)y^3 + 18G(0,-1,r;y)y^3 + 18G(0,0,-1;y)y^3 + 24G(0,0,1;y)y^3 + 6G(0,0,r;y)y^3 + 24G(0,1,1;y)y^3 + 12G(0,1,r;y)y^3 + 18G(0,r,-1;y)y^3 - 12G(0,r,1;y)y^3 - 24G(r,1,1;y)y^3 + 6\zeta(3)y^3 + 12y^3 + (y+1)G^3(0;y)y^2 - \pi^2y^2 - 12G(1;y)G(r;y)y^2 + 12G(r;y)y^2 - 12G(0,1;y)y^2 + 6G(0,r;y)y^2 + 12G(r,1;y)y^2 + 18G(0,-1,r;y)y^2 + 18G(0,0,-1;y)y^2 + 36G(0,0,1;y)y^2 - 6G(0,0,r;y)y^2 + 12G(0,1,r;y)y^2 + 18G(0,r,-1;y)y^2 + 18\zeta(3)y^2 - 36y^2 + \pi^2y + 12G(1;y)G(r;y)y - 12G(r;y)y + 12G(0,1;y)y - 6G(0,r;y)y - 12G(r,1;y)y + 18G(0,-1,r;y)y + 18G(0,0,-1;y)y + 12G(0,0,1;y)y + 12G(0,0,r;y)y + 12G(0,1,r;y)y + 18G(0,r,-1;y)y + 6\zeta(3)y + 36y + 2\pi^2G(1;y) - 3(y-1)G^2(0;y)(y + 2(2y^2+y+2)G(1;y)) + 12G^2(1;y)G(r;y) + 6G(r;y) + 24G(1;y)G(0,1;y) + G(0;y)(3\pi^2y^3 + 12y^3 + \pi^2y^2 - 24G(0,1;y)y^2 - 18y^2 + 2\pi^2y + 12(y-1)G(1;y)y + 6y + 12(y^3-1)G^2(1;y) - 18(y^3+y^2+y+1)G(0,-1;y) - 24G(0,1;y)) - 12G(1;y)G(0,r;y) - (y^3+y^2+y+1)G(-1;y)(-9G^2(0;y) + \pi^2 + 18G(0,r;y)) - 24G(1;y)G(r,1;y) + 18G(0,-1,r;y) + 18G(0,0,-1;y) + 24G(0,0,1;y) - 24G(0,1,1;y) + 12G(0,1,r;y) + 18G(0,r,-1;y) + 12G(0,r,1;y) + 24G(r,1,1;y) + 18\zeta(3) - 12]$

$A^{(1,\mathrm{fin})}_{\text{QCD-EW}}$

$$
\begin{aligned}
A^{(1,\mathrm{fin})}_{\text{QCD-EW}}\left(y, \frac{\mu^2}{s}\right) = & \frac{N_C}{90(y-1)^3}C_1(y) + \frac{1}{N_C[180(y-1)^4]}C_2(y) + \\
& - \log\left(-\frac{s}{\mu^2}\right)\beta_0 A^{(0)}_{\text{QCD-EW}}(y).
\end{aligned}
\tag{C.2}
$$

The functions C_i are listed below.

C_1

$$
C_1(y) = -3y^3G(0;y)^5 - 3y^2G(0;y)^5 + 105y^3G(0;y)^4 + 60y^2G(0;y)^4 + 45yG(0;y)^4 - 195y^3G(-1;y)G(0;y)^4 - 195y^2G(-1;y)G(0;y)^4 -
195yG(-1;y)G(0;y)^4 - 195G(-1;y)G(0;y)^4 + 210y^3G(1;y)G(0;y)^4 - 120y^2G(1;y)G(0;y)^4 + 180yG(1;y)G(0;y)^4 - 150G(1;y)G(0;y)^4 -
10\pi^2y^3G(0;y)^3 - 580y^3G(0;y)^3 - 10\pi^2y^2G(0;y)^3 + 1380y^2G(0;y)^3 - 480y^3G(-1;y)^2G(0;y)^3 - 480y^2G(-1;y)^2G(0;y)^3 -
480yG(-1;y)^2G(0;y)^3 - 480G(-1;y)^2G(0;y)^3 - 810y^3G(1;y)^2G(0;y)^3 + 1050y^2G(1;y)^2G(0;y)^3 - 1050yG(1;y)^2G(0;y)^3 +
810G(1;y)^2G(0;y)^3 - 480yG(0;y)^3 + 1680y^3G(-1;y)G(0;y)^3 + 1680y^2G(-1;y)G(0;y)^3 + 1680yG(-1;y)G(0;y)^3 + 1680G(-1;y)G(0;y)^3 -
2100y^3G(1;y)G(0;y)^3 + 1020y^2G(1;y)G(0;y)^3 - 360yG(1;y)G(0;y)^3 + 960y^3G(-1;y)G(1;y)G(0;y)^3 + 960y^2G(-1;y)G(1;y)G(0;y)^3 +
960yG(-1;y)G(1;y)G(0;y)^3 + 960G(-1;y)G(1;y)G(0;y)^3 + 1440G(1;y)G(0;y)^3 + 780y^3G(0,-1;y)G(0;y)^3 + 780y^2G(0,-1;y)G(0;y)^3 +
780yG(0,-1;y)G(0;y)^3 + 780G(0,-1;y)G(0;y)^3 - 180y^3G(0,1;y)G(0;y)^3 - 180y^2G(0,1;y)G(0;y)^3 + 540yG(0,1;y)G(0;y)^3 +
540G(0,1;y)G(0;y)^3 + 540\pi^2y^3G(0;y)^2 - 3420y^3G(0;y)^2 + 570y^3G(1;y)^3G(0;y)^2 - 3810y^2G(1;y)^3G(0;y)^2 + 3810yG(1;y)^3G(0;y)^2 -
570G(1;y)^3G(0;y)^2 - 330\pi^2y^2G(0;y)^2 + 1380y^2G(0;y)^2 + 3780y^3G(-1;y)^2G(0;y)^2 + 3780y^2G(-1;y)^2G(0;y)^2 + 3780yG(-1;y)^2G(0;y)^2 +
3780G(-1;y)^2G(0;y)^2 + 3960y^3G(1;y)^2G(0;y)^2 + 5220y^2G(1;y)^2G(0;y)^2 - 8190yG(1;y)^2G(0;y)^2 - 990G(1;y)^2G(0;y)^2 +
210\pi^2yG(0;y)^2 + 600yG(0;y)^2 + 30\pi^2y^3G(-1;y)G(0;y)^2 - 3060y^3G(-1;y)G(0;y)^2 + 30\pi^2G(-1;y)G(0;y)^2 + 30\pi^2y^2G(-1;y)G(0;y)^2 +
5940y^2G(-1;y)G(0;y)^2 + 30\pi^2yG(-1;y)G(0;y)^2 + 5940yG(-1;y)G(0;y)^2 - 3060G(-1;y)G(0;y)^2 - 30\pi^2y^3G(1;y)G(0;y)^2 +
7320y^3G(1;y)G(0;y)^2 + 150\pi^2G(1;y)G(0;y)^2 + 570\pi^2y^2G(1;y)G(0;y)^2 - 25920y^2G(1;y)G(0;y)^2 + 2160y^3G(-1;y)^2G(1;y)G(0;y)^2 +
2160y^2G(-1;y)^2G(1;y)G(0;y)^2 + 2160yG(-1;y)^2G(1;y)G(0;y)^2 + 2160G(-1;y)^2G(1;y)G(0;y)^2 - 450\pi^2yG(1;y)G(0;y)^2 +
23760yG(1;y)G(0;y)^2 - 5160G(1;y)G(0;y)^2 + 180y^3G(r;y)G(0;y)^2 - 360y^2G(r;y)G(0;y)^2 + 360y^3G(1;y)^2G(r;y)G(0;y)^2 -
360G(1;y)^2G(r;y)G(0;y)^2 + 360yG(r;y)G(0;y)^2 + 360y^2G(1;y)G(r;y)G(0;y)^2 - 360yG(1;y)G(r;y)G(0;y)^2 - 180G(r;y)G(0;y)^2 -
12600y^3G(0,-1;y)G(0;y)^2 - 12600y^2G(0,-1;y)G(0;y)^2 - 5040yG(0,-1;y)G(0;y)^2 + 720y^3G(-1;y)G(0,-1;y)G(0;y)^2 +
720y^2G(-1;y)G(0,-1;y)G(0;y)^2 + 720yG(-1;y)G(0,-1;y)G(0;y)^2 + 720G(-1;y)G(0,-1;y)G(0;y)^2 - 5040y^3G(1;y)G(0,-1;y)G(0;y)^2 -
5040y^2G(1;y)G(0,-1;y)G(0;y)^2 - 5040yG(1;y)G(0,-1;y)G(0;y)^2 - 5040G(1;y)G(0,-1;y)G(0;y)^2 - 5040G(0,-1;y)G(0;y)^2 -
1620y^3G(0,1;y)G(0;y)^2 - 19260y^2G(0,1;y)G(0;y)^2 - 2520yG(0,1;y)G(0;y)^2 - 3600y^3G(-1;y)G(0,1;y)G(0;y)^2 -
3600y^2G(-1;y)G(0,1;y)G(0;y)^2 - 3600yG(-1;y)G(0,1;y)G(0;y)^2 - 3600G(-1;y)G(0,1;y)G(0;y)^2 - 2880y^2G(1;y)G(0,1;y)G(0;y)^2 -
5040yG(1;y)G(0,1;y)G(0;y)^2 - 7920G(1;y)G(0,1;y)G(0;y)^2 - 4320G(0,1;y)G(0;y)^2 - 180y^2G(0,r;y)G(0;y)^2 + 180yG(0,r;y)G(0;y)^2 +
540y^3G(-1;y)G(0,r;y)G(0;y)^2 + 540y^2G(-1;y)G(0,r;y)G(0;y)^2 + 540yG(-1;y)G(0,r;y)G(0;y)^2 + 540G(-1;y)G(0,r;y)G(0;y)^2 -
360y^3G(1;y)G(0,r;y)G(0;y)^2 + 360G(1;y)G(0,r;y)G(0;y)^2 - 360y^2G(r,1;y)G(0;y)^2 + 360yG(r,1;y)G(0;y)^2 -
720y^3G(1;y)G(r,1;y)G(0;y)^2 + 720G(1;y)G(r,1;y)G(0;y)^2 - 2880y^3G(0,-1,-1;y)G(0;y)^2 + 1440y^2G(0,-1,-1;y)G(0;y)^2 -
2880yG(0,-1,-1;y)G(0;y)^2 + 1440G(0,-1,-1;y)G(0;y)^2 + 1440y^3G(0,-1,1;y)G(0;y)^2 + 5760y^2G(0,-1,1;y)G(0;y)^2 +
1440yG(0,-1,1;y)G(0;y)^2 + 5760G(0,-1,1;y)G(0;y)^2 - 540y^3G(0,-1,r;y)G(0;y)^2 - 540y^2G(0,-1,r;y)G(0;y)^2 -
540yG(0,-1,r;y)G(0;y)^2 - 540G(0,-1,r;y)G(0;y)^2 + 1980y^3G(0,0,-1;y)G(0;y)^2 - 2340y^2G(0,0,-1;y)G(0;y)^2 +
1980yG(0,0,-1;y)G(0;y)^2 - 2340G(0,0,-1;y)G(0;y)^2 + 3600y^3G(0,0,1;y)G(0;y)^2 + 2520y^2G(0,0,1;y)G(0;y)^2 -
360yG(0,0,1;y)G(0;y)^2 - 1440G(0,0,1;y)G(0;y)^2 - 180y^3G(0,0,r;y)G(0;y)^2 + 180y^2G(0,0,r;y)G(0;y)^2 - 360yG(0,0,r;y)G(0;y)^2 +
1440y^3G(0,1,-1;y)G(0;y)^2 + 5760y^2G(0,1,-1;y)G(0;y)^2 + 1440yG(0,1,-1;y)G(0;y)^2 + 5760G(0,1,-1;y)G(0;y)^2 +
1980y^3G(0,1,1;y)G(0;y)^2 + 13500y^2G(0,1,1;y)G(0;y)^2 + 540yG(0,1,1;y)G(0;y)^2 + 12060G(0,1,1;y)G(0;y)^2 - 360y^3G(0,1,r;y)G(0;y)^2 -
360y^2G(0,1,r;y)G(0;y)^2 - 360yG(0,1,r;y)G(0;y)^2 - 360G(0,1,r;y)G(0;y)^2 - 540y^3G(0,r,-1;y)G(0;y)^2 - 540y^2G(0,r,-1;y)G(0;y)^2 -
540yG(0,r,-1;y)G(0;y)^2 - 540G(0,r,-1;y)G(0;y)^2 + 360y^3G(0,r,1;y)G(0;y)^2 - 360G(0,r,1;y)G(0;y)^2 + 720y^3G(r,1,1;y)G(0;y)^2 -
720G(r,1,1;y)G(0;y)^2 - 1800y^2\zeta(3)G(0;y)^2 + 1800y\zeta(3)G(0;y)^2 - 510y^3G(1;y)^4G(0;y) + 1110y^2G(1;y)^4G(0;y) -
1110yG(1;y)^4G(0;y) + 510G(1;y)^4G(0;y) + 107\pi^4y^3G(0;y) - 1740\pi^2y^3G(0;y) + 1920y^3G(0;y) - 2220y^3G(1;y)^3G(0;y) -
3780y^2G(1;y)^3G(0;y) + 3780yG(1;y)^3G(0;y) + 2220G(1;y)^3G(0;y) - 9\pi^4y^2G(0;y) + 3960\pi^2y^2G(0;y) + 1800y^2G(0;y) -
360\pi^2y^3G(-1;y)^2G(0;y) - 360\pi^2G(-1;y)^2G(0;y) - 360\pi^2y^2G(-1;y)^2G(0;y) - 360\pi^2yG(-1;y)^2G(0;y) - 90\pi^2y^3G(1;y)^2G(0;y) -
6960y^3G(1;y)^2G(0;y) + 90\pi^2G(1;y)^2G(0;y) + 330\pi^2y^2G(1;y)^2G(0;y) + 24480y^2G(1;y)^2G(0;y) - 330\pi^2yG(1;y)^2G(0;y) -
24480yG(1;y)^2G(0;y) + 6960G(1;y)^2G(0;y) + 2160y^3G(0,-1;y)^2G(0;y) - 2160y^2G(0,-1;y)^2G(0;y) + 2160yG(0,-1;y)^2G(0;y) -
2160G(0,-1;y)^2G(0;y) + 360y^3G(0,1;y)^2G(0;y) - 9720y^2G(0,1;y)^2G(0;y) + 4680yG(0,1;y)^2G(0;y) - 5400G(0,1;y)^2G(0;y) +
116\pi^4yG(0;y) - 1260\pi^2yG(0;y) - 3720yG(0;y) + 420\pi^2y^3G(-1;y)G(0;y) + 420\pi^2G(-1;y)G(0;y) + 420\pi^2y^2G(-1;y)G(0;y) +
420\pi^2yG(-1;y)G(0;y) - 1500\pi^2y^3G(1;y)G(0;y) + 4680y^3G(1;y)G(0;y) - 480\pi^2G(1;y)G(0;y) - 300\pi^2y^2G(1;y)G(0;y) +
120y^2G(1;y)G(0;y) + 2280\pi^2yG(1;y)G(0;y) - 5880yG(1;y)G(0;y) + 240\pi^2y^3G(-1;y)G(1;y)G(0;y) + 240\pi^2G(-1;y)G(1;y)G(0;y) +
240\pi^2y^2G(-1;y)G(1;y)G(0;y) + 240\pi^2yG(-1;y)G(1;y)G(0;y) + 1080G(1;y)G(0;y) + 720y^3G(r;y)G(0;y) + 960y^3G(1;y)^3G(r;y)G(0;y) -
960G(1;y)^3G(r;y)G(0;y) - 1440y^2G(r;y)G(0;y) + 1440y^2G(1;y)^2G(r;y)G(0;y) - 1440yG(1;y)^2G(r;y)G(0;y) +
1440yG(r;y)G(0;y) + 1440y^2G(1;y)G(r;y)G(0;y) - 1440yG(1;y)G(r;y)G(0;y) - 720G(r;y)G(0;y) + 420\pi^2y^3G(0,-1;y)G(0;y) +
6120y^3G(0,-1;y)G(0;y) - 60\pi^2G(0,-1;y)G(0;y) - 60\pi^2y^2G(0,-1;y)G(0;y) - 11880y^2G(0,-1;y)G(0;y) + 420\pi^2yG(0,-1;y)G(0;y) -
11880yG(0,-1;y)G(0;y) - 15120y^3G(-1;y)G(0,-1;y)G(0;y) - 15120y^2G(-1;y)G(0,-1;y)G(0;y) - 15120yG(-1;y)G(0,-1;y)G(0;y) -
15120G(-1;y)G(0,-1;y)G(0;y) - 8640y^3G(-1;y)G(1;y)G(0,-1;y)G(0;y) - 8640y^2G(-1;y)G(1;y)G(0,-1;y)G(0;y) -
8640yG(-1;y)G(1;y)G(0,-1;y)G(0;y) - 8640G(-1;y)G(1;y)G(0,-1;y)G(0;y) + 6120G(0,-1;y)G(0;y) + 60\pi^2y^3G(0,1;y)G(0;y) -
5040y^3G(0,1;y)G(0;y) - 420\pi^2G(0,1;y)G(0;y) - 2580\pi^2y^2G(0,1;y)G(0;y) + 25920y^2G(0,1;y)G(0;y) + 1440y^3G(-1;y)^2G(0,1;y)G(0;y) +
1440y^2G(-1;y)^2G(0,1;y)G(0;y) + 1440yG(-1;y)^2G(0,1;y)G(0;y) + 1440G(-1;y)^2G(0,1;y)G(0;y) + 3240y^3G(1;y)^2G(0,1;y)G(0;y) +
5400y^2G(1;y)^2G(0,1;y)G(0;y) - 5400yG(1;y)^2G(0,1;y)G(0;y) - 3240G(1;y)^2G(0,1;y)G(0;y) + 2220\pi^2yG(0,1;y)G(0;y) -
$$

$38880yG(0,1;y)G(0;y) - 15120y^3G(-1;y)G(0,1;y)G(0;y) - 15120y^2G(-1;y)G(0,1;y)G(0;y) - 15120yG(-1;y)G(0,1;y)G(0;y) -$
$15120G(-1;y)G(0,1;y)G(0;y) - 5040y^3G(1;y)G(0,1;y)G(0;y) + 2160y^2G(1;y)G(0,1;y)G(0;y) + 5760yG(1;y)G(0,1;y)G(0;y) -$
$8640y^3G(-1;y)G(1;y)G(0,1;y)G(0;y) - 8640y^2G(-1;y)G(1;y)G(0,1;y)G(0;y) - 8640yG(-1;y)G(1;y)G(0,1;y)G(0;y) -$
$8640G(-1;y)G(1;y)G(0,1;y)G(0;y)-2880G(1;y)G(0,1;y)G(0;y)+720y^3G(0,-1;y)G(0,1;y)G(0;y)-5040y^2G(0,-1;y)G(0,1;y)G(0;y)+$
$720yG(0,-1;y)G(0,1;y)G(0;y) - 5040G(0,-1;y)G(0,1;y)G(0;y) + 10320G(0,1;y)G(0;y) - 360y^3G(0,r;y)G(0;y) -$
$2160y^3G(-1;y)^2G(0,r;y)G(0;y) - 2160y^2G(-1;y)^2G(0,r;y)G(0;y) - 2160yG(-1;y)^2G(0,r;y)G(0;y) - 2160G(-1;y)^2G(0,r;y)G(0;y) -$
$2160y^3G(1;y)^2G(0,r;y)G(0;y) + 2160G(1;y)^2G(0,r;y)G(0;y) - 2160y^2G(1;y)G(0,r;y)G(0;y) + 2160yG(1;y)G(0,r;y)G(0;y) +$
$4680y^3G(0,-1;y)G(0,r;y)G(0;y)+360y^2G(0,-1;y)G(0,r;y)G(0;y)+4680yG(0,-1;y)G(0,r;y)G(0;y)+360G(0,-1;y)G(0,r;y)G(0;y)+$
$360G(0,r;y)G(0;y) - 720y^3G(r,1;y)G(0;y) - 4320y^3G(1;y)^2G(r,1;y)G(0;y) + 4320G(1;y)^2G(r,1;y)G(0;y) -$
$4320y^2G(1;y)G(r,1;y)G(0;y) + 4320yG(1;y)G(r,1;y)G(0;y) + 720G(r,1;y)G(0;y) + 15120y^3G(0,-1,-1;y)G(0;y) +$
$15120y^2G(0,-1,-1;y)G(0;y)+15120yG(0,-1,-1;y)G(0;y)+8640y^3G(1;y)G(0,-1,-1;y)G(0;y)+8640y^2G(1;y)G(0,-1,-1;y)G(0;y)+$
$8640yG(1;y)G(0,-1,-1;y)G(0;y) + 8640G(1;y)G(0,-1,-1;y)G(0;y) + 15120G(0,-1,-1;y)G(0;y) + 15120y^3G(0,-1,1;y)G(0;y) +$
$15120y^2G(0,-1,1;y)G(0;y) + 15120yG(0,-1,1;y)G(0;y) - 2880y^3G(-1;y)G(0,-1,1;y)G(0;y) - 2880y^2G(-1;y)G(0,-1,1;y)G(0;y) -$
$2880yG(-1;y)G(0,-1,1;y)G(0;y) - 2880G(-1;y)G(0,-1,1;y)G(0;y) + 8640y^3G(1;y)G(0,-1,1;y)G(0;y) +$
$8640y^2G(1;y)G(0,-1,1;y)G(0;y) + 8640yG(1;y)G(0,-1,1;y)G(0;y) + 8640G(1;y)G(0,-1,1;y)G(0;y) + 15120G(0,-1,1;y)G(0;y) +$
$4320y^3G(-1;y)G(0,-1,r;y)G(0;y) + 4320y^2G(-1;y)G(0,-1,r;y)G(0;y) + 4320yG(-1;y)G(0,-1,r;y)G(0;y) +$
$4320G(-1;y)G(0,-1,r;y)G(0;y) + 40320y^3G(0,0,-1;y)G(0;y) + 40320y^2G(0,0,-1;y)G(0;y) + 10080yG(0,0,-1;y)G(0;y) +$
$2880y^3G(-1;y)G(0,0,-1;y)G(0;y) + 2880y^2G(-1;y)G(0,0,-1;y)G(0;y) + 2880yG(-1;y)G(0,0,-1;y)G(0;y) +$
$2880G(-1;y)G(0,0,-1;y)G(0;y) + 14400y^3G(1;y)G(0,0,-1;y)G(0;y) + 14400y^2G(1;y)G(0,0,-1;y)G(0;y) +$
$14400yG(1;y)G(0,0,-1;y)G(0;y) + 14400G(1;y)G(0,0,-1;y)G(0;y) + 10080G(0,0,-1;y)G(0;y) + 23400y^3G(0,0,1;y)G(0;y) +$
$51480y^2G(0,0,1;y)G(0;y) + 12240yG(0,0,1;y)G(0;y) + 12240y^3G(-1;y)G(0,0,1;y)G(0;y) + 12240y^2G(-1;y)G(0,0,1;y)G(0;y) +$
$12240yG(-1;y)G(0,0,1;y)G(0;y) + 12240G(-1;y)G(0,0,1;y)G(0;y) + 1440y^3G(1;y)G(0,0,1;y)G(0;y) +$
$11520y^2G(1;y)G(0,0,1;y)G(0;y) + 15840yG(1;y)G(0,0,1;y)G(0;y) + 25920G(1;y)G(0,0,1;y)G(0;y) + 8640G(0,0,1;y)G(0;y) +$
$1440y^2G(0,0,r;y)G(0;y) - 1440yG(0,0,r;y)G(0;y) - 1440y^3G(-1;y)G(0,0,r;y)G(0;y) - 1440y^2G(-1;y)G(0,0,r;y)G(0;y) -$
$1440yG(-1;y)G(0,0,r;y)G(0;y) - 1440G(-1;y)G(0,0,r;y)G(0;y) + 2880y^3G(1;y)G(0,0,r;y)G(0;y) - 2880G(1;y)G(0,0,r;y)G(0;y) +$
$15120y^3G(0,1,-1;y)G(0;y) + 15120y^2G(0,1,-1;y)G(0;y) + 15120yG(0,1,-1;y)G(0;y) - 2880y^3G(-1;y)G(0,1,-1;y)G(0;y) -$
$2880y^2G(-1;y)G(0,1,-1;y)G(0;y) - 2880yG(-1;y)G(0,1,-1;y)G(0;y) - 2880G(-1;y)G(0,1,-1;y)G(0;y) +$
$8640y^3G(1;y)G(0,1,-1;y)G(0;y) + 8640y^2G(1;y)G(0,1,-1;y)G(0;y) + 8640yG(1;y)G(0,1,-1;y)G(0;y) +$
$8640G(1;y)G(0,1,-1;y)G(0;y) + 15120G(0,1,-1;y)G(0;y) + 7560y^3G(0,1,1;y)G(0;y) + 4680y^2G(0,1,1;y)G(0;y) +$
$28440yG(0,1,1;y)G(0;y)+11520y^3G(-1;y)G(0,1,1;y)G(0;y)+11520y^2G(-1;y)G(0,1,1;y)G(0;y)+11520yG(-1;y)G(0,1,1;y)G(0;y)+$
$11520G(-1;y)G(0,1,1;y)G(0;y)-6480y^3G(1;y)G(0,1,1;y)G(0;y)+18000y^2G(1;y)G(0,1,1;y)G(0;y)-3600yG(1;y)G(0,1,1;y)G(0;y)+$
$20880G(1;y)G(0,1,1;y)G(0;y) + 9720G(0,1,1;y)G(0;y) + 2880y^3G(-1;y)G(0,1,r;y)G(0;y) + 2880y^2G(-1;y)G(0,1,r;y)G(0;y) +$
$2880yG(-1;y)G(0,1,r;y)G(0;y) + 2880G(-1;y)G(0,1,r;y)G(0;y) + 4320y^3G(-1;y)G(0,r,-1;y)G(0;y) +$
$4320y^2G(-1;y)G(0,r,-1;y)G(0;y)+4320yG(-1;y)G(0,r,-1;y)G(0;y)+4320G(-1;y)G(0,r,-1;y)G(0;y)+2880y^2G(0,r,1;y)G(0;y)-$
$2880yG(0,r,1;y)G(0;y) - 2160y^3G(-1;y)G(0,r,1;y)G(0;y) - 2160y^2G(-1;y)G(0,r,1;y)G(0;y) - 2160yG(-1;y)G(0,r,1;y)G(0;y) -$
$2160G(-1;y)G(0,r,1;y)G(0;y) + 5760y^3G(1;y)G(0,r,1;y)G(0;y) - 5760G(1;y)G(0,r,1;y)G(0;y) + 5760y^2G(r,1,1;y)G(0;y) -$
$5760yG(r,1,1;y)G(0;y) + 11520y^3G(1;y)G(r,1,1;y)G(0;y) - 11520G(1;y)G(r,1,1;y)G(0;y) + 2880y^3G(0,-1,-1,1;y)G(0;y) +$
$2880y^2G(0,-1,-1,1;y)G(0;y) + 2880yG(0,-1,-1,1;y)G(0;y) + 2880G(0,-1,-1,1;y)G(0;y) - 4320y^3G(0,-1,-1,r;y)G(0;y) -$
$4320y^2G(0,-1,-1,r;y)G(0;y) - 4320yG(0,-1,-1,r;y)G(0;y) - 4320G(0,-1,-1,r;y)G(0;y) + 2880y^3G(0,-1,1,-1;y)G(0;y) +$
$2880y^2G(0,-1,1,-1;y)G(0;y) + 2880yG(0,-1,1,-1;y)G(0;y) + 2880G(0,-1,1,-1;y)G(0;y) - 11520y^3G(0,-1,1,1;y)G(0;y) -$
$11520y^2G(0,-1,1,1;y)G(0;y) - 11520yG(0,-1,1,1;y)G(0;y) - 11520G(0,-1,1,1;y)G(0;y) - 2880y^3G(0,-1,1,r;y)G(0;y) -$
$2880y^2G(0,-1,1,r;y)G(0;y) - 2880yG(0,-1,1,r;y)G(0;y) - 2880G(0,-1,1,r;y)G(0;y) - 4320y^3G(0,-1,r,-1;y)G(0;y) -$
$4320y^2G(0,-1,r,-1;y)G(0;y) - 4320yG(0,-1,r,-1;y)G(0;y) - 4320G(0,-1,r,-1;y)G(0;y) + 2160y^3G(0,-1,r,1;y)G(0;y) +$
$2160y^2G(0,-1,r,1;y)G(0;y) + 2160yG(0,-1,r,1;y)G(0;y) + 2160G(0,-1,r,1;y)G(0;y) - 2880y^3G(0,0,-1,-1;y)G(0;y) -$
$2880y^2G(0,0,-1,-1;y)G(0;y) - 2880yG(0,0,-1,-1;y)G(0;y) - 2880G(0,0,-1,-1;y)G(0;y) - 5040y^3G(0,0,-1,1;y)G(0;y) -$
$10800y^2G(0,0,-1,1;y)G(0;y) - 5040yG(0,0,-1,1;y)G(0;y) - 10800G(0,0,-1,1;y)G(0;y) - 7920y^3G(0,0,-1,r;y)G(0;y) +$
$720y^2G(0,0,-1,r;y)G(0;y) - 7920yG(0,0,-1,r;y)G(0;y) + 720G(0,0,-1,r;y)G(0;y) - 21240y^3G(0,0,0,-1;y)G(0;y) +$
$4680y^2G(0,0,0,-1;y)G(0;y) - 21240yG(0,0,0,-1;y)G(0;y) + 4680G(0,0,0,-1;y)G(0;y) - 19800y^3G(0,0,0,1;y)G(0;y) -$
$6840y^2G(0,0,0,1;y)G(0;y) - 10440yG(0,0,0,1;y)G(0;y) + 2520G(0,0,0,1;y)G(0;y) - 1080y^3G(0,0,0,r;y)G(0;y) -$
$1800y^2G(0,0,0,r;y)G(0;y) + 720yG(0,0,0,r;y)G(0;y) - 5040y^3G(0,0,1,-1;y)G(0;y) - 10800y^2G(0,0,1,-1;y)G(0;y) -$
$5040yG(0,0,1,-1;y)G(0;y) - 10800G(0,0,1,-1;y)G(0;y) - 9000y^3G(0,0,1,1;y)G(0;y) - 30600y^2G(0,0,1,1;y)G(0;y) -$
$3240yG(0,0,1,1;y)G(0;y) - 24840G(0,0,1,1;y)G(0;y) + 2880y^2G(0,0,1,r;y)G(0;y) + 2880G(0,0,1,r;y)G(0;y) -$
$7920y^3G(0,0,r,-1;y)G(0;y) + 720y^2G(0,0,r,-1;y)G(0;y) - 7920yG(0,0,r,-1;y)G(0;y) + 720G(0,0,r,-1;y)G(0;y) -$
$2160y^3G(0,0,r,1;y)G(0;y) - 720y^2G(0,0,r,1;y)G(0;y) + 1440yG(0,0,r,1;y)G(0;y) + 2880G(0,0,r,1;y)G(0;y) +$
$2880y^3G(0,1,-1,-1;y)G(0;y) + 2880y^2G(0,1,-1,-1;y)G(0;y) + 2880yG(0,1,-1,-1;y)G(0;y) + 2880G(0,1,-1,-1;y)G(0;y) -$
$11520y^3G(0,1,-1,1;y)G(0;y) - 11520y^2G(0,1,-1,1;y)G(0;y) - 11520yG(0,1,-1,1;y)G(0;y) - 11520G(0,1,-1,1;y)G(0;y) -$
$2880y^3G(0,1,-1,r;y)G(0;y) - 2880y^2G(0,1,-1,r;y)G(0;y) - 2880yG(0,1,-1,r;y)G(0;y) - 2880G(0,1,-1,r;y)G(0;y) +$
$3600y^3G(0,1,0,-1;y)G(0;y) + 720y^2G(0,1,0,-1;y)G(0;y) + 3600yG(0,1,0,-1;y)G(0;y) + 720G(0,1,0,-1;y)G(0;y) +$
$2160y^3G(0,1,0,r;y)G(0;y) + 2160y^2G(0,1,0,r;y)G(0;y) + 2160yG(0,1,0,r;y)G(0;y) + 2160G(0,1,0,r;y)G(0;y) -$
$11520y^3G(0,1,1,-1;y)G(0;y) - 11520y^2G(0,1,1,-1;y)G(0;y) - 11520yG(0,1,1,-1;y)G(0;y) - 11520G(0,1,1,-1;y)G(0;y) +$

$$4320y^3G(0,1,1,1;y)G(0;y) - 48960y^2G(0,1,1,1;y)G(0;y) + 15840yG(0,1,1,1;y)G(0;y) - 37440G(0,1,1,1;y)G(0;y) -$$
$$2880y^3G(0,1,1,r;y)G(0;y) - 2880y^2G(0,1,1,r;y)G(0;y) - 2880yG(0,1,1,r;y)G(0;y) - 2880G(0,1,1,r;y)G(0;y) -$$
$$2880y^3G(0,1,r,-1;y)G(0;y) - 2880y^2G(0,1,r,-1;y)G(0;y) - 2880yG(0,1,r,-1;y)G(0;y) - 2880G(0,1,r,-1;y)G(0;y) +$$
$$1440y^3G(0,1,r,1;y)G(0;y) + 1440y^2G(0,1,r,1;y)G(0;y) + 1440yG(0,1,r,1;y)G(0;y) + 1440G(0,1,r,1;y)G(0;y) -$$
$$4320y^3G(0,r,-1,-1;y)G(0;y) - 4320y^2G(0,r,-1,-1;y)G(0;y) - 4320yG(0,r,-1,-1;y)G(0;y) - 4320G(0,r,-1,-1;y)G(0;y) +$$
$$2160y^3G(0,r,-1,1;y)G(0;y) + 2160y^2G(0,r,-1,1;y)G(0;y) + 2160yG(0,r,-1,1;y)G(0;y) + 2160G(0,r,-1,1;y)G(0;y) -$$
$$4680y^3G(0,r,0,-1;y)G(0;y) - 360y^2G(0,r,0,-1;y)G(0;y) - 4680yG(0,r,0,-1;y)G(0;y) - 360G(0,r,0,-1;y)G(0;y) +$$
$$2160y^3G(0,r,1,-1;y)G(0;y) + 2160y^2G(0,r,1,-1;y)G(0;y) + 2160yG(0,r,1,-1;y)G(0;y) + 2160G(0,r,1,-1;y)G(0;y) -$$
$$7200y^3G(0,r,1,1;y)G(0;y) + 7200G(0,r,1,1;y)G(0;y) - 14400y^3G(r,1,1,1;y)G(0;y) + 14400G(r,1,1,1;y)G(0;y) - 7920y^3\zeta(3)G(0;y) +$$
$$11520y^2\zeta(3)G(0;y) - 3600y\zeta(3)G(0;y) - 4320y^3G(-1;y)\zeta(3)G(0;y) - 4320y^2G(-1;y)\zeta(3)G(0;y) - 4320yG(-1;y)\zeta(3)G(0;y) -$$
$$4320G(-1;y)\zeta(3)G(0;y) + 5040y^3G(1;y)\zeta(3)G(0;y) - 6480y^2G(1;y)\zeta(3)G(0;y) + 6480yG(1;y)\zeta(3)G(0;y) - 5040G(1;y)\zeta(3)G(0;y) +$$
$$95\pi^4y^3 + 540\pi^2y^3 - 2040y^3 - 170\pi^2y^3G(1;y)^3 + 170\pi^2G(1;y)^3 + 370\pi^2y^2G(1;y)^3 - 370\pi^2yG(1;y)^3 + 120y^3G(r;y)^3 - 240y^2G(r;y)^3 +$$
$$240y^3G(1;y)^2G(r;y)^3 - 240G(1;y)^2G(r;y)^3 + 240yG(r;y)^3 + 240y^2G(1;y)G(r;y)^3 - 240yG(1;y)G(r;y)^3 - 120G(r;y)^3 - 480\pi^2 - 236\pi^4y^2 -$$
$$1520\pi^2y^2 + 6120y^2 - 420\pi^2y^3G(-1;y)^2 - 420\pi^2G(-1;y)^2 - 420\pi^2y^2G(-1;y)^2 - 420\pi^2yG(-1;y)^2 + 540\pi^2y^3G(1;y)^2 - 870\pi^2G(1;y)^2 -$$
$$600\pi^2y^2G(1;y)^2 + 930\pi^2yG(1;y)^2 + 1620y^3G(r;y)^2 - 1500y^3G(1;y)^3G(r;y)^2 + 2220y^2G(1;y)^3G(r;y)^2 - 2220yG(1;y)^3G(r;y)^2 +$$
$$1500G(1;y)^3G(r;y)^2 - 3240y^2G(r;y)^2 + 450y^3G(1;y)^2G(r;y)^2 - 5850y^2G(1;y)^2G(r;y)^2 + 5850yG(1;y)^2G(r;y)^2 - 450G(1;y)^2G(r;y)^2 +$$
$$3240yG(r;y)^2 + 1800y^2G(1;y)G(r;y)^2 - 1800yG(1;y)G(r;y)^2 - 1620G(r;y)^2 + 2700y^3G(0,1;y)^2 - 3060y^2G(0,1;y)^2 - 7380yG(0,1;y)^2 -$$
$$2880y^3G(-1;y)G(0,1;y)^2 - 2880y^2G(-1;y)G(0,1;y)^2 - 2880yG(-1;y)G(0,1;y)^2 - 2880G(-1;y)G(0,1;y)^2 + 3600y^3G(1;y)G(0,1;y)^2 -$$
$$720y^2G(1;y)G(0,1;y)^2 - 720yG(1;y)G(0,1;y)^2 - 5040G(1;y)G(0,1;y)^2 + 2700G(0,1;y)^2 + 540y^2G(0,r;y)^2 - 540yG(0,r;y)^2 -$$
$$1620y^3G(-1;y)G(0,r;y)^2 - 1620y^2G(-1;y)G(0,r;y)^2 - 1620yG(-1;y)G(0,r;y)^2 - 1620G(-1;y)G(0,r;y)^2 + 1080y^3G(1;y)G(0,r;y)^2 -$$
$$1080G(1;y)G(0,r;y)^2 + 2160y^2G(r,1;y)^2 - 2160yG(r,1;y)^2 + 4320y^3G(1;y)G(r,1;y)^2 - 4320G(1;y)G(r,1;y)^2 + \pi^4y + 1620\pi^2y - 6120y -$$
$$153\pi^4G(-1;y) - 153\pi^4y^3G(-1;y) + 340\pi^2y^3G(-1;y) + 340\pi^2G(-1;y) - 153\pi^4y^2G(-1;y) - 660\pi^2y^2G(-1;y) - 153\pi^4yG(-1;y) -$$
$$660\pi^2yG(-1;y) - 71\pi^4G(1;y) + 31\pi^4y^3G(1;y) + 1040\pi^2y^3G(1;y) + 9360y^3G(1;y) - 1280\pi^2G(1;y) + 127\pi^4y^2G(1;y) - 3120\pi^2y^2G(1;y) -$$
$$28080y^2G(1;y) - 240\pi^2y^3G(-1;y)^2G(1;y) - 240\pi^2G(-1;y)^2G(1;y) - 240\pi^2y^2G(-1;y)^2G(1;y) - 240\pi^2yG(-1;y)^2G(1;y) -$$
$$167\pi^4yG(1;y) + 3360\pi^2yG(1;y) + 28080yG(1;y) - 9360G(1;y) + 510y^3G(1;y)^4G(r;y) - 1110y^2G(1;y)^4G(r;y) + 1110yG(1;y)^4G(r;y) -$$
$$510G(1;y)^4G(r;y) + 180\pi^2y^3G(r;y) - 5640y^3G(r;y) + 2220y^3G(1;y)^3G(r;y) + 3780y^2G(1;y)^3G(r;y) - 3780yG(1;y)^3G(r;y) -$$
$$2220G(1;y)^3G(r;y) - 180\pi^2G(r;y) - 360\pi^2y^2G(r;y) + 11280y^2G(r;y) + 360\pi^2y^3G(1;y)^2G(r;y) + 6960y^3G(1;y)^2G(r;y) -$$
$$360\pi^2G(1;y)^2G(r;y) - 24480y^2G(1;y)^2G(r;y) + 24480yG(1;y)^2G(r;y) - 6960G(1;y)^2G(r;y) + 360\pi^2yG(r;y) - 11280yG(r;y) -$$
$$1800y^3G(1;y)G(r;y) + 360\pi^2y^2G(1;y)G(r;y) - 3000y^2G(1;y)G(r;y) - 360\pi^2yG(1;y)G(r;y) + 3000yG(1;y)G(r;y) + 1800G(1;y)G(r;y) +$$
$$5640G(r;y) + 420\pi^2y^3G(0,-1;y) - 420\pi^2G(0,-1;y) + 420\pi^2y^2G(0,-1;y) - 420\pi^2yG(0,-1;y) + 960\pi^2y^3G(-1;y)G(0,-1;y) +$$
$$960\pi^2G(-1;y)G(0,-1;y) + 960\pi^2y^2G(-1;y)G(0,-1;y) + 960\pi^2yG(-1;y)G(0,-1;y) + 420\pi^2y^3G(0,1;y) + 6120y^3G(0,1;y) -$$
$$2040y^3G(1;y)^3G(0,1;y) + 4440y^2G(1;y)^3G(0,1;y) - 4440yG(1;y)^3G(0,1;y) + 2040G(1;y)^3G(0,1;y) + 480\pi^2G(0,1;y) +$$
$$1140\pi^2y^2G(0,1;y) - 18840y^2G(0,1;y) + 8460y^3G(1;y)^2G(0,1;y) - 9180y^2G(1;y)^2G(0,1;y) + 9180yG(1;y)^2G(0,1;y) -$$
$$8460G(1;y)^2G(0,1;y) - 2880\pi^2yG(0,1;y) + 18840yG(0,1;y) - 480\pi^2y^3G(-1;y)G(0,1;y) - 480\pi^2G(-1;y)G(0,1;y) -$$
$$480\pi^2y^2G(-1;y)G(0,1;y) - 480\pi^2yG(-1;y)G(0,1;y) + 480\pi^2y^3G(1;y)G(0,1;y) + 13920y^3G(1;y)G(0,1;y) - 720\pi^2G(1;y)G(0,1;y) -$$
$$1440\pi^2y^2G(1;y)G(0,1;y) - 38880y^2G(1;y)G(0,1;y) + 1200\pi^2yG(1;y)G(0,1;y) + 38880yG(1;y)G(0,1;y) - 13920G(1;y)G(0,1;y) +$$
$$2160y^3G(r;y)G(0,1;y) - 4320y^2G(r;y)G(0,1;y) + 4320y^3G(1;y)^2G(r;y)G(0,1;y) - 4320G(1;y)^2G(r;y)G(0,1;y) + 4320yG(r;y)G(0,1;y) +$$
$$4320y^2G(1;y)G(r;y)G(0,1;y) - 4320yG(1;y)G(r;y)G(0,1;y) - 2160G(r;y)G(0,1;y) + 11520y^3G(-1;y)G(0,-1;y)G(0,1;y) +$$
$$11520y^2G(-1;y)G(0,-1;y)G(0,1;y) + 11520yG(-1;y)G(0,-1;y)G(0,1;y) + 11520G(-1;y)G(0,-1;y)G(0,1;y) - 6120G(0,1;y) -$$
$$2520y^3G(0,r;y) + 1020y^3G(1;y)^3G(0,r;y) - 2220y^2G(1;y)^3G(0,r;y) + 2220yG(1;y)^3G(0,r;y) - 1020G(1;y)^3G(0,r;y) -$$
$$180\pi^2y^2G(0,r;y) + 10320y^2G(0,r;y) - 7560y^3G(-1;y)^2G(0,r;y) - 7560y^2G(-1;y)^2G(0,r;y) - 7560yG(-1;y)^2G(0,r;y) -$$
$$7560G(-1;y)^2G(0,r;y) - 7200y^3G(1;y)^2G(0,r;y) + 7560y^2G(1;y)^2G(0,r;y) - 1620yG(1;y)^2G(0,r;y) + 1260G(1;y)^2G(0,r;y) +$$
$$180\pi^2yG(0,r;y) - 8520yG(0,r;y) + 540\pi^2y^3G(-1;y)G(0,r;y) + 6120y^3G(-1;y)G(0,r;y) + 540\pi^2G(-1;y)G(0,r;y) +$$
$$540\pi^2y^2G(-1;y)G(0,r;y) - 11880y^2G(-1;y)G(0,r;y) + 540\pi^2yG(-1;y)G(0,r;y) - 11880yG(-1;y)G(0,r;y) + 6120G(-1;y)G(0,r;y) -$$
$$360\pi^2y^3G(1;y)G(0,r;y) - 9120y^3G(1;y)G(0,r;y) + 360\pi^2G(1;y)G(0,r;y) + 21600y^2G(1;y)G(0,r;y) - 4320y^3G(-1;y)^2G(1;y)G(0,r;y) -$$
$$4320y^2G(-1;y)^2G(1;y)G(0,r;y) - 4320yG(-1;y)^2G(1;y)G(0,r;y) - 4320G(-1;y)^2G(1;y)G(0,r;y) - 17280yG(1;y)G(0,r;y) +$$
$$4800G(1;y)G(0,r;y) - 1080y^3G(r;y)G(0,r;y) + 2160y^2G(r;y)G(0,r;y) - 2160y^3G(1;y)^2G(r;y)G(0,r;y) + 2160G(1;y)^2G(r;y)G(0,r;y) -$$
$$2160yG(r;y)G(0,r;y) - 2160y^2G(1;y)G(r;y)G(0,r;y) + 2160yG(1;y)G(r;y)G(0,r;y) + 1080G(r;y)G(0,r;y) +$$
$$7560y^3G(0,-1;y)G(0,r;y) + 7560y^2G(0,-1;y)G(0,r;y) - 7560yG(0,-1;y)G(0,r;y) - 5760y^3G(-1;y)G(0,-1;y)G(0,r;y) -$$
$$5760y^2G(-1;y)G(0,-1;y)G(0,r;y) - 5760yG(-1;y)G(0,-1;y)G(0,r;y) - 5760G(-1;y)G(0,-1;y)G(0,r;y) - 7560G(0,-1;y)G(0,r;y) -$$
$$2160y^2G(0,1;y)G(0,r;y) + 2160yG(0,1;y)G(0,r;y) + 6480y^3G(-1;y)G(0,1;y)G(0,r;y) + 6480y^2G(-1;y)G(0,1;y)G(0,r;y) +$$
$$6480yG(-1;y)G(0,1;y)G(0,r;y) + 6480G(-1;y)G(0,1;y)G(0,r;y) - 4320y^3G(1;y)G(0,1;y)G(0,r;y) + 4320G(1;y)G(0,1;y)G(0,r;y) +$$
$$3600G(0,r;y) - 6120y^3G(r,1;y) + 2040y^3G(1;y)^3G(r,1;y) - 4440y^2G(1;y)^3G(r,1;y) + 4440yG(1;y)^3G(r,1;y) -$$
$$2040G(1;y)^3G(r,1;y) - 360\pi^2y^2G(r,1;y) + 18840y^2G(r,1;y) - 8460y^3G(1;y)^2G(r,1;y) + 9180y^2G(1;y)^2G(r,1;y) -$$
$$9180yG(1;y)^2G(r,1;y) + 8460G(1;y)^2G(r,1;y) - 720y^2G(r;y)^2G(r,1;y) + 720yG(r;y)^2G(r,1;y) - 1440y^3G(1;y)G(r;y)^2G(r,1;y) +$$
$$1440G(1;y)G(r;y)^2G(r,1;y) + 360\pi^2yG(r,1;y) - 18840yG(r,1;y) - 720\pi^2y^3G(1;y)G(r,1;y) - 13920y^3G(1;y)G(r,1;y) +$$
$$720\pi^2G(1;y)G(r,1;y) + 38880y^2G(1;y)G(r,1;y) - 38880yG(1;y)G(r,1;y) + 13920G(1;y)G(r,1;y) + 720y^3G(r;y)G(r,1;y) -$$
$$5040y^2G(r;y)G(r,1;y) + 10440y^3G(1;y)^2G(r;y)G(r,1;y) - 13320y^2G(1;y)^2G(r;y)G(r,1;y) + 13320yG(1;y)^2G(r;y)G(r,1;y) -$$
$$10440G(1;y)^2G(r;y)G(r,1;y) + 5040yG(r;y)G(r,1;y) - 1800y^3G(1;y)G(r;y)G(r,1;y) + 24840y^2G(1;y)G(r;y)G(r,1;y) -$$
$$24840yG(1;y)G(r;y)G(r,1;y) + 1800G(1;y)G(r;y)G(r,1;y) - 720G(r;y)G(r,1;y) - 4320y^2G(0,1;y)G(r,1;y) + 4320yG(0,1;y)G(r,1;y) -$$

$$8640y^3G(1;y)G(0,1;y)G(r,1;y) + 8640G(1;y)G(0,1;y)G(r,1;y) + 2160y^2G(0,r;y)G(r,1;y) - 2160yG(0,r;y)G(r,1;y) +$$
$$4320y^3G(1;y)G(0,r;y)G(r,1;y) - 4320G(1;y)G(0,r;y)G(r,1;y) + 6120G(r,1;y) - 720\pi^2y^3G(0,-1,-1;y) - 1200\pi^2G(0,-1,-1;y) -$$
$$1200\pi^2y^2G(0,-1,-1;y) - 720\pi^2yG(0,-1,-1;y) - 11520y^3G(0,1;y)G(0,-1,-1;y) - 11520y^2G(0,1;y)G(0,-1,-1;y) -$$
$$11520yG(0,1;y)G(0,-1,-1;y) - 11520G(0,1;y)G(0,-1,-1;y) + 10080y^3G(0,r;y)G(0,-1,-1;y) + 1440y^2G(0,r;y)G(0,-1,-1;y) +$$
$$10080yG(0,r;y)G(0,-1,-1;y) + 1440G(0,r;y)G(0,-1,-1;y) + 720\pi^2y^3G(0,-1,1;y) + 240\pi^2G(0,-1,1;y) + 240\pi^2y^2G(0,-1,1;y) +$$
$$720\pi^2yG(0,-1,1;y) + 5760y^3G(0,1;y)G(0,-1,1;y) + 5760y^2G(0,1;y)G(0,-1,1;y) + 5760yG(0,1;y)G(0,-1,1;y) +$$
$$5760G(0,1;y)G(0,-1,1;y) + 1440y^3G(0,r;y)G(0,-1,1;y) - 7200y^2G(0,r;y)G(0,-1,1;y) + 1440yG(0,r;y)G(0,-1,1;y) -$$
$$7200G(0,r;y)G(0,-1,1;y) - 540\pi^2y^3G(0,-1,r;y) - 6120y^3G(0,-1,r;y) - 540\pi^2G(0,-1,r;y) - 540\pi^2y^2G(0,-1,r;y) +$$
$$11880y^2G(0,-1,r;y) - 540\pi^2yG(0,-1,r;y) + 11880yG(0,-1,r;y) + 15120y^3G(-1;y)G(0,-1,r;y) + 15120y^2G(-1;y)G(0,-1,r;y) +$$
$$15120yG(-1;y)G(0,-1,r;y) + 15120G(-1;y)G(0,-1,r;y) + 8640y^3G(-1;y)G(1;y)G(0,-1,r;y) + 8640y^2G(-1;y)G(1;y)G(0,-1,r;y) +$$
$$8640yG(-1;y)G(1;y)G(0,-1,r;y) + 8640G(-1;y)G(1;y)G(0,-1,r;y) - 6480y^3G(0,1;y)G(0,-1,r;y) - 6480y^2G(0,1;y)G(0,-1,r;y) -$$
$$6480yG(0,1;y)G(0,-1,r;y) - 6480G(0,1;y)G(0,-1,r;y) + 3240y^3G(0,r;y)G(0,-1,r;y) + 3240y^2G(0,r;y)G(0,-1,r;y) +$$
$$3240yG(0,r;y)G(0,-1,r;y) + 3240G(0,r;y)G(0,-1,r;y) - 6120G(0,-1,r;y) - 1380\pi^2y^3G(0,0,-1;y) - 6120y^3G(0,0,-1;y) +$$
$$60\pi^2G(0,0,-1;y) + 60\pi^2y^2G(0,0,-1;y) + 11880y^2G(0,0,-1;y) - 1380\pi^2yG(0,0,-1;y) + 11880yG(0,0,-1;y) +$$
$$15120y^3G(-1;y)G(0,0,-1;y) + 15120y^2G(-1;y)G(0,0,-1;y) + 15120yG(-1;y)G(0,0,-1;y) + 15120G(-1;y)G(0,0,-1;y) +$$
$$8640y^3G(-1;y)G(1;y)G(0,0,-1;y) + 8640y^2G(-1;y)G(1;y)G(0,0,-1;y) + 8640yG(-1;y)G(1;y)G(0,0,-1;y) +$$
$$8640G(-1;y)G(1;y)G(0,0,-1;y) - 4320y^3G(0,-1;y)G(0,0,-1;y) + 4320y^2G(0,-1;y)G(0,0,-1;y) - 4320yG(0,-1;y)G(0,0,-1;y) +$$
$$4320G(0,-1;y)G(0,0,-1;y) - 4320y^3G(0,1;y)G(0,0,-1;y) + 4320y^2G(0,1;y)G(0,0,-1;y) - 4320yG(0,1;y)G(0,0,-1;y) +$$
$$4320G(0,1;y)G(0,0,-1;y) - 6120G(0,0,-1;y) - 60\pi^2y^3G(0,0,1;y) - 13200y^3G(0,0,1;y) + 540\pi^2G(0,0,1;y) + 3780\pi^2y^2G(0,0,1;y) +$$
$$10800y^2G(0,0,1;y) - 7200y^3G(-1;y)^2G(0,0,1;y) - 7200y^2G(-1;y)^2G(0,0,1;y) - 7200yG(-1;y)^2G(0,0,1;y) - 7200G(-1;y)^2G(0,0,1;y) -$$
$$4500y^3G(1;y)^2G(0,0,1;y) - 12060y^2G(1;y)^2G(0,0,1;y) + 12060yG(1;y)^2G(0,0,1;y) + 4500G(1;y)^2G(0,0,1;y) - 3300\pi^2yG(0,0,1;y) +$$
$$28080yG(0,0,1;y) + 15120y^3G(-1;y)G(0,0,1;y) + 15120y^2G(-1;y)G(0,0,1;y) + 15120yG(-1;y)G(0,0,1;y) + 15120G(-1;y)G(0,0,1;y) -$$
$$17280y^3G(1;y)G(0,0,1;y) + 16560y^2G(1;y)G(0,0,1;y) + 3240yG(1;y)G(0,0,1;y) + 8640y^3G(-1;y)G(1;y)G(0,0,1;y) +$$
$$8640y^2G(-1;y)G(1;y)G(0,0,1;y) + 8640yG(-1;y)G(1;y)G(0,0,1;y) + 8640G(-1;y)G(1;y)G(0,0,1;y) - 2520G(1;y)G(0,0,1;y) +$$
$$8640y^3G(0,-1;y)G(0,0,1;y) + 2880y^2G(0,-1;y)G(0,0,1;y) + 8640yG(0,-1;y)G(0,0,1;y) + 2880G(0,-1;y)G(0,0,1;y) +$$
$$5400y^3G(0,1;y)G(0,0,1;y) - 14760y^2G(0,1;y)G(0,0,1;y) + 27000yG(0,1;y)G(0,0,1;y) + 6840G(0,1;y)G(0,0,1;y) -$$
$$10320G(0,0,1;y) - 180\pi^2y^3G(0,0,r;y) + 5520y^3G(0,0,r;y) + 180\pi^2y^2G(0,0,r;y) - 2160y^2G(0,0,r;y) + 7200y^3G(-1;y)^2G(0,0,r;y) +$$
$$7200y^2G(-1;y)^2G(0,0,r;y) + 7200yG(-1;y)^2G(0,0,r;y) + 7200G(-1;y)^2G(0,0,r;y) + 3060y^3G(1;y)^2G(0,0,r;y) +$$
$$19260y^2G(1;y)^2G(0,0,r;y) - 19260yG(1;y)^2G(0,0,r;y) - 3060G(1;y)^2G(0,0,r;y) - 360\pi^2yG(0,0,r;y) - 1080yG(0,0,r;y) +$$
$$7560y^3G(-1;y)G(0,0,r;y) + 7560y^2G(-1;y)G(0,0,r;y) + 7560yG(-1;y)G(0,0,r;y) + 7560G(-1;y)G(0,0,r;y) +$$
$$17640y^3G(1;y)G(0,0,r;y) - 40680y^2G(1;y)G(0,0,r;y) + 20880yG(1;y)G(0,0,r;y) + 4320y^3G(-1;y)G(1;y)G(0,0,r;y) +$$
$$4320y^2G(-1;y)G(1;y)G(0,0,r;y) + 4320yG(-1;y)G(1;y)G(0,0,r;y) + 4320G(-1;y)G(1;y)G(0,0,r;y) + 2160G(1;y)G(0,0,r;y) -$$
$$15840y^3G(0,-1;y)G(0,0,r;y) + 2880y^2G(0,-1;y)G(0,0,r;y) - 15840yG(0,-1;y)G(0,0,r;y) + 2880G(0,-1;y)G(0,0,r;y) -$$
$$13320y^3G(0,1;y)G(0,0,r;y) + 19800y^2G(0,1;y)G(0,0,r;y) - 34920yG(0,1;y)G(0,0,r;y) - 1800G(0,1;y)G(0,0,r;y) - 360G(0,0,r;y) +$$
$$720\pi^2y^3G(0,1,-1;y) + 240\pi^2G(0,1,-1;y) + 240\pi^2y^2G(0,1,-1;y) + 720\pi^2yG(0,1,-1;y) + 5760y^3G(0,1;y)G(0,1,-1;y) +$$
$$5760y^2G(0,1;y)G(0,1,-1;y) + 5760yG(0,1;y)G(0,1,-1;y) + 5760G(0,1;y)G(0,1,-1;y) - 17280y^2G(0,r;y)G(0,1,-1;y) -$$
$$17280G(0,r;y)G(0,1,-1;y) + 180\pi^2y^3G(0,1,1;y) - 13920y^3G(0,1,1;y) + 1140\pi^2G(0,1,1;y) + 1140\pi^2y^2G(0,1,1;y) +$$
$$28800y^2G(0,1,1;y) + 18360y^3G(1;y)^2G(0,1,1;y) - 39960y^2G(1;y)^2G(0,1,1;y) + 39960yG(1;y)^2G(0,1,1;y) - 18360G(1;y)^2G(0,1,1;y) +$$
$$180\pi^2yG(0,1,1;y) - 28800yG(0,1,1;y) - 20520y^3G(1;y)G(0,1,1;y) + 59400y^2G(1;y)G(0,1,1;y) - 59400yG(1;y)G(0,1,1;y) +$$
$$20520G(1;y)G(0,1,1;y) - 17280y^3G(0,-1;y)G(0,1,1;y) - 17280y^2G(0,-1;y)G(0,1,1;y) - 17280yG(0,-1;y)G(0,1,1;y) -$$
$$17280G(0,-1;y)G(0,1,1;y) - 720y^3G(0,1;y)G(0,1,1;y) + 7920y^2G(0,1;y)G(0,1,1;y) + 7920yG(0,1;y)G(0,1,1;y) +$$
$$16560G(0,1;y)G(0,1,1;y) + 13920G(0,1,1;y) - 360\pi^2y^3G(0,1,r;y) - 5520y^3G(0,1,r;y) - 360\pi^2G(0,1,r;y) - 360\pi^2y^2G(0,1,r;y) +$$
$$15840y^2G(0,1,r;y) - 1440y^3G(-1;y)^2G(0,1,r;y) - 1440y^2G(-1;y)^2G(0,1,r;y) - 1440yG(-1;y)^2G(0,1,r;y) -$$
$$1440G(-1;y)^2G(0,1,r;y) - 14760y^3G(1;y)^2G(0,1,r;y) + 13320y^2G(1;y)^2G(0,1,r;y) - 13320yG(1;y)^2G(0,1,r;y) +$$
$$14760G(1;y)^2G(0,1,r;y) - 360\pi^2yG(0,1,r;y) - 2880yG(0,1,r;y) + 15120y^3G(-1;y)G(0,1,r;y) + 15120y^2G(-1;y)G(0,1,r;y) +$$
$$15120yG(-1;y)G(0,1,r;y) + 15120G(-1;y)G(0,1,r;y) + 5760y^3G(1;y)G(0,1,r;y) - 33120y^2G(1;y)G(0,1,r;y) +$$
$$25200yG(1;y)G(0,1,r;y) + 8640y^3G(-1;y)G(1;y)G(0,1,r;y) + 8640y^2G(-1;y)G(1;y)G(0,1,r;y) + 8640yG(-1;y)G(1;y)G(0,1,r;y) +$$
$$8640G(-1;y)G(1;y)G(0,1,r;y) + 2160G(1;y)G(0,1,r;y) + 14400y^3G(0,-1;y)G(0,1,r;y) + 20160y^2G(0,-1;y)G(0,1,r;y) +$$
$$14400yG(0,-1;y)G(0,1,r;y) + 20160G(0,-1;y)G(0,1,r;y) - 4320y^3G(0,1;y)G(0,1,r;y) - 4320y^2G(0,1;y)G(0,1,r;y) -$$
$$4320yG(0,1;y)G(0,1,r;y) - 4320G(0,1;y)G(0,1,r;y) + 240G(0,1,r;y) - 540\pi^2y^3G(0,r,-1;y) - 6120y^3G(0,r,-1;y) - 540\pi^2G(0,r,-1;y) -$$
$$540\pi^2y^2G(0,r,-1;y) + 11880y^2G(0,r,-1;y) - 540\pi^2yG(0,r,-1;y) + 11880yG(0,r,-1;y) + 15120y^3G(-1;y)G(0,r,-1;y) +$$
$$15120y^2G(-1;y)G(0,r,-1;y) + 15120yG(-1;y)G(0,r,-1;y) + 15120G(-1;y)G(0,r,-1;y) + 8640y^3G(-1;y)G(1;y)G(0,r,-1;y) +$$
$$8640y^2G(-1;y)G(1;y)G(0,r,-1;y) + 8640yG(-1;y)G(1;y)G(0,r,-1;y) + 8640G(-1;y)G(1;y)G(0,r,-1;y) -$$
$$4320y^3G(0,-1;y)G(0,r,-1;y) + 4320y^2G(0,-1;y)G(0,r,-1;y) - 4320yG(0,-1;y)G(0,r,-1;y) + 4320G(0,-1;y)G(0,r,-1;y) -$$
$$6480y^3G(0,1;y)G(0,r,-1;y) - 6480y^2G(0,1;y)G(0,r,-1;y) - 6480yG(0,1;y)G(0,r,-1;y) - 6480G(0,1;y)G(0,r,-1;y) +$$
$$3240y^3G(0,r;y)G(0,r,-1;y) + 3240y^2G(0,r;y)G(0,r,-1;y) + 3240yG(0,r;y)G(0,r,-1;y) + 3240G(0,r;y)G(0,r,-1;y) -$$
$$6120G(0,r,-1;y) + 360\pi^2y^3G(0,r,1;y) + 13440y^3G(0,r,1;y) - 360\pi^2G(0,r,1;y) - 20880y^2G(0,r,1;y) + 4320y^3G(-1;y)^2G(0,r,1;y) +$$
$$4320y^2G(-1;y)^2G(0,r,1;y) + 4320yG(-1;y)^2G(0,r,1;y) + 4320G(-1;y)^2G(0,r,1;y) - 9180y^3G(1;y)^2G(0,r,1;y) +$$
$$19980y^2G(1;y)^2G(0,r,1;y) - 19980yG(1;y)^2G(0,r,1;y) + 9180G(1;y)^2G(0,r,1;y) + 7920yG(0,r,1;y) + 15120y^3G(-1;y)G(0,r,1;y) +$$
$$15120y^2G(-1;y)G(0,r,1;y) + 15120yG(-1;y)G(0,r,1;y) + 15120G(-1;y)G(0,r,1;y) + 28080y^3G(1;y)G(0,r,1;y) -$$

$$47520y^2G(1;y)G(0,r,1;y) + 11880yG(1;y)G(0,r,1;y) + 8640y^3G(-1;y)G(1;y)G(0,r,1;y) + 8640y^2G(-1;y)G(1;y)G(0,r,1;y) +$$
$$8640yG(-1;y)G(1;y)G(0,r,1;y) + 8640G(-1;y)G(1;y)G(0,r,1;y) + 7560G(1;y)G(0,r,1;y) + 17280y^2G(0,-1;y)G(0,r,1;y) +$$
$$17280G(0,-1;y)G(0,r,1;y) + 4320y^3G(0,1;y)G(0,r,1;y) - 4320G(0,1;y)G(0,r,1;y) - 2160y^3G(0,r;y)G(0,r,1;y) +$$
$$2160G(0,r;y)G(0,r,1;y) - 480G(0,r,1;y) - 720y^3G(0,r,r;y) - 2520y^2G(0,r,r;y) + 2160y^3G(-1;y)^2G(0,r,r;y) +$$
$$2160y^2G(-1;y)^2G(0,r,r;y) + 2160yG(-1;y)^2G(0,r,r;y) + 2160G(-1;y)^2G(0,r,r;y) + 7380y^3G(1;y)^2G(0,r,r;y) -$$
$$6660y^2G(1;y)^2G(0,r,r;y) + 6660yG(1;y)^2G(0,r,r;y) - 7380G(1;y)^2G(0,r,r;y) + 6840yG(0,r,r;y) - 7560y^3G(-1;y)G(0,r,r;y) -$$
$$7560y^2G(-1;y)G(0,r,r;y) - 7560yG(-1;y)G(0,r,r;y) - 7560G(-1;y)G(0,r,r;y) - 6840y^3G(1;y)G(0,r,r;y) + 20520y^2G(1;y)G(0,r,r;y) -$$
$$8640yG(1;y)G(0,r,r;y) - 4320y^3G(-1;y)G(1;y)G(0,r,r;y) - 4320y^2G(-1;y)G(1;y)G(0,r,r;y) - 4320yG(-1;y)G(1;y)G(0,r,r;y) -$$
$$4320G(-1;y)G(1;y)G(0,r,r;y) - 5040G(1;y)G(0,r,r;y) - 8640y^3G(0,-1;y)G(0,r,r;y) - 8640y^2G(0,-1;y)G(0,r,r;y) -$$
$$8640yG(0,-1;y)G(0,r,r;y) - 8640G(0,-1;y)G(0,r,r;y) - 3600G(0,r,r;y) + 720\pi^2y^3G(r,1,1;y) + 13920y^3G(r,1,1;y) - 720\pi^2G(r,1,1;y) -$$
$$28800y^2G(r,1,1;y) - 18360y^3G(1;y)^2G(r,1,1;y) + 39960y^2G(1;y)^2G(r,1,1;y) - 39960yG(1;y)^2G(r,1,1;y) + 18360G(1;y)^2G(r,1,1;y) +$$
$$1440y^3G(r;y)^2G(r,1,1;y) - 1440G(r;y)^2G(r,1,1;y) + 28800yG(r,1,1;y) + 20520y^3G(1;y)G(r,1,1;y) - 59400y^2G(1;y)G(r,1,1;y) +$$
$$59400yG(1;y)G(r,1,1;y) - 20520G(1;y)G(r,1,1;y) + 1800y^3G(r;y)G(r,1,1;y) - 26280y^2G(r;y)G(r,1,1;y) + 26280yG(r;y)G(r,1,1;y) -$$
$$23760y^3G(1;y)G(r;y)G(r,1,1;y) + 26640y^2G(1;y)G(r;y)G(r,1,1;y) - 26640yG(1;y)G(r;y)G(r,1,1;y) + 23760G(1;y)G(r;y)G(r,1,1;y) -$$
$$1800G(r;y)G(r,1,1;y) + 8640y^3G(0,1;y)G(r,1,1;y) - 8640G(0,1;y)G(r,1,1;y) - 4320y^3G(0,r;y)G(r,1,1;y) + 4320G(0,r;y)G(r,1,1;y) -$$
$$8640y^3G(r,1;y)G(r,1,1;y) + 8640G(r,1;y)G(r,1,1;y) - 13920G(r,1,1;y) - 2880y^3G(r,r,1;y) + 9360y^2G(r,r,1;y) -$$
$$14760y^3G(1;y)^2G(r,r,1;y) + 13320y^2G(1;y)^2G(r,r,1;y) - 13320yG(1;y)^2G(r,r,1;y) + 14760G(1;y)^2G(r,r,1;y) - 9360yG(r,r,1;y) +$$
$$1800y^3G(1;y)G(r,r,1;y) - 29160y^2G(1;y)G(r,r,1;y) + 29160yG(1;y)G(r,r,1;y) - 1800G(1;y)G(r,r,1;y) + 1440y^2G(r;y)G(r,r,1;y) -$$
$$1440yG(r;y)G(r,r,1;y) + 2880y^3G(1;y)G(r;y)G(r,r,1;y) - 2880G(1;y)G(r;y)G(r,r,1;y) + 2880G(r,r,1;y) - 15120y^3G(0,-1,-1,r;y) -$$
$$15120y^2G(0,-1,-1,r;y) - 15120yG(0,-1,-1,r;y) - 8640y^3G(1;y)G(0,-1,-1,r;y) - 8640y^2G(1;y)G(0,-1,-1,r;y) -$$
$$8640yG(1;y)G(0,-1,-1,r;y) - 8640G(1;y)G(0,-1,-1,r;y) - 15120G(0,-1,-1,r;y) - 15120y^3G(0,-1,1,r;y) -$$
$$15120y^2G(0,-1,1,r;y) - 15120yG(0,-1,1,r;y) + 2880y^3G(-1;y)G(0,-1,1,r;y) + 2880y^2G(-1;y)G(0,-1,1,r;y) +$$
$$2880yG(-1;y)G(0,-1,1,r;y) + 2880G(-1;y)G(0,-1,1,r;y) - 8640y^3G(1;y)G(0,-1,1,r;y) - 8640y^2G(1;y)G(0,-1,1,r;y) -$$
$$8640yG(1;y)G(0,-1,1,r;y) - 8640G(1;y)G(0,-1,1,r;y) - 15120G(0,-1,1,r;y) - 15120y^3G(0,-1,r,-1;y) - 15120y^2G(0,-1,r,-1;y) -$$
$$15120yG(0,-1,r,-1;y) - 8640y^3G(1;y)G(0,-1,r,-1;y) - 8640y^2G(1;y)G(0,-1,r,-1;y) - 8640yG(1;y)G(0,-1,r,-1;y) -$$
$$8640G(1;y)G(0,-1,r,-1;y) - 15120G(0,-1,r,-1;y) - 15120y^3G(0,-1,r,1;y) - 15120y^2G(0,-1,r,1;y) - 15120yG(0,-1,r,1;y) -$$
$$8640y^3G(-1;y)G(0,-1,r,1;y) - 8640y^2G(-1;y)G(0,-1,r,1;y) - 8640yG(-1;y)G(0,-1,r,1;y) - 8640G(-1;y)G(0,-1,r,1;y) -$$
$$8640y^3G(1;y)G(0,-1,r,1;y) - 8640y^2G(1;y)G(0,-1,r,1;y) - 8640yG(1;y)G(0,-1,r,1;y) - 8640G(1;y)G(0,-1,r,1;y) -$$
$$15120G(0,-1,r,1;y) + 7560y^3G(0,-1,r,r;y) + 7560y^2G(0,-1,r,r;y) + 7560yG(0,-1,r,r;y) - 4320y^3G(-1;y)G(0,-1,r,r;y) -$$
$$4320y^2G(-1;y)G(0,-1,r,r;y) - 4320yG(-1;y)G(0,-1,r,r;y) - 4320G(-1;y)G(0,-1,r,r;y) + 4320y^3G(1;y)G(0,-1,r,r;y) +$$
$$4320y^2G(1;y)G(0,-1,r,r;y) + 4320yG(1;y)G(0,-1,r,r;y) + 4320G(1;y)G(0,-1,r,r;y) + 7560G(0,-1,r,r;y) - 15120y^3G(0,0,-1,-1;y) -$$
$$15120y^2G(0,0,-1,-1;y) - 15120yG(0,0,-1,-1;y) - 8640y^3G(1;y)G(0,0,-1,-1;y) - 8640y^2G(1;y)G(0,0,-1,-1;y) -$$
$$8640yG(1;y)G(0,0,-1,-1;y) - 8640G(1;y)G(0,0,-1,-1;y) - 15120G(0,0,-1,-1;y) - 15120y^3G(0,0,-1,1;y) - 15120y^2G(0,0,-1,1;y) -$$
$$15120yG(0,0,-1,1;y) - 8640y^3G(-1;y)G(0,0,-1,1;y) - 8640y^2G(-1;y)G(0,0,-1,1;y) - 8640yG(-1;y)G(0,0,-1,1;y) -$$
$$8640G(-1;y)G(0,0,-1,1;y) - 8640y^3G(1;y)G(0,0,-1,1;y) - 8640y^2G(1;y)G(0,0,-1,1;y) - 8640yG(1;y)G(0,0,-1,1;y) -$$
$$8640G(1;y)G(0,0,-1,1;y) - 15120G(0,0,-1,1;y) - 22680y^3G(0,0,-1,r;y) - 22680y^2G(0,0,-1,r;y) + 7560yG(0,0,-1,r;y) -$$
$$2880y^3G(-1;y)G(0,0,-1,r;y) - 2880y^2G(-1;y)G(0,0,-1,r;y) - 2880yG(-1;y)G(0,0,-1,r;y) - 2880G(-1;y)G(0,0,-1,r;y) -$$
$$4320y^3G(1;y)G(0,0,-1,r;y) - 4320y^2G(1;y)G(0,0,-1,r;y) - 4320yG(1;y)G(0,0,-1,r;y) - 4320G(1;y)G(0,0,-1,r;y) +$$
$$7560G(0,0,-1,r;y) - 55440y^3G(0,0,0,-1;y) - 55440y^2G(0,0,0,-1;y) - 10080yG(0,0,0,-1;y) - 7200y^3G(-1;y)G(0,0,0,-1;y) -$$
$$7200y^2G(-1;y)G(0,0,0,-1;y) - 7200yG(-1;y)G(0,0,0,-1;y) - 7200G(-1;y)G(0,0,0,-1;y) - 18720y^3G(1;y)G(0,0,0,-1;y) -$$
$$18720y^2G(1;y)G(0,0,0,-1;y) - 18720yG(1;y)G(0,0,0,-1;y) - 18720G(1;y)G(0,0,0,-1;y) - 10080G(0,0,0,-1;y) -$$
$$21240y^3G(0,0,0,1;y) - 87480y^2G(0,0,0,1;y) - 13680yG(0,0,0,1;y) - 15120y^3G(-1;y)G(0,0,0,1;y) - 15120y^2G(-1;y)G(0,0,0,1;y) -$$
$$15120yG(-1;y)G(0,0,0,1;y) - 15120G(-1;y)G(0,0,0,1;y) - 4680y^3G(1;y)G(0,0,0,1;y) + 9000y^2G(1;y)G(0,0,0,1;y) -$$
$$46440yG(1;y)G(0,0,0,1;y) - 32760G(1;y)G(0,0,0,1;y) - 8640G(0,0,0,1;y) - 25200y^3G(0,0,0,r;y) + 31680y^2G(0,0,0,r;y) -$$
$$1440yG(0,0,0,r;y) - 1440y^3G(-1;y)G(0,0,0,r;y) - 1440y^2G(-1;y)G(0,0,0,r;y) - 1440yG(-1;y)G(0,0,0,r;y) -$$
$$1440G(-1;y)G(0,0,0,r;y) - 2520y^3G(1;y)G(0,0,0,r;y) - 32040y^2G(1;y)G(0,0,0,r;y) + 33480yG(1;y)G(0,0,0,r;y) +$$
$$3960G(1;y)G(0,0,0,r;y) - 15120y^3G(0,0,1,-1;y) - 15120y^2G(0,0,1,-1;y) - 15120yG(0,0,1,-1;y) - 8640y^3G(-1;y)G(0,0,1,-1;y) -$$
$$8640y^2G(-1;y)G(0,0,1,-1;y) - 8640yG(-1;y)G(0,0,1,-1;y) - 8640G(-1;y)G(0,0,1,-1;y) - 8640y^3G(1;y)G(0,0,1,-1;y) -$$
$$8640y^2G(1;y)G(0,0,1,-1;y) - 8640yG(1;y)G(0,0,1,-1;y) - 8640G(1;y)G(0,0,1,-1;y) - 15120G(0,0,1,-1;y) +$$
$$12960y^3G(0,0,1,1;y) - 59760y^2G(0,0,1,1;y) - 24120yG(0,0,1,1;y) + 5760y^3G(-1;y)G(0,0,1,1;y) + 5760y^2G(-1;y)G(0,0,1,1;y) +$$
$$5760yG(-1;y)G(0,0,1,1;y) + 5760G(-1;y)G(0,0,1,1;y) - 16200y^3G(1;y)G(0,0,1,1;y) + 17640y^2G(1;y)G(0,0,1,1;y) -$$
$$40680yG(1;y)G(0,0,1,1;y) - 6840G(1;y)G(0,0,1,1;y) - 9720G(0,0,1,1;y) - 24120y^3G(0,0,1,r;y) + 52920y^2G(0,0,1,r;y) -$$
$$31680yG(0,0,1,r;y) - 26640y^3G(-1;y)G(0,0,1,r;y) - 26640y^2G(-1;y)G(0,0,1,r;y) - 26640yG(-1;y)G(0,0,1,r;y) -$$
$$26640G(-1;y)G(0,0,1,r;y) + 12960y^3G(1;y)G(0,0,1,r;y) - 43200y^2G(1;y)G(0,0,1,r;y) + 41760yG(1;y)G(0,0,1,r;y) -$$
$$14400G(1;y)G(0,0,1,r;y) - 2160G(0,0,1,r;y) - 22680y^3G(0,0,r,-1;y) - 22680y^2G(0,0,r,-1;y) + 7560yG(0,0,r,-1;y) -$$
$$2880y^3G(-1;y)G(0,0,r,-1;y) - 2880y^2G(-1;y)G(0,0,r,-1;y) - 2880yG(-1;y)G(0,0,r,-1;y) - 2880G(-1;y)G(0,0,r,-1;y) -$$
$$4320y^3G(1;y)G(0,0,r,-1;y) - 4320y^2G(1;y)G(0,0,r,-1;y) - 4320yG(1;y)G(0,0,r,-1;y) - 4320G(1;y)G(0,0,r,-1;y) +$$
$$7560G(0,0,r,-1;y) - 46440y^3G(0,0,r,1;y) + 70920y^2G(0,0,r,1;y) - 32400yG(0,0,r,1;y) - 18720y^3G(-1;y)G(0,0,r,1;y) -$$
$$18720y^2G(-1;y)G(0,0,r,1;y) - 18720yG(-1;y)G(0,0,r,1;y) - 18720G(-1;y)G(0,0,r,1;y) + 4680y^3G(1;y)G(0,0,r,1;y) -$$
$$69480y^2G(1;y)G(0,0,r,1;y) + 66600yG(1;y)G(0,0,r,1;y) - 7560G(1;y)G(0,0,r,1;y) - 2160G(0,0,r,1;y) + 14400y^3G(0,0,r,r;y) -$$

$16560y^2G(0,0,r,r;y)+7200yG(0,0,r,r;y)+8640y^3G(-1;y)G(0,0,r,r;y)+8640y^2G(-1;y)G(0,0,r,r;y)+8640yG(-1;y)G(0,0,r,r;y)+$
$8640G(-1;y)G(0,0,r,r;y) - 8280y^3G(1;y)G(0,0,r,r;y) + 15480y^2G(1;y)G(0,0,r,r;y) - 14040yG(1;y)G(0,0,r,r;y) +$
$9720G(1;y)G(0,0,r,r;y) - 15120y^3G(0,1,-1,r;y) - 15120y^2G(0,1,-1,r;y) - 15120yG(0,1,-1,r;y) + 2880y^3G(-1;y)G(0,1,-1,r;y)+$
$2880y^2G(-1;y)G(0,1,-1,r;y) + 2880yG(-1;y)G(0,1,-1,r;y) + 2880G(-1;y)G(0,1,-1,r;y) - 8640y^3G(1;y)G(0,1,-1,r;y) -$
$8640y^2G(1;y)G(0,1,-1,r;y) - 8640yG(1;y)G(0,1,-1,r;y) - 8640G(1;y)G(0,1,-1,r;y) - 15120G(0,1,-1,r;y) -$
$11520y^3G(-1;y)G(0,1,0,-1;y) - 11520y^2G(-1;y)G(0,1,0,-1;y) - 11520yG(-1;y)G(0,1,0,-1;y) - 11520G(-1;y)G(0,1,0,-1;y) -$
$3240y^3G(0,1,0,r;y) + 34920y^2G(0,1,0,r;y) - 24480yG(0,1,0,r;y) - 3600y^3G(-1;y)G(0,1,0,r;y) - 3600y^2G(-1;y)G(0,1,0,r;y) -$
$3600yG(-1;y)G(0,1,0,r;y) - 3600G(-1;y)G(0,1,0,r;y) - 1440y^3G(1;y)G(0,1,0,r;y) - 23040y^2G(1;y)G(0,1,0,r;y) +$
$24480yG(1;y)G(0,1,0,r;y) + 2880G(1;y)G(0,1,0,r;y) - 2160G(0,1,0,r;y) + 24120y^3G(0,1,1,1;y) - 100440y^2G(0,1,1,1;y) +$
$100440yG(0,1,1,1;y) - 61200y^3G(1;y)G(0,1,1,1;y) + 133200y^2G(1;y)G(0,1,1,1;y) - 133200yG(1;y)G(0,1,1,1;y) +$
$61200G(1;y)G(0,1,1,1;y) - 24120G(0,1,1,1;y) - 10440y^3G(0,1,1,r;y) + 9720y^2G(0,1,1,r;y) - 42840yG(0,1,1,r;y) -$
$11520y^3G(-1;y)G(0,1,1,r;y) - 11520y^2G(-1;y)G(0,1,1,r;y) - 11520yG(-1;y)G(0,1,1,r;y) - 11520G(-1;y)G(0,1,1,r;y) +$
$16560y^3G(1;y)G(0,1,1,r;y) - 33840y^2G(1;y)G(0,1,1,r;y) + 19440yG(1;y)G(0,1,1,r;y) - 30960G(1;y)G(0,1,1,r;y) -$
$6840G(0,1,1,r;y) - 15120y^3G(0,1,r,-1;y) - 15120y^2G(0,1,r,-1;y) - 15120yG(0,1,r,-1;y) + 2880y^3G(-1;y)G(0,1,r,-1;y) +$
$2880y^2G(-1;y)G(0,1,r,-1;y) + 2880yG(-1;y)G(0,1,r,-1;y) + 2880G(-1;y)G(0,1,r,-1;y) - 8640y^3G(1;y)G(0,1,r,-1;y) -$
$8640y^2G(1;y)G(0,1,r,-1;y) - 8640yG(1;y)G(0,1,r,-1;y) - 8640G(1;y)G(0,1,r,-1;y) - 15120G(0,1,r,-1;y) - 15840y^3G(0,1,r,1;y)+$
$20160y^2G(0,1,r,1;y) - 32400yG(0,1,r,1;y) - 5760y^3G(-1;y)G(0,1,r,1;y) - 5760y^2G(-1;y)G(0,1,r,1;y) - 5760yG(-1;y)G(0,1,r,1;y) -$
$5760G(-1;y)G(0,1,r,1;y) + 18000y^3G(1;y)G(0,1,r,1;y) - 32400y^2G(1;y)G(0,1,r,1;y) + 20880yG(1;y)G(0,1,r,1;y) -$
$29520G(1;y)G(0,1,r,1;y) - 12240G(0,1,r,1;y) + 5040y^3G(0,1,r,r;y) + 6480y^2G(0,1,r,r;y) + 3600yG(0,1,r,r;y) -$
$4320y^3G(-1;y)G(0,1,r,r;y) - 4320y^2G(-1;y)G(0,1,r,r;y) - 4320yG(-1;y)G(0,1,r,r;y) - 4320G(-1;y)G(0,1,r,r;y) +$
$5760y^3G(1;y)G(0,1,r,r;y) + 2880y^2G(1;y)G(0,1,r,r;y) + 2880yG(1;y)G(0,1,r,r;y) + 5040G(0,1,r,r;y) - 15120y^3G(0,r,-1,-1;y) -$
$15120y^2G(0,r,-1,-1;y) - 15120yG(0,r,-1,-1;y) - 8640y^3G(1;y)G(0,r,-1,-1;y) - 8640y^2G(1;y)G(0,r,-1,-1;y) -$
$8640yG(1;y)G(0,r,-1,-1;y) - 8640G(1;y)G(0,r,-1,-1;y) - 15120G(0,r,-1,-1;y) - 15120y^3G(0,r,-1,1;y) -$
$15120y^2G(0,r,-1,1;y) - 15120yG(0,r,-1,1;y) - 8640y^3G(-1;y)G(0,r,-1,1;y) - 8640y^2G(-1;y)G(0,r,-1,1;y) -$
$8640yG(-1;y)G(0,r,-1,1;y) - 8640G(-1;y)G(0,r,-1,1;y) - 8640y^3G(1;y)G(0,r,-1,1;y) - 8640y^2G(1;y)G(0,r,-1,1;y) -$
$8640yG(1;y)G(0,r,-1,1;y) - 8640G(1;y)G(0,r,-1,1;y) - 15120G(0,r,-1,1;y) + 7560y^3G(0,r,-1,r;y) + 7560y^2G(0,r,-1,r;y) +$
$7560yG(0,r,-1,r;y) - 4320y^3G(-1;y)G(0,r,-1,r;y) - 4320y^2G(-1;y)G(0,r,-1,r;y) - 4320yG(-1;y)G(0,r,-1,r;y) -$
$4320G(-1;y)G(0,r,-1,r;y) + 4320y^3G(1;y)G(0,r,-1,r;y) + 4320y^2G(1;y)G(0,r,-1,r;y) + 4320yG(1;y)G(0,r,-1,r;y) +$
$4320G(1;y)G(0,r,-1,r;y) + 7560G(0,r,-1,r;y) - 7560y^3G(0,r,0,-1;y) - 7560y^2G(0,r,0,-1;y) + 7560yG(0,r,0,-1;y) +$
$5760y^3G(-1;y)G(0,r,0,-1;y) + 5760y^2G(-1;y)G(0,r,0,-1;y) + 5760yG(-1;y)G(0,r,0,-1;y) + 5760G(-1;y)G(0,r,0,-1;y) +$
$7560G(0,r,0,-1;y) - 15120y^3G(0,r,1,-1;y) - 15120y^2G(0,r,1,-1;y) - 15120yG(0,r,1,-1;y) - 8640y^3G(-1;y)G(0,r,1,-1;y) -$
$8640y^2G(-1;y)G(0,r,1,-1;y) - 8640yG(-1;y)G(0,r,1,-1;y) - 8640G(-1;y)G(0,r,1,-1;y) - 8640y^3G(1;y)G(0,r,1,-1;y) -$
$8640y^2G(1;y)G(0,r,1,-1;y) - 8640yG(1;y)G(0,r,1,-1;y) - 8640G(1;y)G(0,r,1,-1;y) - 15120G(0,r,1,-1;y) -$
$41760y^3G(0,r,1,1;y) + 79920y^2G(0,r,1,1;y) - 20520yG(0,r,1,1;y) - 17280y^3G(-1;y)G(0,r,1,1;y) - 17280y^2G(-1;y)G(0,r,1,1;y) -$
$17280yG(-1;y)G(0,r,1,1;y) - 17280G(-1;y)G(0,r,1,1;y) + 30600y^3G(1;y)G(0,r,1,1;y) - 66600y^2G(1;y)G(0,r,1,1;y) +$
$66600yG(1;y)G(0,r,1,1;y) - 30600G(1;y)G(0,r,1,1;y) - 17640G(0,r,1,1;y) + 6840y^3G(0,r,1,r;y) - 19080y^2G(0,r,1,r;y) +$
$7200yG(0,r,1,r;y) + 6480y^3G(-1;y)G(0,r,1,r;y) + 6480y^2G(-1;y)G(0,r,1,r;y) + 6480yG(-1;y)G(0,r,1,r;y) +$
$6480G(-1;y)G(0,r,1,r;y) - 11880y^3G(1;y)G(0,r,1,r;y) + 13320y^2G(1;y)G(0,r,1,r;y) - 13320yG(1;y)G(0,r,1,r;y) +$
$11880G(1;y)G(0,r,1,r;y) + 5040G(0,r,1,r;y) + 7560y^3G(0,r,r,-1;y) + 7560y^2G(0,r,r,-1;y) + 7560yG(0,r,r,-1;y) -$
$4320y^3G(-1;y)G(0,r,r,-1;y) - 4320y^2G(-1;y)G(0,r,r,-1;y) - 4320yG(-1;y)G(0,r,r,-1;y) - 4320G(-1;y)G(0,r,r,-1;y) +$
$4320y^3G(1;y)G(0,r,r,-1;y) + 4320y^2G(1;y)G(0,r,r,-1;y) + 4320yG(1;y)G(0,r,r,-1;y) + 4320G(1;y)G(0,r,r,-1;y) +$
$7560G(0,r,r,-1;y) + 6840y^3G(0,r,r,1;y) - 19080y^2G(0,r,r,1;y) + 7200yG(0,r,r,1;y) - 11880y^3G(1;y)G(0,r,r,1;y) +$
$13320y^2G(1;y)G(0,r,r,1;y) - 13320yG(1;y)G(0,r,r,1;y) + 11880G(1;y)G(0,r,r,1;y) + 5040G(0,r,r,1;y) - 720y^2G(0,r,r,r;y) +$
$720yG(0,r,r,r;y) + 2160y^3G(-1;y)G(0,r,r,r;y) + 2160y^2G(-1;y)G(0,r,r,r;y) + 2160yG(-1;y)G(0,r,r,r;y) +$
$2160G(-1;y)G(0,r,r,r;y) - 1440y^3G(1;y)G(0,r,r,r;y) + 1440G(1;y)G(0,r,r,r;y) - 24120y^3G(r,1,1,1;y) + 100440y^2G(r,1,1,1;y) -$
$100440yG(r,1,1,1;y) + 61200y^3G(1;y)G(r,1,1,1;y) - 133200y^2G(1;y)G(r,1,1,1;y) + 133200yG(1;y)G(r,1,1,1;y) -$
$61200G(1;y)G(r,1,1,1;y) + 26640y^3G(r;y)G(r,1,1,1;y) - 26640y^2G(r;y)G(r,1,1,1;y) + 26640yG(r;y)G(r,1,1,1;y) -$
$26640G(r;y)G(r,1,1,1;y) + 24120G(r,1,1,1;y) - 1800y^3G(r,r,1,1;y) + 26280y^2G(r,r,1,1;y) - 26280yG(r,r,1,1;y) +$
$23760y^3G(1;y)G(r,r,1,1;y) - 26640y^2G(1;y)G(r,r,1,1;y) + 26640yG(1;y)G(r,r,1,1;y) - 23760G(1;y)G(r,r,1,1;y) -$
$2880y^3G(r;y)G(r,r,1,1;y) + 2880G(r;y)G(r,r,1,1;y) + 1800G(r,r,1,1;y) - 1440y^2G(r,r,r,1;y) + 1440yG(r,r,r,1;y) -$
$2880y^3G(1;y)G(r,r,r,1;y) + 2880G(1;y)G(r,r,r,1;y) - 2880y^3G(0,-1,-1,1,r;y) - 2880y^2G(0,-1,-1,1,r;y) -$
$2880yG(0,-1,-1,1,r;y) - 2880G(0,-1,-1,1,r;y) + 8640y^3G(0,-1,-1,r,1;y) + 8640y^2G(0,-1,-1,r,1;y) + 8640yG(0,-1,-1,r,1;y)+$
$8640G(0,-1,-1,r,1;y) + 4320y^3G(0,-1,-1,r,r;y) + 4320y^2G(0,-1,-1,r,r;y) + 4320yG(0,-1,-1,r,r;y) + 4320G(0,-1,-1,r,r;y) -$
$4320y^3G(0,-1,0,-1,r;y)+4320y^2G(0,-1,0,-1,r;y)-4320yG(0,-1,0,-1,r;y)+4320G(0,-1,0,-1,r;y)-2880y^3G(0,-1,1,-1,r;y)-$
$2880y^2G(0,-1,1,-1,r;y) - 2880yG(0,-1,1,-1,r;y) - 2880G(0,-1,1,-1,r;y) + 11520y^3G(0,-1,1,1,r;y) + 11520y^2G(0,-1,1,1,r;y)+$
$11520yG(0,-1,1,1,r;y) + 11520G(0,-1,1,1,r;y) - 2880y^3G(0,-1,1,r,-1;y) - 2880y^2G(0,-1,1,r,-1;y) - 2880yG(0,-1,1,r,-1;y) -$
$2880G(0,-1,1,r,-1;y) + 5760y^3G(0,-1,1,r,1;y) + 5760y^2G(0,-1,1,r,1;y) + 5760yG(0,-1,1,r,1;y) + 5760G(0,-1,1,r,1;y) +$
$4320y^3G(0,-1,1,r,r;y) + 4320y^2G(0,-1,1,r,r;y) + 4320yG(0,-1,1,r,r;y) + 4320G(0,-1,1,r,r;y) + 8640y^3G(0,-1,r,-1,1;y) +$
$8640y^2G(0,-1,r,-1,1;y)+8640yG(0,-1,r,-1,1;y)+8640G(0,-1,r,-1,1;y)+4320y^3G(0,-1,r,-1,r;y)+4320y^2G(0,-1,r,-1,r;y)+$
$4320yG(0,-1,r,-1,r;y)+4320G(0,-1,r,-1,r;y)+8640y^3G(0,-1,r,1,-1;y)+8640y^2G(0,-1,r,1,-1;y)+8640yG(0,-1,r,1,-1;y)+$

$$8640G(0,-1,r,1,-1;y) + 17280y^3G(0,-1,r,1,1;y) + 17280y^2G(0,-1,r,1,1;y) + 17280yG(0,-1,r,1,1;y) + 17280G(0,-1,r,1,1;y) -$$
$$6480y^3G(0,-1,r,1,r;y) - 6480y^2G(0,-1,r,1,r;y) - 6480yG(0,-1,r,1,r;y) - 6480G(0,-1,r,1,r;y) + 4320y^3G(0,-1,r,r,-1;y) +$$
$$4320y^2G(0,-1,r,r,-1;y) + 4320yG(0,-1,r,r,-1;y) + 4320G(0,-1,r,r,-1;y) - 2160y^3G(0,-1,r,r,r;y) - 2160y^2G(0,-1,r,r,r;y) -$$
$$2160yG(0,-1,r,r,r;y) - 2160G(0,-1,r,r,r;y) + 8640y^3G(0,0,-1,-1,1;y) + 8640y^2G(0,0,-1,-1,1;y) + 8640yG(0,0,-1,-1,1;y) +$$
$$8640G(0,0,-1,-1,1;y) - 5760y^3G(0,0,-1,-1,r;y) + 11520y^2G(0,0,-1,-1,r;y) - 5760yG(0,0,-1,-1,r;y) + 11520G(0,0,-1,-1,r;y) +$$
$$4320y^3G(0,0,-1,0,-1;y) - 4320y^2G(0,0,-1,0,-1;y) + 4320yG(0,0,-1,0,-1;y) - 4320G(0,0,-1,0,-1;y) - 16560y^3G(0,0,-1,0,1;y) -$$
$$2160y^2G(0,0,-1,0,1;y) - 16560yG(0,0,-1,0,1;y) - 2160G(0,0,-1,0,1;y) + 16920y^3G(0,0,-1,0,r;y) - 3240y^2G(0,0,-1,0,r;y) +$$
$$16920yG(0,0,-1,0,r;y) - 3240G(0,0,-1,0,r;y) + 8640y^3G(0,0,-1,1,-1;y) + 8640y^2G(0,0,-1,1,-1;y) + 8640yG(0,0,-1,1,-1;y) +$$
$$8640G(0,0,-1,1,-1;y) + 17280y^3G(0,0,-1,1,1;y) + 17280y^2G(0,0,-1,1,1;y) + 17280yG(0,0,-1,1,1;y) + 17280G(0,0,-1,1,1;y) -$$
$$5040y^3G(0,0,-1,1,r;y) + 720y^2G(0,0,-1,1,r;y) - 5040yG(0,0,-1,1,r;y) + 720G(0,0,-1,1,r;y) + 2880y^3G(0,0,-1,r,-1;y) +$$
$$2880y^2G(0,0,-1,r,-1;y) + 2880yG(0,0,-1,r,-1;y) + 2880G(0,0,-1,r,-1;y) + 15840y^3G(0,0,-1,r,1;y) - 1440y^2G(0,0,-1,r,1;y) +$$
$$15840yG(0,0,-1,r,1;y) - 1440G(0,0,-1,r,1;y) + 2160y^3G(0,0,-1,r,r;y) + 2160y^2G(0,0,-1,r,r;y) + 2160yG(0,0,-1,r,r;y) +$$
$$2160G(0,0,-1,r,r;y) + 20160y^3G(0,0,0,-1,-1;y) - 5760y^2G(0,0,0,-1,-1;y) + 20160yG(0,0,0,-1,-1;y) - 5760G(0,0,0,-1,-1;y) -$$
$$10800y^3G(0,0,0,-1,1;y) + 6480y^2G(0,0,0,-1,1;y) - 10800yG(0,0,0,-1,1;y) + 6480G(0,0,0,-1,1;y) + 48960y^3G(0,0,0,-1,r;y) -$$
$$7200y^2G(0,0,0,-1,r;y) + 48960yG(0,0,0,-1,r;y) - 7200G(0,0,0,-1,r;y) + 47160y^3G(0,0,0,0,-1;y) - 4680y^2G(0,0,0,0,-1;y) +$$
$$47160yG(0,0,0,0,-1;y) - 4680G(0,0,0,0,-1;y) + 36720y^3G(0,0,0,0,1;y) + 5760y^2G(0,0,0,0,1;y) + 28800yG(0,0,0,0,1;y) -$$
$$2160G(0,0,0,0,1;y) + 7560y^3G(0,0,0,0,r;y) + 4680y^2G(0,0,0,0,r;y) + 2880yG(0,0,0,0,r;y) - 10800y^3G(0,0,0,1,-1;y) +$$
$$6480y^2G(0,0,0,1,-1;y) - 10800yG(0,0,0,1,-1;y) + 6480G(0,0,0,1,-1;y) - 11160y^3G(0,0,0,1,1;y) + 150840y^2G(0,0,0,1,1;y) -$$
$$140760yG(0,0,0,1,1;y) + 21240G(0,0,0,1,1;y) + 38880y^3G(0,0,0,1,r;y) - 36000y^2G(0,0,0,1,r;y) + 76320yG(0,0,0,1,r;y) +$$
$$1440G(0,0,0,1,r;y) + 48960y^3G(0,0,0,r,-1;y) - 7200y^2G(0,0,0,r,-1;y) + 48960yG(0,0,0,r,-1;y) - 7200G(0,0,0,r,-1;y) +$$
$$43200y^3G(0,0,0,r,1;y) - 26640y^2G(0,0,0,r,1;y) + 71280yG(0,0,0,r,1;y) + 1440G(0,0,0,r,1;y) + 720y^3G(0,0,0,r,r;y) +$$
$$1440y^2G(0,0,0,r,r;y) - 720yG(0,0,0,r,r;y) + 8640y^3G(0,0,1,-1,-1;y) + 8640y^2G(0,0,1,-1,-1;y) + 8640yG(0,0,1,-1,-1;y) +$$
$$8640G(0,0,1,-1,-1;y) + 17280y^3G(0,0,1,-1,1;y) + 17280y^2G(0,0,1,-1,1;y) + 17280yG(0,0,1,-1,1;y) + 17280G(0,0,1,-1,1;y) -$$
$$2160y^3G(0,0,1,-1,r;y) + 20880y^2G(0,0,1,-1,r;y) - 2160yG(0,0,1,-1,r;y) + 20880G(0,0,1,-1,r;y) - 8640y^3G(0,0,1,0,-1;y) -$$
$$2880y^2G(0,0,1,0,-1;y) - 8640yG(0,0,1,0,-1;y) - 2880G(0,0,1,0,-1;y) - 13680y^3G(0,0,1,0,1;y) + 54000y^2G(0,0,1,0,1;y) -$$
$$69840yG(0,0,1,0,1;y) - 2160G(0,0,1,0,1;y) + 15480y^3G(0,0,1,0,r;y) - 10440y^2G(0,0,1,0,r;y) + 26280yG(0,0,1,0,r;y) +$$
$$360G(0,0,1,0,r;y) + 17280y^3G(0,0,1,1,-1;y) + 17280y^2G(0,0,1,1,-1;y) + 17280yG(0,0,1,1,-1;y) + 17280G(0,0,1,1,-1;y) +$$
$$21240y^3G(0,0,1,1,1;y) - 48600y^2G(0,0,1,1,1;y) + 19800yG(0,0,1,1,1;y) - 50040G(0,0,1,1,1;y) + 21960y^3G(0,0,1,1,r;y) +$$
$$73800y^2G(0,0,1,1,r;y) - 18360yG(0,0,1,1,r;y) + 33480G(0,0,1,1,r;y) - 2160y^3G(0,0,1,r,-1;y) + 20880y^2G(0,0,1,r,-1;y) -$$
$$2160yG(0,0,1,r,-1;y) + 20880G(0,0,1,r,-1;y) - 7920y^3G(0,0,1,r,1;y) + 85680y^2G(0,0,1,r,1;y) - 65520yG(0,0,1,r,1;y) +$$
$$28080G(0,0,1,r,1;y) + 1800y^3G(0,0,1,r,r;y) - 19800y^2G(0,0,1,r,r;y) + 16200yG(0,0,1,r,r;y) - 5400G(0,0,1,r,r;y) +$$
$$11520y^3G(0,0,r,-1,-1;y) - 5760y^2G(0,0,r,-1,-1;y) + 11520yG(0,0,r,-1,-1;y) - 5760G(0,0,r,-1,-1;y) + 15840y^3G(0,0,r,-1,1;y) -$$
$$1440y^2G(0,0,r,-1,1;y) + 15840yG(0,0,r,-1,1;y) - 1440G(0,0,r,-1,1;y) + 2160y^3G(0,0,r,-1,r;y) + 2160y^2G(0,0,r,-1,r;y) +$$
$$2160yG(0,0,r,-1,r;y) + 2160G(0,0,r,-1,r;y) + 15840y^3G(0,0,r,0,-1;y) - 2880y^2G(0,0,r,0,-1;y) + 15840yG(0,0,r,0,-1;y) -$$
$$2880G(0,0,r,0,-1;y) + 11160y^3G(0,0,r,0,1;y) - 17640y^2G(0,0,r,0,1;y) + 30600yG(0,0,r,0,1;y) + 1800G(0,0,r,0,1;y) +$$
$$1080y^3G(0,0,r,0,r;y) - 1080y^2G(0,0,r,0,r;y) + 2160yG(0,0,r,0,r;y) + 18720y^3G(0,0,r,1,-1;y) + 18720y^2G(0,0,r,1,-1;y) +$$
$$18720yG(0,0,r,1,-1;y) + 18720G(0,0,r,1,-1;y) - 15480y^3G(0,0,r,1,1;y) + 100440y^2G(0,0,r,1,1;y) - 94680yG(0,0,r,1,1;y) +$$
$$21240G(0,0,r,1,1;y) + 7560y^3G(0,0,r,1,r;y) - 14760y^2G(0,0,r,1,r;y) + 12600yG(0,0,r,1,r;y) - 9720G(0,0,r,1,r;y) +$$
$$2160y^3G(0,0,r,r,-1;y) + 2160y^2G(0,0,r,r,-1;y) + 2160yG(0,0,r,r,-1;y) + 2160G(0,0,r,r,-1;y) + 14040y^3G(0,0,r,r,1;y) -$$
$$16920y^2G(0,0,r,r,1;y) + 16920yG(0,0,r,r,1;y) - 14040G(0,0,r,r,1;y) - 720y^3G(0,0,r,r,r;y) + 720y^2G(0,0,r,r,r;y) -$$
$$1440yG(0,0,r,r,r;y) - 2880y^3G(0,1,-1,-1,r;y) - 2880y^2G(0,1,-1,-1,r;y) - 2880yG(0,1,-1,-1,r;y) - 2880G(0,1,-1,-1,r;y) +$$
$$1440y^3G(0,1,-1,0,r;y) + 10080y^2G(0,1,-1,0,r;y) + 1440yG(0,1,-1,0,r;y) + 10080G(0,1,-1,0,r;y) + 11520y^3G(0,1,-1,1,r;y) +$$
$$11520y^2G(0,1,-1,1,r;y) + 11520yG(0,1,-1,1,r;y) + 11520G(0,1,-1,1,r;y) - 2880y^3G(0,1,-1,r,-1;y) - 2880y^2G(0,1,-1,r,-1;y) -$$
$$2880yG(0,1,-1,r,-1;y) - 2880G(0,1,-1,r,-1;y) + 5760y^3G(0,1,-1,r,1;y) + 5760y^2G(0,1,-1,r,1;y) + 5760yG(0,1,-1,r,1;y) +$$
$$5760G(0,1,-1,r,1;y) + 4320y^3G(0,1,-1,r,r;y) + 4320y^2G(0,1,-1,r,r;y) + 4320yG(0,1,-1,r,r;y) + 4320G(0,1,-1,r,r;y) +$$
$$11520y^3G(0,1,0,-1,-1;y) + 11520y^2G(0,1,0,-1,-1;y) + 11520yG(0,1,0,-1,-1;y) + 11520G(0,1,0,-1,-1;y) +$$
$$11520y^3G(0,1,0,-1,1;y) + 11520y^2G(0,1,0,-1,1;y) + 11520yG(0,1,0,-1,1;y) + 11520G(0,1,0,-1,1;y) - 10800y^3G(0,1,0,-1,r;y) +$$
$$720y^2G(0,1,0,-1,r;y) - 10800yG(0,1,0,-1,r;y) + 720G(0,1,0,-1,r;y) + 11520y^3G(0,1,0,1,-1;y) + 11520y^2G(0,1,0,1,-1;y) +$$
$$11520yG(0,1,0,1,-1;y) + 11520G(0,1,0,1,-1;y) - 10080y^3G(0,1,0,1,1;y) - 18720y^2G(0,1,0,1,1;y) - 18720yG(0,1,0,1,1;y) -$$
$$27360G(0,1,0,1,1;y) + 14040y^3G(0,1,0,1,r;y) + 34200y^2G(0,1,0,1,r;y) - 11880yG(0,1,0,1,r;y) + 8280G(0,1,0,1,r;y) -$$
$$10800y^3G(0,1,0,r,-1;y) + 720y^2G(0,1,0,r,-1;y) - 10800yG(0,1,0,r,-1;y) + 720G(0,1,0,r,-1;y) - 5040y^3G(0,1,0,r,1;y) +$$
$$59760y^2G(0,1,0,r,1;y) - 56880yG(0,1,0,r,1;y) + 7920G(0,1,0,r,1;y) + 3240y^3G(0,1,0,r,r;y) - 18360y^2G(0,1,0,r,r;y) +$$
$$16200yG(0,1,0,r,r;y) - 5400G(0,1,0,r,r;y) + 11520y^3G(0,1,1,-1,r;y) + 11520y^2G(0,1,1,-1,r;y) + 11520yG(0,1,1,-1,r;y) +$$
$$11520G(0,1,1,-1,r;y) + 17280y^3G(0,1,1,0,-1;y) + 17280y^2G(0,1,1,0,-1;y) + 17280yG(0,1,1,0,-1;y) + 17280G(0,1,1,0,-1;y) +$$
$$4680y^3G(0,1,1,0,r;y) + 4680y^2G(0,1,1,0,r;y) - 12600yG(0,1,1,0,r;y) - 12600G(0,1,1,0,r;y) + 85680y^3G(0,1,1,1,1;y) -$$
$$186480y^2G(0,1,1,1,1;y) + 186480yG(0,1,1,1,1;y) - 85680G(0,1,1,1,1;y) - 10080y^3G(0,1,1,1,r;y) + 43200y^2G(0,1,1,1,r;y) -$$
$$10080yG(0,1,1,1,r;y) + 43200G(0,1,1,1,r;y) + 11520y^3G(0,1,1,r,-1;y) + 11520y^2G(0,1,1,r,-1;y) + 11520yG(0,1,1,r,-1;y) +$$
$$11520G(0,1,1,r,-1;y) - 9360y^3G(0,1,1,r,1;y) + 35280y^2G(0,1,1,r,1;y) - 18000yG(0,1,1,r,1;y) + 26640G(0,1,1,r,1;y) -$$
$$720y^3G(0,1,1,r,r;y) + 2160y^2G(0,1,1,r,r;y) + 2160yG(0,1,1,r,r;y) + 5040G(0,1,1,r,r;y) - 2880y^3G(0,1,r,-1,-1;y) -$$
$$2880y^2G(0,1,r,-1,-1;y) - 2880yG(0,1,r,-1,-1;y) - 2880G(0,1,r,-1,-1;y) + 5760y^3G(0,1,r,-1,1;y) + 5760y^2G(0,1,r,-1,1;y) +$$

$5760yG(0,1,r,-1,1;y) + 5760G(0,1,r,-1,1;y) + 4320y^3G(0,1,r,-1,r;y) + 4320y^2G(0,1,r,-1,r;y) + 4320yG(0,1,r,-1,r;y) +$
$4320G(0,1,r,-1,r;y) - 14400y^3G(0,1,r,0,-1;y) - 20160y^2G(0,1,r,0,-1;y) - 14400yG(0,1,r,0,-1;y) - 20160G(0,1,r,0,-1;y) +$
$2160y^3G(0,1,r,0,r;y) + 2160y^2G(0,1,r,0,r;y) + 2160yG(0,1,r,0,r;y) + 2160G(0,1,r,0,r;y) + 5760y^3G(0,1,r,1,-1;y) +$
$5760y^2G(0,1,r,1,-1;y) + 5760yG(0,1,r,1,-1;y) + 5760G(0,1,r,1,-1;y) - 6480y^3G(0,1,r,1,1;y) + 38160y^2G(0,1,r,1,1;y) -$
$15120yG(0,1,r,1,1;y) + 29520G(0,1,r,1,1;y) - 7200y^3G(0,1,r,1,r;y) - 4320y^2G(0,1,r,1,r;y) - 4320yG(0,1,r,1,r;y) -$
$1440G(0,1,r,1,r;y) + 4320y^3G(0,1,r,r,-1;y) + 4320y^2G(0,1,r,r,-1;y) + 4320yG(0,1,r,r,-1;y) + 4320G(0,1,r,r,-1;y) -$
$2880y^3G(0,1,r,r,1;y) + 2880G(0,1,r,r,1;y) - 1440y^3G(0,1,r,r,r;y) - 1440y^2G(0,1,r,r,r;y) - 1440yG(0,1,r,r,r;y) -$
$1440G(0,1,r,r,r;y) + 8640y^3G(0,r,-1,-1,1;y) + 8640y^2G(0,r,-1,-1,1;y) + 8640yG(0,r,-1,-1,1;y) + 8640G(0,r,-1,-1,1;y) +$
$4320y^3G(0,r,-1,-1,r;y) + 4320y^2G(0,r,-1,-1,r;y) + 4320yG(0,r,-1,-1,r;y) + 4320G(0,r,-1,-1,r;y) + 4320y^3G(0,r,-1,0,-1;y) -$
$4320y^2G(0,r,-1,0,-1;y) + 4320yG(0,r,-1,0,-1;y) - 4320G(0,r,-1,0,-1;y) + 8640y^3G(0,r,-1,1,-1;y) + 8640y^2G(0,r,-1,1,-1;y) +$
$8640yG(0,r,-1,1,-1;y) + 8640G(0,r,-1,1,-1;y) + 17280y^3G(0,r,-1,1,1;y) + 17280y^2G(0,r,-1,1,1;y) + 17280yG(0,r,-1,1,1;y) +$
$17280G(0,r,-1,1,1;y) - 6480y^3G(0,r,-1,1,r;y) - 6480y^2G(0,r,-1,1,r;y) - 6480yG(0,r,-1,1,r;y) - 6480G(0,r,-1,1,r;y) +$
$4320y^3G(0,r,-1,r,-1;y) + 4320y^2G(0,r,-1,r,-1;y) + 4320yG(0,r,-1,r,-1;y) + 4320G(0,r,-1,r,-1;y) - 2160y^3G(0,r,-1,r,r;y) -$
$2160y^2G(0,r,-1,r,r;y) - 2160yG(0,r,-1,r,r;y) - 2160G(0,r,-1,r,r;y) - 1440y^3G(0,r,0,-1,-1;y) - 10080y^2G(0,r,0,-1,-1;y) -$
$1440yG(0,r,0,-1,-1;y) - 10080G(0,r,0,-1,-1;y) - 1440y^3G(0,r,0,-1,1;y) - 10080y^2G(0,r,0,-1,1;y) - 1440yG(0,r,0,-1,1;y) -$
$10080G(0,r,0,-1,1;y) + 5400y^3G(0,r,0,-1,r;y) + 5400y^2G(0,r,0,-1,r;y) + 5400yG(0,r,0,-1,r;y) + 5400G(0,r,0,-1,r;y) +$
$5400y^3G(0,r,0,r,-1;y) + 5400y^2G(0,r,0,r,-1;y) + 5400yG(0,r,0,r,-1;y) + 5400G(0,r,0,r,-1;y) + 2160y^3G(0,r,0,r,1;y) -$
$2160G(0,r,0,r,1;y) + 8640y^3G(0,r,1,-1,-1;y) + 8640y^2G(0,r,1,-1,-1;y) + 8640yG(0,r,1,-1,-1;y) + 8640G(0,r,1,-1,-1;y) +$
$17280y^3G(0,r,1,-1,1;y) + 17280y^2G(0,r,1,-1,1;y) + 17280yG(0,r,1,-1,1;y) + 17280G(0,r,1,-1,1;y) - 6480y^3G(0,r,1,-1,r;y) -$
$6480y^2G(0,r,1,-1,r;y) - 6480yG(0,r,1,-1,r;y) - 6480G(0,r,1,-1,r;y) - 17280y^2G(0,r,1,0,-1;y) - 17280G(0,r,1,0,-1;y) +$
$17280y^3G(0,r,1,1,-1;y) + 17280y^2G(0,r,1,1,-1;y) + 17280yG(0,r,1,1,-1;y) + 17280G(0,r,1,1,-1;y) - 42840y^3G(0,r,1,1,1;y) +$
$93240y^2G(0,r,1,1,1;y) - 93240yG(0,r,1,1,1;y) + 42840G(0,r,1,1,1;y) + 13320y^3G(0,r,1,1,r;y) - 13320y^2G(0,r,1,1,r;y) +$
$13320yG(0,r,1,1,r;y) - 13320G(0,r,1,1,r;y) - 6480y^3G(0,r,1,r,-1;y) - 6480y^2G(0,r,1,r,-1;y) - 6480yG(0,r,1,r,-1;y) -$
$6480G(0,r,1,r,-1;y) + 9000y^3G(0,r,1,r,1;y) - 13320y^2G(0,r,1,r,1;y) + 13320yG(0,r,1,r,1;y) - 9000G(0,r,1,r,1;y) +$
$1440y^3G(0,r,1,r,r;y) - 1440G(0,r,1,r,r;y) + 4320y^3G(0,r,r,-1,-1;y) + 4320y^2G(0,r,r,-1,-1;y) + 4320yG(0,r,r,-1,-1;y) +$
$4320G(0,r,r,-1,-1;y) - 2160y^3G(0,r,r,-1,r;y) - 2160y^2G(0,r,r,-1,r;y) - 2160yG(0,r,r,-1,r;y) - 2160G(0,r,r,-1,r;y) +$
$8640y^3G(0,r,r,0,-1;y) + 8640y^2G(0,r,r,0,-1;y) + 8640yG(0,r,r,0,-1;y) + 8640G(0,r,r,0,-1;y) + 9000y^3G(0,r,r,1,1;y) -$
$13320y^2G(0,r,r,1,1;y) + 13320yG(0,r,r,1,1;y) - 9000G(0,r,r,1,1;y) + 1440y^3G(0,r,r,1,r;y) - 1440G(0,r,r,1,r;y) -$
$2160y^3G(0,r,r,r,-1;y) - 2160y^2G(0,r,r,r,-1;y) - 2160yG(0,r,r,r,-1;y) - 2160G(0,r,r,r,-1;y) + 1440y^3G(0,r,r,r,1;y) -$
$1440G(0,r,r,r,1;y) - 85680y^3G(r,1,1,1,1;y) + 186480y^2G(r,1,1,1,1;y) - 186480yG(r,1,1,1,1;y) + 85680G(r,1,1,1,1;y) +$
$8640y^3G(r,1,r,1,1;y) - 8640G(r,1,r,1,1;y) - 720y^3G(r,r,1,1,1;y) + 26640y^2G(r,r,1,1,1;y) - 26640yG(r,r,1,1,1;y) +$
$720G(r,r,1,1,1;y) + 2880y^3G(r,r,r,1,1;y) - 2880G(r,r,r,1,1;y) + 60\pi^2y^3\zeta(3) + 120y^3\zeta(3) + 60\pi^2\zeta(3) + 300\pi^2y^2\zeta(3) -$
$10800y^2\zeta(3) - 2160y^3G(-1;y)^2\zeta(3) - 2160y^2G(-1;y)^2\zeta(3) - 2160yG(-1;y)^2\zeta(3) - 2160G(-1;y)^2\zeta(3) - 6120y^3G(1;y)^2\zeta(3) +$
$11880y^2G(1;y)^2\zeta(3) - 11880yG(1;y)^2\zeta(3) + 6120G(1;y)^2\zeta(3) - 180\pi^2y\zeta(3) + 29520y\zeta(3) + 7560y^3G(-1;y)\zeta(3) + 7560y^2G(-1;y)\zeta(3) +$
$7560yG(-1;y)\zeta(3) + 7560G(-1;y)\zeta(3) + 360y^3G(1;y)\zeta(3) - 24840y^2G(1;y)\zeta(3) + 32760yG(1;y)\zeta(3) + 4320y^3G(-1;y)G(1;y)\zeta(3) +$
$4320y^2G(-1;y)G(1;y)\zeta(3) + 4320yG(-1;y)G(1;y)\zeta(3) + 4320G(-1;y)G(1;y)\zeta(3) - 8280G(1;y)\zeta(3) + 4320y^3G(0,-1;y)\zeta(3) +$
$4320y^2G(0,-1;y)\zeta(3) + 4320yG(0,-1;y)\zeta(3) + 4320G(0,-1;y)\zeta(3) + 2160y^3G(0,1;y)\zeta(3) - 5040y^2G(0,1;y)\zeta(3) + 10800yG(0,1;y)\zeta(3) +$
$3600G(0,1;y)\zeta(3) - 11160\zeta(3) + 1800y^3\zeta(5) + 48960y^2\zeta(5) - 36360y\zeta(5) + 10800\zeta(5) + 2040.$

C_2

$C_2(y) = -6y^4G(0;y)^5 + 6y^2G(0;y)^5 - 15y^4G(0;y)^4 - 30y^3G(0;y)^4 + 15y^2G(0;y)^4 + 30yG(0;y)^4 - 30y^4G(-1;y)G(0;y)^4 +$
$30G(-1;y)G(0;y)^4 + 60y^4G(1;y)G(0;y)^4 - 120y^2G(1;y)G(0;y)^4 + 60G(1;y)G(0;y)^4 - 100\pi^2y^4G(0;y)^3 + 40y^4G(0;y)^3 +$
$1100y^3G(0;y)^3 + 100\pi^2y^2G(0;y)^3 - 750y^2G(0;y)^3 - 960y^4G(-1;y)^2G(0;y)^3 + 960G(-1;y)^2G(0;y)^3 + 540y^4G(1;y)^2G(0;y)^3 -$
$1560y^3G(1;y)^2G(0;y)^3 + 2040y^2G(1;y)^2G(0;y)^3 - 1560yG(1;y)^2G(0;y)^3 + 540G(1;y)^2G(0;y)^3 - 180yG(0;y)^3 - 480y^4G(-1;y)G(0;y)^3 -$
$1920y^3G(-1;y)G(0;y)^3 + 1920yG(-1;y)G(0;y)^3 + 480G(-1;y)G(0;y)^3 + 420y^4G(1;y)G(0;y)^3 + 960y^3G(1;y)G(0;y)^3 -$
$2880y^2G(1;y)G(0;y)^3 + 1200yG(1;y)G(0;y)^3 + 300G(1;y)G(0;y)^3 + 2040y^4G(0,-1;y)G(0;y)^3 - 1920y^2G(0,-1;y)G(0;y)^3 -$
$120G(0,-1;y)G(0;y)^3 - 1320y^4G(0,1;y)G(0;y)^3 + 1440y^3G(0,1;y)G(0;y)^3 - 1440y^2G(0,1;y)G(0;y)^3 + 1440yG(0,1;y)G(0;y)^3 -$
$120G(0,1;y)G(0;y)^3 - 150\pi^2y^4G(0;y)^2 - 3600y^4G(0;y)^2 + 180\pi^2y^3G(0;y)^2 + 5910y^3G(0;y)^2 - 1440y^4G(-1;y)^3G(0;y)^2 +$
$1440G(-1;y)^3G(0;y)^2 - 540y^4G(1;y)^3G(0;y)^2 + 1080y^3G(1;y)^3G(0;y)^2 - 1080y^2G(1;y)^3G(0;y)^2 + 1080yG(1;y)^3G(0;y)^2 -$
$540G(1;y)^3G(0;y)^2 - 450\pi^2y^2G(0;y)^2 - 1740y^2G(0;y)^2 - 1080y^4G(-1;y)^2G(0;y)^2 - 4320y^3G(-1;y)^2G(0;y)^2 + 4320yG(-1;y)^2G(0;y)^2 +$
$1080G(-1;y)^2G(0;y)^2 + 540y^4G(1;y)^2G(0;y)^2 - 4320y^3G(1;y)^2G(0;y)^2 + 8100y^2G(1;y)^2G(0;y)^2 - 5400yG(1;y)^2G(0;y)^2 +$
$1080G(1;y)^2G(0;y)^2 + 420\pi^2yG(0;y)^2 - 570yG(0;y)^2 - 180\pi^2y^4G(-1;y)G(0;y)^2 + 2520y^4G(-1;y)G(0;y)^2 + 5040y^3G(-1;y)G(0;y)^2 +$
$180\pi^2G(-1;y)G(0;y)^2 - 5040yG(-1;y)G(0;y)^2 - 2520G(-1;y)G(0;y)^2 + 420\pi^2y^4G(1;y)G(0;y)^2 - 2100y^4G(1;y)G(0;y)^2 +$
$120\pi^2y^3G(1;y)G(0;y)^2 - 6360y^3G(1;y)G(0;y)^2 + 420\pi^2G(1;y)G(0;y)^2 - 1080\pi^2y^2G(1;y)G(0;y)^2 + 15840y^2G(1;y)G(0;y)^2 +$
$120\pi^2yG(1;y)G(0;y)^2 - 4200yG(1;y)G(0;y)^2 - 3180G(1;y)G(0;y)^2 + 450y^4G(r;y)G(0;y)^2 - 1980y^3G(r;y)G(0;y)^2 +$
$2430y^2G(r;y)G(0;y)^2 + 720y^4G(1;y)^2G(r;y)G(0;y)^2 - 1440y^2G(1;y)^2G(r;y)G(0;y)^2 + 720G(1;y)^2G(r;y)G(0;y)^2 - 1980yG(r;y)G(0;y)^2 -$
$180y^4G(1;y)G(r;y)G(0;y)^2 + 360y^2G(1;y)G(r;y)G(0;y)^2 - 180G(1;y)G(r;y)G(0;y)^2 + 450G(r;y)G(0;y)^2 + 2520y^4G(0,-1;y)G(0;y)^2 +$
$14400y^3G(0,-1;y)G(0;y)^2 - 8640y^2G(0,-1;y)G(0;y)^2 - 5760yG(0,-1;y)G(0;y)^2 + 10080y^4G(-1;y)G(0,-1;y)G(0;y)^2 -$
$10080G(-1;y)G(0,-1;y)G(0;y)^2 - 4320y^4G(1;y)G(0,-1;y)G(0;y)^2 + 8640y^2G(1;y)G(0,-1;y)G(0;y)^2 - 4320G(1;y)G(0,-1;y)G(0;y)^2 -$

$$
2520G(0,-1;y)G(0;y)^2 - 1260y^4G(0,1;y)G(0;y)^2 + 7920y^3G(0,1;y)G(0;y)^2 - 1080y^2G(0,1;y)G(0;y)^2 - 3600yG(0,1;y)G(0;y)^2 +
4320y^4G(-1;y)G(0,1;y)G(0;y)^2 - 4320G(-1;y)G(0,1;y)G(0;y)^2 - 4320y^4G(1;y)G(0,1;y)G(0;y)^2 + 7200y^3G(1;y)G(0,1;y)G(0;y)^2 -
5760y^2G(1;y)G(0,1;y)G(0;y)^2 + 7200yG(1;y)G(0,1;y)G(0;y)^2 - 4320G(1;y)G(0,1;y)G(0;y)^2 - 1980G(0,1;y)G(0;y)^2 +
360y^4G(0,r;y)G(0;y)^2 + 1080y^3G(0,r;y)G(0;y)^2 - 180y^2G(0,r;y)G(0;y)^2 - 1080yG(0,r;y)G(0;y)^2 + 1080y^4G(-1;y)G(0,r;y)G(0;y)^2 -
1080G(-1;y)G(0,r;y)G(0;y)^2 - 720y^4G(1;y)G(0,r;y)G(0;y)^2 + 1440y^2G(1;y)G(0,r;y)G(0;y)^2 - 720G(1;y)G(0,r;y)G(0;y)^2 -
180G(0,r;y)G(0;y)^2 + 180y^4G(r,1;y)G(0;y)^2 - 360y^2G(r,1;y)G(0;y)^2 - 1440y^4G(1;y)G(r,1;y)G(0;y)^2 + 2880y^2G(1;y)G(r,1;y)G(0;y)^2 -
1440G(1;y)G(r,1;y)G(0;y)^2 + 180G(r,1;y)G(0;y)^2 - 5760y^4G(0,-1,-1;y)G(0;y)^2 - 8640y^2G(0,-1,-1;y)G(0;y)^2 +
14400G(0,-1,-1;y)G(0;y)^2 - 8640y^2G(0,-1,1;y)G(0;y)^2 + 8640G(0,-1,1;y)G(0;y)^2 - 1080y^4G(0,-1,r;y)G(0;y)^2 +
1080G(0,-1,r;y)G(0;y)^2 - 11880y^4G(0,0,-1;y)G(0;y)^2 + 11520y^2G(0,0,-1;y)G(0;y)^2 + 360G(0,0,-1;y)G(0;y)^2 +
2880y^4G(0,0,1;y)G(0;y)^2 - 6480y^3G(0,0,1;y)G(0;y)^2 + 10080y^2G(0,0,1;y)G(0;y)^2 - 6480yG(0,0,1;y)G(0;y)^2 -
360y^4G(0,0,r;y)G(0;y)^2 + 360y^2G(0,0,r;y)G(0;y)^2 - 8640y^2G(0,1,-1;y)G(0;y)^2 + 8640G(0,1,-1;y)G(0;y)^2 +
9720y^4G(0,1,1;y)G(0;y)^2 - 7200y^3G(0,1,1;y)G(0;y)^2 + 1440y^2G(0,1,1;y)G(0;y)^2 - 7200yG(0,1,1;y)G(0;y)^2 + 3240G(0,1,1;y)G(0;y)^2 -
720y^4G(0,1,r;y)G(0;y)^2 + 720G(0,1,r;y)G(0;y)^2 - 1080y^4G(0,r,-1;y)G(0;y)^2 + 1080G(0,r,-1;y)G(0;y)^2 + 720y^4G(0,r,1;y)G(0;y)^2 -
1440y^2G(0,r,1;y)G(0;y)^2 + 720G(0,r,1;y)G(0;y)^2 + 1440y^4G(r,1,1;y)G(0;y)^2 - 2880y^2G(r,1,1;y)G(0;y)^2 + 1440G(r,1,1;y)G(0;y)^2 +
1440y^4\zeta(3)G(0;y)^2 - 720y^3\zeta(3)G(0;y)^2 - 720y\zeta(3)G(0;y)^2 - 202\pi^4y^4G(0;y) + 560\pi^2y^4G(0;y) + 1440y^4G(0;y) + 180y^4G(1;y)^4G(0;y) -
360y^3G(1;y)^4G(0;y) + 360y^2G(1;y)^4G(0;y) - 360yG(1;y)^4G(0;y) + 180G(1;y)^4G(0;y) + 44\pi^4y^3G(0;y) + 1180\pi^2y^3G(0;y) + 2940y^3G(0;y) -
360y^4G(1;y)^3G(0;y) + 2160y^3G(1;y)^3G(0;y) - 3600y^2G(1;y)^3G(0;y) + 2160yG(1;y)^3G(0;y) - 360G(1;y)^3G(0;y) + 114\pi^4y^2G(0;y) -
210\pi^2y^2G(0;y) - 10200y^2G(0;y) - 240\pi^2y^4G(-1;y)^2G(0;y) + 240\pi^2G(-1;y)^2G(0;y) - 1140\pi^2y^4G(1;y)^2G(0;y) + 3120y^4G(1;y)^2G(0;y) +
360\pi^2y^3G(1;y)^2G(0;y) + 1920y^3G(1;y)^2G(0;y) - 1140\pi^2G(1;y)^2G(0;y) + 1560\pi^2y^2G(1;y)^2G(0;y) - 10080y^2G(1;y)^2G(0;y) +
360\pi^2yG(1;y)^2G(0;y) + 1920yG(1;y)^2G(0;y) + 3120G(1;y)^2G(0;y) - 4320y^4G(0,-1;y)^2G(0;y) + 8640y^2G(0,-1;y)^2G(0;y) -
4320G(0,-1;y)^2G(0;y) - 5040y^4G(0,1;y)^2G(0;y) - 2880y^3G(0,1;y)^2G(0;y) + 11520y^2G(0,1;y)^2G(0;y) - 2880yG(0,1;y)^2G(0;y) -
720G(0,1;y)^2G(0;y) + 44\pi^4yG(0;y) - 900\pi^2yG(0;y) + 5820yG(0;y) - 120\pi^2y^4G(-1;y)G(0;y) - 480\pi^2y^3G(-1;y)G(0;y) +
120\pi^2G(-1;y)G(0;y) + 480\pi^2yG(-1;y)G(0;y) + 60\pi^2y^4G(1;y)G(0;y) + 4320y^4G(1;y)G(0;y) - 960\pi^2y^3G(1;y)G(0;y) -
4920y^3G(1;y)G(0;y) - 300\pi^2G(1;y)G(0;y) + 1440\pi^2y^2G(1;y)G(0;y) - 1680y^2G(1;y)G(0;y) - 240\pi^2yG(1;y)G(0;y) +
840yG(1;y)G(0;y) + 1440G(1;y)G(0;y) + 840\pi^2y^4G(0,-1;y)G(0;y) - 5040y^4G(0,-1;y)G(0;y) - 10080y^3G(0,-1;y)G(0;y) -
360\pi^2G(0,-1;y)G(0;y) - 480\pi^2y^2G(0,-1;y)G(0;y) + 8640y^4G(-1;y)^2G(0,-1;y)G(0;y) - 8640G(-1;y)^2G(0,-1;y)G(0;y) +
10080yG(0,-1;y)G(0;y) + 4320y^4G(-1;y)G(0,-1;y)G(0;y) + 17280y^3G(-1;y)G(0,-1;y)G(0;y) - 17280yG(-1;y)G(0,-1;y)G(0;y) -
4320G(-1;y)G(0,-1;y)G(0;y) + 5040G(0,-1;y)G(0;y) + 1560\pi^2y^4G(0,1;y)G(0;y) - 1320y^4G(0,1;y)G(0;y) -
960\pi^2y^3G(0,1;y)G(0;y) + 240y^3G(0,1;y)G(0;y) - 600\pi^2G(0,1;y)G(0;y) + 960\pi^2y^2G(0,1;y)G(0;y) - 20520y^2G(0,1;y)G(0;y) +
8640y^4G(-1;y)^2G(0,1;y)G(0;y) - 8640G(-1;y)^2G(0,1;y)G(0;y) + 3600y^4G(1;y)^2G(0,1;y)G(0;y) - 1440y^3G(1;y)^2G(0,1;y)G(0;y) -
4320y^2G(1;y)^2G(0,1;y)G(0;y) - 1440yG(1;y)^2G(0,1;y)G(0;y) + 3600G(1;y)^2G(0,1;y)G(0;y) - 960\pi^2yG(0,1;y)G(0;y) +
12720yG(0,1;y)G(0;y) + 4320y^4G(-1;y)G(0,1;y)G(0;y) + 17280y^3G(-1;y)G(0,1;y)G(0;y) - 17280yG(-1;y)G(0,1;y)G(0;y) -
4320G(-1;y)G(0,1;y)G(0;y) - 720y^4G(1;y)G(0,1;y)G(0;y) + 2880y^3G(1;y)G(0,1;y)G(0;y) - 5760y^2G(1;y)G(0,1;y)G(0;y) +
5760yG(1;y)G(0,1;y)G(0;y) - 2160G(1;y)G(0,1;y)G(0;y) - 7200y^4G(0,-1;y)G(0,1;y)G(0;y) + 17280y^2G(0,-1;y)G(0,1;y)G(0;y) -
10080G(0,-1;y)G(0,1;y)G(0;y) + 6360G(0,1;y)G(0;y) - 900y^4G(0,r;y)G(0;y) + 3960y^3G(0,r;y)G(0;y) - 4860y^2G(0,r;y)G(0;y) -
1440y^4G(1;y)^2G(0,r;y)G(0;y) + 2880y^2G(1;y)^2G(0,r;y)G(0;y) - 1440G(1;y)^2G(0,r;y)G(0;y) + 3960yG(0,r;y)G(0;y) +
360y^4G(1;y)G(0,r;y)G(0;y) - 720y^2G(1;y)G(0,r;y)G(0;y) + 360G(1;y)G(0,r;y)G(0;y) + 2160y^4G(0,-1;y)G(0,r;y)G(0;y) -
2160G(0,-1;y)G(0,r;y)G(0;y) - 900G(0,r;y)G(0;y) - 1800y^4G(r,1;y)G(0;y) + 7920y^3G(r,1;y)G(0;y) - 9720y^2G(r,1;y)G(0;y) -
2880y^4G(1;y)^2G(r,1;y)G(0;y) + 5760y^2G(1;y)^2G(r,1;y)G(0;y) - 2880G(1;y)^2G(r,1;y)G(0;y) + 7920yG(r,1;y)G(0;y) +
720y^4G(1;y)G(r,1;y)G(0;y) - 1440y^2G(1;y)G(r,1;y)G(0;y) + 720G(1;y)G(r,1;y)G(0;y) - 1800G(r,1;y)G(0;y) -
4320y^4G(0,-1,-1;y)G(0;y) - 17280y^3G(0,-1,-1;y)G(0;y) + 17280yG(0,-1,-1;y)G(0;y) - 17280y^4G(-1;y)G(0,-1,-1;y)G(0;y) +
17280G(-1;y)G(0,-1,-1;y)G(0;y) + 4320G(0,-1,-1;y)G(0;y) - 4320y^4G(0,-1,1;y)G(0;y) - 17280y^3G(0,-1,1;y)G(0;y) +
17280yG(0,-1,1;y)G(0;y) - 17280y^4G(-1;y)G(0,-1,1;y)G(0;y) + 17280G(-1;y)G(0,-1,1;y)G(0;y) + 4320G(0,-1,1;y)G(0;y) -
7200y^4G(0,0,-1;y)G(0;y) - 46080y^3G(0,0,-1;y)G(0;y) + 34560y^2G(0,0,-1;y)G(0;y) + 11520yG(0,0,-1;y)G(0;y) -
28800y^4G(-1;y)G(0,0,-1;y)G(0;y) + 28800G(-1;y)G(0,0,-1;y)G(0;y) + 17280y^4G(1;y)G(0,0,-1;y)G(0;y) -
34560y^2G(1;y)G(0,0,-1;y)G(0;y) + 17280G(1;y)G(0,0,-1;y)G(0;y) + 7200G(0,0,-1;y)G(0;y) - 360y^4G(0,0,1;y)G(0;y) -
36000y^3G(0,0,1;y)G(0;y) + 23040y^2G(0,0,1;y)G(0;y) + 7200yG(0,0,1;y)G(0;y) - 18720y^4G(-1;y)G(0,0,1;y)G(0;y) +
18720G(-1;y)G(0,0,1;y)G(0;y) + 11520y^4G(1;y)G(0,0,1;y)G(0;y) - 8640y^3G(1;y)G(0,0,1;y)G(0;y) - 5760y^2G(1;y)G(0,0,1;y)G(0;y) -
8640yG(1;y)G(0,0,1;y)G(0;y) + 11520G(1;y)G(0,0,1;y)G(0;y) + 6120G(0,0,1;y)G(0;y) - 1440y^4G(0,0,r;y)G(0;y) -
4320y^3G(0,0,r;y)G(0;y) + 720y^2G(0,0,r;y)G(0;y) + 4320yG(0,0,r;y)G(0;y) - 4320y^4G(-1;y)G(0,0,r;y)G(0;y) +
4320G(-1;y)G(0,0,r;y)G(0;y) + 2880y^4G(1;y)G(0,0,r;y)G(0;y) - 5760y^2G(1;y)G(0,0,r;y)G(0;y) + 2880G(1;y)G(0,0,r;y)G(0;y) +
720G(0,0,r;y)G(0;y) - 4320y^4G(0,1,-1;y)G(0;y) - 17280y^3G(0,1,-1;y)G(0;y) + 17280yG(0,1,-1;y)G(0;y) -
17280y^4G(-1;y)G(0,1,-1;y)G(0;y) + 17280G(-1;y)G(0,1,-1;y)G(0;y) + 4320G(0,1,-1;y)G(0;y) - 2880y^3G(0,1,1;y)G(0;y) -
10080y^2G(0,1,1;y)G(0;y) + 11520yG(0,1,1;y)G(0;y) - 5760y^4G(-1;y)G(0,1,1;y)G(0;y) + 5760G(-1;y)G(0,1,1;y)G(0;y) -
10080y^4G(1;y)G(0,1,1;y)G(0;y) - 2880y^3G(1;y)G(0,1,1;y)G(0;y) + 25920y^2G(1;y)G(0,1,1;y)G(0;y) - 2880yG(1;y)G(0,1,1;y)G(0;y) -
10080G(1;y)G(0,1,1;y)G(0;y) + 1440G(0,1,1;y)G(0;y) - 1800y^4G(0,r,1;y)G(0;y) - 4320y^3G(0,r,1;y)G(0;y) +
1440y^2G(0,r,1;y)G(0;y) + 4320yG(0,r,1;y)G(0;y) - 4320y^4G(-1;y)G(0,r,1;y)G(0;y) + 4320G(-1;y)G(0,r,1;y)G(0;y) +
5760y^4G(1;y)G(0,r,1;y)G(0;y) - 11520y^2G(1;y)G(0,r,1;y)G(0;y) + 5760G(1;y)G(0,r,1;y)G(0;y) + 360G(0,r,1;y)G(0;y) -
1440y^4G(r,1,1;y)G(0;y) + 2880y^2G(r,1,1;y)G(0;y) + 11520y^4G(1;y)G(r,1,1;y)G(0;y) - 23040y^2G(1;y)G(r,1,1;y)G(0;y) +
11520G(1;y)G(r,1,1;y)G(0;y) - 1440G(r,1,1;y)G(0;y) + 17280y^4G(0,-1,-1,-1;y)G(0;y) - 17280G(0,-1,-1,-1;y)G(0;y) +
$$

$17280y^4G(0,-1,-1,1;y)G(0;y) - 17280G(0,-1,-1,1;y)G(0;y) + 17280y^4G(0,-1,1,-1;y)G(0;y) - 17280G(0,-1,1,-1;y)G(0;y) +$
$5760y^4G(0,-1,1,1;y)G(0;y) - 5760G(0,-1,1,1;y)G(0;y) + 4320y^4G(0,-1,r,1;y)G(0;y) - 4320G(0,-1,r,1;y)G(0;y) +$
$28800y^4G(0,0,-1,-1;y)G(0;y) - 28800G(0,0,-1,-1;y)G(0;y) + 15840y^4G(0,0,-1,1;y)G(0;y) - 15840G(0,0,-1,1;y)G(0;y) +$
$35280y^4G(0,0,0,-1;y)G(0;y) - 34560y^2G(0,0,0,-1;y)G(0;y) - 720G(0,0,0,-1;y)G(0;y) + 3600y^4G(0,0,0,1;y)G(0;y) +$
$14400y^3G(0,0,0,1;y)G(0;y) - 33120y^2G(0,0,0,1;y)G(0;y) + 14400yG(0,0,0,1;y)G(0;y) + 720G(0,0,0,1;y)G(0;y) +$
$2160y^4G(0,0,0,r;y)G(0;y) - 2160y^2G(0,0,0,r;y)G(0;y) + 15840y^4G(0,0,1,-1;y)G(0;y) - 15840G(0,0,1,-1;y)G(0;y) -$
$3600y^4G(0,0,1,1;y)G(0;y) + 14400y^3G(0,0,1,1;y)G(0;y) - 20160y^2G(0,0,1,1;y)G(0;y) + 14400yG(0,0,1,1;y)G(0;y) -$
$5040G(0,0,1,1;y)G(0;y) + 2880y^4G(0,0,1,r;y)G(0;y) - 2880G(0,0,1,r;y)G(0;y) - 1440y^4G(0,0,r,1;y)G(0;y) +$
$4320y^2G(0,0,r,1;y)G(0;y) - 2880G(0,0,r,1;y)G(0;y) + 17280y^4G(0,1,-1,-1;y)G(0;y) - 17280G(0,1,-1,-1;y)G(0;y) +$
$5760y^4G(0,1,-1,1;y)G(0;y) - 5760G(0,1,-1,1;y)G(0;y) - 1440y^4G(0,1,0,-1;y)G(0;y) + 1440G(0,1,0,-1;y)G(0;y) +$
$1440y^4G(0,1,0,r;y)G(0;y) - 1440G(0,1,0,r;y)G(0;y) + 5760y^4G(0,1,1,-1;y)G(0;y) - 5760G(0,1,1,-1;y)G(0;y) +$
$8640y^4G(0,1,1,1;y)G(0;y) + 8640y^3G(0,1,1,1;y)G(0;y) - 43200y^2G(0,1,1,1;y)G(0;y) + 8640yG(0,1,1,1;y)G(0;y) +$
$17280G(0,1,1,1;y)G(0;y) + 2880y^4G(0,1,r,1;y)G(0;y) - 2880G(0,1,r,1;y)G(0;y) + 4320y^4G(0,r,-1,1;y)G(0;y) -$
$4320G(0,r,-1,1;y)G(0;y) - 2160y^4G(0,r,0,-1;y)G(0;y) + 2160G(0,r,0,-1;y)G(0;y) + 4320y^4G(0,r,1,-1;y)G(0;y) -$
$4320G(0,r,1,-1;y)G(0;y) - 8640y^4G(0,r,1,1;y)G(0;y) + 17280y^2G(0,r,1,1;y)G(0;y) - 8640G(0,r,1,1;y)G(0;y) -$
$17280y^4G(r,1,1,1;y)G(0;y) + 34560y^2G(r,1,1,1;y)G(0;y) - 17280G(r,1,1,1;y)G(0;y) - 7200y^3\zeta(3)G(0;y) + 5760y^2\zeta(3)G(0;y) +$
$1440y\zeta(3)G(0;y) - 2880y^4G(-1;y)\zeta(3)G(0;y) + 2880G(-1;y)\zeta(3)G(0;y) + 1440y^4G(1;y)\zeta(3)G(0;y) - 2880y^2G(1;y)\zeta(3)G(0;y) +$
$1440G(1;y)\zeta(3)G(0;y) - 2\pi^4 - 45\pi^4y^4 + 300\pi^2y^4 - 4800y^4 + 190\pi^4y^3 - 1290\pi^2y^3 + 19200y^3 + 160\pi^2y^4G(-1;y)^3 - 160\pi^2G(-1;y)^3 +$
$220\pi^2y^4G(1;y)^3 - 120\pi^2y^3G(1;y)^3 + 220\pi^2G(1;y)^3 - 200\pi^2y^2G(1;y)^3 - 120\pi^2yG(1;y)^3 + 300y^4G(r;y)^3 - 1320y^3G(r;y)^3 +$
$1620y^2G(r;y)^3 + 480y^4G(1;y)^2G(r;y)^3 - 960y^2G(1;y)^2G(r;y)^3 + 480G(1;y)^2G(r;y)^3 - 1320yG(r;y)^3 - 120y^4G(1;y)G(r;y)^3 +$
$240y^2G(1;y)G(r;y)^3 - 120G(1;y)G(r;y)^3 + 300G(r;y)^3 + 60\pi^2 - 325\pi^4y^2 + 1820\pi^2y^2 - 28800y^2 + 120\pi^2y^4G(-1;y)^2 + 480\pi^2y^3G(-1;y)^2 -$
$120\pi^2G(-1;y)^2 - 480\pi^2yG(-1;y)^2 + 60\pi^2y^4G(1;y)^2 + 1200\pi^2y^3G(1;y)^2 - 2580\pi^2y^2G(1;y)^2 + 1320\pi^2yG(1;y)^2 + 1260y^4G(r;y)^2 -$
$3780y^3G(r;y)^2 + 840y^4G(1;y)^3G(r;y)^2 - 720y^3G(1;y)^3G(r;y)^2 - 240y^2G(1;y)^3G(r;y)^2 - 720yG(1;y)^3G(r;y)^2 + 840G(1;y)^3G(r;y)^2 +$
$5040y^2G(r;y)^2 - 180y^4G(1;y)^2G(r;y)^2 + 5400y^3G(1;y)^2G(r;y)^2 - 10440y^2G(1;y)^2G(r;y)^2 + 5400yG(1;y)^2G(r;y)^2 - 180G(1;y)^2G(r;y)^2 -$
$3780yG(r;y)^2 - 1800y^4G(1;y)G(r;y)^2 + 12240y^3G(1;y)G(r;y)^2 - 20880y^2G(1;y)G(r;y)^2 + 12240yG(1;y)G(r;y)^2 - 1800G(1;y)G(r;y)^2 +$
$1260G(r;y)^2 - 3240y^4G(0,1;y)^2 - 4320y^3G(0,1;y)^2 + 2160y^2G(0,1;y)^2 + 4320yG(0,1;y)^2 - 5760y^4G(-1;y)G(0,1;y)^2 +$
$5760G(-1;y)G(0,1;y)^2 + 8640y^4G(1;y)G(0,1;y)^2 - 17280y^2G(1;y)G(0,1;y)^2 + 8640G(1;y)G(0,1;y)^2 + 1080G(0,1;y)^2 -$
$1080y^4G(0,r;y)^2 - 3240y^3G(0,r;y)^2 + 540y^2G(0,r;y)^2 + 3240yG(0,r;y)^2 - 3240y^4G(-1;y)G(0,r;y)^2 + 3240G(-1;y)G(0,r;y)^2 +$
$2160y^4G(1;y)G(0,r;y)^2 - 4320y^2G(1;y)G(0,r;y)^2 + 2160G(1;y)G(0,r;y)^2 + 540G(0,r;y)^2 - 1080y^4G(r,1;y)^2 + 2160y^2G(r,1;y)^2 +$
$8640y^4G(1;y)G(r,1;y)^2 - 17280y^2G(1;y)G(r,1;y)^2 + 8640G(1;y)G(r,1;y)^2 - 1080G(r,1;y)^2 + 182\pi^4y - 890\pi^2y + 19200y + 26\pi^4G(-1;y) -$
$26\pi^4y^4G(-1;y) - 280\pi^2y^4G(-1;y) - 560\pi^2y^3G(-1;y) + 280\pi^2G(-1;y) + 560\pi^2yG(-1;y) + 198\pi^4G(1;y) + 198\pi^4y^4G(1;y) -$
$1180\pi^2y^4G(1;y) + 13080y^4G(1;y) - 28\pi^4y^3G(1;y) + 3880\pi^2y^3G(1;y) - 52320y^3G(1;y) - 1060\pi^2G(1;y) - 340\pi^4y^2G(1;y) -$
$5280\pi^2y^2G(1;y) + 78480y^2G(1;y) - 28\pi^4yG(1;y) + 3640\pi^2yG(1;y) - 52320yG(1;y) + 13080G(1;y) + 450\pi^2y^4G(r;y) - 7260y^4G(r;y) -$
$180y^4G(1;y)^4G(r;y) + 360y^3G(1;y)^4G(r;y) - 360y^2G(1;y)^4G(r;y) + 360yG(1;y)^4G(r;y) - 180G(1;y)^4G(r;y) - 1980\pi^2y^3G(r;y) +$
$21780y^3G(r;y) + 360y^4G(1;y)^3G(r;y) - 2160y^3G(1;y)^3G(r;y) + 3600y^2G(1;y)^3G(r;y) - 2160yG(1;y)^3G(r;y) + 360G(1;y)^3G(r;y) +$
$450\pi^2G(r;y) + 2430\pi^2y^2G(r;y) - 29040y^2G(r;y) + 720\pi^2y^4G(1;y)^2G(r;y) - 3120y^4G(1;y)^2G(r;y) - 1920y^3G(1;y)^2G(r;y) +$
$720\pi^2G(1;y)^2G(r;y) - 1440\pi^2y^2G(1;y)^2G(r;y) + 10080y^2G(1;y)^2G(r;y) - 1920yG(1;y)^2G(r;y) - 3120G(1;y)^2G(r;y) - 1980\pi^2yG(r;y) +$
$21780yG(r;y) - 180\pi^2y^4G(1;y)G(r;y) - 2880y^4G(1;y)G(r;y) + 2040y^3G(1;y)G(r;y) - 180\pi^2G(1;y)G(r;y) + 360\pi^2y^2G(1;y)G(r;y) +$
$1680y^2G(1;y)G(r;y) + 2040yG(1;y)G(r;y) - 2880G(1;y)G(r;y) - 7260G(r;y) - 480\pi^2y^3G(0,-1;y) + 960\pi^2y^2G(0,-1;y) -$
$480\pi^2yG(0,-1;y) + 480\pi^2y^4G(1;y)G(0,-1;y) + 480\pi^2G(1;y)G(0,-1;y) - 960\pi^2y^2G(1;y)G(0,-1;y) - 300\pi^2y^4G(0,1;y) +$
$2160y^4G(0,1;y) - 1440\pi^2y^3G(0,1;y) - 13080y^3G(0,1;y) + 2640y^4G(1;y)^3G(0,1;y) - 1440y^3G(1;y)^3G(0,1;y) - 2400y^2G(1;y)^3G(0,1;y) -$
$1440yG(1;y)^3G(0,1;y) + 2640G(1;y)^3G(0,1;y) + 420\pi^2G(0,1;y) + 1320\pi^2y^2G(0,1;y) + 21840y^2G(0,1;y) + 360y^4G(1;y)^2G(0,1;y) +$
$15120y^3G(1;y)^2G(0,1;y) - 30960y^2G(1;y)^2G(0,1;y) + 15120yG(1;y)^2G(0,1;y) + 360G(1;y)^2G(0,1;y) - 13080yG(0,1;y) -$
$960\pi^2y^4G(-1;y)G(0,1;y) + 960\pi^2G(-1;y)G(0,1;y) + 1920\pi^2y^4G(1;y)G(0,1;y) - 13440y^4G(1;y)G(0,1;y) - 480\pi^2y^3G(1;y)G(0,1;y) +$
$45120y^3G(1;y)G(0,1;y) + 1920\pi^2G(1;y)G(0,1;y) - 2880\pi^2y^2G(1;y)G(0,1;y) - 63360y^2G(1;y)G(0,1;y) - 480\pi^2yG(1;y)G(0,1;y) +$
$45120yG(1;y)G(0,1;y) - 13440G(1;y)G(0,1;y) + 5400y^4G(r;y)G(0,1;y) - 23760y^3G(r;y)G(0,1;y) + 29160y^2G(r;y)G(0,1;y) +$
$8640y^4G(1;y)^2G(r;y)G(0,1;y) - 17280y^2G(1;y)^2G(r;y)G(0,1;y) + 8640G(1;y)^2G(r;y)G(0,1;y) - 23760yG(r;y)G(0,1;y) -$
$2160y^4G(1;y)G(r;y)G(0,1;y) + 4320y^2G(1;y)G(r;y)G(0,1;y) - 2160G(1;y)G(r;y)G(0,1;y) + 5400G(r;y)G(0,1;y) + 2160G(0,1;y) +$
$360\pi^2y^4G(0,r;y) + 1080y^4G(0,r;y) + 1080\pi^2y^3G(0,r;y) + 2940y^3G(0,r;y) + 2880y^4G(-1;y)^3G(0,r;y) - 2880G(-1;y)^3G(0,r;y) -$
$1320y^4G(1;y)^3G(0,r;y) + 720y^3G(1;y)^3G(0,r;y) + 1200y^2G(1;y)^3G(0,r;y) + 720yG(1;y)^3G(0,r;y) - 1320G(1;y)^3G(0,r;y) -$
$180\pi^2G(0,r;y) - 180\pi^2y^2G(0,r;y) - 10920y^2G(0,r;y) + 2160y^4G(-1;y)^2G(0,r;y) + 8640y^3G(-1;y)^2G(0,r;y) - 8640yG(-1;y)^2G(0,r;y) -$
$2160G(-1;y)^2G(0,r;y) + 360y^4G(1;y)^2G(0,r;y) - 8640y^3G(1;y)^2G(0,r;y) + 15480y^2G(1;y)^2G(0,r;y) - 6480yG(1;y)^2G(0,r;y) -$
$720G(1;y)^2G(0,r;y) - 1080\pi^2yG(0,r;y) + 10140yG(0,r;y) + 1080\pi^2y^4G(-1;y)G(0,r;y) - 5040y^4G(-1;y)G(0,r;y) -$
$10080y^3G(-1;y)G(0,r;y) - 1080\pi^2G(-1;y)G(0,r;y) + 10080yG(-1;y)G(0,r;y) + 5040G(-1;y)G(0,r;y) - 720\pi^2y^4G(1;y)G(0,r;y) +$
$5640y^4G(1;y)G(0,r;y) - 20400y^3G(1;y)G(0,r;y) - 720\pi^2G(1;y)G(0,r;y) + 1440\pi^2y^2G(1;y)G(0,r;y) + 31680y^2G(1;y)G(0,r;y) -$
$24720yG(1;y)G(0,r;y) + 7800G(1;y)G(0,r;y) - 2700y^4G(r;y)G(0,r;y) + 11880y^3G(r;y)G(0,r;y) - 14580y^2G(r;y)G(0,r;y) -$
$4320y^4G(1;y)^2G(r;y)G(0,r;y) + 8640y^2G(1;y)^2G(r;y)G(0,r;y) - 4320G(1;y)^2G(r;y)G(0,r;y) + 11880yG(r;y)G(0,r;y) +$
$1080y^4G(1;y)G(r;y)G(0,r;y) - 2160y^2G(1;y)G(r;y)G(0,r;y) + 1080G(1;y)G(r;y)G(0,r;y) - 2700G(r;y)G(0,r;y) -$
$8640y^3G(0,-1;y)G(0,r;y) + 17280y^2G(0,-1;y)G(0,r;y) - 8640yG(0,-1;y)G(0,r;y) + 8640y^4G(1;y)G(0,-1;y)G(0,r;y) -$
$17280y^2G(1;y)G(0,-1;y)G(0,r;y) + 8640G(1;y)G(0,-1;y)G(0,r;y) + 4320y^4G(0,1;y)G(0,r;y) + 12960y^3G(0,1;y)G(0,r;y) -$

$2160y^2G(0,1;y)G(0,r;y) - 12960yG(0,1;y)G(0,r;y) + 12960y^4G(-1;y)G(0,1;y)G(0,r;y) - 12960G(-1;y)G(0,1;y)G(0,r;y) -$
$8640y^4G(1;y)G(0,1;y)G(0,r;y) + 17280y^2G(1;y)G(0,1;y)G(0,r;y) - 8640G(1;y)G(0,1;y)G(0,r;y) - 2160G(0,1;y)G(0,r;y) -$
$3240G(0,r;y) + 180\pi^2y^4G(r,1;y) - 2160y^4G(r,1;y) + 13080y^3G(r,1;y) - 2640y^4G(1;y)^3G(r,1;y) + 1440y^3G(1;y)^3G(r,1;y) +$
$2400y^2G(1;y)^3G(r,1;y) + 1440yG(1;y)^3G(r,1;y) - 2640G(1;y)^3G(r,1;y) + 180\pi^2G(r,1;y) - 360\pi^2y^2G(r,1;y) - 21840y^2G(r,1;y) -$
$360y^4G(1;y)^2G(r,1;y) - 15120y^3G(1;y)^2G(r,1;y) + 30960y^2G(1;y)^2G(r,1;y) - 15120yG(1;y)^2G(r,1;y) - 360G(1;y)^2G(r,1;y) +$
$360y^4G(r;y)^2G(r,1;y) - 720y^2G(r;y)^2G(r,1;y) - 2880y^4G(1;y)G(r;y)^2G(r,1;y) + 5760y^2G(1;y)G(r;y)^2G(r,1;y) -$
$2880G(1;y)G(r;y)^2G(r,1;y) + 360G(r;y)^2G(r,1;y) + 13080yG(r,1;y) - 1440\pi^2y^4G(1;y)G(r,1;y) + 13440y^4G(1;y)G(r,1;y) -$
$45120y^3G(1;y)G(r,1;y) - 1440\pi^2G(1;y)G(r,1;y) + 2880\pi^2y^2G(1;y)G(r,1;y) + 63360y^2G(1;y)G(r,1;y) - 45120yG(1;y)G(r,1;y) +$
$13440G(1;y)G(r,1;y) + 5400y^4G(r;y)G(r,1;y) - 32400y^3G(r;y)G(r,1;y) + 51480y^2G(r;y)G(r,1;y) - 2160y^4G(1;y)^2G(r;y)G(r,1;y) +$
$4320y^3G(1;y)^2G(r;y)G(r,1;y) - 4320y^2G(1;y)^2G(r;y)G(r,1;y) + 4320yG(1;y)^2G(r;y)G(r,1;y) - 2160G(1;y)^2G(r;y)G(r,1;y) -$
$32400yG(r;y)G(r,1;y) - 21600y^3G(1;y)G(r;y)G(r,1;y) + 43200y^2G(1;y)G(r;y)G(r,1;y) - 21600yG(1;y)G(r;y)G(r,1;y) +$
$5400G(r;y)G(r,1;y) + 2160y^4G(0,1;y)G(r,1;y) - 4320y^2G(0,1;y)G(r,1;y) - 17280y^4G(1;y)G(0,1;y)G(r,1;y) +$
$34560y^2G(1;y)G(0,1;y)G(r,1;y) - 17280G(1;y)G(0,1;y)G(r,1;y) + 2160G(0,1;y)G(r,1;y) - 1080y^4G(0,r;y)G(r,1;y) +$
$2160y^2G(0,r;y)G(r,1;y) + 8640y^4G(1;y)G(0,r;y)G(r,1;y) - 17280y^2G(1;y)G(0,r;y)G(r,1;y) + 8640G(1;y)G(0,r;y)G(r,1;y) -$
$1080G(0,r;y)G(r,1;y) - 2160G(r,1;y) - 480\pi^2y^4G(0,-1,-1;y) - 480\pi^2G(0,-1,-1;y) + 960\pi^2y^2G(0,-1,-1;y) -$
$8640y^4G(0,r;y)G(0,-1,-1;y) + 17280y^2G(0,r;y)G(0,-1,-1;y) - 8640G(0,r;y)G(0,-1,-1;y) + 480\pi^2y^4G(0,-1,1;y) -$
$1440\pi^2G(0,-1,1;y) + 960\pi^2y^2G(0,-1,1;y) + 11520y^4G(0,1;y)G(0,-1,1;y) - 11520G(0,1;y)G(0,-1,1;y) -$
$14400y^4G(0,r;y)G(0,-1,1;y) + 17280y^2G(0,r;y)G(0,-1,1;y) - 2880G(0,r;y)G(0,-1,1;y) - 1080\pi^2y^4G(0,-1,r;y) +$
$5040y^4G(0,-1,r;y) + 10080y^3G(0,-1,r;y) + 1080\pi^2G(0,-1,r;y) - 8640y^4G(-1;y)^2G(0,-1,r;y) + 8640G(-1;y)^2G(0,-1,r;y) -$
$10080yG(0,-1,r;y) - 4320y^4G(-1;y)G(0,-1,r;y) - 17280y^3G(-1;y)G(0,-1,r;y) + 17280yG(-1;y)G(0,-1,r;y) +$
$4320G(-1;y)G(0,-1,r;y) - 12960y^4G(0,1;y)G(0,-1,r;y) + 12960G(0,1;y)G(0,-1,r;y) + 6480y^4G(0,r;y)G(0,-1,r;y) -$
$6480G(0,r;y)G(0,-1,r;y) - 5040G(0,-1,r;y) - 1320\pi^2y^4G(0,0,-1;y) + 5040y^4G(0,0,-1;y) + 10080y^3G(0,0,-1;y) +$
$360\pi^2G(0,0,-1;y) + 960\pi^2y^2G(0,0,-1;y) - 8640y^4G(-1;y)^2G(0,0,-1;y) + 8640G(-1;y)^2G(0,0,-1;y) - 10080yG(0,0,-1;y) -$
$4320y^4G(-1;y)G(0,0,-1;y) - 17280y^3G(-1;y)G(0,0,-1;y) + 17280yG(-1;y)G(0,0,-1;y) + 4320G(-1;y)G(0,0,-1;y) +$
$8640y^4G(0,-1;y)G(0,0,-1;y) - 17280y^2G(0,-1;y)G(0,0,-1;y) + 8640G(0,-1;y)G(0,0,-1;y) + 8640y^4G(0,1;y)G(0,0,-1;y) -$
$17280y^2G(0,1;y)G(0,0,-1;y) + 8640G(0,1;y)G(0,0,-1;y) - 5040G(0,0,-1;y) - 2520\pi^2y^4G(0,0,1;y) + 3960y^4G(0,0,1;y) +$
$1440\pi^2y^3G(0,0,1;y) + 10800y^3G(0,0,1;y) + 360\pi^2G(0,0,1;y) - 720\pi^2y^2G(0,0,1;y) + 23040y^2G(0,0,1;y) -$
$8640y^4G(-1;y)^2G(0,0,1;y) + 8640G(-1;y)^2G(0,0,1;y) - 16200y^4G(1;y)^2G(0,0,1;y) + 3600y^3G(1;y)^2G(0,0,1;y) +$
$25200y^2G(1;y)^2G(0,0,1;y) + 3600yG(1;y)^2G(0,0,1;y) - 16200G(1;y)^2G(0,0,1;y) + 1440\pi^2yG(0,0,1;y) - 21360yG(0,0,1;y) -$
$4320y^4G(-1;y)G(0,0,1;y) - 17280y^3G(-1;y)G(0,0,1;y) + 17280yG(-1;y)G(0,0,1;y) + 4320G(-1;y)G(0,0,1;y) +$
$5040y^4G(1;y)G(0,0,1;y) - 23040y^3G(1;y)G(0,0,1;y) + 32400y^2G(1;y)G(0,0,1;y) - 15840yG(1;y)G(0,0,1;y) + 1440G(1;y)G(0,0,1;y) +$
$14400y^4G(0,-1;y)G(0,0,1;y) - 17280y^2G(0,-1;y)G(0,0,1;y) + 2880G(0,-1;y)G(0,0,1;y) + 25200y^4G(0,1;y)G(0,0,1;y) -$
$5760y^3G(0,1;y)G(0,0,1;y) - 2880y^2G(0,1;y)G(0,0,1;y) - 5760yG(0,1;y)G(0,0,1;y) - 10800G(0,1;y)G(0,0,1;y) - 6360G(0,0,1;y) -$
$360\pi^2y^4G(0,0,r;y) + 5700y^4G(0,0,r;y) - 4800y^3G(0,0,r;y) + 360\pi^2y^2G(0,0,r;y) - 3960y^2G(0,0,r;y) - 4320y^4G(-1;y)^2G(0,0,r;y) +$
$4320G(-1;y)^2G(0,0,r;y) + 6120y^4G(1;y)^2G(0,0,r;y) + 2160y^3G(1;y)^2G(0,0,r;y) - 16560y^2G(1;y)^2G(0,0,r;y) +$
$2160yG(1;y)^2G(0,0,r;y) + 6120G(1;y)^2G(0,0,r;y) - 2880yG(0,0,r;y) - 2160y^4G(-1;y)G(0,0,r;y) - 8640y^3G(-1;y)G(0,0,r;y) +$
$8640yG(-1;y)G(0,0,r;y) + 2160G(-1;y)G(0,0,r;y) - 7920y^4G(1;y)G(0,0,r;y) + 23040y^3G(1;y)G(0,0,r;y) - 26640y^2G(1;y)G(0,0,r;y) +$
$15840yG(1;y)G(0,0,r;y) - 4320G(1;y)G(0,0,r;y) - 1440y^4G(0,-1;y)G(0,0,r;y) - 8640y^2G(0,-1;y)G(0,0,r;y) +$
$10080G(0,-1;y)G(0,0,r;y) - 3600y^4G(0,1;y)G(0,0,r;y) + 5760y^3G(0,1;y)G(0,0,r;y) - 23040y^2G(0,1;y)G(0,0,r;y) +$
$5760yG(0,1;y)G(0,0,r;y) + 15120G(0,1;y)G(0,0,r;y) + 900G(0,0,r;y) + 480\pi^2y^4G(0,1,-1;y) - 1440\pi^2G(0,1,-1;y) +$
$960\pi^2y^2G(0,1,-1;y) + 11520y^4G(0,1;y)G(0,1,-1;y) - 11520G(0,1;y)G(0,1,-1;y) - 23040y^4G(0,r;y)G(0,1,-1;y) +$
$17280y^2G(0,r;y)G(0,1,-1;y) + 5760G(0,r;y)G(0,1,-1;y) - 3000\pi^2y^4G(0,1,1;y) + 20640y^4G(0,1,1;y) + 480\pi^2y^3G(0,1,1;y) -$
$94080y^3G(0,1,1;y) - 1320\pi^2G(0,1,1;y) + 3360\pi^2y^2G(0,1,1;y) + 146880y^2G(0,1,1;y) - 18000y^4G(1;y)^2G(0,1,1;y) +$
$12960y^3G(1;y)^2G(0,1,1;y) + 10080y^2G(1;y)^2G(0,1,1;y) + 12960yG(1;y)^2G(0,1,1;y) - 18000G(1;y)^2G(0,1,1;y) + 480\pi^2yG(0,1,1;y) -$
$94080yG(0,1,1;y) + 720y^4G(1;y)G(0,1,1;y) - 73440y^3G(1;y)G(0,1,1;y) + 145440y^2G(1;y)G(0,1,1;y) - 73440yG(1;y)G(0,1,1;y) +$
$720G(1;y)G(0,1,1;y) - 28800y^4G(0,-1;y)G(0,1,1;y) + 28800G(0,-1;y)G(0,1,1;y) - 27360y^4G(0,1;y)G(0,1,1;y) +$
$34560y^2G(0,1;y)G(0,1,1;y) - 7200G(0,1;y)G(0,1,1;y) + 20640G(0,1,1;y) - 720\pi^2y^4G(0,1,r;y) - 6960y^4G(0,1,r;y) +$
$62400y^3G(0,1,r;y) + 720\pi^2G(0,1,r;y) - 80640y^2G(0,1,r;y) - 8640y^4G(-1;y)^2G(0,1,r;y) + 8640G(-1;y)^2G(0,1,r;y) -$
$6480y^4G(1;y)^2G(0,1,r;y) - 4320y^3G(1;y)^2G(0,1,r;y) + 21600y^2G(1;y)^2G(0,1,r;y) - 4320yG(1;y)^2G(0,1,r;y) -$
$6480G(1;y)^2G(0,1,r;y) + 49920yG(0,1,r;y) - 4320y^4G(-1;y)G(0,1,r;y) - 17280y^3G(-1;y)G(0,1,r;y) + 17280yG(-1;y)G(0,1,r;y) +$
$4320G(-1;y)G(0,1,r;y) + 1440y^4G(1;y)G(0,1,r;y) + 23040y^3G(1;y)G(0,1,r;y) - 47520y^2G(1;y)G(0,1,r;y) + 20160yG(1;y)G(0,1,r;y) +$
$2880G(1;y)G(0,1,r;y) + 34560y^4G(0,-1;y)G(0,1,r;y) - 17280y^2G(0,-1;y)G(0,1,r;y) - 17280G(0,-1;y)G(0,1,r;y) -$
$8640y^4G(0,1;y)G(0,1,r;y) + 8640G(0,1;y)G(0,1,r;y) - 14640G(0,1,r;y) - 1080\pi^2y^4G(0,r,-1;y) + 5040y^4G(0,r,-1;y) +$
$10080y^3G(0,r,-1;y) + 1080\pi^2G(0,r,-1;y) - 8640y^4G(-1;y)^2G(0,r,-1;y) + 8640G(-1;y)^2G(0,r,-1;y) - 10080yG(0,r,-1;y) -$
$4320y^4G(-1;y)G(0,r,-1;y) - 17280y^3G(-1;y)G(0,r,-1;y) + 17280yG(-1;y)G(0,r,-1;y) + 4320G(-1;y)G(0,r,-1;y) +$
$8640y^4G(0,-1;y)G(0,r,-1;y) - 17280y^2G(0,-1;y)G(0,r,-1;y) + 8640G(0,-1;y)G(0,r,-1;y) - 12960y^4G(0,1;y)G(0,r,-1;y) +$
$12960G(0,1;y)G(0,r,-1;y) + 6480y^4G(0,r;y)G(0,r,-1;y) - 6480G(0,r;y)G(0,r,-1;y) - 5040G(0,r,-1;y) + 720\pi^2y^4G(0,r,1;y) -$
$7800y^4G(0,r,1;y) + 43440y^3G(0,r,1;y) + 720\pi^2G(0,r,1;y) - 1440\pi^2y^2G(0,r,1;y) - 73440y^2G(0,r,1;y) - 8640y^4G(-1;y)^2G(0,r,1;y) +$
$8640G(-1;y)^2G(0,r,1;y) + 9000y^4G(1;y)^2G(0,r,1;y) - 6480y^3G(1;y)^2G(0,r,1;y) - 5040y^2G(1;y)^2G(0,r,1;y) -$
$6480yG(1;y)^2G(0,r,1;y) + 9000G(1;y)^2G(0,r,1;y) + 50640yG(0,r,1;y) - 4320y^4G(-1;y)G(0,r,1;y) - 17280y^3G(-1;y)G(0,r,1;y) +$

$17280yG(-1;y)G(0,r,1;y) + 4320G(-1;y)G(0,r,1;y) - 3600y^4G(1;y)G(0,r,1;y) + 43200y^3G(1;y)G(0,r,1;y) -$
$72720y^2G(1;y)G(0,r,1;y) + 30240yG(1;y)G(0,r,1;y) + 2880G(1;y)G(0,r,1;y) + 23040y^4G(0,-1;y)G(0,r,1;y) -$
$17280y^2G(0,-1;y)G(0,r,1;y) - 5760G(0,-1;y)G(0,r,1;y) + 8640y^4G(0,1;y)G(0,r,1;y) - 17280y^2G(0,1;y)G(0,r,1;y) +$
$8640G(0,1;y)G(0,r,1;y) - 4320y^4G(0,r;y)G(0,r,1;y) + 8640y^2G(0,r;y)G(0,r,1;y) - 4320G(0,r;y)G(0,r,1;y) - 12840G(0,r,1;y) +$
$4680y^4G(0,r,r;y) - 27360y^3G(0,r,r;y) + 40320y^2G(0,r,r;y) + 4320y^4G(-1;y)^2G(0,r,r;y) - 4320G(-1;y)^2G(0,r,r;y) +$
$3240y^4G(1;y)^2G(0,r,r;y) + 2160y^3G(1;y)^2G(0,r,r;y) - 10800y^2G(1;y)^2G(0,r,r;y) + 2160yG(1;y)^2G(0,r,r;y) +$
$3240G(1;y)^2G(0,r,r;y) - 28800yG(0,r,r;y) + 2160y^4G(-1;y)G(0,r,r;y) + 8640y^3G(-1;y)G(0,r,r;y) - 8640yG(-1;y)G(0,r,r;y) -$
$2160G(-1;y)G(0,r,r;y) - 12960y^3G(1;y)G(0,r,r;y) + 23760y^2G(1;y)G(0,r,r;y) - 8640yG(1;y)G(0,r,r;y) - 2160G(1;y)G(0,r,r;y) -$
$17280y^4G(0,-1;y)G(0,r,r;y) + 8640y^2G(0,-1;y)G(0,r,r;y) + 8640G(0,-1;y)G(0,r,r;y) + 6120G(0,r,r;y) + 1440\pi^2y^4G(r,1,1;y) -$
$20640y^4G(r,1,1;y)+94080y^3G(r,1,1;y)+1440\pi^2G(r,1,1;y)-2880\pi^2y^2G(r,1,1;y)-146880y^2G(r,1,1;y)+18000y^4G(1;y)^2G(r,1,1;y)-$
$12960y^3G(1;y)^2G(r,1,1;y) - 10080y^2G(1;y)^2G(r,1,1;y) - 12960yG(1;y)^2G(r,1,1;y) + 18000G(1;y)^2G(r,1,1;y) +$
$2880y^4G(r;y)^2G(r,1,1;y) - 5760y^2G(r;y)^2G(r,1,1;y) + 2880G(r;y)^2G(r,1,1;y) + 94080yG(r,1,1;y) - 720y^4G(1;y)G(r,1,1;y) +$
$73440y^3G(1;y)G(r,1,1;y) - 145440y^2G(1;y)G(r,1,1;y) + 73440yG(1;y)G(r,1,1;y) - 720G(1;y)G(r,1,1;y) + 720y^4G(r;y)G(r,1,1;y) +$
$21600y^3G(r;y)G(r,1,1;y) - 44640y^2G(r;y)G(r,1,1;y) + 21600yG(r;y)G(r,1,1;y) - 1440y^4G(1;y)G(r;y)G(r,1,1;y) -$
$8640y^3G(1;y)G(r;y)G(r,1,1;y) + 20160y^2G(1;y)G(r;y)G(r,1,1;y) - 8640yG(1;y)G(r;y)G(r,1,1;y) - 1440G(1;y)G(r;y)G(r,1,1;y) +$
$720G(r;y)G(r,1,1;y) + 17280y^4G(0,1;y)G(r,1,1;y) - 34560y^2G(0,1;y)G(r,1,1;y) + 17280G(0,1;y)G(r,1,1;y) -$
$8640y^4G(0,r;y)G(r,1,1;y) + 17280y^2G(0,r;y)G(r,1,1;y) - 8640G(0,r;y)G(r,1,1;y) - 17280y^4G(r,1;y)G(r,1,1;y) +$
$34560y^2G(r,1;y)G(r,1,1;y) - 17280G(r,1;y)G(r,1,1;y) - 20640G(r,1,1;y) - 10800y^4G(r,r,1;y) + 56160y^3G(r,r,1;y) -$
$80640y^2G(r,r,1;y) - 6480y^4G(1;y)^2G(r,r,1;y) - 4320y^3G(1;y)^2G(r,r,1;y) + 21600y^2G(1;y)^2G(r,r,1;y) - 4320yG(1;y)^2G(r,r,1;y) -$
$6480G(1;y)^2G(r,r,1;y) + 56160yG(r,r,1;y) + 2160y^4G(1;y)G(r,r,1;y) + 21600y^3G(1;y)G(r,r,1;y) - 47520y^2G(1;y)G(r,r,1;y) +$
$21600yG(1;y)G(r,r,1;y)+2160G(1;y)G(r,r,1;y)-720y^4G(r;y)G(r,r,1;y)+1440y^2G(r;y)G(r,r,1;y)+5760y^4G(1;y)G(r;y)G(r,r,1;y)-$
$11520y^2G(1;y)G(r;y)G(r,r,1;y) + 5760G(1;y)G(r;y)G(r,r,1;y) - 720G(r;y)G(r,r,1;y) - 10800G(r,r,1;y) + 4320y^4G(0,-1,-1,r;y) +$
$17280y^3G(0,-1,-1,r;y) - 17280yG(0,-1,-1,r;y) + 17280y^4G(-1;y)G(0,-1,-1,r;y) - 17280G(-1;y)G(0,-1,-1,r;y) -$
$4320G(0,-1,-1,r;y) + 4320y^4G(0,-1,1,r;y) + 17280y^3G(0,-1,1,r;y) - 17280yG(0,-1,1,r;y) + 17280y^4G(-1;y)G(0,-1,1,r;y) -$
$17280G(-1;y)G(0,-1,1,r;y) - 4320G(0,-1,1,r;y) + 4320y^4G(0,-1,r,-1;y) + 17280y^3G(0,-1,r,-1;y) - 17280yG(0,-1,r,-1;y) +$
$17280y^4G(-1;y)G(0,-1,r,-1;y) - 17280G(-1;y)G(0,-1,r,-1;y) - 4320G(0,-1,r,-1;y) + 4320y^4G(0,-1,r,1;y) +$
$17280y^3G(0,-1,r,1;y)-17280yG(0,-1,r,1;y)+17280y^4G(-1;y)G(0,-1,r,1;y)-17280G(-1;y)G(0,-1,r,1;y)-4320G(0,-1,r,1;y)-$
$2160y^4G(0,-1,r,r;y) - 8640y^3G(0,-1,r,r;y) + 8640yG(0,-1,r,r;y) - 8640y^4G(-1;y)G(0,-1,r,r;y) + 8640G(-1;y)G(0,-1,r,r;y) +$
$2160G(0,-1,r,r;y)+4320y^4G(0,0,-1,-1;y)+17280y^3G(0,0,-1,-1;y)-17280yG(0,0,-1,-1;y)+17280y^4G(-1;y)G(0,0,-1,-1;y)-$
$17280G(-1;y)G(0,0,-1,-1;y) - 4320G(0,0,-1,-1;y) + 4320y^4G(0,0,-1,1;y) + 17280y^3G(0,0,-1,1;y) - 17280yG(0,0,-1,1;y) +$
$17280y^4G(-1;y)G(0,0,-1,1;y)-17280G(-1;y)G(0,0,-1,1;y)-4320G(0,0,-1,1;y)+2160y^4G(0,0,-1,r;y)+25920y^3G(0,0,-1,r;y)-$
$34560y^2G(0,0,-1,r;y) + 8640yG(0,0,-1,r;y) + 8640y^4G(-1;y)G(0,0,-1,r;y) - 8640G(-1;y)G(0,0,-1,r;y) -$
$17280y^4G(1;y)G(0,0,-1,r;y) + 34560y^2G(1;y)G(0,0,-1,r;y) - 17280G(1;y)G(0,0,-1,r;y) - 2160G(0,0,-1,r;y) +$
$9360y^4G(0,0,0,-1;y) + 63360y^3G(0,0,0,-1;y) - 51840y^2G(0,0,0,-1;y) - 11520yG(0,0,0,-1;y) + 37440y^4G(-1;y)G(0,0,0,-1;y) -$
$37440G(-1;y)G(0,0,0,-1;y) - 25920y^4G(1;y)G(0,0,0,-1;y) + 51840y^2G(1;y)G(0,0,0,-1;y) - 25920G(1;y)G(0,0,0,-1;y) -$
$9360G(0,0,0,-1;y) - 3960y^4G(0,0,0,1;y) + 60480y^3G(0,0,0,1;y) - 52560y^2G(0,0,0,1;y) + 4320yG(0,0,0,1;y) +$
$21600y^4G(-1;y)G(0,0,0,1;y) - 21600G(-1;y)G(0,0,0,1;y) - 720y^4G(1;y)G(0,0,0,1;y) + 7200y^3G(1;y)G(0,0,0,1;y) -$
$12960y^2G(1;y)G(0,0,0,1;y) + 7200yG(1;y)G(0,0,0,1;y) - 720G(1;y)G(0,0,0,1;y) - 8280G(0,0,0,1;y) + 11160y^4G(0,0,0,r;y) +$
$4320y^3G(0,0,0,r;y)-14400yG(0,0,0,r;y)+14400y^4G(-1;y)G(0,0,0,r;y)-14400G(-1;y)G(0,0,0,r;y)-16560y^4G(1;y)G(0,0,0,r;y)-$
$4320y^3G(1;y)G(0,0,0,r;y) + 41760y^2G(1;y)G(0,0,0,r;y) - 4320yG(1;y)G(0,0,0,r;y) - 16560G(1;y)G(0,0,0,r;y) -$
$1080G(0,0,0,r;y) + 4320y^4G(0,0,1,-1;y) + 17280y^3G(0,0,1,-1;y) - 17280yG(0,0,1,-1;y) + 17280y^4G(-1;y)G(0,0,1,-1;y) -$
$17280G(-1;y)G(0,0,1,-1;y) - 4320G(0,0,1,-1;y) + 3600y^4G(0,0,1,1;y) + 74880y^3G(0,0,1,1;y) - 62640y^2G(0,0,1,1;y) -$
$10080yG(0,0,1,1;y) + 34560y^4G(-1;y)G(0,0,1,1;y) - 34560G(-1;y)G(0,0,1,1;y) + 28080y^4G(1;y)G(0,0,1,1;y) -$
$10080y^3G(1;y)G(0,0,1,1;y)-36000y^2G(1;y)G(0,0,1,1;y)-10080yG(1;y)G(0,0,1,1;y)+28080G(1;y)G(0,0,1,1;y)-5760G(0,0,1,1;y)+$
$360y^4G(0,0,1,r;y) - 44640y^3G(0,0,1,r;y) + 23040y^2G(0,0,1,r;y) + 12960yG(0,0,1,r;y) - 24480y^4G(-1;y)G(0,0,1,r;y) +$
$24480G(-1;y)G(0,0,1,r;y) + 8640y^4G(1;y)G(0,0,1,r;y) - 8640y^3G(1;y)G(0,0,1,r;y) - 8640yG(1;y)G(0,0,1,r;y) +$
$8640G(1;y)G(0,0,1,r;y) + 8280G(0,0,1,r;y) + 2160y^4G(0,0,r,-1;y) + 25920y^3G(0,0,r,-1;y) - 34560y^2G(0,0,r,-1;y) +$
$8640yG(0,0,r,-1;y) + 8640y^4G(-1;y)G(0,0,r,-1;y) - 8640G(-1;y)G(0,0,r,-1;y) - 17280y^4G(1;y)G(0,0,r,-1;y) +$
$34560y^2G(1;y)G(0,0,r,-1;y) - 17280G(1;y)G(0,0,r,-1;y) - 2160G(0,0,r,-1;y) + 10800y^4G(0,0,r,1;y) - 40320y^3G(0,0,r,1;y) +$
$29520y^2G(0,0,r,1;y) - 7200yG(0,0,r,1;y) - 8640y^4G(-1;y)G(0,0,r,1;y) + 8640G(-1;y)G(0,0,r,1;y) - 13680y^4G(1;y)G(0,0,r,1;y) -$
$12960y^3G(1;y)G(0,0,r,1;y) + 53280y^2G(1;y)G(0,0,r,1;y) - 12960yG(1;y)G(0,0,r,1;y) - 13680G(1;y)G(0,0,r,1;y) +$
$7200G(0,0,r,1;y) + 12960y^3G(0,0,r,r;y) - 2160y^2G(0,0,r,r;y) - 8640yG(0,0,r,r;y) + 8640y^4G(-1;y)G(0,0,r,r;y) -$
$8640G(-1;y)G(0,0,r,r;y) - 2160y^4G(1;y)G(0,0,r,r;y) + 4320y^3G(1;y)G(0,0,r,r;y) - 4320y^2G(1;y)G(0,0,r,r;y) +$
$4320yG(1;y)G(0,0,r,r;y) - 2160G(1;y)G(0,0,r,r;y) - 2160G(0,0,r,r;y) + 4320y^4G(0,1,-1,r;y) + 17280y^3G(0,1,-1,r;y) -$
$17280yG(0,1,-1,r;y) + 17280y^4G(-1;y)G(0,1,-1,r;y) - 17280G(-1;y)G(0,1,-1,r;y) - 4320G(0,1,-1,r;y) + 3240y^4G(0,1,0,r;y) -$
$21600y^3G(0,1,0,r;y) + 17280y^2G(0,1,0,r;y) - 4320yG(0,1,0,r;y) - 7200y^4G(-1;y)G(0,1,0,r;y) + 7200G(-1;y)G(0,1,0,r;y) +$
$8640y^4G(1;y)G(0,1,0,r;y) - 8640y^3G(1;y)G(0,1,0,r;y) - 8640yG(1;y)G(0,1,0,r;y) + 8640G(1;y)G(0,1,0,r;y) + 5400G(0,1,0,r;y) -$
$2160y^4G(0,1,1,1;y) + 116640y^3G(0,1,1,1;y) - 228960y^2G(0,1,1,1;y) + 116640yG(0,1,1,1;y) + 56160y^4G(1;y)G(0,1,1,1;y) -$
$43200y^3G(1;y)G(0,1,1,1;y) - 25920y^2G(1;y)G(0,1,1,1;y) - 43200yG(1;y)G(0,1,1,1;y) + 56160G(1;y)G(0,1,1,1;y) -$
$2160G(0,1,1,1;y) - 14400y^3G(0,1,1,r;y) + 44640y^2G(0,1,1,r;y) - 28800yG(0,1,1,r;y) + 5760y^4G(-1;y)G(0,1,1,r;y) -$

$5760G(-1;y)G(0,1,1,r;y) + 1440y^4G(1;y)G(0,1,1,r;y) + 8640y^3G(1;y)G(0,1,1,r;y) - 20160y^2G(1;y)G(0,1,1,r;y) +$
$8640yG(1;y)G(0,1,1,r;y) + 1440G(1;y)G(0,1,1,r;y) - 1440G(0,1,1,r;y) + 4320y^4G(0,1,r,-1;y) + 17280y^3G(0,1,r,-1;y) -$
$17280yG(0,1,r,-1;y) + 17280y^4G(-1;y)G(0,1,r,-1;y) - 17280G(-1;y)G(0,1,r,-1;y) - 4320G(0,1,r,-1;y) + 4320y^4G(0,1,r,1;y) -$
$5760y^3G(0,1,r,1;y) + 44640y^2G(0,1,r,1;y) - 37440yG(0,1,r,1;y) + 17280y^4G(-1;y)G(0,1,r,1;y) - 17280G(-1;y)G(0,1,r,1;y) +$
$1440y^4G(1;y)G(0,1,r,1;y) + 8640y^3G(1;y)G(0,1,r,1;y) - 20160y^2G(1;y)G(0,1,r,1;y) + 8640yG(1;y)G(0,1,r,1;y) +$
$1440G(1;y)G(0,1,r,1;y) - 5760G(0,1,r,1;y) - 2880y^4G(0,1,r,r;y) - 8640y^3G(0,1,r,r;y) + 1440y^2G(0,1,r,r;y) + 8640yG(0,1,r,r;y) -$
$8640y^4G(-1;y)G(0,1,r,r;y) + 8640G(-1;y)G(0,1,r,r;y) + 5760y^4G(1;y)G(0,1,r,r;y) - 11520y^2G(1;y)G(0,1,r,r;y) +$
$5760G(1;y)G(0,1,r,r;y) + 1440G(0,1,r,r;y) + 4320y^4G(0,r,-1,-1;y) + 17280y^3G(0,r,-1,-1;y) - 17280yG(0,r,-1,-1;y) +$
$17280y^4G(-1;y)G(0,r,-1,-1;y) - 17280G(-1;y)G(0,r,-1,-1;y) - 4320G(0,r,-1,-1;y) + 4320y^4G(0,r,-1,1;y) +$
$17280y^3G(0,r,-1,1;y) - 17280yG(0,r,-1,1;y) + 17280y^4G(-1;y)G(0,r,-1,1;y) - 17280G(-1;y)G(0,r,-1,1;y) - 4320G(0,r,-1,1;y) -$
$2160y^4G(0,r,-1,r;y) - 8640y^3G(0,r,-1,r;y) + 8640yG(0,r,-1,r;y) - 8640y^4G(-1;y)G(0,r,-1,r;y) + 8640G(-1;y)G(0,r,-1,r;y) +$
$2160G(0,r,-1,r;y) + 8640y^3G(0,r,0,-1;y) - 17280y^2G(0,r,0,-1;y) + 8640yG(0,r,0,-1;y) - 8640y^4G(1;y)G(0,r,0,-1;y) +$
$17280y^2G(1;y)G(0,r,0,-1;y) - 8640G(1;y)G(0,r,0,-1;y) + 4320y^4G(0,r,1,-1;y) + 17280y^3G(0,r,1,-1;y) - 17280yG(0,r,1,-1;y) +$
$17280y^4G(-1;y)G(0,r,1,-1;y) - 17280G(-1;y)G(0,r,1,-1;y) - 4320G(0,r,1,-1;y) + 6480y^4G(0,r,1,1;y) - 69120y^3G(0,r,1,1;y) +$
$114480y^2G(0,r,1,1;y) - 47520yG(0,r,1,1;y) - 28080y^4G(1;y)G(0,r,1,1;y) + 21600y^3G(1;y)G(0,r,1,1;y) + 12960y^2G(1;y)G(0,r,1,1;y) +$
$21600yG(1;y)G(0,r,1,1;y) - 28080G(1;y)G(0,r,1,1;y) - 4320G(0,r,1,1;y) + 360y^4G(0,r,1,r;y) + 17280y^3G(0,r,1,r;y) -$
$22320y^2G(0,r,1,r;y) + 4320yG(0,r,1,r;y) + 4320y^4G(-1;y)G(0,r,1,r;y) - 4320G(-1;y)G(0,r,1,r;y) - 720y^4G(1;y)G(0,r,1,r;y) -$
$4320y^3G(1;y)G(0,r,1,r;y) + 10080y^2G(1;y)G(0,r,1,r;y) - 4320yG(1;y)G(0,r,1,r;y) - 720G(1;y)G(0,r,1,r;y) + 360G(0,r,1,r;y) -$
$2160y^4G(0,r,r,-1;y) - 8640y^3G(0,r,r,-1;y) + 8640yG(0,r,r,-1;y) - 8640y^4G(-1;y)G(0,r,r,-1;y) + 8640G(-1;y)G(0,r,r,-1;y) +$
$2160G(0,r,r,-1;y) - 2880y^4G(0,r,r,1;y) + 4320y^3G(0,r,r,1;y) - 22320y^2G(0,r,r,1;y) + 17280yG(0,r,r,1;y) -$
$8640y^4G(-1;y)G(0,r,r,1;y) + 8640G(-1;y)G(0,r,r,1;y) - 720y^4G(1;y)G(0,r,r,1;y) - 4320y^3G(1;y)G(0,r,r,1;y) +$
$10080y^2G(1;y)G(0,r,r,1;y) - 4320yG(1;y)G(0,r,r,1;y) - 720G(1;y)G(0,r,r,1;y) + 3600G(0,r,r,1;y) + 1440y^4G(0,r,r,r;y) +$
$4320y^3G(0,r,r,r;y) - 720y^2G(0,r,r,r;y) - 4320yG(0,r,r,r;y) + 4320y^4G(-1;y)G(0,r,r,r;y) - 4320G(-1;y)G(0,r,r,r;y) -$
$2880y^4G(1;y)G(0,r,r,r;y) + 5760y^2G(1;y)G(0,r,r,r;y) - 2880G(1;y)G(0,r,r,r;y) - 720G(0,r,r,r;y) + 2160y^4G(r,1,1,1;y) -$
$116640y^3G(r,1,1,1;y) + 228960y^2G(r,1,1,1;y) - 116640yG(r,1,1,1;y) - 56160y^4G(1;y)G(r,1,1,1;y) + 43200y^3G(1;y)G(r,1,1,1;y) +$
$25920y^2G(1;y)G(r,1,1,1;y) + 43200yG(1;y)G(r,1,1,1;y) - 56160G(1;y)G(r,1,1,1;y) + 7200y^4G(r;y)G(r,1,1,1;y) +$
$8640y^3G(r;y)G(r,1,1,1;y) - 31680y^2G(r;y)G(r,1,1,1;y) + 8640yG(r;y)G(r,1,1,1;y) + 7200G(r;y)G(r,1,1,1;y) + 2160G(r,1,1,1;y) -$
$720y^4G(r,r,1,1;y) - 21600y^3G(r,r,1,1;y) + 44640y^2G(r,r,1,1;y) - 21600yG(r,r,1,1;y) + 1440y^4G(1;y)G(r,r,1,1;y) +$
$8640y^3G(1;y)G(r,r,1,1;y) - 20160y^2G(1;y)G(r,r,1,1;y) + 8640yG(1;y)G(r,r,1,1;y) + 1440G(1;y)G(r,r,1,1;y) -$
$5760y^4G(r;y)G(r,r,1,1;y) + 11520y^2G(r;y)G(r,r,1,1;y) - 5760G(r;y)G(r,r,1,1;y) - 720G(r,r,1,1;y) + 720y^4G(r,r,r,1;y) -$
$1440y^2G(r,r,r,1;y) - 5760y^4G(1;y)G(r,r,r,1;y) + 11520y^2G(1;y)G(r,r,r,1;y) - 5760G(1;y)G(r,r,r,1;y) + 720G(r,r,r,1;y) -$
$17280y^4G(0,-1,-1,-1,r;y) + 17280G(0,-1,-1,-1,r;y) - 17280y^4G(0,-1,-1,1,r;y) + 17280G(0,-1,-1,1,r;y) -$
$17280y^4G(0,-1,-1,r,-1;y) + 17280G(0,-1,-1,r,-1;y) - 17280y^4G(0,-1,-1,r,1;y) + 17280G(0,-1,-1,r,1;y) +$
$8640y^4G(0,-1,-1,r,r;y) - 8640G(0,-1,-1,r,r;y) + 8640y^4G(0,-1,0,-1,r;y) - 17280y^2G(0,-1,0,-1,r;y) +$
$8640G(0,-1,0,-1,r;y) - 17280y^4G(0,-1,1,-1,r;y) + 17280G(0,-1,1,-1,r;y) - 5760y^4G(0,-1,1,1,r;y) + 5760G(0,-1,1,1,r;y) -$
$17280y^4G(0,-1,1,r,-1;y) + 17280G(0,-1,1,r,-1;y) - 17280y^4G(0,-1,1,r,1;y) + 17280G(0,-1,1,r,1;y) + 8640y^4G(0,-1,1,r,r;y) -$
$8640G(0,-1,1,r,r;y) - 17280y^4G(0,-1,r,-1,-1;y) + 17280G(0,-1,r,-1,-1;y) - 17280y^4G(0,-1,r,-1,1;y) +$
$17280G(0,-1,r,-1,1;y) + 8640y^4G(0,-1,r,-1,r;y) - 8640G(0,-1,r,-1,r;y) - 17280y^4G(0,-1,r,1,-1;y) + 17280G(0,-1,r,1,-1;y) -$
$4320y^4G(0,-1,r,1,r;y) + 4320G(0,-1,r,1,r;y) + 8640y^4G(0,-1,r,r,-1;y) - 8640G(0,-1,r,r,-1;y) + 8640y^4G(0,-1,r,r,1;y) -$
$8640G(0,-1,r,r,1;y) - 4320y^4G(0,-1,r,r,r;y) + 4320G(0,-1,r,r,r;y) - 17280y^4G(0,0,-1,-1,-1;y) + 17280G(0,0,-1,-1,-1;y) -$
$17280y^4G(0,0,-1,-1,1;y) + 17280G(0,0,-1,-1,1;y) + 8640y^4G(0,0,-1,-1,r;y) - 34560y^2G(0,0,-1,-1,r;y) +$
$25920G(0,0,-1,-1,r;y) - 8640y^4G(0,0,-1,0,-1;y) + 17280y^2G(0,0,-1,0,-1;y) - 8640G(0,0,-1,0,-1;y) - 15840y^4G(0,0,-1,0,1;y) +$
$17280y^2G(0,0,-1,0,1;y) - 1440G(0,0,-1,0,1;y) - 9360y^4G(0,0,-1,0,r;y) + 17280y^2G(0,0,-1,0,r;y) - 7920G(0,0,-1,0,r;y) -$
$17280y^4G(0,0,-1,1,-1;y) + 17280G(0,0,-1,1,-1;y) - 15840y^4G(0,0,-1,1,r;y) + 15840G(0,0,-1,1,r;y) - 8640y^4G(0,0,-1,r,-1;y) +$
$8640G(0,0,-1,r,-1;y) - 8640y^4G(0,0,-1,r,1;y) + 8640G(0,0,-1,r,1;y) + 12960y^4G(0,0,-1,r,r;y) - 17280y^2G(0,0,-1,r,r;y) +$
$4320G(0,0,-1,r,r;y) - 63360y^4G(0,0,0,-1,-1;y) + 51840y^2G(0,0,0,-1,-1;y) + 11520G(0,0,0,-1,-1;y) - 64800y^4G(0,0,0,-1,1;y) +$
$51840y^2G(0,0,0,-1,1;y) + 12960G(0,0,0,-1,1;y) - 10080y^4G(0,0,0,-1,r;y) + 25920y^2G(0,0,0,-1,r;y) - 15840G(0,0,0,-1,r;y) -$
$46800y^4G(0,0,0,0,-1;y) + 46080y^2G(0,0,0,0,-1;y) + 720G(0,0,0,0,-1;y) - 30240y^4G(0,0,0,0,1;y) - 11520y^3G(0,0,0,0,1;y) +$
$54720y^2G(0,0,0,0,1;y) - 11520yG(0,0,0,0,1;y) - 1440G(0,0,0,0,1;y) + 5040y^4G(0,0,0,0,r;y) - 2880y^3G(0,0,0,0,r;y) +$
$720y^2G(0,0,0,0,r;y) - 2880yG(0,0,0,0,r;y) - 64800y^4G(0,0,0,1,-1;y) + 51840y^2G(0,0,0,1,-1;y) + 12960G(0,0,0,1,-1;y) -$
$160560y^4G(0,0,0,1,1;y) + 38880y^3G(0,0,0,1,1;y) + 36000y^2G(0,0,0,1,1;y) + 38880yG(0,0,0,1,1;y) + 46800G(0,0,0,1,1;y) +$
$17280y^4G(0,0,0,1,r;y) - 11520y^3G(0,0,0,1,r;y) + 34560y^2G(0,0,0,1,r;y) - 11520yG(0,0,0,1,r;y) - 28800G(0,0,0,1,r;y) -$
$10080y^4G(0,0,0,r,-1;y) + 25920y^2G(0,0,0,r,-1;y) - 15840G(0,0,0,r,-1;y) + 31680y^4G(0,0,0,r,1;y) - 12960y^3G(0,0,0,r,1;y) +$
$23040y^2G(0,0,0,r,1;y) - 12960yG(0,0,0,r,1;y) - 28800G(0,0,0,r,1;y) - 4320y^4G(0,0,0,r,r;y) + 4320y^2G(0,0,0,r,r;y) -$
$17280y^4G(0,0,1,-1,-1;y) + 17280G(0,0,1,-1,-1;y) + 1440y^4G(0,0,1,-1,r;y) - 1440G(0,0,1,-1,r;y) - 14400y^4G(0,0,1,0,-1;y) +$
$17280y^2G(0,0,1,0,-1;y) - 2880G(0,0,1,0,-1;y) - 47520y^4G(0,0,1,0,1;y) + 20160y^3G(0,0,1,0,1;y) - 14400y^2G(0,0,1,0,1;y) +$
$20160yG(0,0,1,0,1;y) + 21600G(0,0,1,0,1;y) - 720y^4G(0,0,1,0,r;y) - 2880y^3G(0,0,1,0,r;y) + 21600y^2G(0,0,1,0,r;y) -$
$2880yG(0,0,1,0,r;y) - 15120G(0,0,1,0,r;y) + 79920y^4G(0,0,1,1,1;y) + 12960y^3G(0,0,1,1,1;y) - 47520y^2G(0,0,1,1,1;y) +$
$12960yG(0,0,1,1,1;y) - 58320G(0,0,1,1,1;y) + 32400y^4G(0,0,1,1,r;y) + 17280y^3G(0,0,1,1,r;y) - 40320y^2G(0,0,1,1,r;y) +$
$17280yG(0,0,1,1,r;y) - 26640G(0,0,1,1,r;y) + 1440y^4G(0,0,1,r,-1;y) - 1440G(0,0,1,r,-1;y) - 15840y^4G(0,0,1,r,1;y) +$

$$25920y^3G(0,0,1,r,1;y) - 28800y^2G(0,0,1,r,1;y) + 25920yG(0,0,1,r,1;y) - 7200G(0,0,1,r,1;y) + 9360y^4G(0,0,1,r,r;y) -$$
$$8640y^3G(0,0,1,r,r;y) + 14400y^2G(0,0,1,r,r;y) - 8640yG(0,0,1,r,r;y) - 6480G(0,0,1,r,r;y) - 25920y^4G(0,0,r,-1,-1;y) +$$
$$34560y^2G(0,0,r,-1,-1;y) - 8640G(0,0,r,-1,-1;y) - 8640y^4G(0,0,r,-1,1;y) + 8640G(0,0,r,-1,1;y) + 12960y^4G(0,0,r,-1,r;y) -$$
$$17280y^2G(0,0,r,-1,r;y) + 4320G(0,0,r,-1,r;y) + 1440y^4G(0,0,r,0,-1;y) + 8640y^2G(0,0,r,0,-1;y) - 10080G(0,0,r,0,-1;y) -$$
$$720y^4G(0,0,r,0,1;y) - 5760y^3G(0,0,r,0,1;y) + 27360y^2G(0,0,r,0,1;y) - 5760yG(0,0,r,0,1;y) - 15120G(0,0,r,0,1;y) +$$
$$2160y^4G(0,0,r,0,r;y) - 2160y^2G(0,0,r,0,r;y) + 8640y^4G(0,0,r,1,-1;y) - 8640G(0,0,r,1,-1;y) + 15120y^4G(0,0,r,1,1;y) +$$
$$21600y^3G(0,0,r,1,1;y) - 73440y^2G(0,0,r,1,1;y) + 21600yG(0,0,r,1,1;y) + 15120G(0,0,r,1,1;y) + 720y^4G(0,0,r,1,r;y) -$$
$$4320y^3G(0,0,r,1,r;y) + 5760y^2G(0,0,r,1,r;y) - 4320yG(0,0,r,1,r;y) + 2160G(0,0,r,1,r;y) + 12960y^4G(0,0,r,r,-1;y) -$$
$$17280y^2G(0,0,r,r,-1;y) + 4320G(0,0,r,r,-1;y) + 13680y^4G(0,0,r,r,1;y) - 4320y^3G(0,0,r,r,1;y) - 15840y^2G(0,0,r,r,1;y) -$$
$$4320yG(0,0,r,r,1;y) + 10800G(0,0,r,r,1;y) - 1440y^4G(0,0,r,r,r;y) + 1440y^2G(0,0,r,r,r;y) - 17280y^4G(0,1,-1,-1,r;y) +$$
$$17280G(0,1,-1,-1,r;y) + 8640y^4G(0,1,-1,0,r;y) - 8640G(0,1,-1,0,r;y) - 5760y^4G(0,1,-1,1,r;y) + 5760G(0,1,-1,1,r;y) -$$
$$17280y^4G(0,1,-1,r,-1;y) + 17280G(0,1,-1,r,-1;y) - 17280y^4G(0,1,-1,r,1;y) + 17280G(0,1,-1,r,1;y) + 8640y^4G(0,1,-1,r,r;y) -$$
$$8640G(0,1,-1,r,r;y) + 17280y^4G(0,1,0,-1,1;y) - 17280G(0,1,0,-1,1;y) - 4320y^4G(0,1,0,-1,r;y) + 4320G(0,1,0,-1,r;y) +$$
$$17280y^4G(0,1,0,1,-1;y) - 17280G(0,1,0,1,-1;y) + 51840y^4G(0,1,0,1,1;y) - 34560y^2G(0,1,0,1,1;y) - 17280G(0,1,0,1,1;y) +$$
$$19440y^4G(0,1,0,1,r;y) + 14400y^3G(0,1,0,1,r;y) - 31680y^2G(0,1,0,1,r;y) + 14400yG(0,1,0,1,r;y) - 16560G(0,1,0,1,r;y) -$$
$$4320y^4G(0,1,0,r,-1;y) + 4320G(0,1,0,r,-1;y) - 15840y^4G(0,1,0,r,1;y) + 25920y^3G(0,1,0,r,1;y) - 34560y^2G(0,1,0,r,1;y) +$$
$$25920yG(0,1,0,r,1;y) - 1440G(0,1,0,r,1;y) + 6480y^4G(0,1,0,r,r;y) - 8640y^3G(0,1,0,r,r;y) + 17280y^2G(0,1,0,r,r;y) -$$
$$8640yG(0,1,0,r,r;y) - 6480G(0,1,0,r,r;y) - 5760y^4G(0,1,1,-1,r;y) + 5760G(0,1,1,-1,r;y) + 28800y^4G(0,1,1,0,-1;y) -$$
$$28800G(0,1,1,0,-1;y) + 720y^4G(0,1,1,0,r;y) + 8640y^3G(0,1,1,0,r;y) - 8640y^2G(0,1,1,0,r;y) + 8640yG(0,1,1,0,r;y) -$$
$$9360G(0,1,1,0,r;y) - 76320y^4G(0,1,1,1,1;y) + 60480y^3G(0,1,1,1,1;y) + 31680y^2G(0,1,1,1,1;y) + 60480yG(0,1,1,1,1;y) -$$
$$76320G(0,1,1,1,1;y) - 2880y^4G(0,1,1,1,r;y) - 8640y^3G(0,1,1,1,r;y) + 31680y^2G(0,1,1,1,r;y) - 8640yG(0,1,1,1,r;y) -$$
$$11520G(0,1,1,1,r;y) - 5760y^4G(0,1,1,r,-1;y) + 5760G(0,1,1,r,-1;y) + 24480y^4G(0,1,1,r,1;y) - 8640y^3G(0,1,1,r,1;y) -$$
$$2880y^2G(0,1,1,r,1;y) - 8640yG(0,1,1,r,1;y) - 4320G(0,1,1,r,1;y) - 12960y^4G(0,1,1,r,r;y) + 11520y^2G(0,1,1,r,r;y) +$$
$$1440G(0,1,1,r,r;y) - 17280y^4G(0,1,r,-1,-1;y) + 17280G(0,1,r,-1,-1;y) - 17280y^4G(0,1,r,-1,1;y) + 17280G(0,1,r,-1,1;y) +$$
$$8640y^4G(0,1,r,-1,r;y) - 8640G(0,1,r,-1,r;y) - 34560y^4G(0,1,r,0,-1;y) + 17280y^2G(0,1,r,0,-1;y) + 17280G(0,1,r,0,-1;y) +$$
$$4320y^4G(0,1,r,0,r;y) - 4320G(0,1,r,0,r;y) - 17280y^4G(0,1,r,1,-1;y) + 17280G(0,1,r,1,-1;y) + 10080y^4G(0,1,r,1,1;y) -$$
$$8640y^3G(0,1,r,1,1;y) - 2880y^2G(0,1,r,1,1;y) - 8640yG(0,1,r,1,1;y) + 10080G(0,1,r,1,1;y) - 8640y^4G(0,1,r,1,r;y) +$$
$$11520y^2G(0,1,r,1,r;y) - 2880G(0,1,r,1,r;y) + 8640y^4G(0,1,r,r,-1;y) - 8640G(0,1,r,r,-1;y) + 11520y^2G(0,1,r,r,1;y) -$$
$$11520G(0,1,r,r,1;y) - 2880y^4G(0,1,r,r,r;y) + 2880G(0,1,r,r,r;y) - 17280y^4G(0,r,-1,-1,-1;y) + 17280G(0,r,-1,-1,-1;y) -$$
$$17280y^4G(0,r,-1,-1,1;y) + 17280G(0,r,-1,-1,1;y) + 8640y^4G(0,r,-1,-1,r;y) - 8640G(0,r,-1,-1,r;y) - 8640y^4G(0,r,-1,0,-1;y) +$$
$$17280y^2G(0,r,-1,0,-1;y) - 8640G(0,r,-1,0,-1;y) - 17280y^4G(0,r,-1,1,-1;y) + 17280G(0,r,-1,1,-1;y) - 4320y^4G(0,r,-1,1,r;y) +$$
$$4320G(0,r,-1,1,r;y) + 8640y^4G(0,r,-1,r,-1;y) - 8640G(0,r,-1,r,-1;y) + 8640y^4G(0,r,-1,r,1;y) - 8640G(0,r,-1,r,1;y) -$$
$$4320y^4G(0,r,-1,r,r;y) + 4320G(0,r,-1,r,r;y) - 8640y^4G(0,r,0,-1,-1;y) + 17280y^2G(0,r,0,-1,-1;y) - 8640G(0,r,0,-1,-1;y) -$$
$$8640y^4G(0,r,0,-1,1;y) + 8640G(0,r,0,-1,1;y) + 10800y^4G(0,r,0,-1,r;y) - 8640y^2G(0,r,0,-1,r;y) - 2160G(0,r,0,-1,r;y) +$$
$$10800y^4G(0,r,0,r,-1;y) - 8640y^2G(0,r,0,r,-1;y) - 2160G(0,r,0,r,-1;y) + 4320y^4G(0,r,0,r,1;y) - 8640y^2G(0,r,0,r,1;y) +$$
$$4320G(0,r,0,r,1;y) - 17280y^4G(0,r,1,-1,-1;y) + 17280G(0,r,1,-1,-1;y) - 4320y^4G(0,r,1,-1,r;y) + 4320G(0,r,1,-1,r;y) -$$
$$23040y^4G(0,r,1,0,-1;y) + 17280y^2G(0,r,1,0,-1;y) + 5760G(0,r,1,0,-1;y) + 38160y^4G(0,r,1,1,1;y) - 30240y^3G(0,r,1,1,1;y) -$$
$$15840y^2G(0,r,1,1,1;y) - 30240yG(0,r,1,1,1;y) + 38160G(0,r,1,1,1;y) + 3600y^4G(0,r,1,1,r;y) + 4320y^3G(0,r,1,1,r;y) -$$
$$15840y^2G(0,r,1,1,r;y) + 4320yG(0,r,1,1,r;y) + 3600G(0,r,1,1,r;y) - 4320y^4G(0,r,1,r,-1;y) + 4320G(0,r,1,r,-1;y) -$$
$$5040y^4G(0,r,1,r,1;y) + 4320y^3G(0,r,1,r,1;y) + 1440y^2G(0,r,1,r,1;y) + 4320yG(0,r,1,r,1;y) - 5040G(0,r,1,r,1;y) +$$
$$2880y^4G(0,r,1,r,r;y) - 5760y^2G(0,r,1,r,r;y) + 2880G(0,r,1,r,r;y) + 8640y^4G(0,r,r,-1,-1;y) - 8640G(0,r,r,-1,-1;y) +$$
$$8640y^4G(0,r,r,-1,1;y) - 8640G(0,r,r,-1,1;y) - 4320y^4G(0,r,r,-1,r;y) + 4320G(0,r,r,-1,r;y) + 17280y^4G(0,r,r,0,-1;y) -$$
$$8640y^2G(0,r,r,0,-1;y) - 8640G(0,r,r,0,-1;y) + 8640y^4G(0,r,r,1,-1;y) - 8640G(0,r,r,1,-1;y) - 5040y^4G(0,r,r,1,1;y) +$$
$$4320y^3G(0,r,r,1,1;y) + 1440y^2G(0,r,r,1,1;y) + 4320yG(0,r,r,1,1;y) - 5040G(0,r,r,1,1;y) + 2880y^4G(0,r,r,1,r;y) -$$
$$5760y^2G(0,r,r,1,r;y) + 2880G(0,r,r,1,r;y) - 4320y^4G(0,r,r,r,-1;y) + 4320G(0,r,r,r,-1;y) + 2880y^4G(0,r,r,r,1;y) -$$
$$5760y^2G(0,r,r,r,1;y) + 2880G(0,r,r,r,1;y) + 76320y^4G(r,1,1,1,1;y) - 60480y^3G(r,1,1,1,1;y) - 31680y^2G(r,1,1,1,1;y) -$$
$$60480yG(r,1,1,1,1;y) + 76320G(r,1,1,1,1;y) + 17280y^4G(r,1,r,1,1;y) - 34560y^2G(r,1,r,1,1;y) + 17280G(r,1,r,1,1;y) +$$
$$44640y^4G(r,r,1,1,1;y) - 8640y^3G(r,r,1,1,1;y) - 72000y^2G(r,r,1,1,1;y) - 8640yG(r,r,1,1,1;y) + 44640G(r,r,1,1,1;y) +$$
$$5760y^4G(r,r,r,1,1;y) - 11520y^2G(r,r,r,1,1;y) + 5760G(r,r,r,1,1;y) + 1080\pi^2y^4\zeta(3) - 1320y^4\zeta(3) - 480\pi^2y^3\zeta(3) + 21840y^3\zeta(3) -$$
$$120\pi^2\zeta(3) - 15120y^2\zeta(3) - 4320y^4G(-1;y)^2\zeta(3) + 4320G(-1;y)^2\zeta(3) - 5040y^4G(1;y)^2\zeta(3) + 1440y^3G(1;y)^2\zeta(3) + 7200y^2G(1;y)^2\zeta(3) +$$
$$1440yG(1;y)^2\zeta(3) - 5040G(1;y)^2\zeta(3) - 480\pi^2y\zeta(3) + 6480y\zeta(3) - 2160y^4G(-1;y)\zeta(3) - 8640y^3G(-1;y)\zeta(3) + 8640yG(-1;y)\zeta(3) +$$
$$2160G(-1;y)\zeta(3) - 720y^4G(1;y)\zeta(3) + 12960y^3G(1;y)\zeta(3) - 25920y^2G(1;y)\zeta(3) + 15840yG(1;y)\zeta(3) - 2160G(1;y)\zeta(3) +$$
$$11520y^4G(0,-1;y)\zeta(3) - 8640y^2G(0,-1;y)\zeta(3) - 2880G(0,-1;y)\zeta(3) + 7200y^4G(0,1;y)\zeta(3) - 2880y^3G(0,1;y)\zeta(3) - 2880y^2G(0,1;y)\zeta(3) -$$
$$2880yG(0,1;y)\zeta(3) + 1440G(0,1;y)\zeta(3) - 6840\zeta(3) - 7920y^4\zeta(5) - 2880y^3\zeta(5) - 720y^2\zeta(5) - 2880y\zeta(5) + 14400\zeta(5) - 4800.$$

Bibliography

[1] Marco Bonetti, Kirill Melnikov, and Lorenzo Tancredi. “Two-loop electroweak corrections to Higgs–gluon couplings to higher orders in the dimensional regularization parameter”. In: *Nucl. Phys.* B916 (2017), pp. 709–726. DOI: 10.1016/j.nuclphysb.2017.01.020. arXiv: 1610.05497 [hep-ph].

[2] Marco Bonetti, Kirill Melnikov, and Lorenzo Tancredi. “Three-loop mixed QCD-electroweak corrections to Higgs boson gluon fusion”. In: *Phys. Rev.* D97.3 (2018), p. 034004. DOI: 10.1103/PhysRevD.97.034004. arXiv: 1711.11113 [hep-ph].

[3] Marco Bonetti, Kirill Melnikov, and Lorenzo Tancredi. “Higher order corrections to mixed QCD-EW contributions to Higgs boson production in gluon fusion”. In: *Phys. Rev.* D97.5 (2018). [Erratum: Phys. Rev. D97, no.9, 099906 (2018)], p. 056017. DOI: 10.1103/PhysRevD.97.056017,10.1103/PhysRevD.97.099906. arXiv: 1801.10403 [hep-ph].

[4] Marco Bonetti. “NLO mixed QCD-EW corrections to Higgs gluon fusion”. In: *PoS* LL2018 (2018), p. 059. DOI: 10.22323/1.303.0059. arXiv: 1809.02450 [hep-ph].

[5] D. Binosi et al. “JaxoDraw: A Graphical user interface for drawing Feynman diagrams. Version 2.0 release notes”. In: *Comput. Phys. Commun.* 180 (2009), pp. 1709–1715. DOI: 10.1016/j.cpc.2009.02.020. arXiv: 0811.4113 [hep-ph].

[6] Georges Aad et al. “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC”. In: *Phys. Lett.* B716 (2012), pp. 1–29. DOI: 10.1016/j.physletb.2012.08.020. arXiv: 1207.7214 [hep-ex].

[7] Serguei Chatrchyan et al. “Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC”. In: *Phys. Lett.* B716 (2012), pp. 30–61. DOI: 10.1016/j.physletb.2012.08.021. arXiv: 1207.7235 [hep-ex].

[8] Serguei Chatrchyan et al. “Measurement of Higgs boson production and properties in the WW decay channel with leptonic final states”. In: *JHEP* 01 (2014), p. 096. DOI: 10.1007/JHEP01(2014)096. arXiv: 1312.1129 [hep-ex].

[9] Serguei Chatrchyan et al. “Measurement of the properties of a Higgs boson in the four-lepton final state”. In: *Phys. Rev.* D89.9 (2014), p. 092007. DOI: 10.1103/PhysRevD.89.092007. arXiv: 1312.5353 [hep-ex].

[10] Vardan Khachatryan et al. "Constraints on the spin-parity and anomalous HVV couplings of the Higgs boson in proton collisions at 7 and 8 TeV". In: *Phys. Rev.* D92.1 (2015), p. 012004. DOI: 10.1103/PhysRevD.92.012004. arXiv: 1411.3441 [hep-ex].

[11] Vardan Khachatryan et al. "Precise determination of the mass of the Higgs boson and tests of compatibility of its couplings with the standard model predictions using proton collisions at 7 and 8 TeV". In: *Eur. Phys. J.* C75.5 (2015), p. 212. DOI: 10.1140/epjc/s10052-015-3351-7. arXiv: 1412.8662 [hep-ex].

[12] Georges Aad et al. "Study of the spin and parity of the Higgs boson in diboson decays with the ATLAS detector". In: *Eur. Phys. J.* C75.10 (2015). [Erratum: Eur. Phys. J.C76,no.3,152(2016)], p. 476. DOI: 10.1140/epjc/s10052-015-3685-1,10.1140/epjc/s10052-016-3934-y. arXiv: 1506.05669 [hep-ex].

[13] Georges Aad et al. "Measurements of the Higgs boson production and decay rates and coupling strengths using pp collision data at $\sqrt{s} = 7$ and 8 TeV in the ATLAS experiment". In: *Eur. Phys. J.* C76.1 (2016), p. 6. DOI: 10.1140/epjc/s10052-015-3769-y. arXiv: 1507.04548 [hep-ex].

[14] Georges Aad et al. "Test of CP Invariance in vector-boson fusion production of the Higgs boson using the Optimal Observable method in the ditau decay channel with the ATLAS detector". In: *Eur. Phys. J.* C76.12 (2016), p. 658. DOI: 10.1140/epjc/s10052-016-4499-5. arXiv: 1602.04516 [hep-ex].

[15] Albert M Sirunyan et al. "Constraints on anomalous Higgs boson couplings using production and decay information in the four-lepton final state". In: *Phys. Lett.* B775 (2017), pp. 1–24. DOI: 10.1016/j.physletb.2017.10.021. arXiv: 1707.00541 [hep-ex].

[16] CMS Collaboration. "Measurements of properties of the Higgs boson in the four-lepton final state in proton-proton collisions at $\sqrt{s} = 13$ TeV". In: (2019).

[17] Gian Francesco Giudice. *On Future High-Energy Colliders*. 2019. arXiv: 1902.07964 [physics.hist-ph].

[18] Albert Einstein. "Die Grundlage der allgemeinen Relativitätstheorie". In: *Annalen der Physik* 354 (1916), pp. 769–822. DOI: 10.1002/andp.19163540702.

[19] Max Planck. "Über eine Verbesserung der Wienschen Spektralgleichung". In: *Verhandlungen der Deutschen Physikalischen Gesellschaft* 2 (Jan. 1900), pp. 202–204. DOI: 10.1007/978-3-663-13885-3_15.

[20] Wolfgang Pauli. "Relativistic Field Theories of Elementary Particles". In: *Reviews of Modern Physics - REV MOD PHYS* 13 (Jan. 1941), pp. 203–232. DOI: 10.1103/RevModPhys.13.203.

[21] Chen-Ning Yang and Robert L. Mills. "Conservation of Isotopic Spin and Isotopic Gauge Invariance". In: *Phys. Rev.* 96 (1954). [,150(1954)], pp. 191–195. DOI: 10.1103/PhysRev.96.191.

[22] M. Tanabashi et al. “Review of Particle Physics”. In: *Phys. Rev.* D98.3 (2018), p. 030001. DOI: 10.1103/PhysRevD.98.030001.

[23] Jun Gao, Lucian Harland-Lang, and Juan Rojo. “The Structure of the Proton in the LHC Precision Era”. In: *Phys. Rept.* 742 (2018), pp. 1–121. DOI: 10.1016/j.physrep.2018.03.002. arXiv: 1709.04922 [hep-ph].

[24] Charalampos Anastasiou et al. “High precision determination of the gluon fusion Higgs boson cross-section at the LHC”. In: *JHEP* 05 (2016), p. 058. DOI: 10.1007/JHEP05(2016)058. arXiv: 1602.00695 [hep-ph].

[25] D. de Florian et al. *Handbook of LHC Higgs Cross Sections: 4. Deciphering the Nature of the Higgs Sector*. 2016. arXiv: 1610.07922 [hep-ph].

[26] John R. Ellis, Mary K. Gaillard, and Dimitri V. Nanopoulos. “A Phenomenological Profile of the Higgs Boson”. In: *Nucl. Phys.* B106 (1976), p. 292. DOI: 10.1016/0550-3213(76)90382-5.

[27] H. M. Georgi et al. “Higgs Bosons from Two Gluon Annihilation in Proton Proton Collisions”. In: *Phys. Rev. Lett.* 40 (1978), p. 692. DOI: 10.1103/PhysRevLett.40.692.

[28] M. Spira et al. “Higgs boson production at the LHC”. In: *Nucl. Phys.* B453 (1995), pp. 17–82. DOI: 10.1016/0550-3213(95)00379-7. arXiv: hep-ph/9504378 [hep-ph].

[29] Charalampos Anastasiou et al. “Higgs Boson Gluon-Fusion Production in QCD at Three Loops”. In: *Phys. Rev. Lett.* 114 (2015), p. 212001. DOI: 10.1103/PhysRevLett.114.212001. arXiv: 1503.06056 [hep-ph].

[30] Bernhard Mistlberger. “Higgs boson production at hadron colliders at N^3LO in QCD”. In: *JHEP* 05 (2018), p. 028. DOI: 10.1007/JHEP05(2018)028. arXiv: 1802.00833 [hep-ph].

[31] S. Dawson. “Radiative corrections to Higgs boson production”. In: *Nucl. Phys.* B359 (1991), pp. 283–300. DOI: 10.1016/0550-3213(91)90061-2.

[32] D. Graudenz, M. Spira, and P. M. Zerwas. “QCD corrections to Higgs boson production at proton proton colliders”. In: *Phys. Rev. Lett.* 70 (1993), pp. 1372–1375. DOI: 10.1103/PhysRevLett.70.1372.

[33] A. Djouadi, M. Spira, and P. M. Zerwas. “Production of Higgs bosons in proton colliders: QCD corrections”. In: *Phys. Lett.* B264 (1991), pp. 440–446. DOI: 10.1016/0370-2693(91)90375-Z.

[34] Charalampos Anastasiou and Kirill Melnikov. “Higgs boson production at hadron colliders in NNLO QCD”. In: *Nucl. Phys.* B646 (2002), pp. 220–256. DOI: 10.1016/S0550-3213(02)00837-4. arXiv: hep-ph/0207004 [hep-ph].

[35] Robert V. Harlander and William B. Kilgore. “Next-to-next-to-leading order Higgs production at hadron colliders”. In: *Phys. Rev. Lett.* 88 (2002), p. 201801. DOI: 10.1103/PhysRevLett.88.201801. arXiv: hep-ph/0201206 [hep-ph].

[36] V. Ravindran, J. Smith, and W. L. van Neerven. “NNLO corrections to the total cross-section for Higgs boson production in hadron hadron collisions”. In: *Nucl. Phys.* B665 (2003), pp. 325–366. DOI: 10.1016/S0550-3213(03)00457-7. arXiv: hep-ph/0302135 [hep-ph].

[37] Robert V. Harlander and Kemal J. Ozeren. “Finite top mass effects for hadronic Higgs production at next-to-next-to-leading order”. In: *JHEP* 11 (2009), p. 088. DOI: 10.1088/1126-6708/2009/11/088. arXiv: 0909.3420 [hep-ph].

[38] Alexey Pak, Mikhail Rogal, and Matthias Steinhauser. “Finite top quark mass effects in NNLO Higgs boson production at LHC”. In: *JHEP* 02 (2010), p. 025. DOI: 10.1007/JHEP02(2010)025. arXiv: 0911.4662 [hep-ph].

[39] Robert V. Harlander and Kemal J. Ozeren. “Top mass effects in Higgs production at next-to-next-to-leading order QCD: Virtual corrections”. In: *Phys. Lett.* B679 (2009), pp. 467–472. DOI: 10.1016/j.physletb.2009.08.012. arXiv: 0907.2997 [hep-ph].

[40] Robert V. Harlander et al. “Higgs production in gluon fusion at next-to-next-to-leading order QCD for finite top mass”. In: *Eur. Phys. J.* C66 (2010), pp. 359–372. DOI: 10.1140/epjc/s10052-010-1258-x. arXiv: 0912.2104 [hep-ph].

[41] Alexey Pak, Mikhail Rogal, and Matthias Steinhauser. “Beyond Heavy Top Limit In Higgs Boson Production At LHC”. In: *Proceedings, 45th Rencontres de Moriond on QCD and High Energy Interactions: La Thuile, Italy, March 13-20, 2010.* 2010, pp. 203–206. arXiv: 1005.2970 [hep-ph].

[42] Robert Harlander and Philipp Kant. “Higgs production and decay: Analytic results at next-to-leading order QCD”. In: *JHEP* 12 (2005), p. 015. DOI: 10.1088/1126-6708/2005/12/015. arXiv: hep-ph/0509189 [hep-ph].

[43] U. Aglietti et al. “Analytic Results for Virtual QCD Corrections to Higgs Production and Decay”. In: *JHEP* 01 (2007), p. 021. DOI: 10.1088/1126-6708/2007/01/021. arXiv: hep-ph/0611266 [hep-ph].

[44] R. Bonciani, Giuseppe Degrassi, and A. Vicini. “Scalar particle contribution to Higgs production via gluon fusion at NLO”. In: *JHEP* 11 (2007), p. 095. DOI: 10.1088/1126-6708/2007/11/095. arXiv: 0709.4227 [hep-ph].

[45] Charalampos Anastasiou et al. “Two-loop amplitudes and master integrals for the production of a Higgs boson via a massive quark and a scalar-quark loop”. In: *JHEP* 01 (2007), p. 082. DOI: 10.1088/1126-6708/2007/01/082. arXiv: hep-ph/0611236 [hep-ph].

[46] Charalampos Anastasiou, Stefan Bucherer, and Zoltan Kunszt. “HPro: A NLO Monte-Carlo for Higgs production via gluon fusion with finite heavy quark masses”. In: *JHEP* 10 (2009), p. 068. DOI: 10.1088/1126-6708/2009/10/068. arXiv: 0907.2362 [hep-ph].

[47] U. Aglietti et al. “Two loop light fermion contribution to Higgs production and decays”. In: *Phys. Lett.* B595 (2004), pp. 432–441. DOI: 10.1016/j.physletb.2004.06.063. arXiv: hep-ph/0404071 [hep-ph].

[48] Jon Butterworth et al. “PDF4LHC recommendations for LHC Run II”. In: *J. Phys.* G43 (2016), p. 023001. DOI: 10.1088/0954-3899/43/2/023001. arXiv: 1510.03865 [hep-ph].

[49] Alexey Pak, Matthias Steinhauser, and Nikolai Zerf. “Supersymmetric next-to-next-to-leading order corrections to Higgs boson production in gluon fusion”. In: *JHEP* 09 (2012), p. 118. DOI: 10.1007/JHEP09(2012)118. arXiv: 1208.1588 [hep-ph].

[50] Giuseppe Degrassi and Fabio Maltoni. “Two-loop electroweak corrections to Higgs production at hadron colliders”. In: *Phys. Lett.* B600 (2004), pp. 255–260. DOI: 10.1016/j.physletb.2004.09.008. arXiv: hep-ph/0407249 [hep-ph].

[51] U. Aglietti et al. “Master integrals for the two-loop light fermion contributions to $gg \to H$ and $H \to \gamma\gamma$”. In: *Phys. Lett.* B600 (2004), pp. 57–64. DOI: 10.1016/j.physletb.2004.09.001. arXiv: hep-ph/0407162 [hep-ph].

[52] U. Aglietti et al. “Two-loop electroweak corrections to Higgs production in proton-proton collisions”. In: *TeV4LHC Workshop: 2nd Meeting Brookhaven, Upton, New York, February 3-5, 2005.* 2006. arXiv: hep-ph/0610033 [hep-ph].

[53] Stefano Actis et al. “NLO Electroweak Corrections to Higgs Boson Production at Hadron Colliders”. In: *Phys. Lett.* B670 (2008), pp. 12–17. DOI: 10.1016/j.physletb.2008.10.018. arXiv: 0809.1301 [hep-ph].

[54] Stefano Actis et al. “NNLO Computational Techniques: The Cases H —¿ gamma gamma and H —¿ g g”. In: *Nucl. Phys.* B811 (2009), pp. 182–273. DOI: 10.1016/j.nuclphysb.2008.11.024. arXiv: 0809.3667 [hep-ph].

[55] Charalampos Anastasiou, Radja Boughezal, and Frank Petriello. “Mixed QCD-electroweak corrections to Higgs boson production in gluon fusion”. In: *JHEP* 04 (2009), p. 003. DOI: 10.1088/1126-6708/2009/04/003. arXiv: 0811.3458 [hep-ph].

[56] Paulo Nogueira. “Automatic Feynman graph generation”. In: *J. Comput. Phys.* 105 (1993), pp. 279–289. DOI: 10.1006/jcph.1993.1074.

[57] J. Kuipers et al. “FORM version 4.0”. In: *Comput. Phys. Commun.* 184 (2013), pp. 1453–1467. DOI: 10.1016/j.cpc.2012.12.028. arXiv: 1203.6543 [cs.SC].

[58] E. W. Nigel Glover and J. J. van der Bij. “Z boson pair production via gluon fusion”. In: *Nucl. Phys.* B321 (1989), pp. 561–590. DOI: 10.1016/0550-3213(89)90262-9.

[59] E. W. Nigel Glover and J. J. van der Bij. “Vector boson pair production via gluon fusion”. In: *Phys. Lett.* B219 (1989), pp. 488–492. DOI: 10.1016/0370-2693(89)91099-X.

[60] Kirill Melnikov and Matthew Dowling. “Production of two Z-bosons in gluon fusion in the heavy top quark approximation”. In: *Phys. Lett.* B744 (2015), pp. 43–47. DOI: 10.1016/j.physletb.2015.03.030. arXiv: 1503.01274 [hep-ph].

[61] Andreas von Manteuffel and Lorenzo Tancredi. “The two-loop helicity amplitudes for $gg \to V_1V_2 \to 4$ leptons”. In: *JHEP* 06 (2015), p. 197. DOI: 10.1007/JHEP06(2015)197. arXiv: 1503.08835 [hep-ph].

[62] Stefano Catani. “The Singular behavior of QCD amplitudes at two loop order”. In: *Phys. Lett.* B427 (1998), pp. 161–171. DOI: 10.1016/S0370-2693(98)00332-3. arXiv: hep-ph/9802439 [hep-ph].

[63] F. V. Tkachov. “A Theorem on Analytical Calculability of Four Loop Renormalization Group Functions”. In: *Phys. Lett.* 100B (1981), pp. 65–68. DOI: 10.1016/0370-2693(81)90288-4.

[64] K. G. Chetyrkin and F. V. Tkachov. “Integration by Parts: The Algorithm to Calculate beta Functions in 4 Loops”. In: *Nucl. Phys.* B192 (1981), pp. 159–204. DOI: 10.1016/0550-3213(81)90199-1.

[65] A. G. Grozin. “Integration by parts: An Introduction”. In: *Int. J. Mod. Phys.* A26 (2011), pp. 2807–2854. DOI: 10.1142/S0217751X11053687. arXiv: 1104.3993 [hep-ph].

[66] R. N. Lee. “Group structure of the integration-by-part identities and its application to the reduction of multiloop integrals”. In: *JHEP* 07 (2008), p. 031. DOI: 10.1088/1126-6708/2008/07/031. arXiv: 0804.3008 [hep-ph].

[67] S. Laporta. “High precision calculation of multiloop Feynman integrals by difference equations”. In: *Int. J. Mod. Phys.* A15 (2000), pp. 5087–5159. DOI: 10.1016/S0217-751X(00)00215-7,10.1142/S0217751X00002157. arXiv: hep-ph/0102033 [hep-ph].

[68] A. von Manteuffel and C. Studerus. *Reduze 2 - Distributed Feynman Integral Reduction*. 2012. arXiv: 1201.4330 [hep-ph].

[69] Philipp Maierhöfer, Johann Usovitsch, and Peter Uwer. “Kira—A Feynman integral reduction program”. In: *Comput. Phys. Commun.* 230 (2018), pp. 99–112. DOI: 10.1016/j.cpc.2018.04.012. arXiv: 1705.05610 [hep-ph].

[70] A. V. Kotikov. “Differential equation method: The Calculation of N point Feynman diagrams”. In: *Phys. Lett.* B267 (1991). [Erratum: Phys. Lett. B295, 409 (1992)], pp. 123–127. DOI: 10.1016/0370-2693(91)90536-Y,10.1016/0370-2693(92)91582-T.

[71] A. V. Kotikov. “Differential equations method: New technique for massive Feynman diagrams calculation”. In: *Phys. Lett.* B254 (1991), pp. 158–164. DOI: 10.1016/0370-2693(91)90413-K.

[72] A. V. Kotikov. “Differential equations method: The Calculation of vertex type Feynman diagrams”. In: *Phys. Lett.* B259 (1991), pp. 314–322. DOI: 10.1016/0370-2693(91)90834-D.

[73] Zvi Bern, Lance J. Dixon, and David A. Kosower. "Dimensionally regulated pentagon integrals". In: *Nucl. Phys.* B412 (1994), pp. 751–816. DOI: 10.1016/0550-3213(94)90398-0. arXiv: hep-ph/9306240 [hep-ph].

[74] Ettore Remiddi. "Differential equations for Feynman graph amplitudes". In: *Nuovo Cim.* A110 (1997), pp. 1435–1452. arXiv: hep-th/9711188 [hep-th].

[75] T. Gehrmann and E. Remiddi. "Differential equations for two loop four point functions". In: *Nucl. Phys.* B580 (2000), pp. 485–518. DOI: 10.1016/S0550-3213(00)00223-6. arXiv: hep-ph/9912329 [hep-ph].

[76] Nima Arkani-Hamed et al. "Local Integrals for Planar Scattering Amplitudes". In: *JHEP* 06 (2012), p. 125. DOI: 10.1007/JHEP06(2012)125. arXiv: 1012.6032 [hep-th].

[77] Johannes M. Henn. "Multiloop integrals in dimensional regularization made simple". In: *Phys. Rev. Lett.* 110 (2013), p. 251601. DOI: 10.1103/PhysRevLett.110.251601. arXiv: 1304.1806 [hep-th].

[78] Claude Duhr. "Mathematical aspects of scattering amplitudes". In: *Proceedings, Theoretical Advanced Study Institute in Elementary Particle Physics: Journeys Through the Precision Frontier: Amplitudes for Colliders (TASI 2014): Boulder, Colorado, June 2-27, 2014*. 2015, pp. 419–476. DOI: 10.1142/9789814678766_0010. arXiv: 1411.7538 [hep-ph].

[79] Johannes M. Henn. "Lectures on differential equations for Feynman integrals". In: *J. Phys.* A48 (2015), p. 153001. DOI: 10.1088/1751-8113/48/15/153001. arXiv: 1412.2296 [hep-ph].

[80] F. J. Dyson. "The Radiation theories of Tomonaga, Schwinger, and Feynman". In: *Phys. Rev.* 75 (1949), pp. 486–502. DOI: 10.1103/PhysRev.75.486.

[81] Ernst Eduard Kummer. "Über die Transcendenten, welche aus wiederholten Integrationen rationaler Formeln entstehen". In: *J. reine ang. Mathematik* 21 (1840), pp. 74–90, 193–225, 328–371.

[82] Niels Nielsen. "Der Eulersche Dilogarithmus und seine Verallgemeinerungen". In: *Nova Acta Leopoldina* 90 (1909), pp. 121–211.

[83] Alexander B. Goncharov. "Polylogarithms in Arithmetic and Geometry". In: *Proceeding of the International Congress of Mathematicians*. 1994, pp. 374–387.

[84] E. Remiddi and J. A. M. Vermaseren. "Harmonic polylogarithms". In: *Int. J. Mod. Phys.* A15 (2000), pp. 725–754. DOI: 10.1142/S0217751X00000367. arXiv: hep-ph/9905237 [hep-ph].

[85] T. Gehrmann and E. Remiddi. "Two loop master integrals for $\gamma* \to 3$ jets: The Planar topologies". In: *Nucl. Phys.* B601 (2001), pp. 248–286. DOI: 10.1016/S0550-3213(01)00057-8. arXiv: hep-ph/0008287 [hep-ph].

[86] Jens Vollinga and Stefan Weinzierl. "Numerical evaluation of multiple polylogarithms". In: *Comput. Phys. Commun.* 167 (2005), p. 177. DOI: 10.1016/j.cpc.2004.12.009. arXiv: hep-ph/0410259 [hep-ph].

[87] Hjalte Frellesvig, Damiano Tommasini, and Christopher Wever. “On the reduction of generalized polylogarithms to Li_n and $\mathrm{Li}_{2,2}$ and on the evaluation thereof”. In: *JHEP* 03 (2016), p. 189. DOI: 10.1007/JHEP03(2016)189. arXiv: 1601.02649 [hep-ph].

[88] Hjalte Frellesvig. *Generalized Polylogarithms in Maple*. 2018. arXiv: 1806.02883 [hep-th].

[89] Claude Duhr and Falko Dulat. *PolyLogTools - Polylogs for the masses*. 2019. arXiv: 1904.07279 [hep-th].

[90] Vladimir A. Smirnov. “Applied asymptotic expansions in momenta and masses”. In: *Springer Tracts Mod. Phys.* 177 (2002), pp. 1–262.

[91] R. Harlander and M. Steinhauser. “Automatic computation of Feynman diagrams”. In: *Prog. Part. Nucl. Phys.* 43 (1999), pp. 167–228. DOI: 10.1016/S0146-6410(99)00095-2. arXiv: hep-ph/9812357 [hep-ph].

[92] T. Gehrmann, T. Huber, and D. Maitre. “Two-loop quark and gluon form-factors in dimensional regularisation”. In: *Phys. Lett.* B622 (2005), pp. 295–302. DOI: 10.1016/j.physletb.2005.07.019. arXiv: hep-ph/0507061 [hep-ph].

[93] Matthias Steinhauser. “MATAD: A Program package for the computation of MAssive TADpoles”. In: *Comput. Phys. Commun.* 134 (2001), pp. 335–364. DOI: 10.1016/S0010-4655(00)00204-6. arXiv: hep-ph/0009029 [hep-ph].

[94] Roman N. Lee. “Reducing differential equations for multiloop master integrals”. In: *JHEP* 04 (2015), p. 108. DOI: 10.1007/JHEP04(2015)108. arXiv: 1411.0911 [hep-ph].

[95] Christoph Meyer. “Transforming differential equations of multi-loop Feynman integrals into canonical form”. In: *JHEP* 04 (2017), p. 006. DOI: 10.1007/JHEP04(2017)006. arXiv: 1611.01087 [hep-ph].

[96] Christoph Meyer. “Algorithmic transformation of multi-loop Feynman integrals to a canonical basis”. PhD thesis. Humboldt U., Berlin, Jan. 2018. DOI: 10.18452/18763. arXiv: 1802.02419 [hep-ph].

[97] Mario Argeri et al. “Magnus and Dyson Series for Master Integrals”. In: *JHEP* 03 (2014), p. 082. DOI: 10.1007/JHEP03(2014)082. arXiv: 1401.2979 [hep-ph].

[98] Thomas Gehrmann et al. “The two-loop master integrals for $q\bar{q} \to VV$”. In: *JHEP* 06 (2014), p. 032. DOI: 10.1007/JHEP06(2014)032. arXiv: 1404.4853 [hep-ph].

[99] Pascal Wasser. “Analytic properties of Feynman integrals for scattering amplitudes”. master thesis. Johannes Gutenberg-Universität Mainz, 2016.

[100] Amedeo Primo and Lorenzo Tancredi. “On the maximal cut of Feynman integrals and the solution of their differential equations”. In: *Nucl. Phys.* B916 (2017), pp. 94–116. DOI: 10.1016/j.nuclphysb.2016.12.021. arXiv: 1610.08397 [hep-ph].

[101] P. A. Baikov. "Explicit solutions of the three loop vacuum integral recurrence relations". In: *Phys. Lett.* B385 (1996), pp. 404–410. DOI: 10.1016/0370-2693(96)00835-0. arXiv: hep-ph/9603267 [hep-ph].

[102] Hjalte Frellesvig and Costas G. Papadopoulos. "Cuts of Feynman Integrals in Baikov representation". In: *JHEP* 04 (2017), p. 083. DOI: 10.1007/JHEP04(2017)083. arXiv: 1701.07356 [hep-ph].

[103] Oleksandr Gituliar and Vitaly Magerya. "Fuchsia: a tool for reducing differential equations for Feynman master integrals to epsilon form". In: *Comput. Phys. Commun.* 219 (2017), pp. 329–338. DOI: 10.1016/j.cpc.2017.05.004. arXiv: 1701.04269 [hep-ph].

[104] Roman N. Lee. "Libra". Unpublished.

[105] Christoph Meyer. "Algorithmic transformation of multi-loop master integrals to a canonical basis with CANONICA". In: *Comput. Phys. Commun.* 222 (2018), pp. 295–312. DOI: 10.1016/j.cpc.2017.09.014. arXiv: 1705.06252 [hep-ph].

[106] Jakob Ablinger, Johannes Blümlein, and Carsten Schneider. "Harmonic Sums and Polylogarithms Generated by Cyclotomic Polynomials". In: *J. Math. Phys.* 52 (2011), p. 102301. DOI: 10.1063/1.3629472. arXiv: 1105.6063 [math-ph].

[107] Andreas von Manteuffel, Robert M. Schabinger, and Hua Xing Zhu. "The Complete Two-Loop Integrated Jet Thrust Distribution In Soft-Collinear Effective Theory". In: *JHEP* 03 (2014), p. 139. DOI: 10.1007/JHEP03(2014)139. arXiv: 1309.3560 [hep-ph].

[108] Johannes M. Henn, Alexander V. Smirnov, and Vladimir A. Smirnov. "Evaluating Multiple Polylogarithm Values at Sixth Roots of Unity up to Weight Six". In: *Nucl. Phys.* B919 (2017), pp. 315–324. DOI: 10.1016/j.nuclphysb.2017.03.026. arXiv: 1512.08389 [hep-th].

[109] Wolfram Research, Inc. *Mathematica, Version 11.1.* Champaign, IL, 2017.

[110] S. Borowka et al. "SecDec-3.0: numerical evaluation of multi-scale integrals beyond one loop". In: *Comput. Phys. Commun.* 196 (2015), pp. 470–491. DOI: 10.1016/j.cpc.2015.05.022. arXiv: 1502.06595 [hep-ph].

[111] S. Borowka et al. "pySecDec: a toolbox for the numerical evaluation of multi-scale integrals". In: *Comput. Phys. Commun.* 222 (2018), pp. 313–326. DOI: 10.1016/j.cpc.2017.09.015. arXiv: 1703.09692 [hep-ph].

[112] Matteo Becchetti et al. "Planar master integrals for the two-loop light-fermion electroweak corrections to Higgs plus jet production". In: *JHEP* 12 (2018), p. 019. DOI: 10.1007/JHEP12(2018)019. arXiv: 1810.05138 [hep-ph].

[113] Charalampos Anastasiou et al. "Mixed QCD-electroweak corrections to Higgs production via gluon fusion in the small mass approximation". In: *JHEP* 03 (2019), p. 162. DOI: 10.1007/JHEP03(2019)162. arXiv: 1811.11211 [hep-ph].

[114] Wai-Yee Keung and Frank J. Petriello. "Electroweak and finite quark-mass effects on the Higgs boson transverse momentum distribution". In: *Phys. Rev.* D80 (2009), p. 013007. DOI: 10.1103/PhysRevD.80.013007. arXiv: 0905.2775 [hep-ph].

[115] Stefano Catani, Daniel de Florian, and Massimiliano Grazzini. "Higgs production in hadron collisions: Soft and virtual QCD corrections at NNLO". In: *JHEP* 05 (2001), p. 025. DOI: 10.1088/1126-6708/2001/05/025. arXiv: hep-ph/0102227 [hep-ph].

[116] Daniel de Florian and Javier Mazzitelli. "A next-to-next-to-leading order calculation of soft-virtual cross sections". In: *JHEP* 12 (2012), p. 088. DOI: 10.1007/JHEP12(2012)08,10.1007/JHEP12(2012)088. arXiv: 1209.0673 [hep-ph].

[117] Richard D. Ball et al. "Higgs production in gluon fusion beyond NNLO". In: *Nucl. Phys.* B874 (2013), pp. 746–772. DOI: 10.1016/j.nuclphysb.2013.06.012. arXiv: 1303.3590 [hep-ph].

[118] Richard D. Ball et al. "Parton distributions for the LHC Run II". In: *JHEP* 04 (2015), p. 040. DOI: 10.1007/JHEP04(2015)040. arXiv: 1410.8849 [hep-ph].

[119] F. E. Low. "Bremsstrahlung of very low-energy quanta in elementary particle collisions". In: *Phys. Rev.* 110 (1958), pp. 974–977. DOI: 10.1103/PhysRev.110.974.

[120] T. H. Burnett and Norman M. Kroll. "Extension of the low soft photon theorem". In: *Phys. Rev. Lett.* 20 (1968), p. 86. DOI: 10.1103/PhysRevLett.20.86.

[121] Vittorio Del Duca. "High-energy Bremsstrahlung Theorems for Soft Photons". In: *Nucl. Phys.* B345 (1990), pp. 369–388. DOI: 10.1016/0550-3213(90)90392-Q.

[122] D. Bonocore et al. "Non-abelian factorisation for next-to-leading-power threshold logarithms". In: *JHEP* 12 (2016), p. 121. DOI: 10.1007/JHEP12(2016)121. arXiv: 1610.06842 [hep-ph].

[123] V. Del Duca et al. "Universality of next-to-leading power threshold effects for colourless final states in hadronic collisions". In: *JHEP* 11 (2017), p. 057. DOI: 10.1007/JHEP11(2017)057. arXiv: 1706.04018 [hep-ph].

[124] John M. Campbell and R. Keith Ellis. "An Update on vector boson pair production at hadron colliders". In: *Phys. Rev.* D60 (1999), p. 113006. DOI: 10.1103/PhysRevD.60.113006. arXiv: hep-ph/9905386 [hep-ph].

[125] John M. Campbell, R. Keith Ellis, and Ciaran Williams. "Vector boson pair production at the LHC". In: *JHEP* 07 (2011), p. 018. DOI: 10.1007/JHEP07(2011)018. arXiv: 1105.0020 [hep-ph].

[126] John M. Campbell, R. Keith Ellis, and Walter T. Giele. "A Multi-Threaded Version of MCFM". In: *Eur. Phys. J.* C75.6 (2015), p. 246. DOI: 10.1140/epjc/s10052-015-3461-2. arXiv: 1503.06182 [physics.comp-ph].

[127] Radja Boughezal et al. "Color singlet production at NNLO in MCFM". In: *Eur. Phys. J.* C77.1 (2017), p. 7. DOI: 10.1140/epjc/s10052-016-4558-y. arXiv: 1605.08011 [hep-ph].

[128] Jorge C. Romao and Joao P. Silva. "A resource for signs and Feynman diagrams of the Standard Model". In: *Int. J. Mod. Phys.* A27 (2012), p. 1230025. DOI: 10.1142/S0217751X12300256. arXiv: 1209.6213 [hep-ph].

[129] Günther Dissertori, Ian Knowles, and Michael Schmelling. *Quantum Chromodynamics*. Oxford University Press, 2003. ISBN: 9780198505723. URL: https://global.oup.com/academic/product/quantum-chromodynamics-9780199566419.

[130] Nicolas Deutschmann. *ndeutschmann/qgraf-xml-drawer: Qgraf-XML-drawer 1.0*. Nov. 2016. DOI: 10.5281/zenodo.164393. URL: https://doi.org/10.5281/zenodo.164393.

[131] R. N. Lee. *Presenting LiteRed: a tool for the Loop InTEgrals REDuction*. 2012. arXiv: 1212.2685 [hep-ph].

Many Thanks

I am almost finished with writing my thesis, so I guess it is time to say "thank you".

I wish first to thank my supervisor, Prof. Dr. rer. nat. Mr. Herr Sig. Kirill Melnikov. He gave me the opportunity to learn from and work with amazing people, both on the professional and on the personal level. Despite everything, he kept (and keeps) investing time and energies in me. I learnt something every day, and I was brought back (a bit?) to physics, with no small effort. I hope I retained even a tiny bit of what he showed me.

I wish to thank Lorenzo Tancredi, who has been a kind and keen guide, and a font of inspiration. I like to consider him a friend. I do not know where I would have ended without him and Kirill.

I wish to thank my *Korreferent* Prof. Dr. Matthias Steinhauser, for his keen and rapid proofreading of my work. I wish to thank also my mentor, Prof. Dr. Milada M. Mühlleitner; luckily I never had to make use of her institutional role until now.

I wish to thank Prof. Pierpaolo Mastrolia, my master thesis supervisor from Padova: without his advice and guide this adventure would not have even started. I wish to thank also Prof. Ettore Remiddi and Prof. Stefano Rigolin.

Many thanks go to the people from the *TTP* that helped me during these years, both in physics and outside of physics (the order is not meaningful): Amedeo, Chris, Christoph, Florian, Hjalte, Kirill(ka), Konstantin, Marta, and Simon (they also joined me in other adventures besides physics); Thomas (who also introduced me to the KIT karate group); Aliaksei, David, Marvin, and Matthias (my office mates); Daniel, *Frau* Schorn, Max, Martin, Raoul, Rebecca, and Robbert.

Un grazie va ai miei amici, che malgrado io me ne sia andato non mi hanno lasciato andare: Federico, Gianluca, Lorenza, Thomas.

Un grande grazie va a chi non demorde mai, e ai miei genitori: mi hanno appoggiato e continuano ad appoggiarmi nella mia scelta di andare lontano da casa. Aggiungo sempre problemi, ma fortunatamente loro aggiungono sempre soluzioni.

I care more about people who do not care about me than about people who care about me. This page is a poor thing to start mending for this behavior and to show how grateful I am to all these people, but it is a starting point, at least.

Danke.