

Evidence for $B^+ \rightarrow h_c K^+$ and observation of $\eta_c(2S) \rightarrow p\bar{p}\pi^+\pi^-$

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A search for the decays $B^+ \rightarrow h_c K^+$ and $B^0 \rightarrow h_c K_S^0$ is performed. Evidence for the decay $B^+ \rightarrow h_c K^+$ is found; its significance is 4.8σ . No evidence is found for $B^0 \rightarrow h_c K_S^0$. The branching fraction for $B^+ \rightarrow h_c K^+$ is measured to be $(3.7^{+1.0}_{-0.9} {}^{+0.8}_{-0.8}) \times 10^{-5}$; the upper limit for the $B^0 \rightarrow h_c K_S^0$ branching fraction is 1.4×10^{-5} at 90% C.L. In addition, a study of the $p\bar{p}\pi^+\pi^-$ invariant mass distribution in the channel $B^+ \rightarrow (p\bar{p}\pi^+\pi^-)K^+$ results in the first observation of the decay $\eta_c(2S) \rightarrow p\bar{p}\pi^+\pi^-$ with 12.1σ significance. The analysis is based on the 711 fb^{-1} data sample collected by the Belle detector at the asymmetric-energy e^+e^- collider KEKB at the $\Upsilon(4S)$ resonance.

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I. INTRODUCTION

The decays $B^+ \rightarrow \chi_{c0} K^+$, $B^+ \rightarrow \chi_{c2} K^+$, and $B^+ \rightarrow h_c K^+$ are suppressed by factorization [1,2]. The decays $B^+ \rightarrow \chi_{cJ} K^+$ have been observed; the current world-average branching fractions are $\mathcal{B}(B^+ \rightarrow \chi_{c0} K^+) = (1.49^{+0.15}_{-0.14}) \times 10^{-4}$ and $\mathcal{B}(B^+ \rightarrow \chi_{c2} K^+) = (1.1 \pm 0.4) \times 10^{-5}$ [3]. While $\mathcal{B}(B^+ \rightarrow \chi_{c0} K^+)$ is smaller than the branching fraction of the factorization-allowed process $\mathcal{B}(B^+ \rightarrow \chi_{c1} K^+) = (4.84 \pm 0.23) \times 10^{-4}$, it is not strongly suppressed. Before the first experimental searches, this resulted in an assumption that the process $B^+ \rightarrow h_c K^+$ may also have a large branching fraction $\mathcal{B}(B^+ \rightarrow h_c K^+) \approx \mathcal{B}(B^+ \rightarrow \chi_{c0} K^+)$ [2].

However, the decay $B^+ \rightarrow h_c K^+$ has not been observed experimentally yet. Neither the Belle [4] nor the BABAR [5] collaboration has found a statistically significant signal of $B^+ \rightarrow h_c K^+$ using the decay mode $h_c \rightarrow \eta_c \gamma$. The current branching-fraction upper limit of $\mathcal{B}(B^+ \rightarrow h_c K^+) < 3.8 \times 10^{-5}$ at 90% C.L. [3] was obtained in the h_c search by Belle [4].

Also, the LHCb Collaboration searched for the process $B^+ \rightarrow h_c(\rightarrow p\bar{p})K^+$ [6] and set the upper limit on the branching fraction product $\mathcal{B}(B^+ \rightarrow h_c K^+) \times \mathcal{B}(h_c \rightarrow p\bar{p}) < 6.4 \times 10^{-8}$ (95% C.L.). However, this measurement does not result in a stronger restriction on $\mathcal{B}(B^+ \rightarrow h_c K^+)$ because the decay $h_c \rightarrow p\bar{p}$ has never been observed and the upper limit on its branching fraction is $\mathcal{B}(h_c \rightarrow p\bar{p}) < 1.5 \times 10^{-4}$ (90% C.L.) [3]. Note that a newer LHCb analysis of the same channel performed in Ref. [7] does not update the upper limit on $\mathcal{B}(B^+ \rightarrow h_c K^+) \times \mathcal{B}(h_c \rightarrow p\bar{p})$.

Several new theoretical predictions of $\mathcal{B}(B^+ \rightarrow h_c K^+)$ were made after the experimental upper limit was set. The branching fraction has been calculated in the QCD factorization approach to be 2.7×10^{-5} [8]. A calculation using perturbative QCD was performed in Ref. [9]; the result is $\mathcal{B}(B^+ \rightarrow h_c K^+) = 3.6 \times 10^{-5}$. Another calculation performed in Ref. [10] results in $\mathcal{B}(B^+ \rightarrow h_c K^+)$ [$\mathcal{B}(B^0 \rightarrow h_c K^0)$] in the interval from 3.1×10^{-5} to 5.7×10^{-5} (from 2.9×10^{-5} to 5.3×10^{-5}), depending on the assumed value of the c quark mass. All the results mentioned above are close to each other, and the theoretical values of $\mathcal{B}(B^+ \rightarrow h_c K^+)$ are slightly below the current experimental upper limit. This motivates an updated study of the decays $B^+ \rightarrow h_c K^+$, which may be able to find them.

Here, we present such an updated search for the decays $B^+ \rightarrow h_c K^+$ and also include a search for the decays $B^0 \rightarrow h_c K_S^0$. The analysis is performed using the 711 fb^{-1} data sample collected by the Belle detector at the asymmetric-energy e^+e^- collider KEKB [11]. The data sample was collected at the $\Upsilon(4S)$ resonance and contains 772×10^6 $B\bar{B}$ pairs. The integrated luminosity is 2.8 times greater than the luminosity used in the previous analysis [4]. For further improvement of the sensitivity, the new analysis uses ten η_c decay channels to reconstruct the decay $h_c \rightarrow \eta_c \gamma$; only two channels were used in the old one. The new h_c decay channel $h_c \rightarrow p\bar{p}\pi^+\pi^-$ observed recently by the BESIII Collaboration [12] is also used for its reconstruction; in addition, we study the decays of other charmonium states to $p\bar{p}\pi^+\pi^-$. The discrimination of the signal and background events is improved by performing a multivariate analysis.

II. BELLE DETECTOR

The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC),

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a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter (ECL) composed of CsI(Tl) crystals located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux return located outside of the coil is instrumented to detect K_L^0 mesons and to identify muons. The detector is described in detail elsewhere [13]. Two inner detector configurations were used. A 2.0 cm radius beam pipe and a three-layer silicon vertex detector were used for the first sample of 140 fb^{-1} , while a 1.5 cm radius beam pipe, a four-layer silicon detector, and a small-cell inner drift chamber were used to record the remaining data [14].

We use a GEANT-based Monte Carlo (MC) simulation [15] to model the response of the detector, identify potential backgrounds, and determine the acceptance. The MC simulation includes run-dependent detector performance variations and background conditions. Signal MC events are generated with EVTGEN [16] in proportion to the relative luminosities of the different running periods.

III. EVENT SELECTION

We select events of the type $B^+ \rightarrow h_c K^+$ and $B^0 \rightarrow h_c K_S^0$. Inclusion of charge-conjugate modes is implied hereinafter. The reconstruction is performed with a conversion from the Belle to the Belle II data format [17].

All tracks are required to originate from the interaction point region: we require $dr < 0.2 \text{ cm}$ and $|dz| < 2 \text{ cm}$, where dr and dz are the cylindrical coordinates of the point of the closest approach of the track to the beam axis (the z axis of the laboratory reference frame coincides with the positron-beam axis).

Charged π , K mesons and protons are identified using likelihood ratios $R_{h_1/h_2} = \mathcal{L}_{h_1}/(\mathcal{L}_{h_1} + \mathcal{L}_{h_2})$, where h_1 and h_2 are the particle-identification (PID) hypotheses (π , K , or p) and \mathcal{L}_{h_i} are their corresponding likelihoods. The likelihoods are calculated from the combined time-of-flight information from the TOF, the number of photoelectrons from the ACC, and dE/dx measurements in the CDC. We require $R_{K/\pi} > 0.6$ for K candidates, $R_{\pi/K} > 0.6$ for π candidates, and $R_{p/\pi} > 0.6$, $R_{p/K} > 0.6$ for p candidates. The identification efficiency of the above requirements varies in the ranges (94–99)%, (84–93)%, and (90–98)% for π , K , and p , respectively, depending on the h_c or η_c decay channel. The misidentification probability for the background particles that are not π , K , and p , varies in the ranges (25–49)%, (4.9–11.3)%, and (0.5–1.9)%, respectively. Without the electron background, which is rejected as described below, the π fake rate drops to (20–35)%.

Electron candidates are identified as CDC charged tracks that are matched to electromagnetic showers in the ECL. The track and ECL cluster matching quality, the ratio of the electromagnetic shower energy to the track momentum, the transverse shape of the shower, the ACC light yield, and

the track dE/dx ionization are used in our electron-identification criteria. A similar likelihood ratio is constructed: $R_e = \mathcal{L}_e/(\mathcal{L}_e + \mathcal{L}_h)$, where \mathcal{L}_e and \mathcal{L}_h are the likelihoods for electrons and charged hadrons (π , K and p), respectively [18]. An electron veto ($R_e < 0.9$) is imposed on π , K , and p candidates. It is not applied for the K_S^0 and Λ daughter tracks because they have independent selection criteria. For the h_c or η_c decay channels other than $\eta_c \rightarrow K_S^0 K_S^0 \pi^0$ and $\eta_c \rightarrow \Lambda \bar{\Lambda}$, the electron veto rejects from 3.5% to 15% of the background events, while its signal efficiency is no less than 97.5%.

Photons are identified as ECL electromagnetic showers that have no associated charged tracks detected in the CDC. The shower shape is required to be consistent with that of a photon.

The π^0 candidates are reconstructed via their decay to two photons. The photon energies in the laboratory frame are required to be greater than 30 MeV. The π^0 invariant mass is required to satisfy $|M_{\pi^0} - m_{\pi^0}| < 15 \text{ MeV}/c^2$. Here and elsewhere, M_{particle} denotes the reconstructed invariant mass of the specified particle, and m_{particle} stands for the nominal mass of this particle [3]. This requirement corresponds approximately to a 3σ mass window around the nominal mass.

The V^0 -particle (K_S^0 and Λ) candidates are reconstructed from pairs of oppositely charged tracks that are assumed to be $\pi^+ \pi^-$ and $p \pi^-$ for K_S^0 and Λ , respectively. We require $|M_{K_S^0} - m_{K_S^0}| < 20 \text{ MeV}/c^2$ and $|M_{\Lambda} - m_{\Lambda}| < 10 \text{ MeV}/c^2$, corresponding approximately to 5.5σ mass windows in both cases. The V^0 candidates are selected by a neural network using the following input variables: the V^0 candidate momentum, decay angle, flight distance in the xy plane, the angle between the V^0 momentum and the direction from the interaction point to the V^0 vertex, the shortest z distance between the two daughter tracks, their radial impact parameters, and numbers of hits in the SVD and CDC. The separation of the K_S^0 and Λ candidates is performed by another neural network. The input variables of this network are the momenta and polar angles of the daughter tracks in the laboratory frame, their likelihood ratios $R_{\pi/p}$, and the V^0 candidate invariant mass for the Λ hypothesis.

The η candidates are reconstructed in $\gamma\gamma$ and $\pi^+ \pi^- \pi^0$ channels. The reconstructed η candidates are denoted by the η decay channel as $\eta_{2\gamma}$ and $\eta_{3\pi}$. The η invariant mass is required to satisfy $|M_{\eta_{2\gamma}} - m_{\eta}| < 30 \text{ MeV}/c^2$ and $|M_{\eta_{3\pi}} - m_{\eta}| < 15 \text{ MeV}/c^2$; these requirements correspond to 2.5σ and 4σ mass windows, respectively.

The η' candidates are reconstructed in the $\eta \pi^+ \pi^-$ decay mode. The invariant mass window is $|M_{\eta'} - m_{\eta'}| < 15 \text{ MeV}/c^2$, corresponding to a 4σ mass window.

The η_c candidates are reconstructed in ten decay channels: $K^+ K_S^0 \pi^-$, $K^+ K^- \pi^0$, $K_S^0 K_S^0 \pi^0$, $K^+ K^- \eta$, $K^+ K^- K^+ K^-$,

$\eta'(\rightarrow \eta\pi^+\pi^-)\pi^+\pi^-$, $p\bar{p}$, $p\bar{p}\pi^0$, $p\bar{p}\pi^+\pi^-$, and $\Lambda\bar{\Lambda}$. The selected η_c candidates are required to satisfy $|M_{\eta_c} - m_{\eta_c}| < 50 \text{ MeV}/c^2$; the mass-window width is about 1.6 widths of the η_c .

The h_c candidates are reconstructed in the $h_c \rightarrow \eta_c\gamma$ and $h_c \rightarrow p\bar{p}\pi^+\pi^-$ decay channels. The invariant mass of the h_c candidates is not restricted for the channel $\eta_c\gamma$; for the channel $p\bar{p}\pi^+\pi^-$, it is required to be greater than $2.7 \text{ GeV}/c^2$. The lower mass limit is selected to be very low to study other charmonium states decaying to the same final state.

The B -meson candidates are reconstructed via the decay modes $B^+ \rightarrow h_c K^+$ and $B^0 \rightarrow h_c K_S^0$. The B candidates are selected by their energy and the beam-energy-constrained mass. The difference between the B -meson and beam energies is defined as $\Delta E = \sum_i E_i - E_{\text{beam}}$, where E_i are the energies of the B decay products in the center-of-mass frame and E_{beam} is the beam energy in the same frame. The beam-energy-constrained mass is defined as $M_{\text{bc}} = \sqrt{E_{\text{beam}}^2 - (\sum_i \vec{p}_i)^2}$, where \vec{p}_i are the momenta of the B decay products in the center-of-mass frame. We retain B candidates satisfying the conditions $5.2 < M_{\text{bc}} < 5.3 \text{ GeV}/c^2$ and $|\Delta E| < 0.2 \text{ GeV}$. A mass-constrained fit is applied to the selected B -meson candidates.

In addition, for the channel $h_c \rightarrow \eta_c\gamma$, the h_c daughter γ energy is required to be greater than 200 MeV in the B rest frame. This requirement removes the background from low-energy photons, including the peaking backgrounds from B decays to the same final state without the photon. The signal efficiency of this requirement is 100% because the $\eta_c\gamma$ invariant mass of all excluded events is smaller than the h_c mass.

Also, the ratio of the Fox-Wolfram moments [19] F_2/F_0 is required to be less than 0.3. This requirement reduces the continuum background, rejecting from 18% to 53% of background events, depending on the h_c or η_c decay channel. Its signal efficiency is from 93.3% to 96.3%.

IV. MULTIVARIATE ANALYSIS AND OPTIMIZATION OF THE SELECTION REQUIREMENTS

A. General analysis strategy and data samples

To improve the separation of the signal and background events, we perform a multivariate analysis followed by an optimization of selection requirements. The first stages of the analysis are performed individually for $h_c \rightarrow p\bar{p}\pi^+\pi^-$, and each η_c decay channel for the h_c candidates reconstructed in the $\eta_c\gamma$ mode [the channels $\eta_c \rightarrow K^+K^-\eta$ and $\eta_c \rightarrow \eta'(\rightarrow \eta\pi^+\pi^-)\pi^+\pi^-$ are optimized separately for $\eta_{2\gamma}$ and $\eta_{3\pi}$]. They include the determination of two-dimensional $(\Delta E, M_{\text{bc}})$ resolution and the distribution of the background in $(\Delta E, M_{\text{bc}})$ and the multivariate-analysis stage. The optimization of the selection requirements uses the results of all initial stages as its input. The resolution is

used to determine the expected number of the signal events, and the distribution of the background in $(\Delta E, M_{\text{bc}})$ is used to determine the expected number of the background events in the signal region. The optimization is performed individually for the channel $h_c \rightarrow p\bar{p}\pi^+\pi^-$ and globally for all η_c decay channels for the channel $h_c \rightarrow \eta_c\gamma$. The data selected using the resulting channel-dependent criteria are merged into a single sample for the $h_c \rightarrow \eta_c\gamma$ channel. The final fit is performed simultaneously to the $h_c \rightarrow \eta_c\gamma$ and $h_c \rightarrow p\bar{p}\pi^+\pi^-$ samples.

The experimental data are used for the determination of the $(\Delta E, M_{\text{bc}})$ distribution, selection of the background samples for the neural network, and final fit to the selected events. During the development of the analysis procedure, the h_c signal region was excluded to avoid bias of the h_c significance. The final fit described in Sec. V was performed on MC pseudoexperiments generated in accordance with the fit result without the h_c mixed with the $B^+ \rightarrow h_c K^+$ ($B^0 \rightarrow h_c K_S^0$) signal MC. The h_c signal region is defined by

$$\sqrt{\left(\frac{\Delta E}{\sigma_{\Delta E}}\right)^2 + \left(\frac{M_{\text{bc}} - m_B}{\sigma_{M_{\text{bc}}}}\right)^2} < 3, \quad (1)$$

where $\sigma_{\Delta E} = 18 \text{ MeV}$ and $\sigma_{M_{\text{bc}}} = 2.5 \text{ MeV}/c^2$ are the approximate resolutions in ΔE and M_{bc} , respectively, and

$$\begin{aligned} 3.50 < M_{h_c} < 3.55 \text{ GeV}/c^2 & \text{ for } h_c \rightarrow \eta_c\gamma, \\ 3.515 < M_{h_c} < 3.535 \text{ GeV}/c^2 & \text{ for } h_c \rightarrow p\bar{p}\pi^+\pi^-. \end{aligned} \quad (2)$$

After completion of the analysis procedure development, this requirement is no longer used.

The signal MC is used for the determination of the resolution and the selection of the signal samples for the neural network. The signal MC is generated using the known information about the angular or invariant-mass distributions of the decay products if it is possible; otherwise, uniform distributions are assumed. The angular distribution is known for the channel $h_c \rightarrow \eta_c\gamma$. It does not have any free parameters and is proportional to $\sin^2\theta_{h_c}$, where θ_{h_c} is the h_c helicity angle that is defined as the angle between $-\vec{p}_B$ and \vec{p}_{η_c} , where \vec{p}_B and \vec{p}_{η_c} are the momenta of the B and η_c in the h_c rest frame, respectively. In addition, the η_c decay resonant structure is taken into account if it is known. The distributions for the channels $K^+K_S^0\pi^-$, $K^+K^-\pi^0$, $K_S^0K_S^0\pi^0$, $K^+K^-\eta_{2\gamma}$, and $K^+K^-\eta_{3\pi}$ are based on the results of a Dalitz plot analysis performed in Ref. [20]. The contributions of intermediate ϕ resonances are taken into account for the channel $K^+K^-K^+K^-$ based on the world-average branching fractions from Ref. [3].

B. Resolution

The resolution is parametrized by the function

$$S(\Delta E, M_{bc}) = N_{CB} F_{CB}(x_1) G_a^{(12)}(y_1) + N_{G1} G_a^{(21)}(x_2) G_a^{(22)}(y_2) + N_{G2} G_a^{(31)}(x_3) G_a^{(32)}(y_3), \quad (3)$$

where F_{CB} is an asymmetric Crystal Ball function [21]; $G_a^{(ij)}$ are asymmetric Gaussian functions; N_{CB} , N_{G1} , and N_{G2} are normalizations; and x_i and y_i ($i = 1, 2, 3$) are rotated variables that are given by

$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} = \begin{pmatrix} \cos \alpha_i & \sin \alpha_i \\ -\sin \alpha_i & \cos \alpha_i \end{pmatrix} \begin{pmatrix} \Delta E - (\Delta E)_0 \\ M_{bc} - (M_{bc})_0 \end{pmatrix}. \quad (4)$$

Here, $((\Delta E)_0, (M_{bc})_0)$ is the central point, and α_i is the rotation angle. The central point is the same for all three components. The resolution is determined from a binned maximum likelihood fit to signal MC events. Example resolution fit results [for the channel $B^+ \rightarrow h_c K^+$ with $h_c \rightarrow \eta_c (\rightarrow K^+ K^- \pi^0) \gamma$] are shown in Fig. 1.

C. Fit to the $(\Delta E, M_{bc})$ distribution

The $(\Delta E, M_{bc})$ distribution is fitted in order to estimate the expected number of the background events in the signal region. The distribution is fitted to the function

$$N_S S(\Delta E, M_{bc}) + B(\Delta E, M_{bc}), \quad (5)$$

where N_S is the number of signal events and B is the background density function that is given by

$$B(\Delta E, M_{bc}) = \sqrt{m_0 - M_{bc}} \exp[-a(m_0 - M_{bc})] \times P_3(\Delta E, M_{bc}), \quad (6)$$

where m_0 is the threshold mass, a is a rate parameter, and P_3 is a two-dimensional third-order polynomial. The region with $\Delta E < -0.12$ GeV is excluded for the channel $h_c \rightarrow p \bar{p} \pi^+ \pi^-$ because of the presence of peaking backgrounds from partially reconstructed B decays with an additional π meson.

Example $(\Delta E, M_{bc})$ fit results [for the channel $B^+ \rightarrow h_c K^+$ with $h_c \rightarrow \eta_c (\rightarrow K^+ K^- \pi^0) \gamma$] are shown in Fig. 2.

D. Multivariate analysis

To improve the separation of signal and background events, we perform a multivariate analysis for each individual channel. The algorithm used for the multivariate analysis is the multilayer perceptron (MLP) neural network implemented in the TMVA library [22]. The following variables are always included in the neural network: the angle between the thrust axes of the B candidate and the remaining particles in

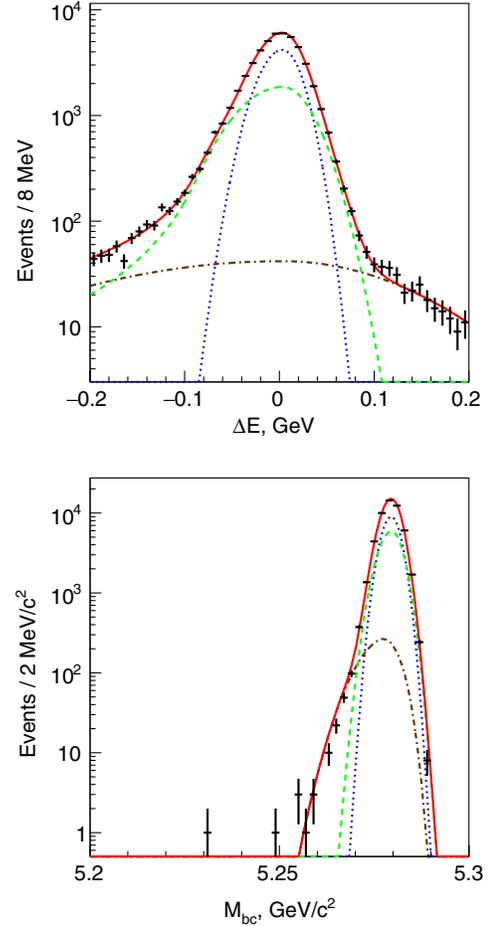


FIG. 1. Projections of the resolution fit results onto ΔE and M_{bc} for the channel $B^+ \rightarrow h_c K^+$ with $h_c \rightarrow \eta_c (\rightarrow K^+ K^- \pi^0) \gamma$. The red solid line is the fit result, the green dashed line is the Crystal Ball component, the blue dotted line is the first Gaussian component, and the brown dash-dotted line is the second Gaussian component.

the event, the angle between the thrust axes of all tracks and all photons in the event, the ratio of the Fox-Wolfram moments F_2/F_0 , the B production angle, and the vertex fit quality. For the h_c candidates reconstructed in the $\eta_c \gamma$ channel, the MLP also includes the h_c helicity angle, the η_c mass, and the number of π^0 candidates that include the h_c daughter photon as one of their daughters (separately for two groups of π^0 candidates with the energy of another photon less and greater than 100 MeV).

For the channels $\eta_c \rightarrow K^+ K_S^0 \pi^-$, $\eta_c \rightarrow K^+ K^- \pi^0$, and $\eta_c \rightarrow K_S^0 K_S^0 \pi^0$, two invariant masses of the η_c daughter particle pairs (both $K\pi$ combinations) are added to the neural network.

The following PID variables are included in the neural network if there are corresponding charged particles in the final state: the minimum likelihood ratio $R_{K/\pi}$ of the h_c daughter kaons, the minimum of the two likelihood ratios $R_{p/K}$, $R_{p/\pi}$ of the h_c daughter protons, and $R_{K/\pi}$ for the B

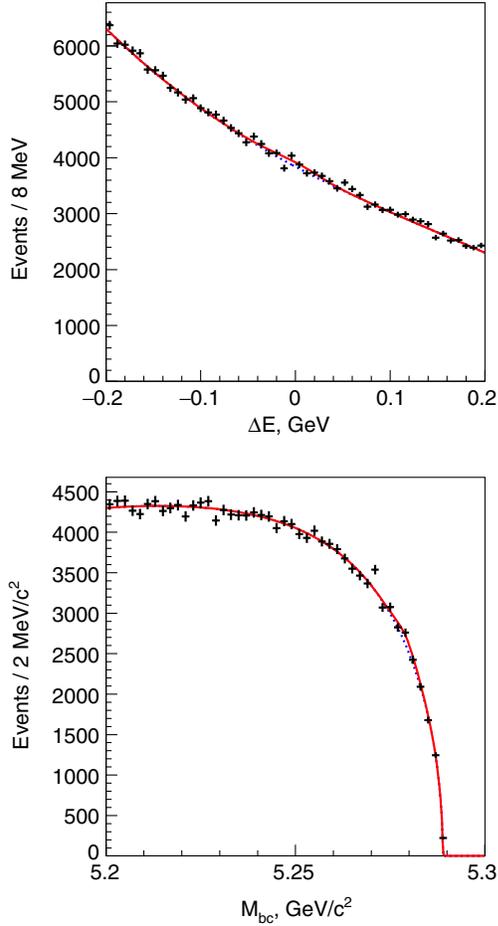


FIG. 2. Projections of the results of the fit to the $(\Delta E, M_{bc})$ distribution onto ΔE (with $M_{bc} > 5.272 \text{ GeV}/c^2$) and M_{bc} (with $|\Delta E| < 20 \text{ MeV}$) for the channel $B^+ \rightarrow h_c K^+$ with $h_c \rightarrow \eta_c(\rightarrow K^+ K^- \pi^0)\gamma$. The red solid line is the fit result, and the blue dotted line is the background. Since there is no significant signal before the optimization of the selection requirements and for the entire $\eta_c\gamma$ mass range, the two lines almost coincide.

daughter K^+ (for the channel $B^+ \rightarrow h_c K^+$). Here, the h_c daughters may be either direct (from the decay $h_c \rightarrow p\bar{p}\pi^+\pi^-$) or indirect (the η_c daughters for the h_c candidates reconstructed in the $\eta_c\gamma$ mode).

If there is a π^0 or η decaying to $\gamma\gamma$ in the final state, four additional variables are added: the π^0 (η) mass, the minimal energy of the π^0 (η) daughter photons in the laboratory frame, and the number of π^0 candidates that include a π^0 (η) daughter photon as one of their daughters [for each of the π^0 (η) daughter photons]. If there is an η reconstructed in the $\pi^+\pi^-\pi^0$ decay mode, then only its mass is added to the MLP. If the η_c has a daughter η' , then the mass of the η' candidate is also included to the neural network.

The training and testing signal samples are taken from the signal MC. The background sample is taken from a two-dimensional $(\Delta E, M_{bc})$ sideband. For the channel $B^+ \rightarrow h_c K^+$, the sideband is defined as

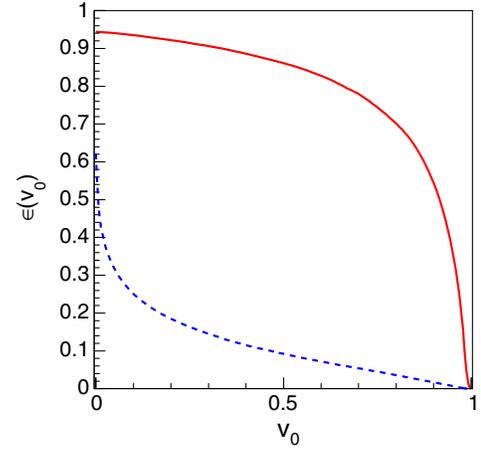


FIG. 3. Efficiency of the MLP output requirement ($v > v_0$) for the channel $B^+ \rightarrow h_c K^+$ with $h_c \rightarrow \eta_c(\rightarrow K^+ K^- \pi^0)\gamma$. The red solid line is the signal efficiency, and the blue dashed line is the background efficiency.

$$3 < \sqrt{\left(\frac{\Delta E}{\sigma_{\Delta E}}\right)^2 + \left(\frac{M_{bc} - m_B}{\sigma_{M_{bc}}}\right)^2} < 8. \quad (7)$$

The background sample is divided into training and testing samples of equal size.

The channel $B^0 \rightarrow h_c K_S^0$ has a small number of background events. In order to avoid overtraining, the background region for this channel is redefined. It includes all selected events except the central region defined by Eq. (1). In addition, the MLP internal architecture is changed. Instead of the default TMVA neural network with two hidden layers, only one hidden layer is used.

The resulting efficiency of the requirement ($v > v_0$) on the MLP output variable v for the training sample is shown in Fig. 3 for the channel $B^+ \rightarrow h_c K^+$ with $h_c \rightarrow \eta_c(\rightarrow K^+ K^- \pi^0)\gamma$. Note that the efficiency is given by

$$\epsilon(v_0) = \epsilon_{\text{MLP}}(v_0) \times \epsilon_{\text{multiple}}(v_0), \quad (8)$$

where ϵ is the full efficiency, ϵ_{MLP} is the raw MLP output requirement efficiency and $\epsilon_{\text{multiple}}$ is the best-candidate selection efficiency. Because of the correction by $\epsilon_{\text{multiple}}$, the efficiency at the minimal MLP output value v_{min} is not 1 but rather $\epsilon_{\text{multiple}}(v_{\text{min}})$.

The best-candidate selection is performed for each of the multivariate-analysis channels separately in the following way. The selected $(\Delta E, M_{bc})$ region is subdivided into three bins in both ΔE and M_{bc} . The selection is performed for each of the bins separately. The candidate with the largest MLP output is selected. One of the bins ($-67 < \Delta E < 67 \text{ MeV}$, $5.267 < M_{bc} < 5.3 \text{ GeV}/c^2$) always contains the entire signal region selected by the optimization procedure as described in Sec. IV E. Thus, the signal region of the final data sample does not contain multiple

candidates that originate from the same h_c channel. However, multiple candidates from different h_c channels are possible.

The best-candidate selection efficiency increases for larger values of the MLP output cutoff value v_0 . For the v_0 values obtained as the result of the optimization of the selection requirements as described in Sec. IV E, the selection procedure removes from 3% to 15% of data events, depending on the multivariate-analysis channel.

E. Optimization of the selection requirements

Optimization of the selection requirements is performed by maximizing the value

$$F_{\text{opt}} = \frac{\sum_i N_{\text{sig}}^{(i)}}{\frac{a}{2} + \sqrt{\sum_i N_{\text{bg}}^{(i)}}}, \quad (9)$$

where i is the channel index, $N_{\text{sig}}^{(i)}$ is the expected number of the signal events for the i th channel, $N_{\text{bg}}^{(i)}$ is the expected number of the background events in the signal region, and $a = 3$ is the target significance. This optimization method is based on Ref. [23].

The signal region is defined as

$$\left(\frac{\Delta E}{R_{\Delta E}^{(i)}}\right)^2 + \left(\frac{M_{\text{bc}} - m_B}{R_{M_{\text{bc}}}^{(i)}}\right)^2 < 1. \quad (10)$$

where $R_{\Delta E}^{(i)}$ and $R_{M_{\text{bc}}}^{(i)}$ are the half-axes of the signal region ellipse. The parameters determined by the optimization are $R_{\Delta E}^{(i)}$, $R_{M_{\text{bc}}}^{(i)}$, and the minimal value of the MLP output ($v_0^{(i)}$) for each channel.

The expected number of signal events for $B^+ \rightarrow h_c K^+$ is calculated as

$$N_{\text{sig}}^{(i)} = 2N_{\Upsilon(4S)} \mathcal{B}(\Upsilon(4S) \rightarrow B^+ B^-) \mathcal{B}(B^+ \rightarrow h_c K^+) \times \mathcal{B}(h_c \rightarrow i) \epsilon_{\text{SR}}^{(i)} \epsilon_S^{(i)}(v_0^{(i)}), \quad (11)$$

where $N_{\Upsilon(4S)}$ is the number of $\Upsilon(4S)$ events, $\mathcal{B}(h_c \rightarrow i)$ is the branching fraction of the h_c to its i th decay channel, $\epsilon_{\text{SR}}^{(i)}$ is the reconstruction efficiency for the specific signal region SR, and $\epsilon_S^{(i)}(v_0^{(i)})$ is the efficiency of the requirement ($v > v_0$) on the MLP output variable v for the signal events. The number of $\Upsilon(4S)$ events is assumed to be equal to the number of $B\bar{B}$ pairs; the branching fraction $\mathcal{B}(\Upsilon(4S) \rightarrow B^+ B^-)$ is calculated under the same assumption [3]. The signal-region-dependent reconstruction efficiency is calculated as

$$\epsilon_{\text{SR}}^{(i)} = \epsilon_R^{(i)} \int_{\text{SR}} S_i(\Delta E, M_{\text{bc}}) d\Delta E dM_{\text{bc}}, \quad (12)$$

where $\epsilon_R^{(i)}$ is the reconstruction efficiency and S_i is the signal probability density function for i th h_c decay channel (the integral of S_i over the signal region is the efficiency of the signal region selection). The unknown branching fraction $\mathcal{B}(B^+ \rightarrow h_c K^+)$ can be set to an arbitrary value because the maximum of F_{opt} does not depend on it. The expected number of signal events for $B^0 \rightarrow h_c K_S^0$ is calculated similarly.

The expected number of background events is calculated as

$$N_{\text{bg}}^{(i)} = \epsilon_B^{(i)}(v_0^{(i)}) \frac{N_{h_c \text{ region}}}{N_{\text{full}}} \int_{\text{SR}} B_i(\Delta E, M_{\text{bc}}) d\Delta E dM_{\text{bc}}, \quad (13)$$

where $\epsilon_B^{(i)}(v_0^{(i)})$ is the efficiency of the MLP output requirement for the background events, $N_{h_c \text{ region}}$ is the number of background events in the h_c region defined by Eq. (2), N_{full} is the full number of the background events, and B_i is the background density function defined in Eq. (6) for the i th h_c decay channel.

The optimization is performed separately for two channel groups. The first group includes the multivariate-analysis channels corresponding to the decay $h_c \rightarrow \eta_c \gamma$; the index i runs over all η_c decay channels. The second group consists of the single channel $h_c \rightarrow p\bar{p}\pi^+\pi^-$. The separate optimization is required by the difference of further data processing: the data from the first group of channels are combined into a single $h_c \rightarrow \eta_c \gamma$ data sample, while the $h_c \rightarrow p\bar{p}\pi^+\pi^-$ data are fitted with another function, as described below in Sec. V. The optimization results are shown in Table I. We also check the improvement achieved by MLP usage by changing the selection method to rectangular cuts. The values of F_{opt} are found to be about 30% and 10% smaller for the channels $h_c \rightarrow \eta_c \gamma$ and $h_c \rightarrow p\bar{p}\pi^+\pi^-$, respectively.

After the optimization, the resulting selection criteria are applied. The selected events for the channel $h_c \rightarrow \eta_c \gamma$ are merged. The resolution and distribution in $(\Delta E, M_{\text{bc}})$ are determined again for the $h_c \rightarrow p\bar{p}\pi^+\pi^-$ sample, since the knowledge of the background distribution in $(\Delta E, M_{\text{bc}})$ is necessary for the final fit described in Sec. V. The $(\Delta E, M_{\text{bc}})$ fit results are shown in Fig. 4.

F. Resolution in M_{h_c}

The resolution in M_{h_c} is determined from a fit to the combined $h_c \rightarrow \eta_c \gamma$ or $h_c \rightarrow p\bar{p}\pi^+\pi^-$ signal MC samples with η_c decaying to the reconstructed channels only. All final selection criteria are applied. The distribution of the difference of the reconstructed and true masses is fitted to the sum of an asymmetric Gaussian and asymmetric double-sided Crystal Ball functions,

$$R_{h_c}(\Delta M) = N[F_{\text{CB}}(\Delta M)f_{\text{CB}} + G_a(\Delta M)(1 - f_{\text{CB}})], \quad (14)$$

where ΔM is the difference of the reconstructed and true h_c masses, N is the common normalization, and f_{CB} is the

TABLE I. Results of the optimization of the selection requirements. The signal-region half-axes $R_{\Delta E}^{(i)}$ ($R_{M_{bc}}^{(i)}$) are in MeV (MeV/ c^2); all other values are dimensionless.

Channel	$B^+ \rightarrow h_c K^+$						$B^0 \rightarrow h_c K_S^0$					
	Parameters			Efficiency			Parameters			Efficiency		
	$R_{\Delta E}^{(i)}$	$R_{M_{bc}}^{(i)}$	$v_0^{(i)}$	$\epsilon_{SR}^{(i)}$	$\epsilon_S^{(i)}(v_0^{(i)})$	$\epsilon_B^{(i)}(v_0^{(i)})$	$R_{\Delta E}^{(i)}$	$R_{M_{bc}}^{(i)}$	$v_0^{(i)}$	$\epsilon_{SR}^{(i)}$	$\epsilon_S^{(i)}(v_0^{(i)})$	$\epsilon_B^{(i)}(v_0^{(i)})$
Channel group 1: $h_c \rightarrow \eta_c \gamma$												
$\eta_c(\rightarrow K^+ K_S^0 \pi^-) \gamma$	32.7	4.82	0.804	6.27%	59.5%	5.08%	34.2	4.97	0.702	4.31%	69.8%	8.55%
$\eta_c(\rightarrow K^+ K^- \pi^0) \gamma$	36.2	3.90	0.958	4.27%	31.7%	0.56%	43.5	4.54	0.942	3.38%	39.3%	0.85%
$\eta_c(\rightarrow K_S^0 K_S^0 \pi^0) \gamma$	42.3	4.49	0.976	1.79%	17.8%	0.18%	35.9	4.07	0.954	1.05%	35.9%	0.64%
$\eta_c(\rightarrow K^+ K^- \eta_{2\gamma}) \gamma$	34.4	4.16	0.977	4.21%	20.2%	0.22%	37.8	4.49	0.967	3.10%	28.0%	0.41%
$\eta_c(\rightarrow K^+ K^- \eta_{3\pi}) \gamma$	24.9	3.59	0.978	1.75%	29.7%	0.23%	33.2	4.55	0.986	1.50%	23.7%	0.17%
$\eta_c(\rightarrow K^+ K^- K^+ K^-) \gamma$	25.3	4.13	0.770	4.89%	53.2%	6.69%	29.9	4.80	0.734	3.71%	56.9%	8.49%
$\eta_c(\rightarrow \eta'(\rightarrow \eta_{2\gamma} \pi^+ \pi^-) \pi^+ \pi^-) \gamma$	30.5	4.21	0.958	2.69%	40.5%	0.63%	32.0	4.50	0.946	1.87%	45.6%	0.96%
$\eta_c(\rightarrow \eta'(\rightarrow \eta_{3\pi} \pi^+ \pi^-) \pi^+ \pi^-) \gamma$	26.8	4.16	0.990	1.01%	29.2%	0.13%	24.6	3.87	0.986	0.59%	32.3%	0.26%
$\eta_c(\rightarrow p\bar{p}) \gamma$	38.9	5.48	0.654	17.70%	75.7%	10.66%	42.5	5.85	0.513	12.47%	82.8%	15.43%
$\eta_c(\rightarrow p\bar{p}\pi^0) \gamma$	30.2	3.75	0.954	4.65%	30.3%	0.50%	31.8	4.01	0.934	3.33%	39.4%	0.93%
$\eta_c(\rightarrow p\bar{p}\pi^+\pi^-) \gamma$	24.1	4.03	0.912	6.31%	30.0%	1.53%	24.3	4.09	0.860	4.23%	41.8%	3.26%
$\eta_c(\rightarrow \Lambda\bar{\Lambda}) \gamma$	40.4	5.66	0.727	4.04%	70.6%	6.79%	41.5	5.19	0.586	2.65%	76.3%	11.24%
Channel group 2: $h_c \rightarrow p\bar{p}\pi^+\pi^-$												
$p\bar{p}\pi^+\pi^-$	13.5	4.36	0.598	14.81%	64.6%	18.40%	13.8	4.56	0.519	10.30%	71.2%	24.20%

Crystal Ball fraction. Example resolution fit results (for the channel $B^+ \rightarrow h_c K^+$ with $h_c \rightarrow \eta_c \gamma$) are shown in Fig. 5.

V. FIT TO THE DATA

A. Default model

For the $h_c \rightarrow \eta_c \gamma$ final sample, the distribution in the h_c mass in the $(\Delta E, M_{bc})$ sideband cannot be used to constrain the background level in the signal region because of the presence of peaking backgrounds, such as the background from B decays to a similar final state with a π^0 instead of the h_c daughter γ . If the second photon from this π^0 has a small energy, then the ΔE and M_{bc} values are close to 0 and the B mass, respectively. Thus, the fit is based on the signal distribution only for the channel $h_c \rightarrow \eta_c \gamma$.

For the channel $h_c \rightarrow p\bar{p}\pi^+\pi^-$, there is a signal from B decays to the same final state ($B^+ \rightarrow p\bar{p}\pi^+\pi^- K^+$ or $B^0 \rightarrow p\bar{p}\pi^+\pi^- K_S^0$) that do not proceed via any charmonium state, called the noncharmonium signal hereinafter. Because of the possible interference of the charmonium and noncharmonium signals, the distribution of the noncharmonium signal in the $p\bar{p}\pi^+\pi^-$ invariant mass needs to be determined by the fit. Thus, both signal and background

distributions are included in the fit for the channel $h_c \rightarrow p\bar{p}\pi^+\pi^-$.

We perform a simultaneous extended unbinned maximum likelihood fit to the $h_c \rightarrow \eta_c \gamma$ signal, $h_c \rightarrow p\bar{p}\pi^+\pi^-$ background, and $h_c \rightarrow p\bar{p}\pi^+\pi^-$ signal distributions. The charmonium states are represented by the Breit-Wigner amplitude

$$A_R(M_R) = \frac{1}{M_R^2 - m_R^2 + iM_R\Gamma_R}, \quad (15)$$

where M_R is the invariant mass, m_R is the nominal mass, and Γ_R is the width of the resonance R . The signal-region density function for the channel $h_c \rightarrow \eta_c \gamma$ is given by

$$S_{\eta_c \gamma}(M) = (N_{h_c} |A_{h_c}(M)|^2) \otimes R_{h_c}^{(\eta_c \gamma)}(\Delta M) + P_2(M), \quad (16)$$

where N_{h_c} is the number of signal events, $R_{h_c}^{(\eta_c \gamma)}$ is the h_c mass resolution for the channel $\eta_c \gamma$, and P_2 is a second-order polynomial. The background density function $B_{p\bar{p}\pi^+\pi^-}(M)$ for the channel $h_c \rightarrow p\bar{p}\pi^+\pi^-$ is a third-order polynomial. The signal density function for the channel $h_c \rightarrow p\bar{p}\pi^+\pi^-$ is given by

$$S_{p\bar{p}\pi^+\pi^-}(M) = \left(|P_3(M) + \sum_{R=\eta_c, \chi_{c0}, \eta_c(2S)} \sqrt{N_R} e^{i\varphi_R} A_R(M)|^2 + \sum_{R=J/\psi, \chi_{c1}, h_c, \chi_{c2}, \psi(2S)} N_R |A_R(M)|^2 \right) \otimes R_{h_c}^{(p\bar{p}\pi^+\pi^-)}(\Delta M), \quad (17)$$

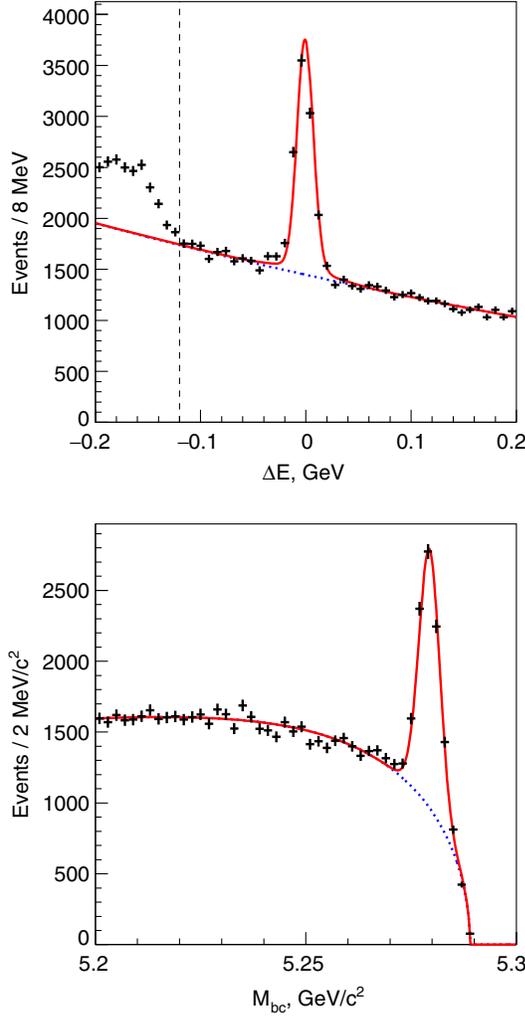


FIG. 4. Projections of the results of the fit to the $(\Delta E, M_{bc})$ distribution onto ΔE (with $M_{bc} > 5.272 \text{ GeV}/c^2$) and M_{bc} (with $|\Delta E| < 20 \text{ MeV}$) for the channel $B^+ \rightarrow h_c K^+$ with $h_c \rightarrow p\bar{p}\pi^+\pi^-$ after the application of the final MLP output selection criterion. The red solid line is the fit result, and the blue dotted line is the background. The region with $\Delta E < -0.12 \text{ GeV}$ is excluded from the fit because of the presence of peaking backgrounds from partially reconstructed B decays with an additional π meson. The cutoff value is marked by a vertical dashed line.

where P_3 is a third-order polynomial representing the noncharmonium signal. The wide states are added coherently to the signal density function, while the states that are narrower than the resolution are added incoherently. The amplitudes are normalized in such a way that all the parameters N_R represent the yields of the corresponding states. The signal distribution is fitted to the function

$$S_{p\bar{p}\pi^+\pi^-}(M) + wB_{p\bar{p}\pi^+\pi^-}(M), \quad (18)$$

where w is the weight of the background events in the signal region that is calculated as the ratio of integrals of the

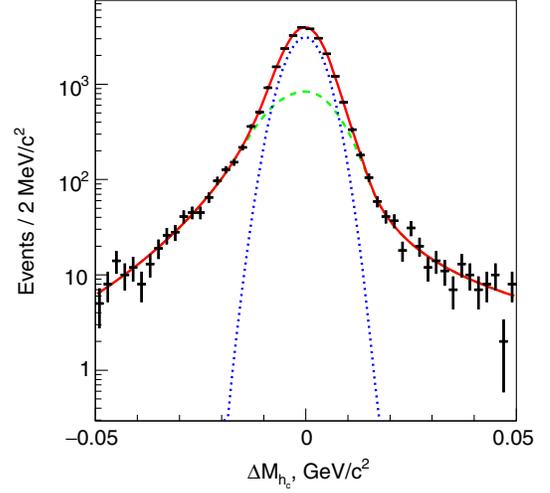


FIG. 5. Resolution in M_{h_c} for the channel $B^+ \rightarrow h_c K^+$ with $h_c \rightarrow \eta_c \gamma$. The red solid line is the fit result, the green dashed line is the Crystal Ball component, and the blue dotted line is the Gaussian component.

background distribution in $(\Delta E, M_{bc})$ over the signal and background regions. The model described above is the default one; additional models are considered to study systematic uncertainties. In the default model, the masses and widths of all resonances are fixed to their world-average values [3]; all other parameters are free.

The best-candidate selection procedure described in Sec. IV D guarantees that there are no multiple candidates in the $h_c \rightarrow p\bar{p}\pi^+\pi^-$ signal sample, but multiple candidates in the $h_c \rightarrow \eta_c \gamma$ signal sample are possible if they originate from different η_c decay channels. However, the fraction of the events with multiple candidates is found to be negligibly small. No events with multiple candidates (for the h_c masses within the default fitting regions) are observed for both $B^+ \rightarrow h_c K^+$ and $B^0 \rightarrow h_c K_S^0$ channels.

The fit results are shown in Fig. 6 for the channel $B^+ \rightarrow h_c K^+$ and in Fig. 7 for the channel $B^0 \rightarrow h_c K_S^0$. The signal yields and phases are listed in Table II. The statistical significance of the decays $B^+ \rightarrow h_c K^+$ and $B^0 \rightarrow h_c K_S^0$, as well as the significances of other charmonium states in the channel $p\bar{p}\pi^+\pi^-$, are calculated from the difference of $(-2 \ln L)$, where L is the maximum likelihood, between the models with and without these states taking the number of degrees of freedom into account. The significances of the decays $B^+ \rightarrow h_c K^+$ and $B^0 \rightarrow h_c K_S^0$ in the default model are found to be 5.0σ and 0.8σ , respectively. The significances of the decays $B^+ \rightarrow h_c K^+$ and $B^0 \rightarrow h_c K_S^0$ with the systematic error taken into account are 4.8σ and 0.7σ , respectively; the procedure of the calculation of the systematic uncertainty is described in Sec. V B. Thus, we find evidence for the decay $B^+ \rightarrow h_c K^+$ but do not find evidence for $B^0 \rightarrow h_c K_S^0$. The significances of charmonium states in the channel $p\bar{p}\pi^+\pi^-$ (except the h_c , which is

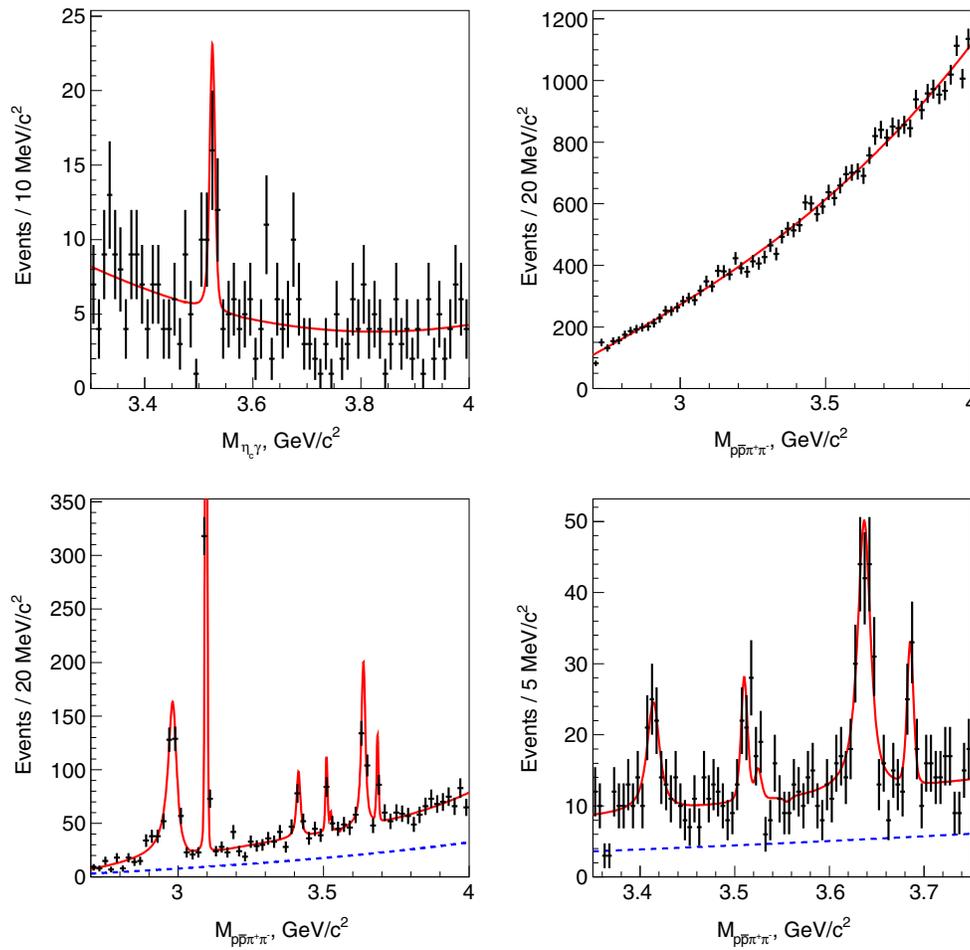


FIG. 6. Fit results in the $B^+ \rightarrow h_c K^+$ channel: $h_c \rightarrow \eta_c \gamma$ signal (top left), $h_c \rightarrow p\bar{p}\pi^+\pi^-$ background (top right), and $h_c \rightarrow p\bar{p}\pi^+\pi^-$ signal in the entire fitting region (bottom left) and in the χ_{cJ} region (bottom right). The red solid line is the fit result, and the blue dashed line is the background. The maximum of the $p\bar{p}\pi^+\pi^-$ signal fit result at the J/ψ peak is more than two times greater than the number of data events in the corresponding bin because the J/ψ peak is narrower than the bin size. Thus, a part of the fit result at the J/ψ peak is not shown.

reconstructed in two decay channels) are shown in Table III. The significances of the $\eta_c(2S)$ in the default model are 12.3σ and 5.9σ for the processes $B^+ \rightarrow (c\bar{c})(\rightarrow p\bar{p}\pi^+\pi^-)K^+$ and $B^0 \rightarrow (c\bar{c})(\rightarrow p\bar{p}\pi^+\pi^-)K_S^0$, respectively. The significances including the systematic error are 12.1σ and 5.8σ , respectively. Consequently, the decay $\eta_c(2S) \rightarrow p\bar{p}\pi^+\pi^-$ is observed for the first time in both $B^+ \rightarrow (c\bar{c})(\rightarrow p\bar{p}\pi^+\pi^-)K^+$ and $B^0 \rightarrow (c\bar{c})(\rightarrow p\bar{p}\pi^+\pi^-)K_S^0$ processes.

B. Systematic uncertainty: Model dependence

For a systematic-uncertainty study, we consider additional models. They include the models with free masses and widths of the h_c and all other charmonium states (with Gaussian constraints in accordance with the errors of their current world-average values), with increased order (3) of the background probability density function polynomial, different fitting ranges, scaled resolution, and variation of

the relative fraction of the channels $h_c \rightarrow \eta_c \gamma$ and $h_c \rightarrow p\bar{p}\pi^+\pi^-$. For the model with scaled resolution, the resolution function $R_{h_c}(\Delta M)$ is changed to

$$R_{h_c}(\Delta M) \rightarrow \frac{1}{S} R_{h_c}\left(\frac{\Delta M}{S}\right), \quad (19)$$

where S is the resolution scaling parameter. The variation of the relative fraction of the channels $h_c \rightarrow \eta_c \gamma$ and $h_c \rightarrow p\bar{p}\pi^+\pi^-$ is performed by changing the expected yields in these channels by $\pm 1\sigma$, where the error is due to the error of the corresponding branching fractions. The results are listed in Table IV.

C. Branching fraction

Using the number of reconstructed events, we calculate the branching fractions of the decays $B^+ \rightarrow h_c K^+$ and $B^0 \rightarrow h_c K_S^0$, as well as the branching fraction products

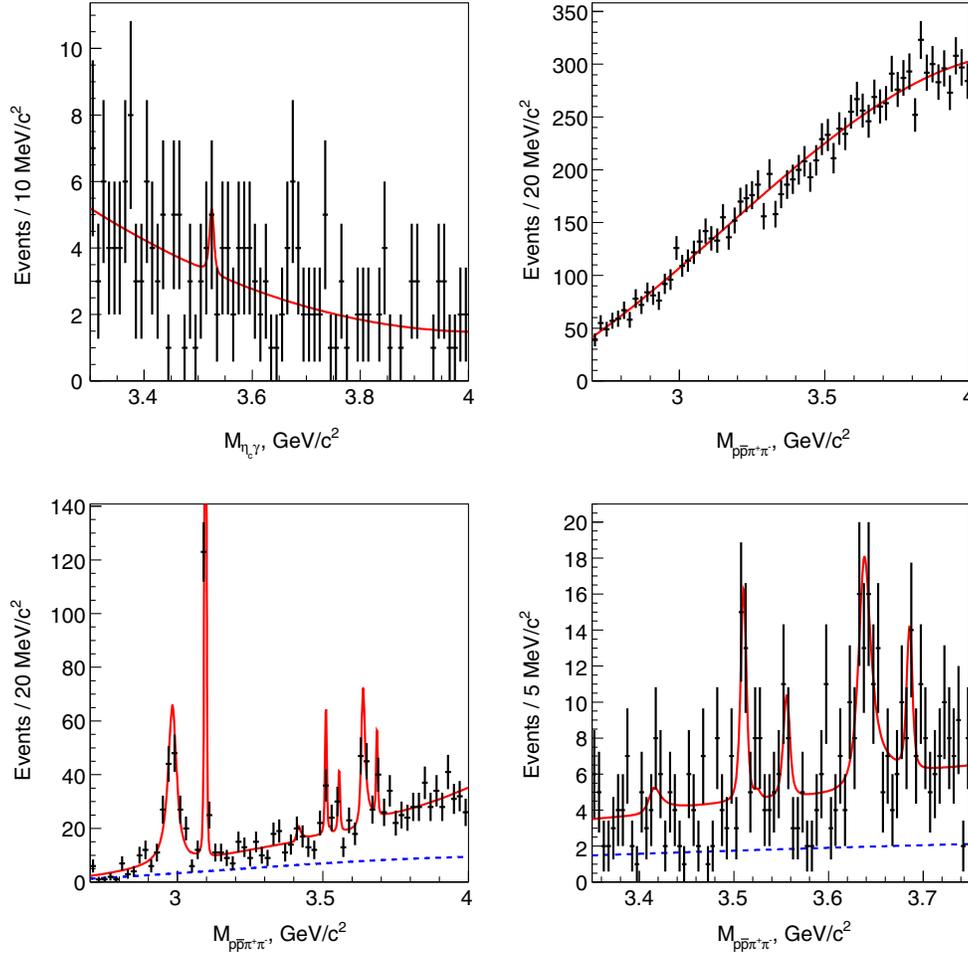


FIG. 7. Fit results in the $B^0 \rightarrow h_c K_S^0$ channel: $h_c \rightarrow \eta_c \gamma$ signal (top left), $h_c \rightarrow p\bar{p}\pi^+\pi^-$ background (top right), and $h_c \rightarrow p\bar{p}\pi^+\pi^-$ signal in the entire fitting region (bottom left) and in the χ_{cJ} region (bottom right). The red solid line is the fit result, and the blue dashed line is the background. The maximum of the $p\bar{p}\pi^+\pi^-$ signal fit result at the J/ψ peak is more than two times greater than the number of data events in the corresponding bin because the J/ψ peak is narrower than the bin size. Thus, a part of the fit result at the J/ψ peak is not shown.

$\mathcal{B}(B^+ \rightarrow (c\bar{c})K^+) \times \mathcal{B}((c\bar{c}) \rightarrow p\bar{p}\pi^+\pi^-)$ and $\mathcal{B}(B^0 \rightarrow (c\bar{c})K_S^0) \times \mathcal{B}((c\bar{c}) \rightarrow p\bar{p}\pi^+\pi^-)$. The decay $\psi(2S) \rightarrow p\bar{p}\pi^+\pi^-$ can proceed via the J/ψ : $\psi(2S) \rightarrow J/\psi(\rightarrow p\bar{p})\pi^+\pi^-$.

TABLE II. The resulting signal yields and phases in the default model. The errors are statistical only.

Parameter	$B^+ \rightarrow h_c K^+$	$B^0 \rightarrow h_c K_S^0$
N_{η_c}	229 ± 18	96 ± 11
φ_{η_c}	-1.12 ± 0.09	-1.47 ± 0.14
$N_{J/\psi}$	345 ± 19	128 ± 12
$N_{\chi_{c0}}$	25.5 ± 7.1	0.9 ± 1.6
$\varphi_{\chi_{c0}}$	-1.51 ± 0.23	-2.05 ± 1.20
$N_{\chi_{c1}}$	34.5 ± 8.8	21.2 ± 6.1
N_{h_c}	32.6 ± 8.0	3.1 ± 3.8
$N_{\chi_{c2}}$	-1.6 ± 6.3	11.6 ± 5.5
$N_{\eta_c(2S)}$	86.1 ± 11.9	24.0 ± 6.8
$\varphi_{\eta_c(2S)}$	-1.41 ± 0.14	-1.90 ± 0.23
$N_{\psi(2S)}$	36.9 ± 8.8	13.0 ± 5.6

To remove the events with a J/ψ , the $\psi(2S)$ yield is taken from an alternative fit with an additional J/ψ veto defined as $|M_{p\bar{p}} - m_{J/\psi}| > 50 \text{ MeV}/c^2$.

The sources of the systematic uncertainty of the branching fractions include the model dependence (the same set of alternative fit models is used as in Sec. VB), overtraining

TABLE III. Significances of the charmonium states decaying to $p\bar{p}\pi^+\pi^-$ except the h_c in the default model.

State	$B^+ \rightarrow (c\bar{c})K^+$	$B^0 \rightarrow (c\bar{c})K_S^0$
η_c	20.1σ	12.5σ
J/ψ	33.9σ	20.8σ
χ_{c0}	6.0σ	0.6σ
χ_{c1}	4.9σ	4.5σ
χ_{c2}	0.3σ	2.5σ
$\eta_c(2S)$	12.3σ	5.9σ
$\psi(2S)$	5.0σ	2.8σ

TABLE IV. Model dependence of the h_c and $\eta_c(2S) \rightarrow p\bar{p}\pi^+\pi^-$ significance.

Model	h_c significance		$\eta_c(2S) \rightarrow p\bar{p}\pi^+\pi^-$ significance	
	$B^+ \rightarrow h_c K^+$	$B^0 \rightarrow h_c K_S^0$	$B^+ \rightarrow h_c K^+$	$B^0 \rightarrow h_c K_S^0$
Default	5.0σ	0.8σ	12.3σ	5.9σ
Free masses and widths	5.0σ	0.8σ	12.3σ	6.0σ
Polynomial order ($h_c \rightarrow \eta_c\gamma$)	4.8σ	0.8σ	12.3σ	5.9σ
Polynomial order ($h_c \rightarrow p\bar{p}\pi^+\pi^-$ background)	5.0σ	0.8σ	12.2σ	5.9σ
Polynomial order ($h_c \rightarrow p\bar{p}\pi^+\pi^-$ signal)	5.0σ	0.9σ	12.2σ	5.9σ
Fitting range variation ($h_c \rightarrow \eta_c\gamma$)	5.0σ	0.9σ	12.3σ	5.9σ
Fitting range variation ($h_c \rightarrow p\bar{p}\pi^+\pi^-$)	5.0σ	0.8σ	12.1σ	5.8σ
Scaled resolution	5.0σ	0.8σ	12.3σ	6.0σ
Fraction of $h_c \rightarrow \eta_c\gamma$ and $h_c \rightarrow p\bar{p}\pi^+\pi^-$	4.9σ	0.7σ	12.2σ	5.9σ

(the difference between the efficiency in the training and testing samples), the error of the difference of the PID requirements efficiency between the data and MC, the difference of the MLP efficiency between the data and MC, tracking efficiency, number of $\Upsilon(4S)$ events, the $\eta_c \rightarrow K^+ K_S^0 \pi^-$ and $\Upsilon(4S) \rightarrow B^+ B^-$ or $\Upsilon(4S) \rightarrow B^0 \bar{B}^0$ branching fractions. All systematic error sources are listed in Table V for the channel $B^+ \rightarrow (c\bar{c})K^+$ [$B^+ \rightarrow h_c K^+$ for the h_c and $B^+ \rightarrow (c\bar{c})(\rightarrow p\bar{p}\pi^+\pi^-)K^+$ for all other charmonium states] and in Table VI for the channel $B^0 \rightarrow (c\bar{c})K_S^0$ [$B^0 \rightarrow h_c K_S^0$ for the h_c and $B^0 \rightarrow (c\bar{c})(\rightarrow p\bar{p}\pi^+\pi^-)K_S^0$ for all other charmonium states]. The errors of the tracking efficiency and the difference of the efficiency of the PID requirements depend on the multivariate-analysis channel in the case of the calculation of $\mathcal{B}(B^+ \rightarrow h_c K^+)$ and $\mathcal{B}(B^0 \rightarrow h_c K_S^0)$; the errors related to the MLP efficiency and overtraining are estimated separately for $h_c \rightarrow \eta_c\gamma$ and $h_c \rightarrow p\bar{p}\pi^+\pi^-$. The values presented in Tables V and VI are weighted averages.

The difference of the PID requirements efficiency between the data and MC is estimated from several control samples, such as $D^{*+} \rightarrow D^0(\rightarrow K^-\pi^+)\pi^+$ for K and π , $\Lambda \rightarrow p\pi^+$ for p , $\Lambda_c^+ \rightarrow \Lambda\pi^+$ for Λ ,

$D^0 \rightarrow K_S^0 \pi^+ \pi^-$ for K_S^0 , and $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$ for π^0 . The resulting overall efficiency ratio depends on the final state and the momenta of the decay products; thus, it is different for all branching fractions. For example, for the h_c , it is found to be $(95.0 \pm 3.8)\%$ for the channel $B^+ \rightarrow h_c K^+$ and $(93.0 \pm 4.5)\%$ for the channel $B^0 \rightarrow h_c K_S^0$.

The error caused by the difference of the MLP efficiency between the data and MC is estimated for the channel $h_c \rightarrow \eta_c\gamma$ using the decay mode $B^0 \rightarrow \eta_c\pi^-K^+$. This decay is reconstructed using selection criteria that are as similar as possible to the signal mode $B^+ \rightarrow h_c K^+$. The same MLP optimized for $B^+ \rightarrow h_c K^+$ is applied to the control channel. Some MLP input variables used for the signal channel are undefined for $B^0 \rightarrow \eta_c\pi^-K^+$, for example, the number of π^0 candidates that include the h_c daughter γ as one of their daughters. Such variables are held constant. The ratio of the number of signal candidates before and after the application of the MLP selection requirements is used to measure the difference of the MLP efficiency between the data and MC:

$$r_{\text{MLP}} = \frac{N(\text{all } \eta_c \text{ channels, with MLP cut})}{N(\eta_c \rightarrow K^+ K_S^0 \pi^-, \text{ no MLP cut})}. \quad (20)$$

TABLE V. Relative systematic uncertainties of the branching fractions for the channel $B^+ \rightarrow (c\bar{c})K^+$ [$B^+ \rightarrow h_c K^+$ for the h_c and $B^+ \rightarrow (c\bar{c})(\rightarrow p\bar{p}\pi^+\pi^-)K^+$ for all other charmonium states].

Error source	h_c	η_c	J/ψ	χ_{c0}	χ_{c1}	χ_{c2}	$\eta_c(2S)$	$\psi(2S)$
Model dependence	$(^{+9.09}_{-9.16})\%$	$(^{+3.45}_{-1.47})\%$	$(^{+1.96}_{-0.04})\%$	$(^{+4.59}_{-5.32})\%$	$(^{+7.10}_{-2.98})\%$	$(^{+21.13}_{-65.93})\%$	$(^{+1.75}_{-4.62})\%$	$(^{+1.74}_{-7.07})\%$
PID	3.99%	3.64%	3.62%	3.50%	3.50%	3.51%	3.52%	3.53%
Overtraining	0.41%	0.14%	0.14%	0.14%	0.14%	0.14%	0.14%	0.14%
Tracking	1.60%	1.75%	1.75%	1.75%	1.75%	1.75%	1.75%	1.75%
MLP efficiency	12.73%	0.25%	0.25%	0.25%	0.25%	0.25%	0.25%	0.25%
Number of π^0 candidates	11.60%
η_c mass and width	0.99%
h_c branching fraction	10.22%
$\mathcal{B}(\Upsilon(4S) \rightarrow B^+ B^-)$	1.17%	1.17%	1.17%	1.17%	1.17%	1.17%	1.17%	1.17%
Number of $\Upsilon(4S)$ events	1.37%	1.37%	1.37%	1.37%	1.37%	1.37%	1.37%	1.37%
Total	$(^{+22.51}_{-22.54})\%$	$(^{+5.62}_{-4.67})\%$	$(^{+4.83}_{-4.41})\%$	$(^{+6.30}_{-6.86})\%$	$(^{+8.31}_{-5.25})\%$	$(^{+21.57}_{-66.07})\%$	$(^{+4.68}_{-6.34})\%$	$(^{+4.68}_{-8.29})\%$

TABLE VI. Relative systematic uncertainties of the branching fractions for the channel $B^0 \rightarrow (c\bar{c})K_S^0$ [$B^0 \rightarrow h_c K_S^0$ for the h_c and $B^0 \rightarrow (c\bar{c})(\rightarrow p\bar{p}\pi^+\pi^-)K_S^0$ for all other charmonium states].

Error source	h_c	η_c	J/ψ	χ_{c0}	χ_{c1}	χ_{c2}	$\eta_c(2S)$	$\psi(2S)$
Model dependence	$^{+30.94}_{-24.05}\%$	$^{+4.91}_{-24.37}\%$	$^{+1.45}_{-1.80}\%$	$^{+134.01}_{-24.56}\%$	$^{+6.79}_{-2.28}\%$	$^{+8.63}_{-4.78}\%$	$^{+4.51}_{-5.36}\%$	$^{+10.89}_{-9.01}\%$
PID	4.86%	3.93%	3.93%	3.87%	3.87%	3.86%	3.83%	3.81%
Overtraining	0.15%	0.19%	0.19%	0.19%	0.19%	0.19%	0.19%	0.19%
Tracking	1.95%	2.10%	2.10%	2.10%	2.10%	2.10%	2.10%	2.10%
MLP efficiency	12.79%	0.25%	0.25%	0.25%	0.25%	0.25%	0.25%	0.25%
Number of π^0 candidates	11.66%
η_c mass and width	0.96%
h_c branching fraction	10.27%
$\mathcal{B}(\Upsilon(4S) \rightarrow B^0 \bar{B}^0)$	1.23%	1.23%	1.23%	1.23%	1.23%	1.23%	1.23%	1.23%
Number of $\Upsilon(4S)$ events	1.37%	1.37%	1.37%	1.37%	1.37%	1.37%	1.37%	1.37%
Total	$^{+37.33}_{-31.86}\%$	$^{+6.89}_{-24.85}\%$	$^{+5.04}_{-5.16}\%$	$^{+134.10}_{-25.02}\%$	$^{+8.31}_{-5.30}\%$	$^{+9.87}_{-6.76}\%$	$^{+6.56}_{-7.17}\%$	$^{+11.88}_{-10.18}\%$

Only the channel $\eta_c \rightarrow K^+ K_S^0 \pi^-$ is used before the MLP selection because only this channel is sufficiently clean for the determination of the number of the signal events without the MLP selection. The ratio r_{MLP} is extracted from a simultaneous fit to the η_c mass distribution before and after the application of the MLP selection. The relative difference between the values of r_{MLP} in data and MC is found to be $(14.4 \pm 9.0)\%$. For conservative treatment, the statistical error is added in quadrature to the central value of the difference. The resulting systematic uncertainty caused by the MLP selection efficiency difference in data and MC is 17.0%.

The estimation of the MLP efficiency error for the hadronic channel $p\bar{p}\pi^+\pi^-$ is done by performing the fit using the $p\bar{p}\pi^+\pi^-$ data without the MLP selection and comparing the resulting branching fraction products $\mathcal{B}(B^+ \rightarrow J/\psi K^+) \times \mathcal{B}(J/\psi \rightarrow p\bar{p}\pi^+\pi^-)$ with the results of the default procedure. Their relative difference is found to be 0.2%.

The same estimates of the MLP efficiency uncertainty for the channels $h_c \rightarrow \eta_c \gamma$ and $h_c \rightarrow p\bar{p}\pi^+\pi^-$ are used for both $B^+ \rightarrow h_c K^+$ and $B^0 \rightarrow h_c K_S^0$. The final value of the MLP efficiency uncertainty is calculated as a weighted average of the errors for the two h_c decay channels. The result is slightly different for the channels $B^+ \rightarrow h_c K^+$ and $B^0 \rightarrow h_c K_S^0$ because of the difference in the relative number of the expected $h_c \rightarrow \eta_c \gamma$ and $h_c \rightarrow p\bar{p}\pi^+\pi^-$ events. The MLP efficiency error for the branching fraction products for the channels $B^+ \rightarrow (c\bar{c})(\rightarrow p\bar{p}\pi^+\pi^-)K^+$ and $B^0 \rightarrow (c\bar{c})(\rightarrow p\bar{p}\pi^+\pi^-)K_S^0$ is equal to the error for the channel $p\bar{p}\pi^+\pi^-$, since all charmonium states other than the h_c are reconstructed in this channel only.

The MLP efficiency uncertainty for the channel $h_c \rightarrow \eta_c \gamma$ does not include the uncertainty caused by the difference between the data and MC in the distributions of the variables that are not defined for the channel $B^0 \rightarrow \eta_c \pi^- K^+$. There are four such variables: the η_c mass, the h_c helicity angle, and two numbers of π^0 candidates that include the h_c

daughter photon as one of their daughters. The distribution of the h_c helicity angle for the signal events is known precisely; thus, there is no additional uncertainty caused by the difference of its distribution in data and MC. The difference of the numbers of π^0 candidates is taken into account by removing these variables from the neural network for the channel $B^+ \rightarrow h_c K^+$, performing an alternative optimization, and comparing the resulting h_c branching fractions in the channel $h_c \rightarrow \eta_c \gamma$. The relative difference is found to be 15.6%. The error due to the η_c mass distribution uncertainty is estimated by varying the η_c mass and width by $\pm 1\sigma$ and reweighting the selected MC events in accordance with the relative difference between the modified and default η_c mass distributions. The largest resulting efficiency difference is considered as the systematic uncertainty related to the η_c mass distribution. This uncertainty is estimated to be 1.3% for both the $B^+ \rightarrow h_c K^+$ and $B^0 \rightarrow h_c K_S^0$ channels.

The ratio r_{MLP} used for determination of the MLP efficiency uncertainty includes the number of reconstructed events relatively to the number of events in $h_c \rightarrow \eta_c (\rightarrow K^+ K_S^0 \pi^-) \gamma$. The total expected number of events can be calculated as

$$2N_{\Upsilon(4S)} \left(\sum_i \epsilon_i \mathcal{B}_i \right) = 2N_{\Upsilon(4S)} \mathcal{B}_{h_c \rightarrow \eta_c \gamma} \left(\sum_i \epsilon_i \frac{\mathcal{B}_i}{\mathcal{B}_{h_c \rightarrow \eta_c \gamma}} \right), \quad (21)$$

where ϵ_i and \mathcal{B}_i are the efficiency and branching fraction for i th channel, respectively. The last term in Eq. (21) is proportional to r_{MLP} . Consequently, for the channel $h_c \rightarrow \eta_c \gamma$, one needs to take into account only the error of $\mathcal{B}(h_c \rightarrow \eta_c (\rightarrow K^+ K_S^0 \pi^-) \gamma)$. Errors of the branching fractions of all other channels relative to $h_c \rightarrow \eta_c (\rightarrow K^+ K_S^0 \pi^-) \gamma$ enter the MLP efficiency error.

The final h_c branching fraction error is calculated as a weighted average of the errors for the channels $h_c \rightarrow \eta_c \gamma$

TABLE VII. Measured branching fractions and branching fraction products and their comparison with the current world-average values [3].

Branching fraction	Value or confidence interval (90% C.L.)	World-average value
$\mathcal{B}(B^+ \rightarrow h_c K^+)$	$(3.7_{-0.9}^{+1.0+0.8}) \times 10^{-5}$	$< 3.8 \times 10^{-5}$
$\mathcal{B}(B^+ \rightarrow \eta_c K^+) \times \mathcal{B}(\eta_c \rightarrow p\bar{p}\pi^+\pi^-)$	$(39.4_{-3.9}^{+4.1+2.2}) \times 10^{-7}$	$(57.8 \pm 20.2) \times 10^{-7}$
$\mathcal{B}(B^+ \rightarrow J/\psi K^+) \times \mathcal{B}(J/\psi \rightarrow p\bar{p}\pi^+\pi^-)$	$(56.4_{-3.2}^{+3.3+2.7}) \times 10^{-7}$	$(60.6 \pm 5.3) \times 10^{-7}$
$\mathcal{B}(B^+ \rightarrow \chi_{c0} K^+) \times \mathcal{B}(\chi_{c0} \rightarrow p\bar{p}\pi^+\pi^-)$	$(3.7_{-1.0}^{+1.2+0.2}) \times 10^{-7}$	$(3.1 \pm 1.1) \times 10^{-7}$
$\mathcal{B}(B^+ \rightarrow \chi_{c1} K^+) \times \mathcal{B}(\chi_{c1} \rightarrow p\bar{p}\pi^+\pi^-)$	$(4.7_{-1.2}^{+1.3+0.4}) \times 10^{-7}$	$(2.4 \pm 0.9) \times 10^{-7}$
$\mathcal{B}(B^+ \rightarrow \chi_{c2} K^+) \times \mathcal{B}(\chi_{c2} \rightarrow p\bar{p}\pi^+\pi^-)$	$< 1.9 \times 10^{-7}$	$(0.15 \pm 0.06) \times 10^{-7}$
$\mathcal{B}(B^+ \rightarrow \eta_c(2S) K^+) \times \mathcal{B}(\eta_c(2S) \rightarrow p\bar{p}\pi^+\pi^-)$	$(11.2_{-1.6}^{+1.8+0.5}) \times 10^{-7}$	Not seen
$\mathcal{B}(B^+ \rightarrow \psi(2S) K^+) \times \mathcal{B}(\psi(2S) \rightarrow p\bar{p}\pi^+\pi^-)$	$[0.5, 3.5] \times 10^{-7}$	$(3.7 \pm 0.3) \times 10^{-7}$
$\mathcal{B}(B^0 \rightarrow h_c K_S^0)$	$< 1.4 \times 10^{-5}$	Not seen
$\mathcal{B}(B^0 \rightarrow \eta_c K_S^0) \times \mathcal{B}(\eta_c \rightarrow p\bar{p}\pi^+\pi^-)$	$(19.0_{-2.9}^{+3.2+1.3}) \times 10^{-7}$	$(20.9 \pm 7.8) \times 10^{-7}$
$\mathcal{B}(B^0 \rightarrow J/\psi K_S^0) \times \mathcal{B}(J/\psi \rightarrow p\bar{p}\pi^+\pi^-)$	$(24.3_{-2.2}^{+2.3+1.2}) \times 10^{-7}$	$(26.2 \pm 2.4) \times 10^{-7}$
$\mathcal{B}(B^0 \rightarrow \chi_{c0} K_S^0) \times \mathcal{B}(\chi_{c0} \rightarrow p\bar{p}\pi^+\pi^-)$	$< 1.3 \times 10^{-7}$	$(1.5 \pm 0.6) \times 10^{-7}$
$\mathcal{B}(B^0 \rightarrow \chi_{c1} K_S^0) \times \mathcal{B}(\chi_{c1} \rightarrow p\bar{p}\pi^+\pi^-)$	$(3.7_{-1.0}^{+1.2+0.3}) \times 10^{-7}$	$(1.0 \pm 0.4) \times 10^{-7}$
$\mathcal{B}(B^0 \rightarrow \chi_{c2} K_S^0) \times \mathcal{B}(\chi_{c2} \rightarrow p\bar{p}\pi^+\pi^-)$	$[0.7, 3.8] \times 10^{-7}$	Not seen
$\mathcal{B}(B^0 \rightarrow \eta_c(2S) K_S^0) \times \mathcal{B}(\eta_c(2S) \rightarrow p\bar{p}\pi^+\pi^-)$	$(4.2_{-1.2}^{+1.4+0.3}) \times 10^{-7}$	Not seen
$\mathcal{B}(B^0 \rightarrow \psi(2S) K_S^0) \times \mathcal{B}(\psi(2S) \rightarrow p\bar{p}\pi^+\pi^-)$	$< 1.9 \times 10^{-7}$	$(1.7 \pm 0.2) \times 10^{-7}$

and $h_c \rightarrow p\bar{p}\pi^+\pi^-$. Since branching fraction products are measured for all other charmonium states, they do not have a similar systematic error source.

The resulting branching fractions with both statistical and systematic errors are listed in Table VII. For insignificant decays or decay chains, the confidence intervals are calculated in the frequentist approach [24] using an asymmetric Gaussian as the branching-fraction probability density function and the measured central value and errors as its parameters. This is the first measurement of $\mathcal{B}(B^0 \rightarrow h_c K_S^0)$. Also, the branching fraction products $\mathcal{B}(B^+ \rightarrow (c\bar{c})K^+) \times \mathcal{B}((c\bar{c}) \rightarrow p\bar{p}\pi^+\pi^-)$ and $\mathcal{B}(B^0 \rightarrow (c\bar{c})K_S^0) \times \mathcal{B}((c\bar{c}) \rightarrow p\bar{p}\pi^+\pi^-)$ are measured directly in B decays for the first time. The current world-average values of the same branching fractions [3] are also presented in Table VII for comparison if they are known. The values of the branching fraction products are calculated by multiplying the individual branching fractions listed in Ref. [3] assuming uncorrelated errors. The measured branching fractions are consistent with the world averages; the largest deviation is observed for the branching-fraction product $\mathcal{B}(B^0 \rightarrow \chi_{c1} K_S^0) \times \mathcal{B}(\chi_{c1} \rightarrow p\bar{p}\pi^+\pi^-)$, which differs from the world-average value by 2.4σ taking its error into account. The results for $\mathcal{B}(B^+ \rightarrow \eta_c K^+) \times \mathcal{B}(\eta_c \rightarrow p\bar{p}\pi^+\pi^-)$ and $\mathcal{B}(B^0 \rightarrow \eta_c K_S^0) \times \mathcal{B}(\eta_c \rightarrow p\bar{p}\pi^+\pi^-)$ have a better precision than the world-average values.

VI. CONCLUSIONS

A search for the decays $B^+ \rightarrow h_c K^+$ and $B^0 \rightarrow h_c K_S^0$ has been performed. Evidence for the decay $B^+ \rightarrow h_c K^+$ is

found; its significance is 4.8σ . No evidence is found for $B^0 \rightarrow h_c K_S^0$. The branching fraction of $B^+ \rightarrow h_c K^+$ is measured to be $(3.7_{-0.9}^{+1.0+0.8}) \times 10^{-5}$; the upper limit for the $B^0 \rightarrow h_c K_S^0$ branching fraction is 1.4×10^{-5} at 90% C.L. The measured value of $\mathcal{B}(B^+ \rightarrow h_c K^+)$ is consistent with the existing upper limit of 3.8×10^{-5} (90% C.L.) obtained in the previous Belle analysis [4] and supersedes it. The resulting branching fraction $\mathcal{B}(B^+ \rightarrow h_c K^+)$ agrees with the existing theoretical predictions [8–10]. In addition, a study of the $p\bar{p}\pi^+\pi^-$ invariant mass distribution in the channel $B^+ \rightarrow (p\bar{p}\pi^+\pi^-)K^+$ results in the first observation of the decay $\eta_c(2S) \rightarrow p\bar{p}\pi^+\pi^-$ with 12.1σ significance.

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