Group: Transmutation

Analytical Modeling of the Emergency Draining Tank for a Molten Salt Reactor

Mattia Massone, Shisheng Wang, Andrei Rineiski

Abstract

The Molten Salt Fast Reactor is a reactor concept developed by the European Union based on a liquid fuel salt circulating through the reactor core. A peculiar emergency system, which takes advantage of the liquid fuel state, is represented by a tank located underneath the core, where the fuel can be passively drained and cooled; its geometry ensures that the fuel remains in subcritical conditions.

In the framework of the SAMOFAR project, a design for the Emergency Draining Tank has been proposed: the tank shall be equipped with vertical cooling elements, arranged in a hexagonal grid; the liquid fuel salt, which heats up due to decay heat, will fill the gaps between the elements.

In this work, analytical methods (Green's functions and orthogonal decomposition) are employed to study the transient heat transfer associated with the proposed design and to perform a preliminary dimensioning of the system, such that overheating is avoided in any moment of the transient and the fuel salt is kept in a liquid state and in safe conditions for a long time. The models are constituted by multilayer monodimensional slabs and cylinders, with a pure heat conduction model. The assessment of the available grace time and preliminary considerations about fuel salt freezing and its influence on the system effectiveness are also included.

Key Words: MSFR, reactor safety, transient heat conduction, emergency draining, Green's function

Introduction

The Molten Salt Reactor concept, originally developed at the Oak Ridge Laboratories [1] is currently under study by many public and private actors. Among them, the European Union is developing the Molten Salt Fast Reactor Concept (MSFR) with the SAMOFAR project [2], in the framework of the HORIZON-2020 program.

The used fuel is a binary salt, composed of lithium and actinides fluorides. At the operating temperature (~725 ºC), the fuel is in liquid state and continuously flows through the core and the intermediate heat exchangers. The geometrical configuration ensures that criticality is achieved only in the core region, where the liquid heats up due to the fission reaction; a fraction of the nominal power (3 GWth) is, however, released outside of the core in form of decay heat [3], [4].

Taking advantage of the liquid state of the fuel, an Emergency Draining System has been included in the design: in case the fuel overheats it can be passively relocated (through freeze plugs) in the Emergency Draining Tank (EDT), where it is kept in safe conditions. The geometry of the tank is specifically designed to keep the fuel salt in subcritical conditions, but the decay heat has to be somehow evacuated to ensure long-term safety.

Gérardin et al. [5] investigated in the past the design of the EDT from the neutronic point of view, to ensure the system subcriticality, and performed preliminary calculations on the heat transfer. The present work aims at studying different options from the heat transfer point of view with the intention to suggest a design solution that assures the safety of the system and meets the EDT specific objectives.

Model Description

Geometry

The proposed EDT is a cavity filled with hexagonal cooling assemblies, arranged in a hexagonal grid. The drained fuel is transferred from the core to the EDT through a funnel-shaped draining shaft [6] ending above the central assembly, which does not have cooling capability but helps achieving an equal distribution of the fuel salt in the tank [4], [5].

Each assembly, as shown in Fig. 1, is constituted by a water pipe surrounded by a layer of an inert material enclosed in a steel casing; the fuel fills the gaps between the assemblies [[4]][[5]].

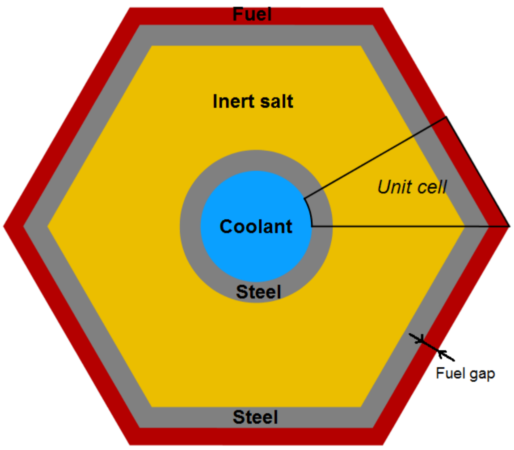

Fig. 1. EDT cooling assembly scheme and model unit cell

This work will consider different options for the inert and fuel salt thickness; the 3-cm-steel layer, instead, will not be changed, as it is considered a needed safety measure to keep a robust structure that prevents the fuel salt from mixing with the inert material. The influence of the steel components on the temperature profile is, however, very limited, due to their high thermal conductivity and low thermal capacity.

It is clearly possible (and advisable) modeling only the unit cell of the assembly, i.e. the rectangular triangle representing 1/12 of the whole assembly and depicted in Fig. 1.

Design Objectives

The EDT aims at ensuring the coolability of the fuel salt that, as a result of the accumulation of fission products and transuranics due to fission, has an internal heat generation due to decay heat; such time-dependent generation is provided by a fitted curve [[4]], based on the reactor isotopic composition.

The design should be capable of preventing the fuel salt from boiling. In addition, as the structural integrity of the tank should be kept, a safety limit should be set to avoid any deterioration of the steel mechanical properties. The maximum admissible temperature is set to 1200°C, which is considered a safe margin before the steel mechanical properties are compromised [4]. Being such value lower than fuel boiling point, above 2000 K [7], the temperature limit is entirely defined by the steel.

The EDT is intended to be used, in principle, only in case of emergency [4]; anyhow, one cannot exclude that spurious discharge can occur or that, in some cases, the system can be employed in other circumstances. In all cases, it is not acceptable that the fuel salt becomes useless once drained; this means that the draining operation should be reversible. Such point requires basically two conditions: the fuel salt in the EDT does not form a homogeneous mixture with other materials; the salt is kept in a liquid state for a long time, so that it can be pumped back into the core as soon as the cause of the draining has been removed.

In order to increase the safety level provided by the design, this should be able to provide additional grace time [5], i.e. the intervention of the operators should not be required immediately to ensure the fuel coolability. The study of the coolant system, or whether it should be operated in natural or forced circulation, is not within the scope of this paper; however it is important to estimate the maximum time allowed before the coolant flow is available, compatible with the compliance with the safety limits of the EDT.

Main Assumptions

Convection

The convective motion is not considered in the model. The establishment of natural circulation cannot be excluded, as the cooling on the steel side combined with the internal heating and symmetry condition on the other side will lead to a temperature gradient. Nevertheless, the space between the two plates is very small and the flow can be disturbed by local fuel solidification and by the initial conditions of the draining; also, the circulation scheme could be more complicated than expected, involving more unit cells in a systemic circulation. However, as it is considered that the natural convection can only improve the heat transfer by favoring remixing of the fluid, the exclusion of the phenomenon can only make the calculation of the system maximum temperature more conservative; on the contrary, for the same reason, the fuel temperature at the interface with the cooling assembly might be higher than calculated. Hence the match with the safety limit will be checked with the maximum system temperature rather than the maximum steel one.

More precisely, with regards to its motion, the fuel salt is considered to be completely at rest, as if it were a solid. This conflicts with the chaotic movement expected by the fuel salt during the draining, which will take some time before it settles down. Anyhow, the advantages provided by the exclusion of the convective motion (namely, the model is more conservative) applies also in this case.

Initial temperature

We hypothesize that the flushing is instantaneous as the reactor stops operating: no fuel energy is lost or gained during the draining and the initial temperature is the average operating temperature in the core, i.e. 725 ºC [4]. Similarly, the cooling assemblies are not preheated, and they are initially at room temperature, assumed to be 20 ºC.

This constitutes an approximation because, actually, the draining can take a few minutes to be completed, and it has an influence on the fuel initial temperature and on the generated heat flux. The initial temperature, in fact should reflect the time required for the freeze plugs of the core to open, the heating of the collector, the energy released through the draining shaft to the surrounding air… Similarly, the decay heat generated starts dropping as soon as the system is not critical anymore, i.e. before the fuel reaches the EDT.

Material properties

The temperature dependence of the material properties is neglected; the introduction of temperature-variable properties, in fact, would change radically the problem characterization, which would become non-linear [8]. The analytical solution of a non-linear problem is much more complicated and not always feasible; in particular, in a non-linear system the superposition of effects, which is at the base of the Green's function approach, cannot be applied. For similar reason, the latent heat could also not be taken into account.

However, as the internal (coolant tube) and external (casing) steel walls are expected to have very different temperatures, their constant properties (related to the same kind of steel) are sampled for the temperature ranges which are most close to the expected conditions (as shown in Table I). Being the dimensioning of the tank still in a preliminary phase, specific steel has not been selected; the 316 stainless steel [[9]] has then been used. This choice has hardly an effect on the results of our study, as the thermal transfer is dominated by the other materials, which have much lower thermal conductivities and higher heat capacity.

The material inside the cooling assembly has the function to conduct the heat to the cooling tube during its functioning and provide thermal inertia to the system during the grace time. As a large amount of heat can be stored as fusion latent heat, the ideal material should be solid at room temperature, but melt at a temperature below the maximum allowable temperature, i.e. 1200 ºC, assumed as safety limit for the steel. Hence, FLiNaK [10] has been chosen.

Analytical Formulation

The basic relation to describe the heat conduction is the diffusion equation [8]

$$(\boldsymbol{x})\nabla^2 T(\boldsymbol{x},t) + \frac{\alpha(x)}{\lambda(x)}\dot{q}(\boldsymbol{x},t) = \frac{\partial T(x,t)}{\partial t}, \quad (1)$$

being α the thermal diffusivity, λ the thermal conductivity and $\dot{q}$ the internal volumetric heat generation rate of the material.

The space dependence of the material properties can be removed by applying the equation separately for each of the M homogenous regions R_i

$$\alpha_i \nabla^2 T_i(\boldsymbol{x},t) + \frac{\alpha_i}{\lambda_i}\dot{q}_i(t) = \frac{\partial T_i(x,t)}{\partial t} \qquad i = 1..M \quad (2)$$

and imposing the temperature and heat flux continuity on the boundary surfaces S_i (perfect thermal contact) as boundary conditions (BCs)

$$T_i(\boldsymbol{x},t) = T_{i+1}(\boldsymbol{x},t) \quad \forall \boldsymbol{x} \in S_i, i \in (0,M) \quad (3)$$

$$\lambda_i \frac{\partial T_i(x,t)}{\partial \boldsymbol{n}} = \lambda_{i+1}\frac{\partial T_{i+1}(x,t)}{\partial \boldsymbol{n}} \quad \forall \boldsymbol{x} \in S_i, i \in (0,M). \quad (4)$$

The problem is closed with the initial conditions defined in §2.3.2 and imposing a symmetry condition on the fuel side and a fixed temperature condition on the water side

$$\frac{\partial T_M(x,t)}{\partial \boldsymbol{n}} = 0 \qquad \forall \boldsymbol{x} \in S_M \quad (5)$$

$$T_1(\boldsymbol{x},t) = T_\infty \qquad \forall \boldsymbol{x} \in S_0. \quad (6)$$

This latter condition implies that the water is considered as a perfect heat sink, kept at constant temperature.

The solution of the presented problem [8] is a convolution of the Green's functions G_{ij}

$$T_i(\boldsymbol{x},t) = \sum_{j=1}^{M}\left[\int_{R_i} G_{ij}(\boldsymbol{x},t|\boldsymbol{\xi},\tau)\Big|_{\tau=0} \cdot \tilde{T}_j(\boldsymbol{\xi})\, d\boldsymbol{\xi} + \frac{\alpha_i}{\lambda_i}\int_0^t \dot{q}_j(\tau)\int_{R_i} G_{ij}(\boldsymbol{x},t|\boldsymbol{\xi},\tau)\, d\boldsymbol{\xi}\, d\tau\right]. \quad (7)$$

While the first term considers the initial conditions, which can be regarded as an instantaneous impulse, the second one takes into account the effect of the internal heat generation, composed by the contribution of all the impulses from the initial time, appropriately weighted depending on the elapsed time. It is

Table I. Materials thermal properties

	Fuel salt [[7]]	Inert salt [[10]] (FLiNaK)	Steel [9]	
			Coolant tube (~100 ºC)	Casing (~700 ºC)
$\lambda\ (W \cdot m^{-1} \cdot s^{-1})$	1.7	0.905	15.1	24.5
$\alpha\ (m^2 \cdot s^{-1})$	3.70 10^{-7}	2.40 10^{-7}	3.72 10^{-6}	5.39 10^{-6}

important noticing that the Green's function depends on two regions: the function represents the way the region *j* affects the temperature distribution in *i*.

Assessment of the Mathematical Model

The height of each cooling assembly is much larger (in the order of meters) than the transversal dimensions (dm): hence the reduction from a 3D to a 2D geometry is straightforward and has a negligible effect on the results accuracy, at least as far as convection is excluded.

The 2D section described in §2.1, instead can be easily approximated by a multilayer trapezoid but, even if the X direction is larger than the Y one, the difference is not large enough to assume the discrepancy between 2D and 1D models would be negligible. This poses an issue from the mathematical point of view: in fact, none of the 11 orthogonal coordinate systems listed by Morse and Feshbach [11] as separable for the Helmholtz equation can be used to represent the trapezoid-like geometry. This means that the eigenfunctions of the Helmholtz equation, i.e. the homogeneous heat conduction problem in space, are not a product of functions depending each one on a single coordinate. In other words, the separation of the space coordinates is not a viable path, and eigenfunctions depending on both coordinates at the same time should be looked for.

The problem of eigenfunctions in non-separable space domains is an extremely interesting topic, still discussed in applied mathematics [12], [13], but the presentation and application of these methods goes beyond the scope of this paper, aiming at a preliminary dimensioning of the EDT elements. Hence, we will approximate the system as either a slab or a cylinder in the X direction (i.e. on the vertical sections of the assembly passing through the symmetry center). The discrepancy with the actual 2D temperature profile is expected to be very small in the vicinity of the hexagon apothem and increase while approaching the hexagon circumradius.

The Green's Function

The separation of variables can anyhow be applied to the space and time parameters and the solution, once applied the initial and BCs, yields the Green's function [8] to be used in (7)

$$G_{ij}(x,t|\xi,\tau) = \frac{\lambda_j}{\alpha_j}\sum_{n=1}^{\infty}\left\{\begin{array}{c}\left[\sum_{j=1}^{M}\frac{\lambda_j}{\alpha_j}\int_{x_{j-1}}^{x_j}\psi_{j,n}^2(\xi)\,d\xi\right]^{-1} \\ e^{-\beta_n^2(t-\tau)}\psi_{i,n}(x)\psi_{j,n}(\xi)\end{array}\right\} \tag{8}$$

The normalization integral depends on the adopted geometry, and includes an additional *x* term for the cylindrical geometry. In (8) appear the eigenfunctions ψ_n, which are solutions of the eigenvalue problem associated to the homogeneous diffusion equation (1) and the BCs: the shape of the solutions depends on the system geometry

$$\psi_{i,n}^{\text{slab}}(x) = A_{i,n}\sin\left(\frac{\beta_n}{\sqrt{\alpha_i}}x\right) + C_{i,n}\cos\left(\frac{\beta_n}{\sqrt{\alpha_i}}x\right) \tag{9}$$

$$\psi_{i,n}^{\text{cylinder}}(x) = A_{i,n}\,\mathrm{J}_0\left(\frac{\beta_n}{\sqrt{\alpha_i}}x\right) + C_{i,n}\,\mathrm{Y}_0\left(\frac{\beta_n}{\sqrt{\alpha_i}}x\right), \tag{10}$$

while the values of the coefficients A_i, C_i and of the eigenvalues $\boldsymbol{\beta_n}$ are determined using the BCs.

The Eigenvalues

By imposing the BCs (3)-(6) one obtains a system of $2M$ equations and $2M$ unknowns ($A_{i,n}$ and $C_{i,n}$); in order to exclude the trivial solution and get the needed eigenvalues and eigenvectors, an arbitrary unknown is set to a non-zero value. The eigenvalues of the problem are those for which the determinant of the associated matrix is null [8].

Fig. 2 shows the value of the associated matrix determinant depending on β_n; the zeros of the functions can be found using a numerical method (e.g. the bisection method). In principle, for slab geometry the function is periodic, as it is a composition of periodic functions, but the period is often so large that it includes more β_n than actually needed to have a good representation; hence, its determination has no practical use.

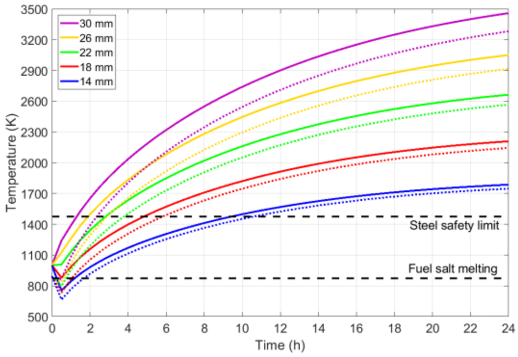

Fig. 3. Maximum (solid line) and minimum (dotted line) fuel temperature along the transient depending on the fuel layer thickness.

Results

The procedure described above has been coded into the MATLAB software. The following results are calculated using such software and truncate the series in (8) after the first 1000 β_n.

Fuel layer

As a first step in the assessment of the EDT element dimension, we test the effectiveness of the system in the transient depending on the thickness of the fuel layer. In order to have the highest thermal inertia, the initial thickness of the inert salt is set to 15 cm, and will be later reduced. As a first approximation, a slab geometry is considered.

As shown in Fig. 3 the thickness of the fuel salt has a huge impact on the maximum temperature of the system; in fact, the larger the amount of fuel, the larger the heat generated in the system that has to be removed by each cooling assembly. With all tested fuel gaps the temperature keeps increasing even after one day; in addition, none of the systems is able to stay below the steel safety limit. However, it is not considered a valid option reducing the gap below 28 mm due to the high risk of blockage in the draining procedure. We will then proceed with such fuel gap, reducing the inert salt thickness instead.

Inert salt layer

In order to comply with the limit, we consider reducing the inert salt thickness, which hinders

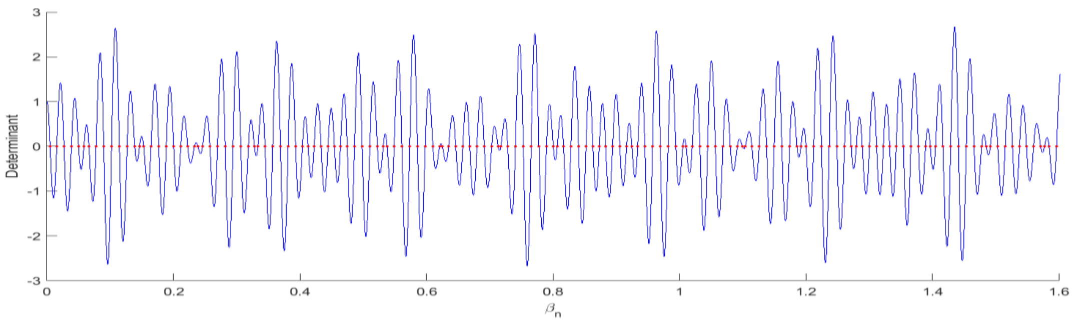

Fig. 2. Eigenvalue search for a slab case.

the heat transfer due to its low thermal conductivity, especially in the later phases of the transient, when the thermal capacity is saturated.

One can see in Fig. 4 that the temperature trend of the different options start to diverge after a couple of hours, i.e. when the thermal capacity of the 7 cm inert salt layer approaches saturation; in a temperature profile, like Fig. 5, a measure of the remaining available heat capacity is given by the curvature of the profile (its second derivative): for a non-heat-generating material, a positive curvature indicates that some energy can still be stored, while a negative one denotes an excess of energy stored, e.g. due to the reduction of the temperature of the heated side.

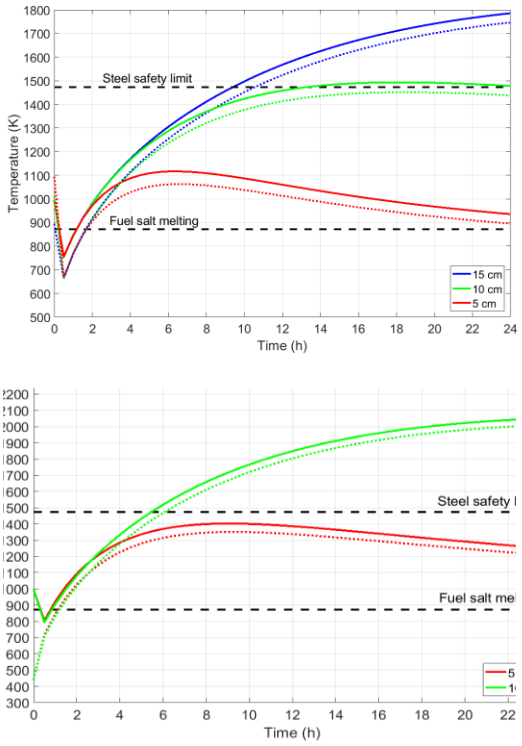

Fig. 4. Maximum (solid line) and minimum (dotted line) fuel temperature along the transient depending on the inert salt layer thickness (top: slab; bottom: cylindrical geometry).

The two figures show also that with the 5 cm configuration, a maximum temperature is achieved after 4-8 h (depending on the geometry); after this moment, the heat generated is lower than the energy removed by the cooling. As such maximum temperature is below the steel safety limit, we can conclude that the system is able to keep the fuel in a long-lasting safe condition.

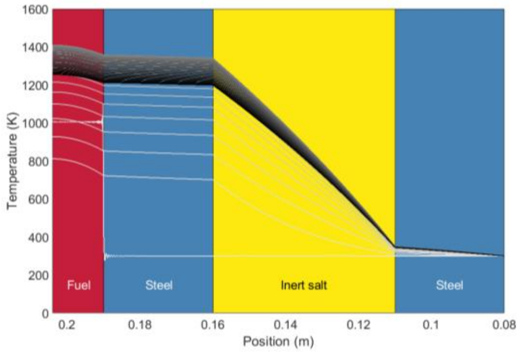

Fig. 5. Temperature profile (cylinder) of the 14/50 mm EDT element between 0 and 24 h, with intervals of 30 minutes (lines color is darker as time progresses).

Grace time

The large thermal inertia provided by the FLiNaK layer allows using the EDT as a completely passive system for some time before switching on the coolant flow.

Fig. 6 shows the effect of the delayed activation of the cooling on the maximum temperature of the fuel salt, i.e. the main safety issue considered in this paper. It is clear that a delay of up to 2 hours has almost no influence on the fuel temperature, and one can also wait for 3 hours with no concern on the temperature limits. In case the grace time is extended to 4 hours, instead, the temperature increases enough to slightly exceed the steel safety limit.

In all cases, the plot shows that there is about 30 minutes delay between the cooling activation and the first visible effect on the fuel maximum temperature.

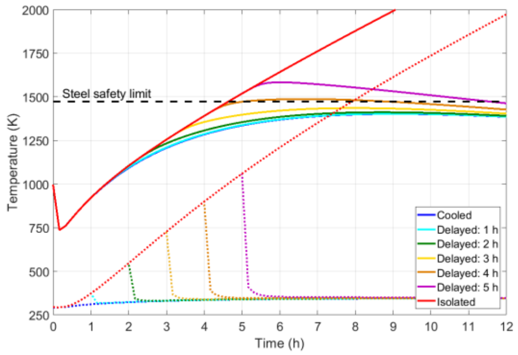

Fig. 6. Fuel maximum temperature (solid lines) and inert salt minimum temperature (dotted line) along the transient.

Fuel salt solidification

The assessment of the solidification behavior of the frozen salt represents an important matter for the EDT design. In the draining phase, a quick solidification should be avoided, as it could block the gaps and create an obstacle to a uniform and complete discharge. In addition, the fuel salt liquid state is required to ensure reversibility of the draining operation.

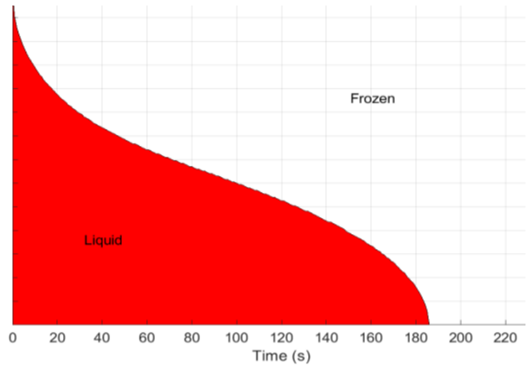

Fig. 7. Available fuel gap for salt flow in the first minutes after draining.

The freezing of part of the fuel on the cooling assembly walls cannot be completely avoided if the EDT is initially at room temperature. Fig. 7 one shows that the fuel temperature drops below the freezing point (872 K) after 3 minutes; hence, if the draining were completed within such time, the salt could be distributed uniformly in the tank without any blockage risks. On the contrary, if more time were required, a detailed transient 2D analysis, including also the chance of stratification of the solid fuel in the Z-direction, would be needed to assess the expected distribution of the salt and be sure that the draining can be completed safely. Actually, the time required to freeze the inter-assembly gap is longer, as the simulation does not take into account the effect of the latent heat, that should be removed too before freezing actually occurs

Reversibility is guaranteed by the fact that complete remelting of the fuel salt, due to the decay heat, is achieved within 1 h 30’, as shown in Fig. 4. It has been already shown that the maximum fuel temperature reaches a peak between 8 and 12 h after the discharge. Once the generated heat is lower than what the coolant is capable of removing, an adequate handling of the coolant flow allows to keep under control the energy evacuated and so the temperature of fuel, which hence can be kept above the melting point for a long time. This ensures the reversibility of the system in the long term.

However, it should be taken into account that, due to the solidification/remelting behavior, any draining process forces the reactor to stop for a couple of hours, being such time needed for the fuel salt to melt again. Also, one should consider the effect of the thermal expansion in freezing/melting processes to exclude the possibility of structural damage.

The solidification issue is one of the reasons why using more conductive materials (e.g. lead) is not advised: the heat transfer through the layers, in fact, is so fast that a remelting of the fuel salt could not be guaranteed. In addition, lead has a thermal inertia much lower than the FLiNaK, hence making difficult providing grace time, as discussed in §4.3.

Conclusions

This study aims at analyzing the performance of the emergency draining tank of the molten salt reactor based on the temperature profiles calculated analytically using a Green's functions and orthogonal decomposition in a 1D geometry. The analytical approach proved to be very suitable for this kind of study, as it can be easily applied to simplified geometries and provide results which are accurate enough but can be obtained in very short time and with limited computational resources. It is considered of interest for future studies the extension into a 2D geometry, employing the state-of-the-art knowledge regarding the computation of non-separable spaces eigenfunctions.

The results show that it is possible to find a layout of the proposed tank design that is able to keep the drained fuel temperature below 1200 °C, representing the structural integrity limit. The same layout allows a grace time up to 3 hours after the draining, during which no cooling at all is required. Once passed a few hours (between 8 and 12) from the draining, the generated heat is lower than the evacuated heat, assuring the safety of the design; from this point on, a careful tuning of the removed heat flux can be used to keep the fuel salt in a liquid state for a very long time, ensuring the reversibility of the draining operation. It is however not possible to avoid the solidification of the fuel during the draining in a containment which is kept at room temperature; fuel gaps take at least 3 minutes to become completely blocked, but this time does not take into account the effect of the latent heat, which should instead be considered in future studies. The solidified fuel, however, melts again completely within 90 minutes; this however means that any discharge results in a reactor stop, to allow the fuel to melt again.

The proposed design proved to be able to comply with the objectives required in very conservative conditions. Future analyses should start from this point, focusing on removing the assumptions made for this analytical modeling (e.g. by including the temperature-dependence of the material properties and assessing the convective motion) and so refining the design.

Acknowledgements

The authors thank the anonymous reviewer for his/her accurate review and insightful comments.

The authors would like to acknowledge the financial support by the Euratom research and training programme 2014-2018, under the Grant Agreement Number 661891 – SAMOFAR.

Partners

We thank Paul Servell, Grenoble INP – Phelma, for his contribution to EDT studies, including code development and preliminary calculations.

References

[1] Nuttin et al.; “Potential of thorium molten salt reactors: detailed calculations and concept evolution with a view to large scale energy production”, Progress in Nuclear Energy, 46, pp. 77-99 (2005).

[2] “A Paradigm Shift in Reactor Safety with the Molten Salt Fast Reactor”, https://cordis.europa.eu/project/rcn/196909_en.html (2017).

[3] Wang, S., et al.: “A Passive Decay Heat Removal System for Emergency Draining Tanks of Molten Salt Reactors”, *Proceedings of the Int. Topical Meeting NURETH-17*, Xi'an (China), 3-8 September 2017.

[4] Allibert, M., et al.: “D1.1 Description of initial reference design and identification of safety aspects”, *A Paradigm Shift in Nuclear*

Reactor Safety with the Molten Salt Fast Reactor (grant agreement number: 661891) – SAMOFAR (2017).

[5] Gérardin, D., et al.; "Design Evolutions of the Molten Salt Fast Reactor", *Proceedings of the Int. Conf. FR17*, Yekaterinburg (Russian Federation), 26-29 June 2017.

[6] Wang, S., et al.; "Analytical Investigation of the Draining System for a Molten Salt Fast Reactor", *Proceedings of the Int. Topical Meeting NUTHOS-11*, Gyeongju (Korea), 9-13 October 2016.

[7] Beneš, O., et al.; "D3.2 Physico-Chemical properties of the MSFR fuel salt", *Evaluation and Viability of Liquid Fuel Fast Reactor System (grant agreement number: 249696) – EVOL* (2013).

[8] Necati Özişik, M.; *Heat Conduction – Second Edition*, Wiley, New York, USA (1993).

[9] Waltar, V.; Reynolds, A.; *Fast Breeder Reactors*, Pergamon Press, New York, USA (1980).

[10] Beneš, O.; Konings, R.J.M.; "Thermodynamic properties and phase diagrams of fluoride salts for nuclear applications", *Journal of Fluorine Chemistry*, **130**, pp. 22-29 (2009).

[11] Morse, P. M.; Feshbach, H.; *Methods of Theoretical Physics, Part I*, McGraw-Hill, New York, USA (1953).

[12] Betcke, T.; Trefethen, L. ; "Reviving the Method of Particular Solutions", *SIAM Rev.*, **47(3)**, pp 469-491 (2005).

[13] Descloux, J.; Tolley, M.; "An accurate algorithm for computing the eigenvalues of a polygonal membrane", *Comput. Methods Appl. Mech. Engrg.*, **39**, pp. 37-53 (1983)