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PERTURBATION OF SYNCHROTRON MOTION IN THE MICRO-BUNCHING INSTABILITY

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Abstract

Short electron bunches in a storage ring are subject to complex longitudinal dynamics due to self-interaction with their own CSR. Above a particular threshold current, this leads to the formation of dynamically changing micro-structures within the bunch, generally known as the micro-bunching instability. The longitudinal dynamics of this phenomenon can be simulated by solving the Vlasov-Fokker-Planck equation, where the CSR self-interaction can be added as a perturbation to the Hamiltonian. This contribution particularly focuses on the comprehension of synchrotron motion in the micro-bunching instability and how it relates to the formation of the occurring micro-structures. Therefore, we adopt the perspective of a single particle and comment on its implications for collective motion. We explicitly show how the shape of the parallel plates CSR wake potential breaks homogeneity in longitudinal phase space and propose a quadrupole-like mode as potential seeding mechanism of the micro-bunching instability. The gained insights are verified using the passive particle tracking method of the Vlasov-Fokker-Planck solver Inovesa.

INTRODUCTION

In order to increase the emission of coherent radiation, modern synchrotron light sources are deliberately operating with short electron bunches. The KIT storage ring KARA thus has a dedicated short-bunch mode providing picosecond-long bunches that result in the emission of coherent synchrotron radiation (CSR) up to the THz frequency range. Yet, due to self-interaction with its own radiation field, the increased CSR strength also leads to complex longitudinal dynamics within the electron bunch. At low bunch currents, the resulting potential well distortion mainly causes a slight deformation of the still fairly stationary electron distribution. However, above a particular threshold current I_{th} depending on the specific machine settings of the accelerator, it leads to the formation of dynamically changing micro-structures within the bunch. As the longitudinal charge distribution varies over time, this in turn results in major fluctuations of the emitted CSR power and is thus called micro-bunching or micro-wave instability. The underlying longitudinal dynamics can be simulated to high qualitative agreement by numerically solving the Vlasov-Fokker-Planck equation (VFP) [1–4].

In this contribution, we explicitly focus on the understanding of synchrotron motion below the instability threshold. To that end, we first consider single particle motion in the absence of collective effects, where the system can be modeled as a simple one-dimensional harmonic oscillator. By introducing CSR self-interaction (considering the wake potential of the entire bunch) as a perturbation, the dynamics below the threshold I_{th} can easily be illustrated in the single particle picture. Furthermore, we show how the specific shape of the CSR wake potential breaks homogeneity in longitudinal phase space and propose a quadrupole-like mode to initially drive the micro-bunching instability.

VFP EQUATION

The longitudinal dynamics of an electron bunch in a storage ring are conveniently described in the phase space spanned by the longitudinal position z and particle energy E. By introducing the generalized coordinates $q \doteq (z-z_{\mathrm{s}})/\sigma_{z,0}$ and $p \doteq (E-E_{\mathrm{s}})/\sigma_{E,0}$, the resulting phase space is dimensionless and its origin marks the synchronous particle. Here, z_{s} and E_{s} denote position and energy of the synchronous particle, $\sigma_{z,0}$ the natural bunch length and $\sigma_{E,0}$ the natural energy spread. The temporal evolution of the electron distribution $\psi(q,p,t)$ in the longitudinal phase space can be described by the Vlasov-Fokker-Planck equation (following the notation in [1])

$$\frac{\partial\psi}{\partial\theta} + \frac{\partial\mathcal{H}}{\partial p}\frac{\partial\psi}{\partial q} - \frac{\partial\mathcal{H}}{\partial q}\frac{\partial\psi}{\partial p} = \frac{1}{f_{\mathrm{s},0}\tau_{\mathrm{d}}}\frac{\partial}{\partial p}\left(p\psi + \frac{\partial\psi}{\partial p}\right), \quad (1)$$

with the time given in multiples of nominal synchrotron periods $\theta = f_{\mathrm{s},0}\,t$, the Hamiltonian $\mathcal{H}$ and the damping time τ_{d}. The inhomogeneous part on the right hand side describes the influence of radiation damping and diffusion. In the absence of collective effects and assuming linear accelerating voltage V_{RF} and linear momentum compaction factor α_{c}, the Hamiltonian is given as

$$\mathcal{H}_0(q,p,t) = \frac{1}{2}\left(q^2+p^2\right) . \quad (2)$$

The unperturbed system is thus a one-dimensional harmonic oscillator. Collective effects such as CSR self-interaction can be included as a perturbation to the Hamiltonian

$$\mathcal{H}_{\mathrm{c}}(q,p,t) = \int_q^\infty Q_{\mathrm{c}}\,V_{\mathrm{c}}(q',t)\mathrm{d}q' , \quad (3)$$

where Q_{c} denotes the charge involved in the perturbation and $V_{\mathrm{c}}(q,t)$ is the potential due to collective effects. In order to calculate the CSR-induced wake potential, it is useful to express the potential in terms of an impedance $Z_{\mathrm{CSR}}(\omega)$

$$V_{\mathrm{CSR}}(q,t) = \int_{-\infty}^{\infty}\widetilde{\rho}(\omega,t)Z_{\mathrm{CSR}}(\omega)e^{i\omega q}\mathrm{d}\omega , \quad (4)$$

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where $\widetilde{\rho}(\omega)$ denotes the Fourier-transformed longitudinal bunch profile. In general, the exact impedance of a storage ring is not known. However, in the case of CSR-driven dynamics, the approximation of modeling the shielding effect of the beam pipe by two parallel plates has proven to yield quite reasonable results, e.g. [4]. The full Hamiltonian is finally given by

$$\mathcal{H}(q,p,t) = \mathcal{H}_0(q,p,t) + \mathcal{H}_c(q,p,t)\,. \tag{5}$$

SINGLE PARTICLE MOTION

Neglecting radiation damping and diffusion, we now consider single particle motion.

One-dimensional Harmonic Oscillator

In the absence of collective effects, the Hamiltonian in Eq. (5) takes the form of a simple one-dimensional harmonic oscillator. Classically, such systems are defined by their linear restoring force

$$F = -kx\,, \tag{6}$$

which leads to the well known equation of motion

$$m\ddot{x} + kx = 0\,, \tag{7}$$

and their solution

$$x(t) = a_0 \cos(\omega t + \varphi_0) \tag{8}$$

$$\dot{x}(t) = -a_0\,\omega \sin(\omega t + \varphi_0)\,, \tag{9}$$

with $\omega = \sqrt{k/m}$, the amplitude a_0 and the initial phase φ_0. By choosing the generalized coordinates

$$q \doteq \sqrt{m}\omega x \quad \text{and} \quad p \doteq \sqrt{m}\dot{x}\,, \tag{10}$$

the Hamiltonian is equal to Eq. (2) and motion in the corresponding phase space is perfectly circular. This corresponds to the longitudinal motion of electrons in the absence of collective effects, where the RF potential acts as a linear restoring force. By introducing a small perturbation

$$k' = k - \varepsilon \quad \text{with} \quad \varepsilon > 0\,, \tag{11}$$

the system remains a harmonic oscillator, but with the altered solution

$$x'(t) = a_0 \cos(\omega' t + \varphi_0) \tag{12}$$

$$\dot{x}'(t) = -a_0\,\omega' \sin(\omega' t + \varphi_0)\,, \tag{13}$$

with $\omega' = \sqrt{k'/m} = \sqrt{(k-\varepsilon)/m}$. Particle motion in the phase space spanned by the original q and p is thus elliptical, as illustrated in Fig. 1, and of altered periodicity.

Perturbation of the RF Potential

Given the parallel plates impedance Z_{CSR} (see [5]), the wake potential of a Gaussian bunch profile takes the form depicted in the upper part of Fig. 2. While such a perfectly

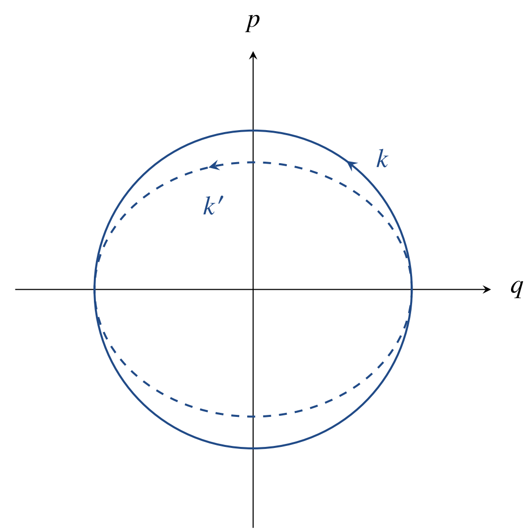

Figure 1: A small perturbation of the restoring force leads to an elliptical particle trajectory in the phase space spanned by the generalized coordinates q and p.

Gaussian electron distribution exists in the zero current limit, higher bunch current leads to an increased perturbation strength and thus distortion of the Gaussian shape. Yet, below the threshold current, the distribution still remains fairly stationary $\psi(q,p,t) \approx \psi(q,p)$, which corresponds to a stationary wake potential as shown in the lower part of Fig. 2 for a range of different bunch currents. It should be noted, the general shape of the wake potential is very similar to that of a Gaussian shaped bunch up until right below the threshold current of $I_{\mathrm{th}} = 260\,\mu\mathrm{A}$, where the wake potential is no longer stationary.

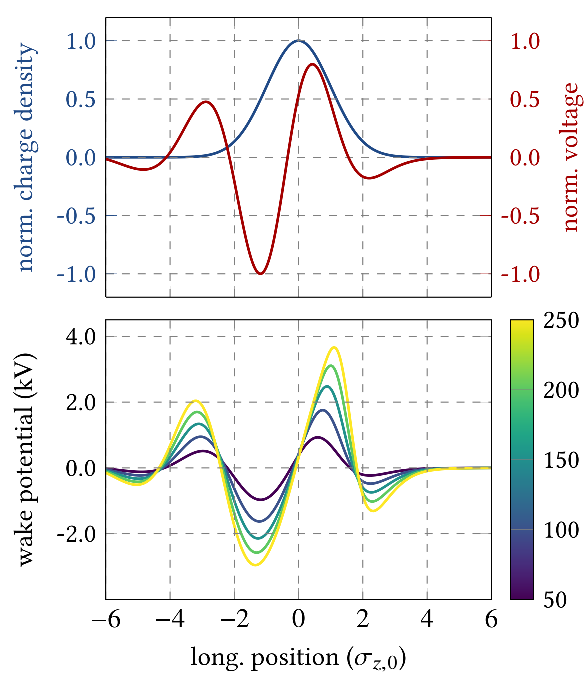

Figure 2: CSR wake potential for a Gaussian bunch profile (red and blue, top) and for $I = (50, 100, 150, 200, 250)\,\mu\mathrm{A}$ below the instability threshold of $I_{\mathrm{th}} = 260\,\mu\mathrm{A}$ (bottom).

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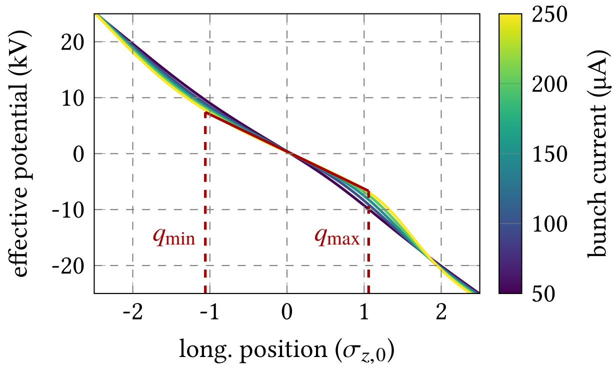

Figure 3: Linear approximation (solid red line) of the effective potential below the threshold current I_{th}.

In order to investigate its effects on single particle motion, we introduce the effective potential

$$V_{eff}(q) = V_{RF}(q) + V_{CSR}(q)\,, \tag{14}$$

combining the linear RF potential $V_{RF}(q)$ and the CSR wake potential $V_{CSR}(q)$ (average over the ring). A single particle moving in phase space is now subject to the effective potential $V_{eff}(q)$ over the interval $[q_{min}, q_{max}]$, where q_{min} and q_{max} denote the maximum deviations from the longitudinal position of the synchronous particle. By approximating $V_{eff}(q)$ as a linear function on the given interval

$$V_{eff}(q) \approx -k'\,q\,, \quad q \in [q_{min}, q_{max}]\,, \tag{15}$$

as illustrated in Fig. 3, single particle motion is still harmonic below the threshold current, with the strength of the restoring force k' being dependent on q_{min} and q_{max}. According to equations (11-13), this results in a position-dependent ellipticity of particle trajectories in phase space. This statement can be verified using the passive particle tracking method of Inovesa [6]. As illustrated in Fig. 4, the simulated particle trajectories clearly show the expected position-dependent elliptical shape. After an initial increase, the amplitude difference between the space and energy dimension $q_{max} - p_{max}$ decreases again, indicating a trend towards a more circular shape for larger amplitudes.

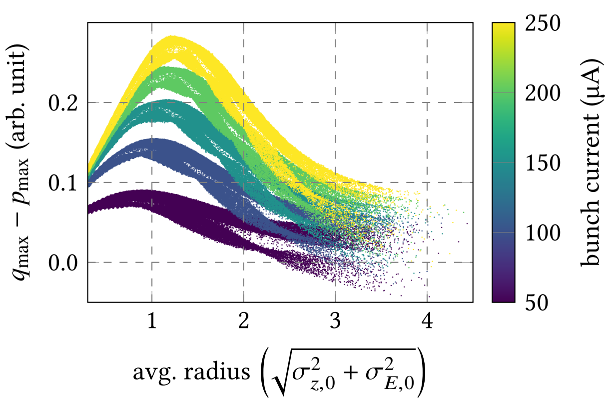

Figure 4: Shown is the amplitude difference of single particle trajectories in phase space for $n = 100\,000$ particles.

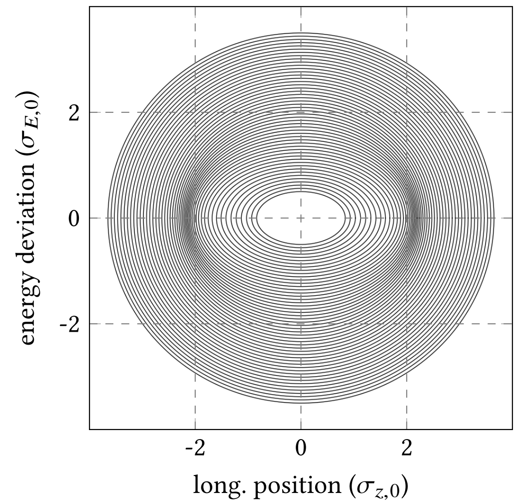

Figure 5: Visualization of the effect of the position-dependent elliptical trajectories in phase space on the charge density. Shown are trajectories for particles with equidistant average radius and an ellipticity comparable to Fig. 4 (effect is magnified for better visibility).

Initial Quadrupole-like Mode

Considering an ensemble of particles, the altered trajectories lead to a concentration of particles at specific locations in the longitudinal phase space, as is illustrated qualitatively in Fig. 5. The CSR-induced perturbation of the RF potential thus breaks the homogeneity in phase space and creates local particle densities that form a quadrupole-like mode. This inhomogeneity may initially seed the formation of micro-structures and thereby kick off the micro-bunching instability. Measurements of such a quadrupole-like deformation of the charge distribution have also been reported in [7] for currents above the instability threshold.

SUMMARY

The CSR self-interaction leads to a perturbation of the single particle synchrotron motion within short electron bunches in a storage ring. As derived above, the particle trajectories take an approximately elliptical form in the longitudinal phase space and are dependent on the particle's deviation from the synchronous position. This leads to a non-uniform concentration of particle trajectories in phase space forming a quadrupole-like mode, which may act as seeding mechanism for the micro-bunching instability.

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