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# Oscillations in a system of two coupled self-regulating spool valves with switching properties

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In hydraulic systems, valves can be considered as fundamental components. They serve as control elements to regulate hydraulic power transmission. In order to minimize control effort, self-regulating spool valves enjoy great popularity. However, their disadvantage is a possible loss of stability, caused by the coupling between hydraulic and mechanical degrees of freedom via pressure feedback areas. So far, the self-excited oscillations, evoked from the operating point's loss of stability, have mostly been investigated using minimal models of individual valves. In real world applications, for example in automotive transmissions, typically several valves are employed which are coupled by hydraulic pipes. Here, it is expected, that dynamical phenomena occur, which cannot be portrayed by simple models of individual valves. Within this contribution, the oscillatory behaviour of a system employing two coupled self-regulating valves is discussed. The resulting non-stationary solutions are characterized by using Floquet theory and computing Lyapunov-Exponents.

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### 1 Introduction

Because of their self-regulating property, spool valves with pressure feedback areas enjoy great popularity for the task of pressure and volume flow regulation in hydraulic circuits. Their working principle either allows for operation with volume flow supply or with pressure supply, whereby in the first configuration the valve regulates or limits line pressure and in the second configuration serves as an actuation or pressure reducing valve, see Fig. 1. The self-regulation relates to the desired pressure level and is realized via the stationary equilibrium between pressure induced force on pressure feedback area and control input, so that the spool automatically positions for the control edge flow across the valve into the reservoir to compensate for excessive power supply. Herewith, the control input sets the desired pressure level, which is then independent of the current power supply and load.

Turbulent orifice flows and non-smooth switching transitions are the dominant non-linearities in hydraulic systems involving spool valves with pressure feedback areas. These promise interesting dynamic behaviour, as it is a well-known phenomenon, that, even in simple systems involving one individual valve, self-excited oscillations arise at certain operating points or in case of unfavourable parametrization, see e.g. [1,2].

#### 2 Physical Model

In order to analyse the dynamical interaction between two coupled valves, the system under investigation employs two structurally different and differently parametrized valves with ideal power supply (constant pump volume flow  $q_P$ ) and ideal consumer (constant load pressure  $p_{C2}$ ), see Fig. 2. The first valve is supplied with volume flow  $q_P$  and at the same time sets – via control input  $f_1$  – a desired line pressure level  $p_1$ . This pressure can be considered as the power supply for the second (actuation) valve, which, via control input  $f_2$ , defines the pressure level  $p_2$  for the actuation of a consumer. The system thus contains both valve configurations presented in Fig. 1. The governing system equations are derived from force balances on valve spools and volume flow balances on capacitances. By neglecting hydraulic inductances, friction, laminar pipe resistance



Fig. 1: Possible valve configurations.

Fig. 2: System under investigation: Two coupled structurally different valves.

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and flow forces they read

$$m_1 \ddot{x}_1 + d_1 \dot{x}_1 + c_1 x_1 = r_1^2 \pi p_1 - f_1, \qquad C_{h1} \dot{p}_1 = q_P - q_{V2_S} - q_{V1} - r_1^2 \pi \dot{x}_1, m_2 \ddot{x}_2 + d_2 \dot{x}_2 + c_2 x_2 = r_2^2 \pi p_2 + f_2, \qquad C_{h2} \dot{p}_2 = q_{V2_S} - q_{C2} - q_{V2_T} - r_2^2 \pi \dot{x}_2$$
(1)

with the turbulent orifice and non-smooth control edge volume flows

$$q_{V1} = \gamma_V 2r_1 \pi \begin{cases} (x_1 - u_1)\sqrt{p_1 - p_0} & \text{if } x_1 \ge u_1 \\ 0 & \text{if } x_1 < u_1 \end{cases}, \quad q_{V2_S} = \gamma_V 2r_2 \pi \begin{cases} (x_2 - u_{S2})\sqrt{p_1 - p_2} & \text{if } x_2 \ge u_{S2} \\ 0 & \text{if } x_2 < u_{S2} \end{cases}$$

$$q_{C2} = \gamma_V A_{C2} \sqrt{p_2 - p_{C2}}, \qquad \qquad q_{V2_T} = \gamma_V 2r_2 \pi \begin{cases} 0 & \text{if } x_2 > u_{T2} \\ (u_{T2} - x_2) \sqrt{p_2 - p_0} & \text{if } x_2 \le u_{T2} \end{cases}$$

Parameter  $\gamma_V$  represents the turbulent flow discharge coefficient,  $C_{h1}$  and  $C_{h2}$  describe the hydraulic capacitances and  $u_1$ ,  $u_{S2}$  and  $u_{T2}$ ,  $u_{S2} > u_{T2}$  are the corresponding valve overlaps representing the three different switching borders of the system (leading to six different switching modes), where the system equations (1) are continuous, but not continuously differentiable.

#### **3** Investigation of non-stationary solutions

From Fig. 2 it follows intuitively, that the control edge flow  $q_{V2_S}$  and therefore radius  $r_2$  determines the coupling strength between the two valves. Thus, in order to analyse the dynamical interaction between the two valves, a parameter study is performed by varying radius  $r_2$ . The system equations (1) are numerically integrated by using event detection of the different switching borders. The results in the upper part of Fig. 3 represent the crossings of the Poincaré section  $\Sigma = \{\dot{x}_1 = 0, \, \dot{x}_1 < 0\}$  and shows the corresponding spool position  $x_1$  of the pressure regulation valve. The lower part of Fig. 3 shows the associated four largest Lyapunov-Exponents  $\Lambda$ .

As observable in the bifurcation diagram, a period-1 limit cycle oscillation exists for weak coupling, whereas in the case of medium and strong coupling higher period limit cycle oscillations coexist with quasi-periodic and chaotic oscillations, indicated by two zero- respectively one positive Lyapunov-Exponent. By applying a path following algorithm and computing the corresponding Floquet-Multipliers [3], the existence and stability (green: stable, red: unstable) of the period-1 limit cycle is investigated indicating a torus-bifurcation at  $r_2 \approx 1.75$  mm, which confirms the quasi-periodicity of the solutions obtained right after this bifurcation point. A stability analysis of the two individual valves reveals that the torus bifurcation point coincides with the point on which the equilibrium position of the actuation valve loses stability,



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Fig. 3: Bifurcation diagram and corresponding four largest Lyapunov-Exponents.

whereas the equilibrium position of the line pressure regulation valve is always unstable in the considered parameter range.

## 4 Conclusions

In order to investigate the dynamical interaction of self-excited oscillations between different valves in a hydraulic system, a model employing two different valves has been presented. The non-stationary solutions evoked from the self-excitation mechanism of the valves have been analysed by calculating Lyapunov-Exponents and by using Floquet theory. As anticipated, if both valves are driven in an unstable operation mode, complex oscillatory behaviour is observed.

#### References

- M. A. Köster, and A. Fidlin, Nonlinear Dynamics of a Hydraulic Pressure Control Valve, Proceedings of 11th Conference on Vibration Problems, Lisbon, Portugal (2013)
- [2] H.E. Merrit, Hydraulic Control Systems (John Wiley & Sons, Inc., New York, 1967)
- [3] A.H. Nayfeh, and B. Balachandran, Applied Nonlinear Dynamics (WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim, 2004)