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PolyChaos.jl – An open source Julia package for polynomial chaos expansion

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github.com/timueh/PolyChaos.jl

Abstract

Polynomial chaos expansion (PCE) is a Hilbert space technique for random variables that alleviates uncertainty propagation. Random variables are expanded in terms of polynomials that are orthogonal relative to a given probability density function. The applicability of PCE hinges on software that allows, among others, to construct orthogonal polynomials. We offer a package for (intrusive) PCE written in the Julia programming language, a trending programming language dedicated to scientific computing.

1. Polynomial Chaos

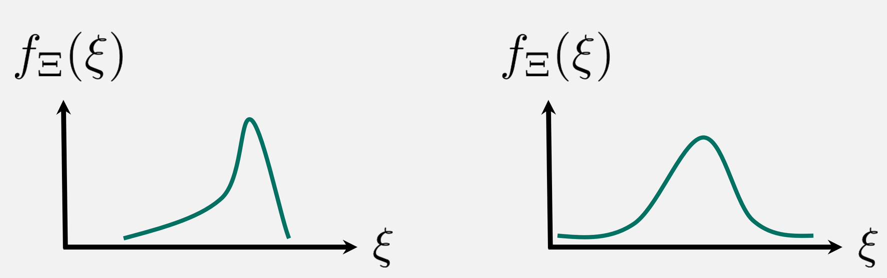
Hilbert space method for random variables.

Orthogonal basis	$\{\phi_k(\xi)\}_{k=0}^{\infty}$ with deg $\phi_k(\xi)=k$
Hilbert space	$\mathsf{L}^2(\mathbb{R}) \equiv \mathsf{L}^2(\Omega, \mu; \mathbb{R})$ with $(\Omega, \mathcal{F}, \mu)$
Scalar product	$\langle \mathsf{x}, \mathsf{y} \rangle_{\mathsf{L}^2} = \mathsf{Cov}[\mathsf{x}, \mathsf{y}]$
Norm	$\Vert \mathsf{x} \Vert_{\mathsf{L}^2} = \sqrt{\langle \mathsf{x}, \mathsf{x} \rangle_{\mathsf{L}^2}}$
Expansion	$\mathsf{X} = X(\xi) = \sum_{l=0}^{\infty} x_l \phi_l(\xi)$
Truncation	$\tilde{\mathsf{x}} = \tilde{x}(\xi) = \sum_{l=0}^{L} x_l \phi_l(\xi)$
Coefficients	$x_l = \dfrac{\langle \mathsf{x}, \phi_l(\xi) \rangle_{\mathsf{L}^2}}{\langle \phi_l(\xi), \phi_l(\xi) \rangle_{\mathsf{L}^2}}$
Optimality	$\Vert \mathsf{x} - \tilde{\mathsf{x}} \Vert_{\mathsf{L}^2} = \min_{\mathsf{y} \in \mathfrak{Y}} \Vert \mathsf{x} - \mathsf{y} \Vert_{\mathsf{L}^2}$ $\mathfrak{Y} = \mathsf{span}\{\phi_l(\xi)\}_{l=0}^{L}$

Several well-known analytic bases (Askey).

Distribution	Polynomial basis
Normal	Hermite
Uniform	Legendre
Beta	Jacobi
Gamma	Laguerre

→ Arbitrary densities?



2. Desired Features

- Compute orthogonal polynomials for arbitrary densities
- Provide scalar products for intrusive PCE

$$\langle \phi_{i_1}, \phi_{i_2} \cdots \phi_{i_m} \rangle$$

- Multivariate support
- Comprehensible documentation

3. Existing Software

Name	Features
UQLab	– Matlab – BSD 3-clause license – Classic and arbitrary distributions – Stieltjes procedure – Gauss and sparse quadrature – Basis-adaptive sparse PCE – Least-angle regression
Chaospy	– Python – MIT license – Classic and arbitrary distributions – Gram-Schmidt – Stieltjes procedure – Gauss quadrature – Clenshaw-Curtis
MUQ	– C++, Python – Classic distributions – Gauss quadrature
UQToolkit	– C++, Python – GNU LGP license – Classic distributions – Gauss quadrature

4. Julia

- "Walks like Python, runs like C"
- Solves the two-language problem
- Easy syntax
- Multiple dispatch
- Dynamically-typed
- Metaprogramming
- Package management
- Open source
- Unicode support

julia

5. Details

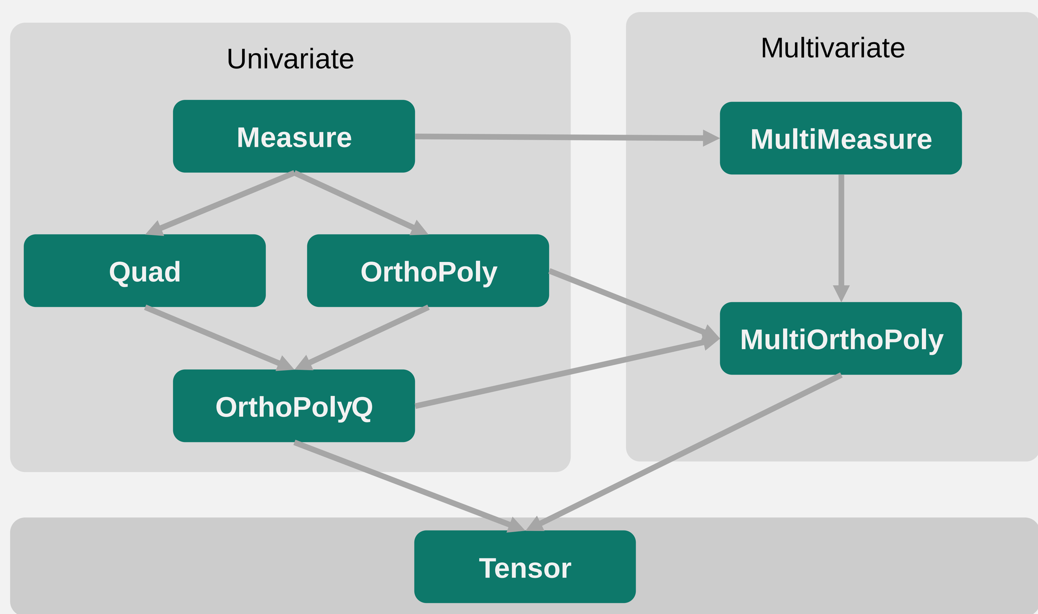
Given an absolutely continuous nonnegative measure, PolyChaos.jl allows

- To compute the coefficients for the monic three-term recurrence relation
- To evaluate orthogonal polynomials at arbitrary points
- To compute the quadrature rule
- To compute tensors of scalar products
- To do all of the above in a multivariate setting

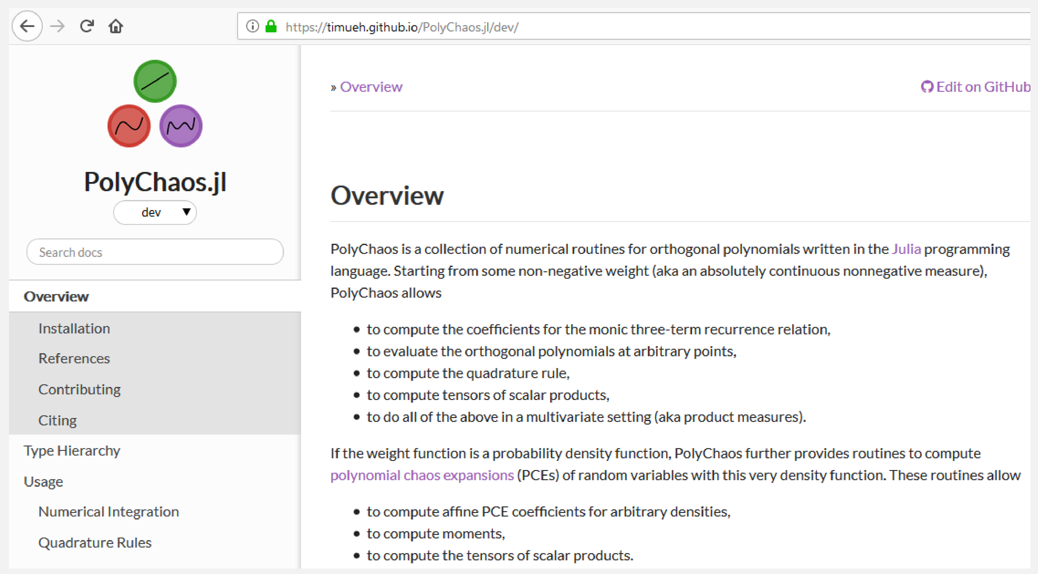
Methods

- Stieltjes procedure
- Lanczos procedure
- Gauss quadrature (+ Lobatto, Radau)
- Fejér's rules, Clenshaw-Curtis
- Sparse computation of scalar products

Type hierarchy



Documentation & Examples



Contributors welcome

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