PolyChaos.jl – An open source Julia package for polynomial chaos expansion

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Abstract
Polynomial chaos expansion (PCE) is a Hilbert space technique for random variables that alleviates uncertainty propagation. Random variables are expanded in terms of polynomials that are orthogonal relative to a given probability density function. The applicability of PCE hinges on software that allows, among others, to construct orthogonal polynomials. We offer a package for (intrusive) PCE written in the Julia programming language, a trending programming language dedicated to scientific computing.

1. Polynomial Chaos
Hilbert space method for random variables.

Orthogonal basis \( \{ \phi_k(\xi) \}_{k=0}^{\infty} \) with deg \( \phi_k(\xi) = k \)

Hilbert space \( L^2(\Omega) = L^2(\Omega, \mu, \mathcal{F}, \mathbb{P}) \) with \( (\Omega, \mathcal{F}, \mathbb{P}) \)

Scalar product \( (x, y)_{L^2} = \text{Cov}[x, y] \)

Norm \( \|x\|_{L^2} = \sqrt{(x, x)_{L^2}} \)

Expansion \( X = X(\xi) = \sum_{k=0}^{\infty} x_k \phi_k(\xi) \)

Truncation \( \tilde{x} = \tilde{x}(\xi) = \sum_{k=0}^{\tilde{N}} x_k \phi_k(\xi) \)

Coefficients \( x_k = \frac{(x, \phi_k)_{L^2}}{(\phi_k, \phi_k)_{L^2}} \)

Optimality \( \| x - \tilde{x} \|_{L^2} = \min_{\tilde{x}} \| x - \tilde{x} \|_{L^2} \) \( \mathfrak{g} = \text{span}\{\phi_k(\xi)\}_{k=0}^{\infty} \)

Several well-known analytic bases (Askey).

Distribution | Polynomial basis
--- | ---
Normal | Hermite
Uniform | Legendre
Beta | Jacobi
Gamma | Laguerre

→ Arbitrary densities?

2. Desired Features

- Compute orthogonal polynomials for arbitrary densities
- Provide scalar products for intrusive PCE \( \langle \phi_1, \phi_2, \ldots, \phi_m \rangle \)
- Multivariate support
- Comprehensible documentation

3. Existing Software

<table>
<thead>
<tr>
<th>Name</th>
<th>Features</th>
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| UQLab | - Matlab  
- BSD 3-clause license  
- Classic and arbitrary distributions  
- Stieltjes procedure  
- Gauss and sparse quadrature  
- Basis-adaptive sparse PCE  
- Least-angle regression |
| Chaospy | - Python  
- MIT license  
- Classic and arbitrary distributions  
- Gram-Schmidt  
- Clenshaw-Curtis  
- Fejér's rules  |
| MUQ | - C++, Python  
- Classic distributions  
- Gauss quadrature  |
| UQToolkit | - C++, Python  
- GNU LGPL license  
- Classic distributions  
- Gauss quadrature |

4. Julia

- "Walks like Python, runs like C"
- Solves the two-language problem
- Easy syntax
- Multiple dispatch
- Dynamically-typed
- Metaprogramming
- Package management
- Open source
- Unicode support

Contributors welcome

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