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PolyChaos.jl – An open source Julia package for polynomial chaos expansion

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Polynomial chaos expansion (PCE) is a Hilbert space technique for random variables that alleviates uncertainty propagation. Random variables are expanded in terms of polynomials that are orthogonal relative to a given probability density function. The applicability of PCE hinges on software that allows, among others, to construct orthogonal polynomials. We offer a package for (intrusive) PCE written in the Julia programming language, a trending programming language dedicated to scientific computing.

1. Polynomial Chaos

Hilbert space method for random variables.

Orthogonal basis $\{\phi_k(\xi)\}_{k=0}^{\infty}$ with deg $\phi_k(\xi) = k$ Hilbert space $L^2(\mathbb{R}) \equiv L^2(\Omega, \mu; \mathbb{R})$ with $(\Omega, \mathcal{F}, \mu)$ Scalar product $\langle x, y \rangle_{L^2} = Cov[x, y]$

- Compute orthogonal polynomials for arbitrary densities
- Provide scalar products for intrusive PCE

 $\langle \phi_{i_1}, \phi_{i_2} \cdots \phi_{i_m} \rangle$

- Multivariate support
- Comprehensible documentation

3. Existing Software

- Name Features
 - Matlab
 - BSD 3-clause license
 - Classic and arbitrary distributions
- UQLab
 - Stieltjes procedure
 - Gauss and sparse quadrature
 - Basis-adaptive sparse PCE
 - Least-angle regression

Given an absolutely continuous nonnegative measure, PolyChaos.jl allows

- To compute the coefficients for the monic three-term recurrence relation
- To evaluate orthogonal polynomials at arbitrary points
- To compute the quadrature rule
- To compute tensors of scalar products
- To do all of the above in a multivariate setting

Methods

- Stieltjes procedure
- Lanczos procedure
- Gauss quadrature (+ Lobatto, Radau)
- Fejér's rules, Clenshaw-Curtis
- Sparse computation of scalar products

Type hierarchy

| Norm | $\ \mathbf{x}\ _{L^2} = \sqrt{\langle \mathbf{x}, \mathbf{x} \rangle_{L^2}}$ |
|--------------|--|
| Expansion | $X = X(\xi) = \sum_{l=0}^{\infty} x_l \phi_l(\xi)$ |
| Truncation | $\tilde{\mathbf{x}} = \tilde{x}(\xi) = \sum_{l=0}^{L} x_l \phi_l(\xi)$ |
| Coefficients | $x_{l} = \frac{\langle \mathbf{x}, \phi_{l}(\xi) \rangle_{L^{2}}}{\langle \phi_{l}(\xi), \phi_{l}(\xi) \rangle_{L^{2}}}$ |
| Optimality | $\ \mathbf{x} - \tilde{\mathbf{x}}\ _{L^2} = \min_{\mathbf{y} \in \mathfrak{Y}} \ \mathbf{x} - \mathbf{y}\ _{L}$ |
| | $\mathfrak{Y} = \operatorname{span}\{\phi_l(\xi)\}_{l=0}^L$ |

Several well-known analytic bases (Askey).

| Distribution | Polynomial basis |
|--------------|------------------|
| Normal | Hermite |
| Uniform | Legendre |
| Beta | Jacobi |
| Gamma | Laguerre |

| Chaospy | Python MIT license Classic and arbitrary distributions Gram-Schmidt Stieltjes procedure Gauss quadrature Clenshaw-Curtis |
|-----------|--|
| MUQ | C++, Python Classic distributions Gauss quadrature |
| UQToolkit | C++, Python GNU LGP license Classic distributions Gauss quadrature |
| | |

4. Julia

- "Walks like Python, runs like C"
- Solves the two-language problem



Documentation & Examples

| $\leftarrow \rightarrow$ C \triangle | 🛈 🔒 https://timueh.github.io/PolyChaos.jl/dev/ | |
|--|--|--|
| | » Overview Celit on GitHub | |
| PolyChaos.jl | Overview | |
| Search docs | PolyChaos is a collection of numerical routines for orthogonal polynomials written in the Julia programming language. Starting from some non-negative weight (aka an absolutely continuous nonnegative measure), | |
| Overview | PolyChaos allows | |
| Installation | to compute the coefficients for the monic three-term recurrence relation, | |
| References | to evaluate the orthogonal polynomials at arbitrary points, | |
| Contributing | to compute the quadrature rule, | |
| Citing | to compute tensors of scalar products, to do all of the above in a multivariate setting (aka product measures). | |
| Type Hierarchy | | |
| туре піетаї спу | If the weight function is a probability density function, PolyChaos further provides routines to compute | |
| Usage | porynomial chaos expansions (PCLS) of random variables with this very density function. These routines allow | |
| Numerical Integration | to compute affine PCE coefficients for arbitrary densities, | |
| Ouadrature Rules | to compute moments, | |
| | to compute the tensors of scalar products. | |

→ Arbitrary densities?



- Easy syntax
- Multiple dispatch
- Dynamically-typed
- Metaprogramming
- Package management
- Open source
- Unicode support



Contributors welcome

github.com/timueh/PolyChaos.jl

timueh.github.io/PolyChaos.jl/stable/

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