Optimal Adaptive Power Flow Linearizations:
Expected Error Minimization using Polynomial Chaos Expansion

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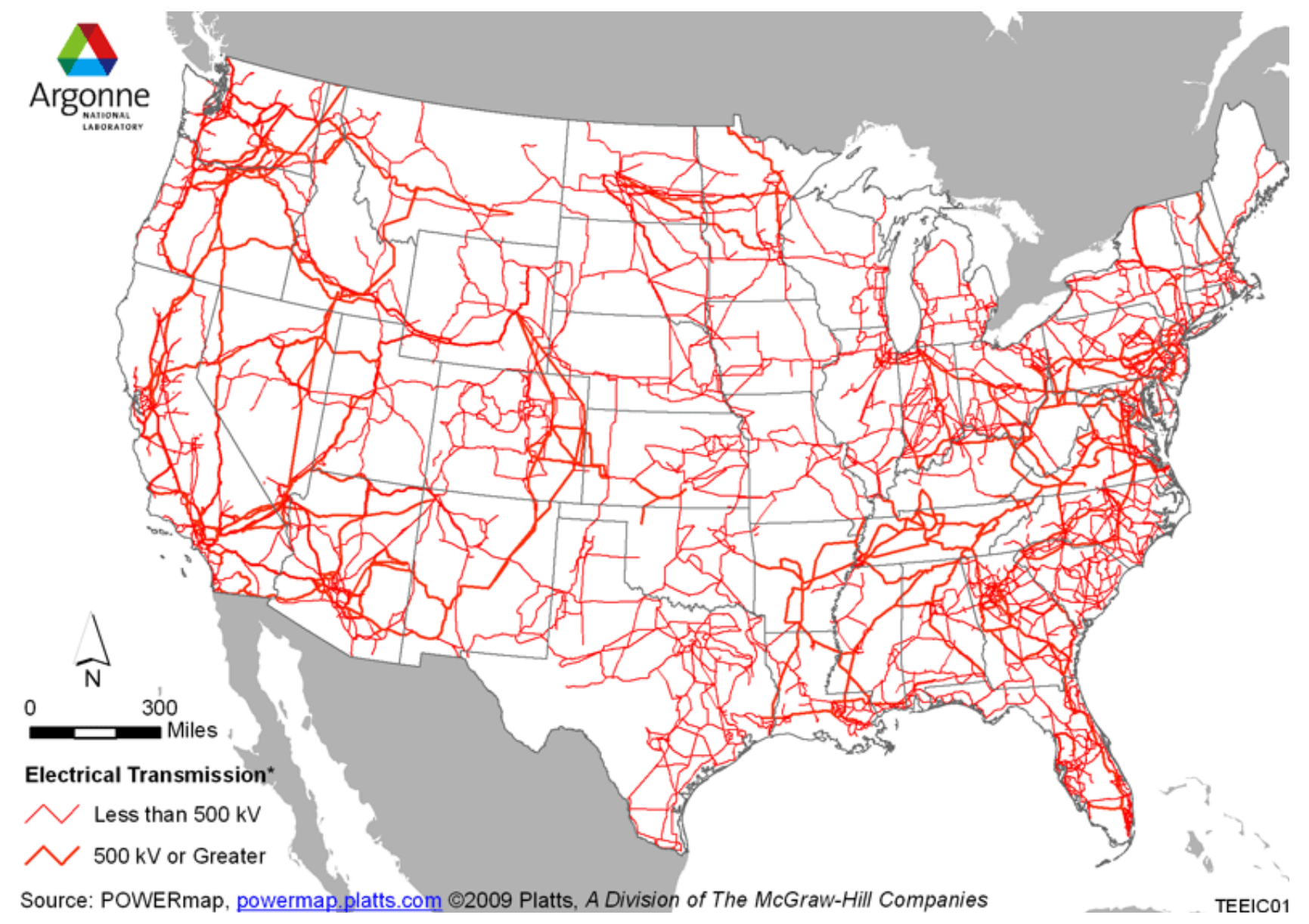
Challenges in Power System Optimization

- Large-scale

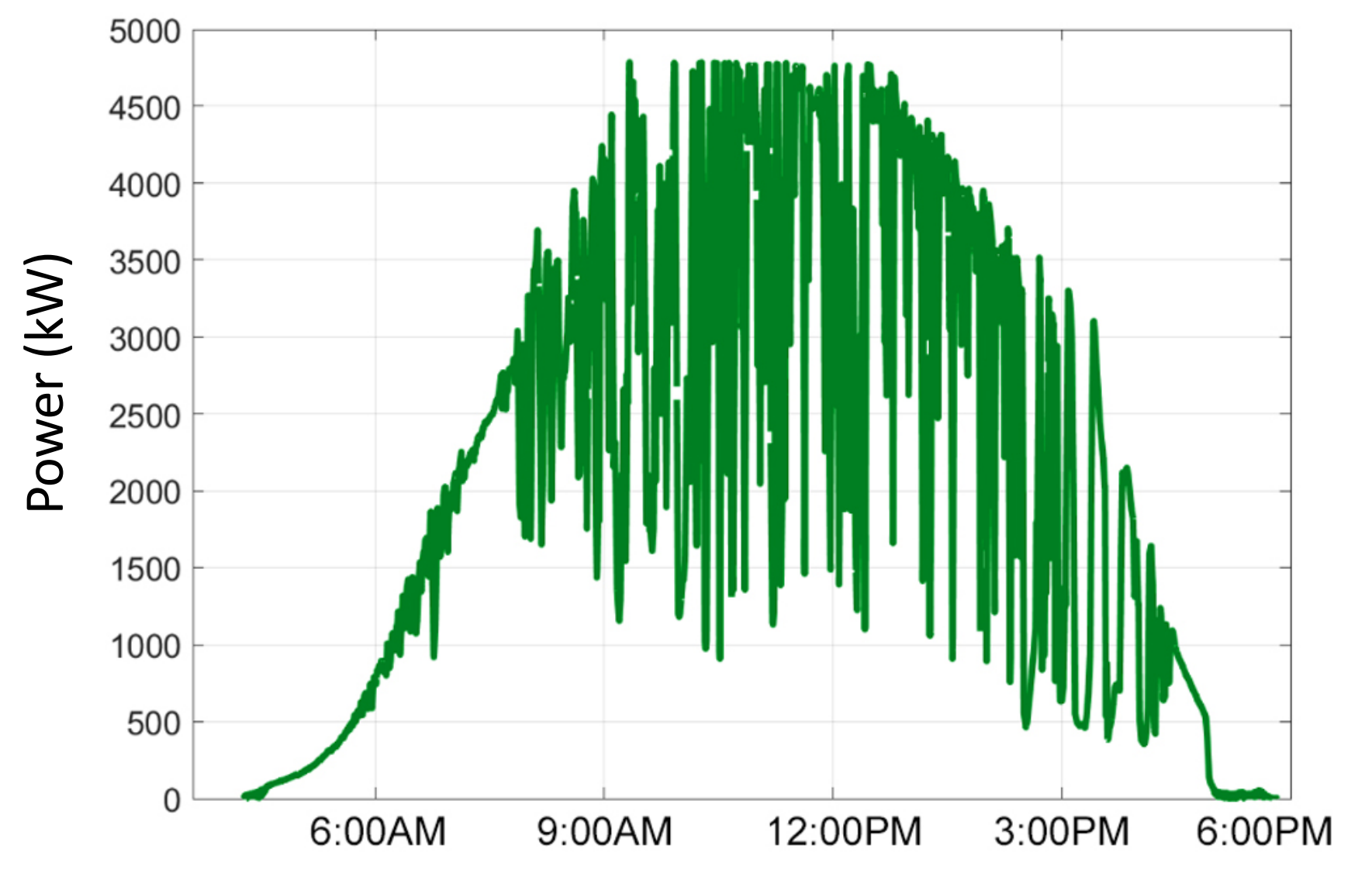
The High-Voltage Transmission System



Source: POWERmap, powermap.platts.com ©2009 Platts, *A Division of The McGraw-Hill Companies*

- Uncertain

A Typical Daily Solar PV Generation Profile



EPRI Report "Flicker Measurements at Photovoltaic Plants," Sept. 2018

- Nonlinear (nonconvex)

The Power Flow Equations

Polar Voltage Coordinates $V_i = |V_i|\angle\theta_i,\ \theta_1 = 0$

$$P_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik}\cos(\theta_i - \theta_k) + \mathbf{B}_{ik}\sin(\theta_i - \theta_k)\right)$$

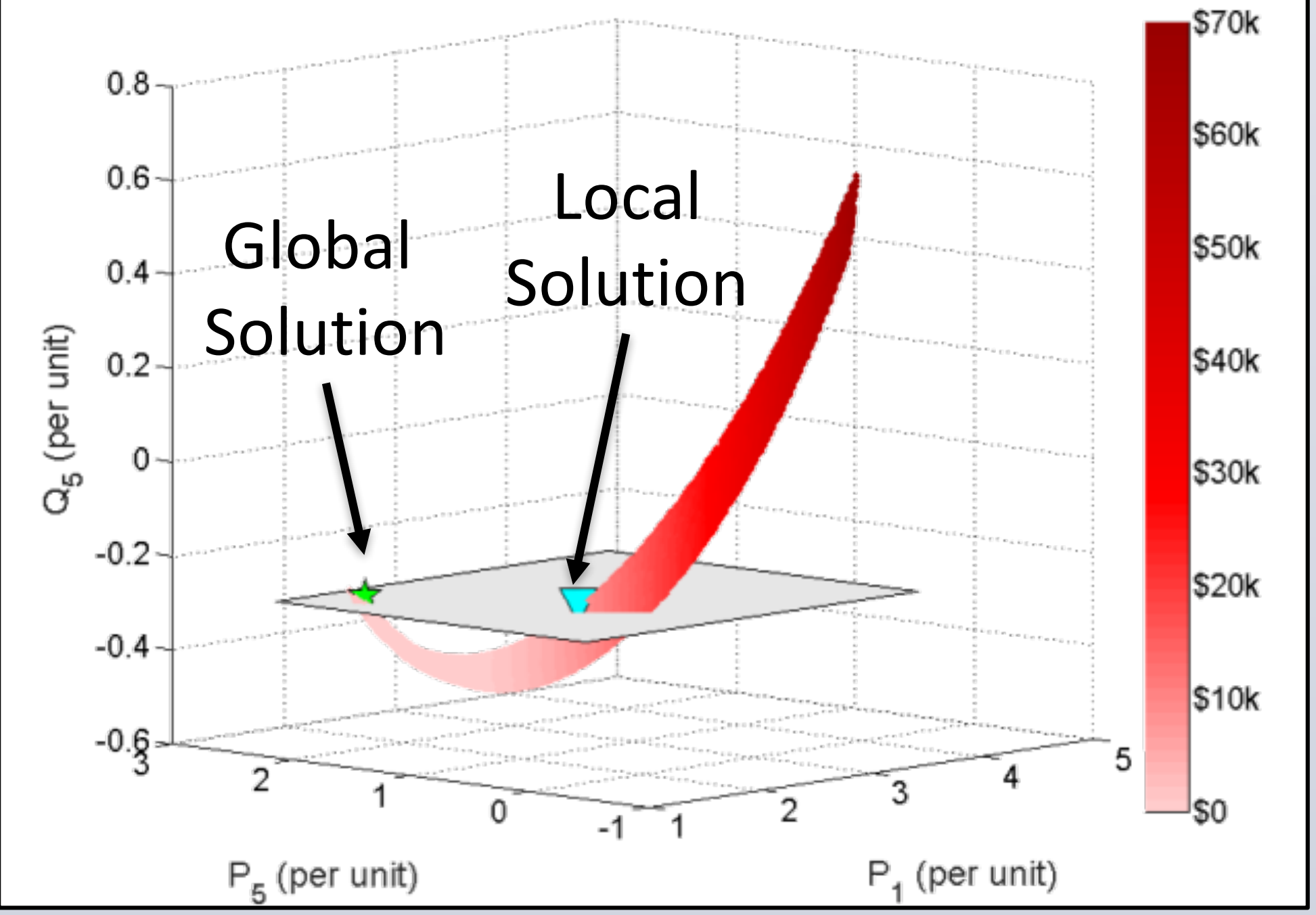
$$Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik}\sin(\theta_i - \theta_k) - \mathbf{B}_{ik}\cos(\theta_i - \theta_k)\right)$$

Rectangular Voltage Coordinates $V_i = V_{di} + jV_{qi},\ V_{q1} = 0$

$$P_i = \sum_{k=1}^{n} V_{di}\left(\mathbf{G}_{ik}V_{dk} - \mathbf{B}_{ik}V_{qk}\right) + V_{qi}\left(\mathbf{B}_{ik}V_{dk} + \mathbf{G}_{ik}V_{qk}\right)$$

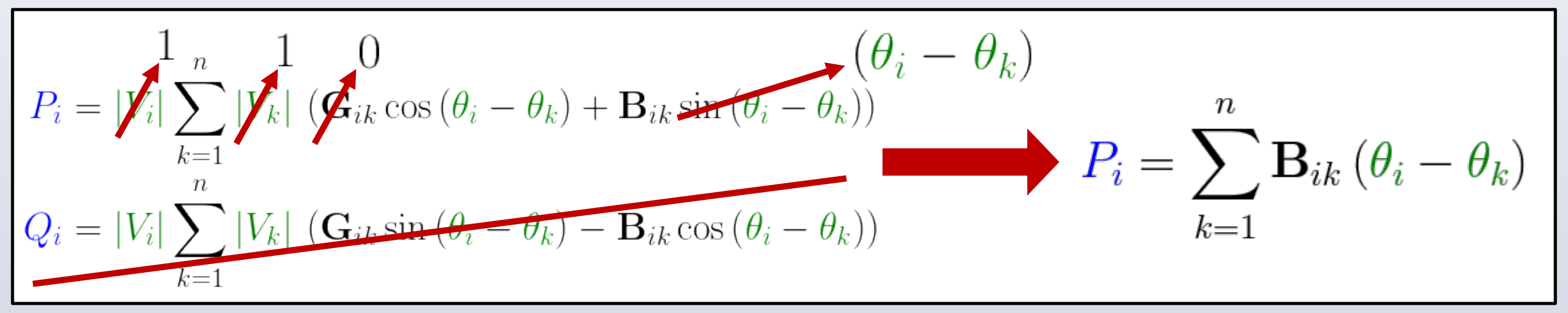
$$Q_i = \sum_{k=1}^{n} -V_{di}\left(\mathbf{B}_{ik}V_{dk} + \mathbf{G}_{ik}V_{qk}\right) + V_{qi}\left(\mathbf{G}_{ik}V_{dk} - \mathbf{B}_{ik}V_{qk}\right)$$

Feasible Region for a Small Test Case



Often use linear approximations to address these challenges.

Traditional Approach: Linearizations use assumptions regarding typical system characteristics based on a single operating point forecast.



$$P_i = |\overset{1}{\cancel{V_i}}| \sum_{k=1}^{n} |\overset{1}{\cancel{V_k}}| \left(\overset{0}{\cancel{\mathbf{G}}}_{ik}\cos(\theta_i - \theta_k) + \mathbf{B}_{ik}\overset{(\theta_i - \theta_k)}{\cancel{\sin(\theta_i - \theta_k)}}\right)$$

$$\cancel{Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik}\sin(\theta_i - \theta_k) - \mathbf{B}_{ik}\cos(\theta_i - \theta_k)\right)}$$

$$\Longrightarrow P_i = \sum_{k=1}^{n} \mathbf{B}_{ik}(\theta_i - \theta_k)$$

Problem: Excessively large **linearization errors** lead to inefficient and unreliable operation.

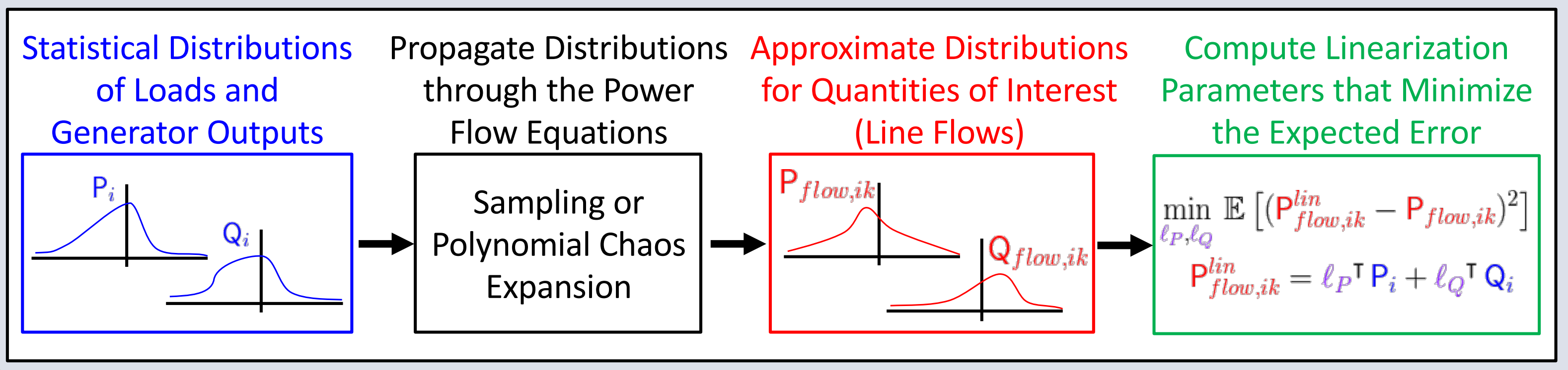
Optimal Adaptive Power Flow Approximations

Specifically tailored to a particular system and operating range.

Minimize a specified error metric.

Computed using algorithms adapted from **machine learning**:

Minimize Expected Linearization Errors



Statistical Distributions of Loads and Generator Outputs → Propagate Distributions through the Power Flow Equations → Approximate Distributions for Quantities of Interest (Line Flows) → Compute Linearization Parameters that Minimize the Expected Error

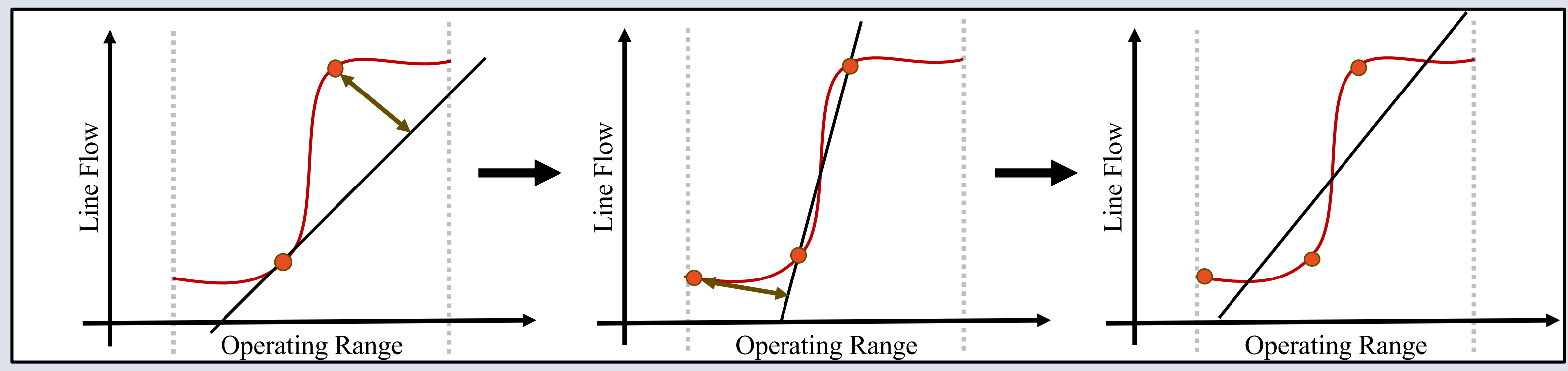
$$\min_{\ell_P,\ell_Q} \mathbb{E}\left[\left(P^{lin}_{flow,ik} - P_{flow,ik}\right)^2\right]$$

$$P^{lin}_{flow,ik} = {\ell_P}^{\intercal} P_i + {\ell_Q}^{\intercal} Q_i$$

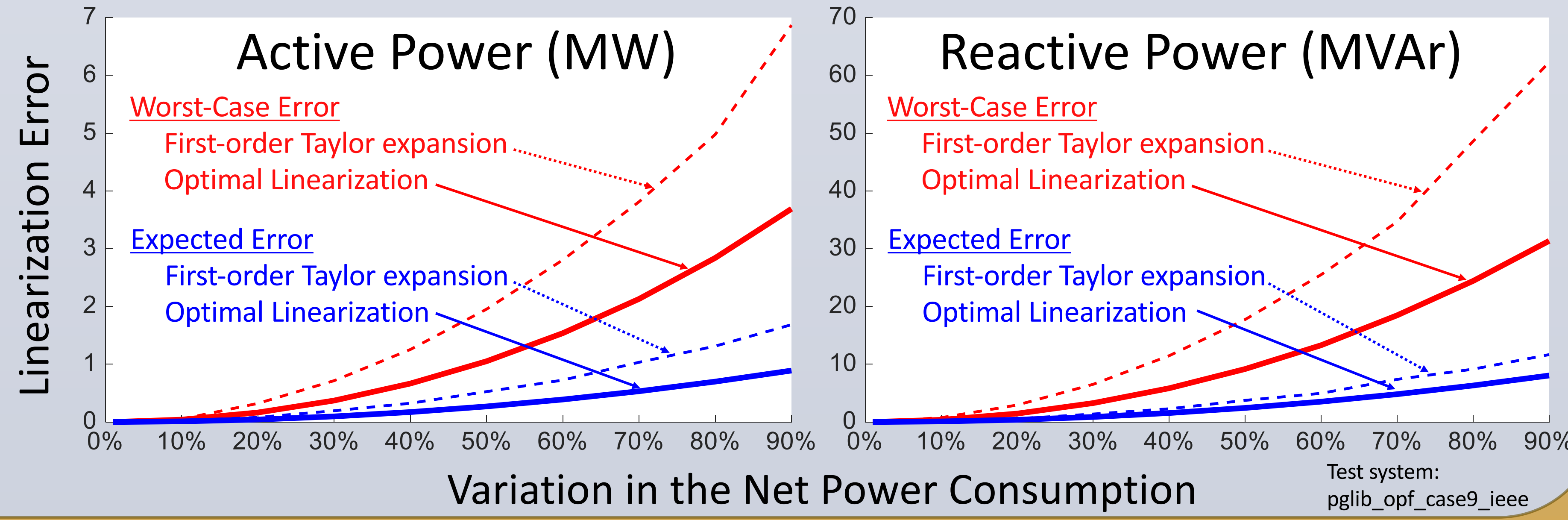
- Regularize estimators to exploit prior information and network sparsity.
- Sample complexity bounds establish performance characteristics.

Minimize Worst-Case Linearization Errors

Constraint generation algorithm based on importance sampling.



Advantage: Smaller linearization errors.



Test system: pglib_opf_case9_ieee

References

- S. Misra and D.K. Molzahn, "Sampling-Based Optimal Adaptive Linearizations of the Power Flow Equations," in preparation for submission to *IEEE Transactions on Power Systems.*
- T. Mühlpfordt, D.K. Molzahn, V. Hagenmeyer, and S. Misra, "Optimal Adaptive Power Flow Linearizations: Expected Error Minimization using Polynomial Chaos Expansion," *IEEE PowerTech 2019*.
- S. Misra, D.K. Molzahn, and K. Dvijotham, "Optimal Adaptive Linearizations of the AC Power Flow Equations," *Power Systems Computation Conf. (PSCC),* June 2018.
- K. Dvijotham and D.K. Molzahn, "Error Bounds on the DC Power Flow Approximation: A Convex Relaxation Approach," *IEEE Conf. Decis. Control (CDC)*, Dec. 2016.