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The effects of $O(\alpha^2)$ initial state QED corrections to $e^+e^- \to \gamma^*/Z^*$ at very high luminosity colliders

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ABSTRACT

We present numerical results on the recently completed $O(\alpha^2)$ initial state corrections to the process $e^+e^- \to \gamma^*/Z^*$, which is a central process at past and future high energy and high luminosity colliders for precision measurements of the properties of the Z-boson, the Higgs boson, and the top quark. We observe differences to an earlier result [1] in the non-logarithmic contributions at $O(\alpha^2)$. The new result leads to a 4 MeV shift in the Z width considering the lower end $s_0 = 4m_\tau^2$ of the radiation region, which is larger than the present accuracy. A corresponding cut to $s_0/s = 0.01$ only implies a shift of 0.2 MeV. We present predictions on the radiative corrections to the central processes $e^+e^- \to \gamma^*/Z^*$, $e^+e^- \to ZH$ and $e^+e^- \to t\bar{t}$ planned at future colliders like the ILC, CLIC, FCC_ee and CEPC to measure the mass and the width of the Z boson, the Higgs boson and the top quark, for which the present corrections are significant.

An important ingredient to precision measurements at e^+e^- colliders is the precise knowledge of the QED initial state corrections (ISR). The $O(\alpha^2)$ corrections have been completed very recently. Already in 1987 a first calculation to $O(\alpha^2)$ has been performed [1] for the process $e^+e^- \to \gamma^*/Z^*$. These corrections have been used in the analysis of the LEP1 data, cf. [2] and are implemented in fitting codes like TOPAZ0 [3] and ZFITTER [4]. In 2011, using the light cone expansion and assuming the factorization of the massive Drell-Yan process, the corrections for the same process have been calculated in [5] and disagreement was found with the results of [1] for the non-logarithmic terms at $O(\alpha^2)$.

We have repeated the calculation using conventional methods without performing any approximation and expanded the final results in the mass ratio m_e^2/s to obtain compact analytic expressions for the respective radiators, cf. [6,7]. The calculation has been accompanied by controlling the results using high precision numerics. We confirm the results presented in [5]. Both calculations are completely independent in the methods which have been used. The one in [5] has assumed the factorization of the Drell–Yan process with external massive fermions in addition and has been performed for vector couplings only. Furthermore, in Ref. [1] no account was given on the axialvector terms, which have different corrections than the vector terms in some cases. Also some processes only contributing to the non-logarithmic order known from [8,9] were missing, which we have recalculated and added, completing the $O(\alpha^2)$ QED ISR corrections. Here we include both photon and e^+e^- pair emission up to $O(\alpha^2)$. The initial state QED corrections can be written in terms of the following functions

$$H\left(z, \alpha, \frac{s}{m^2}\right) = \delta(1-z) + \sum_{k=1}^{\infty} \left(\frac{\alpha}{4\pi}\right)^k C_k\left(z, \frac{s}{m^2}\right) \quad (1)$$

$$C_k\left(z, \frac{s}{m^2}\right) = \sum_{l=0}^{k} \ln^{k-l}\left(\frac{s}{m^2}\right) c_{k,l}(z), \quad (2)$$

which yield the respective differential cross sections by

$$\frac{d\sigma_{e^+e^-}}{ds'} = \frac{1}{s}\sigma_{e^+e^-}(s') H\left(z, \alpha, \frac{s}{m^2}\right), \quad (3)$$

with $\sigma_{e^+e^-}(s')$ the scattering cross section without the ISR QED corrections, $\alpha \equiv \alpha(s)$ the fine structure constant and $z = s'/s$, where s' is the invariant mass of the produced (off-shell) γ/Z boson.

These results are of phenomenological importance for the precision measurements of the Z resonance, high luminosity ZH production, and $t\bar{t}$ production at LEP1, and for future planned e^+e^-

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Table 1
Shifts in the Z-mass and the width due to the different contributions to the ISR QED radiative corrections for a fixed width of $\Gamma_Z = 2.4952$ GeV and s-dependent width using $M_Z = 91.1876$ GeV [15] and $s_0 = 4m_\tau^2$, cf. [2].

	Fixed width		s dep. width	
	Peak (MeV)	Width (MeV)	Peak (MeV)	Width (MeV)
$O(\alpha)$ correction	210	603	210	602
$O(\alpha^2)$ correction	-109	-187	-109	-187
$O(\alpha^2)$: γ only	-110	-215	-110	-215
Soft exp. beyond the $O(\alpha^2)$ correction	17	23	17	23
Difference to $O(\alpha^2)$ [1]		4		4

colliders such as ILC and CLIC [10], the FCC_ee [11,12], the CEPC [13], and also for muon colliders [14].

In this letter we detail the phenomenological results for the impact of the ISR QED corrections up to $O(\alpha^2)$ and also include soft resummation beyond this order, cf. e.g. [1], studying their effect on the Z peak, ZH- and $t\bar{t}$-production. These processes will serve to perform highly precise measurements of the Z and Higgs boson, H, and the top quark mass in the future. Likewise, we reconsider the measurement at LEP1. A detailed account on the analytic calculation will be given in [7], providing also all the radiation functions needed in the analyses, which are too voluminous to be presented here.

1. The Z peak and its surrounding

For this production channel we consider the measurement of the inclusive cross section of a $\mu^+\mu^-$ state above a certain threshold s_0 of its invariant mass squared, while all the radiation products due to ISR are integrated. The theoretical value for s_0 is $4m_\mu^2$, while in the measurements a series of cuts are used and then one extrapolates again to a value of s_0. In the LEP1 analysis, examples are $s_0 = 4m_\tau^2$ or $s_0 = 0.01M_Z^2$ [2]. We will discuss effects for these values and also consider values down to the theoretical boundary.

In Table 1 we summarize the effect of the different order ISR corrections on the shift of the Z peak and the modification of the half-width performing the difference from a given order to the previous one. Very similar values are obtained in the case of a fixed width or the s-dependent width. At $O(\alpha^2)$ we distinguish the cases of either pure photon emission or including also e^+e^- pair production. While the peak shift comes out the same in both cases, there is a shift on the width of 28 MeV by including the emission of e^+e^- pairs. Finally, soft photon exponentiation from $O(\alpha^3)$ onward leads to a peak shift of 17 MeV and to a 23 MeV width shift. The numbers are quite comparable to those given in [1], where at $O(\alpha^2)$ only the photon emission has been considered and the integration was performed from $s_0 = 4m_\mu^2$.

At $s_0 = 4m_\tau^2$ the corrected expressions w.r.t. Ref. [1] are too small to be visible at the peak position. However, a 4 MeV shift is obtained in the width, in comparison with the present result. This is of relevance since the current error is $\Delta\Gamma_Z = \pm 2.3$ MeV [15]. For $s_0 = 0.01M_Z^2$, on the other hand, the shift amounts to 0.2 MeV, which is relevant at Giga-Z and FCC_*ee* [10,11], where resolutions of a few hundred keV can be reached for both M_Z and Γ_Z, see also [16]. If s_0 would have been chosen as low as 1 GeV^2, the width would shift by 18 MeV and the peak position by 3 keV, while for larger cuts the effect on the peak shift cannot be resolved. The effects would even be larger for $s_0 = 4m_\mu^2$. To clarify this further, we show in Fig. 1 the relative difference of the correction for a series of s_0 values in the vicinity of the Z peak.

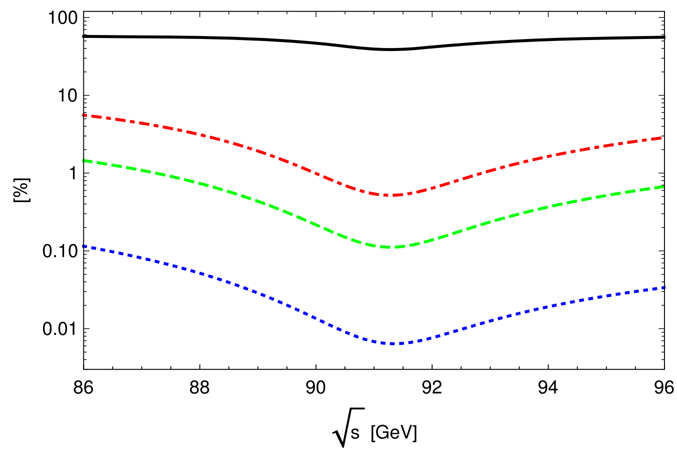

Fig. 1. Relative difference between the $O(\alpha^2)$ results of [1] and the present paper as a function of $\sqrt{s}$ in dependence of s_0. Dotted line $s_0 = 0.01M_Z^2$; Dashed line $s_0 = 4m_\tau^2$; Dash-dotted line $s_0 = 1$ GeV^2; Full line $s_0 = 4m_\mu^2$.

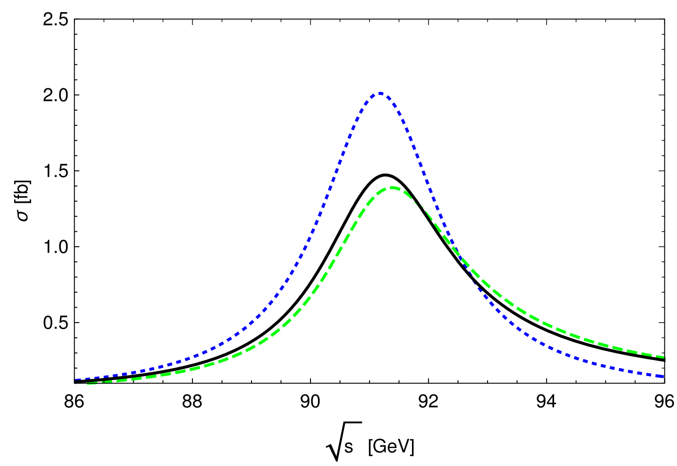

Fig. 2. The Z-resonance in $e^+e^- \to \mu^+\mu^-$. Dotted line: Born cross section; Dashed line: $O(\alpha)$ ISR corrections; Full line: $O(\alpha^2)$ + soft resummation ISR corrections, with $s_0 = 4m_\tau^2$.

The shifts in the width are majorly caused by the discrepancies in the pure singlet terms (process 3 in [1]) containing $1/z$ contributions, cf. [6].

Between the cases of a constant width and the s-dependent width we find a peak shift of 34.2 MeV and a shift of the width of 1 MeV, irrespective of the applied ISR corrections, in accordance with Refs. [17]. In Fig. 2 we illustrate the different QED ISR corrections to $e^+e^- \to Z^*/\gamma^*$ around the Z peak. The ISR corrections change the profile of the resonance, i.e. the peak position, height and the half width. The lines for the $O(\alpha^2)$ correction and the one including soft resummation are nearly identical. In Fig. 3 the region of $\sqrt{s}$ is extended to [10, 200] GeV. The individual contributions of the fixed order corrections at low order show growing effects off the Z peak. The soft resummation corrections stay nearly constant in the whole range, except in the region around the Z peak. The full $O(\alpha^2)$ corrections prove to be already important in the analysis of the LEP1. The difference to the previous results [1] has an effect when analyzing the LEP1 data, applying a lower cut of the size $s_0 = 4m_\tau^2$, and likewise $s_0 = 0.01$, for the future measurements at Giga-Z and FCC_*ee*.

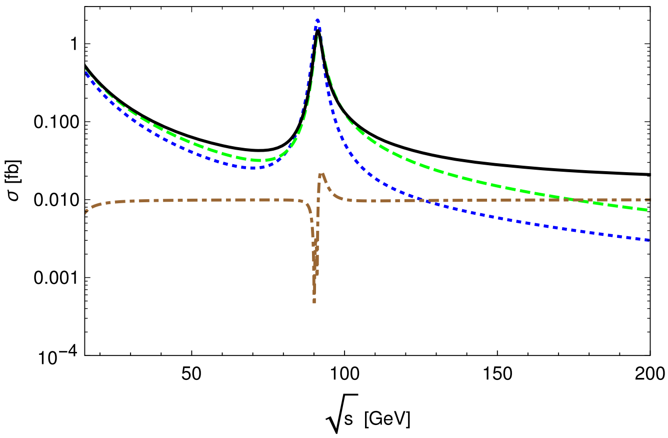

Fig. 3. The Z-resonance in $e^+e^- \to \mu^+\mu^-$. Dotted line: Born cross section; Dashed line: $O(\alpha)$ ISR corrections; Full line: $O(\alpha^2)$ + soft resummation ISR corrections; Dash-dotted line: individual contribution of soft resummation.

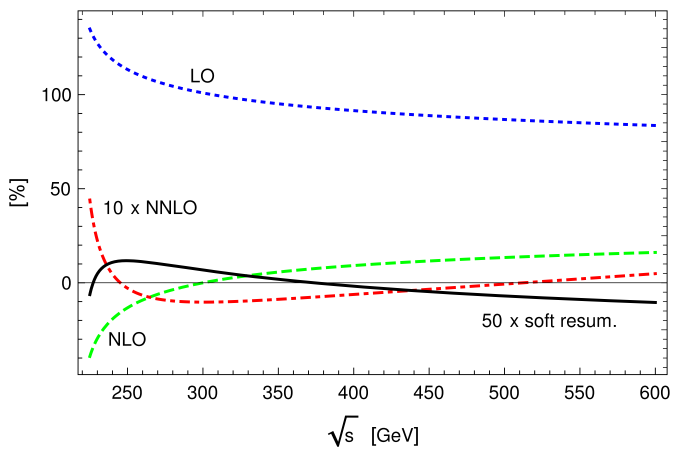

Fig. 4. Relative contributions of the ISR QED corrections to the cross section for $e^+e^- \to ZH$ in %. Dotted line: $O(\alpha^0)$; Dashed line: $O(\alpha)$; Dash-dotted line: $O(\alpha^2)$; Full line: soft resummation beyond $O(\alpha^2)$, with $s_0 = 4m_t^2$.

2. The process $e^+e^- \to ZH$

For the study of the radiative corrections we refer to the Born cross section given in Ref. [18]. The accuracy of the cross section measurement has been estimated to reach 1% [16] at future colliders like the ILC, CLIC, and 0.4% at the FCC_*ee* [19]. In Fig. 4 we show the relative contributions of the Born and the different ISR radiative corrections to ZH-production.

The NNLO corrections vary between +4.8% and -1% and are larger or of the size of the expected experimental errors. The corrections due to soft resummation are of $O(\pm 0.2\%)$ and reach half of the projected accuracy.

3. The $t\bar{t}$-production at threshold and in the continuum

For the process of $e^+e^- \to t\bar{t}$ we consider the ISR effects both in the threshold and the continuum region. In the former case they are applied to the cross section based on including the N^{3}LO QCD corrections implemented in the code `QQbar_threshold` [20–22], while in the continuum case for $\sqrt{s} > 500$ GeV we use the Born cross section [1] for a first numerical illustration. The anticipated accuracy to measure this scattering cross section at future e^+e^- colliders has been estimated to be ±2% [23,24].

Leading order QED corrections and a part ot the $O(\alpha^2)$ terms, based on the results of [1], are implemented in the code described in [20–22]. Their effect has been illustrated in part in [24], however, also including electro–weak effects. We have only illustrated the QED ISR corrections. However, thorough studies up to $O(\alpha^2)$ precision are only possible based on the results we obtained. Therefore a direct comparison to the ISR radiative corrections of [24] is not possible.

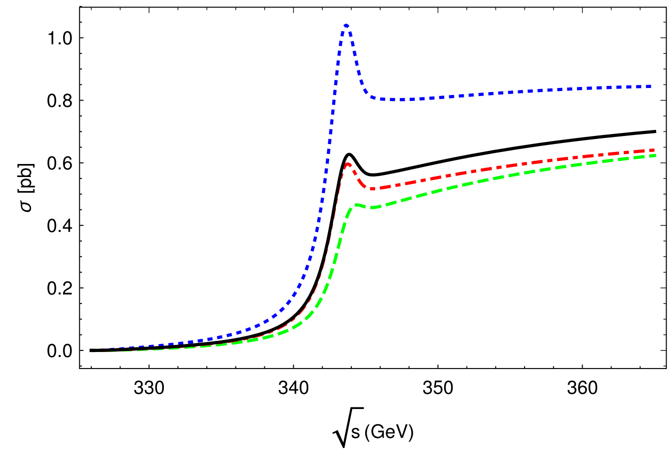

Fig. 5. The QED ISR corrections to $e^+e^- \to t\bar{t}$ (s-channel photon exchange) in the threshold region far a PS-mass of $m_t = 172$ GeV. Dotted line $O(\alpha^0)$; Dashed line $O(\alpha)$; Dash-dotted line $O(\alpha^2)$; Full line $O(\alpha^2)$ + soft resummation.

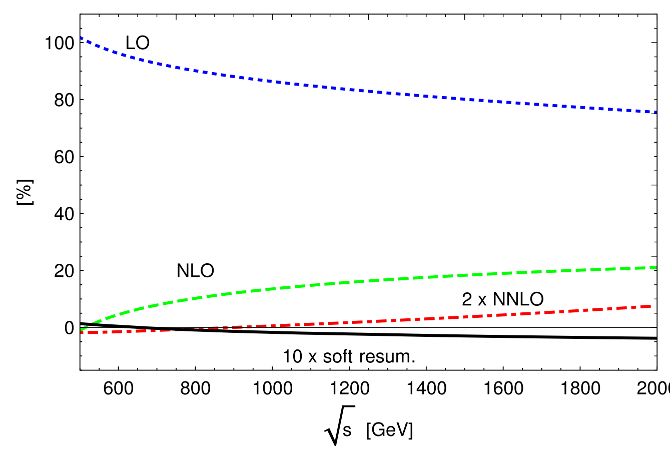

Fig. 6. Relative contributions of the continuum cross section of $t\bar{t}$ production including the NNLO ISR corrections. Dotted line: $O(\alpha^0)$; Dashed line: $O(\alpha)$; Dash-dotted line: $O(\alpha^2)$ scaled by 2; Full line: soft resummation beyond $O(\alpha^2)$ scaled by 10.

For the top-quark mass we refer to the PS mass of 172 GeV. The corrections in the threshold regions are shown in Fig. 5. The different corrections change the profile of the cross section significantly. Up to $\sqrt{s} \sim 344$ GeV the contributions due to soft resummation agree with the NNLO corrections. Above they deliver an additional contribution. Adding soft exponentiation implies a correction between 2 and 8%. Both the $O(\alpha^2)$ and soft resummation corrections have effects of the size of the expected experimental accuracy and larger.

In the continuum region the relative size of the ISR corrections to $t\bar{t}$ production are shown in Fig. 6. The $O(\alpha^2)$ corrections vary between -1 and 4% and soft resummation yields further corrections of 0.13 to -0.38%.

Whether or not the difference to the results given in [1] is visible depends on the range in z over which is integrated. Only at small values of z the effect is visible. At higher cuts in $\sqrt{s}$, as the case for ZH- and $t\bar{t}$-production, the numerical effects are very small, given the respective collider energies.

We designed the FORTRAN-code RC2.f for the numerical calculations of the ISR corrections [which can be compiled together with other FORTRAN- and C-codes by gfortran [25]] for data analyses. We also used an implementation in mathematica.

In conclusion, the numerical investigation of the Z boson production, as well as ZH and $t\bar{t}$ production has shown the relevance of these effects for LEP1 and at future e^+e^- colliders. The new results, compared with [1] imply a relative shift in the Z-width by $\sim$4 MeV for $s_0 = 4m_\tau^2$.

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