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Karlsruher Institut für Technologie (KIT)
Fakultät für Wirtschaftswissenschaften
Institut für Volkswirtschaftslehre (ECON)

Kaiserstraße 12
76131 Karlsruhe

KIT – Die Forschungsuniversität in der Helmholtz-Gemeinschaft

Working Paper Series in Economics
No. 137, October 2019

ISSN 2190-9806

econpapers.wiwi.kit.edu

Institute of Economic Theory and Operations Research
Karlsruhe Institute of Technology

Tackling the Bundestag Growth by Introducing Fraction-Valued Votes

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Working paper Nr. 137

October 2019

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Abstract

Currently, only China has a parliament larger than the German Bundestag, which continues to grow due to the increasing number of overhang mandates. In 2016, Norbert Lammert, then president of the Bundestag, proposed to restrict it to 630 members by allocating mandates according to quotas for each of the German states (Länder), which should be proportional to their population. This idea found no approval among the German parties, neither large nor small [Finthammer 2018]. Only in October 2019, after predictions that the next Bundestag could exceed 800 seats, did some 100 German experts in constitutional law write an open letter suggesting to constrain its size by reducing the number of effective constituencies, and the Bundestag vice-president, Thomas Oppermann, called for such a reform without delay.

These and other proposals require a profound change in the existing election system. But a mathematical solution to the problem does not require such changes and is much simpler. We can prevent unfettered growth of the Bundestag — caused by allotting too many direct mandates to parties that received too few second votes — by replacing the principle of 'one man, one vote' with a new concept: *fraction-valued votes* for Bundestag members. Such a practice could make overhang mandates unnecessary and the basic 598 Bundestag seats sufficient under all circumstances. For this purpose, the members of the overrepresented parties (because they receive too many direct mandates) should have vote power < 1 and the members of other parties should have vote power > 1 . We explain the vote power adjustments using the example of the 2017 Bundestag.

Keywords: Representative democracy, elections, theory of voting, proportional representation.

JEL Classification: D71

1 Introduction

The German two-vote electoral system embodies two major historical concepts of political representation coined during the American and French Revolutions. The *descriptive concept* (leading to proportional representation) — that is, the parliament portrays the society in miniature¹ — is implemented in the first vote (*Erststimme*), with which local candidates are elected within constituencies and delegated to the federal parliament (Bundestag). These *direct mandate* holders from 299 German constituencies fill 299 Bundestag seats.

The *agent concept* (leading to majoritarianism) — that is, the parliament is a committee of political experts who make majority decisions as the people’s trustees and not simply as their fellow countrymen² — is embodied in the second vote (*Zweitstimme*) for a party. The second vote serves two purposes: (1) to qualify parties receiving at least 5% of the second votes nationwide for seats in the Bundestag, and (2) to apportion the number of seats allotted to each qualifying party in accordance with the second votes, including the direct mandate holders. For this purpose, another 299 Bundestag seats are allocated. If the required proportion between party factions is unattainable within the regular $299 + 299 = 598$ seats, some extra seats called overhang mandates (*Überhangmandate*) are added.

Currently, only China has a parliament larger than the German Bundestag, which continues to grow due to the increasing number of overhang mandates: the 2005, 2009, 2013 and 2017 Bundestags had 16, 24, 33 and 111 overhangs, respectively, and the 2017 Bundestag has as many as 709 members. Such growth makes the Bundestag more expensive for taxpayers: its annual budget is already approaching a billion Euros [Finthammer 2018]. In 2016, Norbert Lammert, then president of the Bundestag, proposed to restrict it to 630 members by allocating mandates according to quotas for each of the German states (Länder), which should be proportional to their population [Roßner 2016]. This idea found no approval among the German parties, neither large nor small [Finthammer 2018]. Only in October 2019, after predictions that the next Bundestag could exceed 800 seats, did some 100 Ger-

¹The descriptive concept was defended in America by John Adams (1735–1826), one of the key Founding Fathers, the first Vice President and the Second President of the United States from 1797–1801. In France, the same viewpoint was shared by Honore Gabriel Riqueti, comte de Mirabeau (1749–1791), a statesman, a moderate revolutionary and promoter of a British-like constitutional monarchy [Tangian 2014, pp. 167–168].

²The agent concept was promoted by American Federalists, particularly by Alexander Hamilton (1755?–1804), one of key Founding Father of the United States and James Madison (1751–1836), the fourth President of the USA from 1809–1817. In France, the concept of political representative as professional was developed by Emmanuel Joseph Sieyès (1748–1836), clergyman and political writer [Tangian 2014, pp. 165–166, 168–169].

man experts in constitutional law write an open letter suggesting to constrain its size by reducing the number of effective constituencies, and the Bundestag vice-president, Thomas Oppermann, called for such a reform without delay [Spiegel online 2019, Zeit online 2019].

These and other proposals require a profound change in the existing election system. But a mathematical solution to the problem does not require such changes and is much simpler. We can prevent unfettered growth of the Bundestag — caused by allotting too many direct mandates to parties that received too few second votes — by replacing the principle of ‘one man, one vote’ with a new concept: *fraction-valued votes* for Bundestag members. Such a practice could make overhang mandates unnecessary and the basic 598 Bundestag seats sufficient under all circumstances. For this purpose, the members of the overrepresented parties (because they receive too many direct mandates) should have vote power < 1 and the members of other parties should have vote power > 1 .

Deviations from ‘the ideal of one man, one vote’ [Balinski and Young 1982] are in fact not that uncommon. For instance, the chairman of a committee with an even number of members may be given one and a half votes to avoid a tie. In joint-stock companies, the vote power of each shareholder is proportional to his/her percentage of shares, etc. In parliaments like the Bundestag, the situation is only complicated by a possible mismatch between the number of the parties’ direct mandates and the faction’s weights derived from the second votes. We explain the vote power adjustments in such cases using an example.

In Section 2, ‘Fraction-Valued Votes for the 598-seat Bundestag’, the vote power adjustments are explained using the example of the 2017 Bundestag.

In Section 3, ‘Bundestag with Restricted Fraction-Valued Votes and a Few Overhangs’, the deviation from the principle ‘one voter, one vote’ is made small at the price of adding a few overhang mandates.

Section 4, ‘Compatibility with the Current Bundestag Allocation Rules’, shows that in the case of 709 seats, as in the actual 2017 Bundestag, our model computes exactly the same allocation of Bundestag seats as it has.

In Section 5, ‘Conclusion’, the main findings are recapitulated and put into context.

Section 6, ‘Annex: D’Hondt and Saint-Laguë Methods for Allocating Parliamentary Seats’, explains two main methods for allocating parliamentary seats.

2 Fraction-Valued Votes for the 598-seat Bundestag

The 2017 Bundestag election outcomes [Bundestwahlleiter 2017] are displayed in the first section of Table 1 (Columns 1–3). Column 1 shows the number of direct mandates received by the seven parties eligible for the Bundestag seats — in total 299 mandates from 299 constituencies. Column 2 contains the percentages of the second votes received by these parties. Since 5% of second votes were cast for other parties, the grand total of Column 2 is equal to 95%. Column 3 contains the values of the second column normalized — brought up to 100%, prescribing the weights of the Bundestag factions.

The second section of the table (Columns 4–6) describes the Bundestag with the regular 598 seats. As shown in Columns 4–5, the CDU’s 185 direct mandates occupy 30.9% of the 598 Bundestag seats, exceeding the CDU’s faction weight of 28.2%. To bring the total CDU’s voting power in line with the CDU’s weight, the CDU faction member’s vote power v_{CDU} is reduced. Since we consider 598 Bundestag seats, we solve the following equation:

$$\text{CDU member's vote power: } \frac{185}{598} \times v_{\text{CDU}} \times 100\% = 28.2\% \Rightarrow v_{\text{CDU}} \approx 0.91 .$$

This CDU faction members’ vote power is shown at the top of Column 6.

Similarly, the 46 CSU direct mandate holders occupy 7.7% of the Bundestag seats instead of the prescribed 6.5%. Proceeding in the same way as above, we obtain the CSU faction member’s vote power v_{CSU} from the following equation:

$$\text{CSU member's vote power: } \frac{46}{598} \times v_{\text{CSU}} \times 100\% = 6.5\% \Rightarrow v_{\text{CSU}} \approx 0.84 .$$

A simple check shows that the direct mandate holders of the other five parties do not fill their shares of Bundestag seats, so the corresponding factions have to be completed using the remaining

$$\underbrace{598}_{\substack{\text{All} \\ \text{Bundestag} \\ \text{seats}}} - \underbrace{185}_{\substack{\text{Seats of CDU} \\ \text{direct mandate} \\ \text{holders}}} - \underbrace{46}_{\substack{\text{Seats of CSU} \\ \text{direct mandate} \\ \text{holders}}} = 367 \text{ seats} .$$

These 367 seats are allocated to the five parties according to the German election law — using the Sainte-Laguë method; see Part IV, Section 6.2 — and the results are listed in Column 4.

Comparing Columns 3 and 5, we see that these five parties have fewer seats than prescribed by their faction weights. Correspondingly, the vote powers v of these faction

Table 1: Fractional vote power of 2017 Bundestag parties to avoid overhang mandates

Party	Election results			Bundestag with 598 seats			Bundestag with 630 seats			Bundestag with 638 seats			Bundestag with 709 seats		
	Direct	2nd	Faction	Vote			Vote			Vote			Vote		
	man-	votes	weight	Seats	Seats	Vote	Seats	Seats	Vote	Seats	Seats	Vote	Seats	Seats	Vote
	dates	%	%	%	v		%	v		%	v		%	v	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
CDU	185	26.8	28.17	185	30.9	0.91	185	29.4	0.96	185	29.0	0.97	200	28.2	1.00
SPD	59	20.5	21.59	121	20.2	1.07	133	21.1	1.02	135	21.2	1.02	153	21.6	1.00
AfD	3	12.6	13.30	75	12.5	1.06	81	12.9	1.03	83	13.0	1.02	94	13.3	1.00
FDP	0	10.7	11.31	63	10.5	1.07	69	11.0	1.03	71	11.1	1.02	80	11.3	1.00
LINKE	5	9.2	9.72	55	9.2	1.06	59	9.4	1.04	60	9.4	1.03	69	9.7	1.00
GRÜNE	1	8.9	9.41	53	8.9	1.06	57	9.0	1.04	58	9.1	1.04	67	9.4	1.00
CSU	46	6.2	6.49	46	7.7	0.84	46	7.3	0.89	46	7.2	0.90	46	6.5	1.00
Total	299	95.0	100	598	100	598	630	100	630	638	100	638	709	100	709

members are increased to make the total voting power of every faction equal to its weight in Column 3. For instance, the SPD faction member’s vote power v_{SPD} is found from the following equation:

$$\text{SPD member's vote power: } \frac{121}{598} \times v_{\text{SPD}} \times 100\% = 21.6\% \Rightarrow v_{\text{SPD}} \approx 1.07 .$$

The bottom row of the table consists of control sums, except for Columns 6, 9, and 12, with the total vote power of the Bundestag. For instance, the total vote power of the 598-seat Bundestag — the scalar product of Columns 4 and 6 — is equal to 598 votes:

$$\underbrace{185 \times 0.91}_{\text{Vote power of CDU}} + \underbrace{121 \times 1.07}_{\text{Vote power of SPD}} + \dots + \underbrace{46 \times 0.84}_{\text{Vote power of CSU}} = \underbrace{598}_{\text{Vote power of Bundestag}} \text{ votes.}$$

3 Bundestag with Restricted Fraction-Valued Votes and a Few Overhangs

The third section of Table 1 (Columns 7–9) describes the Bundestag with 630 seats (= with 32 overhangs), as suggested by Norbert Lammert. The CDU and CSU are still overrepresented because they have too many direct mandates, but not as much as in the case of the Bundestag with 598 seats. Due to overhang mandates, the degree of parties’ over-/underrepresentation is smaller than in the Bundestag with 598 seats, requiring smaller deviations from 1 in the faction members’ vote power. Now, the CDU and CSU faction members’ vote powers are equal to 0.96 and 0.89 instead of 0.91 and 0.84, respectively. The vote powers of the other Bundestag members are of the range 1.02–1.04 instead of 1.06–1.07.

Relaxations of the principle ‘one man, one vote’ can be moderated by overhang mandates. If the maximal relaxation is, say, ‘one man, one vote $\pm 10\%$ ’, i.e. the vote powers are restricted to $0.9 \leq v \leq 1.1$, then the Bundestag must have at least 638 seats (40 overhangs), see Columns 10–12.

4 Compatibility with the Current Bundestag Allocation Rules

The last section of Table 1 shows our computations for the Bundestag with 709 seats (111 overhangs). All vote powers are equal to 1.00, and the allocation of seats to the eligible parties is 1–1 as in the actual 2017 Bundestag with its 709 seats;³ see [Bundestwahlleiter 2017]. Therefore, our model is compatible with the German election rules.

5 Conclusion

To conclude, we make a note on our terminology. The ‘faction sizes’ characterize the faction’s *physical* volume, which is measured, for instance, in the percentage of parliamentary seats. The ‘faction weights’, which are proportional to the electoral votes received by the *parliamentary* parties, are not physical but *electoral* characteristics which prescribe the factions’ total *legislative* ‘vote power’. As long as the Bundestag seats are allocated with added overhang mandates — to respect the principle of ‘one man, one vote’ — the factions’ physical size and the equal legislative vote power will be close to its electoral weight, although slightly different. These interrelations can be schematized by the following ‘equation’:

Factions in a parliament with numerous overhang mandates:

$$\text{Electoral weight} \approx \text{Legislative weight} = \text{Physical size} .$$

In our consideration, the faction’s legislative vote power is made precisely equal to its electoral weight, and both can differ from the faction’s physical size. The vote power of individual faction members is adjusted correspondingly — at the price of relaxing the principle of ‘one man, one vote’. These interrelations can be summarized as follows:

Factions in a parliament with a few overhang mandates:

$$\text{Electoral weight} = \text{Legislative weight} \approx \text{Physical size} ,$$

³The actual fractional vote powers in Column 15 are not seen within two decimals.

Factions in a parliament with no overhang mandates:

$$\text{Electoral weight} = \text{Legislative weight} \neq \text{Physical size} .$$

Thus, the German election rules remain intact, only the counting of parliamentary votes is modified. The overhang mandates, which regulate the relative influence of parliament factions, are no longer necessary, being replaced by adjustments to the vote power of faction members. Moreover, the overhang mandates alone, even as numerous as in the 2017 Bundestag, cannot make the faction ratio absolutely precise. Conversely, the use of fractional vote powers always guarantees the true proportions between faction weights.

6 Annex: D'Hondt and Saint-Laguë Methods for Allocating Parliamentary Seats

6.1 D'Hondt Method

In 1882, Belgian lawyer and mathematician Victor d'Hondt (1841–1902) published his method [D'Hondt 1882] for allocating parliamentary seats to parties in proportion to the votes they received at elections. The eponymous method is used in many countries and, in particular, was applied in the German Bundestag until 1985 [D'Hondt-Verfahren 2019]. The D'Hondt method has numerous mathematical advantages but is also known for slightly favoring large parties over small ones [Balinski and Young 1979, Lijphart 2003, Pukelsheim 2007, D'Hondt-Verfahren 2019].

The idea of the method is as follows. The party with the most electoral votes ‘purchases’ its first parliamentary seat by ‘spending’ a half of the total votes it received in the election. At each successive step, the currently ‘richest’ party acquires a seat. For its first seat, the party ‘pays’ an amount that leaves it with only $1/2$ of its original number of votes; then for its next seat it pays an amount that leaves it with only $1/3$ of its original number of votes, then $1/4$, and so on. At every round, however, the next seat goes up to the ‘highest bidder’ — the party with the most votes to spend — until the seats are exhausted. In this way, the biggest winners can acquire several seats before a minor party ever gets to make its first ‘purchase’.

Thus, to allocate the next available seat, the algorithm finds the party i with the *largest*

remainder of votes:

$$\text{while } \sum_{i=1}^n s_i < S \quad \text{find } i : \max_{i=1, \dots, n} \underbrace{\left(\frac{V_i}{s_i + 1} \right)}_{\substack{\textit{i} \text{th party's} \\ \textit{remainder} \\ \textit{of votes}}} \Rightarrow s_i = s_i + 1, \quad (1)$$

where

$i = 1, \dots, n$ are labels of n parties eligible for parliamentary seats,

s_i is the number of seats that have already been allocated to the i party (initially $s_i = 0$),

S is the total number of parliamentary seats to be allocated, and

V_i is the total number of electoral votes that party i received at the election.

6.2 Webster/Sainte-Laguë Method

The Webster/Sainte-Laguë method strives to complete the same task as the D'Hondt method and is very similar to it. It is named after the American statesman Daniel Webster (1782–1852), who proposed it in 1832 for proportional allocation of seats in the United States congressional apportionment [Balinski and Young 1982], and the French mathematician André Sainte-Laguë (1882–1950), who independently rediscovered it and studied its properties [Sainte-Laguë 1910]. Together with the D'Hondt method, it is widely used worldwide, sometimes interchangeably.

In 1980, the German physicist and electoral expert Hans Schepers (1928), having studied the D'Hondt method used by the German Bundestag, discovered that it disadvantaged smaller parties and suggested an improved version equivalent to the Sainte-Laguë method [Pukelsheim 2002]. At first it was adopted only for certain Bundestag commissions, but since 2009 it has been used to allocate seats both in the German Bundestag and the European Parliament [Sainte-Laguë-Verfahren 2019].

The idea of the method is the same as that of d'Hondt, but the progression of 'payments' for the seats is different. The party with most electoral votes 'purchases' its first parliamentary seat by 'spending' $2/3$ of its votes. At each successive step, the seat goes to the currently 'richest' party, who 'pays' at first an amount that leaves it with only $1/3$ of its original votes, then an amount that leaves it with only $1/5$ of its original votes, then $1/7$, etc. The procedure continues as long as there are still seats to be apportioned. As one can

see, the biggest winners ‘spend’ their votes much faster than under the D’Hondt method, thereby giving way to smaller parties.

Correspondingly, the allocation algorithm is slightly modified. In its loop (1), the divisor $s_i + 1$ is replaced by $2s_i + 1$ as follows:

$$\text{while } \sum_{i=1}^n s_i < S \quad \text{find } i : \max_{i=1, \dots, n} \left(\underbrace{\frac{V_i}{2s_i + 1}}_{\substack{\textit{ith party's} \\ \textit{remainder} \\ \textit{of votes}}} \right) \Rightarrow s_i = s_i + 1 .$$

References

- [Balinski and Young 1979] Balinski M, Young HP (1979) Criteria for proportional representation. *Operations Research* 27(1): 80–95
- [Balinski and Young 1982] Balinski M, Young HP (1982) Fair representation: meeting the ideal of one man, one vote. Yale University Press, New Haven
- [Bundestwahlleiter 2017] Bundeswahlleiter [Federal Returning Officer] (2017) Bundestagswahl 2017: Ergebnisse der Wahl zum 19. Bundestag (Flyer). https://www.bundeswahlleiter.de/dam/jcr/3f3d42ab-faef-4553-bdf8-ac089b7de86a/btw17_ergebnisse_flyer.pdf. Cited 12 Oct 2019
- [D’Hondt 1882] D’Hondt V (1882) *Système pratique et raisonné de représentation proportionnelle*. Librairie C Muq quardt, Merzbach et Falk, éditeurs, libraires du roi et du cte de Flandre, Bruxelles
- [D’Hondt method 2019] D’Hondt method (2019). Wikipedia. https://en.wikipedia.org/wiki/D’Hondt_method. Cited 12 Aug 2019
- [D’Hondt-Verfahren 2019] D’Hondt-Verfahren (2019). Wikipedia (DE). <https://de.m.wikipedia.org/wiki/D’Hondt-Verfahren>. Cited 12 Aug 2019
- [Deutscher Bundestag 2019] Deutscher Bundestag [German Bundestag] (2019). Wikipedia (DE). http://de.wikipedia.org/wiki/Deutscher_Bundestag. Cited 13 Aug 2019
- [Finthammer 2018] Finthammer V (2018) Bericht des Rechnungshofs: Bundestag wird immer größer und teuer. Deutschlandfunk, 8 Oct 2018. https://www.deutschlandfunk.de/bericht-des-rechnungshof-wird-immergroesser-und.1766.de.html?dram:article_id=429962. Cited 13 Oct 2019

- [Lijphart 2003] Lijphart A (2003) Degrees of proportionality of proportional representation formulas. In: Grofman B, Lijphart A (eds) Electoral laws and their political consequences. Algora Publishing, New York: 170–179
- [Pukelsheim 2002] Pukelsheim F (2002) Die Väter der Mandatzuteilungsverfahren. Spectrum der Wissenschaft, Sep 2002: 83. Internet version: Die Drei in Deutschland verwendeten Mandatzuteilungsmethoden und ihre Namenspatrone. math.uni-augsburg.de/htdocs/emeriti/pukelsheim/2002g.html. Cited 14 Oct 2019
- [Pukelsheim 2007] Pukelsheim F (2007) Seat bias formulas in proportional representation systems. Paper at the ECPR General Conference, Pisa, September 6–8, 2007.
- [Roßner 2016] Roßner S (2016) Die Grenzen des Wachstums. Legal Tribune Online 02.11.2016 ito.de/recht/hintergruende/h/bundestag-mitglied-begrenzung-wahlrecht-landeslisten-zweitstimmen/. Cited 12 Oct 2019
- [Sainte-Laguë 1910] Sainte-Laguë A (1910) La représentation proportionnelle et la méthode des moindres carrées. Annales scientifiques de l'École normale supérieure 27: 529–542. Abstract in: Lijphart A, Gibberd RW (1977) Thresholds and payoffs in list systems of proportional representation, Appendix 2. Eur J Pol Res 5: 219–244
- [Sainte-Laguë-Verfahren 2019] Sainte-Laguë-Verfahren (2019) Wikipedia (DE). <http://de.m.wikipedia.org/wiki/Sainte-Lagu%C3%AB-Verfahren>
- [Spiegel online 2019] Spiegel online (2019, 20 Sep) Staatsrechtler halten Bundestag für zu groß. spiegel.de/politik/deutschland/bundestag-staatsrechtler-halten-709-abgeordnete-fuer-zu-viel-a-1287739.html. Cited 12 Oct 2019
- [Tangian 2014] Tangian A (2104) Mathematical theory of democracy. Springer, Berlin–Heidelberg.
- [Webster/Sainte-Laguë method 2019] Webster/Sainte-Laguë method (2019) Wikipedia. https://en.wikipedia.org/wiki/Webster/Sainte-Lagu%C3%AB_method. Cited 13 Oct 2019
- [Zeit online 2019] Zeit online (2019, 21 Sep) Thomas Oppermann will Wahlrechtsreform noch dieses jahr. zeit.de/politik/deutschland/2019-09/bundestag-thomas-oppermann-einigung-wahlrechtsreform-verkleinerung?print. Cited 12 Oct 2019

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