Voting over Resource Allocation:
Nash Equilibria and Costly Participation

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Abstract

The subject of the present thesis is voting over resource allocation with single-peaked preferences, using the aggregation rules of the mean and median. An important and well-known property of the median rule is its strategy-proofness, which implies that truth-telling is a (weakly) dominant strategy. However, there exists a multitude of other Nash equilibria, including Pareto-inefficient outcomes. The mean rule involves strategic voting, but contrary to the median rule, each Nash equilibrium is unique and Pareto-efficient.

In part I of this thesis, we show that the above-mentioned characteristics of the voting rules are only valid for one-dimensional allocation problems. In multi-dimensional allocation settings, the median rule is indeed strategy-proof; however, the resource constraint might not be satisfied and necessary adaptations might result in inefficient equilibria or induce strategic voting. Moreover, voting for multi-dimensional allocations under the mean rule might also entail Pareto-inefficient Nash equilibria. Besides a theoretical analysis of Nash equilibria, we conduct an experimental study of voting behavior. The results indicate strategic voting behavior under both rules, which occurs more frequently with an increased number of repetitions as well as a higher degree of information.

Parts II and III of the thesis address costly participation. Prior to a potential voting decision, individuals face a decision on participation or abstention from the election. The introduction of participation costs and the two-stage decision process of each individual increases the complexity of strategic decision making.

Part II demonstrates the existence of multiple Nash equilibria, frequently with different sets of participants, and distinguishes between the mean and the median rule. For either rule, we derive a complete classification of all Nash equilibria depending on participation costs.

Due to the existence of multiple Nash equilibria and the complexity of determining these equilibria, we hypothesize that the individual participation decisions are in practice not driven by equilibrium considerations. We rather hypothesize in part III of this thesis that individuals base their decision on an approximate estimate of their impact on the social outcome and that participation decisions are also driven by risk attitude. In a field experiment on voting over resource allocation, we study participation depending on the voting rule and elicit the participants' beliefs about the impact on the outcome.

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1 Introduction

The social coexistence of human beings involves decision making in everyday life. The process and result of decision making often does not only comprise single individuals but larger groups or even a society as a whole. Examples include a population that votes for a parliament in their country, a community that decides on where to build a new shopping mall or committees that determine future investments. Even in small groups like families, friends or colleagues collective decisions are made on a daily basis - for example on where to spend the next holiday or where to go for lunch. In economics, social choice theory is the field that refers to the question on voting over alternatives and on the aggregation of individual preferences in a collective decision making process.

This thesis addresses voting over resource allocations. In this setting, subjects may vote on the allocation of a resource by submitting a fraction to be used for a given public project. We assume that projects are neutral: no additional valuation is given when a project is allocated no resource or all of it and the resource is not wasted when a project is allocated only a small fraction of it. Voting over resource allocation implies that subjects, which are part of an electorate, may vote for a specific allocation and theoretically, there exists an infinite number of possible votes. As soon as at least two subjects submit a vote, one has to think of how to aggregate these votes and how to constitute a social outcome. We concentrate on two 'focal' voting rules: the mean and the median rule. Under the mean rule, the social outcome is given as the sum of all votes divided by the number of voters. By contrast, the median rule allocates to each project the median value of all submitted votes. Probably the most intuitive resource to be allocated on public projects is money and in this thesis we refer to a budget allocation problem. However, there exist several other applications to use the mean or the median rule, like voting over locations or the size of an investment. As Louis et al. (2019) point out, the LIBOR, which is an important financial benchmark and serves as the key index in the global financial markets, is determined in an election and calculated by a trimmed mean.

In part I of this thesis, we study mainly the voting decision. We explain that the optimal voting behavior is different depending on the underlying rule. We show that voting over resource allocation on multiple projects comes with a set of challenges, like the requirement of satisfying the budget constraint and the existence of multiple or inefficient Nash equilibria. In two laboratory experiments, we analyze individuals' voting behavior in small groups and consider effects of (i) the distribution of most preferred allocations, (ii) the degree of information subjects are provided before the vote, (iii) the number of periods in repeated votes, and (iv) the voting rule.

In a next step, we address to participation in voting over simple resource allocation. With the knowledge on the individual voting decisions, especially strategic voting in Nash equilibria, we extend the voting game by a participation decision. We assume that voting is costly and, for simplicity, that participation costs are identical for all subjects. Introducing participation costs into the model further increases the complexity of the theoretical analysis.

Part II of this thesis approaches the extended model of costly participation. Subjects face two simultaneous decisions: the participation decision and - if it is positive - the voting decision. We show that the corresponding participation games in general have multiple Nash equilibria, frequently with different sets of participants and the equilibria differ for the voting rules. For the mean and the median rule, we derive a complete classification by participation costs for all Nash equilibria.

Due to the existence of multiple equilibria and the complexity of the task to determine the equilibria, we hypothesize in part III of this thesis that the individual participation decision is in practice not driven by equilibrium considerations but by other factors: the impact of a vote on the social outcome (or the belief about the impact) and the risk attitude. While the impact of a vote on the social outcome under the mean rule is small for large electorates, it is certain and always greater than zero. By contrast, the impact under the median rule has large variance. This raises the question whether the difference in the individual impact has effects on the participation rates under different voting rules. In a field experiment, we test whether, and how, voter turnout varies with the voting rule. To this end, we conduct a vote using either the mean or the median rule to determine the allocation of a donation on two public projects on the university campus. Our focus lies on the voter turnout under either rule and we additionally consider the role of impact and risk attitude. Subsequent to the vote, we implement a survey in order to elicit beliefs about the allocation result, about the participation rate and about the impact of the individual's vote on the social outcome. The field experiment on costly participation finalizes this thesis, which provides new theoretical and experimental insights on voting and participation in voting over resource allocation.

Part I

Voting over Resource Allocation

2 Motivation

Imagine the following situation: A volleyball team drew profit over the previous season and is planning to spend it on a number of projects. These projects are public, meaning that everyone in the team has access to them, like new balls, jerseys or training devices.[1] Since it is important to the volleyball team that there are no disputes, they want to let everyone in the team take part in the decision on how to allocate the budget. Supposing that each team member has an opinion on how the budget should be allocated and one allocation is preferred the most, the team conducts a vote. Each team member might reveal the most preferred allocation of the total budget on each project and these votes will be aggregated to result in a social outcome. In this situation, two questions arise: How should the votes be aggregated, i.e. which voting rule should the team use? Given the voting rule, are there incentives not to vote for the true most preferred allocation?

One way to answer these questions is to consider the *efficiency* of voting rules. A feasible rule to aggregate votes over resource allocation is the median rule. Under the assumption of single-peaked preferences[2], Cason et al. (2006) state that "Many [...] strategy-proof mechanisms [...] have Nash equilibrium outcomes that do not coincide with the dominant strategy equilibrium outcome. These Nash equilibrium outcomes are frequently socially undesirable."

Consider three team members that vote on the allocation of 100 Euros on two public projects: balls and jerseys. Assume that their most preferred allocations for balls are 25, 40 and 65 Euros, respectively. This automatically involves that the most preferred allocations for jerseys are 75, 60 and 35, as a total of 100 Euros should be distributed. A strategy-proof voting rule implies that truth-telling is a (weakly) dominant strategy. It is well known that the median is a strategy-proof voting rule (Moulin, 1980) and might be used for aggregating the votes. This implies that under the median rule, truth-telling is a possible Nash equilibrium. Given that each of the three team members reveals the true most preferred allocation, the social choice under the median rule is to allocate 40 Euros on balls and 60 Euros on jerseys, respectively. In this equilibrium, one team member receives exactly the most preferred outcome and therefore given truth-telling of the others, this person has no incentive to deviate from revealing the true most preferred allocation. The team member who wants to allocate 25 Euros on balls would favor a social outcome closer to the most preferred allocation. Nevertheless, he or she is only able to change the social outcome by voting for a higher allocation on balls than the median-value. However, this strategy deteriorates the situation, as the outcome would move further away from the most preferred allocation. The same applies to the third team member, who might only influence the social outcome by increasing its distance to the most preferred allocation. Hence, truth-telling is a weakly dominant strategy resulting in a Nash equilibrium.

Nevertheless, truth-telling is not the only Nash equilibrium. As long as the team members with most preferred allocations unequal to the median stay within their rank, i.e. vote for an allocation equal to or higher than the median if the most preferred allocation exceeds it and an equal or lower allocation if the allocation undercuts it, every combination of these votes and truth-telling of the median voter results in

[1] Please note that we are not addressing on a typical public goods problem, meaning that thresholds are immaterial.
[2] We assume single-peaked preferences throughout the thesis. The property of metric single-peaked preferences will be explained in detail in chapter 3.3.

a Nash equilibrium with the same outcome as truth-telling of all team members. Therefore, voting 10, 40 and 70 Euros for balls would be a Nash equilibrium as well.

Since the median minimizes the sum of the absolute distances between the most preferred allocations and the social outcome, the median should also maximize the total utility for single-peaked preferences. But what about the socially undesirable Nash equilibria Cason et al. (2006) were writing about? Suppose that each one of the three team members votes for an allocation of no budget on balls and the total budget on jerseys. The social outcome is a median of 0 Euros on balls and 100 Euros on jerseys, which is worse for all team members as compared to the outcome under truth-telling. However, no team member has an incentive to deviate from his or her vote, as the median remains unchanged given the other votes; denoting that 'bad' or Pareto-inefficient Nash equilibria exist when using the median rule. In fact, every situation in which all three team members vote for the same allocation constitutes a Nash equilibrium under the median rule, including several Pareto-inefficient outcomes.

Yamamura and Kawasaki (2013) are providing a solution to this problem: "If there exist a multitude of bad Nash equilibria for a given [...] median rule, then using [...] average rules can be a comparable alternative [...]." So, again, if we consider our example from the volleyball team and use the average or mean rule to aggregate the votes, a unique and Pareto-efficient Nash equilibrium under complete information exists, in which at most one team member votes for the true most preferred allocation (Renault and Trannoy, 2005). In this equilibrium, the team member with the lowest preferred allocation on balls votes for 0 Euros, whereas the one with the highest preferred allocation votes for 100 Euros. In order to receive a Nash equilibrium outcome that corresponds to the most preferred allocation of the 'middle-voter', this team member has to vote for 20 Euros on balls, resulting in an average of $\frac{0+20+100}{3} = 40$ Euros.

So far, this is what common literature tells us about the mean or the median rule. But what happens, if we increase the number of projects and address to multi-dimensional voting problems? When using the median rule, truth-telling is once more a Nash equilibrium. In addition, as long as the team member with the median-value of all most preferred allocations for one project states the truth and the others stay within their rank, every combination of votes results in a Nash equilibrium. However, another efficiency problem might occur: Even though the sum of absolute deviations from the most preferred allocation is minimized, the social outcome does not necessarily satisfy the total budget. Besides, 'bad' Nash equilibria that are worse for every team member as compared to the social outcome under truth-telling do not disappear when using the median rule on more than two public projects.

Therefore, when following the suggestion of Yamamura and Kawasaki (2013) again and making use of the mean rule, we already stated that the allocation problem on two public projects yields a unique and Pareto-efficient Nash equilibrium. What might be surprising is the fact that in multi-dimensional budget allocation problems, there might exist multiple Nash equilibria when using the mean rule, including 'bad' ones.

The following chapters of this part of the thesis are organized as follows. We begin by introducing the theoretical model, including the set of feasible allocations, the voter's preferences as well as the voting rules. We focus on the mean and median-based rules. The theoretical part also includes a description of the individual strategies and provides for each of the voting rules the Nash equilibria in a complete information context. We further classify the social outcome and thereby define among others criteria for efficiency. The theoretical part is followed by two experiments, which seek to test hypotheses on voting behavior within rules and also differences in voting across rules. While on the aggregated level, we do find very low shares of outcomes that correspond to the outcome in a Pareto-efficient Nash equilibrium, the individual shares of strategic voting are high for the mean and the median rule.

3 Theoretical Model of the Voting Game

In this chapter, we describe the general budget allocation problem in the form of a voting game. We define the mean and the median rule and present two different modifications of the median such that the budget constraint is satisfied. We further depict the individual strategies of the voting rules on multi-dimensional budget allocation problems that result in Nash equilibria for single-peaked preferences. These equilibria are evaluated regarding efficiency and social welfare.

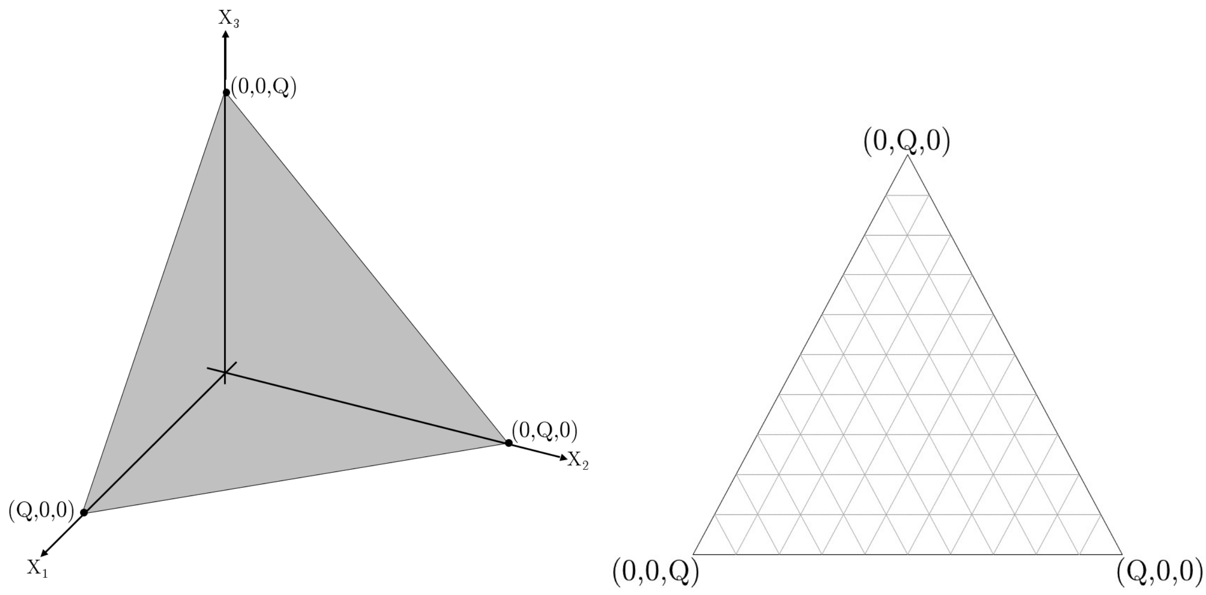
3.1 Basic Definitions

Consider a set of individuals $I = \{1, ..., n\}$ that decide in a voting game on the allocation of a budget Q on m public projects $J = \{1, ..., m\}$. The set of feasible allocations of Q on the projects is given by $\mathcal{B} := \{x \in \mathbb{R}^m_{\geq 0} | \sum\limits_{j \in J} x^j = Q\}$, which contains non-negative values and allocations for which the sum of the values for every project corresponds to the total budget. We call an allocation $(p_i)_{i \in I}$ the peak of individual i, if it is i's most preferred allocation. The peak of individual i comprises preferred allocations for all projects j: $p_i = (p_i^1, \ldots, p_i^m) = (p_i^j)_{j \in J} \in \mathcal{B}$. The peak distribution contains the peaks of all individuals and is defined by the vector of peaks $p = (p_1, \ldots, p_n) = (p_i)_{i \in I} \in \mathcal{B}$.

The preferences of the individuals are metric single-peaked, as described in detail later in chapter 3.3. In order to decide on the social outcome of the allocation, every individual submits a vote $q_i = (q_i^1, \ldots, q_i^m) = (q_i^j)_{j \in J} \in \mathcal{B}$ that might differ from his or her peak. The vector of all votes is $q = (q_1, \ldots, q_n) = (q_i)_{i \in I}$. We define the distance between individual i's peak and the social outcome $x(q) = (x^1(q), \ldots, x^m(q)) \in \mathcal{B}$ as the sum of their absolute deviation in every project: $d(p_i, x(q)) := \sum\limits_{j \in J} |p_i^j - x^j(q)|$. As we claim $p_i, q_i, x(q) \in \mathcal{B}$, we exclude negative or budget-exceeding peaks, votes and social outcomes and make sure that the budget constraint is satisfied.

3.2 Feasible Allocations for $m = 3$: The Simplex

For the following chapters, let the number of public projects be $m = 3$. The budget constraint is graphically given by a triangle in a 3-dimensional space, see figure 3.1a. This simplex contains all possible allocations, at which the budget constraint is satisfied, i.e. all allocations in $\mathcal{B}$. The vertices are the allocations, at which the total budget Q is distributed on exactly one project and no budget on the other two projects. On the edges between the vertices, a budget of zero is allocated on one project. As can be seen in figure 3.1b, it is possible to represent the 3-dimensional simplex in a 2-dimensional space. By doing so, it is obvious that a shift from one point to another inside the simplex means reallocating the budget between the projects, such that the budget constraint is not violated.



(a) 3-dimensional space

(b) 2-dimensional space

Figure 3.1: The simplex

3.3 The Voters' Preferences

In order to determine the best strategy of an individual in the voting game, the preferences need to be specified. We use the Manhattan or L_1 distance function that sums up the project-wise absolute differences between two allocations $a, b \in \mathcal{B}$. This sum is defined as distance $d(a, b)$ between the two allocations a and b. The following definition is based on Lindner (2011).

Definition 1 (**Metric single-peaked preferences**). *Preference rankings are* ***metric single-peaked*** *if there exists a unique peak p_i and for any two allocations $a, b \in \mathcal{B}$ with $d(p_i, a) \leq d(p_i, b)$, voter i (weakly) prefers a over b.*

In the following, we assume that every voter i has a metric single-peaked preference ranking. This implies that voters try to minimize the distance between the social outcome and their true peak, $d(p_i, x(q))$ meaning that a lower distance from the peak results in a higher utility and the utility maximum is reached when $p_i = x(q)$. Moreover, we make sure that the budget allocation of every project is reflected symmetrically in the utility function. Thus, a deviation of $x(q)$ from p_i in project 1 has the same effect on the utility as an equally high deviation from p_i in project 2 or 3. Figure 3.2 shows the indifference curves of individual i given the peak p_i. The highest payoff results from a social outcome equal to p_i and it decreases symmetrically in every coordinate with a higher distance from the peak, which is represented by the hexagon-shaped curves. A possible utility function of voter i is given by the negation of the distance between the social outcome and the peak:

$$u_i(p_i, x(q)) = -d(p_i, x(q)). \tag{3.1}$$

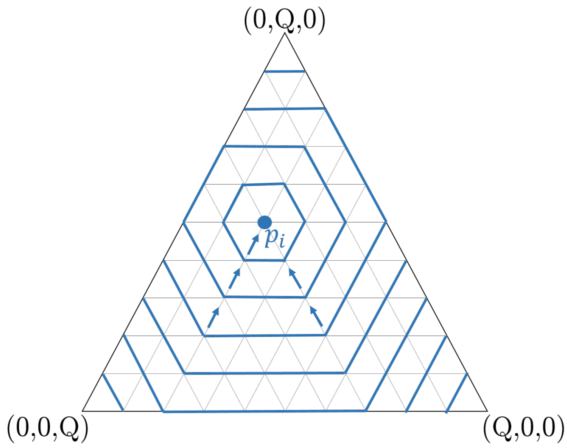


Figure 3.2: Metric single-peaked preferences

3.4 The Mean Rule

In a next step, we describe the voting rules that aggregate the votes q and provide the social outcome $x(q)$. The first rule to consider is the mean rule. The social outcome under the mean rule is calculated by adding up the votes of all individuals separately for each project and dividing these sums by the number of votes:

$$Mean(q) = \frac{1}{n}\sum_{i=1}^{n} q_i. \tag{3.2}$$

Example 1. $Q = 100;\ q_1 = (20, 50, 30);\ q_2 = (10, 40, 50);\ q_3 = (0, 0, 100)$
The social outcome of the voting game calculated by the mean rule is $Mean(q) = (10, 30, 60)$.

By construction, the social outcome under the arithmetic mean always satisfies the budget constraint, i.e. the sum of the mean-values per project corresponds to the budget that should be allocated. Another property of the mean rule is that voters may manipulate the outcome. Given different peaks, at most one individual votes for the true preferred allocation in a Nash equilibrium, as is shown later in chapter 7.2.3. While example 1 helps to understand the calculation of the outcome, we anticipate already at this stage that not all votes are part of a Nash equilibrium in the voting game.

3.5 The Median Rule

The median rule selects for every project the middle vote of all increased ordered votes $q^j_{[i]}$ if the number of individuals is odd or the average of the two middle votes if there is an even number of voters. Thus, the median $Med(q)$ consists of m coordinate-by-coordinate median-values:

$$Med^j(q) = \begin{cases} q^j_{[\frac{n+1}{2}]}, & \text{if } n \text{ is odd} \\ \frac{1}{2}\cdot(q^j_{[\frac{n}{2}]} + q^j_{[\frac{n}{2}+1]}), & \text{if } n \text{ is even.} \end{cases} \tag{3.3}$$

Example 2. $Q = 100;\ m = 3;\ q_1 = (70, 30, 0);\ q_2 = (10, 40, 50);\ q_3 = (20, 60, 20)$
The social outcome of the voting game calculated by the median rule is $Med(q) = (20, 40, 20)$.

A restriction to the median rule is the possibility that the coordinate-by-coordinate median-values do not satisfy the total budget, i.e. $\sum_{j=1}^{3} Med^j(q) \neq Q$. In the previous example 2, the total budget is undercut and therefore an adaptation of the median outcome is necessary.

An interesting study by Ehrhart et al. (2007) refers to budget processes and considers differences in government spending dependent on the sequence of budgeting decisions. The bottom-up process allocates the budget to each project first and then calculates the sum of the total budget. By contrast, the top-down process determines the total size of the budget in a first step - making sure that a potential budget constraint is satisfied. This study offers some interesting similarities to our model even though voters decide on the allocation using majority voting.

In this study, we focus on the normalized and the sequential median rule in order to satisfy the budget constraint. Further adaptations like the Condorcet-center rule or the inertial median rule can be found in Lindner (2011).

3.5.1 The Normalized Median Rule

The normalized median rule, as suggested by Nehring et al. (2008), chooses the element on the simplex, at which the values of the coordinates are in the same proportion to each other compared to the values of the median. Graphically, the normalized median for $m = 3$ is the intersection of the simplex and the line through the origin and the non-adapted median, as shown in figure 3.3. Computationally, the normalized median per project is determined by a multiplication of the corresponding median-value with the total budget divided by the sum of the median-values for all projects before adaptation. As the calculation of the normalized median is only possible if at least one project-wise median value is strictly positive, we adapt the rule by Nehring et al. (2008) and provide a case differentiation.

$$NMed^j(q) := \begin{cases} Med^j(q) \cdot \frac{Q}{\sum\limits_{j \in J} Med^j(q)}, & \text{if } \sum\limits_{j \in J} Med^j(q) > 0 \\ \frac{Q}{m}, & \text{else.} \end{cases} \tag{3.4}$$

Example 3. $Q = 100;\ m = 3;\ Med(q) = (20, 40, 20)$
After normalization, we get a social outcome of $NMed(q) = (20, 40, 20) \cdot \frac{100}{80} = (25, 50, 25)$.

The second case of equation 3.4 applies if $Med^j(q) = 0$ for all projects j, which is displayed in the following example.

Example 4. $Q = 100;\ m = 3;\ q_1 = (0, 0, 100);\ q_2 = (0, 100, 0);\ q_3 = (100, 0, 0)$
The median of all votes is $Med(q) = (0, 0, 0)$*, such that after normalization we get a social outcome of* $NMed(q) = (\frac{100}{3}, \frac{100}{3}, \frac{100}{3})$.

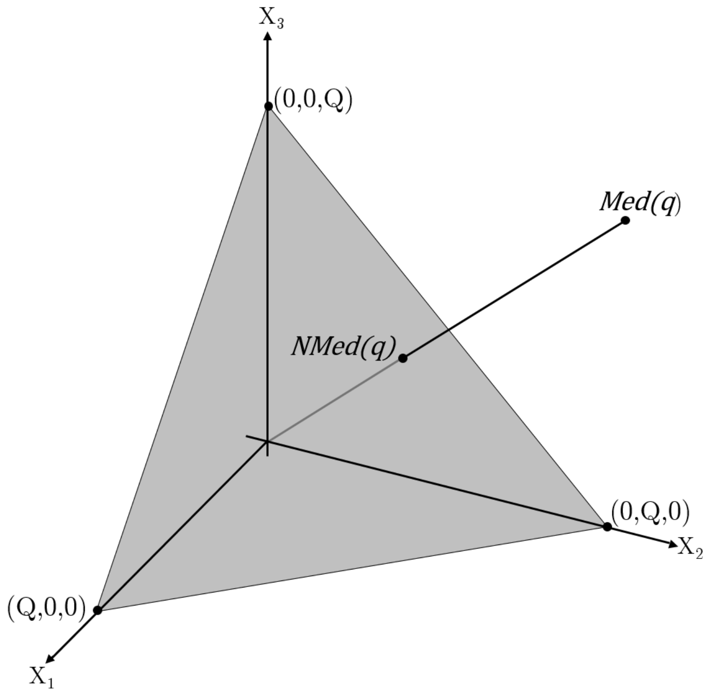


Figure 3.3: The normalized median

3.5.2 The Sequential Median Rule

If the sum of the median-values does not satisfy the total budget, another possible adaptation is given by the sequential median rule, as described in Lindner (2011). Here, the median-values for each project are adapted until the budget is satisfied according to a defined sequence. We fix the sequence for m by adapting the outcome first for project m and then, if necessary for project $(m-1)$, and so on.

If the sum of the median-values *undercuts* the budget, the remainder has to be added to the last project m, where the first elements of the sequence will obtain the original median-value:

$$\sum_{k=1}^{m} Med^k(q) < Q: \; SMed^j(q) := \begin{cases} Q - \sum\limits_{k=1}^{m} Med^k(q), & \text{if } j = m \\ Med^j(q), & \text{else.} \end{cases} \tag{3.5}$$

Example 5. $Q = 100;\; m = 3;\; Med(q) = (20, 5, 50)$
The sum of all median values is $\sum\limits_{k=1}^{3} Med^k(q) = 75 < Q$. *According to the sequential median rule we adapt the value of the third project and get a social outcome of* $SMed(q) = (20, 5, 75)$.

If the sum of the median-values *exceeds* the budget, the values will be assigned the original median-values (beginning with project 1) as long as the budget is satisfied. As soon as the original median-values exceed the budget, the sequential median assigns the appropriate value, such that the budget is satisfied. The sequential median rule might therefore adapt the values of all projects besides project 1 to zero. However, the adapted median-value of project 1 always corresponds to the original value, since all votes have to lie within the set of feasible allocations.

$$\sum_{k=1}^{m} Med^k(q) > Q: \; SMed^j(q) := \begin{cases} 0, & \text{if } \sum\limits_{k=1}^{j-1} Med^k(q) \geq Q \\ Q - \sum\limits_{k=1}^{j-1} Med^k(q), & \text{if } \sum\limits_{k=1}^{j-1} Med^k(q) < Q < \sum\limits_{k=1}^{j} Med^k(q) \\ Med^j(q), & else. \end{cases} \tag{3.6}$$

Example 6. $Q = 100;\ m = 3;\ Med(q) = (40, 75, 20)$
The sum of all median values is $\sum\limits_{k=1}^{3} Med^k(q) = 135 > Q$. *According to the sequential median rule we adapt the values of the third project to zero, because* $\sum\limits_{k=1}^{2} Med^k(q) = 115 > Q$. *For the second project,* $\sum\limits_{k=1}^{1} Med^k(q) = 40 < Q < 135 = \sum\limits_{k=1}^{3} Med^k(q)$*, such that* $SMed^2(q) = 100 - 40 = 60$*. The outcome under the sequential median rule is therefore* $SMed(q) = (40, 60, 0)$.

4 Individual Strategies

With the knowledge on the voting rules, one should think of possible strategies during the voting process. A simple and straightforward strategy is truth-telling: stating a vote q_i that is equal to the peak p_i. Nevertheless, for some individuals, there might be incentives to deviate from the truth and thereby affecting the social outcome to their benefit.

Definition 2 (**Strategic voting**). *A vote q_i that differs from the peak p_i and thereby reduces the distance between the peak and the social outcome $d(p_i, x(q))$ is called* ***strategic****.*

Definition 3 (**Nash strategy**). *A vote q_i is a* ***Nash strategy****, if it minimizes the distance between the peak and the social outcome given the votes of the other individuals. We denote a Nash strategy q_i^*.*

While a strategic vote always distinguishes from the true peak, a Nash strategy might include truth-telling as a Nash strategy is the best response to a given vector of votes q_{-i}. For the analysis of strategies that not only reduce but also minimize the distance between the peak and the social outcome, we assume perfect information on the true peaks of the other participants. Moreover, we premise that there are no manipulation costs, which among others implies that strategic voting itself does not affect the payoff, whereas a shift in the social outcome due to a changed vote does affect the individual utility.[3]

A Nash strategy is not necessarily unique but might comprise ranges of allocations. Due to the construction of the voters' preferences, given the votes of the others, there might be several best responses of an individual that result in the same utility. We will refer to voting according to the Nash strategy as 'Nash play'.

4.1 Strategic Voting under the Mean Rule

Under the mean rule, strategic voting is possible for the most peak distributions. Exceptions include equal peaks of all voters, where given truth-telling of the others voting for the true peak is a best response, or peaks that allocate zero budget to one or two projects such that due to the budget constraint no strategic voting is possible. Suppose there exist two individuals with peaks $p_1 = (30, 60, 10)$ and $p_2 = (20, 30, 50)$. Given that both state their true most preferred allocation, the social outcome is $Mean(q) = (25, 45, 30)$, resulting in distances from the peaks of 40 for each individual. Supposing that the first individual reallocates 10 units from the third to the second project, i.e. $q_1 = (30, 70, 0)$ the mean outcome changes to $Mean(q) = (25, 50, 25)$, decreasing the distance from the first individual's peak to 30 (and increasing the distance from the second one's peak to 50). Analogically, the second individual has an incentive to deviate from truth-telling and is able to vote strategically by shifting units from the first or second to the third project. We can not only state that strategic voting is possible, but also in which direction and to what extent.

[3] One might argue that the act of lying decreases the utility not directly but in a more subtle and moral way that is not observable or not quantifiable. We exclude negative utility effects through strategic voting.

Given the votes of the other subjects, q_{-i}, individual i can influence the social outcome within the option set $\mathcal{OS}_i(q_{-i})$, which is determined by every possible outcome given q_{-i} and any feasible vote q_i:

$$\mathcal{OS}_i(q_{-i}) = \{\beta \in \mathcal{B} \mid \exists\, q_i \in \mathcal{B} : x(q_i, q_{-i}) = \beta\}\,. \tag{4.1}$$

Obviously, the option set varies for different voting rules. For the mean rule, the option set depends on the 'weight' of individual i's vote and therefore on the number of total voters n in the voting game:

$$\mathcal{OS}_i(q_{-i}) = \left\{\beta \in \mathcal{B} \mid \exists\, q_i \in \mathcal{B} : \beta = \frac{1}{n} \cdot \left(\sum_{h \in I \setminus \{i\}} q_h + q_i\right)\right\}. \tag{4.2}$$

The option set can be displayed as a triangular in the simplex, figure 4.1 provides an example for $n = 5$.

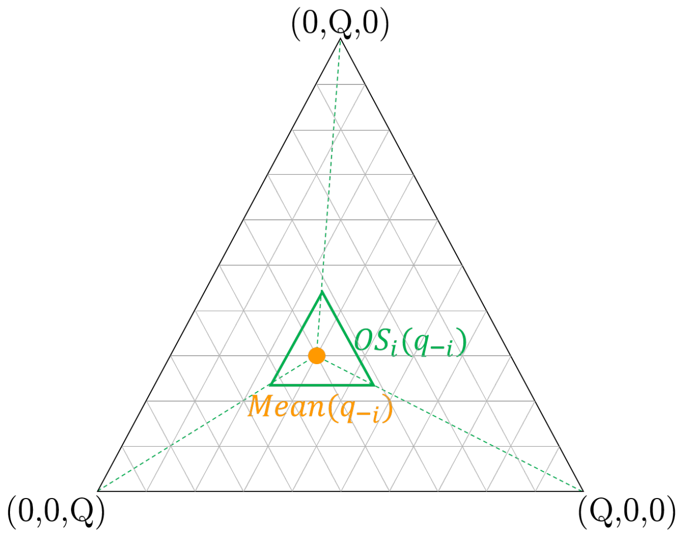
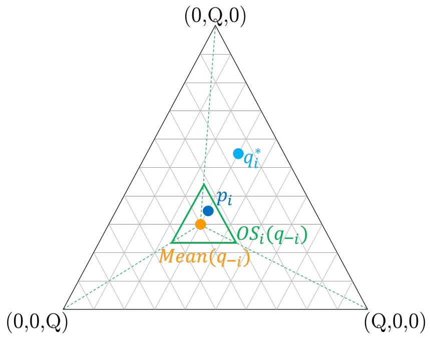


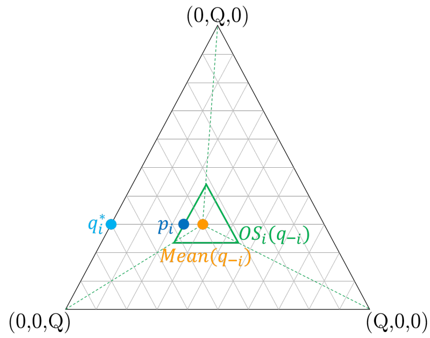
Figure 4.1: The option set

Individual i might face two different scenarios. In the first one, $p_i \in \mathcal{OS}_i(q_{-i})$, such that i's vote can equalize the social outcome and i's peak. Therefore, the triangular-shaped option set is stretched by the factor n on the size of the simplex. Given $Mean(q_{-i})$, individual i's best response q_i^* is to vote for an allocation such that $Mean(q) = p_i$. In this situation, q_i^* is strictly positive for every project as long as p_i lies on the inside of the option set (see figure 4.2a), zero for one project, if p_i lies on an edge of the option set (see figure 4.2b), and zero for two projects, if p_i lies on a vertex of the option set (see figure 4.2c). In summary, if $p_i \in \mathcal{OS}_i(q_{-i})$, $p_i = Mean(q)$ can be achieved by voting either $(q_i^j)^* > 0\ \forall j \in J$, $(q_i^j)^* = 0$ for one j or $(q_i^j)^* = 0$ for two j, depending on the location of the peak in the option set. If $p_i \in \mathcal{OS}_i(q_{-i})$, the best response of i is to vote for $q_i^* = p_i \cdot n - Mean(q_{-i})$.

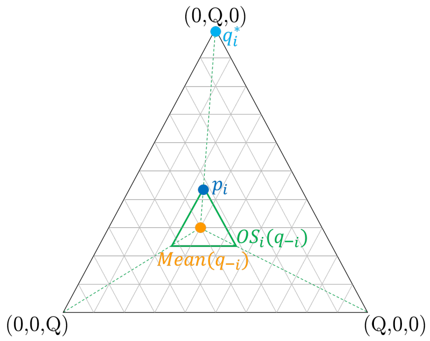
In the second scenario, $p_i \notin \mathcal{OS}_i(q_{-i})$, as displayed in figure 4.2d. Individual i might now vote strategically such that the distance between the social outcome and the peak is minimized but given $Mean(q_{-i})$ the peak will always differ from the outcome. In order to achieve the lowest possible distance, individual i submits a vote q_i^*, which leads to a mean outcome that is tangent to i's highest possible hexagon-shaped indifference curve. The best response is unique, if the closest tangent is a vertex of the option set. In this case, $(q_i^j)^* = Q$ for one j. If the intersection between the indifference curve and the option set is a line segment, the optimal choice of individual i is not unique, since he or she might vote for a set of allocations that all result in social outcomes yielding equal utility. Here, a best response is voting zero for at least one project. In any case, the option set comprises a multitude of allocations. Therefore, each vote has an impact on the social outcome and even a slight deviation of a vote changes the social outcome under the mean rule.



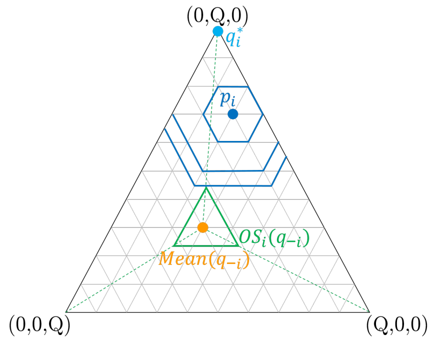
(a) Peak within option set



(b) Peak on edge of option set



(c) Peak on vertex of option set



(d) Peak outside of option set

Figure 4.2: Option sets and Nash strategies

4.2 Strategic Voting under the Median Rule

In order to determine the Nash strategies of the median rule, a definition of pivotality and its distinctions is necessary. We give a definition for pivotality regarding votes but it applies to peaks as well.

Definition 4 (Pivotal vote). *Given a vector of votes $q = (q_i)_{i \in I}$ and an allocation rule that yields $x(q)$. The vote of individual i is* ***pivotal*** *regarding this vector if any deviation from q_i affects the social outcome.*

Example 7. $Q = 100;\ q = (q_1, q_2, q_3) = (50, 70, 90)$
Given the median rule, any deviation $\varepsilon > 0$ from q_2, i.e. $q_2 \pm \varepsilon$, will change the social outcome, whereas $q_1 \pm \varepsilon$ and $q_3 \pm \varepsilon$ will not affect the outcome for $\varepsilon \leq 20$. Hence, q_2 is pivotal regarding q.

Definition 5 (Semi-pivotal vote). *Given a vector of votes $q = (q_i)_{i \in I}$ and an allocation rule that yields $x(q)$. The vote of individual i is* ***semi-pivotal*** *regarding this vector if the change of the social outcome due to any deviation from q_i is limited to one direction.*

Example 8. $Q = 100;\ q = (q_1, q_2, q_3) = (50, 70, 70)$
Given the median rule, the social outcome is only affected by $q_2 - \varepsilon$ and not by $q_2 + \varepsilon$ for any deviation $\varepsilon > 0$. The same holds for q_3, such that q_2 and q_3 are semi-pivotal regarding q.

Note that under the mean rule, each vote is always semi-pivotal and always pivotal if $0 < x(q_{-i}) < Q$. With the definitions on pivotality, we are able to classify votes under the median rule as strategic and find the Nash strategies. For a one-dimensional budget allocation problem and an odd number of voters, none of the Nash strategies under the median rule is a strategic vote. Since the median rule for odd voter quantities is strategy-proof and therefore truth-telling is a weakly dominant strategy (Moulin, 1980), strategic voting is never part of an equilibrium as it is not possible given the votes of the others. While truth-telling is always a Nash strategy for $m = 2$ and an odd number of voters, strategic voting might be possible for $m = 3$ and the normalized median rule.

Proposition 1. *The normalized median rule is not strategy-proof for $m = 3$ (Lindner, 2011).*

Proof. Assume that strategic voting is not possible. The following example provides a contradiction. Let $Q = 100, p_1 = (40, 20, 40), p_2 = (30, 10, 60), p_3 = (30, 70, 0), p_4 = (60, 10, 30)$, and $p_5 = (0, 70, 30)$. p_1 is pivotal regarding p in the second coordinate, whereas p_2 and p_3 are semi-pivotal in the first, and p_4 and p_5 in the last coordinate. Truth-telling of all individuals leads to a social outcome of $Med(q) = (30, 20, 30)$, which becomes $NMed(q) = (37.5, 25, 37.5)$ after normalization. However, truth-telling is not a Nash strategy for each individual. Given truth-telling of the others, individual 1 is able to vote strategically. Since p_1 is pivotal in the second project, voting for 15 instead of 20 and for a value equal to or greater than 30 for the other projects, i.e. $q_1 = (\geq 30, 15, \geq 30)$ with $q_1 \in \mathcal{B}$, yields a social outcome $Med(q) = (30, 15, 30)$, or $NMed(q) = (40, 20, 40)$, which is equal to the peak p_1. Hence, truth-telling is not a (weakly) dominant strategy for voter 1 and the normalized median rule is not strategy-proof. For the other individuals in this example, strategic voting is not possible. □

For the sequential median rule, the following proposition and its proof can be found in Lindner (2011).

Proposition 2. *Under metric single-peaked preferences, strategic voting is not possible under the sequential median rule.*

Given an even number of voters, strategic voting might be possible even before normalization if a vote is pivotal regarding the vector of all votes. The median rule determines the outcome by the average of the two middle votes, leading to best responses different from truth-telling for distinct peaks. Strategic voting is similar to the mean rule but with a limitation on the next ranked voters and not the extremes. In part I, we focus on odd numbers of voters, such that strategic voting under the median rule without adaptation is excluded. In part II we describe in detail the strategies and Nash concepts for an even number of voters under the median rule and $m = 2$.

5 Nash Equilibria of the Voting Game

After defining Nash strategies for each individual i and a given vector of votes q_{-i}, the next logical step is to define the Nash equilibria of the voting game.

Definition 6 (**Nash equilibrium**). *A vector of votes q constitutes a **Nash equilibrium** in the voting game, if it contains solely Nash strategies. We denote a Nash equilibrium $q^* = (q_1^*, \ldots, q_n^*)$.*

We study the voting game in a complete information setting and provide propositions on the structure of the Nash equilibria under both voting rules.

5.1 Nash Equilibria of the Mean Rule Voting Game

We already know that for $m = 2$ and full information the mean rule yields a unique Nash equilibrium (Renault and Trannoy, 2005). Block (2014) develops an algorithm to calculate the Nash strategies. She finds that for $m = 2$, at most one individual votes for a strictly positive allocation on both projects. We will proof that this likewise holds for $m = 3$.

Proposition 3. *Given a mean rule voting game on $m = 3$ public projects. In every Nash equilibrium, if all individuals have different peaks, at most one individual votes for a strictly positive amount of every public project, i.e. for at most one voter i, $(q_i^j)^* > 0$ for all j.*

Proof. In order to proof proposition 3, consider a situation, in which *two* individuals vote for a strictly positive amount of every public project. Since we assume different peaks, at most one individual's peak is equal to the social outcome. If the social outcome is unequal to the peak, the mean value deviates from the preferred allocation in at least two projects. Moreover, in at least one project the mean-value exceeds the own preferred allocation, since the total budget has to be allocated by the peak as well as by the social outcome ($p_i, Mean(q) \in \mathcal{B}$). In this situation, strategic voting is possible by a shift of the allocation and voting for a smaller amount for these projects. The other individual, which also voted for a strictly positive amount of every project, can also vote strategically and will shift the vote accordingly. As long as the outcome is not equal to one of the peaks, both individuals have an incentive to vote for a smaller amount in the project where $p_i^j < Mean^j(q)$, until the smallest possible allocation on this project is reached, namely zero. Therefore, in a Nash equilibrium, there is maximal one individual, who does not vote zero for at least one project: the individual, that can achieve a social outcome equal to his or her most preferred allocation for every project. Thus, given distinct peaks of all individuals, at most one individual votes $(q_i^j)^* > 0$ for all j in a Nash equilibrium and at least $(n-1)$ individuals vote zero for at least one project. $\square$

We showed that in every Nash equilibrium at least $(n-1)$ individuals do vote for zero in at least one project. In games with multiple Nash equilibria, Schelling (1980) introduces the concept of focal points. Focal points are solutions with outstanding character, such that players expect of others to play the prominent strategy. In terms of our concrete voting game, we say that a Nash strategy is focal if it allocates the highest possible amount Q to exactly the project with the highest peak value.

Definition 7 (Focal strategy). *Given a peak* $p_i = (p_i^j)_{j \in J}$. *Let* $j = [m]$ *be the project with the highest value in* p_i *where* $[m]$ *is distinct, i.e.* $p_i^{[m]} > p_i^k \ \forall k \in J \setminus [m]$. *A Nash strategy* q_i^* *is* ***focal****, if it allocates the total budget on the project* $j = [m]$*, i.e. if* $(q_i^{[m]})^* = Q$ *and* $(q_i^k)^* = 0 \ \forall k \in J \setminus [m]$.

Obviously, focal strategies only exist for specific peak distributions. While one might construct constellations of q_{-i} in which voting for Q in any project is a Nash strategy of individual i, the highest value might not be distinct and thus, no focal strategy exists.

In the following, we describe Nash equilibria for specific peak distributions and provide a classification of these equilibria for the underlying peak distributions. We consider three individuals that have a most preferred allocation that is greater than $\frac{Q}{3}$ in different projects.

Proposition 4. *Consider three individuals* $f, g, h \in I$, $f \neq g \neq h$ *with distinct peaks for three projects* $m = \{1, 2, 3\}$. *Let the most preferred allocation for each individual exceed* $\frac{Q}{3}$ *in different projects. W.l.o.g. let* $p_f^1, p_g^2, p_h^3 > \frac{Q}{3}$. *Further, let the most preferred allocations of all other projects be below* $\frac{Q}{3}$*, i.e.* $p_f^{2,3}, p_g^{1,3}, p_h^{1,2} < \frac{Q}{3}$.

a) The outcome in every Nash equilibrium is $Mean(q^*) = (\frac{Q}{3}, \frac{Q}{3}, \frac{Q}{3})$.

b) The Nash equilibrium is unique and each individual has a focal strategy.

Proof. W.l.o.g. let $p_1 = (< \frac{Q}{3}, < \frac{Q}{3}, > \frac{Q}{3})$, $p_2 = (< \frac{Q}{3}, > \frac{Q}{3}, < \frac{Q}{3})$ and $p_3 = (> \frac{Q}{3}, < \frac{Q}{3}, < \frac{Q}{3})$.

a) Suppose that there exists a Nash equilibrium with an outcome $Mean(q^*) \neq (\frac{Q}{3}, \frac{Q}{3}, \frac{Q}{3})$. This implies that the outcome has to be strictly greater than $\frac{Q}{3}$ in at least one project. W.l.o.g. let $Mean^1(q^*) > \frac{Q}{3}$.

Given that $Mean^1(q^*) > \frac{Q}{3}$, it follows out of the budget constraint that the outcome for at least one other project has to be lower than $\frac{Q}{3}$, w.l.o.g. suppose that $Mean^2(q^*) < \frac{Q}{3}$, such that $(q_i^2)^* < Q \ \forall \ i \ \in I$. However, as long as $Mean^1(q^*) > \frac{Q}{3}$ and $Mean^2(q^*) < \frac{Q}{3}$, individual 2 may vote strategically and increase the allocation of project 2 and decrease the allocation of project 1 until $(q_2^2)^* = Q$ and $(q_2^1)^* = 0$. Given that $(q_2^2)^* = Q$, $Mean^2(q^*) < \frac{Q}{3}$ is never the outcome in any Nash equilibrium for project 2 and due to the budget constraint, $Mean^1(q^*) > \frac{Q}{3}$ is never an outcome for project 1 in any Nash equilibrium.

We showed that $Mean^1(q^*) > \frac{Q}{3}$ is never an outcome in any Nash equilibrium and because of the symmetry of the peaks, $Mean^2(q^*) > \frac{Q}{3}$ and $Mean^3(q^*) > \frac{Q}{3}$ is also never an outcome in any Nash equilibrium. Therefore, the only Nash outcome that is possible for the given peak distribution is $Mean(q^*) = (\frac{Q}{3}, \frac{Q}{3}, \frac{Q}{3})$.

b) We showed that the Nash outcome is $Mean(q^*) = (\frac{Q}{3}, \frac{Q}{3}, \frac{Q}{3})$ in any Nash equilibrium. W.l.o.g. suppose that individual 1 does not have a focal strategy such that $(q_1^3)^* < Q$.

As long as $Mean(q^*) = (\frac{Q}{3}, \frac{Q}{3}, \frac{Q}{3})$, individual 1 improves by voting for a lower allocation on project 1 and/or project 2 and a higher allocation on project 3. This argument holds for any $(q_1^3)^* < Q$, such that individual 1 has the possibility to vote strategically by a higher allocation for project 3. Since $Mean^3(q^*) = \frac{Q}{3}$ is the outcome in any Nash equilibrium, the best response of individual 1 for any other Nash votes is playing the focal strategy and voting for $(q_1^3)^* = Q$. This implies that $(q_1^1)^* = (q_1^2)^* = 0$ and $(q_2^3)^* = (q_3^3)^* = 0$. Following the same argumentation, we can show that also individuals 2 and 3 have a focal strategy. Therefore, given that the outcome is $Mean(q^*) = (\frac{Q}{3}, \frac{Q}{3}, \frac{Q}{3})$ in every equilibrium, we show that the corresponding Nash strategies are unique. Obviously, if the Nash strategies are unique and a Nash equilibrium exists, it has to be unique. □

Figure 5.1 displays the conditions for the peaks of proposition 4 and the focal equilibrium under the mean rule. The proof in part a) holds for even larger sets of peak allocations, which are indicated by the dashed lines: $p_1 = (<\frac{Q}{2}, <\frac{Q}{2}, >\frac{Q}{3})$, $p_2 = (<\frac{Q}{2}, >\frac{Q}{3}, <\frac{Q}{2})$ and $p_3 = (>\frac{Q}{3}, <\frac{Q}{2}, <\frac{Q}{2})$. If the three peaks are located within these areas, then the Nash equilibrium is identical, however it is not necessarily focal, as focal strategies in this setting may not exist for peaks outside of the solid lines.

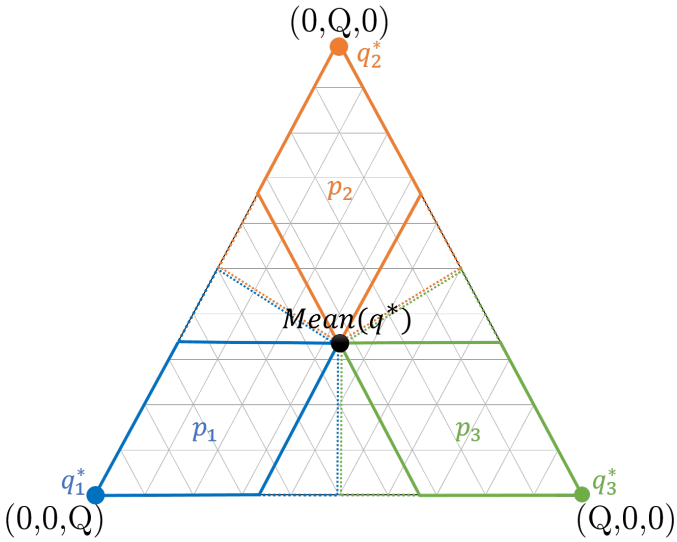


Figure 5.1: Focal Nash strategies under the mean rule

Block (2014) shows that under the mean rule, there exist Nash equilibria that are unique and efficient. By contrast, if the mean rule is applied to multi-dimensional allocation problems, this statement does not hold anymore. Under the mean rule and a voting game on more than two public projects, there might exist several Nash equilibria that lead to different social outcomes, even including some that are Pareto-inefficient and a Pareto-improvement for all voters would be possible. The following proposition is therefore a considerable contribution to the existing literature on mean voting for $m = 2$.

Proposition 5. *Given a mean rule voting game on $m = 3$ public projects. There exist Nash equilibria that are Pareto-inefficient.*

Proof. Consider a budget of $Q = 100$ that is allocated on $m = 3$ public projects in a voting game by using the mean rule. Let the peaks be as follows: $p_1 = (30, 10, 60)$, $p_2 = (10, 30, 60)$ and $p_3 = (20, 20, 60)$. A Nash equilibrium consists of the votes $q_1^* = (10, 0, 90)$, $q_2^* = (0, 10, 90)$ and $q_3^* = (50, 50, 0)$, with a social outcome equal to the preferred allocation of the third voter, $Mean(q^*) = (20, 20, 60)$. In this situation, no voter can improve by deviating from the votes stated before. The social outcome corresponds to the peak of voter 3, such that this individual cannot further improve. For individuals 1 and 2, the allocation of the third project corresponds to their most preferred allocation and the project in which the outcome exceeds the peak is already allocated the minimal vote of zero. Hence, no further improvement is possible and the votes constitute a Nash equilibrium. The sum of absolute deviations is 40, as $d(p_1, x(q^*)) = 20$, $d(p_2, x(q^*)) = 20$ and $d(p_3, x(q^*)) = 0$. However, there exists another Pareto-inefficient Nash equilibrium: $q_1^* = (90, 0, 10)$, $q_2^* = (0, 90, 10)$ and $q_3^* = (0, 0, 100)$, leading to a social outcome of $Mean(q^*) = (30, 30, 40)$. For individuals 1 and 2, the social outcome corresponds to their peak in project 1 and project 2, respectively. Therefore, no further improvement is possible in these projects. p_1^2 and p_2^1 are strictly smaller than the outcome for the corresponding projects. However, given the votes $q_1^2 = 0$ and $q_2^1 = 0$, no further decrease is possible and the votes are Nash strategies. Individual 3 would like to increase the allocation of project 3 and decrease the allocations for projects 1 and 2. Since individual 3 already votes for the extreme allocation, no further improvement is possible and the vote is a Nash strategy. In this Nash equilibrium, the absolute deviation from the peaks is higher for every voter (it is 40 for each voter), with a total sum of 120. Hence, a Pareto-improvement for all voters is possible and the Nash equilibrium is thus Pareto-inefficient. □

5.2 Nash Equilibria of the Median Rule Voting Game

Even though truth-telling is a (weakly) dominant strategy under the median rule with an odd number of voters (Moulin, 1980), the Nash equilibria are in general not unique. Moreover, inefficient equilibria exist as referred to earlier and stated e.g. by Cason et al. (2006). For $m = 3$ public projects and $n > 2$ voters, there exist multiple equilibria under the median rule as well. We will now provide the conditions for a truth-telling Nash equilibrium.

Proposition 6. *If there exists a peak that is (coordinate-wise) pivotal or semi-pivotal regarding the vector of peaks, then truth-telling is a Nash equilibrium of the median rule voting game.*

Proof. Consider a situation in which one voter is (semi-)pivotal regarding the peaks in the allocation of every public project. Given truth-telling of every individual, the median outcome is equal to the peak of the (semi-)pivotal voter. Since the peaks satisfy the budget, in this case, the same is true for the median outcome and an adaptation is not necessary. For the (semi-)pivotal voter, the social outcome is equal to the peak, leading to the highest possible payoff, such that revealing the true preferred allocation is the best strategy. For the non-pivotal or (other) semi-pivotal voters, strategic voting is not possible. The only way to change the social outcome is to vote for an allocation that increases the distance from the own peak, such that these individuals do not have an incentive to deviate from truth-telling. Provided the impossibility for every voter to improve the payoff by voting for a different allocation than the peak, truth-telling is a Nash equilibrium if one voter is (semi-)pivotal. □

Truth-telling of all voters is indeed a Nash equilibrium under the above-mentioned assumptions. However, any constellation of votes in which the individual with a (semi-)pivotal peak votes for the true peak and the other individuals vote within their rank constitutes a Nash equilibrium. Additionally, equilibria with inefficient outcomes exist, as defined in chapter 6. Under the median rule for $n > 2$, any outcome on the simplex is possible in a Nash equilibrium regardless of the peak distribution. In order to classify outcomes and strategies, we give a refinement of the Nash equilibrium concept and introduce Nash equilibria that are partially honest.

Dutta and Sen (2012) describe the concept of partial honesty. The authors define a partially honest player as someone who has a strict preference for truth-telling over lying when truth-telling does not lead to a worse outcome. We make use of the concept of partial honesty and refine the regular Nash equilibrium definition for our voting game. A partially honest subject is someone who tells the truth when truth-telling is a Nash strategy but the Nash strategy is not necessarily distinct. In a partially honest Nash equilibrium all subjects are partially honest.

Definition 8 (**Partial honesty**). *A **partially honest** Nash equilibrium is a Nash equilibrium, in which each non-truth-telling subject deteriorates by truth-telling.*

Under the median rule, best responses are in most cases not unique and Pareto-inefficient Nash equilibria in which all voters state the same allocation exist. When using the concept of partially honest voters, we are able to eliminate all Pareto-inefficient social outcomes. Another issue is the necessity to distinguish between odd and even numbers of voters. At this stage, we focus only on odd numbers of voters, where the median outcome is determined by the allocation of the median vote. We will later also describe the case for an even number of voters. Let truth-telling be a Nash equilibrium of the median rule voting game, i.e. all voters have the weakly dominant strategy to tell the truth. Since partially honest individuals always tell the truth when they are not deteriorated, the weakly dominant strategy of truth-telling is sufficient for arguing that truth-telling of all individuals is the unique partially honest Nash equilibrium. We further know that given the uniqueness of the equilibrium, the median outcome in a partially honest Nash equilibrium for an odd number of voters corresponds always to the peak of the median voter - and the peak of a voter is never a Pareto-inefficient outcome.

For multi-dimensional budget allocation problems, it might be necessary to adapt the median outcome. If the sum of the median-values does not satisfy the budget, an adaptation might be done by normalization as explained earlier in chapter 3.5. We showed in chapter 4.2 that strategic voting is possible in some situations under the normalized median rule.

Corollary 1. *When using the normalized median rule, truth-telling of all voters is not always a Nash equilibrium.*

Proof. The corollary is shown by the following counterexample.
Let $Q = 100, p_1 = (30, 50, 20), p_2 = (20, 30, 50)$, and $p_3 = (50, 20, 30)$. Truth-telling would yield a normalized median outcome of $NMed = (\frac{100}{3}, \frac{100}{3}, \frac{100}{3})$. Individual 1 and due to the symmetry of the peaks, also individuals 2 and 3, may vote strategically, e.g. by voting $q_1^1 = 26$ instead of 30 which leads to a social outcome of $Med(q) = (26, 30, 30)$ and therefore decreases the allocation result for project 1, as $NMed(q) \approx (30, 35, 35)$. Thus, truth-telling of all individuals is not a Nash equilibrium. Nevertheless, there exists a Nash equilibrium with the same outcome as under truth-telling, which results from the vector of votes $q^* = ((20, 60, 20), (20, 20, 60), (60, 20, 20))$. □

Since strategic voting under the sequential median rule is not possible, truth-telling is a weakly dominant strategy that results in a Nash equilibrium. Similar to the normalized median rule, the Nash strategies are in most cases not unique and may contain several allocations. Hence, there might exist a multiplicity of Nash equilibria with equal outcomes under the sequential median rule.

6 Classification of Social Outcomes

In our voting game, the only restriction on any outcome $x(q)$ is that it is feasible: $x(q) \in \mathcal{B}$. Theoretically, any allocation on the simplex is a possible outcome since the votes are also only restricted in lying on the simplex. In order to evaluate the outcomes of the voting game, we classify them according to the categories truth-telling, Nash, efficiency and welfare optimality. These categories are independent on the actual votes.

6.1 Outcomes under Truth-telling

We classify a social outcome as an outcome under truth-telling, if the aggregated outcome coincides with the one that would result if all individuals vote for their true most preferred allocation.

Definition 9 (**Truth-telling outcome**). *A social outcome $x^T(q)$ is called* ***outcome under truth-telling****, if the allocation is equal to the aggregated allocation of all peaks: $x^T(q) = x(p)$.*

It is important to note that an outcome might be classified as truth-telling outcome even if none of the votes is equal to the corresponding true peak. As long as the outcome $x(q)$ is identical to $x(p)$, the votes q may take any value in the set of feasible allocations. Obviously, the outcome under truth-telling of a given peak distribution depends on the aggregation rule. This definition implies that truth-telling outcomes are by chance higher under the median rules. As long as all non-pivotal voters stay within their rank, the truth-telling outcome is determined only by the (semi-)pivotal voters. Therefore, there exist more combinations in which voters may deviate from truth-telling but the outcome remains a truth-telling outcome. In fact, the (semi-)pivotal votes do not have to come from the same individuals as the (semi-)pivotal peaks. Consider an example for $m = 2$, $Q = 100$ and $p_1 = 10$, $p_2 = 50$ and $p_3 = 60$. The truth-telling outcome under the median rule is 50, which is derived from any situation in which any individual $f \in \{1, 2, 3\}$ votes $q_f = 50$, another individual $g \in \{1, 2, 3\}, g \neq f$ votes for any allocation between zero and 50, and the other individual $h \in \{1, 2, 3\}, h \neq f, g$ votes for any allocation between 50 and 100. Note also that under the median rule the outcome under truth-telling is not necessarily identical to the median of all peaks as the median might have been adapted. While under the median rule an outcome may remain a truth-telling outcome even if one voter does not vote for his or her true peak, a deviation from truth-telling of any voter has a direct effect on the outcome under the mean rule and needs to be compensated by a deviation of another voter such that the outcome remains a truth-telling outcome.

6.2 Nash Outcomes

In definition 6 we clarify when a set of votes constitutes a Nash equilibrium. We showed in the previous chapters that the voting rules we consider yield different Nash strategies and therefore the outcomes may vary across the rules. Moreover, multiple Nash equilibria exist for some peak distributions even under the mean rule. Each Nash equilibrium generates a unique allocation result, where a Nash outcome is an allocation that equals the allocation result of a Nash equilibrium.

Definition 10 (**Nash outcome**). *A social outcome $x^N(q)$ is called* ***Nash outcome****, if the allocation is equal to the aggregated allocation in a Nash equilibrium: $x^N(q) = x(q^*)$.*

Analogue to truth-telling outcomes, Nash outcomes depend only on the social outcome and for the classification, it is irrelevant whether all of the votes are Nash strategies. One should keep in mind that multiple Nash equilibria might exist and these equilibria might have different outcomes. One distinction for outcomes is thus to consider only partially honest Nash outcomes, i.e. Nash outcomes under partial honesty. Additionally, Nash outcomes and truth-telling outcomes may coincide for some peak distributions.

6.3 Efficient Outcomes

As already mentioned, we evaluate the voting rules by their efficiency. In this context, we do not consider the complexity of calculating the social outcome but we measure efficiency solely by the outcome. Therefore, we need a more detailed definition and classification of how efficient the social outcome is. We distinguish between outcomes that are peak-bounded as minimal requirement for efficiency and further restrict this set of allocations by defining Pareto-efficient outcomes.

Definition 11 (**Peak-bounded outcome**). *Let $p^j_{[1]}$ be the peak with the lowest value on j and $p^j_{[n]}$ be the peak with the highest value on $j \in J$. A social outcome $x^B(q)$ is called a **peak-bounded outcome**, if $x^B(q)$ is not smaller than the lowest ranked peak and not larger than the highest ranked peak for each project j, i.e. $p^j_{[1]} \leq x^{B^j}(q) \leq p^j_{[n]}$ for all $j \in J$.*

Contrary to the previous definitions, the classification of peak-bounded outcomes is independent on the voting rule and fixed for a given distribution of peaks. When considering a budget allocation problem on two public projects, a social outcome is peak-bounded, if it lies within the convex hull of all peaks. In multi-dimensional budget allocation problems, the convex hull comprises indeed also only peak-bounded outcomes, however, the set of peak-bounded outcomes is even larger, i.e. the set of the convex hull of all peaks is a subset of the set of peak-bounded allocations.

Figure 6.1 displays the difference between the convex hull and the set of peak-bounded outcomes for $m = 3$. Moreover, when voting on a budget allocation on more than two public projects, the set of peak-bounded outcomes as well as the convex hull of the peaks might include outcomes that are Pareto-inefficient. An allocation a is Pareto-efficient, if there exists no other allocation a', such that the distance between a' and p_i is smaller than or equal to the distance between a and p_i for all individuals i and strictly smaller for some individuals h.

Definition 12 (**Pareto-efficient outcome**). *A social outcome $x^P(q)$ is called a **Pareto-efficient outcome**, if $\nexists x'(q) : d(x'(q), p_i) \leq d(x^P(q), p_i)\ \forall i \in I$ and $d(x'(q), p_h) < d(x^P(q), p_h))$ for some $h \in I$.*

By definition, every Pareto-efficient outcome is peak-bounded. For any allocation that is not peak-bounded, there exists a peak-bounded allocation that reduces the distance between the outcome and the peak for at least one voter without deteriorating the other voter(s). Moreover, any outcome that corresponds to a peak of any voter is Pareto-efficient as a deviation from this outcome increases the distance from this peak and therefore has a negative effect on this voter's utility. A Pareto-efficient outcome must not necessarily be the optimal social outcome for the group in total. There exists a smaller set of outcomes that minimizes the total distance sum over all individuals and thus represents a welfare optimum, as can also be seen in figure 6.1. However, any welfare optimal outcome is Pareto-efficient as it would not be an optimum if at least one peak-outcome-distance could be reduced without increasing the other distances. Welfare optimality is described more detailed in chapter 6.4.

We showed in chapter 5 that Pareto-inefficient equilibria exist under the mean and the median rule and under the median rule even outcomes that are not peak-bounded exist in equilibrium. By construction of the voting rules, the mean and the median rule have in common that the truth-telling outcome is always peak-bounded.

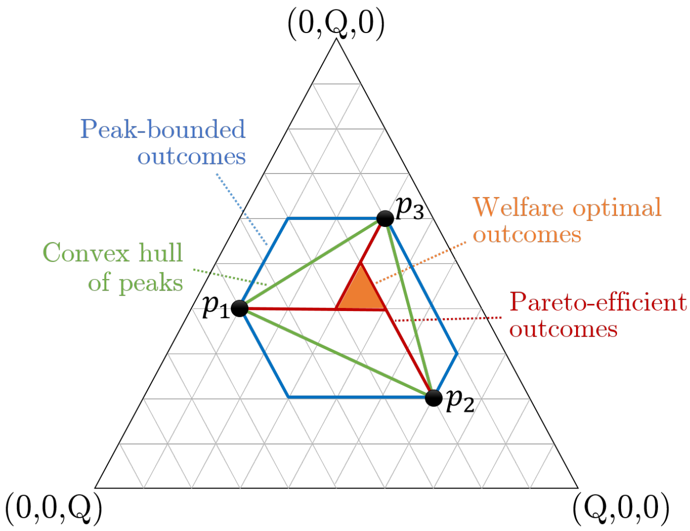


Figure 6.1: Efficiency and welfare optimality

6.4 Welfare Optimal Outcomes

In chapter 3.3, we described that the preferences of each voter are metric single-peaked. The only relevant factor that determines the utility is the distance between the peak and the social outcome. A social outcome that coincides with the peak yields the highest utility for a voter. We now classify the outcome by its social welfare or the sum of utilities that all voters gain from the aggregated votes.

Definition 13 (**Welfare optimal outcome**). *A social outcome $x^W(q)$ is called a* ***welfare optimal outcome****, if the allocation maximizes the sum of utilities:* $x^W(q) = \arg\max_{x(q)} \sum_{i \in I} u_i(p_i, x(q))$.

Since the welfare optimality condition uses the sum of all utilities, we need to imply interpersonal comparable utility information. Similar to efficient outcomes, welfare optimal outcomes may be classified independently from the voting rule. Since in our model utility is determined solely by the distance between the peak and the social outcome, we determine welfare optimality by minimizing the total distance sum of all voters. A welfare optimal outcome might therefore be either unique or within ranges. In a two-dimensional case by contrast, the median of the peaks is the unique welfare optimum if the number of voters is odd. Welfare optimality is the most stringent condition of our analysis and truth-telling outcomes are not necessarily welfare optimal (but always peak-bounded).

We classify the social outcome of concrete peak distributions according to the above categories in chapter 8.

7 Experiments on the Mean and the Median Rule

In two laboratory experiments, we analyze the voting behavior in a multi-dimensional budget allocation problem. A special focus lies on how the voting strategies of the mean and the median rule differ both, in theory and in the lab. The first experiment accounts for voting behavior under the mean rule, the normalized and the sequential median rule. The second experiment additionally captures differences in the voting behavior between the mean and the normalized median.

7.1 General setup

Both experiments took place at Karlsruhe Institute of Technology (KIT) in 2015. Eleven sessions were conducted in January (pilot session), April and June (sessions one to ten), but due to technical problems in the last session in June, only ten of them were finalized. The sessions of this first experiment lasted about 1.5 hours (the pilot session 1 hour and 40 minutes) and the laboratory accommodated twelve participants. The recruitment of the participants was made by ORSEE (Greiner, 2015). As our experiment is designed in cohorts of five, we invited two groups, i.e. ten plus three spare participants in case of no-shows. In total, 143 individuals were invited out of which 110 participated the first experiment.

After a new design was implemented, another experiment with eight sessions was conducted at the newly constructed KD2Lab in October 2015. As the KD2Lab accommodates more workplaces, each session could be conducted with three groups of five participants. For each of the eight sessions, 15 participants (120 in total) and four spare participants were recruited via ORSEE. One session of the second experiment lasted on average about 1 hour and 15 minutes.

The average age of the participants of the first experiment is 22.9, with a 25.0% share of women. 54.0% study Business Engineering and 73.0% have an economic part in their field of study. The demographic data in the second experiment looks quite similar. The average age of the participants is 24.1, the share of women 24.2%. 52.5% study Business Engineering and more than 70% have an economic part in their field of study. These shares correspond to the ORSEE subject pool of the KIT. According to the statistics of KIT students from 2018, the overall share of female students is comparable with 29.0%, but the percentage that study Business Engineering is only 13.5%. It is therefore worthwhile to notice that the ORSEE subject pool is no exact representation of all KIT students but more a representation of students with an economic subject (Karlsruher Institut für Technologie (KIT), 2018). The software used for both experiments is z-Tree (Fischbacher, 2007).

7.1.1 Laboratory Procedure

In the first experiment (pilot session and sessions one to ten), 131 out of the 143 invited individuals showed up. Each individual that showed up in time received a show-up fee of 5.00 Euros. Individuals, who showed up for a session in time but after 10 participants already were seated, received a show-up fee of 5.00 Euros but could not participate. On average, each participant earned 17.12 Euros in the sessions one to nine.[4] Directly after arrival, the participants were allocated randomly to the workplaces and it was

[4] Due to a technical breakdown in the middle of session ten, each participant received 15.00 Euros blanket independent on his or her choice during the session. The average payoff of the pilot session was 15.10 Euros.

made sure that they do not interact with each other. The workplaces were equipped with a computer, paper, a pencil and a calculator, as can be seen in figure 7.1.

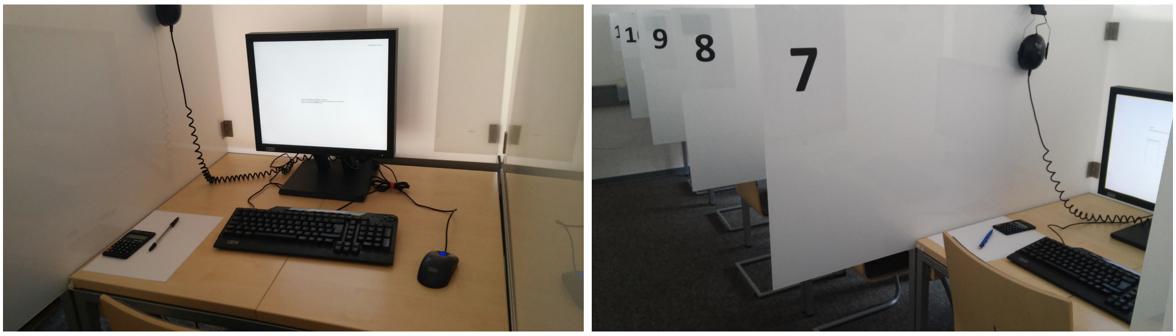


Figure 7.1: Laboratory workplace first experiment

After all participants were seated, the instructions were distributed and read aloud by audiotape to avoid variation in the readings across sessions. Beside mathematical information on the calculation of the mean and adapted median rules, the instructions include the session procedure. The handout with the instructions of the first experiment can be found in appendix A. Subsequently, participants answered a short quiz to make sure the instructions were understood properly. The participants could reread the instructions on the handout during the entire experiment. After the voting process, participants were asked to fill out a questionnaire on demographic data and the strategy underlying their decisions. To make sure that they cannot obey the earned profit of their competitors, participants were payed one after another in a separate room.

During the second experiment at the KD2Lab, out of the 152 invited people, 145 showed up to eight sessions. The procedure was similar to the first experiment: directly after arrival, the participants were allocated randomly to the cabins. Figure 7.2 shows some of the cabins. As soon as all 15 participants (three groups of five) were sitting at their workplaces, the instruction was read aloud by audiotape to avoid variation in the readings across sessions. The average payoff of the participants of the second experiment amounted to 13.98 Euros, including a show-up fee of 5.00 Euros. Since the workplaces at the KD2Lab are located in soundproof cabins, the payment could be given directly at the workplaces.

Figure 7.2: Laboratory workplace second experiment

7.1.2 Treatment Variables

The treatment variables of both experiments are the voting rule, the degree of information the participants receive and the peak distribution.

The treatment variable **rule** determines the voting rule that is used for aggregating the individual votes to a social outcome. **Rule** may attain the values *mean*, *normalized median* or *sequential median*. In the first experiment, **rule** is a within-subject treatment variable, since each participant votes under the same voting rule that changes during the experiment. We modify the design at the second experiment, with **rule** being a between-subject treatment variable.

Info is a within-subject treatment variable that indicates the degree of information each participant receives prior to the voting decision. We vary the stated information from *no info*, where the participants obtain solely their own peak, to *full info*, where the peaks of the other four participants are displayed. The degree of information is identical for each participant and all voters have either full information on the entire peak distribution or no information on the other peaks. In both experiments, the participants play for five periods under the no info treatment and for three periods under the full info treatment.

Peak distribution is our last treatment variable including 13 different distributions that are labeled from *A to M*. Each distribution indicates the peaks of all five participants. In the first experiment, we use nine distributions (A, B, and C in the pilot session and D, E, F, G, H, and I in sessions 1 to 10) as a within-subject treatment variable. In the second experiment, we use the four peak distributions J, K, L, and M but we use the peak distribution additionally as a between-subject variable. Dependent on the sessions, each participant plays three out of the four distributions. All peak distributions are provided in chapter 7.2.2.

The detailed procedure of both experiments including all treatment variables is displayed in the tables B.1, B.2, and B.3 in the appendix B.

7.2 The Design

In both experiments, the attendees are told in the instructions that they participate in a vote together with four other anonymous participants at which the funding of three public projects is determined. The aggregation of the votes is done either by mean or by a median-based rule. Five participants vote on the allocation of 100 monetary units on the three projects. Every participant receives a peak that represents the most preferred funding of the projects. Thereby, we make sure that the 'true' peaks are determined. These peak distributions, i.e. five different peaks that belong to the voters of one group, remain identical for several rounds, depending on the degree of information. The vote is executed anonymously and in several rounds. In a first step, the participants get to know the voting rule. This information is only relevant in the first experiment where the voting rule is a within-subject treatment variable. One may argue when the voting rule is a within-subject variable, the sequence of the voting rules has an effect on the voting behavior. Block (2014) do find a sequence effect for the median rule with no information, however this treatment is only tested in the pilot session.

Subsequently, subjects get to know their own peaks and under full information the peaks of the four other participants of their group. Once all group members voted, the subjects get to know the social outcome and their payoff.

Since the second experiment is designed as between-subjects, the aggregation of the votes either is done by mean (the first four sessions) or normalized median rule (the last four sessions). The peak distributions under the no info treatment are played for five periods and under the full info treatment for three periods. The peak distributions, information and number of rounds are identical for both voting rules, such that a direct comparison with respect to the treatment variable rule is possible.

7.2.1 Payoff

In both experiments, the underlying individual payoff function in the unit ECU^5 is the following:

$$f_i(p_i, x) = 10 + \frac{760}{4 + \sum_{j=1}^{3} |p_i^j - x^j(q)|} \ , \tag{7.1}$$

where p_i^j denotes the peak of individual i for one of the three projects j and $x(q)$ the social outcome, either calculated by the mean or (adapted) median of all five votes. One vote represents the share of 100 monetary units that is allocated to three projects and therefore consists of three natural numbers that have to add up to the total budget of 100. The minimum payoff per period is about 13.73 ECU, because the highest possible distance is 200, e.g. between a social outcome $x(q) = (100, 0, 0)$ and a peak $p_i = (0, 100, 0)$. Once the social outcome corresponds to the given peak, the maximal payoff of 200 ECU is reached. Figure 7.3 shows the payoff function, which is also displayed during the experiment. 'Distance' adds up the absolute distance between the own peak and the social choice in every project, i.e. $\sum_{j=1}^{3} |p_i^j - x^j(q)|$.

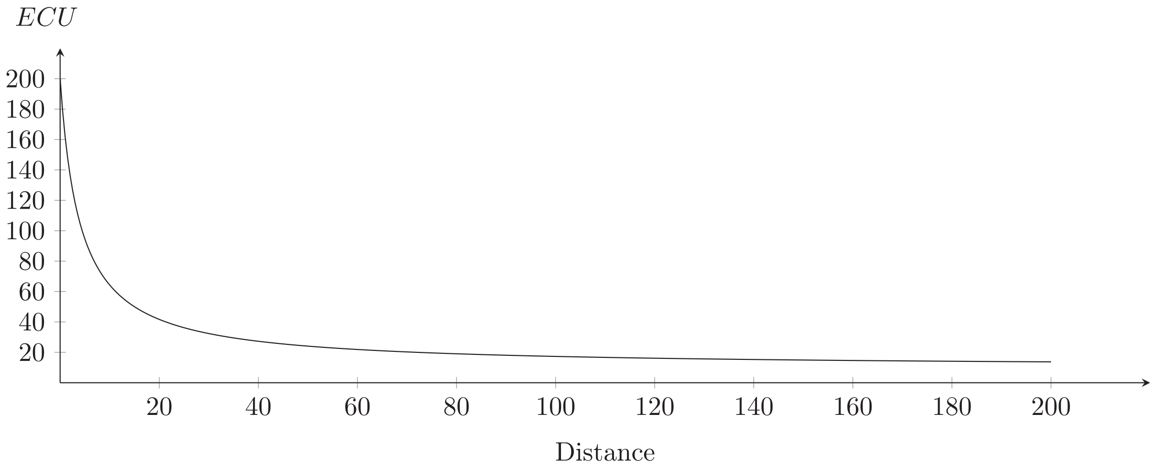


Figure 7.3: The payoff function

7.2.2 Peak Distributions

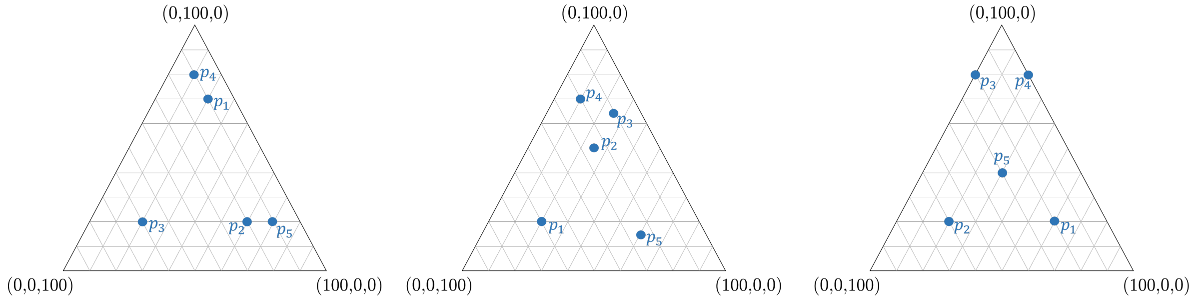
We use a total of 13 different peak distributions in both experiments. Obviously, the optimal strategies and Nash equilibria depend not only on the voting rule but also on the peak distribution and individual peaks.

Pilot Session: A, B, C

The pilot study makes use of three different peak distributions (A, B, C) which are played five times in the *no info* treatment and three times in the *full info* treatment of the mean and normalized median. The treatment with sequential adaptation of the median is played only with full information and for three periods based on only one peak distribution (C). It is made sure that no participant receives the same peak of one distribution when using different voting rules. Accordingly, each participant has to make 51 decisions, consisting of three natural numbers that add up to 100. The detailed procedure of the pilot session is displayed in table B.1 in the appendix B.

[5] Experimental Currency Unit; 100 *ECU* corresponds to 1.00 Euro in the pilot session, 1.50 Euros in the sessions one to ten, and 1.00 Euro in the second experiment.

Figure 7.4 visualizes the three peak distributions A, B, and C that we use in the first session. We give an overview of all peak distributions together with the corresponding Nash equilibria later in chapter 7.2.3.



(a) A (b) B (c) C

Figure 7.4: Peak distributions A,B,C

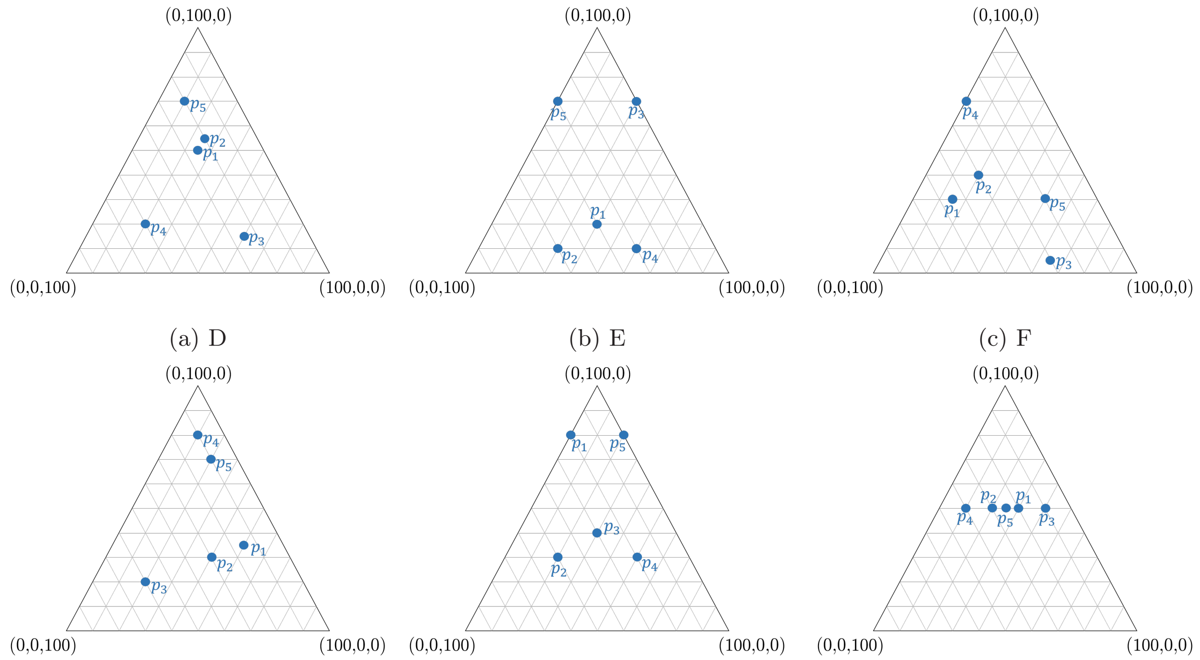
Sessions One to Ten: D, E, F, G, H, I

From the pilot session, we got some important organizational insights that caused us to revise the design of our experiment. The session lasted 1 hour and 40 minutes, which turned out to be too long for participants to spend in the laboratory. Further, the average payoff was 15.10 Euros, which we considered too low when converted into hourly earnings.

In the next 10 sessions, we dropped the sequential adaptation of the median rule, played the normalized median rule only with full information and the mean rule only without information. In order to increase the payoffs further, the conversion of 100 ECU was increased from 1.00 Euro to 1.50 Euros. Additionally, we considered specific (in terms of optimal voting behavior) peak distributions that varied according to the voting rule. In total, we used six different peak distributions (D-I), one of them (D) was played under both rules. Every participant thus had to vote for an allocation in 29 decisions. Figure 7.5 displays the peak distributions used in the second set of sessions. The peak distributions and the procedure of the second set of sessions are detailed in the appendix B in table B.2.

The Second Experiment: J, K, L, M

With the first experiment, we were able to analyze the behavior of participants within the voting rules mean, sequential and normalized median. After getting the results, that are discussed in chapters 8 and 9, another experiment was designed in order to make a comparison between the voting rules mean and normalized median. Therefore, we had a between-subject design where the first half of the participants played solely the mean rule and the other half solely the normalized median rule. The peak distributions, information and number of rounds were identical in the two groups. In total, four peak distributions are used, of which three different ones are played in every session. The peak distributions remain identical for five periods in the no info treatment and for three periods in the full info treatment. Accordingly, each participant has to make 24 decisions, consisting of three natural numbers that add up to 100 monetary units. Figure 7.6 displays the peak distributions of the second experiment. The detailed peak distributions and the procedure of each session are displayed in table B.3 in the appendix B, where the voting rules of sessions one to four is the mean and of sessions five to eight the normalized median rule.



(a) D (b) E (c) F

(d) G (e) H (f) I

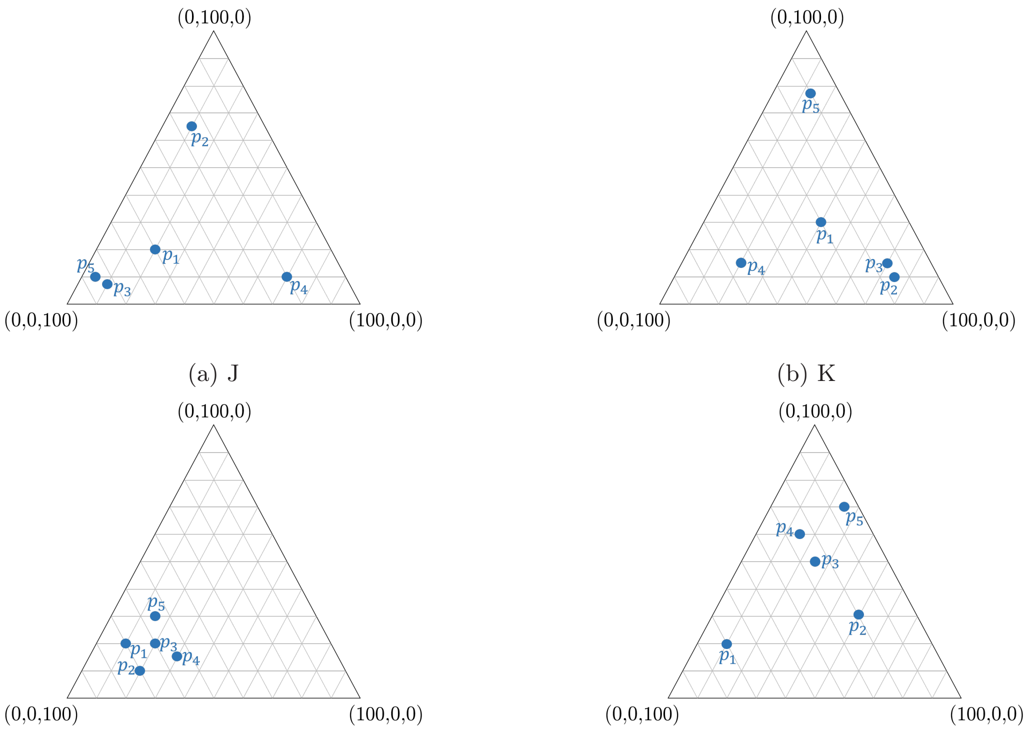
Figure 7.5: Peak distributions D,E,F,G,H,I

7.2.3 Nash Equilibria

In chapter 5, we gave a theoretical analysis of the Nash equilibria under the mean and median-based rules. We chose the peak distributions according to the Nash strategies and these might include focal strategies, truth-telling and strategic voting even under the median rule. The following sections provide the Nash equilibria for all peak distributions that were used in both experiments. One should keep in mind that there exists a multitude of equilibria under the median rule, including Pareto-inefficient equilibrium outcomes. In our equilibrium analysis and also later in the data evaluation we therefore focus on partially honest Nash equilibria under the median rule.

Nash Equilibria A, B, C

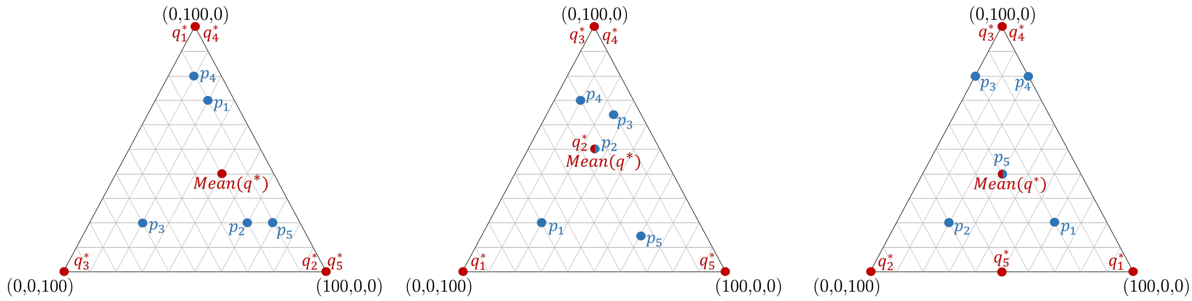
We chose these distributions for the pilot session because of their different characteristics when it comes to the Nash equilibria. The Nash equilibria under the mean rule are displayed in figure 7.7. In the Nash equilibrium of distribution A, there is no vote that allocates a positive amount to more than one project, which means that all Nash votes are located in the corners of the simplex and the total budget is allocated to one project. More specifically, each Nash strategy is a focal strategy. In this distribution, the social outcome under Nash play is different from all peaks. The Nash strategy under peak distribution B is no focal strategy but different for one voter. In figure 7.7b it is visualized that exemplary individual 2 may get his or her most preferred allocation as social outcome by stating the true peak in the Nash equilibrium. The other Nash strategies are focal, i.e. voting for an allocation of the total budget to the project that has the highest value in the peak. Distribution C is used to examine another Nash equilibrium in which the social outcome corresponds to the peak of an individual, in figure 7.7c exemplary individual 5. In contrast to distribution B, the voter does deviate from the true peak in equilibrium and states a vote that allocates a positive budget to two projects and zero budget to the project with the highest value in the peak.



(a) J (b) K

(c) L (d) M

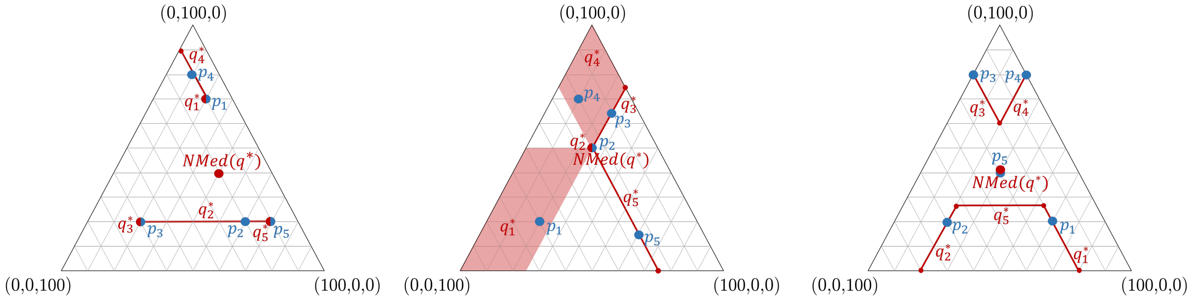
Figure 7.6: Peak distributions J,K,L,M



(a) A (b) B (c) C

Figure 7.7: Nash equilibria A,B,C, mean rule

The Nash equilibria under median-based rules are in general not unique and most of the time the best responses consist of an entire section on the simplex. If the Nash strategy is not unique, we visualize it by either a line, which indicates that the best response in one project is fixed, or a shaded section, indicating that the best response is a range for every project. Figure 7.8 displays the Nash equilibria under the normalized median rule. As shown in figure 7.8a, the best response under distribution A is to state the true allocation value for coordinate-wise median-voters and to vote for a value that is equal to or greater (lower) than the median for peaks that are above (below) the median value. After normalization, the social outcome does not correspond to any of the peaks and strategic voting is not possible. Figure 7.8b displays distribution B in which individual 2 is pivotal in the second project and semi-pivotal in projects 1 and 3. Truth-telling is again a weakly dominant strategy and the social outcome corresponds to the most preferred allocation of voter 2 such that normalization is not necessary. The Nash strategies under peak distribution C differ from truth-telling. Individual 5 is pivotal in the second coordinate and strategic voting is possible. Theoretically, the strategic vote for project 2 would be to allocate $26\frac{2}{3}$, such that the normalized median is exactly the peak of voter 5. However, the participants in the experiment were only allowed to vote for integers, such that the Nash strategy is to vote for 27 and the social outcome slightly differs from p_5.



(a) A (b) B (c) C

Figure 7.8: Nash equilibria A,B,C, normalized median rule

Table 7.1 provides an overview of peak distributions A, B, and C with the corresponding Nash equilibria under the mean, the normalized and the sequential median rule. In some cases under the median rule, the coordinate-wise values for a Nash strategy differ from the multi-dimensional allocation problem with three coordinates. For example, $q_4^{*(NMed)}$ in distribution A is to vote for an allocation of at least 70 for the second project. If one considers only the second project, a value of at least 20 would be a best response to the other votes. Due to the budget constraint of 100 monetary units however, when the vote for the first project is at most 20 and exactly 10 for the third project, it results that the allocated value for the second project has to be at least 70. Note that we only represent the Nash strategies of distributions that were actually played in the experiment, such that for the sequential median rule we only represent distribution C. As already stated in chapter 3, the sequential median rule is strategy-proof and therefore truth-telling is a (weakly) dominant strategy in every Nash equilibrium, as displayed in figure 7.9 and table 7.1.

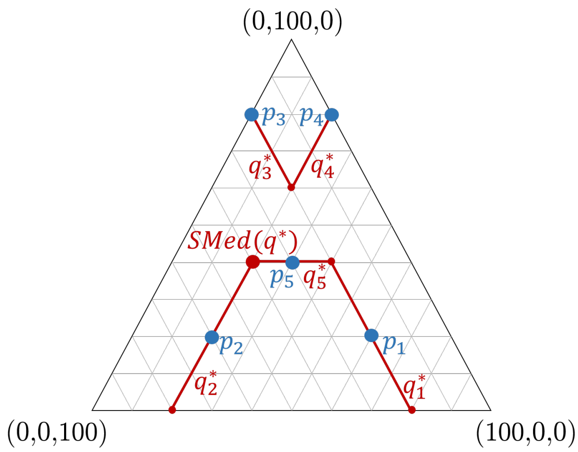


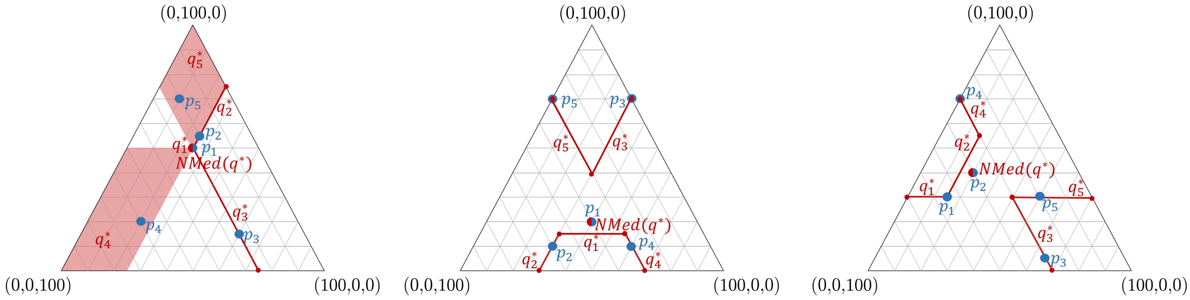
Figure 7.9: Nash equilibrium C, sequential median rule

	Participant					
A	**1**	**2**	**3**	**4**	**5**	$\boldsymbol{x(q^*)}$
p_i	20	60	20	10	70	
	70	20	20	80	20	
	10	20	60	10	10	
$q_i^{*(Mean)}$	0	100	0	0	100	40
	100	0	0	100	0	40
	0	0	100	0	0	20
$q_i^{*(NMed)}$	=20	≥20	=20	≤20	=70	40
	=70	=20	=20	≥70	=20	40
	=10	≥10	=60	=10	=10	20
B						
p_i	20	25	25	10	60	
	20	50	65	70	15	
	60	25	10	20	25	
$q_i^{*(Mean)}$	0	25	0	0	100	25
	0	50	100	100	0	50
	100	25	0	0	0	25
$q_i^{*(NMed)}$	≤25	=25	=25	≤25	≥25	25
	≤50	=50	≥50	≥50	≤50	50
	≥25	=25	≤25	≤25	=25	25
C						
p_i	60	20	0	20	30	
	20	20	80	80	40	
	20	60	20	0	30	
$q_i^{*(Mean)}$	100	0	0	0	50	30
	0	0	100	100	0	40
	0	100	0	0	50	30
$q_i^{*(NMed)}$	≥53	=20	≤20	=20	≥20	29.85
	≤27	≤27	≥60	≥60	=27	40.30
	= 20	≥53	=20	≤20	≥20	29.85
$q_i^{*(SMed)}$	≥40	=20	≤20	=20	≥20	20
	≤40	≤40	≥60	≥60	=40	40
	= 20	≥40	= 20	≤20	≥20	40

Table 7.1: Nash equilibria A,B,C

Nash Equilibria D, E, F, G, H, I

The characteristics of the next peak distributions also emerge when considering the Nash equilibria. Figure 7.10 illustrates the Nash equilibria under the normalized median rule. One set of Nash strategies in distribution D contains truth-telling of all voters resulting in a social outcome that corresponds to the peak of individual 1 who is pivotal in project 2 and semi-pivotal in the other projects. In distributions E and F, the pivotal voter (individual 1 is pivotal in the second project in E and individual 2 in the first project in F) affects the social outcome in equilibrium by strategic voting. In the Nash equilibrium, the normalized median is identical to the peak of the pivotal voter.

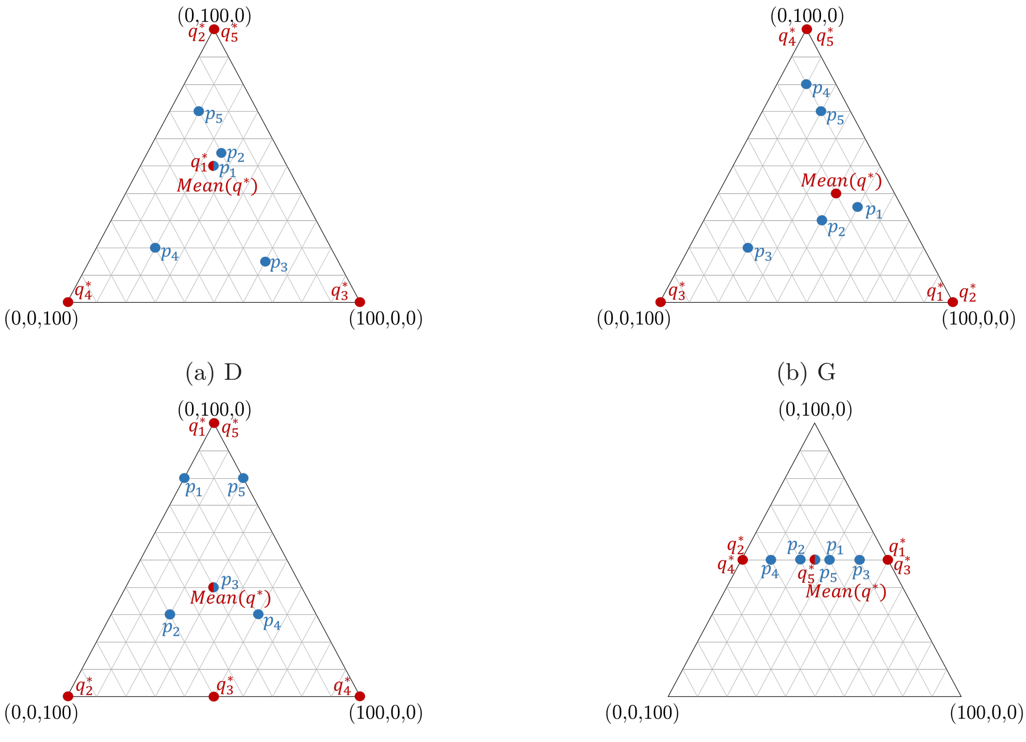


(a) D (b) E (c) F

Figure 7.10: Nash equilibria D,E,F, normalized median rule

Distributions D, G, H, and I were played using mean voting. Figure 7.11 illustrates the Nash equilibria for these distributions. The Nash strategies for D are comparable to distribution B with one voter stating the true most preferred allocation and the Nash outcome corresponds to this peak. Distribution G is special since the best response of individual 2 is not unique and thus multiple Nash equilibria with different outcomes exist. Dependent on q_2^*, the outcome is either closer to the peak of individual 1 or to the peak of individual 2. The Nash strategies of the other individuals are focal and figure 7.11b displays the focal Nash equilibrium. Distribution H is comparable to distribution C.

Distribution I is special in the way that the peaks of all voters allocate the same value (50) to the second project. This peak distribution yields a variety of Nash equilibria. The most prominent one is an equilibrium in which all agents state the truth in the second project and allocate the remainder to the project with the second highest peak value. Individual 5 votes for an allocation equal to the peak and the outcome is identical to this peak. Remember that given distinct peaks, it is never a Nash equilibrium under the mean rule if more than one individual vote for a strictly positive amount in every project. This Nash equilibrium is the most prominent, since all individuals agree about the second project and thus the Nash strategies correspond to those in the one-dimensional budget allocation problem. However, several Nash equilibria exist. We provide some of the most interesting equilibria in table 7.2. Note that all of them yield different outcomes. The outcomes under the second and third Nash equilibria are identical to the peaks of individuals 1 and 2, respectively. The forth Nash equilibrium yields an outcome that is Pareto-inefficient, since distances from the prominent Nash equilibrium to all of the five peaks is lower. Since all distributions under the mean rule are played under no information, we find it especially interesting to observe the voting behavior under distribution I. Table 7.2 summarizes the peak distributions and Nash equilibria of sessions one to ten.



(a) D (b) G

(c) H (d) I

Figure 7.11: Nash equilibria D,G,H,I, mean rule

	Participant					
D	**1**	**2**	**3**	**4**	**5**	$x(q^*)$
	25	25	60	20	10	
p_i	50	55	15	20	70	
	25	20	25	60	20	
	25	0	100	0	0	25
$q_i^{*(Mean)}$	50	100	0	0	100	50
	25	0	0	100	0	25
	=25	=25	≥25	≤25	≤25	25
$q_i^{*(NMed)}$	=50	≥50	≤50	≤50	≥50	50
	=25	≤25	=25	≥25	≤25	25
E						
	40	30	30	60	0	
p_i	20	10	70	10	70	
	40	60	0	30	30	
	≥30	=30	=30	≥55	≤30	40
$q_i^{*(NMed)}$	=15	≤15	≥40	≤15	≥40	20
	≥30	≥55	≤30	=30	=30	40
F						
	15	20	65	0	50	
p_i	30	40	5	70	30	
	55	40	30	30	20	
	≤15	= 15	≥40	≤15	≥40	20
$q_i^{*(NMed)}$	=30	≥30	≤30	≥55	=30	40
	≥55	≥30	=30	=30	≤30	40
G						
	50	40	20	10	20	
p_i	35	30	20	80	70	
	15	30	60	10	10	
	100	≥ 50	0	0	0	30-40
$q_i^{*(Mean)}$	0	0	0	100	100	40
	0	≤ 50	100	0	0	20-30
H						
	0	20	30	50	20	
p_i	80	30	40	30	80	
	20	50	30	20	0	
	0	0	50	100	0	30
$q_i^{*(Mean)}$	100	0	0	0	100	40
	0	100	50	0	0	30

I						
	30	20	40	10	25	
p_i	50	50	50	50	50	
	20	30	10	40	25	
	50	0	50	0	25	25
$q_i^{*(Mean)}$	50	50	50	50	50	50
	0	50	0	50	25	25
	50	0	100	0	0	30
$q_i^{*(Mean)}$	50	0	0	100	100	50
	0	100	0	0	0	20
	100	0	0	0	0	20
$q_i^{*(Mean)}$	0	0	100	50	100	50
	0	100	0	50	0	30
	75	0	75	0	0	30
$q_i^{*(Mean)}$	25	25	25	25	100	40
	0	75	0	75	0	30

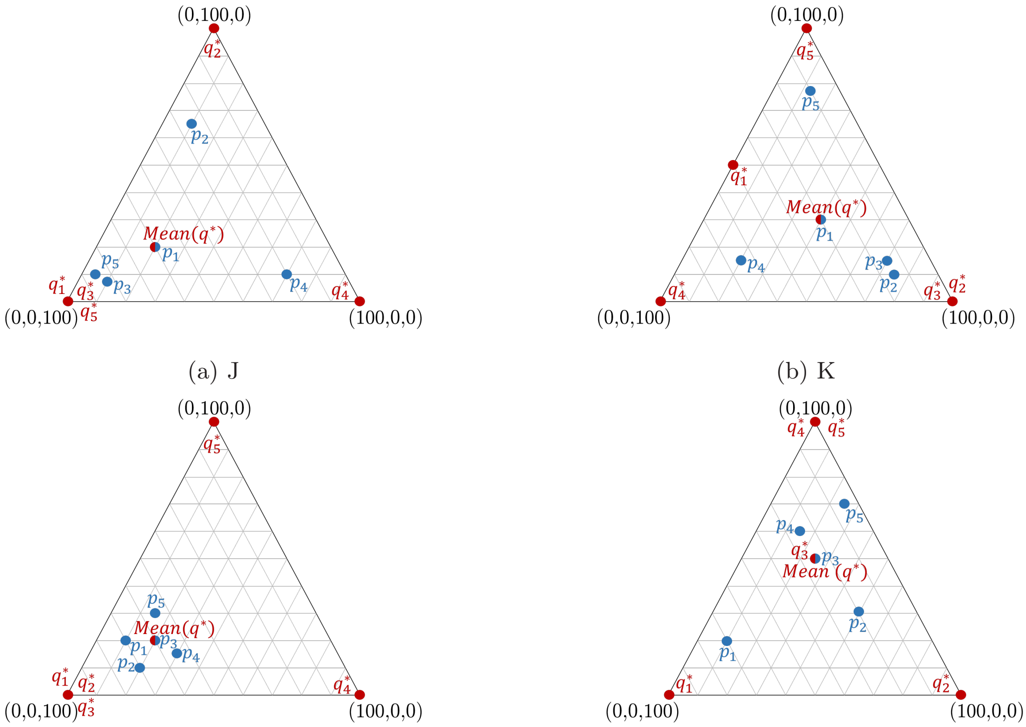
Table 7.2: Nash equilibria D,E,F,G,H,I

Nash Equilibria J, K, L, M

The last four peak distributions J, K, L, and M are used as between-subject variable in the second experiment. Interestingly, the Nash equilibria under the mean rule for distribution J and L are of the same type, as can be seen in figures 7.12a and 7.12c. Three participants vote for an allocation of the total budget on the third project, while the other two voters allocate the total budget to projects 1 or 2. This results in a social outcome that is identical to the peak of one of the first three voters. However, it is obvious that especially under no information, the Nash strategy for the two voters 4 and 5 in distributions L is more difficult to find, since in equilibrium these individuals allocate the total budget to a project where their most preferred value is only the second highest. By contrast, the Nash strategies under distribution J are focal strategies for all individuals. The Nash strategies under distribution K are similar to H and C, where one voter allocates half of the budget each to two projects and zero to the project with the highest peak value. Distribution M is comparable to B and D, where one voter states the true most preferred allocation and the Nash outcome corresponds to this peak with all other individuals playing a focal strategy.

For the normalized median rule, figure 7.13 displays the Nash equilibria of all four peak distributions. Under distributions J and K, strategic voting is possible. As observable in figures 7.13a and 7.13b, the Nash strategy of voter 1 is to deviate from truth-telling to obtain a social outcome that corresponds to his or her peak. This strategic voting under the median rule is only possible because voter 1 may exploit the normalized adaptation of the coordinate-wise median values. Figures 7.13c and 7.13d display the Nash equilibria for distributions L and M. Both have in common that one voter is pivotal in every project and thus truth-telling is this voter's Nash strategy. For the other voters, truth-telling is a weakly dominant strategy and the Nash outcome results in the pivotal voter's peak, such that normalization is redundant.

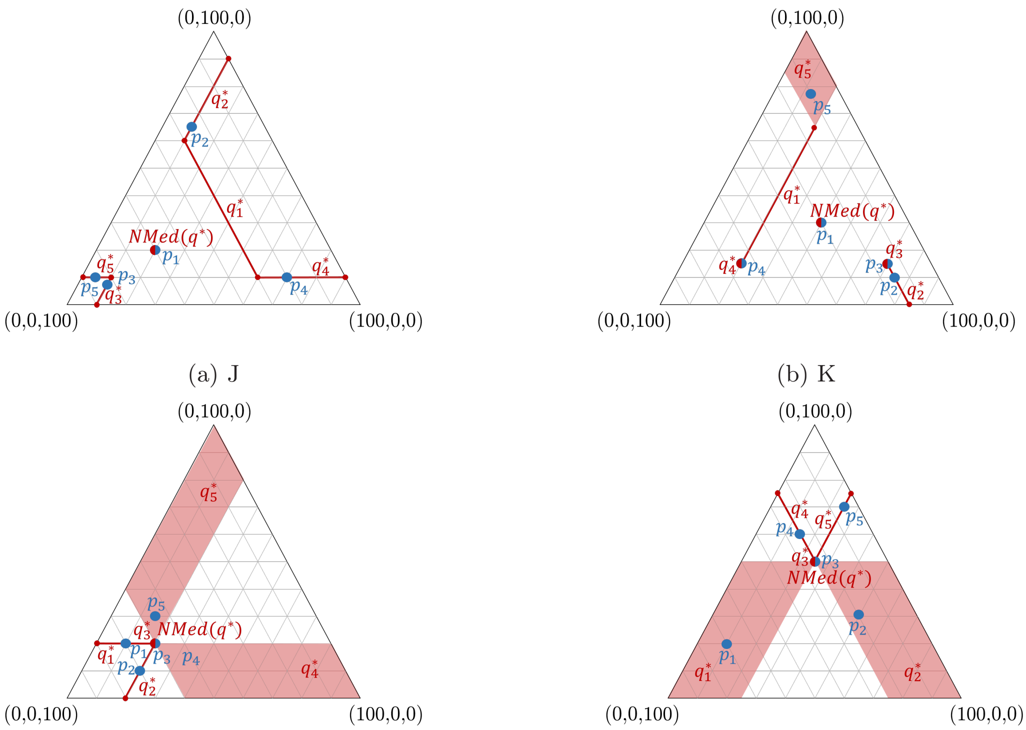
Table 7.3 provides a detailed overview of the peak distributions and Nash equilibria of the second experiment.



(a) J (b) K

(c) L (d) M

Figure 7.12: Nash equilibria J,K,L,M, mean rule



(a) J (b) K

(c) L (d) M

Figure 7.13: Nash equilibria J,K,L,M, normalized median rule

	Participant					
J	**1**	**2**	**3**	**4**	**5**	$\boldsymbol{x(q^*)}$
	20	10	10	70	5	
p_i	20	65	8	10	10	
	60	25	82	20	85	
	0	0	0	100	0	20
$q_i^{*(Mean)}$	0	100	0	0	0	20
	100	0	100	0	100	60
	≥10	=10	=10	≥60	≤10	20
$q_i^{*(NMed)}$	≥10	≥60	≤10	=10	=10	20
	=30	≤30	≥80	≤30	≥80	60
K						
	40	75	70	20	12	
p_i	30	10	15	15	78	
	30	15	15	65	10	
	0	100	100	0	0	40
$q_i^{*(Mean)}$	50	0	0	0	100	30
	50	0	0	100	0	30
	=20	≥70	=70	=20	≤20	40
$q_i^{*(NMed)}$	≥15	≤15	=15	=15	≥65	30
	≥15	=15	=15	=65	≤15	30
L						
	10	20	20	30	15	
p_i	20	10	20	15	30	
	70	70	60	55	55	
	0	0	0	100	0	20
$q_i^{*(Mean)}$	0	0	0	0	100	20
	100	100	100	0	0	60
	≤20	=20	=20	≥20	≤20	20
$q_i^{*(NMed)}$	=20	≤20	=20	≤20	≥20	20
	≥60	≥60	=60	≤60	≤60	60
M						
	10	50	25	15	25	
p_i	20	30	50	60	70	
	70	20	25	25	5	
	0	100	25	0	0	25
$q_i^{*(Mean)}$	0	0	50	100	100	50
	100	0	25	0	0	25
	≤25	≥25	=25	≤25	=25	25
$q_i^{*(NMed)}$	≤50	≤50	=50	≥50	≥50	50
	≥25	≤25	=25	=25	≤25	25

Table 7.3: Nash equilibria J,K,L,M

7.3 Research Questions

With the two laboratory experiments on the voting behavior under different rules, degree of information and peak distributions, several research questions may be formulated. We cluster the hypotheses according to the tested voting rule. Since we use different designs in our two experiments, they strive for testing different hypotheses. Due to the between-subject design, the last set of hypotheses is only captured by the second experiment.

7.3.1 Mean Rule

Hypothesis (H1.1). ***Truth-telling under the mean rule occurs less with full information.***

We compare the voting behavior for both values of the treatment variable info and hypothesize that truth-telling is lower under full information. Given that participants observe the peaks of the other voters and that the Nash strategies are in most of the cases focal, we expect votes to have lower shares of truth-telling compared to the no information treatment.

Hypothesis (H1.2). ***Under the mean rule, the (Pareto-efficient) Nash equilibrium will be played.***

We expect that participants understand the possibility of strategic voting under the mean rule and play the according Nash strategy. Given that all group members play their Nash strategy, the social outcome corresponds to the Nash outcome. We further capture participants that play not the exact Nash strategy but a tendency to Nash play. In order to get insights on the dependent factors of Nash play, we regress the distance of the vote from Nash strategy on independent variables like the distance between the peak and the Nash strategy or the type of the Nash strategy (e.g. focal strategy, truth-telling or strategic voting). We also analyze one peak distribution that entails a Pareto-inefficient Nash equilibrium and expect that this Nash equilibrium is not played.

Hypothesis (H1.3). ***Nash play increases over time under the mean rule.***

We believe that individuals adapt their voting behavior over time and the quantity of Nash strategies increases with the number of periods per peak distribution and with the number of total rounds over all distributions. We examine learning effects by testing whether individuals play a best response to the result of the previous round.

7.3.2 Median Rule

Hypothesis (H2.1). ***Under median-based rules, truth-telling prevails.***

In the theoretical analysis, we show that strategic voting is possible under some peak distributions using the normalized median rule. Nevertheless, strategic voting is difficult and presumes the corresponding behavior of the other voters. In the one-dimensional budget allocation problem, the median rule is strategy-proof and therefore truth-telling is a weakly dominant strategy. The same holds for the sequential median rule. We hypothesize that participants announce their true peak under both median-based rules.

Hypothesis (H2.2). ***Under the normalized median rule, a best response to truth-telling is played.***

Given that truth-telling is a dominant strategy, it is only weakly dominant for any non-pivotal or semi-pivotal voter. Therefore, a best response is often not unique. We expect that participants play a best response to truth-telling of the other voters. This implies that only the pivotal voter (if existent) states the true peak, whereas the others vote for an allocation that is within their rank with respect to the median.

7.3.3 Mean versus Median Rule

The second experiment is designed to test further hypotheses that compare both voting rules.

Hypothesis (H3.1). ***The normalized median rule leads to more truth-telling as compared to the mean rule.***

Since truth-telling is a weakly dominant strategy for most of the individuals under the median rule and strategic voting is possible for at least four voters under the mean rule, we hypothesize that the shares of truth-telling are higher under the median rule as compared to the mean rule.

Hypothesis (H3.2). ***The distance of the votes from the true peak is higher under the mean as compared to the normalized median rule.***

The expectation that participants deviate more from truth-telling under the mean rule is derived from the previous hypotheses. Since we expect Nash play under the mean rule and truth-telling or best-response-to-truth under the median rule, we hypothesize that subjects that play the mean rule deviate more from truth-telling. We measure the distance between the peak and the vote and regress it on the voting rule and further variables like info or the peak distribution.

8 Experimental Results - Social Outcomes

This chapter presents the results of both laboratory experiments with respect to the aggregated group outcomes. We categorize the social outcomes according to the classification from chapter 6, including truth-telling, Nash, efficiency and welfare optimality. Obviously, these categories might vary for the peak distributions and voting rules but remain constant over the periods. In total, we have 1,200 social outcomes that might be analyzed: 102 outcomes from the pilot session, 522 from sessions one to nine (remember that we could not use the data of the tenth session) and 576 from the second experiment. For the mean rule, we observe 696 outcomes, for the normalized median rule 498 and for the sequential median rule six. 780 outcomes are derived under no information and 420 under full information.

Whereas in this chapter, we only consider aggregated results, we go into more detail in the subsequent chapters. This is especially interesting since e.g. an outcome might be classified as Nash outcome even though no individual played a Nash strategy. Table 8.1 displays for each peak distribution the concrete conditions we used to classify the social outcomes. For most of the peak distributions, the condition Pareto efficiency may not be classified within a minimum and maximum allocation, such that a list of all allocations would be necessary and we do not display the condition in the table. As already stated, all Pareto-efficient allocations are a subset of the set of peak-bounded outcomes and must include the set of welfare optimal allocations. Figure 8.1 provides a graphic illustration of peak distribution G, including the truth-telling outcome $x^T(q)$ and the Nash outcome $x^N(q)$ for the mean rule. It also displays the set of peak-bounded allocations $x^B(q)$, the set of Pareto-efficient allocations $x^P(q)$, as well as the set of welfare optimal allocations $x^W(q)$. The most important results are summarized in table 8.2.

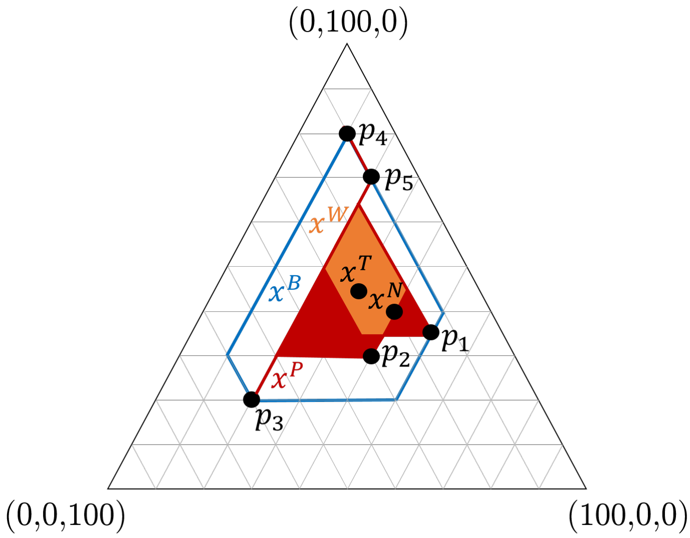


Figure 8.1: Outcome conditions for peak distribution G

Peak	Truth-telling Outcome			Nash Outcome			Peak-bounded		Welfare Opt.	
Distr.	Mean	NMed	SMed	Mean	NMed	SMed	min	max	min	max
	36	40		40	40		10	70	20	60
A	42	40	-	40	40	-	20	80	20	70
	22	20		20	20		10	60	10	20
	28	25		25	25		10	60	25	25
B	44	50	-	50	50	-	15	70	50	50
	28	25		25	25		10	60	25	25
	26	25	20	30	29.85	20	0	60	20	30
C	48	50	40	40	40.30	40	20	80	40	60
	26	25	40	30	29.85	40	0	60	20	30
	28	25		25	25		10	60	25	25
D	42	50	-	50	50	-	15	70	50	50
	30	25		25	25		20	60	25	25
		37.50			40		0	60	30	40
E	-	25	-	-	20	-	10	70	20	40
		37.50			40		0	60	30	40
		25			20		0	65	20	40
F	-	37.50	-	-	40	-	5	70	30	40
		37.50			40		20	55	30	40
	30			40			10	50	20	40
G	45.83	-	-	40	-	-	20	80	35	65
	24.17			20			10	60	15	30
	25			30			0	50	20	30
H	50	-	-	40	-	-	30	80	40	60
	25			30			0	50	20	30
	25			25			10	40	25	25
I	50	-	-	50	-	-	50	50	50	50
	25			25			10	40	25	25
	22.14	12.50		20	20		5	70	10	20
J	21.86	12.50	-	20	20	-	8	65	10	20
	56	75		60	60		20	85	60	80
	42.43	57.14		40	40		12	75	40	70
K	29.71	21.43	-	30	30	-	10	78	15	30
	27.86	21.43		30	30		10	65	15	30
	19.17	20		20	20		10	30	20	20
L	19.17	20	-	20	20	-	10	30	20	20
	61.67	60		60	60		55	70	60	60
	25	25		25	25		10	50	25	25
M	46.67	50	-	50	50	-	20	70	50	50
	28.33	25		25	25		5	70	25	25

Table 8.1: Classification of social outcomes

8.1 Outcomes under Truth-telling

A social outcome is called outcome under truth-telling, if the aggregated outcome coincides with the one that would result if all individuals voted for their true most preferred allocation. The truth-telling outcome is therefore either the mean, the normalized or the sequential median of the peaks. As already mentioned in chapter 6.1, the chances of an outcome to be classified as a truth-telling outcome are higher under the median rule. It is possible to analyze and compare the outcomes of the mean and the median rules, nevertheless one should keep in mind the different preconditions of the rules.

In the pilot session, 10.78% of all outcomes (eleven out of 102) are outcomes under truth-telling. All of the truth-telling outcomes occur under median-based rules but there seems to be no obvious pattern regarding the peak distribution, the degree of information or the period. If we omit the four cases in which the truth-telling outcome corresponds to the Nash outcome, which is considered in chapter 8.2 and is the case for distributions A and B, seven outcomes (6.86%) remain outcomes under truth-telling.

In sessions one to nine, we have only one outcome out of 522 that might be classified as truth-telling outcome, which is less than one percent. This outcome occurs under the mean rule (and therefore under no information), using peak distribution H in the third period. Anticipating the individual results in the subsequent chapters: in this situation, that produces a truth-telling outcome, none of the five subjects votes for the true most preferred allocation, but four play a Nash strategy.

The second experiment yields 39 out of 576 or 6.71% truth-telling outcomes. All of these outcomes occur under the normalized median rule. If we consider only the normalized median rule, we have 13.54% truth-telling outcomes that divide into 7.22% of all outcomes under the no information and 24.07% under the full info treatment. Peak distribution K yields no truth-telling outcome, distribution J two. The most frequent truth-telling outcomes occur under distributions L (23) and M (14), which are distributions where the normalized median coincides with the semi-pivotal peak. With the additional restriction that a truth-telling outcome is no Nash outcome (see chapter 8.2), both distributions L and M need to be removed, shrinking the truth-telling outcomes to less than one percent. Interestingly, the mean outcome in the second experiment is never identical to the theoretical median outcome under truth-telling and vice versa.

Across all sessions, we classify 4.25% of all outcomes as outcomes under truth-telling. Our data shows different percentages of truth-telling outcomes for different sessions. Nevertheless, one may conclude that truth-telling outcomes do not occur frequently. Under the mean rule, we have only one out of 696 outcomes (0.14%) that is classified as truth-telling outcome, such that we need to look into the individual data before judging about hypothesis H1.1. Truth-telling outcomes do emerge more often under the median rule (46 times) but the percentages (9.13%) are still low, especially when recalling the high chances of an outcome being classified as truth-telling outcome. With respect to the outcomes, we do not find strong support for H2.1 or H3.1 yet. We do observe more truth-telling outcomes under the full information treatment (8.81%) as compared to no info (1.79%); however even under full information on the peak distribution, this share is rather low.

8.2 Nash Outcomes

A Nash outcome is a group result that is consistent with the outcome if all five voters would play the corresponding Nash strategy. Since Nash equilibria are in some cases not unique and voters have different Nash strategies, we restrict the set of Nash outcomes. Under the median rule, we focus only on partially honest Nash outcomes, which eliminates Pareto-inefficient Nash equilibria. Comparable to truth-telling outcomes, the possibilities of Nash outcomes are much higher under the median rules than under mean voting. In chapter 7.2.3, we gave a detailed overview of all (partially honest) Nash strategies for the relevant peak distributions and for most of the peaks, the corresponding Nash strategy under the median rules are ranges of allocations. The Nash outcomes however are always unique allocations and partial honesty yields unique strategies when the number of voters is odd. Under the mean rule, multiple Nash equilibria are also possible. For peak distributions with multiple Nash equilibria, we focus on the outcome under focal strategies (distribution G) and the welfare optimal Nash outcome (distribution I).

In the pilot session, we have a total of nine Nash outcomes, which is 8.82%. Under the mean rule, the aggregated values are Nash outcomes five times, compared to four times under the normalized median rule. The sequential rule yields no Nash outcomes. Due to the limited data of the pilot session, we do not observe clear patterns at this stage. We do observe Nash outcomes under all peak distributions, information levels, under both, the mean and the normalized median rule, as well as in each period. Interestingly, under the mean rule all Nash outcomes are derived from Nash play of all five subjects, whereas under the normalized median in none of the four cases all subjects played the Nash strategy.

The number of Nash outcomes is very low in sessions one to nine. Only one out of the 522 outcomes is classified as Nash outcome. This situation occurs for distribution H, the mean rule and in period 3, which is the same setting as for the above-mentioned truth outcomes. However, since the truth outcome and the Nash outcome differ for distribution H (see table 8.1), it cannot be the same group of individuals. It is not only the case that Nash outcomes never occur in distributions D, E, F, G, and I but it is also surprising that it never occurs even though the Nash outcome corresponds to the truth outcome. In the only situation with Nash outcome, all five individuals play a Nash strategy, i.e. the strategies constitute a Nash equilibrium. For distribution I, we find that an allocation of 50 in the second project (which is the most preferred allocation of all individuals) occurs only once. The inefficient Nash outcome also never occurs.

In the second experiment, a total of 47 Nash outcomes occur, which is 2.78% of all outcomes under the mean rule and 13.54% under the normalized median. Under the mean rule, all eight Nash outcomes are given for distribution K, whereas under the normalized median rule we have one for each of the distributions J and K, and since the conditions for truth-telling and Nash outcomes are identical for the median rule in distributions L and M, the frequencies (37) are the same. Under full information, 13.89% of all outcomes are Nash outcomes and therefore more frequently as compared to 4.72% under no information.

Similar to the truth-telling outcomes, one may infer that Nash outcomes are rather seldom. In total, 4.75% of all outcomes are classified as Nash outcomes, under the mean rule the share is 2.01%, under the normalized median rule 8.23% and the sequential median rule no Nash outcomes occur. More outcomes are classified as Nash outcomes under full information (8.33%) as compared to no information (2.82%). It is not necessary that each Nash outcome comes from a Nash equilibrium but the very low shares of Nash outcomes do not give evidence to support our hypothesis H1.2 or H1.3. Yet before judging about the outcomes, one should consider wider ranges of classifications. A truth-telling or Nash outcome is only one specific allocation that might be lost by deviation of one single individual. When regarding our simplex, these outcomes represent only one point, meaning that all three coordinates need to satisfy the outcome condition. In a next step, we analyze efficiency and welfare optimality of the outcome, which are in most cases ranges and therefore should produce higher percentages.

8.3 Efficient Outcomes

In chapter 6.3, we define a minimal requirement for efficiency by the set of peak-bounded allocations. A social outcome is peak-bounded, if it is at least as high as the lowest ranked peak and not larger than the highest ranked peak for each project. In order to classify an outcome as peak-bounded, we have a lower and an upper bound for each project and check whether the social outcome lies within all of these boundaries. Given truth-telling of all agents, the mean and the median rule yield outcomes that are peak-bounded. If an outcome is not peak-bounded, this implies that at least one agent did not state the true most preferred allocation. An outcome that is not peak-bounded does not imply that an untruthful vote was a non-strategic vote, since the criterion of peak-bounded allocations is a measure of the aggregated outcome. An outcome might therefore be not peak-bounded but yield a higher utility for single individuals as compared to a peak-bounded outcome. The more stringent condition for efficiency is the set of Pareto-efficient allocations. We already stated that all Pareto-efficient outcomes must be peak-bounded, as displayed in figure 6.1 in chapter 6.3 and can also be observed in figure 8.1 for peak distribution G. Since the set of Pareto-efficient allocations is a subset of peak-bounded allocations, we first provide the data for outcomes that are peak-bounded and subsequently the outcomes that additionally satisfy Pareto efficiency.

In the pilot session, 89 outcomes out of 102 are peak-bounded, which is 87.25%. Six outcomes that are not peak-bounded emerge under the mean rule, under both information treatments and all three peak distributions but not in the first and fourth period. The other seven outcomes that are not peak-bounded occur under the normalized median rule with no information and peak distribution A in each of the five periods. The sequential median rule yields only peak-bounded outcomes. The results for Pareto efficiency are similar in the pilot session: 82.23% of all outcomes are Pareto-efficient, which divides into 83.33% under the mean rule, 79.17% under the normalized median rule and 100% (six out of six outcomes) under the sequential rule. The differences in the results between beak-bounded and Pareto-efficient outcomes occur for the mean and the normalized median rule solely in peak distribution C, where the set of allocations that are classified as peak-bounded but not Pareto-efficient is the largest.

For the mean rule in sessions one to nine, 247 outcomes out of 360 are peak-bounded, which is 68.61%. The percentage for the median rule is higher: 88.89% or 144 out of 162 outcomes are peak-bounded. Over both rules in these sessions, 74.90% of all outcomes are classified as peak-bounded. The total percentage of Pareto-efficient outcomes is 56.90%, divided into 52.50% under the mean rule and 66.67% under the median rule. Whereas the efficiency results are similar for both voting rules for peak distributions D, E, F, G, and H, the result for peak distribution I under the mean rule is prominent. There is only one social outcome that is classified as peak-bounded and therefore Pareto-efficient. Distribution I is special since the most preferred allocation of all agents is equal for the second project and the peak distribution is in a way one-dimensional. Thus, the peak-bounded allocations correspond to the convex hull of all peaks and to the set of Pareto-efficient allocations. We use this distribution under the mean rule and the no information treatment and it occurs only once that the social outcome in the second project coincides with the most preferred allocation of all voters. It seems natural that under full information voters might coordinate in a peak-bounded outcome. For the other peaks, we do observe a slight decline in the peak-bounded results among periods, which might be an indicator that truth-telling of voters decreases over periods.

In the second experiment, a total of 86.81% of all outcomes are peak-bounded and 73.26% are Pareto-efficient. Again, the percentages for the median rule are higher (88.89% and 76.74%) as compared to the peak-bounded and Pareto-efficient outcomes under the mean rule (84.72% and 69.79%). Since the efficiency levels are rather high, we focus on the outcomes that do not satisfy the peak-bounded criterion. We find outcomes that are not peak-bounded in all periods and do not identify a trend over time. Under the mean rule, most of the 44 outcomes that are not peak-bounded occur under distribution L, for which the share of not peak-bounded outcomes is 47.22%. In this distribution, all peaks are clustered in the same area of the simplex, as all peaks have the highest value in the same project. Interestingly, for distribution L and the mean rule, outcomes that are not peak-bounded occur at 44.44% under the no information treatment but with 51.85% even more frequently under full information. Regarding the degree of information among the entire second experiment, the results are as one would expect: outcomes that are not peak-bounded occur at 15.00% under no information but less frequently under full information with 10.19%. The results for Pareto efficiency under the mean rule are similar. While for distributions J, K, and M the shares of Pareto-efficient outcomes are around 80%, under distribution L, there are only 40.28% of all outcomes Pareto-efficient and the share is higher under no information (44.44% as compared to 33.33%). Under the median rule, the degree of information plays a more prominent role: outcomes are not peak-bounded at 16.11% under no information but only at 2.78% under full information. The normalized median rule produces only 32 outcomes that are not peak-bounded. Surprisingly, under the median rule outcomes that are not peak-bounded occur the most frequently under distribution J, where the range of peak-bounded outcomes is very large. However, only one outcome that is not peak-bounded under distribution J occurs under full information. The results for Pareto efficiency are similar. While under the median rule and full information 88.89% of all outcomes are Pareto-efficient, the share under no information is 69.44%. The shares of Pareto-efficient outcomes under the median rule are comparably high for all peak distributions and range from 72.22% for distribution J to 78.17% for distributions L and M.

Even though the frequencies of truth-telling and Nash outcomes over all sessions are low, only few outcomes are inefficient. Summarizing the results for efficiency, we classify 81.67% of the total outcomes as peak-bounded and 68.58% as Pareto-efficient. In total, we have a share of 88.55% of peak-bounded outcomes under the normalized median rule and 76.58% under the mean rule. We classify 73.69% of all median outcomes as Pareto-efficient and 61.78% of all mean outcomes. Under the sequential median rule, all outcomes are peak-bounded and Pareto-efficient but the number of total outcomes is very small. Peak-bounded outcomes occur under the no info treatment in 77.31% of the cases and under full information at a very high level of 89.76%. These results show that all of our tested rules yield high levels of efficiency for the given peak distributions. Efficiency is a desirable feature of social outcomes. However, when it comes to the utility sum of all agents, we further restrict the efficient outcomes to find the welfare optimal outcomes for the group in total.

8.4 Welfare Optimal Outcomes

We defined earlier in chapter 6.4 an outcome as welfare optimal, if it maximizes the sum of utilities, i.e. minimizes the total distance sum of peaks and outcome. Given that welfare optimality is a more stringent condition than efficiency, lower frequencies of welfare optimal outcomes are predicted. As already mentioned, truth-telling outcomes are not necessarily welfare optimal (but always peak-bounded), as can be seen in table 8.1 for the mean rule and distributions B, D, J, L, and M.

In the pilot session, 22 outcomes or 21.57% are welfare optimal. Interestingly, under peak distribution C the outcomes are never distance minimizing and therefore never under the sequential median rule. Consider the conditions for welfare optimality of the pilot session (distributions A, B, and C) in table 8.1. The welfare optimal outcome under distribution B is unique and therefore more difficult to be reached. Nevertheless, the frequency of welfare optimal outcomes is the highest under peak distribution B. Another interesting occurrence is that under no information the outcomes are never welfare optimal in the last period. Except for the fact that no outcomes are welfare optimal under the sequential median rule, we do not find obvious differences in the rules: there are seven welfare optimal outcomes under the mean rule and 15 under the normalized median rule.

In sessions one to nine, the numbers vary a lot across the rules. Under the mean rule, we have 30.28% (109 out of 306) welfare optimal outcomes that are evenly distributed among the periods. All welfare optimal outcomes under the mean rule occur at distributions G (68) and H (41), never D and I, where the welfare optimum is a unique allocation. The outcomes under the median rule are welfare optimal in only 5.56% of all cases (nine out of 162). All distance minimal outcomes occur under distribution F and in the last period. In contrast to distribution D, the lack of welfare optimal outcomes under distribution E may not be explained by the tight boundaries of the classified allocations, since the size of the allocation condition is the same as for peak distribution H. Additionally, the truth outcome under distribution E and the normalized median rule is welfare optimal. The data hints that welfare optimal allocations are less frequent under the normalized median rule.

The total percentage of outcomes that is classified as welfare optimal is 21.18% in the second experiment. Due to the between-subject design, we are able to compare the outcomes under the mean and the normalized median rule directly. In this setting, we find a higher share of welfare optimal outcomes under the median rule: 31.60% as compared to 10.76% under the mean rule. The degree of information leads to differences in the welfare optimality levels as well: under full information, we classify 27.31% of the outcomes as welfare optimal, whereas under no info the percentage is only at 17.50%. Under the median rule, welfare optimal outcomes occur with all peak distributions even under distributions L and M (23 and 14 outcomes) where the condition for a welfare optimal outcome is a unique allocation. This is not true for the mean rule, where welfare optimal outcomes only occur under distributions J and K (two and 29 outcomes). We do find welfare optimal outcomes in all periods, where under the normalized median rule with full information a negative trend is observable, to the extent that the occurrence of welfare optimality halves itself in the third period as compared to the first period.

Welfare optimal outcomes comprise a limited range and in five out of 13 peak distributions only a single allocation. Nevertheless, we classify 21.83% of all outcomes as welfare optimal. The percentage is slightly higher for the normalized median rule (23.09%) as compared to the mean rule (21.12%). None of the six outcomes under the sequential median rule is welfare optimal. What is surprising is that for the overall data, we classify a higher percentage of outcomes as welfare optimal under the no info treatment (23.33%) than under full information (19.05%).

8.5 Summary of the Outcome Results

Table 8.2 summarizes the outcome results and provides the shares of truth-telling and Nash outcomes as well as the percentage of outcomes that are classified as efficient (peak-bounded and Pareto-efficient) and welfare optimal. Besides the total shares, the table gives the shares by rule and the degree of information.[6] The sequential rule yields remarkable results, however only six outcomes were analyzed and thus we do only give limited attention to the results. One should also keep in mind that the peak distributions, degree of information and number of periods varied across the rules in the pilot session and in sessions one to nine. For the mean and the normalized median rule, we observe the highest percentage for peak-bounded outcomes, which is not surprising as the classification in table 8.1 comprises the widest range of allocations. Especially in the light of the low shares of truth-telling and Nash outcomes, we find it worth mentioning that the groups manage to coordinate on efficient outcomes. In particular, under full information we find a very large share of efficient outcomes. The share of welfare optimal outcomes is also not negligible for both the mean and the normalized median rule and surprisingly it is even higher under no information as under the full info treatment.

[6] Since the outcome numbers differ by rule and also by info, the column-wise aggregated values of the rules do not deliver the same percentages as the total percentage.

Unexpected are also the low shares of outcomes under truth-telling. Even though the percentage of truth-telling outcomes is higher under median based rules, we have only 9.13% of truth-telling outcomes under the normalized median rule. For five out of the ten peak distributions that were played under the normalized median rule, strategic voting is not possible. For all non-pivotal voters, truth-telling is a weakly dominant strategy and it is a strictly dominant strategy for the pivotal voter. For the other five peak distribution, strategic voting is hard to calculate and truth-telling is a focal strategy. Nevertheless, the share of truth-telling outcomes is far from fifty percent.

Comparably low are the shares of Nash outcomes. Given the low shares of truth-telling outcomes and the complicated calculation of Nash strategies under the normalized median rule, it is not surprising that Nash outcomes occur less frequently as compared to truth-telling outcomes. For the mean rule however, we expected higher shares of Nash outcomes, especially when Nash strategies are focal. Therefore, we find it surprising that more than half of the Nash outcomes occur under distribution K, where not even all Nash strategies are focal.

	Total	**Mean**	**NMed**	**SMed**	**No Info**	**Full Info**
Truth-telling	4.25	0.14	9.13	66.67	1.79	8.81
Nash	4.75	2.01	8.23	0	2.82	8.33
Peak-bounded	81.67	76.58	88.55	100	77.31	89.76
Pareto-efficient	68.58	61.78	73.69	100	70.51	82.62
Welfare optimal	21.83	21.12	23.09	0	23.33	19.05
No. of outcomes	1,200	696	498	6	780	420

Table 8.2: Outcome results (percentages)

9 Experimental Results - Individual Decisions

In the previous chapter, we analyzed the social outcome as an aggregated group result. In order to get further insights, we need to consider individual results for each voter. As mentioned earlier, at an aggregated level we cannot rule out the possibility that group outcomes are classified as truth-telling or Nash without any voter having stated the true most preferred allocation or a Nash strategy. We gave a hint that the truth-telling outcome under distribution H occurred without anyone voting for the true peak. Vice versa, we did not state so far whether an outcome was classified as either non-truth-telling or non-Nash because of only one non-truthful or non-Nash vote. In any case, it is urgently required to consider individual voting behavior. We give the results for the pilot session, sessions one to nine and the second experiment separately as the treatment variables varied across these experiments. The total number of individual decisions that we analyze adds up to 6,000: 510 for the pilot session, 2.610 for sessions one to nine, and 2.880 for the second experiment.

9.1 Results under the Mean Rule

We analyze the individual results first by the voting rules. For the mean rule, we are interested in the shares of truth-telling and Nash play. We also consider whether individuals play a best response to the result of the previous period and a best response to a uniform distribution of the other votes. For the mean rule, we analyze 240 votes from the pilot session, 1.800 from sessions one to nine and 1.440 from the second experiment, which is a total of 3.180 individual decisions.

9.1.1 Truth-telling

Our first analysis of individual results covers truth-telling. Hence, we examine the individual votes and compare them to the peaks. Truth-telling is given if the vote corresponds exactly to the most preferred allocation. However, we exclude votes for those truth-telling is a Nash strategy. For the mean rule, we therefore exclude one peak in each of the distributions B, D, I, and M. Since we already described in chapter 8.1 that for all sessions, the truth-telling outcome under the mean rule occurred only once and none of the five subjects did vote for the true most preferred allocation, we do expect low shares of individual truth-telling. Moreover, we hypothesize in H1.1 that the shares of truth-telling are even lower when information on the other peaks is disclosed.

In the pilot session, only seven decisions may be classified as truth-telling, six of them under no information and mainly in the first period and distribution A, which was the first distribution that the participants played. We do not have enough data from the pilot session to get a clear insight but the low shares of true votes give a tendency on what to expect from the following sessions.

In sessions one to nine a total of 125 votes are equal to the true most preferred allocation and not equal to Nash play. Since we played the mean rule voting game for the distributions G, D, H, and I only under no information, we cannot make a distinction to the full info treatment. Truth-telling prevails under each peak distribution and among all true votes, the shares range from 20.00% under distribution D and 30.40% under distribution H. We find a clear tendency that truth-telling decreases over time: out of the 125 true votes, 73 were played in the first period. Figure 9.1 displays for all four peaks the shares of truth-telling under the mean rule by period.

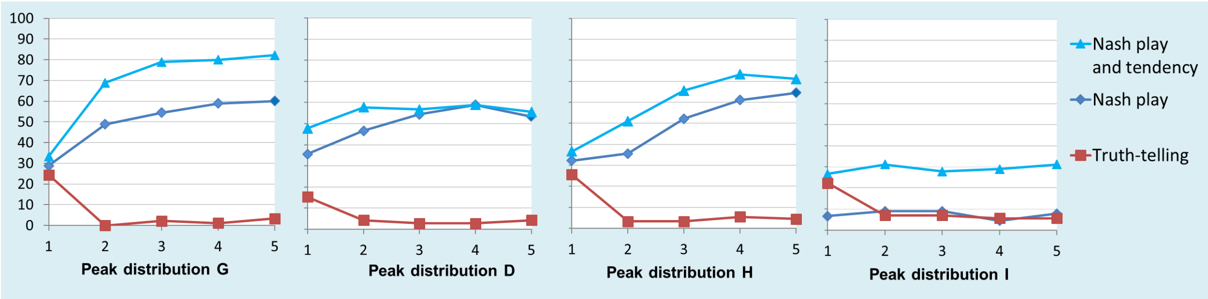


Figure 9.1: Results of the mean rule; G,D,H,I; no info (percentages by period)

A similar picture occurs in the second experiment. When using the mean rule, we find that participants rarely reveal their true peak. Only 5.70% of all votes are equal to the peaks and truth-telling is low over all four peak distributions. Moreover, truth-telling decreases over time, especially without any information on the peaks of the other participants. While 21.05% of the votes are equal to the individual peaks in the first period, this number declines to 2.34% in the fifth period.

In order to get a better insight of the influencing factors on truth-telling, we run a regression of the 'Peak-Vote-Distance', i.e. the absolute deviation of the vote from the true peak, on a variety of independent variables. We use the data of sessions one to nine and the second experiment. The regressions differ as we have both information treatments solely in the second experiment. The results of the regressions can be found in tables 9.1 and 9.2. We consider the voting behavior over periods and rounds, where round describes the decision number over all periods, and we expect that the distance to the peak increases over time. Three further distance measures are given: 'Peak-Nash-Distance', 'Nash-Vote-Distance' and the distance between the peak and the result of the previous period ('P-PR-Distance'), which we expect to have a positive effect on the deviation from truth-telling. We construct different dummy variables depending on the position of the Nash strategy: on the edge of the simplex, truth-telling or either one. In the regressions of the second experiment, we include also a dummy variable for the peak distribution and information level and run an additional regression including the demographic data from the questionnaire.

As stated before, we find a positive and significant correlation with the variable 'period', indicating a higher degree of deviation from truth-telling over time. Since the coefficient of 'round', which labels the total decisions over all periods from 1 to 20 or 1 to 24, is positive and significant, there is not only a higher degree of lying over periods but also over the entire decision-making process. As anticipated, truth-telling decreases slightly with an increasing distance between the true peak and the theoretical Nash strategy, since a greater 'Peak-Nash-Distance' indicates that participants have to deviate more from their true peak to play their Nash strategy.

Contrary to our expectations, the deviation from the peak is positively affected by the distance between the theoretical Nash strategy and the actual vote, i.e. 'Nash-Vote-Distance'. This implies that the higher the deviation of the vote from the Nash strategy, the more participants tend to lie. In other words, voters lie more if they are not voting according to the predicted Nash strategy. Given Nash play of the other four voters, this strategy results in a lower payoff. After the experiment, we asked the participants about their approach of voting. Some argued that they tried to deceive the others by votes that lead to a lower payoff in order to receive a higher payoff in the next period. This behavior might explain the results that seem non-strategic at first appearance. Participants also tend to significantly lower truth-telling with a higher distance between the peak and the result of the previous round ('P-PR-Distance'). Thereby, we are able to observe a learning effect over periods of increasing strategic voting. Although in absolute numbers the difference of truth-telling across the peak distributions is low, we find a significant and high difference in the extent of truth-telling dependent on the theoretical Nash strategy. The dummy variable 'edge_d' takes the value 1 if the individual Nash strategy is voting zero for exactly one project (distributions H and I) and 0 else. The dummy variable ‘truth_d’ takes the value 1 if the Nash strategy corresponds to truth-telling and else 0. The dummy variable ‘edgetruth_d’ takes the value 1 if the individual Nash strategy is either truth-telling (peak distribution M) or voting zero for exactly one project (peak distribution K) and the value 0 if Nash play is to vote zero for two projects, as this captures all cases for our peak distributions.

VARIABLES	Peak-Vote-Distance
period	3.651***
	(0.637)
subject	0.718***
	(0.266)
round	0.438**
	(0.199)
Peak-Nash-Distance	0.108***
	(0.0357)
Nash-Vote-Distance	0.153***
	(0.0171)
P-PR-Distance	0.192***
	(0.0339)
edge_d	-9.600***
	(2.896)
truth_d	-7.059*
	(4.042)
Constant	18.70***
	(4.003)
Observations	1,800
R-squared	0.143

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 9.1: Regression results sessions one to nine: peak-vote-distance

We find that deviation from the true peak (the extent of lying) is significantly lower if the theoretical Nash strategy is to vote zero for only one project, compared to voting zero for two projects and also lower if it corresponds to truth-telling as compared to strategic voting. We conclude that the degree of lying is lower if the theoretical Nash strategy is not focal, like it would be for a vote on an allocation with the total budget in the project with the highest share of the true peak. Since the Nash strategy is not straightforward, it is reasonable to vote for an allocation close to the peak.

The dummy variable ‘peak_d’ we use in the second regression takes the value 1 if the votes belong to peak distribution L and 0 else. For two peaks in this distribution, the Nash strategy is to vote zero for the project with the highest peak value and therefore not focal. We find a negative and significant coefficient for the peak dummy, indicating that for this distribution the deviation from the peak is lower. Subjects may face more difficulties in finding the Nash strategy and therefore it is natural to vote for an allocation closer to the peak as compared to the other peak distributions.

We run an unpaired t test with the data from the second experiment in order to test our hypothesis about truth-telling under the mean rule dependent on the degree of information. We hypothesize in H1.1 that subjects reveal their true preferences less under full information as compared to no information. We are able to reject the null hypothesis of identical shares of truth-telling and find significant support for the alternative hypothesis H1.1: Under the mean rule, the average share of truth-telling is higher under no information as compared to full information (two-sample t test, p<0.001).

	(1)	(2)
VARIABLES	Peak-Vote-Distance	Peak-Vote-Distance
period	3.091***	3.147***
	(0.849)	(0.842)
subject	-0.0633	-0.222
	(0.193)	(0.214)
round	0.697**	0.569*
	(0.331)	(0.334)
Peak-Nash-Distance	0.171***	0.160***
	(0.0322)	(0.0321)
Nash-Vote-Distance	0.181***	0.194***
	(0.0154)	(0.0156)
P-PR-Distance	0.0279**	0.0311***
	(0.0115)	(0.0114)
edgetruth_d	-9.873***	-9.248***
	(3.024)	(3.006)
peak_d	-26.18***	-24.40***
	(2.957)	(3.009)
info_d	5.862	7.725
	(4.668)	(4.693)
male_d		3.019
		(2.184)
understandrule_d		3.778*
		(2.123)
econstudy_d		5.495**
		(2.431)
wingstudy_d		3.824*
		(2.313)
Constant	17.69***	8.257**
	(3.577)	(4.057)
Observations	1,440	1,440
R-squared	0.183	0.201

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 9.2: Regression results second experiment: peak-vote-distance

9.1.2 Nash Play

Hypotheses H1.2 and H1.3 state that we expect Nash play under the mean rule and that the quantity of Nash strategies increases over time. In chapter 7.2.3, we provided the theoretical Nash equilibria of all peak distributions we used in the experiments. We showed that for one subject under distribution G, the Nash strategy is a range of allocations. For the analysis of Nash play, we consider only the focal Nash strategy. Peak distribution I is another special case with several equilibria that we have to keep in mind when evaluating Nash play. For distribution I, we evaluate Nash play using the most prominent Nash strategies, which are voting for the true allocation in the second project. While in the last chapter we excluded true votes when truth-telling is a Nash strategy, we do count true votes as Nash play when truth-telling is a Nash strategy.

In the pilot session, we have a total of 137 votes that are classified as Nash play under the mean rule, which is 57.08% of all decisions. While the shares are already high in the first periods and even under no information, the numbers increase over time and shares begin at higher levels under the full info treatment. There was one decision of an individual with peak $p_i = (30, 40, 30)$ under distribution C where the Nash strategy $q_i^* = (50, 0, 50)$ is not focal but played in the last period. We already see from the pilot session that the shares of Nash play are relatively high and we do expect similar shares for the other peak distributions.

In sessions one to nine, a total of 674 votes are Nash play, which is a share of 37.44%. Nash play increases over time from 25.83% in the first period to 43.89% in the last period. Without the results from distribution I, the share is 47.48%. Beside the shares of truth-telling, figure 9.1 displays the fractions of Nash play for the mean rule and distributions G, D, H, and I. The shares of Nash play are above 30.00% in every period under distributions G, D, and H and increase with the number of periods. Including a tendency to Nash play, which comprises votes with a maximal absolute distance of 10 between the Nash strategy and the vote in every project, the shares are especially high under distribution G, where all Nash strategies are focal. Even though we classify 919 votes, i.e. 51.06% of all decisions, to be Nash play or a tendency to Nash play, only once a Nash equilibrium was played. We further conclude that Nash play is heavily affected by the peak distribution since we have very low shares of Nash play for distribution I. We already stated that there was only one social outcome under distribution I that is classified as efficient, indicating that subjects face a coordination problem when all of them prefer the same allocation on one project and no info on the peak distribution is revealed.

In the second experiment, we also observe a high ratio of votes (35.76%) that correspond to the theoretical Nash equilibrium. When adding a tendency to Nash play, 48.54% of all votes are Nash play and Nash tendency. Further, we perceive a learning effect over periods, both with and without information on the other peaks, indicating a convergence to the group Nash equilibrium. Nevertheless, the group Nash equilibrium, i.e. the situation in which all of the five group members choose the individual Nash strategy, arises in only 2.78% of the social outcomes. Compared to the no info treatment, Nash play is higher under full information in every period when using peak distributions K, L and M. This outcome is in line with the fact that we observed less truth-telling with full disclosure of the other peaks. Figures 9.2 and 9.3 summarize the truth-telling, Nash play and tendency to Nash play shares of total possible votes over periods for each peak distribution under no and full information.

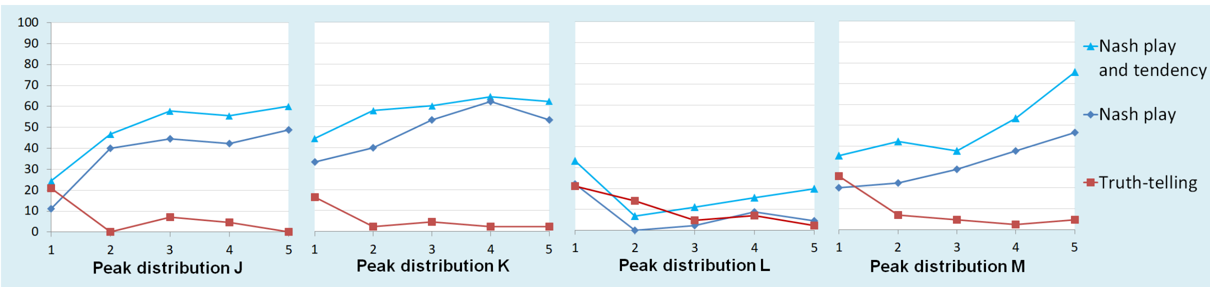


Figure 9.2: Results of the mean rule; J,K,L,M; no info (percentages by period)

A further interesting result can be found with peak distribution L. We created a situation at which all peaks allocate the highest amount on the third project. Only a small proportion of the overall votes are Nash strategies (10.83% in comparison to the other three peak distributions with an average of 44.07%) and even under full information, the share is with 16.30% relatively low. Another conspicuousness of peak distribution L is the result that the average share of Nash play decreases by periods under no information.

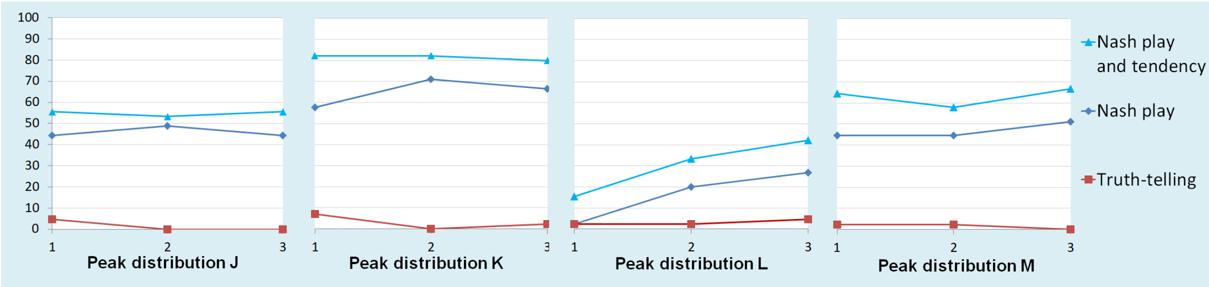


Figure 9.3: Results of the mean rule; J,K,L,M; full info (percentages by period)

We run a regression of the Nash-Vote-Distance to get a better insight on the deviation from votes to the Nash strategy. Table 9.3 reveals the regression results for sessions one to nine and table 9.4 for the second experiment. The explanatory variables are the same as in the previous analysis for truth-telling.

VARIABLES	Nash-Vote-Distance
period	-4.300***
	(0.863)
subject	-1.519***
	(0.358)
round	-0.282
	(0.270)
Peak-Nash-Distance	0.490***
	(0.0470)
Peak-Vote-Distance	0.279***
	(0.0312)
P-PR-Distance	-0.0563
	(0.0462)
edge_d	38.01***
	(3.820)
truth_d	53.18***
	(5.320)
Constant	5.678
	(5.439)
Observations	1,800
R-squared	0.161

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 9.3: Regression results session one to nine: Nash-vote-distance

The coefficients of 'period' and 'round' have the anticipated negative sign but statistical significance is only given for one at the same time. The negative sign supports the expectation of a learning effect in playing the Nash strategy over the session: the more advanced the voting game, the closer are the votes to the Nash strategy. Therefore, we confirm Hypothesis H1.3: Nash play increases over time under the mean rule. Another indicator for a change in voting behavior is the negative and in the second experiment significant coefficient of the distance between the peak and the result of the previous period ('P-PR-Distance'), which reflects the higher gain in utility by Nash play if the peak is distant from the social outcome of the last round. The positive and significant correlation of the deviation of the vote to the theoretical Nash strategy and the 'Peak-Nash-Distance' highlights the growing difficulties of finding the corresponding Nash equilibrium the more remote the Nash strategy is from the peak. The distance between Nash play and vote increases with a higher 'Peak-Vote-Distance', indicating that manipulation occurs for subjects with a Nash strategy that is more 'difficult' to predict but not towards the Nash equilibrium. We also find a higher deviation from the Nash strategy if this theoretical strategy is to vote zero for one project (edge) or to vote for the actual peak (truth), denoted by the positive coefficients of the dummy-variables 'edge_d', 'truth_d', and 'edgetruth_d'. These positive coefficients indicate that focal strategies are easier to find and we could see in the last chapter that subjects with non-focal Nash strategies tend to truth-telling. It is especially interesting that given the Nash strategy is truth-telling (the truth dummy takes the value 1), the distance between the actual vote and the Nash strategy is much higher as compared to Nash strategies that are non-truthful. Subjects do also vote for allocations more that deviate more from the Nash strategy at peak distribution L, compared to the other peak distributions, see the coefficient of the dummy-variable 'peak_d' in table 9.4.

VARIABLES	(1) Nash-Vote-Distance	(2) Nash-Vote-Distance
period	-0.854	-1.029
	(1.397)	(1.364)
subject	-0.173	0.335
	(0.316)	(0.345)
round	-2.032***	-1.770***
	(0.541)	(0.537)
Peak-Nash-Distance	0.376***	0.389***
	(0.0524)	(0.0512)
Peak-Vote-Distance	0.485***	0.504***
	(0.0414)	(0.0406)
P-PR-Distance	-0.0644***	-0.0691***
	(0.0188)	(0.0184)
edgetruth_d	17.04***	15.73***
	(4.952)	(4.843)
peak_d	34.31***	30.61***
	(4.891)	(4.894)
info_d	2.933	-0.632
	(7.651)	(7.572)
male_d		-16.03***
		(3.497)
understandrule_d		-9.904***
		(3.416)
econstudy_d		7.594*
		(3.921)
wingstudy_d		-19.54***
		(3.696)
Constant	23.78***	41.76***
	(5.876)	(6.454)
Observations	1,440	1,440
R-squared	0.217	0.259

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 9.4: Regression results second experiment: Nash-vote-distance

Surprisingly, we are not able to find a significant effect of the degree of information on the distance to the Nash strategy. Only when including dummy variables with demographic data from the questionnaire, the coefficient of the dummy variable 'info_d' is negative but still insignificant. Nevertheless, we are able to reject the hypothesis that the mean of votes that are classified as Nash strategies is similar for rounds without any and under full information. Instead, we find that the mean share of Nash strategies is higher under full information compared to no information (two-sample t test, $p<0.001$).

What remains unspecified is the answer to our hypothesis about playing the Nash equilibrium under the mean rule, H1.2. We observe high shares of Nash play in both experiments at the individual level; however, the percentages of Nash outcomes are low. The hypothesis that under the mean rule the (Pareto-efficient) Nash equilibrium will be played is not supported by our data as rarely all group members play a Nash strategy at the same time. The results for peak distribution I reveal that when multiple equilibria exist, subjects fail to coordinate under no information. Nevertheless, at an individual level, Nash play is a prominent strategy and we observe a tendency to Nash play.

9.1.3 Best Response to the Result of the Previous Period

When analyzing the voting behavior, we also find some further interesting strategies. Besides truth-telling or Nash play, subjects might vote for an allocation that is a best response to the social outcome of the previous period (BRP). Therefore, we first calculate the theoretical best response ($tBRP$) in period t, where t takes the values 2 to 5 in the no info and 2 and 3 in the full info treatment. Given the votes of the other subjects in the previous period, q_{-i}^{t-1}, under the assumption that the mean of the other subjects remains the same in the current period t, the theoretical best response is to vote for an allocation such that the social outcome is equal to the own peak:

$$p_i = Mean^{t-1}(q) - \frac{1}{5} \cdot q_i^{t-1} + \frac{1}{5} \cdot tBRP_i^t. \tag{9.1}$$

Solving for the theoretical best response to the result of the previous period gives the following optimization problem:

$$tBRP_i^t = 5 \cdot (p_i - Mean^{t-1}(q)) + q_i^{t-1}. \tag{9.2}$$

A nice feature of this computation is the fact that the sum of $tBRP_i^t$ over all j is equal to the total budget $Q = 100$. Nevertheless, there might exist allocations that include project-wise votes we prohibited in the experiment, like non-negative votes or ones that exceed the total budget. Hence, by using the $tBRP$, we calculate the BRP, again, separately for all j projects, but only with feasible allocations $\in \mathcal{B}$ by 'cutting off' the unfeasible ones:

$$BRP_i^{j,t} = \begin{cases} 100, & if\ tBRP_i^{j,t} > 100 \\ 0, & if\ tBRP_i^{j,t} < 0 \\ tBRP_i^{j,t} & else. \end{cases} \tag{9.3}$$

This time, we do not get any project-wise prohibited votes, i.e. the allocation for each project is a natural number between zero and 100, as we demanded in the experiment. However, there might be allocations at which the sum of $BRP_i^{j,t}$ over all projects exceeds the total budget. Note that by construction of $BRP_i^{j,t}$, it is not possible that the sum of $BRP_i^{j,t}$ over all projects undercuts the total budget. Since the sum of $tBRP_i^t$ over all j is equal to the total budget Q, the positive values have to add up to at least Q. Thus, in a further step, we create ranges of best responses to the previous period result that reach from a lower ($lBRP$) to an upper boundary of the BRP ($uBRP$). Within these ranges, the social outcomes result in equal payoffs due to the hexagon-shaped indifference curves. The ranges for each project $j, k, l \in J$ are calculated exemplary for j by the following equations:

$$lBRP_i^{j,t} = \begin{cases} 100 - BRP_i^{k,t} - BRP_i^{l,t}, j \neq k \neq l, & if\ BRP_i^{j,t} \neq 0 \\ BRP_i^{j,t} & else. \end{cases} \tag{9.4}$$

$$uBRP_i^{j,t} = BRP_i^{j,t}. \tag{9.5}$$

In the pilot session, we get a total of 117 votes that are a best response to the result of the previous period, which are 65.00% of all decisions from the second period on. Interestingly, the shares are higher under full info (75.00%) as compared to no information (60.00%), but the no info treatment has two more periods. The shares do not vary much across the peaks nor periods; there is only a slight increase by periods in the no info treatment.

We find that in sessions one to nine a total share of 53.89% of votes are classified as BRP. For all peak distributions, we find an increase from the second to the fifth period. The shares of BRP are the highest for peak distribution G (67.50%) and lowest for distribution I (34.17%). However, the relatively low shares for distribution I are still higher than the shares of truth-telling or Nash play for this distribution. Therefore, these votes seem not to be random but a strategical factor appears to play a role.

In the second experiment, we also find that with 46.94% a relatively high share of votes are a best response to the result of the previous period. Over all peaks and degrees of information, the share of BRP is higher in the last period ($t = 5$ or $t = 3$) as compared to the second ($t = 2$). This result complies with the observation of the increase in Nash play over time. Once a Nash equilibrium is reached, the BRP in the next period is always equal to the Nash play, indicating the stability of the Nash equilibrium. When distinguishing between the different info treatments and the peak distributions, we find results comparable to Nash play. The share of votes that are classified as a BRP is higher under full information and peak distribution L has a considerably lower share of BRP votes as compared to the other distributions. Nevertheless, within peak distribution L the share of votes that are a BRP amount to 27.78%, which constitutes a higher share than Nash play with 10.83%. Therefore, we conclude that even if the theoretical Nash equilibrium is 'hard' to achieve, a considerable share of votes is strategical.

In summary, the strategy of playing a best response to the outcome of the previous period has large explanatory power for the voting behavior under the mean rule. Especially under the no info treatment, where the result of the previous period is the only information participants receive before voting, it is rational to adapt the own strategy according to this result.

9.1.4 Best Response to a Uniform Distribution of the Other Votes

One might argue that without any information, a possible assumption on the other subjects' peaks is a uniform distribution on the feasible allocations. This results in an expected mean of the other peaks that allocates a third of the budget on every project. A strategy might now be to play a best response to uniformly distributed votes of the other subjects ($BRUD$). Table 9.5 displays for the mean rule all peaks where Nash play in equilibrium and $BRUD$ differ. As the conditions of Nash play and $BRUD$ are identical for the other peaks, the results are identical: each time a subject plays a Nash strategy, the vote is also a best response to uniformly distributed votes. The peak distributions in table 9.5 that are marked with a star (*) indicate that the $BRUD$ is a range of allocations that includes Nash play. Since we are only interested in the cases where $BRUD$ and Nash play are distinct, we exclude the overlapping allocations of six peaks. Due to the restriction on natural numbers of the votes, we consider for the peaks of distributions C, G, H, and K all feasible combinations of votes. Exemplary for peak $p_i = (30, 40, 30)$ of distribution C we count the following votes as $BRUD$: $q_i = (17, 66, 17)$, $q_i = (16, 67, 17)$ and $q_i = (17, 67, 16)$.

Table 9.5 indicates that there exist peaks for which $BRUD$ and Nash play differ. One should also consider the dissimilarities of the peak distributions. While for some peaks the $BRUD$ is also a tendency to Nash play, the distances between Nash play and $BRUD$ are rather high for other peaks.

For the two peaks of the pilot session, where 32 votes may be classified as $BRUD$, we do not find any vote that corresponds to a $BRUD$. The allocations are unique and therefore the chances that a vote is classified as $BRUD$ are lower as for ranges.

In sessions one to nine there are 990 possible decisions that may be classified as $BRUD$ and not Nash play. We find a total of 132 votes that are a best response to a uniform distribution, which is 13.33%. As one might expect, the shares are highest for peaks where the $BRUD$ is a range of allocations. Especially high shares of $BRUD$ can be found for the peaks $p_i = (40, 50, 10)$ (31 votes) and $p_i = (10, 50, 40)$ (35 votes) of distribution I. Since for distribution I there exists a multitude of Nash equilibria, it is also not clear whether the votes are classified as $BRUD$ or Nash play of a different Nash equilibrium. Another peak with high shares of $BRUD$ is $p_i = (50, 35, 15)$ of distribution G. Here, $BRUD$ is also a tendency to Nash play and moreover, the Nash strategy of this peak is focal such that the classification of votes to be a $BRUD$ may be driven by a tendency to Nash play.

In the second experiment, there are five peaks for which Nash play and $BRUD$ differ. Therefore, 360 votes are possible candidates to be classified as $BRUD$ and not Nash play. The total share of $BRUD$ is 9.44% and is derived mainly from peak $p_i = (50, 30, 20)$ of distribution M. For this peak, playing a $BRUD$ is also a tendency to Nash play. Moreover, the Nash strategy for this peak is focal, such that the high numbers of $BRUD$ may also be explained by a tendency to Nash play.

The total share of decision that are a $BRUD$ adds up to 12.01% across all peaks. The results show that theoretical Nash play and $BRUD$ are identical for the most peaks and therefore subjects might play a Nash strategy automatically because they intend to play a $BRUD$. Nevertheless, the very low shares of $BRUD$ in cases where it is not identical to Nash play indicate that voting according to a $BRUD$ does not have large explanatory power for subjects with these peaks.

Peak Distribution	p_i	Nash play	$BRUD$
	25	25	0
B	50	50	100
	25	25	0
	30	50	$16.\overline{6}$
C	40	0	$66.\overline{6}$
	30	50	$16.\overline{6}$
	25	25	0
D	50	50	100
	25	25	0
	50	100	$\geq 58.\overline{3}$
G*	35	0	$\leq 41.\overline{6}$
	15	0	0
	40	100	$66.\overline{6}$
G	30	0	$16.\overline{6}$
	30	0	$16.\overline{6}$
	20	0	0
H*	30	0	$\leq 16.\overline{6}$
	50	100	$\geq 83.\overline{3}$
	30	50	$16.\overline{6}$
H	40	0	$66.\overline{6}$
	30	50	$16.\overline{6}$
	50	100	$\geq 83.\overline{3}$
H*	30	0	$\leq 16.\overline{6}$
	20	0	0
	30	50	$\leq 16.\overline{6}$
I	50	50	$\geq 83.\overline{3}$
	20	0	0
	20	0	0
I	50	50	$\geq 83.\overline{3}$
	30	50	$\leq 16.\overline{6}$
	40	50	$\leq 66.\overline{6}$
I*	50	50	$\geq 33.\overline{3}$
	10	0	0
	10	0	0
I*	50	50	$\geq 33.\overline{3}$
	40	50	$\leq 66.\overline{6}$
	25	25	0
I	50	50	100
	25	25	0
	40	0	$66.\overline{6}$
K	30	50	$16.\overline{6}$
	30	50	$16.\overline{6}$
	30	100	$\leq 16.\overline{6}$
L	15	0	0
	55	0	$\geq 83.\overline{3}$
	15	0	0
L	30	100	$\leq 16.\overline{6}$
	55	0	$\geq 83.\overline{3}$
	50	100	$\geq 83.\overline{3}$
M*	30	0	$\leq 16.\overline{6}$
	20	0	0
	25	25	0
M	50	50	100
	25	25	0

Table 9.5: $BRUD$ conditions mean

9.2 Results under the Median Rule

Under the median rule, we are also interested in the shares of truth-telling and Nash play. As described earlier, Nash play includes truth-telling in many cases. We also analyze whether individuals play a best response to truth-telling of the other individuals. From the pilot study, we have 240 individual decisions for the normalized median rule and 30 decisions for the sequential median rule. Sessions one to nine yield 810 decisions and the second experiment 1.440, both for the normalized median rule only. Since we mainly focus on the normalized median rule, the results are indicated for this rule when not explicitly stated otherwise.

9.2.1 Truth-telling

Similar to the mean rule, we analyze the shares of truth-telling and Nash play also for the median-based rules. Under the sequential median rule, truth-telling is always part of the Nash strategy. Under the normalized median rule, however, Nash play includes strategic voting for one peak in distributions C, E, F, J, K, and L. For these peaks, truth-telling and Nash play differ. Since truth-telling is part of the Nash strategy for all other peaks, we additionally provide the results of truth-telling for those peaks. Compared to the mean rule, the normalized median is more difficult to understand and therefore the voting behavior is more subtle and not as straightforward to predict. Hence, as stated in hypothesis H2.1, we anticipate truth-telling since voting for the true preferred allocation is a weakly dominant strategy under the median rule without adaptation.

For the normalized median rule in the pilot session, we have 51 votes that are identical to the peak, which is a share of 21.25%. Interestingly, the share is higher under full information (30.00%) as compared to the no info treatment (16.00%). There is no clear pattern observable regarding the differences in the peak distributions or the number of periods. Only for one peak in distribution C, the Nash strategy is different from truth-telling. The vote is identical to the peak in this case once under no information in period 1 and twice under full info in periods 1 and 3. The subject that reveals the true peak is different in all three cases.

The sequential median rule is only tested in the pilot study and solely for peak distribution C under full information; therefore, we include the results at this stage. Out of the 30 votes, ten are identical to the true allocation (33.33%) and truth-telling occurred in every period. What might be surprising is the fact that half of the true votes come from the two subjects with peak $p_i = (30, 40, 30)$.

Sessions one to nine yield 171 true votes including Nash play, which is a share of 21.11%. Truth-telling occurs under all peak distributions and periods and a slight trend regarding decreasing shares of truth-telling over time is observable. The results for truth-telling under the distributions D, E, and F are displayed in figure 9.4. There exist peaks that include strategic voting, meaning that truth-telling is not a Nash strategy: $p_i = (40, 20, 40)$ in distribution E and $p_i = (20, 40, 40)$ in distribution F. Surprisingly, the shares of truth-telling for these peaks are relatively high: 58.33% of decisions from subjects with these peaks are truth-telling and thus not Nash play. Most of these true votes occur in the first period and truth-telling declines over time.

Even though truth-telling is part of the Nash strategy in many cases, only 18.96% of all votes are equal to the true peaks in the second experiment. We also find a tendency of less truth-telling with increasing periods over all peak distributions and degrees of information, i.e. the share of truth-telling is lower in period five than in period one. Figure 9.5 displays the results under no info and figure 9.6 under full information. Interestingly, there are increases in truth-telling in the last periods in some peak distributions. This strategy might reveal failure in strategic voting and therefore going back to truth-telling. Some participants argue in the questionnaire that voting for an extreme allocation in order to irritate the other voters and benefit in the next period was part of their voting behavior. If we consider truth-telling only for the peaks $p_i = (20, 20, 60)$ from peak distribution J and $p_i = (40, 30, 30)$ from distribution K, we observe an even higher share of 19.44% true votes. The Nash strategies for these peaks are strategic and exclude truth-telling. However, 22.22% for distribution J and 16.67% for distribution K of the votes from peaks with strategic Nash play are truth-telling.

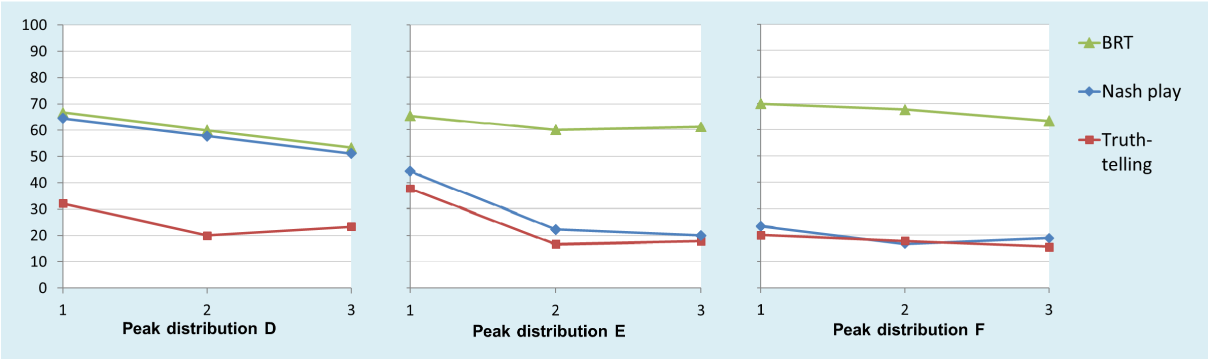


Figure 9.4: Results of the median rule; D,E,F; full info (percentages by period)

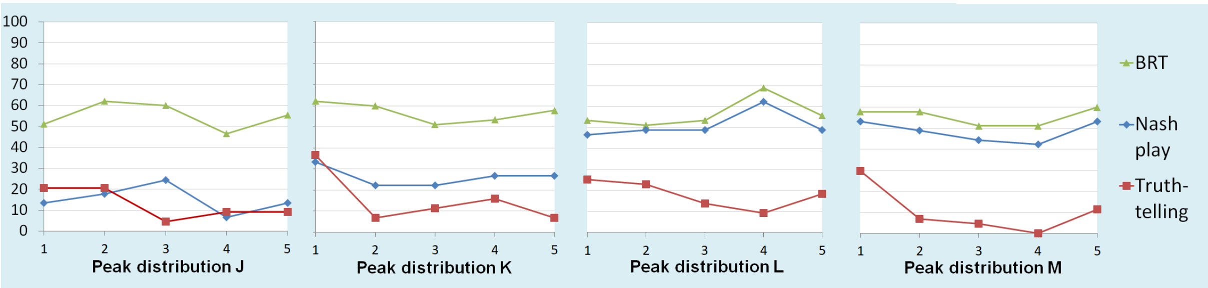


Figure 9.5: Results of the median rule; J,K,L,M; no info (percentages by period)

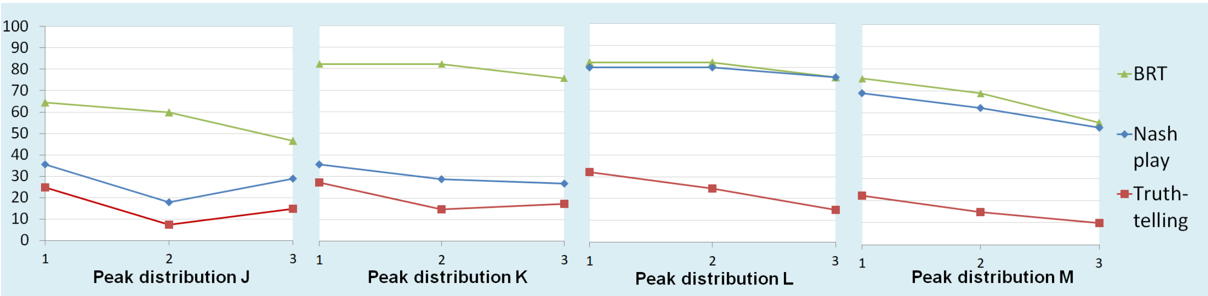


Figure 9.6: Results of the median rule; J,K,L,M; full info (percentages by period)

Our hypothesis H2.1 states that we expect truth-telling under median-based rules. The shares of truth-telling are around 20.00% over all sessions for the normalized median rule and even though they are higher for the sequential rule, we do not find the numbers high enough to support our hypothesis. We perform an unpaired t test to compare truth-telling under both information levels in the second experiment. We are able to reject the hypothesis of equal mean shares of truth-telling under full and no information and find a higher share of truth-telling with a higher level of information over all periods and rounds (two-sample t test, p<0.001). Since truth-telling is only a weakly dominant strategy in the median rule without adaptation and within-rank deviation of the non-pivotal voters has no effect on the social outcome, the low shares of true votes come not as a big surprise.

9.2.2 Best Response to Truth-Telling

In a further step, we expand the strategy of truth-telling to a best response to the true peaks of the other participants (BRT). A BRT of an individual who is pivotal in every project is to vote for the true preferred allocation. By contrast, the BRT of semi-pivotal or non-pivotal voters is to stay within the rank, i.e. voting for an equal or higher (lower) value than the pivotal voter if the own true value is higher (lower) in this project. As we assume truth-telling of the other individuals, the BRT might differ from the Nash strategies under the normalized median rule, where strategic voting of the (semi-)pivotal voter is sometimes possible. The detailed BRT conditions of all voters for peak distributions of the normalized median rule are shown in table 9.6. For the sequential median rule, the BRT is identical to Nash play and 14 out of 30 votes are a BRT.

An interesting peculiarity is given with respect to the peak distributions C, E, F, J, and K. The voter that is pivotal in only one project is able to vote strategically given truth-telling of the other voters. The median outcome under BRT results in a normalized median of the peak of this participant. Thus, the BRT of the other subjects is in general not identical to Nash play.

In the pilot session, the shares of BRT are already high. 69.17% of all decisions are a BRT under the normalized median rule and the share is higher under the no information treatment (72.00%) as compared to the full info treatment (64.44%). BRT is played in similar shares across the peak distributions and over the periods.

The fractions of BRT of sessions one to nine are comparably high and are displayed in figure 9.4. A total of 511 votes, which is a fraction of 63.09%, are a best response to truth-telling of the other voters. The shares are high among all three peak distributions and we observe a decline over periods (besides the slight increase in the third period under distribution E). We observe that subjects tend to vote for extreme allocations, comparable to voting under the mean rule.

A consideration of the experimental results from the second experiment also confirms hypothesis H2.2: a relatively high share of votes (59.72%) are a best response to true revelation of the other peaks. This value fluctuates slightly but remains high over all periods and peak distributions. The shares of BRT are high under both treatments, we observe 56.00% under no info and 65.93% under full information, and we are able to reject the hypothesis that there is no difference between the treatments. The shares of BRT under full information are significantly higher compared to no information (two-sample t test, $p<0.001$), which stands to reason as the individuals in the experiment can only play a BRT if the entire peak distribution is disclosed.

	Participant				
A	**1**	**2**	**3**	**4**	**5**
	$\leq$20	$\geq$20	$\leq$20	$\leq$20	$\geq$70
BRT_i	$\geq$70	$\leq$20	$\leq$20	$\geq$70	$\leq$20
	$\leq$10	$\geq$10	$\geq$60	$\leq$10	$\leq$10
B					
	$\leq$25	=25	$\geq$25	$\leq$25	$\geq$25
BRT_i	$\leq$50	=50	$\geq$50	$\geq$50	$\leq$50
	$\geq$25	=25	$\leq$25	$\leq$25	$\geq$25
C					
	$\geq$40	$\leq$20	$\leq$20	$\leq$20	$\geq$20
BRT_i	$\leq$40	$\leq$40	$\geq$60	$\geq$60	=27
	$\leq$20	$\geq$40	$\leq$20	$\leq$20	$\geq$20
D					
	$\geq$25	$\geq$25	$\geq$25	$\leq$25	$\leq$25
BRT_i	= 50	$\geq$50	$\leq$50	$\leq$50	$\geq$50
	$\geq$25	$\leq$25	$\geq$25	$\geq$25	$\leq$25
E					
	$\geq$30	$\leq$30	$\leq$30	$\geq$50	$\leq$30
BRT_i	=15	$\leq$20	$\geq$40	$\leq$20	$\geq$40
	$\geq$30	$\geq$50	$\leq$30	$\leq$30	$\geq$30
F					
	$\leq$20	= 15	$\geq$40	$\leq$20	$\geq$40
BRT_i	$\leq$30	$\geq$30	$\leq$30	$\geq$50	$\leq$30
	$\geq$50	$\geq$30	$\leq$30	$\leq$30	$\leq$30
J					
	$\geq$10	$\leq$10	$\leq$10	$\geq$30	$\leq$10
BRT_i	$\geq$10	$\geq$30	$\leq$10	$\leq$10	$\leq$10
	=30	$\leq$60	$\geq$80	$\leq$60	$\geq$80
K					
	=20	$\geq$70	$\geq$70	$\leq$40	$\leq$40
BRT_i	$\geq$15	$\leq$15	$\leq$15	$\leq$15	$\geq$45
	$\geq$15	$\leq$15	$\leq$15	$\geq$45	$\leq$15
L					
	$\leq$ 20	$\geq$ 20	$\geq$ 20	$\geq$ 20	$\leq$ 20
BRT_i	$\geq$ 20	$\leq$ 20	$\geq$ 20	$\leq$ 20	$\geq$ 20
	$\geq$ 60	$\geq$ 60	= 60	$\leq$ 60	$\leq$ 60
M					
	$\leq$ 25	$\geq$ 25	$\geq$ 25	$\leq$ 25	$\geq$ 25
BRT_i	$\leq$ 50	$\leq$ 50	= 50	$\geq$ 50	$\geq$ 50
	$\geq$ 25	$\leq$ 25	$\geq$ 25	$\geq$ 25	$\leq$ 25

Table 9.6: BRT conditions median

9.2.3 Nash Play

In chapter 7.2.3, we provide the peak distributions we use in the experiment and the corresponding Nash equilibria of the normalized median rule. A comparison to table 9.6 reveals the differences to the votes that are a best response to truth-telling of the other subjects. While the BRT of semi-pivotal voters is to stay within their rank, truth-telling corresponds to Nash play. Therefore, the possibility of votes that are a theoretical Nash play is more limited as compared to BRT. Since pivotal voters are able to affect the social outcome, Nash play of the other voters takes this strategic voting into consideration and adjusts the margins of the ranks, as in peak distributions C, E, F, J, and K.

The results of the pilot session for the normalized rule show that Nash play occurs in 36.25% of all decisions and the shares are higher under full information. The data reveals no clear variations in Nash play over time or for the different peak distributions. As already mentioned, the sequential rule yields 14 out of 30 Nash votes.

The share of Nash play is comparably high in sessions one to nine, where 287 votes (35.43%) correspond to Nash strategies. Also observable in figure 9.4 and quite surprising is the fact that for all peak distributions the share of Nash play is lower in the last as compared to the first period. It is also worth noticing that Nash play varies across the peak distributions. However, we may not preclude the possibility that the decline in Nash play from peak distributions D to F is driven by a sequence effect as peak distribution D was always played before E and F. Considering strategic voting of subjects with peaks $p_i = (40, 20, 40)$ under distribution E and $p_i = (20, 40, 40)$ under distribution F, we observe that none of the decisions were Nash play. This result is not surprising as the Nash strategies under the normalized median rule are not straightforward when deviating from truth-telling is optimal. Another interesting fact is that in no round a Nash equilibrium was played, even when all Nash strategies were truth-telling.

The proportion of votes that are Nash play in the second experiment amounts to a total of 40.56% and remains stable at a level between 34.44% and 36.67% over periods under no information. Under full information, Nash play decreases unexpectedly from 55.00% in period one to 46.11% in the third period. An analysis of Nash play under different information treatments leads to a rejection of the hypothesis of equal average shares in favor of the alternative hypothesis that the average percentage of Nash play under full information exceeds the one under no information (two-sample t test, $p<0.001$).

When considering only subjects that have a possibility of strategic voting under the normalized median rule, i.e. one subject in peak distribution J and K, we find that their votes are never Nash play. We conclude that strategic voting is not exploited if it is complex but the high values of total Nash play and BRT show that subjects have strategical voting behavior.

Figures 9.5 and 9.6 provide an overview of the votes as percentage of the total possible decisions under the median rule that are a best response to truth-telling of the other subjects, Nash play as well as truth-telling per peak distribution and period.

9.3 Mean versus Median Rule

In this chapter, we examine the differences in mean and median voting using the results from the second experiment. In order to compare the mean with the median rule, we analyze the influence of the voting rule on the parameters truth-telling, distance between peak and vote as well as Nash play. We are able to reject the hypothesis of equal average shares of truth-telling in both voting rules and find a significantly higher share of true votes under the normalized median rule (paired t test, $p<0.001$), which confirms hypothesis H3.1. The fact that the median rule is a strategy-proof voting mechanism if we disregard adaptation plays a crucial role in the different voting behaviors of the two rules. The mean rule is highly manipulable and as mentioned in chapter 3, in any Nash equilibrium at most one individual votes for a strictly positive amount of every project. By contrast, with the normalized median rule, strategic voting is seldom possible and very difficult to realize.

Going further into detail, we consider not only truth-telling but also the degree of lying. As stated in hypothesis H3.2, we expect a higher deviation of votes from the true peaks under the mean rule, which is indeed what we find in our analysis. The absolute value of the average peak-vote-distance under the mean rule adds up to 52, as against 32 under the median rule. A paired t test confirms our hypothesis: the average distance between the peak and the vote is higher under the mean rule (paired t test, $p<0.001$). Table 9.7 shows the regression results for the peak-vote-distance but this time for both rules in combination. Including a dummy variable for the voting rule, 'rule_d' which takes the value 1 for the mean rule and 0 for the normalized median rule, we observe that using the mean rule yields a significantly higher distance from the peak. Another insight that we get is the impact of including the rule dummy on the information level: the coefficient of 'info_d' is now significant, meaning that full information increases the distance between the peak and the vote.

VARIABLES	Peak-Vote-Distance
period	3.440***
	(0.562)
subject	-0.122
	(0.150)
round	-0.181
	(0.241)
peak_d	-11.44***
	(2.004)
info_d	8.366**
	(3.428)
rule_d	20.26***
	(1.248)
male_d	0.997
	(1.477)
understandrule_d	-0.331
	(1.452)
econstudy_d	2.284
	(1.885)
wingstudy_d	-2.192
	(1.745)
Constant	25.03***
	(2.828)
Observations	2,808
R-squared	0.122

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 9.7: Regression results mean and median: peak-vote-distance

A comparison between the two voting rules concerning Nash play is possible, but has to be done cautiously. While the Nash strategies of the peak distributions we used in the experiment were mostly unique under the mean rule, there exist several Nash strategies under the median rule that lead to the same social outcome. Like stated in chapter 3, there are ranges of allocations that belong to a Nash equilibrium and therefore theoretically votes are more often categorized as Nash play under the median rule as compared to the mean rule. The experimental results show indeed a significantly higher average share of Nash play under the median rule (paired t test, $p=0.004$), but since the difference between the shares is not so high, one might argue that the mean rule leads to relatively higher shares. Nevertheless, the average shares of Nash outcomes are considerably higher under the median rule.

We further analyze the influence of the degree of information on truth-telling and Nash play of both voting rules. Regarding truth-telling, we are not able to find a significant influence of information on the aggregated results of the mean and the median rule (two-sample t test, $p=0.293$). As stated in the previous chapters, the average share of truth-telling under the median rule is higher with full information, what might come as a surprise. In contrast, under the mean rule, we find a higher average share of truth-telling when the peaks of the other subjects are unknown.

We are able to reject the hypothesis of equal frequencies of Nash play under both information treatments. Like already stated before, the average shares of Nash play are higher under full information for both voting rules. Comprising results for all voting rules in an unpaired t test, we also find statistical evidence that full information is accompanied by higher mean shares of Nash play compared to no information (two-sample t test, $p<0.001$).

10 Summary and Conclusion

In part I of this thesis, we develop a theoretical model on a multi-dimensional resource allocation problem and determine individual strategies as well as Nash equilibria of the mean and the median rule voting game. The mean rule is highly manipulable and we show that under single-peaked preferences, at most one individual votes for a strictly positive amount of every public project in a Nash equilibrium. Moreover, we state that in multi-dimensional allocation problems, Pareto-inefficient equilibria exist even under the mean rule. Strategic voting under median-based rules is more difficult but it might be possible using normalization. In a laboratory experiment, we empirically test the mean and the median rule on the allocation of three public projects. In particular, we are interested in the occurrence of truth-telling and strategic voting.

We observe low shares of truth-telling under the mean rule and a strong tendency of playing the individual Nash equilibrium strategy. Nevertheless, Nash equilibria of the entire group rarely emerge. Whereas a large fraction of votes under the mean rule are a best response to the result of the previous period, the concept of best response to uniformly distributed votes doesn't provide large explanatory power.

The normalized median rule yields contrary results. While most subjects play a best response to truth-telling of the other voters, only a small fraction votes truthfully themselves. However, strategic voting is never exploited in the experiment. A comparison of the rule-dependent voting behavior reveals higher shares of truth-telling as well as less absolute deviation from the peak under the median rule. Even though the frequency of Nash play is higher under the median rule, this result does not provide us detailed information due to the different Nash conditions. The findings of the degree of information and its influence on truth-telling are quite surprising. While the shares of truth-telling under the mean rule are higher without any information on the peak distribution, under the median rule, subjects tend to vote truthfully more often if information on the detailed peak distribution is provided. Even though the social outcomes are very seldom truth-telling or Nash outcomes, it is worth noticing that a high share of outcomes is classified as efficient.

Further research should be done in both fields, on the theoretical model and in experimental studies. The implementation of manipulation costs as well as further adaptations of the median rule are interesting topics to be covered. In our model, the utility function is based on the L_1 distance function and thus the indifference curves are hexagon-shaped. Due to this modeling, we show that in some cases the best response of an individual is not unique as several allocations in the option set yield equal utility levels. This is different when using a Euclidean distance function, where the indifference curves are shaped circular. Under this assumption, the best response under the mean rule is always unique as the tangent of the indifference curve that yields the highest utility value for a subject with this individual's option set is a unique allocation. We make use of the L_1 distance function because we find it more intuitive and easier to calculate for subjects with regard to our lab experiments. However, a unique best response under the mean rule would simplify the theoretical analysis.

Part II

Participation in Resource Allocation Voting

11 Motivation

A main characteristic of a democracy is letting the population take part in the decision process. People all over the world participate in political decisions and thus affect the environment they are living in. However, the participation rates vary a lot across countries. The International Institute for Democracy and Electoral Assistance (International IDEA) (2019) provides data on voter turnout for several years in different countries. The data on parliamentary elections during the years 2016 until 2019 reveal voter turnout levels from only 23% in Benin to countries like Vietnam where the participation rate is above 99%. In some countries, participating in elections is mandatory and enforced especially in countries in Latin America. Among countries with compulsory voting, Argentina has the lowest turnout rates with 77% in a 2017 parliamentary election and the highest participation rate is 86% in Turkey for the election in 2018. In Brazil, the constitution of 1988 states in Article 14 §1 that voting is compulsory for persons over 18 years of age and optional for illiterates, people over 70 as well as between 16 and 18 years of age. If a subject does not participate in an election, he or she has to give an accepted excuse to the government as otherwise a fine has to be paid. Even though monetary penalties are low, if abstention without excuse is repeated several times, then it is possible to lose the right to vote or not be able to apply for a new passport (Presidência da República Casa Civil Subchefia para Assuntos Jurídicos, 1988). In this thesis, we focus on voluntary voting in the context of a simple resource allocation problem.

Part I addressed voting over resource allocation in a voting game without the question of abstention or participation. We assumed that all subjects of the voting game participate and we analyzed the optimal strategies for the mean and the median rule. Another interpretation of disregarding abstention in elections is the assumption that voting is costless. Given that participation comes without any cost and the voting rule satisfies monotonicity, an individual is never worse off as compared to abstention when he or she participates and tells the truth or votes strategically. The situation differs when voting is costly. In contrast to part I, the following parts II and III include a participation decision to the voting game and cover the topic of participation costs. Depending on the peak distribution and the voting rule, positive costs imply that it is not always optimal for all individuals to attend the voting game. If the costs of participation are sufficiently high, the only Nash equilibria of the participation game are those where exactly one subject participates and all others abstain. By contrast, sufficiently low costs yield Nash equilibria in which all subjects participate.

The following chapters cover participation in voting over resource allocation. We give an overview of the existing literature and show the distinction of the literature on participation to our topic of voting over resource allocation. The theoretical model contains some definitions of the voting game from part I, however there appear several major differences when the participation decision is included. The voting rules we consider are the mean and the median rule. We will explain in detail the participation game, including the individual strategies and provide different equilibrium concepts in preparation for the Nash equilibria of the participation game. We develop a simultaneous and a sequential participation game, which differ in the degree of information a participant receives when submitting a vote. Subsequently, we analyze the participation game Nash equilibria for the mean and the median rule dependent on the participation costs and, as will become apparent, therefore several case distinctions are necessary. For both rules, we derive a complete classification by participation costs for all Nash equilibria.

12 Literature

With their work on rational choice theory and the calculus of voting, Downs (1957), Tullock (1967) as well as Riker and Ordeshook (1968) provide a decision-theoretic model of participation in elections. The question these models face is why a rational individual would vote if the return from voting is often outweighed by the costs that emerge in the voting process. Even if the costs to participate in an election are rather small, the probability that a single vote affects the outcome is almost zero in large electorates. As an example, Gelman et al. (1998) estimate the ex post probability of a single vote being decisive in the 1992 U.S. presidential election to be 1 in 10 million. Therefore, the rational choice model predicts turnout levels that are far below the actual participation rates in elections. This discrepancy is referred to as the paradox of voting.

The calculus of voting has been tested empirically in a variety of studies in the 70s and 80s, whereas Aldrich (1993) points out that using aggregate data like Barzel and Silberberg (1973), Settle and Abrams (1976), as well as Silberman and Durden (1975) yields a correlation between pivotality and turnout while survey data as in studies by Ferejohn and Fiorina (1975) or Foster (1984) does not. Enos and Fowler (2014) review 70 articles on voter turnout regarding the relation between pivotality and participation rates. They find that in a majority of studies pivotality is an important driving force for turnout and that most models on turnout focus mainly on pivotality. In their own study, the authors describe a rare circumstance of an exact tie in electing a candidate for the Massachusetts State House in 2010, which engendered a reelection and gave the opportunity for a field experiment that measures the effect of pivotality on turnout. They inform subjects on the closeness of the election but find a significant increase in turnout only among a subgroup of frequent voters. Thus, Enos and Fowler (2014) conclude that pivotality is not as relevant for the turnout decision as the calculus of voting would predict.

Ferejohn and Fiorina (1974, 1975) also argue that subjects are often not expected utility maximizers since the probabilities of the relevant terms in the model are unknown. Instead of making their decisions regarding voting or abstention based upon pivotality, subjects rather follow a strategy based on minimax regret. Under this premise, potential participants evaluate the maximal regret they would experience for all possible actions and choose the action that yields the minimum of these maximal regrets. Since for sufficiently low costs, voting for the favorite candidate yields a lower maximal regret than voting for the opponent or abstaining, a subject should participate and vote for his or her favorite candidate. Consequently, the minimax regret approach results in a higher participation rate.

Palfrey and Rosenthal (1983, 1985) and Ledyard (1984) formulate the pivotal voter model in a game-theoretic approach. In the Palfrey-Rosenthal participation game, two groups of subjects prefer either one or another candidate. Each subject may vote for his or her preferred candidate (voting for the opponent is strictly dominated in the two-candidate case) or abstain. Participation is costly, while abstention is free. The candidate that polls the majority of votes wins. The payoff that results from a candidate receiving the majority of votes is split equally among the group members that prefer the winning candidate. In their equilibrium analysis, Palfrey and Rosenthal (1983) show that there exist not only low turnout levels but also equilibria with substantial turnout when participants face identical costs and complete information on the distribution of preferences. Ledyard (1984) endogenizes pivotality and highlights that the participation decision of all subjects is made simultaneously. His model implements uncertainty about preferences as well as costs, and turnout levels lie somewhere in-between zero and full participation in equilibrium. Palfrey and Rosenthal (1985) admit that the requirement of full information that yields multiple equilibria in their previous model is very demanding. Building on Ledyard (1984), Palfrey and Rosenthal (1985) implement uncertainty about the individual voting costs and show that this lack of information causes individuals to abstain even though participation is optimal under full information. Hence, they show that in large electorates a unique Bayesian equilibrium with low turnout exists under incomplete information.

Blais (2000) analyzes numerous empirical studies and provides a literature review on rational choice models. He concludes that the rational choice model has limited explanatory power for empirical turnout rates. Dhillon and Peralta (2002) provide a complementary survey on the existing models and theoretical literature on participation.

The Palfrey-Rosenthal participation game is used widely in the literature for testing the pivotal voter model especially in experimental studies. A lab experiment conducted by Levine and Palfrey (2007) tests the voter turnout predictions of the Palfrey-Rosenthal model with asymmetric information, where participation costs are individual and only known privately. The authors find a size effect, meaning that in large elections, participation rates are lower as compared to small electorates. The data also reveal a competition effect, i.e. elections that are expected to be close are associated with a higher voter turnout. Another finding regarding participation is an underdog effect: groups that support a less popular alternative have higher turnout rates as compared to the supporters of the popular alternative.

Duffy and Tavits (2008) perform a lab experiment on the complete information pivotal voter model and additionally elicit the subjects' beliefs about the probability of a close election. Therefore, the authors are able to directly test the pivotal voter model and focus on the correlation of beliefs about being pivotal and the participation decision. The study finds that a higher belief about the probability of being pivotal increases the likelihood of participation and that subjects tend to overestimate this probability of pivotality. Nevertheless, the authors admit the lack of an important factor in their study, namely different types of risk preferences. They assume risk neutrality and state that risk-averse subjects might abstain despite high beliefs about pivotality, whereas risk-seeking individuals might participate with low beliefs.

In another lab experiment, Agranov et al. (2017) test the effect of information on the voter turnout using an election over two alternatives in groups of nine subjects. In the first treatment subjects receive solely the information on their own preferred alternative. Additional information on the distribution of supporters is provided before the election in the second treatment. In the third treatment, a poll was conducted before the election and subjects were reported the results of the poll including the voting intentions of the other subjects. The study finds that pre-election polls influence participation, where the effects depend on the expectation on the closeness of the election. When a poll reveals that the election is expected to be close, bandwagon effects appear (higher voter turnout among the majority), whereas when landslide victories are predicted, the authors find underdog effects. Further, the authors find that landslide elections occur more often in treatments with more information and that voter turnout is higher, the more likely subjects expect the preferred alternative to win.

Grillo (2017) presents a decision-theoretic and a game-theoretic model that takes risk aversion into account. He implements an adaptation of the pivotal voter model and makes the utility function of subjects concave in order to model risk aversion. With this adaptation, he is able to explain a bandwagon effect in both models, where the pivotal voter model would otherwise predict an underdog effect.

Another lab experiment to test the Palfrey-Rosenthal participation game is conducted by Schram and Sonnemans (1996). They study a game in which subjects are split into two groups and individually decide on buying an imaginary disc at a cost (participation) or not (abstention). An extension to the original model, where the winning position (outcome of the election) is determined by winner-takes-all, is made by considering additionally proportional representation. The payoffs therefore depend on the number of discs each group buys relative to the other group. The authors' main finding is that participation is higher under majority voting than under proportional representation and that group size does not affect voter turnout.

Blais et al. (2014) also study the rational choice model in the lab but the authors frame the game explicitly as an election. In their experiment, 21 subjects are located at distinct positions on a zero to 20 scale and may decide to vote either for party A (positioned at 5), or for B (positioned at 15), or to abstain. The outcome is determined by first-past-the-post or proportional representation depending on the treatment. The subjects' payoff depends on the distance between the winning position and the position of the subject as well as the cost of voting given participation. The authors show that using first-past-the-post implies a large impact on the social outcome but only with low probability, whereas the impact is small but certain under proportional representation. Although the authors find that turnout under first-past-the-post is slightly higher than under proportional representation, the data does not confirm their hypothesis that it yields substantially higher turnout rates as predicted by their equilibrium analysis. Moreover, when looking at individual rationality, they find that a tremendous share of subjects (62%) make the 'wrong' decision, i.e. they vote when they should have abstained and vice versa. Even when controlling for beliefs of the opponents' behavior, the rational choice model fails to explain the decision of voting and abstaining, as subjects do not maximize their payoff.

Börgers (2004) develops the costly voting model based on Ledyard (1984) and Palfrey and Rosenthal (1983, 1985) and focuses on the costs of participation. Voting is costly as the process of gathering information on which alternative to vote for is complicated and might be time consuming. He assumes that the costs of voting differ for individuals and are observed privately. Börgers (2004) compares compulsory to voluntary voting and finds that, using the majority rule, compulsory voting is Pareto-dominated by voluntary voting. Krasa and Polborn (2009) also assume individual costs of voting for each subject and their model advises a social planner to provide subsidies for participants in order to prevent 'wrong' electoral decisions and increase social welfare by increasing the electorate. Another extension of the costly voting model with majority rule is done by Arzumanyan and Polborn (2017) who consider not only two but a multitude of candidates. The interesting difference is that strategic voting, i.e. participation and not voting for the top candidate, might be possible when this candidate is in the subset of 'irrelevant' candidates who do not have a positive winning probability. By contrast, strategic voting is never optimal in the two-candidate setting. However, the authors find that for three candidates, all equilibria include solely sincere voting which is driven by the characteristic that voting is costly.

Osborne et al. (2000) consider elections in meetings with costly participation where the participants vote on a policy within a compact convex subset of $\mathbb{R}^{\ell}$. The authors find equilibria in which the number of participants is small and state that in these equilibria the subjects with moderate positions abstain. They compare the mean and the median rule but do not account for strategic voting which is a crucial aspect especially for small meetings. Ehlers et al. (2004) consider manipulation in large elections with a multi-dimensional alternative space. They find that in large electorates, the maximal utility gain by strategic voting is restricted. Moreover, they state that the mean rule provides a minimal number of agents that is needed for a strategy-proof voting rule and that this rule "seems the best one to use if manipulation biases should be only small" (Ehlers et al. (2004), p.105).

The field of political science has grown steadily over the last decades and the vast majority of both the theoretical foundations and the empirical tests of the pivotal voter model has assumed two alternatives or candidates and simple majority voting as the decision rule. We contribute to the literature by studying the pivotal voter model in the context of a simple resource allocation problem with single-peaked preferences using the mean or the median rule and account for strategic voting.

13 Theoretical Model

Our model of participation is nested in the context of voting on budget allocation problems. Therefore, the main definitions are the same as in the last chapters: Consider a set of individuals $I = \{1, ..., n\}$ that decide on the allocation of a budget Q on m public projects $J = \{1, ..., m\}$. We have a budget restriction such that no negative budget might be allocated as well as the total budget has to be spent. The set of feasible allocations is therefore given by $\mathcal{B} := \{x \in \mathbb{R}^m_{\geq 0} | \sum_{j \in J} x^j = Q\}$. The following chapters describe further definitions that are relevant for including participation decisions.

13.1 Basic Definitions

Since an individual has the possibility to decide whether to participate in the voting process or not, each individual i faces a participation decision $\vartheta_i \in \{0, 1\}$ that takes the value 1 for participation and 0 in case of abstention. An individual that decides to participate, i.e. $\vartheta_i = 1$, submits a vote that is taken into account in the calculation of the social outcome and therefore this individual is called voter or participant. The set of participants is $I^{-*} := \{i \in I | q_i \neq *\}$, where the number of participants is $k = |I^{-*}|$. Each participant $i \in I^{-*}$ submits a vote $q_i = (q_i^1, \ldots, q_i^m) \in \mathcal{B}$. The vector of all votes is given by $q = (q_1, \ldots, q_k) = (q_i)_{i \in I^{-*}}$ and determines the social outcome. If an individual abstains, i.e. $\vartheta_i = 0$, no vote is submitted and we define $q_i = *$. We define the set of abstainers by $A := \{a \in I | q_a = *\}$.

The social outcome $x(q) = \big(x^1(q), \ldots, x^m(q)\big) \in \mathcal{B}$ is calculated by either the mean or the median rule. Under the mean rule, all votes are added up separately for every project and divided by the number of votes:

$$Mean(q) = \frac{1}{k} \sum_{i \in I^{-*}} q_i. \tag{13.1}$$

Obviously, the calculation of the mean outcome is only possible for $k > 0$. By construction, the social outcome under the arithmetic mean always satisfies the budget constraint, such that $x(q) \in \mathcal{B}$ is always true.

The median rule selects of all ascending ordered votes $q^j_{[i]}$ for every project the one in the middle if the number of participants is odd or the average of the two middle votes if it is even. Thus, the median $Med(q)$ consists of m coordinate-by-coordinate median-values:

$$Med^j(q) = \begin{cases} q^j_{[\frac{k+1}{2}]}, & \text{if } k \text{ is odd} \\ \\ \frac{1}{2} \cdot (q^j_{[\frac{k}{2}]} + q^j_{[\frac{k}{2}+1]}), & \text{if } k \text{ is even.} \end{cases} \tag{13.2}$$

The median outcome may also only be calculated for $k > 0$. We will explain the implication of $k = 0$ in chapter 13.2. A restriction to the median rule is the possibility that the coordinate-by-coordinate median-values do not satisfy the total budget in multi-dimensional allocation problems, i.e. $\sum_{j=1}^m Med^j(q) \neq Q$ is a possible outcome for $m > 2$. Therefore, an adaptation of the median outcome may be necessary, as already described in detail in chapter 3. In the following, we restrict the number of projects to $m = 2$, such that no adaptation of the median is necessary and $j \in \{1, 2\}$. For two public projects and a given budget, it is sufficient to indicate the vote or social outcome for only one project - the value for the other project results directly from the budget constraint: $x^1(q) + x^2(q) = Q$. With the restriction to $m = 2$, we resign definitions that include $j = 2$ and always imply that $j = 1$ if no index is used.

We assume that participation is costly. Each participant faces a cost $c > 0$ if $\vartheta_i = 1$. Abstention is free. Without participation costs, we already analyzed the full participation Nash equilibria under the mean and the median rule in the last part. We assume that there are no differences in individual costs among participants. This is crucial for the calculation of Nash equilibria as one could adapt the individual costs and create Nash equilibria for each scenario if costs differ. We also assume that the participation costs do not exceed the maximal distance on the budget interval, i.e. $c \leq Q$. We will see that if the costs were higher, only equilibria with one single participant would occur.

13.2 Preferences

Every individual i is assumed to have a preference ranking that includes a unique peak p_i, which is the most preferred allocation of i. Individual i (weakly) prefers allocation a over b if and only if the distance between a and p_i is smaller than (or equal to) the distance between b and p_i. Since $m = 2$, it is sufficient to calculate the distance of two allocations a, b by their absolute deviation in the first of the two projects: $d(a, b) := |a^1 - b^1|$. Due to the budget constraint, using the second project yields the same distance. Preference rankings that satisfy these conditions are single-peaked. The most preferred outcome of i is a social outcome that is equal to i's most preferred allocation. We define $u_i(\cdot)$ as i's single-peaked utility function over the allocation to the first project. The lower the distance between p_i and the social outcome, $x(q)$ for $\vartheta_i = 1$ and $x(q_{-i})$ with $q_{-i} = (q_1, \ldots, q_{i-1}, q_{i+1}, q_k)$ for $\vartheta_i = 0$, the higher is i's utility. When defining the voting rules, we stated that the mean and the median outcomes $x(q)$ are only calculable for $k > 0$. We assume that given no subject participates in the vote, none of the projects is funded and that this is the worst possible case. The individual utility of each subject is given by:

$$u_i(\cdot) = \begin{cases} -d(p_i, x(q)) - c, & \text{if } \vartheta_i = 1 \text{ and } k > 0 \\ -d(p_i, x(q_{-i})), & \text{if } \vartheta_i = 0 \text{ and } k > 0 \\ -\infty, & \text{if } k = 0. \end{cases} \tag{13.3}$$

Each individual maximizes the utility by the decision of participation and abstention. An individual participates in the voting process if the utility from participation exceeds the utility from non-participation. We therefore adapt the original pivotal voter model and state that an individual participates iff

$$-d(p_i, x(q_{-i}, q_i^*)) - c > -d(p_i, x(q_{-i})). \tag{13.4}$$

If the utility from participation is equal to the utility derived from abstention, the individual is indifferent. The inequality induces that participation is dependent on the optimal choice of the vote, q_i^*, which is optimal in the sense that it minimizes the distance between the social outcome and the peak. Given that each individual maximizes the utility by the optimal choice of the participation decision ϑ_i and given participation the optimal choice of the vote q_i^*, an important aspect that is included in the optimization problem is the impact of individual i's vote on the social outcome. In cases where the social outcome with and without the vote of i is identical, it does not make a difference on the outcome whether i participates or not. Therefore, the impact is zero and the optimal vote is not decisive. The actual impact of i's vote may also only be calculated ex post and i has to form beliefs about the number of participants and the distribution of votes which gives him an expected belief about the impact. The impact of i's vote further depends on the voting rule and differs for the mean and the median rule. More details on the impact are given in chapter 19.

14 The Participation Game

In the following chapters, we focus on the game theoretical analysis of participation in a budget allocation context. As a first step, we define the strategies of each individual, including strategic and Nash voting. The *participation game* consists of two elements: the participation decision and the voting decision. We call the latter one the *voting game*, which was addressed to in part I of this thesis. For the participation game we distinguish between a simultaneous participation game, in which both, the participation and the voting decision are made at the same time, and a sequential participation game, where subjects decide first on the participation decision and afterwards submit a vote for a given set of participants. In chapter 15, we examine Nash equilibrium concepts including partial honesty and coalition-proofness. These concepts are necessary for the final chapter of part II, in which we derive for the mean and the median rule of complete classification of all Nash equilibria depending on participation costs.

14.1 Individual Strategies

In our setting, each individual faces a participation decision ϑ_i. Let k be the number of individuals that decide to participate and thus submit a vote $q_i \neq *$. In a simultaneous participation game, both decisions are made at the same time. This implies particularly that the voting decision has to be made without any information on the set of participants. An example for a simultaneous participation game is an online election on how to allocate a public budget. No information on the number of participants or their peaks is provided. Here, subjects that decide on participation submit their vote without certain knowledge on how many other subjects (and which subjects) participate.

The sequential game differs by the degree of information the participants receive after the participation decision. All subjects begin with the same information level (no information) but subsequent to the participation decision, the participants get to know the set of participants. For a given set of participants, the best response in the voting decision is identical in both games. What is different is the behavior in a Nash equilibrium, since abstention of one voter will lead to an adaptation of the voting decision in the sequential game. Hence, each individual has different strategies depending on the type of game. Situations where sequential participation games occur are votes in committee meetings. As soon as the meeting begins, each individual observes not only the number of individuals that attend the meeting but also who the participants are.

14.1.1 The Simultaneous Participation Game

When the participation and the voting decision are made simultaneously, the individual strategies are independent of the participation decision of the others. One strategy is $q_i = *$, which induces $\vartheta_i = 0$ and is abstention. The other strategies $q_i \in \mathcal{B}$ are given for participation and thus for all feasible values of q_i. Hence, the total number of strategies for each individual is one (abstention) plus the quantity of feasible votes. Figure 14.1 displays the game tree for $n = 2$ including the strategies 'abstention' and 'voting for a feasible allocation'. The latter combines all strategies of voting and its depiction is simplified by $q_i \in \mathcal{B}$. The total game comprises one subgame, which is the game itself. Therefore, each Nash equilibrium constitutes a subgame perfect Nash equilibrium.

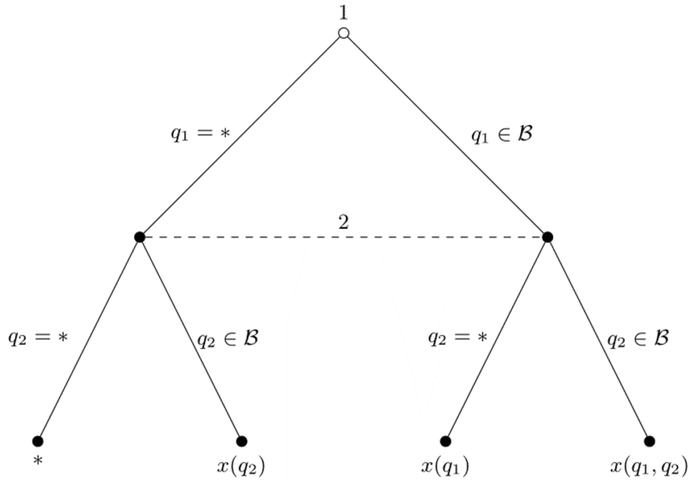


Figure 14.1: Simultaneous participation game, $n = 2$

14.1.2 The Sequential Participation Game

In the sequential participation game, strategies in the voting game are conditioned on the participation decision of the others and therefore more complex. More precisely, the voting decision is conditioned on the other individuals' participation decisions. In a two-player game with $I = \{h, i\}$ for example, one action of subject i is abstention, i.e. $\vartheta_i = 0$ and $q_i = *$. However, if individual i decides to participate, the voting decision is conditioned on the participation decision of h: $\{(q_i \in \mathcal{B}|\vartheta_h)\}$ for any feasible vote q_i and participation decision ϑ_h.

The total number of individual strategies is two (abstention and participation) times the squared quantity of feasible votes to the power of $(n-1)$: $2\mathcal{B}^{(2^{n-1})}$. Figure 14.2 shows the game tree for $n = 2$. As one may observe, the game has four subgames including the total game tree.

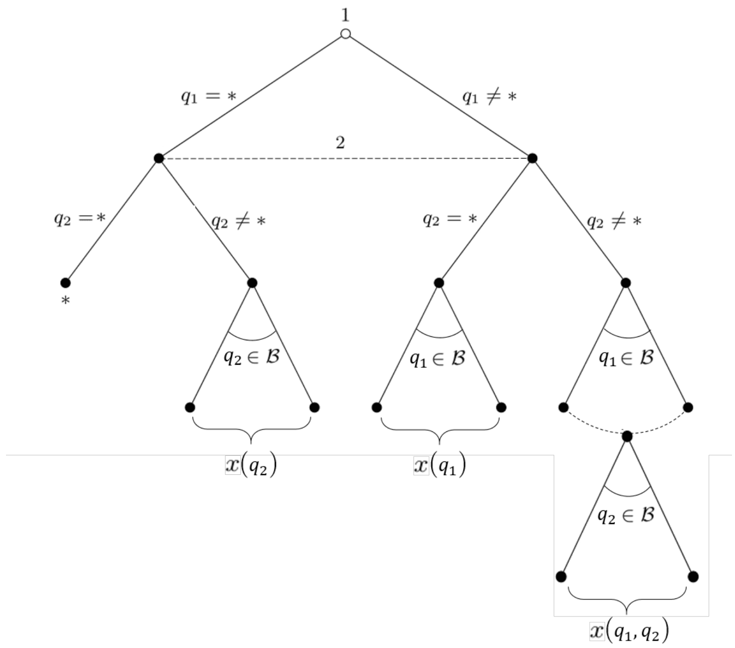


Figure 14.2: Sequential participation game, $n = 2$

This thesis focuses on the simultaneous participation game. More details on the sequential participation game can be found in Müller and Puppe (2020).

14.2 Strategic Voting

Recall definition 2 from chapter 4: A vote q_i is strategic, if it differs from i's true peak p_i and thereby reduces the distance between p_i and the social outcome $x(q)$.

Previous studies show that different voting rules in allocation problems induce different voting behavior. As shown for example in Renault and Trannoy (2005), the Nash equilibrium predicts extremist voting behavior under the mean rule. Marchese and Montefiori (2011) confirm these predictions and find strategic voting in an experimental study on mean voting in the lab. Experiments run by Block (2014) as well as the experiments from part I of this thesis also find high shares of non-truthful voting and Nash play under the mean rule. As also stated in Ehlers et al. (2004), the theoretical impact of each vote on the social outcome is high under the mean rule, but it decreases to zero if the group size becomes large. The authors argue further that manipulation is too costly in very large voting problems because the preferences of the other participating subjects are unknown and this incomplete information produces high risk of voting untruthfully.

It is always possible to change the social outcome under the mean rule, but the extent of strategic voting is greater when $Mean(q_{-i})$ and the interval boundaries are initially more distant. Another factor is the limitation of the option set due to the restrictions on the set of feasible allocations. The more extreme $Mean(q_{-i})$ and q_i are in the sense that both allocations are distant from $\frac{Q}{2}$ to the same direction, the less possibilities for strategic voting are given, especially if q_i is more extreme i.e. closer to zero or Q as compared to $Mean(q_{-i})$. The following example provides an explanation. Assume that $Q = 100$, $n = 2$, $Mean(q_{-i}) = 80$ and $p_i > 95$. If $q_i = 100$, the social outcome is $Mean(q) = 90$ and the maximal alteration of the outcome by strategic voting is 10. For $Mean(q_{-i}) = 85$, the alteration is only 7.5 and for $Mean(q_{-i}) = 90$ it declines to 5. Since $Mean(q_{-i})$ and q_i are close together, distant from $\frac{Q}{2}$ and q_i is closer to Q, the range of strategic voting is limited by $\mathcal{B}$. Similar to the approach for maximal impact, which is described later in chapter 19.1, it is possible to construct situations where the change of the social outcome is marginal. However, in these constructs, the worst-case scenario originates from a peak that is already close to the social outcome.

By contrast, the median rule for an odd number of voters is strategy-proof, meaning that truth-telling is a (weakly) dominant strategy.[7] Depending on the calculation of the median for an even number of participants, the voters with middle ranked peaks might be able to vote strategically. Given our definition of the median as the average of the two middle votes, the extent of strategic voting is limited by the position of the next ranked votes.

Strategic voting is possible under the mean rule and may also be under the median rule - with the restriction that k is even. For an *even number of participants*, voter i may vote strategically if p_i is positioned between $Med(q_{-i})$ and the next ranked vote. Let q_{Med-} be the vote that is ranked one position left of the median vote for $j = 1$ and q_{Med+} the vote ranked one position right of the median vote.[8] If p_i is located more distant from $Med(q_{-i})$ than the next ranked vote, the social outcome also changes with q_i but truth-telling is a weakly dominant strategy. For an *odd number of participants*, p_i is either equal to the social outcome without i or not. Given $p_i = Med(q_{-i})$, truth-telling, i.e. $q_i = p_i$, is a weakly dominant strategy. Truth-telling is a strictly dominant strategy if no other vote is equal to p_i, because then $Med(q_{-i})$ is derived by the average of the two (different) middle votes and the social

[7] For median-based rules that are not strategy-proof, see Lindner (2011) and the explanations in part I, chapter 3.5.
[8] Note that the next ranked vote might be identical to the median vote or not be distinct.

outcome with i corresponds exactly the vote of i. Strategic voting of i is not possible for $p_i \neq Med(q_{-i})$. Given that p_i is positioned between q_{Med-} and q_{Med+} and these votes are distinct, individual i obtains the most preferred allocation as outcome by truth-telling. When q_{Med-} and q_{Med+} are identical, no q_i will change the social outcome, making truth-telling a weakly dominant strategy. Let p_i be positioned outside of the two middle votes, w.l.o.g. let $p_i < q_{Med-}$. Every $q_i > q_{Med-}$ results in a greater distance to the social outcome as compared to truth-telling and every $q_i < q_{Med-} \neq p_i$ results in the same social outcome $Med(q_{-i}) = q_{Med-}$ as truth-telling. The same argumentation holds for $p_i > q_{Med+}$, which implies strategy-proofness.

We conclude that strategic voting is possible under both voting rules but the range of impact may be limited. We give a formal definition of impact later in chapter 19. Strategic voting is possible under the mean rule for any voter whose peak is not identical to $Mean(q_{-i})$, impossible under the median rule when k is odd and might be possible when k is even.

15 Nash Equilibria of the Voting Game: Equilibrium Concepts

With the knowledge on strategies and strategic voting in the voting game, the next step is to find combinations of strategies that constitute a Nash equilibrium of the participation game. Omit the participation decision and consider only full participation in a budget allocation setting with costless participation. We already determined the Nash equilibria of the voting game for $m = 3$ in chapter 7.2.3. For the mean rule in a one-dimensional setting ($m = 2$), Renault and Trannoy (2005) and later Block (2014) show that a Nash equilibrium exists and that it is unique if preferences are single-peaked and peaks are distinct. More precisely, in every equilibrium at most one participant does not vote for an extreme allocation, i.e. zero or Q. The one who does not vote for the extremes in equilibrium is the one subject whose peak is equal to the mean outcome, but not necessarily induced by truth-telling. All other subjects vote for an allocation of either zero or the total budget for one project. Block (2014) provides an algorithm that calculates the voting decision for each individual in equilibrium under the mean rule and shows that all Nash equilibria are Pareto-efficient.

Under the median rule, the costless voting game yields a truth-telling equilibrium if the number of voters is odd (Moulin, 1980), but several Nash equilibria exist for a fixed set of participants. As already mentioned earlier, Cason et al. (2006) describe that the median rule yields not only several Nash equilibria but also Nash equilibrium outcomes that are Pareto-inefficient and thus socially undesirable.

15.1 Partially Honest Nash Equilibria

Recall definition 8 from chapter 5. A voter is called partially honest if he or she votes for the true allocation when truth-telling is a Nash strategy. A Nash equilibrium in which all subjects are partially honest is called a partially honest Nash equilibrium.

15.1.1 Partial Honesty under the Mean Rule

Since the Nash equilibrium under the mean rule voting game is unique for single-peaked preferences and distinct peaks, it is straightforward to show that the equilibrium is also partially honest. Using the mean rule in this setting, the Nash strategy of each voter is unique. Uniqueness of the Nash strategy implies that no voter can be in a situation of a tie between lying and truth-telling, since either of it is strictly preferred. Each voter who plays the unique Nash strategy is therefore partially honest. He or she either already tells the truth or truth-telling would yield a worse social outcome, which is the definition of a Nash strategy.

Observation 1. *The unique Nash equilibrium under the mean rule voting game for $m = 2$ is partially honest.*

The outcome in every mean rule Nash equilibrium is $(0, Q)$ if $k > 1$ and the peaks are distinct. It might be zero or Q if $k = 1$ or more than one peak is allocated at zero or Q. In any case, the mean outcome is a Pareto-efficient allocation, as described already in part I. Pareto efficiency in one-dimensional budget allocation problems implies that the outcome has to be within the convex hull of all peaks, i.e. the outcome may not be lower than the lowest ranked peak with respect to project j, denoted by $p_{[1]}$, and not higher than the highest ranked peak with respect to the same project j, $p_{[k]}$. The outcome range of the mean rule is thus $Mean(q^*) \in [p_{[1]}, p_{[k]}]$ in every partially honest Nash equilibrium.

15.1.2 Partial Honesty under the Median Rule

For the median rule, the equilibria vary widely since Nash strategies are in most cases not unique. Using the concept of partially honest voters, we are able to eliminate Pareto-inefficient social outcomes. Another issue is the necessity to distinguish between odd and even numbers of voters. We begin with the easier case: an odd number of voters. For an *odd number of participants* (remember, we are still considering costless voting with full participation), the median outcome is determined by the allocation of the median vote. All voters have the weakly dominant strategy to tell the truth, as we already analyzed in chapter 14.2. Since partially honest individuals always tell the truth when they are not deteriorated, the weakly dominant Nash strategy of truth-telling is sufficient for arguing that truth-telling of all individuals is the unique partially honest Nash equilibrium.

Observation 2. *For an odd number of voters, truth-telling is the unique partially honest Nash equilibrium of the median rule voting game. Therefore, the outcome in any partially honest Nash equilibrium under the median rule is $Med(q^*) = p_{Med}$ if k is odd.*

It is well-known that truth-telling is a weakly dominant strategy for the median rule with an odd number of voters and thus, truth-telling constitutes a Nash equilibrium. Given that all voters tell the truth, this Nash equilibrium is also partially honest and the outcome corresponds to the median peak. Moreover, the peak of the median voter is never a Pareto-inefficient outcome.

For an *even number of participants*, several partially honest Nash equilibria exist. Consider figure 15.1. The first row displays for one project j a budget line from zero to Q and the peaks p^j of $k = 8$ participants. The two red peaks are the median peaks, such that the median outcome given truth-telling is positioned halfway between them, as denoted by the black **x**. We split the budget line into peaks that are lower and that are higher as the median under truth-telling. Contrary to the odd case, truth-telling of all voters including the pivotal red voters is no partially honest Nash equilibrium. Since the median is determined by the mean of the two red votes, strategic voting of the voters with the median peaks is possible in the range of the next ranked green peaks. Therefore, the red participants vote strategically and drift apart until in this case the median outcome corresponds to the left red peak in equilibrium. This combination of Nash strategies in the second row is a partially honest Nash equilibrium in which all non-pivotal voters reveal their true most preferred allocation and the two participants with pivotal red peaks vote strategically. By the definition of strategic voting, truth-telling would yield a worse outcome and since all non-pivotal voters are telling the truth, this Nash equilibrium is partially honest. Interestingly, there exist several partially honest Nash equilibria using the median with an even number of participants and distinct peaks.[9] All rows in figure 15.1 display Nash equilibria in which voters either tell the truth or vote strategically. We observe that in the equilibria of rows 2-4, the left red voter approaches the left green voter and makes sure that the median outcome corresponds to his or her peak. The red and green voters on the right side vote for the same allocation in equilibrium. If their votes are not equal, e.g. if the red is to the left of the green vote and the social outcome is to the left of the right median voter, the red voter may improve by voting for a lower allocation. In equilibrium, the red and the green voter on the right hand side only differ if the median outcome is identical to the peak of the red voter. In all partially honest Nash equilibria, voters either tell the truth or vote for an allocation, which is more extreme regarding their side of the median under truth-telling. In the last row of figure 15.1, all left-sided

[9] In the extreme case where all peaks are identical, truth-telling is the only partially honest Nash equilibrium.

participants vote for the extreme left allocation zero and all right-sided participants vote for the extreme right allocation Q. In this situation, no voter reveals the true most preferred allocation. Since a deviation of any voter to truth-telling yields a more distant median outcome, these Nash strategies also constitute a partially honest Nash equilibrium. We conclude by the following observation:

Observation 3. *Let the number of voters be even and their peaks be distinct. In the median rule voting game, multiple Nash equilibria exist; however, truth-telling is never a partially honest Nash equilibrium.*

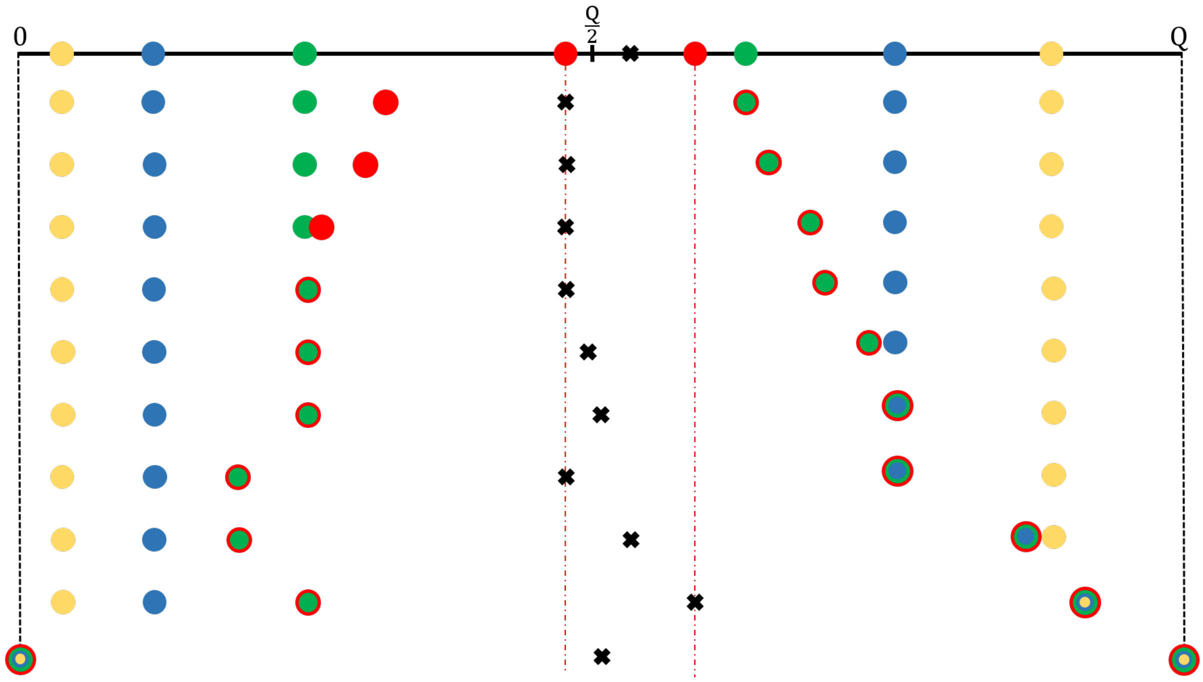


Figure 15.1: Partially honest Nash equilibria, median, k even

We observe that several partially honest Nash equilibria exist under an even number of participants, such that one may ask whether the concept of partially honest voters is helpful at all. Similar to the odd case, we are able to omit Pareto-inefficient equilibria, like every participant submitting a vote of an extreme allocation. Here, each partially honest player deviates by telling the truth as the outcome would not be worse. Furthermore, in all partially honest Nash equilibria for even k, the median outcome always lies within the two pivotal peaks.

Proposition 7. *Consider a median rule voting game with k even. In every partially honest Nash equilibrium $Med(q^*) \in [p_{Med-}, p_{Med+}]$.*

Proof. Suppose the voting game with k even constitutes a partially honest Nash equilibrium. We already stated that if individuals vote strategically, then the votes are smaller than the corresponding peaks for all $p_i \leq p_{Med-}$ and greater than the corresponding peaks for all $p_i \geq p_{Med+}$. If an outcome is smaller than p_{Med-}, this participant could vote strategically with a vote more to the right, i.e. q_{Med-} increases until $Med(q) = p_{Med-}$. The same holds for an outcome higher than the right median peak, where the right pivotal voter could vote strategically and decrease q_{Med+} until $Med(q) = p_{Med+}$. Hence, a partially honest Nash equilibrium always yields an outcome between the median peaks, otherwise strategic voting is possible and the strategies in the voting game do not constitute a Nash equilibrium. □

Even though for an even k we do not get uniqueness of the equilibrium, it is very helpful that the median outcome in any partially honest Nash equilibrium is positioned between the two pivotal peaks and thus never Pareto-inefficient.

15.2 Strong and Coalition-Proof Nash Equilibria

Aumann (1959) introduces the notion of a strong Nash equilibrium. A strong Nash equilibrium is stable to deviations of single players (i.e. it is a Nash equilibrium) and stable to deviations by any conceivable coalition of players, if there exists no coalition of players that may jointly deviate such that the deviation benefits all members of the coalition. This definition implies that every strong Nash equilibrium is also Pareto-efficient.

Bernheim et al. (1987) argue that the concept of strong Nash equilibria is too strong because often no such equilibria exist. The authors introduce another weaker concept representing a variation of Nash equilibria: coalition-proofness. The authors refine the set of Nash equilibria by making use of "the notion of an efficient self-enforcing agreement for environments with unlimited, but nonbinding, pre-play communication" (Bernheim et al., 1987, p.3). They further define coalition-proofness as follows: "An agreement is coalition-proof if and only if it is Pareto efficient within the class of self-enforcing agreements. In turn, an agreement is self-enforcing if and only if no proper subset (coalition) of players, taking the actions of its complement as fixed, can agree to deviate in a way that makes all of its members better off" (Bernheim et al., 1987, p.3). The weakening with regard to strong Nash equilibria is that under coalition-proofness, the deviations must be self-enforcing, such that no further self-enforcing and improving deviation is possible for the coalition. In other words, if a coalition of voters agrees on a deviation, it must be excluded that another profitable deviation from the deviation is possible for any sub-coalition. If a Nash equilibrium is strong, then it is also coalition-proof. By contrast, there might exist coalition-proof Nash equilibria that are not strong. Bernheim et al. (1987) provide a nice example to highlight the difference.

In our setting, we restrict coalition-proofness to the *voting game* and thus the set of participants. We do not consider coalitions of abstainers but the participation decision is made individually.

15.2.1 Strong and Coalition-Proof Nash Equilibria under the Mean Rule

For the mean rule, these refinements imply that the only contemplable strong or coalition-proof Nash equilibrium is the unique Nash equilibrium. In every mean rule equilibrium, some participants vote for the lower bound of the budget set (zero), some for the upper bound (Q) and at most one votes for an allocation that is not extreme. All of the voters positioned at the lower bound either reach their most preferred allocation or prefer a social outcome that is lower than the mean outcome. For the first set of voters, the minimal distance is already reached and thus no coalition of voters can further reduce it. The voters who prefer a social outcome that is lower than the mean outcome in equilibrium would improve by a deviation of any voter (and a coalition of voters) further away from the current mean. This deviation, however, is impossible due to the restricted set of feasible allocations. The same argumentation holds for the voters at the upper bound of the budget set. As the individual who votes neither for zero nor for Q already reaches his or her peak and cannot further improve, the Nash equilibrium is stable to any deviations of coalitions. The following proposition is adapted from Renault and Trannoy (2011).

Proposition 8. *The unique Nash equilibrium under the mean rule voting game for $m = 2$ is a strong Nash equilibrium and thus coalition-proof.*

Renault and Trannoy (2005, 2011) state that each mean rule Nash equilibrium is Pareto-efficient, thus the outcome is Pareto-efficient in every strong and coalition-proof mean rule Nash equilibrium. Therefore, the set of outcomes in any coalition-proof Nash equilibrium under the mean rule is restricted to $Mean(q^*) = [p_{[1]}, p_{[k]}]$.

15.2.2 Strong and Coalition-Proof Nash Equilibria under the Median Rule

Under the median rule, several Nash equilibria exist for any number of k. Consider the voting game for an *odd number of participants* with distinct peaks. Truth-telling of all agents is a Nash equilibrium that is also coalition-proof. The median outcome of an odd number of truth-tellers corresponds to the median peak. This voter has no incentive to change the social outcome as he or she might not further reduce the distance to the peak, which is already zero. By construction, the number of peaks to the left and to the right of the median is equal. All subjects with peaks lower than the median prefer an outcome that is smaller; all subjects with peaks to the right of the median prefer a higher outcome. Since the median outcome is fixed to the median voter - he or she has no incentive to deviate from truth-telling - no coalition of voters is able to change the social outcome to their benefit. The argumentation of coalition-proof Nash equilibria with the outcome of the median peak is the same as for Nash equilibria. There exist several coalition-proof Nash equilibria in which the subject with the median peak reveals the true most preferred allocation, the voters positioned to the left of the median peak vote for any allocation that is smaller and the voters to the right of the median peak vote for any allocation that is higher than the median peak. While for an odd k the partially honest equilibrium is unique, there exist several coalition-proof equilibria.

Observation 4. *There exist several coalition-proof Nash equilibria under the median rule voting game for an odd number of participants.*

At first glance, it seems that we do not get further insights by introducing the concept of coalition-proofness. However, similar to partial honesty, a crucial refinement occurs to the social outcome in equilibrium:

Proposition 9. *In every coalition-proof Nash equilibrium of the median rule voting game with an odd number of voters, the social outcome corresponds to the median peak.*

Proof. Suppose, by way of contradiction, that the social outcome is unequal to the median peak. Let $Med(q^*) < p_{Med}$. The coalition of all voters with peaks greater than or equal to the median peak improves by voting for a higher allocation until the outcome corresponds to the median peak. Similarly, if $Med(q^*) > p_{Med}$, the coalition of all voters with peaks smaller than or equal to the median peak improves by voting for a lower allocation until the outcome corresponds to the median peak. Such a coalition exists in every Nash equilibrium with an outcome different from the median peak, so that no Nash equilibrium with $Med(q^*) \neq p_{Med}$ is coalition-proof. □

Hence, in contrast to the unrefined Nash concept, coalition-proofness prevents from Pareto-inefficient outcomes under the median rule, which is a crucial feature. Pareto efficiency also implies that these equilibria are strong Nash equilibria. Note that in each of these equilibria the voter with the median peak has to vote truthfully for p_{Med}.

Next, consider the median rule for an *even number of participants*, where the social outcome is determined as the average of the two middle votes. Given distinct peaks, this implies that half of the peaks are positioned to the left and half of the peaks are positioned to the right of the median under truth-telling. Based on the median of all peaks, $\frac{k}{2}$ of the participants prefer a median outcome that is positioned closer to zero and $\frac{k}{2}$ prefer an outcome closer to Q as compared to $Med(q) = \frac{q_{Med-}+q_{Med+}}{2}$. Truth-telling of all individuals is therefore never a strong nor coalition-proof Nash equilibrium for distinct peaks - it is not even a Nash equilibrium. Reconsider the peak distribution in the first line of figure 15.1. For every median outcome that is smaller than the left median peak, there exists a coalition of five voters that prefers the left red peak to any lower allocation. The argumentation for median outcomes to the right of the right red peak is analogous, such that in any coalition-proof Nash equilibrium, the median outcome has to lie between the two median peaks. In fact, the social outcome under coalition-proofness is unique and even more: under some distributions the equilibrium itself is unique. Depending on the peak distribution or more precisely the distance between the median peaks and the boundaries of the budget set, three variations of coalition-proof Nash outcomes exist, which are displayed in figure 15.2.

In any case, the median outcome is either equal to one of the median peaks or positioned in-between them, which means that it is Pareto-efficient. We will now show that the median outcome is the median of the three allocations p_{Med-}, p_{Med+} and $\frac{Q}{2}$.

Proposition 10. *The outcome in every coalition-proof Nash equilibrium under the median rule voting game for an even number of voters is the median of* $\{p_{Med-}, p_{Med+}, \frac{Q}{2}\}$. *If the equilibrium outcome is* $\frac{Q}{2}$, *the coalition-proof Nash equilibrium is unique.*

Proof. We distinguish the following three exhaustive cases:

(i) $Median\{p_{Med-}, p_{Med+}, \frac{Q}{2}\} = \frac{Q}{2}$
The peak distribution in figure 15.2a yields an outcome that is exactly half of the budget set, $\frac{k}{2}$ of the participants vote for the lower boundary of the budget set (zero) and $\frac{k}{2}$ of the participants for the upper boundary (Q). In this situation, no other coalition-proof Nash equilibrium exists. Every peak distribution for which $\frac{Q}{2}$ lies in-between the two median peaks yields the unique coalition-proof Nash equilibrium with a social outcome of $\frac{Q}{2}$. Suppose that k is even, $Median\{p_{Med-}, p_{Med+}, \frac{Q}{2}\} = \frac{Q}{2}$ and $Med(q^*) \neq \frac{Q}{2}$. Since the number of voters is even and $p_{Med-} \leq \frac{Q}{2} \leq p_{Med+}$, there exist $\frac{k}{2}$ voters that prefer p_{Med-} to p_{Med+} and $\frac{k}{2}$ voters that prefer p_{Med+} to p_{Med-}. The only coalition-proof Nash strategy is $q_i^* = 0$ for all voters with peaks $p_i \leq p_{Med-}$ and $q_i^* = Q$ for all voters with peaks $p_i \geq p_{Med-}$, otherwise the coalition would improve by voting for the extremes. Since the same number of voters votes for zero and for Q and k is even, this implies that the outcome has to be $\frac{Q}{2}$. By implication, if the median outcome in a coalition-proof Nash equilibrium is $\frac{Q}{2}$, the maximal impact of an additional participant is also $\frac{Q}{2}$, what will be explained in detail in part III.[10]

(ii) $Median\{p_{Med-}, p_{Med+}, \frac{Q}{2}\} = p_{Med-}$
The second and third peak distributions (figures 15.2b and 15.2c) yield median outcomes different from $\frac{Q}{2}$: either $p_{Med-} > \frac{Q}{2}$ or $p_{Med+} < \frac{Q}{2}$. If the outcome under a coalition-proof Nash equilibrium is p_{Med-} like for the peak distribution of figure 15.2b, all of the subjects with peaks higher than p_{Med-} vote for Q. The individual with peak p_{Med-} votes for an allocation that is generated by $q^*_{Med-} = 2p_{Med-} - Q$, such that the social outcome is equal to the peak. For the other individuals with peaks left of p_{Med-}, any $q_i^* \leq q^*_{Med-}$ is part of a coalition-proof Nash equilibrium, i.e. there exist several coalition-proof Nash equilibria with outcome $Med(q^*) = p_{Med-}$.

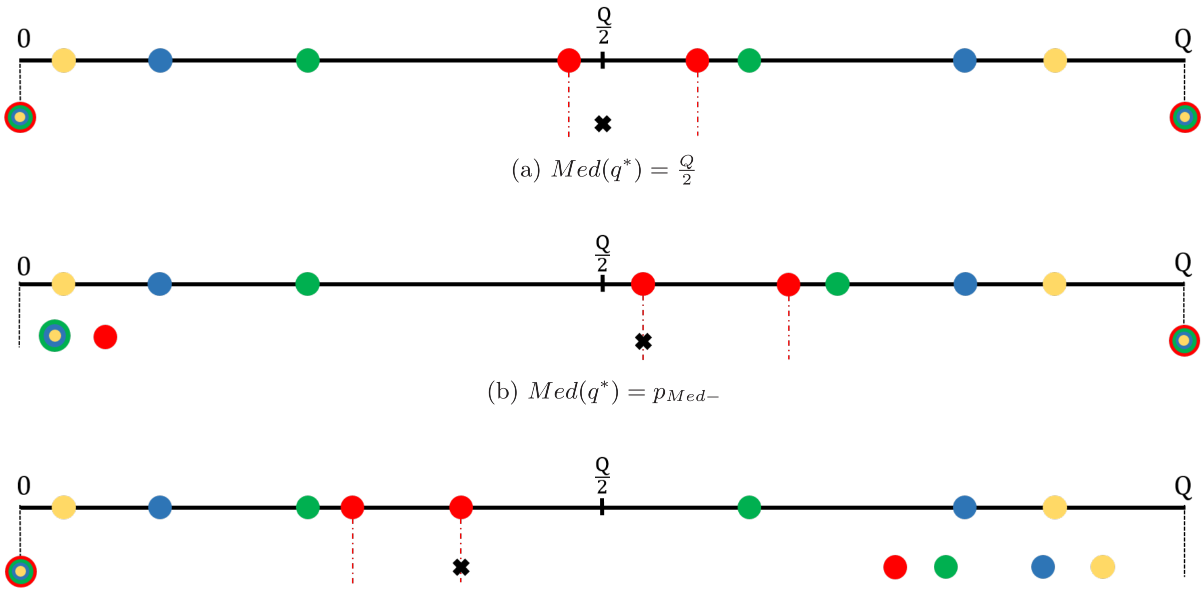
(iii) $Median\{p_{Med-}, p_{Med+}, \frac{Q}{2}\} = p_{Med+}$
Figure 15.2c shows another peak distribution and one of its coalition-proof Nash equilibrium. The social outcome is p_{Med+}, all of the individuals with peaks lower than p_{Med+} vote for an allocation of zero, the right median peak votes for $q^*_{Med+} = 2p_{Med+}$ and the individuals with peaks higher

[10] Note that one of the red peaks might as well be positioned exactly at $\frac{Q}{2}$.

than p_{Med+} vote for any allocation $q_i^* \geq q_{Med+}^*$. Again, several coalition-proof Nash equilibria exist with outcome $Med(q^*) = p_{Med+}$.

□

While the unique Nash equilibrium under the mean rule (for distinct peaks and single-peaked preferences) is identical to the strong and coalition-proof Nash equilibrium, the refinements are crucial for the median rule. Several strong and coalition-proof Nash equilibria exist in the k-odd and also in the k-even case if $\frac{Q}{2}$ is not positioned within the median peaks, but the social outcome is determined: p_{Med}, $\frac{Q}{2}$, p_{Med-} or p_{Med+}. The concepts of strong and coalition-proof Nash equilibria therefore provide a solution to Pareto-inefficient Nash equilibria under the median rule for any number of participants.



(a) $Med(q^*) = \frac{Q}{2}$

(b) $Med(q^*) = p_{Med-}$

(c) $Med(q^*) = p_{Med+}$

Figure 15.2: Coalition-proof Nash equilibria, median, k even

Table 15.1 provides an overview of the equilibrium concepts Nash, partial honesty, and coalition-proofness in the voting game. The table displays whether the equilibrium in the voting game is unique and whether the outcome is Pareto-efficient in *every* equilibrium. It also provides the allocation range of the social outcome under the corresponding equilibrium concept. The last column describes whether the outcome is unique in *every* equilibrium given that multiple equilibria exist.

The broadest concept of Nash equilibria, which includes partially honest and coalition-proof Nash equilibria, possesses nice features under the mean rule, as the equilibrium is unique and efficient for distinct peaks and single-peaked preferences. The outcome depends on the peak distribution but since the Nash equilibrium is unique, the equilibrium outcome is unique as well.

For the median rule, however, restrictions are only made if the Nash concepts are refined. A great advantage is that Pareto-inefficient outcomes disappear and the allocation of the social outcome may be restricted to the range within the two median peaks. Given partial honesty, the Nash equilibrium under the median rule with k even is not unique if $k > 2$, whereas it is unique and identical to the mean rule equilibrium if $k = 2$ and peaks are distinct. We also showed that several outcomes under partial honest equilibria are possible, such that the outcome in equilibrium is not unique. By contrast, for an odd number of voters, the equilibrium and thus the outcome is unique under the median rule and the concept of partial honesty.

The coalition-proof Nash equilibrium under the median rule with an odd number of voters is not unique; however, the outcome always corresponds to the median peak. For an even number of voters, the coalition-proof equilibrium is unique under the median rule if the social outcome in the equilibrium is $\frac{Q}{2}$. It is not unique if the median of $\{p_{Med-}, p_{Med+}, \frac{Q}{2}\}$ is not $\frac{Q}{2}$. However, the outcome is identical in every equilibrium with the same median. Any coalition-proof Nash equilibrium under the median rule is also a strong Nash equilibrium.

Nash	**Equilibrium Uniqueness**	**Outcome Efficiency**	**Outcome Range**	**Outcome Uniqueness**
Mean	✓	✓	$[p_{[1]}, p_{[k]}]$	✓
Median, k odd	✗	✗	$[0, Q]$	✗
Median, k even	✗	✗	$[0, Q]$	✗
Partially Honest				
Mean	✓	✓	$[p_{[1]}, p_{[k]}]$	✓
Median, k odd	✓	✓	p_{Med}	✓
Median, k even	✗	✓	$[p_{Med-}, p_{Med+}]$	✗
Coalition-Proof				
Mean	✓	✓	$[p_{[1]}, p_{[k]}]$	✓
Median, k odd	✗	✓	p_{Med}	✓
Median, k even	✓✗	✓	$Median\{p_{Med-}, p_{Med+}, \frac{Q}{2}\}$	✓

Table 15.1: Equilibrium concepts

15.3 Further Refinement Concepts for the Median Voting Game

The refinements of Nash equilibria in the voting game using the concepts of partial honesty and coalition-proofness help us to eliminate bad Nash equilibria under the median rule and provide unique equilibria in some cases. However, we described in the last chapter that the median voting game yields unique equilibria and unique outcomes only in some cases. In the sequential participation game, the outcome is the only relevant factor for finding equilibria of the participation game. The focus of this thesis lies on the simultaneous participation game, where uniqueness of the voting game equilibrium is required. We explain in detail how the participation game equilibria are determined in the next chapter. In preparation for the participation game equilibria, we further refine the median rule voting game equilibrium concepts by using the following concepts:

1. **Partial honesty with minimal lying**: All subjects vote truthfully. If k is even, the subjects with pivotal peaks regarding the peaks of all participants vote strategically but lie minimally.

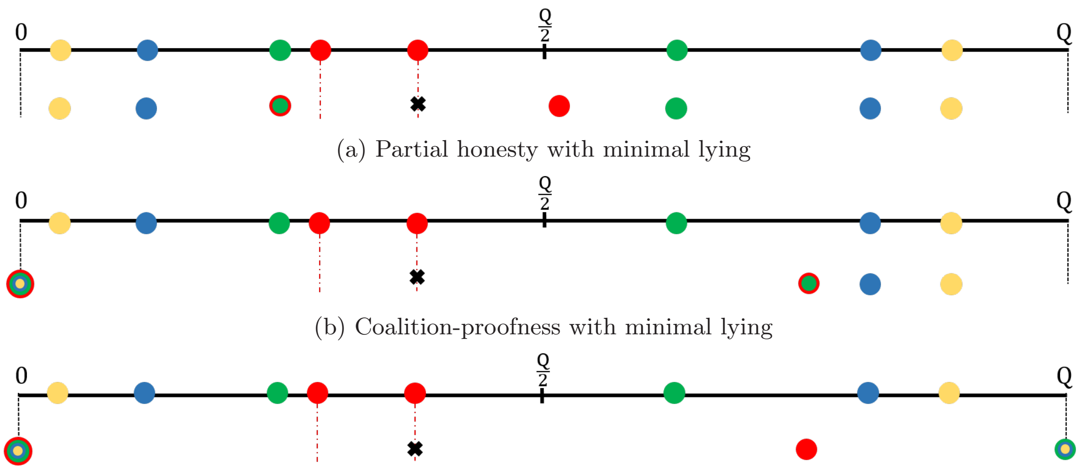
2. **Coalition-proofness with minimal lying**: Choose the coalition-proof equilibrium where the distances between all peaks and the corresponding votes are minimal.

3. **Coalition-proofness with extreme voting**: Choose the coalition-proof equilibrium with the maximal number of extreme votes (zero and Q).

Since uniqueness of Nash equilibria is given under the mean rule, we consider only the median rule. For an *odd number of participants*, partial honesty yields a unique equilibrium. This equilibrium also satisfies coalition-proofness, which gives us a first unique Nash equilibrium of the voting game for k odd under the median when applying both concepts: every participant states the true most preferred allocation. Given that truth-telling is a weakly dominant strategy under the median with an odd number of participants, the Nash equilibrium that implies truth-telling of all voters and thus combines partial honesty and coalition-proofness is prominent.

Considering coalition-proofness separately, there exists another equilibrium with salient strategies: the pivotal median voter p_{Med} reveals the true peak and the other participants vote either for zero if their peak is smaller than or equal to p_{Med} or Q if their peak is greater than or equal to p_{Med}. Since the idea of coalition-proofness is that no coalition may coordinate and obtain a better outcome for this coalition, the boundaries of the budget interval are focal allocations. Both equilibria (truth-telling of all voters and truth-telling of only the median voter together with extreme voting of the others) yield the same outcome but the impact of a potential participant is much higher in the second case. Therefore, it is crucial to distinguish between different concepts and keep their structure in mind when evaluating the equilibria.

We use the same logic for an *even number of participants*. Since partial honesty yields several equilibria, we focus on the equilibrium where only the voters who are pivotal regarding the peaks possibly deviate from truth-telling. An example for a peak distribution is given in figure 15.3a. All non-pivotal voters tell the truth and the pivotal red voters choose their optimal (strategic) vote. If the optimal vote of a subject is a range of votes, this subject chooses the optimal vote with minimal distance to the peak, i.e. he or she lies minimally. We therefore call this refinement partial honesty with minimal lying.



(a) Partial honesty with minimal lying

(b) Coalition-proofness with minimal lying

(c) Coalition-proofness with extreme voting

Figure 15.3: Unique Nash equilibria, median, k even

Combining honesty of all non-pivotal voters and coalition-proofness yields a second concept with a unique equilibrium. Depending on the median of $\{p_{Med-}, p_{Med+}, \frac{Q}{2}\}$, the uniqueness of an equilibrium results from the following. If the median is $\frac{Q}{2}$, the equilibrium is unique and every voter allocates either zero if the peak is lower than $\frac{Q}{2}$ or Q if it is higher. If p_{Med-} is the median, every voter with a peak greater than p_{Med-} votes for Q. The voter with peak p_{Med-} votes $q^*_{Med-} = 2p_{Med-} - Q$ and the other voters vote truthfully if their peak is smaller than or equal to q^*_{Med-} or, if their peak is greater, they vote q^*_{Med-}. This equilibrium is coalition-proof with the additional assumption of minimal lying: individuals either tell the truth or vote for an allocation that has minimal distance to their peak but still satisfies coalition-proofness. Analogously, we determine the voting behavior if the median is p_{Med+}, as in the peak distribution of figure 15.3b. Voters with peaks lower than p_{Med+} vote for zero, the voter with peak p_{Med+} votes $q^*_{Med+} = 2p_{Med+}$ and voters with peaks greater than p_{Med+} vote truthfully if their peak is greater than q^*_{Med+} or, if it is smaller than or equal to q^*_{Med+}, they vote q^*_{Med+}. This refinement is called coalition-proofness with minimal lying.

The third concept we use is coalition-proofness in combination with extreme voting. The equilibrium structure is the same as under the mean rule: at most one voter does not vote for zero nor for Q, namely the voter who reaches the most preferred allocation. The others vote according to the mean rule algorithm given by Renault and Trannoy (2005) and Block (2014). We call this refinement coalition-proofness with extreme voting. An example is provided in figure 15.3c.

For an odd number of voters, partial honesty and coalition-proofness with minimal lying yield the same distribution of votes. Given these three further refinements of equilibrium concepts, we are now able to provide the equilibria not only of the voting game but also of the participation game.

16 Nash Equilibria of the Participation Game

After defining the strategies for the simultaneous and sequential participation game, analyzing the Nash strategies in the voting games and refining the equilibrium concepts, the next step is to find the equilibria of the participation game. Obviously, each equilibrium of the participation game has to include an equilibrium of the voting game. For the latter, we already determined the Nash strategies of each participant in different refinements of the Nash equilibrium. To verify whether a combination of the participation decisions and optimal votes constitutes a Nash equilibrium, we calculate for each voter the utility in the present situation and examine incentives to deviate.

Definition 14 (Nash equilibrium participation game). *A* ***participation game*** *constitutes a* ***Nash equilibrium*** *if and only if the participants play a Nash equilibrium in the voting game, no participant has an incentive to abstain, and no abstainer has an incentive to participate.*

Since the Nash equilibria under the median rule voting game are in general not unique, we use the concepts of partial honesty with minimal lying, coalition-proofness with minimal lying and with extreme voting to solve for the equilibria in the participation game. Now it becomes more obvious why we need to apply uniqueness of the equilibria in the voting game: In the median rule participation game, the alteration of the social outcome by each vote depends on the distribution of votes and thus does the decision on participation and abstention for given costs. The utility from voting is positive, i.e. participation yields a higher utility for an individual as compared to abstention, if the alteration of the outcome induced by an optimal vote exceeds the cost of participation. We restrict the budget set to $Q = 100$ and the cost to $c \leq 100$, as the maximal alteration of the social outcome is theoretically Q, induced by a change from zero to Q or vice versa. If no individual participates, i.e. $k = 0$, we define the utility of each subject as in equation 13.3: $u_i(\cdot) = -\infty$ if $k = 0$. We justify this negative utility by the following consideration: In the context of voting over resource allocation, our model describes a situation where subjects vote on how a resource like budget is allocated on public projects. Since the outcome is only defined for $k > 1$, the meaning is that the projects will not be allocated any budget if $q_i = *$ for all $i \in I$. As we assume that individuals have single-peaked preferences over the allocations and voting is costly, we define the utility of each individual for $k = 0$ to be lower than any other case, even lower as if the outcome is positioned at the other extreme of the budget set, the individual participated and thereby faced very high costs. For simplicity, we define the worst-case utility to be minus infinity. This restriction implies that abstention of all subjects is never an equilibrium and every Nash equilibrium includes at least one participant.

16.1 Nash Equilibria for Given Peak Distributions

In this chapter, we provide examples of equilibria for the mean and the median rule participation game with different refinements of the voting game Nash equilibria. We make use of a computer program to find for both voting rules all Nash equilibria for a given peak distribution with any cost and aim to get first insights in the general structure of equilibria. The program works as follows.

1. Fix the costs of participation to $c = 0$.
   a) Start with full participation $k = n$.
   b) Find the unique Nash equilibrium (with refinements) of the voting game.
   c) Check for each participant whether participation is a best response given the optimal votes of the others. If yes for all n: Nash equilibrium found (*unique full-participation Nash equilibrium*).
   d) Decrease the number of participants step-wise by one until $k = 0$ and find the unique Nash equilibrium (with refinements) of the voting game for every combination of participants.
   e) For every combination, check whether participation/abstention is a best response given the optimal votes of the other participants. If yes for all n: Nash equilibrium found with $k < n$.
2. Increase the costs step-wise by 0.001 until $c = 100$ and repeat 1.a)-e).
3. Find cost ranges that include the same Nash equilibria and sort equilibria by increasing c and k.

The program provides for any peak distribution a list of increasing cost ranges and the corresponding Nash equilibria for the mean and median rule with different equilibrium refinements. Since all combinations of participants need to be checked, the program needs more time, the more peaks are analyzed and should only serve as insight in equilibria before obtaining general results. Table 16.1 provides the Nash equilibria for the mean rule with three individuals and peaks $p = (20, 40, 80)$. For each cost range, the table displays the number of participants (k), which individuals participate (k-ID) and the social outcome $x(q^*)$ in equilibrium: k:k-ID($x(q^*)$). The cost range is a single value if for this cost value additional Nash equilibria exist compared to the subsequent lower and higher cost ranges.

Cost Range	**Nash Equilibria**
$[0, 10)$	3:123(40)
10	2:13(50) | 3:123(40)
$(10, 16.\bar{6}]$	2:13(50)
$[16.\bar{6}, 20]$	2:13(50) | 2:23(50)
$[20, 30]$	2:12(40) | 2:13(50) | 2:23(50)
$[30, 40)$	1:2(40) | 2:12(40) | 2:13(50) | 2:23(50)
40	1:1(20) | 1:2(40) | 1:3(80) | 2:12(40) | 2:13(50) | 2:23(50)
$(40, 50]$	1:1(20) | 1:2(40) | 1:3(80) | 2:13(50) | 2:23(50)
$[50, 100]$	1:1(20) | 1:2(40) | 1:3(80)

Table 16.1: Nash equilibria, mean, $n = 3$, $p = (20, 40, 80)$

For reasons of clarity, we do not display the votes directly but print the votes for specific costs in a separate table. Exemplary for the same peak distribution, table 16.2 displays the votes under the mean rule for the two Nash equilibria with $c = 10$.

Participants	**Votes and Nash Outcome**
$k = 3$	$q_1^* = 0$, $q_2^* = 20$, $q_3^* = 100$, $x(q^*) = 40$
$k = 2$	$q_1^* = 0$, $q_2^* = *$, $q_3^* = 100$, $x(q^*) = 50$

Table 16.2: Nash equilibria, mean, $p = (20, 40, 80)$, $c = 10$

For the median rule, a refinement of the Nash concept is necessary if $k > 2$. Table 16.3 displays the partially honest Nash equilibria with minimal lying for the previous peak distribution. Since $n = 3$ and the number of participants is at most n, the equilibria correspond to the coalition-proof Nash equilibria with minimal lying. Even more, the number of participants, the outcome, and the cost range is identical to the equilibria of the coalition-proof concept with extreme voting. The entries of table 16.3 are therefore identical for all three equilibrium concepts.

Cost Range	**Nash Equilibria**
$[0, 10)$	3:123(40)
10	2:13(50) | 3:123(40)
$(10, 30]$	2:13(50)
$[30, 40)$	1:2(40) | 2:13(50) | 2:23(50)
40	1:1(20) | 1:2(40) | 1:3(80) | 2:12(40) | 2:13(50) | 2:23(50)
$(40, 50]$	1:1(20) | 1:2(40) | 1:3(80) | 2:13(50) | 2:23(50)
$[50, 100]$	1:1(20) | 1:2(40) | 1:3(80)

Table 16.3: Partially honest Nash equilibria with minimal lying, median, $n = 3$, $p = (20, 40, 80)$

Although the Nash equilibrium outcomes of table 16.3 are identical for all three equilibrium concepts, the votes differ for $k = 3$. Table 16.4 displays details of the partially honest Nash equilibria with minimal lying under the median rule for specific costs of $c = 10$. As mentioned earlier, the equilibria are congruent with the coalition-proof concept with minimal lying. In addition, the optimal votes are equal under partially honest equilibria with minimal lying and coalition-proofness with minimal lying. Coalition-proofness with extreme voting yields different votes for participants with non-pivotal peaks, as indicated in table 16.5 for $k = 3$. In contrast to the partially honest equilibria, where non-pivotal voters tell the truth, the individuals with the highest and lowest peak vote for the extreme allocations of zero and 100.

Participants	**Votes and Nash Outcome**
$k = 3$	$q_1^* = 20$, $q_2^* = 40$, $q_3^* = 80$, $x(q^*) = 40$
$k = 2$	$q_1^* = 0$, $q_2^* = *$, $q_3^* = 100$, $x(q^*) = 50$

Table 16.4: Partially honest Nash equilibria with minimal lying, median, $p = (20, 40, 80)$, $c = 10$

Participants	**Votes and Nash Outcome**
$k = 3$	$q_1 = 0$, $q_2 = 40$, $q_3 = 100$, $x(q) = 40$
$k = 2$	$q_1 = 0$, $q_2 = *$, $q_3 = 100$, $x(q) = 50$

Table 16.5: Coalition-proof Nash equilibria with extreme voting, median, $p = (20, 40, 80)$, $c = 10$

So far, we observe that for the peak distribution $p = (20, 40, 80)$ equilibria exist for any cost range under both rules. Full participation is an equilibrium for $c \leq 10$ and with increasing costs, the number of participants in equilibrium decreases. For $c \geq 40$ the equilibria are identical for both rules, since the median and mean outcomes for $k = 1$ and $k = 2$ are the same as the rules are identical. Interestingly, the outcomes are equal for most cost ranges even though the votes for $k = 3$ differ among the rules and equilibrium concepts, as can be seen when comparing the votes of tables 16.2, 16.4 and 16.5.

Comparing the rules, it is observable that participation of only individuals 1 and 2 is an equilibrium in the mean rule for $20 \leq c \leq 40$ (see table 16.1), whereas it is under the median rule for $c = 40$ only (see table 16.3). Another interesting fact is that under the median rule there is a large cost range between 10 and 30 where individual 2 abstains. Since the outcome is $x(q^*_{-2}) = 50$ and given $q_2^* \neq *$ it would be $x(q^*) = 40$, individual 2 abstains when costs exceed the difference of $|x(q^*_{-2}) - x(q^*)| = 10$. The cost range under the mean rule with $q_2 = *$ is smaller, since there exists another equilibrium including participation of individuals 2 and 3 for $c \geq 16.\bar{6}$.

In a next step, we consider a more divers peak distribution and point out conspicuous findings. We adhere to $n = 3$ but the peaks are closer to the extremes of the allocation range: $p = (5, 90, 95)$.

Tables 16.6 and 16.7 present the equilibria under the mean and the median rule with minimal lying partial honesty, respectively.

Cost Range	**Nash Equilibria**
$[0, 16.\bar{6}]$	3:123(66.$\bar{6}$)
$[16.\bar{6}, 45]$	2:12(50) | 2:13(50)
$[45, 47.5]$	1:2(90) | 2:12(50) | 2:13(50)
$[47.5, 50]$	1:1(5) | 1:2(90) | 1:3(95) | 2:12(50) | 2:13(50)
$[50, 100]$	1:1(5) | 1:2(90) | 1:3(95)

Table 16.6: Nash equilibria, mean, $n = 3$, $p = (5, 90, 95)$

Cost Range	**Nash Equilibria**
$[0, 2.5]$	3:123(90)
$[2.5, 10)$	
10	2:23(90)
$(10, 40]$	
$[40, 45]$	2:13(50)
$[45, 47.5]$	1:2(90) | 2:12(50) | 2:13(50)
$[47.5, 50]$	1:1(5) | 1:2(90) | 1:3(95) | 2:12(50) | 2:13(50)
$[50, 100]$	1:1(5) | 1:2(90) | 1:3(95)

Table 16.7: Partially honest Nash equilibria with minimal lying, median, $n = 3$, $p = (5, 90, 95)$

Again, since $n = 3$, the concepts of partial honesty and coalition-proofness with minimal lying yield the same equilibria because the voting behavior under the median rule is identical. The full participation equilibrium given coalition-proofness with extreme voting holds for costs $c \leq 5$ (as compared to $c \leq 2.5$, see the first entry in table 16.7), the other Nash equilibria are the same.

Another insight that we observe for this specific peak distribution is that there exist cost ranges without any equilibrium under the median rule. In contrast, under the mean rule equilibria do exist for any cost range. We show in Puppe and Rollmann (2019) that for $n = 3$, $Q = 100$, and distinct peaks there exists an equilibrium in the mean rule participation game for all $c \geq 0$. Further, the Nash equilibria vary widely. While full participation given the peak distribution $p = (5, 90, 95)$ is an equilibrium under the mean rule for cost of $c \leq 16.\bar{6}$, it is under the median rule only a partially honest or coalition-proof Nash equilibrium with minimal lying for $c \leq 2.5$ or for $c \leq 5$ given coalition-proofness with extreme voting. The asymmetry of the peak distribution reveals another difference among the voting rules. Individual 1 might be regarded as an outsider given the proximity to zero and at the same time the high distance to the other clustered peaks. Besides full participation, individual 1 participates under the median rule in any equilibrium concept only for cost $c \geq 40$. Compared to the mean rule, where equilibria exist for any cost range with participation of individual 1, we observe that even for low costs individual 1 has an incentive to abstain under the median rule given that individuals 2 and 3 participate.

16.2 Full Participation Nash Equilibria

After we gave examples of specific peak distributions and the differences among the participation game Nash equilibria under mean and median voting, we will now provide general results for any number of individuals. To begin with, let us consider Nash equilibria of the voting game with full participation. Each full participation equilibrium of the voting game is an equilibrium of the participation game if no participant has an incentive to abstain from the vote. If we do not state otherwise, we always imply that peaks are distinct.

16.2.1 Full Participation under the Mean Rule

We will explain later in chapter 19 that the impact under the mean rule is independent of the peak distribution but solely varies with the number of votes. For any distribution, the theoretical impact of an additional vote is $\frac{Q}{k}$. The theoretical impact of an absent vote is $\frac{Q}{k-1}$. We already know from chapter 15 that in every mean rule Nash equilibrium at most one voter does not vote for the extreme allocations of zero or Q. A Nash vote only differs from zero or Q if a social outcome is obtained that is equal to the most preferred allocation of this individual - otherwise the vote is not a best response and the voting game does not constitute a Nash equilibrium. An individual might also achieve the most preferred allocation by voting for zero or Q. With the knowledge on the Nash strategies, the following proposition provides a restriction on the mean outcome for full participation Nash equilibria with strictly positive costs.

Proposition 11. *The outcome under any full participation Nash equilibrium under the mean rule is never $Mean(q^*) = 0$ nor $Mean(q^*) = Q$ if $c > 0$ and $n > 1$.*

Proof. Suppose that $n > 1$ peaks are distinct and $c > 0$. By way of contradiction, let $Mean(q^*) = 0$. This implies that the vector of all votes has to include solely extreme votes, $q^* = (0, \ldots, 0)$, otherwise, the outcome is always greater than zero. However, given that peaks are distinct, at most one peak is equal to zero and any voter whose peak is different from zero improves by increasing his or her vote. Thus, the outcome increases and $Mean(q^*) = 0$ is never an equilibrium outcome. The similar holds for $Mean(q^*) = Q$, which is only the outcome if all voters vote for Q but with distinct peaks this is a Nash strategy for at most one voter and deviation of the other voter(s) decreases the outcome. Suppose that $n > 1$ peaks are not distinct but identical and the vector of peaks is $p = (0, \ldots, 0)$ (or $p = (Q, \ldots, Q)$). Let $Mean(q^*) = 0$, i.e. $q^* = (0, \ldots, 0)$. Full participation is never an equilibrium for $c > 0$ as the outcome under abstention of any voter i remains $Mean(q^*_{-i}) = 0$ but participation is costly. Hence, each voter improves by abstention and $Mean(q^*_{-i}) = 0$ is no Nash equilibrium outcome. □

In this chapter, we define the voter whose peak corresponds to the equilibrium outcome as individual i. In order to determine the cost ranges for all full participation Nash equilibria under the mean rule we make a case analysis. The cases comprise three different outcomes. Cases 1 and 2 include the existence of a voter i that obtains the peak by a specific vote. Case 3 distinguishes between an even and an odd number of votes.

<u>Case 1:</u> $Mean(q^*) \in [0, \frac{Q}{2})$

a) $\exists i \in I : q^*_i \in (0,\ 2Mean(q^*))$

We have already shown that given an individual exists who votes for $q^*_i \in (0,\ 2Mean(q^*))$ in a Nash equilibrium, this individual has to obtain the peak as mean outcome: $p_i = Mean(q^*)$. Otherwise, the vote would not be a Nash vote and the voting game would not constitute a Nash equilibrium.[11] As displayed in figure 16.1a, we define by section I the change of the outcome if a voter who votes for Q abstains, by section II the change if a voter who votes for zero abstains, and by section III the change if voter i abstains. Under the above assumptions, the vote of individual i has minimal distance to the mean outcome when comparing the distances of all other votes to the mean. Since $Mean(q^*)$ is smaller than half of the budget set, the distance between the left extreme voters (at zero) to the mean outcome

[11] Note that $2Mean(q^*) = 0$ only holds if $Mean(q^*) = 0$. From proposition 11 we know that this is only possible if $c = 0$ or $n = 1$. We already stated that full participation is optimal if no participation costs exist or the set of individuals comprises only a single subject.

is smaller as compared to the distance between the right extreme voters (at Q) to the mean outcome. Hence, section I is larger than section II. Given that $q_i \in (0,\ 2Mean(q^*))$, the distance between q_i and the mean is the smallest of all votes and relevant for the cost range of full participation as the abstention of voter i changes the outcome at most to the extent that the abstention of any other voter would, i.e. section III is at most as large as section II and smaller than section I. This implies that the cost range is dependent on the distance between $Mean(q^*)$ and $Mean(q^*_{-i})$, which is calculated as follows: $\frac{1}{n-1} \cdot \big|Mean(q^*) \cdot (n-1) - (Mean(q^*) \cdot n - q_i^*)\big| = \frac{1}{n-1} \cdot \big|q_i^* - Mean(q^*)\big|$.

Observation 5. *Given a Nash equilibrium of the mean rule voting game under case 1a). Full participation is a Nash equilibrium if*

$$c \leq \frac{1}{n-1} \cdot d\big(q_i^*, Mean(q^*)\big).$$

b) $\nexists i \in I : q_i^* \in (0,\ 2Mean(q^*))$

If no voter exists who votes for $q_i^* \in (0,\ 2Mean(q^*))$ (the outcome might still be equal to i's peak) and the outcome is smaller than half of the budget set, the voter $q^* = 0$ have minimal distance to $Mean(q^*)$.

The cost range is not identical for both sides of the mean outcome but section I to the left of $Mean(q^*)$ is larger than section II as the alteration of the outcome is larger if a voter who voted for Q abstains. Section II is thus relevant for all participants as it is the smallest section. Figure 16.1b provides an example. The crucial maximal costs that imply full participation are therefore determined by the length of section II, which is calculated by $\frac{1}{n-1} \cdot Mean(q^*) \cdot n - Mean(q^*)$.

Observation 6. *Given a Nash equilibrium of the mean rule voting game under case 1b). Full participation is a Nash equilibrium if*

$$c \leq \frac{1}{n-1} \cdot Mean(q^*).$$

Note that an outcome of $Mean(q^*) = 0$ is only a Nash equilibrium outcome for $k > 1$ if the peaks of all participants are equal to zero and thus are not distinct, see proposition 11.

<u>Case 2:</u> $Mean(q^*) \in (\frac{Q}{2}, Q]$

a) $\exists i \in I : q_i^* \in (2Mean(q^*) - Q,\ Q)$

Analogously to case 1a), there exists an individual i with a vote q_i^* that has minimal distance of all votes to the mean outcome and the outcome is identical to this voter's peak: $p_i = Mean(q^*)$. The condition on the cost range remains the same as in case 1a).

Observation 7. *Given a Nash equilibrium of the mean rule voting game under case 2a). Full participation is a Nash equilibrium if*

$$c \leq \frac{1}{n-1} \cdot d\big(q_i^*, Mean(q^*)\big).$$

b) $\nexists i \in I : q_i^* \in (2Mean(q^*) - Q,\ Q)$

This case is symmetric to case 1b), the voters who vote for Q now have minimal distance to the outcome and the relevant section is section I. The size of section I is calculated by $Mean(q^*) - \frac{1}{n-1} \cdot (Mean(q^*) \cdot n - Q)$ and is therefore different from case 1b) due to the distinct boundary values of the budget set.

Observation 8. *Given a Nash equilibrium of the mean rule voting game under case 2b). Full participation is a Nash equilibrium if*

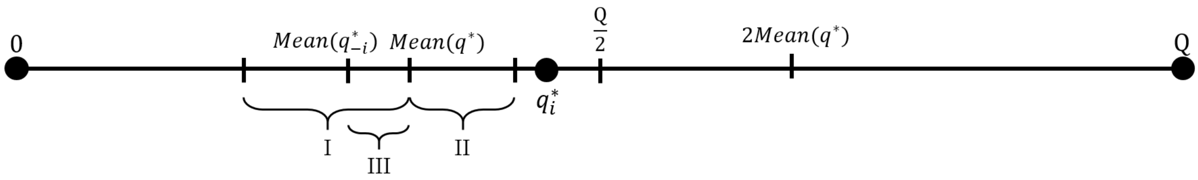
$$c \leq \frac{1}{n-1} \cdot \big(Q - Mean(q^*)\big).$$

Note that an outcome of $Mean(q^*) = Q$ is only a Nash equilibrium outcome for $k > 1$ if the peaks of all participants are equal to Q and thus are not distinct, see proposition 11.

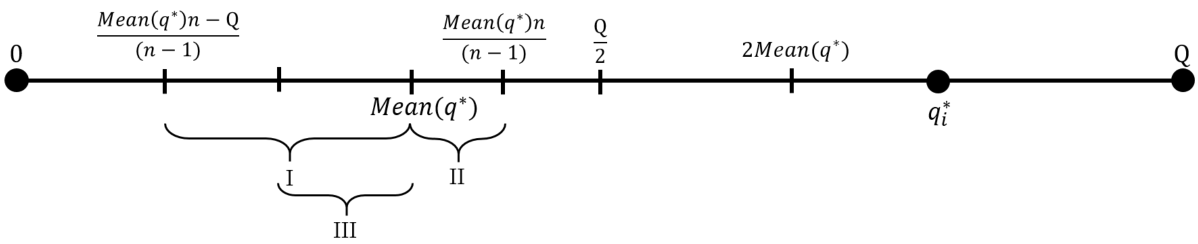
<u>Case 3:</u> $Mean(q^*) = \frac{Q}{2}$

a) k is even

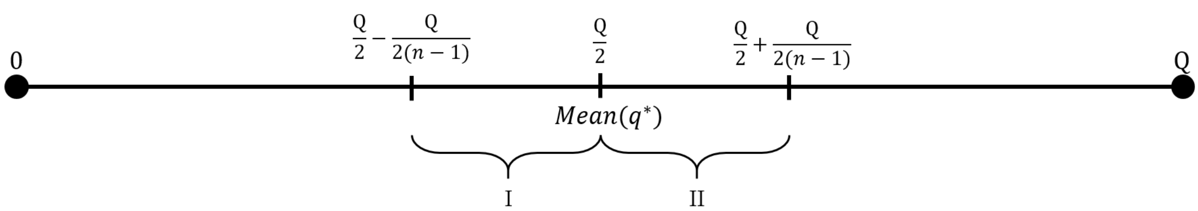
This case describes a distribution of votes where $n = k$ is even and therefore the same number of votes is positioned at the lower and upper bound of the budget set. In this case, all votes have the same distance to the outcome, which means that the size of section I equals the size of section II, as displayed in figure 16.1c. The cost range for full participation Nash equilibria may be either calculated by the inequality of observation 6 or of observation 8.



(a) Case 1a)



(b) Case 1b)



(c) Case 3a)

Figure 16.1: Full participation conditions, mean

b) k is odd and $\exists i \in I : q_i^* = \frac{Q}{2}$

If $n = k$ is odd and the mean outcome equal to $\frac{Q}{2}$, there has to exist one voter i that votes for $q_i^* = \frac{Q}{2}$ such that this is the last case we analyze. In this case, however, full participation is never an equilibrium for $c > 0$. Individual i will always abstain in equilibrium if costs are positive as the outcome will not change.

Observation 9. *Given a Nash equilibrium of the mean rule voting game under case 3b). Full participation is a Nash equilibrium if*

$$c = 0.$$

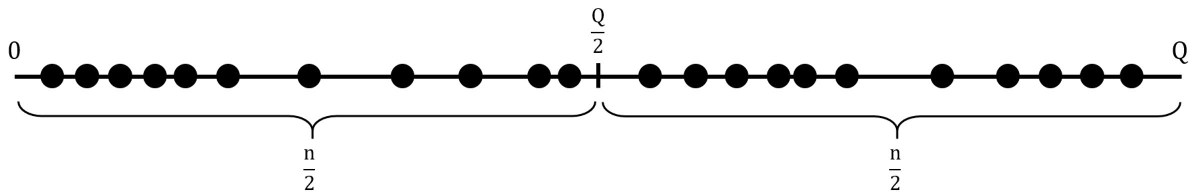
Directly from observation 9 follows:

Observation 10. *If $d(q_i^*, Mean(q^*)) = 0$ for any voter i, then full participation is never a Nash equilibrium of the mean rule for $c > 0$.*

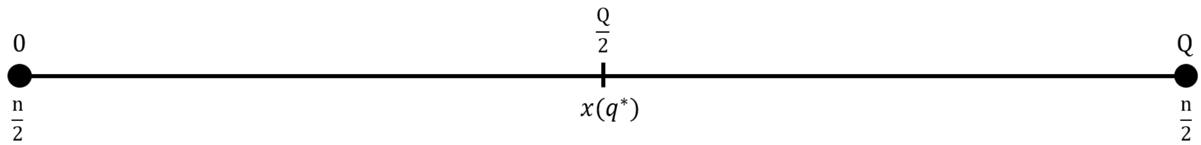
Considering all full participation conditions on the cost range, we obtain the highest cost range for $Mean(q) = \frac{Q}{2}$ and if no individual votes for $q_i = \frac{Q}{2}$. All peak distributions with the same number of voters that have peaks to the left and to the right of $\frac{Q}{2}$ satisfy this condition, such that full participation is an equilibrium for a maximum cost of $c_{Mean}^{max} = \frac{Q}{2(n-1)}$. This cost range decreases with the total number of subjects in the participation game, as the impact of each absent vote decreases in n, making participation less effective.

Observation 11. *The maximal cost such that full participation is a Nash equilibrium of the participation game is $c_{Mean}^{max} = \frac{Q}{2(n-1)}$. This cost range holds if n is even, $Mean(q^*) = \frac{Q}{2}$, and if there does not exist any voter i who votes for $q_i^* = \frac{Q}{2}$.*

Figure 16.2 illustrates an example for a peak distribution (16.2a) and the corresponding full participation Nash equilibrium (16.2b), where the cost range for full participation is maximal.



(a) Peak distribution



(b) Nash equilibrium

Figure 16.2: Maximal cost range for full participation

16.2.2 Full Participation under the Median Rule

The last chapter provided all cost conditions for full participation Nash equilibria using the mean rule and we will now refer to the median rule participation game. If a participant abstains, the median outcome is determined by another vote even though there are circumstances in which the impact of each vote is zero and thus the outcome does not change. To begin with, consider the case with an even number of individuals and full participation $k = n$.

Even Number of Participants

With an even number of voters, the outcome is determined by the average of the two median voters. Figure 16.3a displays a voting game for $n = 2$. The conditions on the participation cost are applicable for any even number of individuals but since Nash equilibria under the median rule are in general not unique and we apply several refinements, we use $n = 2$ for simplicity. If the individual who votes for q^*_{Med+} abstains, the social outcome changes from $Med(q^*)$ to q^*_{Med-} which is the size of section I or $\frac{1}{2}(q^*_{Med-} + q^*_{Med+}) - q^*_{Med-}$. The effect of abstention of q^*_{Med-} is $q^*_{Med+} - \frac{1}{2}(q^*_{Med-} + q^*_{Med+})$ or section II. Since $Med(q^*)$ is positioned exactly in the middle of the interval $[q^*_{Med-}, q^*_{Med+}]$, sections I and II are of the same size and we may define a unique cost condition dependent on the positions of q^*_{Med-} and q^*_{Med+}.

Observation 12. *Consider a Nash equilibrium of the median rule voting game with n even. Full participation is a Nash equilibrium of the participation game if*

$$c \leq \frac{1}{2}(q^*_{Med+} - q^*_{Med-}) \ .$$

The highest cost range for $n = 2$ is therefore given for $q^*_{Med-} = 0$ and $q^*_{Med+} = Q$. In this case, full participation is a Nash equilibrium (given that the votes are Nash strategies) for a maximum cost of $c^{max}_{Med,even} = \frac{Q}{2}$. The corresponding peaks for $n = 2$ are $p_{Med-} \leq \frac{Q}{2}$ and $p_{Med+} \geq \frac{Q}{2}$ such that $q^*_{Med-} = 0$ and $q^*_{Med+} = Q$ is a partially honest and coalition-proof Nash equilibrium of the voting game. We exclude the case of both peaks being equal to $\frac{Q}{2}$, since voting for the extremes is only one possible set of Nash strategies under both concepts, coalition-proofness and partial honesty, and maximizing the cost range may not be induced for equal peaks. For sufficiently low costs $c > 0$, full participation is an equilibrium of the participation game for any peak distribution with distinct peaks since abstention will always change the social outcome. The upper cost boundary is dependent on the distribution of peaks and might be close to zero though.

Observation 13. *The maximal cost such that full participation is a Nash equilibrium of the participation game with n even is $c^{max}_{Med,even} = \frac{Q}{2}$. This cost range holds if peaks are distinct, $p_{Med-} \leq \frac{Q}{2}$ and $p_{Med+} \geq \frac{Q}{2}$, and the concept coalition-proofness with extreme voting.*

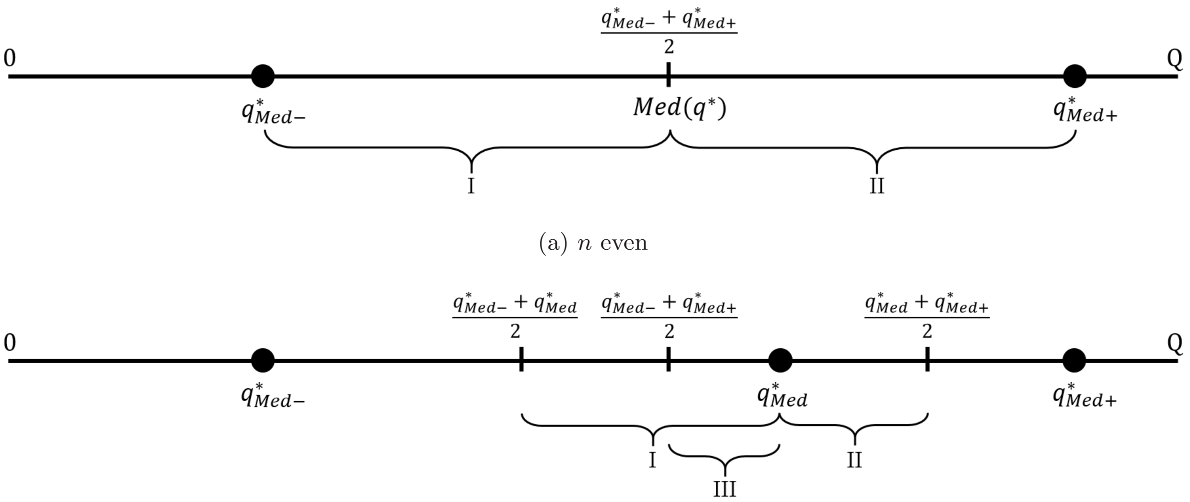
An exemplary peak distribution is that of figure 16.2a, which we also used for the mean rule. The corresponding Nash equilibrium under coalition-proofness with extreme voting is identical to that of the mean rule and displayed in figure 16.2b.

As already shown, the maximum of c depends on the distance between the votes ranked next to the median and their average, such that full participation for $n = 2$ is an equilibrium if c is at most as high as the length of sections I or II. The closer the peaks of the individuals, the closer are the votes in equilibrium since there exist equilibria in which one individual does not vote for an extreme allocation when the median of p_{Med-}, p_{Med+} and $\frac{Q}{2}$ is different from $\frac{Q}{2}$. The closer the votes, the smaller the sections I and II and thus decreases the maximal cost of participation that generates a full participation equilibrium. In the extreme case that the median peaks are identical and therefore $q^*_{Med-} = q^*_{Med+}$ are possible Nash strategies, full participation is never an equilibrium for strictly positive costs:

Observation 14. *If k is even and $q^*_{Med-} = q^*_{Med+}$, then full participation is never a Nash equilibrium of the median rule participation game for $c > 0$.*

Odd Number of Participants

We will now consider Nash equilibria with full participation, where the number of individuals that participate is odd. Figure 16.3b displays an example for $n = 3$. The median outcome is determined by q^*_{Med} and the strategies might result from a partially honest Nash equilibrium.



(a) n even

(b) n odd

Figure 16.3: Full participation conditions, median

Section I displays the alteration of the median outcome if the individual abstains who votes for an allocation of q^*_{Med+} (or any individual who votes at least q^*_{Med+} if n is odd and greater than three), which is $q^*_{Med} - \frac{1}{2}(q^*_{Med-} + q^*_{Med})$. Abstention of a voter who votes for (at most) q^*_{Med-} affects the median outcome by the size of section II, calculated by $\frac{1}{2}(q^*_{Med} + q^*_{Med+}) - q^*_{Med}$. If the median voter abstains, the outcome alters by $|q^*_{Med} - \frac{1}{2}(q^*_{Med-} + q^*_{Med+})|$, as indicated by section III. The size of section III is therefore determined by the absolute deviation of the sizes of sections I and II. With the knowledge on how the median outcome varies for abstention of each participant, we are able to calculate the cost range for the full participation equilibrium. As long as the cost of participation is at least as high as the minimal modification of the outcome under abstention, no participant has an incentive to abstain.

Observation 15. *Consider a Nash equilibrium of the median rule voting game with n odd. Full participation is a Nash equilibrium of the participation game if*

$$c \leq \frac{1}{2} \min \left\{ q^*_{Med} - q^*_{Med-} \ , \ q^*_{Med+} - q^*_{Med} \ , \ \left| q^*_{Med-} - 2q^*_{Med} + q^*_{Med+} \right| \right\}.$$

The last observation indicates that the Nash equilibria with full participation of an odd number of voters are more stable in the sense that they remain equilibria with increasing costs if sections I, II, and III are large. Sections I and II are affected by the remoteness of q^*_{Med-} and q^*_{Med+} from the median vote q^*_{Med}. The higher the distance $d(q^*_{Med}, q^*_{Med+})$, the larger section I and the higher $d(q^*_{Med-}, q^*_{Med})$, the larger section II. However, this remoteness is not the only determinant of the minimum function in observation 15. Section III could have length zero even though q^*_{Med-} and q^*_{Med+} have maximal distance to q^*_{Med}, namely when sections I and II are of identical size. This implies that $\frac{1}{2}(q^*_{Med-} + q^*_{Med+})$ corresponds to q^*_{Med} and therefore section III has minimal length. Thus, different to the n even case, there exist situations in which full participation is an equilibrium only if $c = 0$ - even with distinct peaks.

In order to maximize the minimum function and obtain the highest possible cost range, $d(q^*_{Med}, q^*_{Med+})$ and $d(q^*_{Med-}, q^*_{Med})$ have to be large but at the same time of sufficiently different size.

For $n = 3$ the maximum cost range is specified as follows. In order to maximize the length of sections I and II and since we know that $q^*_{Med-} \leq q^*_{Med}$ and $q^*_{Med+} \geq q^*_{Med}$, it ensues that $q^*_{Med-} = 0$ and $q^*_{Med+} = Q$. The remaining allocation that needs to be specified is q^*_{Med}. For maximization, none of the sections I, II, and III should have length zero, such that $q^*_{Med} \neq 0 \neq Q \neq \frac{Q}{2}$. We make a case analysis for the allocation of the median vote.

Case 1: $q^*_{Med} \in (0, \frac{Q}{2})$

Since $d(q^*_{Med-}, q^*_{Med}) < d(q^*_{Med}, q^*_{Med+})$, section I is smaller than section II, such that we need to maximize the size of sections I and III for the given values of $q^*_{Med-} = 0$ and $q^*_{Med+} = Q$ (remember, we maximize the minimum function of observation 15). Section I increases with q^*_{Med}, so we equalize sections I and III to get the maximum of the minimum function:

$$\frac{q^*_{Med} - 0}{2} \stackrel{!}{=} \frac{Q + 0}{2} - q^*_{Med}. \tag{16.1}$$

Thus, we get

$$q^*_{Med} = \frac{1}{3}Q. \tag{16.2}$$

Case 2: $q^*_{Med} \in (\frac{Q}{2}, Q)$

Since $d(q^*_{Med-}, q^*_{Med}) > d(q^*_{Med}, q^*_{Med+})$, section I is larger than section II, such that we need to maximize the size of sections II (which decreases with q^*_{Med}) and III for the given values of $q^*_{Med-} = 0$ and $q^*_{Med+} = Q$:

$$\frac{Q - q^*_{Med}}{2} \stackrel{!}{=} q^*_{Med} - \frac{Q - 0}{2}, \tag{16.3}$$

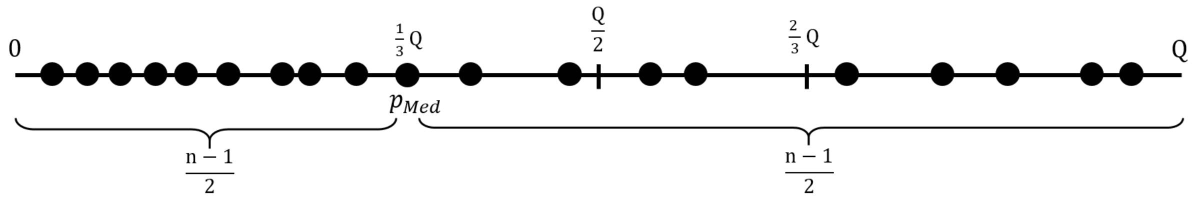
which yields

$$q^*_{Med} = \frac{2}{3}Q. \tag{16.4}$$

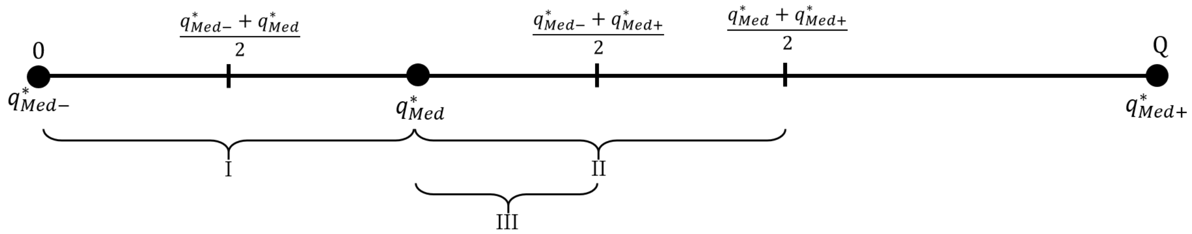
To sum up, we get that the cost range for a full participation equilibrium is maximal with $c^{max}_{Med,odd} = \frac{1}{6}Q$ for an odd number of individuals if $q^*_{Med-} = 0$, $q^*_{Med+} = Q$ and $q^*_{Med} = \frac{1}{3}Q$ or $q^*_{Med} = \frac{2}{3}Q$. An open question is for which peak distributions these votes constitute a Nash equilibrium in the voting game. Since in both of the equilibrium concepts partial honesty and coalition-proofness the median outcome in equilibrium is p_{Med} for k odd, we get that $p_{Med} = q^*_{Med}$, i.e. $p_{Med} = \frac{1}{3}Q$ or $p_{Med} = \frac{2}{3}Q$. For partially honest equilibria or coalition-proofness with minimal lying, $q^*_{Med-} = 0$ and $q^*_{Med+} = Q$ are only Nash strategies if $p_{Med-} = 0$ and $p_{Med+} = Q$. In equilibria of coalition-proofness with extreme voting, $q^*_{Med-} = 0$ and $q^*_{Med+} = Q$ are Nash strategies for any $p_{Med-} \leq q^*_{Med}$ and any $p_{Med+} \geq q^*_{Med}$. Therefore, the cost range for full participation is maximal when using the concept of coalition-proofness with extreme voting and for an odd number of voters if $q^*_{Med} = \frac{1}{3}Q$ or $q^*_{Med} = \frac{2}{3}Q$.

Observation 16. *The maximal cost such that full participation is a Nash equilibrium of the participation game with n odd is $c^{max}_{Med,odd} = \frac{1}{6}Q$. This cost range holds if peaks are distinct, $p_{Med} = q^*_{Med} = \frac{1}{3}Q$ or $p_{Med} = q^*_{Med} = \frac{2}{3}Q$, and the concept coalition-proofness with extreme voting.*

An example for a peak distribution that yields the maximal cost range for full participation under the median rule with an odd number of voters is provided in figure 16.4a. The corresponding Nash equilibrium is displayed in figure 16.4b and emerges from the concept coalition-proofness with extreme voting.



(a) Peak distribution



(b) Nash equilibrium

Figure 16.4: Maximal cost range for full participation, median, n odd

Another insight that we gain is that given the extreme votes of the individuals with peaks ranked next to the median peak, full participation is never an equilibrium for $c > 0$ if $p_{Med} = \frac{1}{2}Q$, as abstention of the median voter does not change the outcome. More general, for equilibria in which $d(q^*_{Med-}, q^*_{Med}) = d(q^*_{Med}, q^*_{Med+})$ (sections I and II have the same length), $k = n$ is no Nash equilibrium of the median rule participation game for positive participation costs. In this case, the median peak is located at exactly half of the distance between the two next ranked votes and therefore abstention of the median voter does not change the social outcome. If participation costs are strictly positive, the median voter has an incentive to abstain. Thus, for any symmetric peak distribution, there does never exist a full participation Nash equilibrium with an odd number of participants if participation is costly. We also do not get a full participation Nash equilibrium under the median rule with an odd number of voters if either q^*_{Med-} or q^*_{Med+} is identical to q^*_{Med}. If the median vote occurs more than once and n is odd, then the outcome remains the median vote when any non-pivotal voter abstains.

Observation 17. *Full participation is never a Nash equilibrium of the median rule for $c > 0$ if*

(i) $d(q^*_{Med-}, q^*_{Med}) = d(q^*_{Med}, q^*_{Med+})$ or

(ii) $q^*_{Med-} = q^*_{Med}$ or $q^*_{Med+} = q^*_{Med}$.

16.2.3 Full Participation: Mean vs. Median

For either rule, we derived a complete classification of full participation Nash equilibria by the maximal cost range and determined the conditions when full participation is never a Nash equilibrium for strictly positive costs. We will now compare our results for both voting rules.

The first finding that is identical under both rules is the following: For any odd number of participants $k = n > 1$ with a symmetric peak distribution, the outcome in any Nash equilibrium is never $x(q^*) = \frac{Q}{2}$ if $c > 0$.

Observation 18. *If $c > 0$ and the number of participants $k = n > 1$ is odd, full participation is never a Nash equilibrium for symmetric peak distributions.*

For the mean rule, symmetry of the peak distribution in the n-odd case requires that one peak is located at $\frac{Q}{2}$ and the same number of peaks are located on the left and on the right side of $\frac{Q}{2}$, which was described in case 3b). The symmetry condition for the median rule is more stringent, as it requires that the median outcome is located at exactly half of the distance between the vote ranked left of the median and the vote ranked right of the median. Under the concept of coalition-proofness with extreme voting, this is always the case when $p_{Med} = \frac{Q}{2}$. The concepts including minimal lying require that p_{Med-} and p_{Med+} have identical distance to p_{Med}.

We demonstrated that the cost conditions for full participation Nash equilibria vary for the mean and the median rule. We calculated the maximal cost for full participation and provided the corresponding peak distributions. A comparison of both rules yields the following proposition:

Proposition 12. *For any $n \geq 4$, the maximum cost range for full participation Nash equilibria is higher under the median rule as compared to the mean rule.*

Proof. The maximal cost conditions for an *even* number of voters with distinct peaks unequal to $\frac{Q}{2}$ are directly comparable for the mean and the median rule. For the mean rule, we have $c^{max}_{Mean} = \frac{Q}{2(n-1)}$ (Observation 11), whereas the maximal cost for full participation under the median rule for the same case is $c^{max}_{Med,even} = \frac{Q}{2}$ (Observation 13), which implies that the maximal cost is higher under the median rule for any n. For an *odd* number of voters, the maximal cost for full participation under the median rule is $c^{max}_{Med,odd} = \frac{1}{6}Q$ (Observation 16) and therefore smaller as under an even number. However, the maximal cost for the mean rule is still smaller as long as the number of voters is $n > 4$ and odd. For $n = 3$, the maximal cost for full participation under the mean rule is identical to the maximal cost under the median rule, which is $c^{max}_{Med,odd} = \frac{1}{6}Q$. If $n \leq 2$, both rules are identical and thus are the maximal cost conditions for full participation. □

Since the maximum cost range is higher for the median rule, this proposition implies that the full participation Nash equilibria are more stable as they hold for more costs. Given that the votes in equilibrium are identical for both rules, like under coalition-proofness with extreme voting for an even number of voters with distinct peaks, we therefore conclude that full participation is a Nash equilibrium of the participation game for higher costs under the median rule as compared to the mean rule.

The constellations under which full participation is never an equilibrium if costs are strictly positive are identical for both rules. If there exists a voter i such that $x(q^*_{-i}) = x(q^*)$, i.e. abstention of this voter does not change the social outcome, then full participation is only a Nash equilibrium if $c = 0$. For an even number of voters, this situation is only possible if the median peaks are identical, as otherwise the voting game does not constitute a Nash equilibrium.

When comparing the full participation conditions for the mean and the median rule, we observe that in some constellations, there do not exist full participation equilibria for $c > 0$ under the mean rule but they do exist under the median rule for any equilibrium concept.

Observation 19. *If n is even, peaks are distinct, and $c > 0$, there exist constellations in which full participation is not a Nash equilibrium under the mean rule but it is a Nash equilibrium under the median rule.*

Consider the following example to show that the existence of full participation equilibria can differ with the voting rule.

Example 9. *Let $n = 6$ and the vector of peaks $p = (5, 12, 15, 40, 50, 70)$.*
Under full participation, the unique vector of Nash votes under the mean rule is $q^* = (0, 0, 0, 40, 100, 100)$ with a social outcome of $Mean(q^*) = 40$. Since one vote is not strategical but includes truth-telling (let this voter be i), the outcome remains identical if this voter abstains: $q^*_{-i} = (0, 0, 0, 100, 100)$ and $Mean(q^*_{-i}) = 40$. Hence, if $c > 0$, voter i abstains and full participation is never an equilibrium.
The Nash strategies with full participation under the median rule and coalition-proofness with extreme voting are different: $q^* = (0, 0, 0, 80, 100, 100)$, which also yields an outcome of $Med(q^*) = 40$, however abstention of any voter shifts the social outcome by 40. Therefore, full participation is an equilibrium under the median rule and coalition-proofness with extreme voting if $c \leq 40$. It can be shown that the cost range for coalition-proofness with minimal lying is also $c \leq 40$ and $c \leq 19$ for partial honesty with minimal lying.

We detect another interesting peculiarity when considering full participation under asymmetric peak distributions. Consider the peak distribution of all n individuals in figure 16.5a. In this distribution, the median peak is located closer to zero than to Q and there is a large gap between the median peak and the peaks located closer to Q. Under full participation, the Nash equilibrium for the median rule and partial honesty or coalition proofness with minimal lying is that all individuals vote according to their peaks, as displayed in figure 16.5b. With increasing costs, the voters that will have the first incentive to abstain are the individuals with peaks higher than the median peak, as their vote changes the outcome only slightly given the votes of the others. Starting from full participation, with increasing costs the median outcome shifts further to zero as voters close to Q will abstain. This is also the case under coalition-proofness with extreme voting.

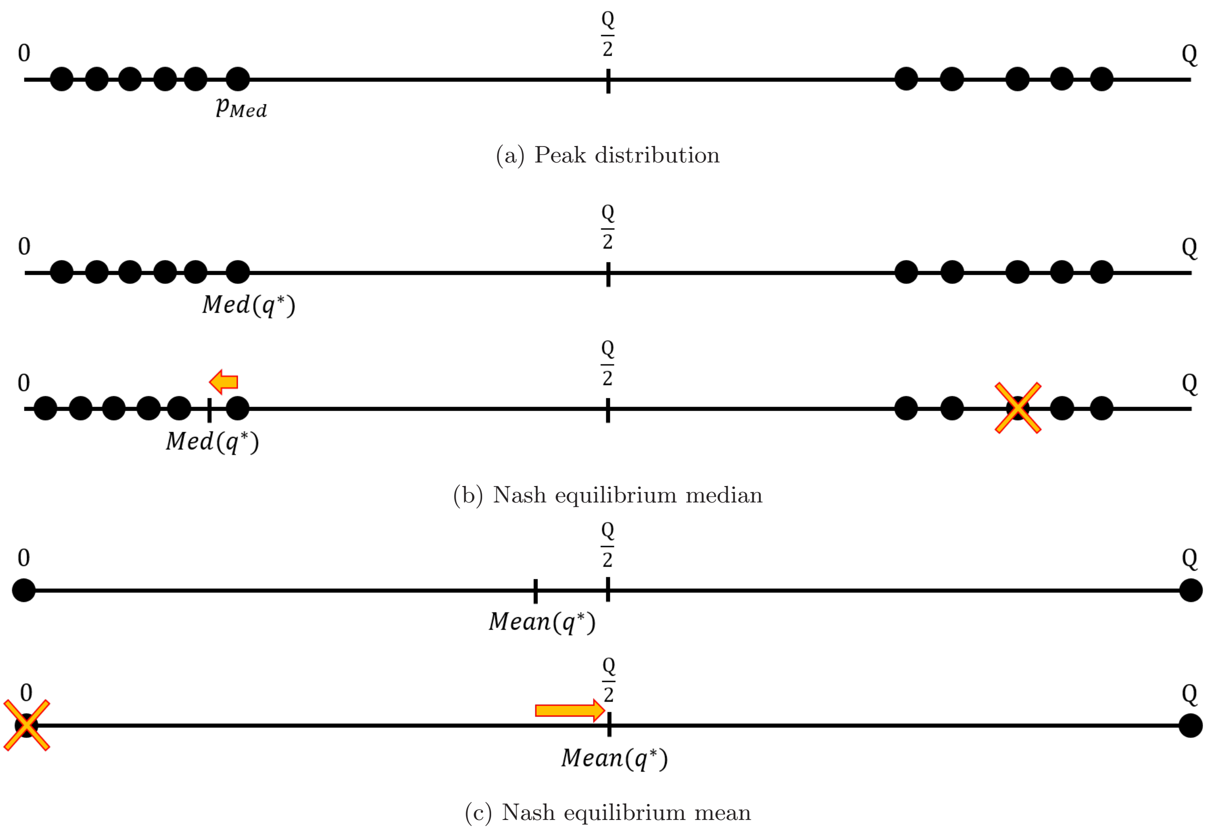


Figure 16.5: Full participation mean vs. median

Now consider the full participation Nash equilibrium for the same peak distribution under the mean rule in figure 16.5c. When participation costs increase, those individuals whose vote affects the outcome the least will abstain first. As we know from chapter 16.2.1 and case 1b), individuals that vote for zero will first abstain, which shifts the outcome closer to $\frac{Q}{2}$. This phenomenon under the mean rule is referred to as 'minority protection' by Renault and Trannoy (2005). We observe that for asymmetric peak distributions like the one in figure 16.5a, the full participation Nash equilibria yield differences in the outcome for the median and the mean rule. While the median outcome is located closer to the extreme of the budget set, the mean outcome is more moderate and takes the votes of the 'minority' with opponent peaks more into account. Moreover, increasing costs strengthen this asymmetry in the outcome, as under the median rule, the voters of the minority side will abstain first.

The median of an odd number of peaks minimizes the total distance sum to all peaks and according to our utility definition, the median peak is welfare optimal when voting is costless. However, the median outcome in this example yields very large distances for a minority of voters and with increasing costs, these voters are incentivized to abstain which shifts the outcome even more to the opposed extreme. Even if the median rule is welfare optimal, it is questionable whether the median rule is a socially desirable rule as in bipolar societies, minority voters are suppressed and the minority may differ from the majority by only one subject.[12]

16.3 Single Participation Nash Equilibria

After the examination of the cost ranges such that full participation is a Nash equilibrium, we now consider situations in which only one subject participates and give the restrictions on the cost range for these Nash equilibria with $k = 1$. Since abstention of every voter is defined to be the worst-case scenario for each subject, $k = 0$ is never a Nash equilibrium and it is never optimal for the single participant in a $k = 1$ scenario to abstain. When considering single participation Nash equilibria, we therefore only need to examine incentives for abstainers to participate in the vote. Single participation equilibria are further equal for the mean and the median rule, as the rules are identical for $k \leq 2$. We also know that for both rules, the participant in a single participation equilibrium votes for his or her true most preferred allocation, $q_i^* = p_i$. Let individual i be a single participant and a an abstainer. An abstainer has an incentive to participate if the alteration of the outcome given an optimal vote exceeds the cost: $c \leq |x(p_i) - x(p_i, q_a^*)|$. The maximal change of the outcome by q_a^* is $\frac{Q}{2}$, namely when p_i is positioned at either extreme of the budget set and the peak of the abstainer is positioned beyond half of the budget set. Given q_a^* is optimal, the alteration will be at most $d(p_i, p_a)$ and at the same time at most $\frac{1}{2}\max\{p_i, Q - p_i\}$, induced by voting for an extreme allocation. Therefore, as long as the costs of participation exceed either the distance between p_i and any other peak or the maximal alteration, single participation is a Nash equilibrium of the participation game.

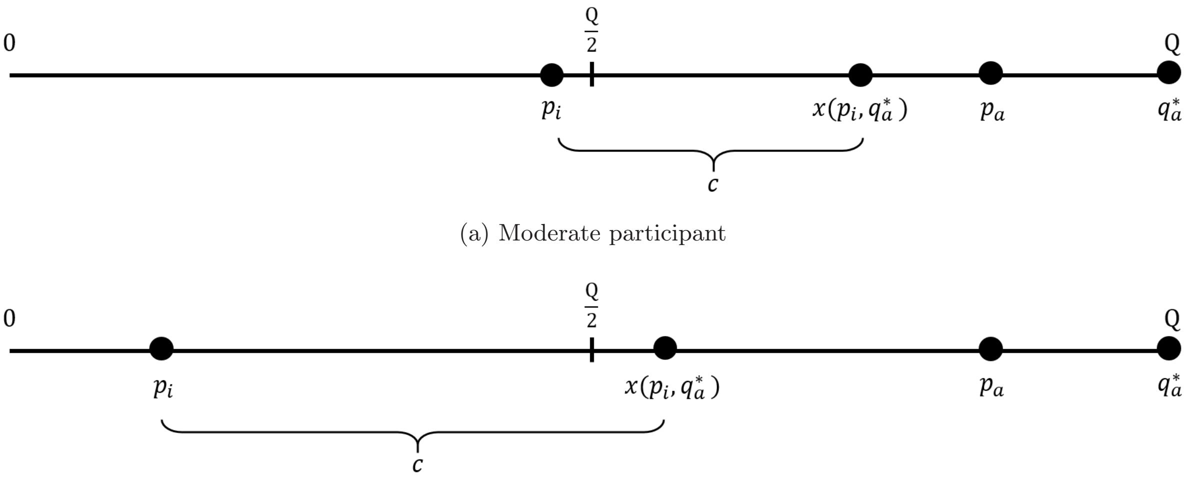
Observation 20. *Single participation is a Nash equilibrium if the single participant i votes for his or her true peak (i.e. the voting game constitutes a Nash equilibrium) and*

$$c \geq \frac{1}{2} \max_{q_a^*} |p_i - q_a^*| \quad \forall a \in I \setminus \{i\},$$

$$\text{where } q_a^* = \begin{cases} 0, & \text{if } p_a \leq \frac{p_i}{2} \\ 2p_a - p_i, & \text{if } \frac{p_i}{2} < p_a < \frac{Q+p_i}{2} \\ Q, & \text{if } p_a \geq \frac{Q+p_i}{2}. \end{cases}$$

[12] If the minority differs from the majority by more than one subject, the argumentation for the median rule is valid under the concept of coalition-proofness with extreme voting.

Note that single participation is always a Nash equilibrium if $c \geq \frac{Q}{2}$, which is the maximal change of the outcome by participation. Therefore, several single participation Nash equilibria exist with each individual being the single participant. The above observation also reveals that the cost range for single participation is small if the peak of the participant is close the optimal votes of the abstainers, meaning that all peaks are close. By contrast, single participation Nash equilibria are more stable, meaning that these equilibria hold for larger cost ranges, when the single participant has a moderate peak, i.e. one that is close to $\frac{Q}{2}$. Consider figure 16.6. In both situations there exists an abstainer a with peak p_a whose optimal vote given p_i is $q_a^* = Q$. Figure 16.6a reflects a situation in which the single participant i has a moderate peak, whereas p_i is (more) extreme in figure 16.6b. In both single participation situations, the optimal vote of the abstainer is supposed to have the maximal distance to p_i, such that single participation of i is a Nash equilibrium if $c \geq \frac{1}{2}|p_i - q_a^*|$ with $q_a^* = Q$. The minimal cost for the single participation Nash equilibrium is indicated by the change from the outcome under single participation, p_i, to the outcome if the abstainer participates. The figures show that the minimal cost is smaller for moderate participants, which means that the cost range is larger. There exist costs for which single participation of individual i is a Nash equilibrium when i is moderate, whereas it is no Nash equilibrium and the abstainer a has an incentive to participate when i is extreme. The single participation Nash equilibrium is thus more stable when the single participant has a moderate peak as the change of the social outcome induced by an additional participant is smaller.



(a) Moderate participant

(b) Extreme participant

Figure 16.6: Single participation

16.4 Nash Equilibria of the Participation Game for $1 < k < n$

The last chapters provided for all full participation and single participation Nash equilibria the conditions of the participation cost. Full participation yields the upper boundary for c or the maximal cost range under which no participant has an incentive to abstain. The cost range for the single participation equilibria gives the lowest cost for which the abstainers do not have an incentive to participate. For any number of participants $1 < k < n$, we combine both conditions since abstainers as well as participants need to choose their best response in equilibrium. While the conditions for participants remain identical to the full participation conditions, the single participation condition holds only for $k = 1$ and needs to be adapted according to the number of participants and the voting rule.

16.4.1 Nash Equilibria in the Mean Rule Participation Game

Consider a mean rule participation game with $1 < k < n$ in which all participants play a Nash strategy in the voting game with outcome $Mean(q^*)$. Any abstainer a that participates and plays a best response q_a^* changes the social outcome by $|Mean(q^*) - Mean(q_a^*, q^*)|$. In Nash equilibria for any number of participants $1 < k < n$ in which no abstainer has an incentive to participate, the lower boundary of the cost range has to be at least as high as the alteration of the social outcome induced by the abstainer whose optimal vote yields the maximum alteration: $d(Mean(q^*), Mean(q^*, q_a^*)) = \big|\frac{1}{k}\sum_{i=1}^k q_i^* - \frac{1}{k+1}(\sum_{i=1}^k q_i^* + q_a^*)\big|$, where $\sum_{i=1}^k q_i^*$ denotes the sum of all votes by k participants. Rearranging the terms provides the cost range for abstainers, which is identical for all cases.

Observation 21. *No abstainer a has an incentive to participate if*

$$c \geq \frac{1}{k+1} \max_{q_a^*} d(Mean(q^*), q_a^*),$$

$$\textit{where } q_a^* = \begin{cases} 0, & \textit{if } p_a \leq \frac{1}{k+1}\sum_{i=1}^k q_i^* \\ (k+1)p_a - \sum_{i=1}^k q_i^*, & \textit{if } \frac{1}{k+1}\sum_{i=1}^k q_i^* < p_a < \frac{1}{k+1}(\sum_{i=1}^k q_i^* + Q) \\ Q, & \textit{if } p_a \geq \frac{1}{k+1}(\sum_{i=1}^k q_i^* + Q). \end{cases}$$

The conditions for participants were defined for full participation in chapter 16.2.1 and we make the same case analysis to get the cost ranges for any $k < n$.

Case 1: $Mean(q^*) \in [0, \frac{Q}{2})$

a) $\exists i \in I : q_i^* \in (0,\ 2Mean(q^*))$

Again, in this case voter i obtains the peak as social outcome, otherwise the voting game would not constitute a Nash equilibrium: $p_i = Mean(q^*)$. The full participation condition may be applied on participants $k < n$ by the upper bound of the cost range $c \leq \frac{1}{k-1}\Big(d\big(q_i^*, Mean(q^*)\big)\Big)$. Combining this upper boundary of the cost for participants and the conditions for abstainers, i.e. the lower boundary of c, yields the cost range for the participation Nash equilibrium.

Observation 22. *Given a Nash equilibrium of the mean rule voting game with $1 < k < n$ under case 1a). The participation game constitutes a Nash equilibrium if*

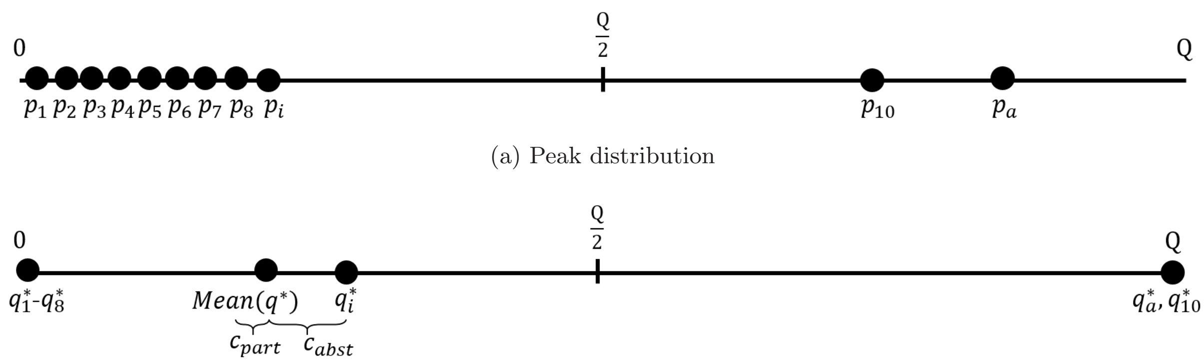
$$c \in \left[\frac{1}{k+1} \max_{q_a^*} d\big(Mean(q^*), q_a^*\big)\ ,\ \frac{1}{k-1}\Big(d\big(q_i^*, Mean(q^*)\big)\Big)\right],$$

where q_a^* is a Nash vote of any abstainer a.

We get the same cost condition for the analogous case 2a) where $Mean(q^*) \in (\frac{Q}{2}, Q]$ and there exists an individual i that obtains the most preferred allocation by voting for $q_i^* \in (2Mean(q^*) - Q, Q)$. Therefore, as soon as $Mean(q^*) \neq \frac{Q}{2}$ and the outcome corresponds to the peak of a participant i, the cost condition of observation 22 applies. Note that an outcome of $Mean(q^*) = 0$ or $Mean(q^*) = Q$ is only a Nash equilibrium outcome for $k > 1$ if the peaks of all participants are equal to zero or Q and thus are not distinct, see proposition 11.

An interesting insight that we get is that there exist peak distributions that do not yield a cost range, meaning that for specific constellations of participants and abstainers no Nash equilibrium exists under case 1a). Consider the following example.

Example 10. *Let $n > k = 10$, an individual i with peak $p_i = 12$ and the vector of Nash votes $q^* = (0,0,0,0,0,0,0,0,20,100)$, where $q_i^* = 20$, and thus $Mean(q^*) = 12$. Suppose that there exists an abstainer a with peak $p_a = 80$, such that the optimal vote given participation is $q_a^* = 100$. For better understanding, a possible peak distribution is provided in figure 16.7a and the corresponding Nash votes in figure 16.7b. The upper cost condition gives the maximal cost such that no participant has an incentive to abstain. The figure denotes this cost by c_{part}. In this example, $c_{part} \leq \frac{1}{k-1}\Big(d\big(q_i^*, Mean(q^*)\big)\Big) = \frac{8}{9}$. The lower cost condition in observation 22 provides the minimal cost such that no abstainer has an incentive to participate, c_{abst}. In this example, $c_{abst} \geq \frac{1}{11} \cdot |12 - 100| = 8$. We see that the condition for participants is $c_{part} \leq \frac{8}{9}$ and at the same time for the abstainers $c_{abst} \geq 8$. We do not find any cost that satisfies both conditions at the same time. Therefore in this example, there does not exist a Nash equilibrium in the participation game in which exactly the above-mentioned $k = 10$ individuals participate and all other $(n-k)$ individuals abstain.*



(b) Nash votes

Figure 16.7: Non-existence of Nash equilibria

b) $\nexists i \in I : q_i^* \in (0,\ 2Mean(q^*))$

The upper cost condition for $k < n$ we get from full participation is $c \leq \frac{1}{k-1} Mean(q^*)$.

Observation 23. *Given a Nash equilibrium of the mean rule voting game with $1 < k < n$ under case 1b). The participation game constitutes a Nash equilibrium if*

$$c \in \left[\frac{1}{k+1} \max_{q_a^*} d\big(Mean(q^*), q_a^*\big)\ ,\ \frac{1}{k-1} Mean(q^*)\right],$$

where q_a^* is a Nash vote of any abstainer a.

The conditions under this case also include peak distributions with specific constellations of participants and abstainers for which no Nash equilibrium of the participation game exists, as demonstrated in the following example.

Example 11. *Let $n > k = 50$ and the vector of Nash votes $q^* = (0, \ldots, 0, 100)$, and thus $Mean(q^*) = 2$. Suppose that there exists an abstainer a with peak $p_a = 80$, such that the optimal vote given participation is $q_a^* = 100$. The upper cost condition gives the maximal cost such that no participant has an incentive to abstain. In this example, $c \leq \frac{1}{k-1} Mean(q^*) = \frac{2}{49}$. The lower cost condition in observation 23 provides the minimal cost such that no abstainer has an incentive to participate. In this example, $c \geq \frac{1}{51} \cdot |2 - 100| = \frac{98}{51}$. We see that while for participants $c \leq \frac{2}{49}$ and at the same time for the abstainers $c \geq \frac{98}{51}$, we do not find any cost that satisfies both conditions at the same time. Therefore in this example, there does not exist a Nash equilibrium in the participation game in which exactly the above-mentioned $k = 50$ individuals participate and all other $(n - k)$ individuals abstain.*

Case 2: $Mean(q^*) \in (\frac{Q}{2}, Q]$

a) $\exists i \in I : q_i^* \in (2Mean(q^*) - Q,\ Q)$

Case 2a) is analogous to case 1a) and yields the same cost condition, therefore see observation 22 for the cost range.

b) $\nexists i \in I : q_i^* \in (2Mean(q^*) - Q,\ Q)$

Case 2b) is analogous to case 1b), which yields a symmetric (but different) cost condition to the one in observation 23:

Observation 24. *Consider a Nash equilibrium of the mean rule voting game with $1 < k < n$ under case 2b). The participation game constitutes a Nash equilibrium if*

$$c \in \left[\frac{1}{k+1} \max_{q_a^*} d\big(Mean(q^*), q_a^*\big)\ ,\ \frac{1}{k-1} \big(Q - Mean(q^*)\big) \right],$$

where q_a^* is a Nash vote of any abstainer a.

Case 3: $Mean(q^*) = \frac{Q}{2}$

a) k is even

Given that the number of voters is even and the outcome is the allocation of half of the budget, the same number of participants votes for zero and for Q. The upper cost condition is identical for all participants and is the same as the upper cost conditions in observation 23 and observation 24. The lower cost condition for abstainers is the same as in any of the previous cases, such that we get the following observation.

Observation 25. *Consider a Nash equilibrium of the mean rule voting game with $1 < k < n$ under case 3a). The participation game constitutes a Nash equilibrium if*

$$c \in \left[\frac{1}{k+1} \max_{q_a^*} d\Big(\frac{Q}{2}, q_a^*\Big)\ ,\ \frac{Q}{2(k-1)} \right],$$

where q_a^* is a Nash vote of any abstainer a.

b) k is odd and $\exists i \in I : q_i^* = \frac{Q}{2}$

Following the same logic as for full participation, if k is odd at the mean outcome is the allocation of half of the budget, there has to exist one voter i that votes for $q_i^* = \frac{Q}{2}$. The individual that votes for $q_i^* = \frac{Q}{2}$ will abstain for any strictly positive cost, as the outcome is not affected.

Observation 26. *There does not exist a $1 < k < n$ Nash equilibrium of the participation game under case 3b) if $c > 0$.*

From the conditions above we derive a complete classification of all Nash equilibria under the mean rule depending on participation costs. An important insight that is worth highlighting is the following. The larger the cost range for a specific constellation in the voting game, the more stable is the Nash equilibrium of the participation game, as the possibilities for Nash equilibria of a specific number of voters k are high. This means that the lower cost condition has to be low and the upper cost condition has to be high. From cases 1b) and 2b) we observe that the lower cost condition is minimal, when the most distant abstainer is close to the mean outcome. At the same time, the upper cost condition is high when the mean outcome is close to $\frac{Q}{2}$. These conditions imply that Nash equilibria are more stable if the abstainers have moderate peaks and the votes are balanced on both sides of the budget set. In other words, Nash equilibria are more likely to occur when individuals with extreme peaks participate and the quantities of the participants' peaks are not too different for the areas below and above $\frac{Q}{2}$. Figure 16.8 illustrates an example for a peak distribution and Nash votes that comply with the above conditions. Note that for k we get contrary conditions, as k is supposed to be large such that the lower cost condition yields small costs but at the same time k is supposed to be small such that the upper cost condition yields large costs. The stability of Nash equilibria in terms of the cost range is therefore not depended on the number of participants but on the peak distribution.

In cases 1a) and 2a), the additional assumption for the upper cost condition - besides that the outcome is close to $\frac{Q}{2}$ - is that the voter that obtains the most preferred allocation by participation has an optimal vote that is close to zero or Q, indicating an extreme peak.

If the outcome is exactly $\frac{Q}{2}$ as for case 3, we observe again that for the stability of the Nash equilibria, the abstainers must have moderate peaks when the number of participants is even. For an odd number of voters, a costly participation equilibrium never exists, what we already identified for full participation Nash equilibria.

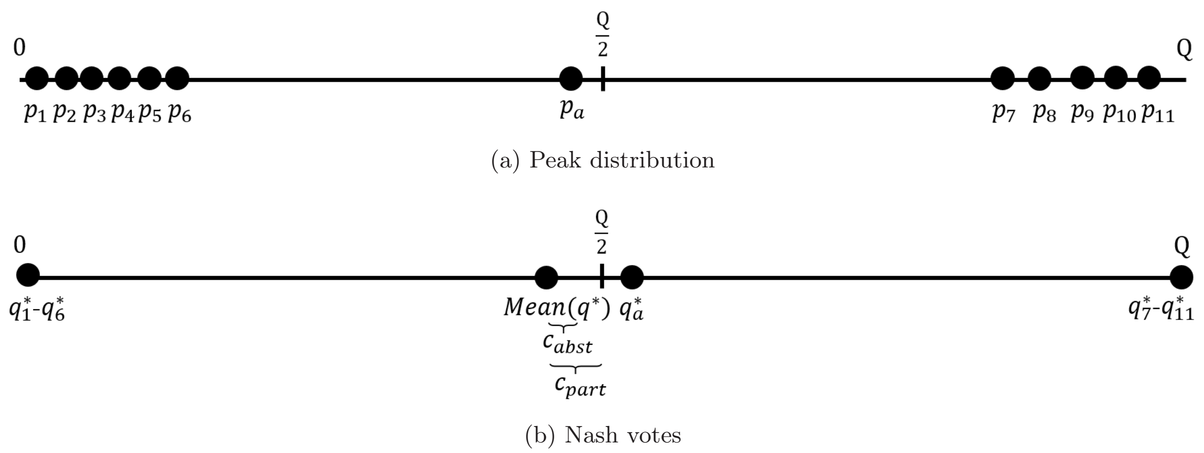


Figure 16.8: Stability of Nash equilibria

16.4.2 Nash Equilibria in the Median Rule Participation Game

In chapter 16.2.2, we developed the upper cost conditions such that no participant has an incentive to abstain and full participation is an equilibrium. Additionally, we know the conditions for Nash equilibria with a single participant. In this chapter, we evolve conditions for any number of participants $1 < k < n$ for the median rule and distinguish between an even and an odd number of voters. As before, we imply that peaks are distinct if not explicitly stated otherwise.

Even Number of Participants

From the full participation Nash equilibria, we know that if k is even, no participant has an incentive to abstain if $c \leq \frac{1}{2}(q^*_{Med+} - q^*_{Med-})$. Since under full participation we only need to consider participants, this condition holds for any even number of participants, regardless whether k is equal to n or smaller. The upper bound of the cost range for any even number of participants is therefore fixed.

The lower bound of the cost range, which emerges from the condition that no abstainer has an incentive to participate still needs to be specified for any even $k < n$. In order to obtain the full cost range, a case analysis is necessary. Consider a Nash equilibrium of the voting game with an even number of participants, where q^*_{Med-} and q^*_{Med+} are the median votes in any Nash equilibrium concept, a is an abstainer and A the set of abstainers. We distinguish between the positions of an abstainer's peak, whereas in this setting the declaration 'extreme' refers to the relative position with respect to the median votes.

<u>Case 1:</u> $\exists a \in A : p_a \in [0, q^*_{Med-}] \cup [q^*_{Med+}, Q]$ (*'extreme abstainer'*)

If there exists an abstainer a who has a peak to the left of q^*_{Med-} or to the right of q^*_{Med+}, participation of this individual given optimal voting changes the outcome by $Med(q^*) - q^*_{Med-}$ or $q^*_{Med+} - Med(q^*)$. Due to the calculation of the median outcome if k is even, both terms are identical and provide a lower cost range of $\frac{1}{2}(q^*_{Med+} - q^*_{Med-})$. Any other abstainer b with peak $p_b \in (q^*_{Med-}, q^*_{Med+})$ is the median voter if he or she participates and thus voting for the true peak is optimal. Individual b does not participate as long as the costs exceed the distance between the median outcome and the peak p_b. However, this distance is always smaller as compared to $\frac{1}{2}(q^*_{Med+} - q^*_{Med-})$, such that a unique lower bound for the cost range of all abstainers remains determined by any abstainer a.

We merge the conditions for participants and abstainers and get the following result.

Observation 27. *Given a Nash equilibrium of the median rule voting game with $k < n$ even under case 1. The participation game constitutes a Nash equilibrium if*

$$c = \frac{1}{2}(q^*_{Med+} - q^*_{Med-}).$$

The cost condition is very strict in cases where individuals with more extreme peaks than the median votes abstain. The unique cost implies that Nash equilibria are unlikely and case 1 occurs more often if the median votes are close as under partially honest equilibria with minimal lying. By contrast, the median votes are extreme under coalition-proof concepts.

<u>Case 2:</u> $\nexists a \in A : p_a \in [0, q^*_{Med-}] \cup [q^*_{Med+}, Q]$ (*'no extreme abstainer'*)

Given that no abstainer has a peak in the relevant intervals and $k < n$, all peaks of the abstainers have to be positioned within (q^*_{Med-}, q^*_{Med+}). Therefore, any abstainer a votes for the true peak p_a if he or she participates, as the vote will be the median vote and determine the social outcome. To make sure that no abstainer has an incentive to participate, we need to find the absent peak with maximum distance to the outcome. This distance yields the lower bound of the cost range, as the alteration of the social outcome would be maximal for this abstainer. Any abstainer a does not have an incentive to participate if $c \geq \max_{p_a} d\big(p_a, \frac{1}{2}(q^*_{Med-} + q^*_{Med+})\big)$.

Observation 28. *Given a Nash equilibrium of the median rule voting game with $k < n$ even under case 2. The participation game constitutes a Nash equilibrium if*

$$c \in \Big[\max_{p_a} d\Big(p_a, \frac{1}{2}(q^*_{Med-} + q^*_{Med+})\Big)\ ,\ \frac{1}{2}(q^*_{Med+} - q^*_{Med-})\Big].$$

While in case 1 the cost range is a unique value and only dependent on the position of the median voters, case 2 requires the knowledge on the detailed peak distribution. Since the alteration of the social outcome by an absent peak is smaller in case 2, the equilibrium for k even holds for smaller costs as well, as the incentives for any abstainer to participate are not as high. This implies that a cost range and thus a Nash equilibrium always exists for the specific constellation of participants and abstainers.

We conclude that the positions of the abstainers are crucial for the cost ranges. If all individuals with peaks more extreme than q^*_{Med-} and q^*_{Med+} participate, then the cost range is wider (i.e. Nash equilibria are more stable) than in constellations where some of these individuals abstain. The cost range is maximal if q^*_{Med-} and q^*_{Med+} are extreme, as under coalition-proof with extreme voting, and all peaks of the abstainers are close to the median (or identical to the median).

Odd Number of Participants

We already analyzed the full participation Nash equilibria for $k = n$ odd. For the determination of equilibria for any $1 < k < n$ where the number of participants is odd, we use the upper cost condition of full participation. If $c \leq \frac{1}{2} \min\big\{q^*_{Med} - q^*_{Med-}, q^*_{Med+} - q^*_{Med}, |q^*_{Med-} - 2q^*_{Med} + q^*_{Med+}|\big\}$, no participant has an incentive to abstain, where the last term of the minimum function is the absolute value of the difference between the first two terms. Therefore, we also already know that $c = 0$ if the first two terms are identical, which is the case if the median vote is positioned exactly at half of the distance between q^*_{Med-} and q^*_{Med+} or if q^*_{Med-} or q^*_{Med+} is identical to q^*_{Med}.

In order to determine the lower boundary of the cost range, we consider the lowest cost for which no abstainer has an incentive to participate. Therefore, it is necessary to make a case analysis that minds the position of the absent peaks. The cases describe situations in which there exist abstainers a, b at both 'extremes', any abstainer a at only one 'extreme' side, and constellation of peaks where no 'extreme' abstainer exists. The definition of 'extreme' is given by intervals and refers to the relative position with respect to the votes next to the median vote.

Case 1: $\exists a \in A : p_a \in [0, \frac{q^*_{Med-}+q^*_{Med}}{2}]$ and $\exists b \in A : p_b \in [\frac{q^*_{Med}+q^*_{Med+}}{2}, Q]$
(*'extreme abstainers on either side'*)

This case describes a situation in which there exist two abstainers a and b in the above intervals. For a and b, the optimal vote given participation is to allocate $q_a \leq q^*_{Med-}$ or $q_b \geq q^*_{Med+}$. Thus, the maximum alteration of the social outcome is either $\frac{1}{2}(q^*_{Med} - q^*_{Med-})$ or $\frac{1}{2}(q^*_{Med+} - q^*_{Med})$, whatever is greater. None of the abstainers a, b has an incentive to participate if $c \geq \frac{1}{2}\max\{q^*_{Med} - q^*_{Med-}, q^*_{Med+} - q^*_{Med}\}$. In a next step, we calculate the cost range such that an odd k is a Nash equilibrium for peaks under case 1.

Therefore, we need to find the union of the minimum function and the condition for abstainers (the maximum function) and solve for the following inequalities:

$$\frac{1}{2} max\{q^*_{Med} - q^*_{Med-}, q^*_{Med+} - q^*_{Med}\} \leq c$$

$$c \leq \frac{1}{2} \min\{q^*_{Med} - q^*_{Med-}, q^*_{Med+} - q^*_{Med}, |q^*_{Med-} - 2q^*_{Med} + q^*_{Med+}|\},$$

thus

$$max\{q^*_{Med} - q^*_{Med-}, q^*_{Med+} - q^*_{Med}\} \leq c \leq \min\{q^*_{Med} - q^*_{Med-}, q^*_{Med+} - q^*_{Med}\}.$$

We observe that the only cost range that satisfies both the upper and the lower boundary is $c = 0$ if there exist 'extreme' abstainers on either side of the interval boundaries. Given that $c = 0$, every Nash equilibrium under case 1 claims that $q_{Med-} = q_{Med} = q_{Med+}$, which is an equilibrium for an odd number of voters and the refinement concepts of chapter 15.3 only if the peaks are equal. Assuming that peaks are distinct, we derive the following.

Observation 29. *There does not exist a $1 < k < n$ Nash equilibrium of the participation game under case 1 if $c > 0$ and k is odd.*

Case 2: $\exists a \in A : p_a \in [0, \frac{q^*_{Med-}+q^*_{Med}}{2}]$ and $\nexists b \in A : p_b \in [\frac{q^*_{Med}+q^*_{Med+}}{2}, Q]$
(*'extreme abstainer on left side'*)

In this case, there exists an abstainer only at the left side but not on the right side of the budget interval. Therefore, we consider the cost range for the abstainer a and find that any abstainer a has no incentive to participate if $c \geq \frac{1}{2}(q^*_{Med} - q^*_{Med-})$.

Observation 30. *Given a Nash equilibrium of the median rule voting game with $1 < k < n$ odd under case 2. The participation game constitutes a Nash equilibrium if*

$$c \in \left[\frac{1}{2}(q^*_{Med} - q^*_{Med-})\ ,\ \frac{1}{2}\min\left\{q^*_{Med} - q^*_{Med-}, q^*_{Med+} - q^*_{Med}, \left|q^*_{Med-} - 2q^*_{Med} + q^*_{Med+}\right|\right\}\right].$$

The previous condition reveals that if $d(q_{Med-}, q_{Med}) \leq d(q_{Med}, q_{Med+})$, i.e. the vote left of the median is closer to the median as compared to the vote right of the median, then the cost condition yields a unique value: $c = \frac{1}{2}(q^*_{Med} - q^*_{Med-})$. The cost range is larger and thus the Nash equilibrium is more stable, if the opposite is true ($d(q_{Med-}, q_{Med}) > d(q_{Med}, q_{Med+})$).

Case 3: $\nexists a \in A : p_a \in [0, \frac{q^*_{Med-}+q^*_{Med}}{2}]$ and $\exists b \in A : p_b \in [\frac{q^*_{Med}+q^*_{Med+}}{2}, Q]$
(*'extreme abstainer on right side'*)

Case 3 describes the symmetric analog constellation of case 2: there exists an abstainer only at the right side but not on the left side of the budget interval. Analogously to case 2, we consider the cost range for abstainer b. Any abstainer b has no incentive to participate if $c \geq \frac{1}{2}(q^*_{Med+} - q^*_{Med})$.

Observation 31. *Given a Nash equilibrium of the median rule voting game with $1 < k < n$ odd under case 3. The participation game constitutes a Nash equilibrium if*

$$c \in \left[\frac{1}{2}(q^*_{Med+} - q^*_{Med})\ ,\ \frac{1}{2}\min\left\{q^*_{Med} - q^*_{Med-}, q^*_{Med+} - q^*_{Med}, \left|q^*_{Med-} - 2q^*_{Med} + q^*_{Med+}\right|\right\}\right].$$

We find for both cases 2 and 3 that Nash equilibria are possible with extreme abstainers on only one side; however, the cost range might be very low. The cost condition is a single value if the extreme abstainer's peak is positioned at the same side relative to the median as the next ranked median vote (q_{Med-} or q_{Med+}) with minimal distance to the median.

Case 4: $\nexists a \in A : p_a \in [0, \frac{q^*_{Med-}+q^*_{Med}}{2}] \cup [\frac{q^*_{Med}+q^*_{Med+}}{2}, Q]$ (*'no extreme abstainer'*)

The last case 4 describes a situation in which there do not exist extreme abstainers, i.e. all abstainers have peaks within $(\frac{q^*_{Med-}+q^*_{Med}}{2}, \frac{q^*_{Med}+q^*_{Med+}}{2})$. If any of the abstainers participates, strategic voting yields a social outcome that is identical to the peak. For a constellation with k odd to be a Nash equilibrium, even the abstainer who generates the greatest alteration of the outcome must have no incentive to participate. Therefore, the lower cost boundary is determined by the maximal distance of any absent peak and q^*_{Med}. No abstainer a has an incentive to participate if $c \geq \max_{p_a} d(p_a, q^*_{Med})$.

Observation 32. *Given a Nash equilibrium of the median rule voting game with $1 < k < n$ odd under case 4. The participation game constitutes a Nash equilibrium if*

$$c \in \left[\max_{p_a} d(p_a, q^*_{Med}), \frac{1}{2}\min\left\{q^*_{Med} - q^*_{Med-}, q^*_{Med+} - q^*_{Med}, \left|q^*_{Med-} - 2q^*_{Med} + q^*_{Med+}\right|\right\}\right].$$

The last observation reveals that the cost range for Nash equilibria without extreme abstainers is maximal if $q^*_{Med} = \frac{1}{3}Q$ or $q^*_{Med} = \frac{2}{3}Q$, the concept of coalition-proofness with extreme voting and if the peaks of the abstainers are close to the median outcome. Further, we observe that under other equilibrium concepts, there exist peak distributions in which no such cost range exist, as shown in the following example.

Example 12. *Let $n = 3$ and the vector of Nash votes $q^* = (25, 40, 50)$. Suppose that there exists exactly one abstainer a with peak $p_a = 35$. The upper cost condition gives the highest cost such that no participant has an incentive to abstain. In this example, $c \leq \frac{1}{2}\min\{40-25, 50-40, |25-2\cdot 40+50|\} = 2.5$.*
The lower cost condition such that the abstainer has no incentive to participate is given by
$c \geq \max_{p_a} d(p_a, q^*_{Med}) = |35-40| = 5$. We see that while for the participants $c \leq 2.5$, at the same time for the abstainer $c \geq 5$, such that we do not find any c that satisfies both conditions. Therefore in this example, there does not exist a Nash equilibrium in the participation game in which exactly the above-mentioned $k = 3$ individuals participate with the given votes (as possible under coalition-proofness with minimal lying or partial honesty with minimal lying) and a abstains.

17 Summary and Conclusion

In part II of this thesis, we address the participation game of a simple resource allocation problem when voting is costly. We begin with an analysis of the voting game and describe the differences of the mean and the median rule Nash equilibria. While the Nash equilibria in mean rule voting games are unique and efficient under the mean rule, there exist several Nash equilibria under the median rule voting game and inefficient equilibria occur. Thus, in a next step we refer to the equilibrium concepts of partial honesty and coalition-proofness in order to prepare for the median rule Nash equilibria of the participation game where uniqueness of the median votes is necessary. We show that these concepts come with helpful properties, as the outcome range can be restricted. However, further refinements of the equilibrium concepts are essential for the median voting game, as the uniqueness of the equilibrium is not ensured for all cases. Based on partial honesty and coalition-proofness, we develop three Nash equilibrium concepts that produce unique equilibria under the median rule voting game: partial honesty with minimal lying, coalition-proofness with minimal lying and coalition-proofness with extreme voting.

Our analysis of the participation game Nash equilibria begins with specific peak distributions. Therefore, we develop a computer program that calculates the Nash equilibria for the mean rule and for the median rule with refinements. We provide two examples of peak distributions for three individuals. While under the median rule cost ranges exist for which some peak distributions do not constitute a Nash equilibrium under any of the refinement concepts, we find that there always exist Nash equilibria of the mean rule participation game. The structure of the equilibria may also vary, with different combinations of participants and different cost ranges.

In a next step, we provide results that are more general for any number of subjects. We analyze full participation equilibria and find that for the mean rule the conditions on the cost range for full participation depend on the total number of individuals, the position of the peaks and therefore the optimal votes and the mean outcome. Since the optimal votes under the mean rule are unique in the one-dimensional voting game, we generalize the results and give classes of peak distributions for which the Nash equilibria hold. The full participation cost conditions for the median rule differ depending on whether the total number of subjects is even or odd. However for both cases, the conditions depend solely on the position of the median (vote) and of the votes ranked next to the median (vote). Both rules have in common that full participation with symmetric peak distributions is never a Nash equilibrium for strictly positive costs if the number of individuals is odd. We further compare both rules by calculating the maximum cost range for full participation and find that for at least four subjects, the maximum cost range is higher under the median rule. Moreover, we demonstrate that for strictly positive costs, peak distributions exist in which full participation is not a Nash equilibrium under the mean rule but under the median rule. We also show that the mean rule protects minorities, by inducing a Nash outcome that is more moderate as compared to the Nash outcome under the median rule. If minorities exist, these individuals are the first to abstain under median voting when participation costs increase, which shifts the median outcome even further from the peak.

The cost condition for single participation is identical for both rules and depends on the distance between the peak of the single participant (which corresponds to this subject's vote) and the optimal votes of all abstainers. For large enough costs $c \geq \frac{Q}{2}$, single participation is always an equilibrium under either rule.

The final step is to find the cost conditions for Nash equilibria that include any number of participants $1 < k < n$. While the upper cost conditions are identical to those for full participation (such that none of the participants has an incentive to abstain), the lower cost conditions also depend on the position of the abstainers' peaks. An interesting insight that we gain is that under the median rule Nash equilibria in which subjects with extreme peaks abstain are either only stable for very specific costs or there do not exist such Nash equilibria. A similar pattern occurs under the mean rule: Nash equilibria are more stable when the outcome is moderate and the abstainers have moderate peaks.

In summary, part II demonstrates the existence of multiple Nash equilibria in the participation game, frequently with different sets of participants, and distinguishes between the mean and the median rule. For either rule, we derive a complete classification of all Nash equilibria depending on participation costs.

Part III

A Field Experiment on Budget Allocation Voting

18 Motivation

The subject of this thesis is voting over resource allocation using the aggregation rules of the mean and the median. In the previous parts, we showed that without participation costs, truth-telling is a weakly dominant strategy under the median rule (Moulin, 1980), and there is a unique Nash equilibrium under the mean rule with complete information (Renault and Trannoy, 2005). By contrast, the game-theoretic analysis of both the median and the mean rule participation game becomes complex if voting is costly. In particular, we demonstrated in part II that the corresponding participation games in general have multiple equilibria, frequently with different sets of participants. Due to the existence of multiple equilibria and the complexity of the task to determine the equilibria, we hypothesize that the individual participation decision is in practice not driven by equilibrium considerations but that individuals base their decision on an approximate estimate of their impact on the social outcome.

We measure the *theoretical impact* of a vote on the outcome by the length of the option set, i.e. the distance between the interval boundaries, which are achieved by the minimal and maximal value of a subject's vote. Although the theoretical impact of a vote on the social outcome under the mean rule is small for large electorates, it is certain and always greater than zero. By contrast, the theoretical impact under the median rule has large variance. We theoretically analyze the individual impact on the social outcome for different preference distributions under the mean and the median rule. For some preference distributions, the *expected impact* on the social outcome is larger under the median rule than under the mean rule. This raises the question whether the difference in the individual impact has effects on the participation rates under different voting rules.

In a field experiment, we test whether, and how, voter turnout in a simultaneous participation game varies with the voting rule. To this end, we conduct a vote using either the mean or the median rule to determine the allocation of a donation on two public projects on the university campus. Our focus lies on the voter turnout under either rule, as well as on the role of impact and risk attitude. Subsequent to the vote, we implement a survey in order to elicit beliefs about the allocation result, about the participation rate and about the *real impact* of the individual's vote on the social outcome. Additionally, we ask for strategic voting behavior and elicit risk preferences via a standard lottery choice procedure.

The mean rule is in many ways 'simpler' than the median rule. In particular, the individual impact can be computed without knowledge of the votes of the other participants (it depends only on the *number* of the participants). By contrast, the impact under the median rule is highly sensitive on the distribution of the votes of all other participants. Moreover, with an odd number of participants, ex-post generically only *one* vote determines the outcome under the median rule. Since strategic voting is possible under the mean rule and truth-telling is a Nash strategy for one subject at most (assuming that the peaks are distinct), we anticipate the belief about the real impact to be high under the mean rule. The median rule, by contrast, is strategy-proof. As only the median vote (or the two median votes if the number of participants is even) is decisive, we expect that subjects underestimate their real impact of participation.

We therefore have the hypothesis that the belief about the impact is higher for mean rule participants as compared to median rule participants. Related to the previous point, while the impact under the mean rule is certain and always strictly positive, the impact is uncertain under the median rule and its expectation can be either smaller or larger than the corresponding impact under the mean rule with the same number of participants. We thus hypothesize a 'selection effect,' i.e. the participants under the mean rule are more risk averse as compared to the participants under the median rule. Together with standard assumptions about individual preferences, in particular about risk preferences, we obtain the following hypothesis about voter turnout: The actual number of participants is higher under the mean rule as compared to the median rule.

The following chapters of the final part III of this thesis are organized as follows. Chapter 19 defines the impact of participation. We distinguish between the theoretical impact, which is measured by the size of the option set, the maximal impact and minimal impact, as well as the expected impact of participation. Since the expected impact under most peak distributions varies for the voting rules, we give the linkage between participation decisions and risk preferences. In chapter 20, we describe the setup and design of the field experiment and provide the research questions that we seek to answer. Chapter 21 provides the experimental results, focusing on impact, beliefs, and participation before concluding in chapter 22.

19 Theoretical Impact and Risk

As already mentioned, an aspect that brings about a decision on i's participation is how i's vote affects the social outcome and therefore how restricted i is in voting strategically. We define the *theoretical impact* of participation as any possible alteration of the social outcome induced by participation. If individual i abstains from voting ($\vartheta_i = 0$) and the number of participants is $k > 0$, the social outcome is denoted by $x(q_{-i})$. As already stated in part II, no social outcome is defined for $k = 0$. With this definition, the theoretical impact of individual i's vote is independent of the most preferred allocation or the actual submitted vote.

Remember the definition of the option set in equation 4.1 in part I of this thesis:

$$\mathcal{OS}_i(q_{-i}) = \{\beta \in \mathcal{B} \mid \exists\, q_i \in \mathcal{B} : x(q_i, q_{-i}) = \beta\}\,.$$

The option set describes the set of all possible social outcomes for every $q_i \in \mathcal{B}$ given $x(q_{-i})$. The theoretical impact of a vote is defined as the size of the option set. Since we restrict the number of projects to $m = 2$ in the participation game, it is sufficient to provide the definition for the theoretical impact for only one project j. Under both, the mean and the median rule the minimum vote is zero and the maximum vote is Q. By construction, voting for an allocation higher than zero never decreases the mean nor the median outcome as compared to voting for zero. Similarly, voting for an allocation lower than Q never increases the social outcome as compared to voting for Q if everything else remains constant. The option set of i depends on the voting rule, the distribution of votes and the minimal and maximal value of i's vote. Using the mean and the median rule the option set is calculated as follows:

$$\mathcal{OS}_i = \big[x(0, q_{-i})\,,\, x(q_{-i}, Q)\big], \tag{19.1}$$

where the option set is identical regardless of whether considering $j = 1$ or $j = 2$.

We measure the theoretical impact imp_i by the length of the option set, i.e. the distance between the interval boundaries, which are achieved by the minimal and maximal value of i's vote, zero and Q:

$$imp_i = d\Big(x(0, q_{-i})\,,\, x(q_{-i}, Q)\Big). \tag{19.2}$$

Therefore, we ensure that the alteration of the social outcome is not limited to one direction towards the interval boundaries. The option set depends on $x(q_{-i})$ and implicitly the voting rule that determines the social outcome. In use of the mean rule, an important factor determining the extent of the theoretical impact is the number of voters k. Since every vote under the mean rule has the same weight of $\frac{1}{k}$, the higher k, the smaller is a voter's option set. Take the mean rule and let $Q = 100$, $Mean(q_{-i}) = 20$ and $k = 4$. Individual i's option set is $\mathcal{OS}_i = [15, 40]$, resulting in a theoretical impact of $imp_i = 25$. With an additional voter, the option set is $\mathcal{OS}_i = [16, 36]$, which reduces the theoretical impact to $imp_i = 20$. For $k = 100$, individual i's theoretical impact decreases to $imp_i = 1$.

If the median rule is used, the distribution of votes determines the option set, more precisely the positions of the votes that are ranked next to the median, i.e. q_{Med-} as the vote that is ranked one position left of the median and q_{Med+} as the vote ranked one position right of the median. Given the different calculations under the median rule, we additionally need the information on whether $Med(q_{-i})$

results from an even or odd number of voters. Consider the following example for the median rule[13]: $Q = 100$, $Med(q_{-i}) = 44$, $q_{Med-} = 18$, $q_{Med+} = 70$ and $(k-1)$ is an even number. Including individual i, there is an odd number of voters k and thus, the median outcome is the vote that is ranked at the middle position $[\frac{k+1}{2}]$. For any $q_i \leq 18$, the social outcome is $Med(q_i) = 18$ and for any $q_i \geq 70$, the social outcome is $Med(q_i) = 70$. Therefore, $imp_i = 52$. Take the same example but let $(k-1)$ be an odd number, which means there exists a median voter who votes for $q_{Med} = 44$. For any $q_i \leq 18$, the social outcome is $Med(q_i) = \frac{1}{2} \cdot \big(q_{Med-} + Med(q_{-i})\big) = 31$. For any $q_i \geq 70$, $Med(q_i) = \frac{1}{2} \cdot \big(Med(q_{-i}) + q_{Med+}\big) = 57$, resulting in $imp_i = 26$ or half the size of the theoretical impact in the even case.

Given that the participation decision depends on the impact of participation, it is plausible that a higher impact is accompanied by a higher chance of participation. We will show in the following chapters that the maximal impact, the expected impact and the minimal impact vary for both voting rules depending on the impact factors.

19.1 Maximal Impact of Participation

The participation decision and the impact of individual i are assumed positively correlated. Keeping everything else constant, our utility maximization problem implies that a higher impact on the social outcome goes along with an increase in participation. We calculate the *maximal impact* of participation under the mean and the median rule and define the maximal impact to be the maximal change of the social outcome induced by participation. Therefore, the maximal impact describes the longest option set independent of the exact positions of $x(q_{-i})$ or q_{-i}. In other words, the maximal impact is the maximum of all theoretical impact values for any vector of votes.

Under the mean rule, the maximal impact of participation is fixed for every distribution of votes by

$$imp_{max}^{Mean} = \frac{Q}{k}. \tag{19.3}$$

With an increase in the number of votes, the theoretical and the maximal impact of participation decreases. Let us assume that $Q = 100$, $k = 2$ and since we only allow for two public projects, $m \in \{1, 2\}$ holds for all assumptions, where we consider $j = 1$. Individual 1 votes for the allocation $q_1 = 0$. The option set of the second voter is $\mathcal{OS}_2 = [0, 50]$, resulting in a maximal impact of $imp_{max}^{Mean} = 50 = \frac{Q}{k}$. With an increase in the number of participants, the impact decreases, i.e. the highest impact possible is given for $k = 2$.[14]

By contrast, the maximal impact under the median rule is constant for every number of participants but dependent on whether k is even or odd. The maximal impact one can construct under the median rule is given by

$$imp_{max}^{Med} = \begin{cases} Q, & \text{if } k \text{ is odd} \\ \\ \frac{Q}{2}, & \text{if } k \text{ is even.} \end{cases} \tag{19.4}$$

Equation 19.4 states that for any number of voters, one might create a situation in which the maximal impact under the median rule is equal to Q if the total number of voters is odd and $\frac{Q}{2}$ if it is even. One example for constructing the maximal impact under the median rule first for an odd and then for an even number of participants is provided in the following. Let $Q = 100$, $k = 3$, $q_1 = 0$, and $q_2 = 100$. The

[13] Remember that we indicate all values for $j = 1$.
[14] Since we do not specifically define the outcome under $k = 0$, we exclude social outcomes of single participants.

option set of individual 3 is $\mathcal{OS}_3 = [0, 100]$, resulting in a maximal impact of $imp_{max}^{Med} = 100 = Q$. For $k = 2$ and $q_1 = 0$, the option set of individual 2 is $\mathcal{OS}_2 = [0, 50]$, which implies $imp_{max}^{Med} = 50 = \frac{Q}{2}$.

Comparing the mean and the median rule, we see that the maximal impact is equal for $k = 2$ and strictly larger under the median rule for $k \geq 3$. It is especially worth noticing that under the median rule even for a large number of participants, a theoretical impact of Q, which is the total size of the budget interval, is possible.

19.2 Minimal Impact of Participation

Along with the maximal impact of participation, we analyze the worst-case scenario. Since the utility function implies that the participation decision is driven by the impact of an additional vote, we examine all possible vectors of votes for which the option set of a voter has minimal length. We call the minimum of all theoretical impacts values *minimal impact*.

Given that the length of the option set under the mean rule is independent of the distribution of $(k-1)$ votes, the theoretical impact of individual i's vote is small for large k but always positive. Therefore, the minimal impact of all votes is also greater than zero:

$$imp_{min}^{Mean} > 0. \tag{19.5}$$

The minimal impact under the median rule is zero. For $k > 2$, situations may occur where $Med(q_{-i})$ is identical to the vote of at least two individuals other than i. In these cases, $imp_i = 0$ is possible. Consider $Q = 100$, $k = 5$, $q_1 = 10$ and $q_2 = q_3 = q_4 = 60$. Therefore, $Med(q_{-i}) = 60$ and the option set of an additional voter i is $\mathcal{OS}_i = [60, 60]$ for every $q_i \in \mathcal{B}$. Thus, the minimal impact under the median rule is given by

$$imp_{min}^{Med} = 0. \tag{19.6}$$

19.3 Expected Impact of Participation and Risk Preferences

Since the number of voters, the distribution of votes and therefore $x(q_{-i})$ is in general unknown when individuals make their participation decision, they have to form beliefs about the *expected impact* of their vote on the social outcome. The expected impact of participation also differs for our two voting rules considered. The theoretical impact under the mean rule is independent of the exact distribution of $(k-1)$ votes, it hinges only on $Mean(q_{-i})$ and k. Since the theoretical impact under the median rule depends on $Med(q_{-i})$ and the position of the next ranked votes, we need to distinguish between different distributions of votes that form the social outcome $Med(q_{-i})$. We consider symmetric distributions, which means that the social outcome $x(q_{-i})$ is $\frac{Q}{2}$ in expectation and the same number of votes is located to the left and to the right of $\frac{Q}{2}$.

If we consider a uniform distribution of $(k-1)$ votes, the expected impact under the median rule is identical to the one under the mean rule:

$$imp_{exp,unif} = \frac{Q}{k}. \tag{19.7}$$

The reason for the identical expected impact is that a uniform distribution of $(k-1)$ votes implies that in expectation the distance of two adjacent votes is exactly $\frac{Q}{k}$ and this distance is relevant for the theoretical impact under the median rule. Consider the following example: $Q = 100$, $(k-1) = 3$, and a uniform distribution of votes where $q_1 = 25$, $q_2 = 50$, and $q_3 = 75$. Under both rules, $x(q_{-i}) = 50$. The expected impact of an additional vote q_i under the mean rule is identical to the one under the median rule ($imp_{exp} = 25$) and is calculated by the length of the option set: $\mathcal{OS}_i = [37.5, 62.5]$. For the expected impact, we do not need to distinguish between even and odd numbers of voters as the number of participants is already included in the definition.

A distribution under which the expected impact differs for both rules is the normal distribution. Such as the uniform distribution, the normal distribution is symmetric, so that $x(q_{-i}) = \frac{Q}{2}$ in expectation under both rules. Where the expected impact under the mean rule remains $imp^{Mean}_{exp,norm} = \frac{Q}{k}$, the expected impact under the median rule is smaller. Under the normal distribution with an expected value of $Med(q_{-i}) = \frac{Q}{2}$, the position of the next ranked vote is with a higher probability closer to $Med(q_{-i})$ as compared to the uniform distribution. Therefore, the expected impact is more limited to a range close the social outcome without i.

The contrary holds for bimodal distributions of $(k-1)$ votes. The probability that the next ranked vote is close to $Med(q_{-i})$ is smaller and therefore the expected impact on the social outcome is higher under the median rule as compared to $imp^{Mean}_{exp,bimod} = \frac{Q}{k}$.

Figure 19.1 shows the expected probability density functions of votes for $j = 1$ and $Q = 100$ for a uniform, normal and bimodal distribution. While the mean and median outcomes in expectation are equal under symmetric vote distributions, the impact of an additional vote under the median rule depends on the exact distribution.

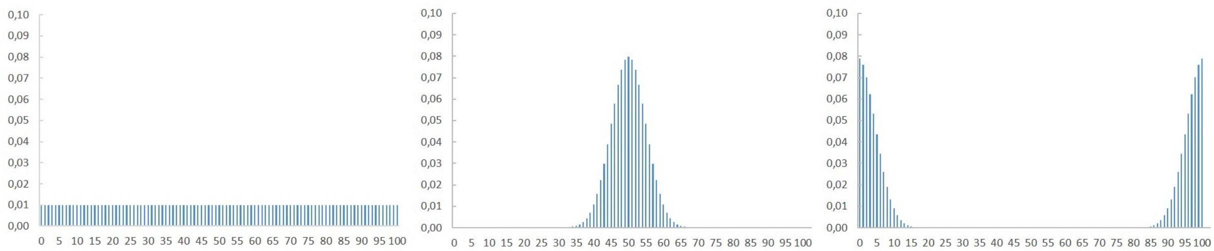
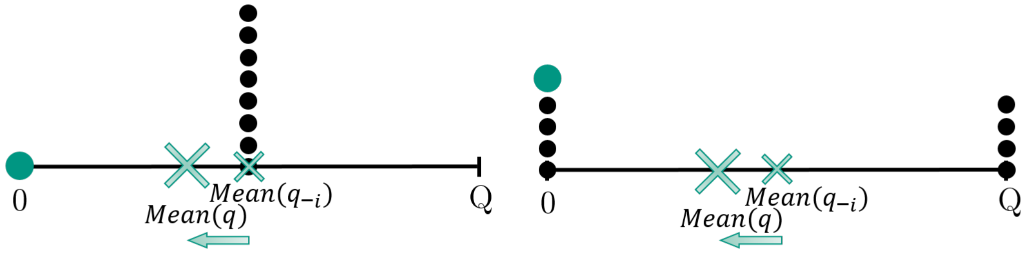


Figure 19.1: Probability density functions of votes for $j = 1$, $sd = 5$

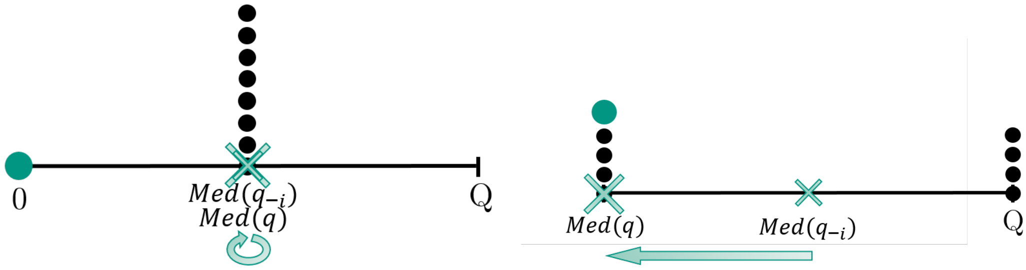
Figures 19.2 and 19.3 provide an example of the impact for $k = 9$ voters for two extreme allocations. Consider a set of eight voters, depicted by the black circles, that vote all for the allocation of $\frac{Q}{2}$ (figures 19.2a and 19.3a) or half of them votes for zero and the other half for Q (figures 19.2b and 19.3b). These examples are extreme cases of a normal distribution (with small θ) and of a bimodal distribution. Under both rules, the outcome is identical and denoted by $Mean(q_{-i})$ and $Med(q_{-i})$, respectively. The green circle displays the vote of participant i, which is $q_i = 0$. A vote $q_i = Q$ yields symmetric results. The mean outcome including voter i is denoted by $Mean(q)$ and it is observable in both figures 19.2a and 19.2b, that the expected impact of i's vote on the mean outcome is identical for both distribution of votes. The (expected) impact under the mean rule depends only on the number of participants and not on the distribution of votes. For the median rule, however, the impact can be zero as under the distribution of figure 19.3a, where participation of voter i has no effect on the median outcome. By contrast, the impact can be high for a distribution as in figure 19.3b, where the theoretical impact is $imp_i = Q$ under the median rule.

Where the impact under the mean rule is small for large k but certain, the impact under the median rule ranges from values below to above the impact under the mean rule and is determined by the votes positioned next to $Med(q_{-i})$.



(a) Mean rule: impact (1) (b) Mean rule: impact (2)

Figure 19.2: Impact of vote q_i under the mean rule



(a) Median rule: no impact (b) Median rule: high impact

Figure 19.3: Impact of vote q_i under the median rule

In general, individuals are not completely informed about the votes of the other subjects and therefore have to form beliefs about the distribution of votes and the number of participants in the simultaneous participation game. We will show in the following example that the variance of the impact for a fixed number of participants is higher under the median rule as compared to the mean rule.

Example 13.
Consider the following distributions of $(k-1)$ *votes:*

(1) $q = (20, 40, 60, 80)$
(2) $q = (40, 45, 55, 90)$
(3) $q = (10, 25, 75, 90)$
(4) $q = (10, 30, 30, 80)$.

The theoretical impact of an additional vote q_i *under the mean rule is* $imp_i^{Mean} = \frac{Q}{k} = 20$ *for all of the four distributions. For a fixed number of voters, the variance of the theoretical impact is zero under the mean rule, as the theoretical impact is not dependent on the distribution of votes.*

Now consider the theoretical impact under the median rule. The votes are uniformly distributed in the first distribution and the calculation of the theoretical impact results in $imp_i^{(1)} = \frac{Q}{k} = 20$ *as under the mean rule. The theoretical impact for the other distributions varies and is given by* $imp_i^{(2)} = 10$, $imp_i^{(3)} = 50$, *and* $imp_i^{(4)} = 0$. *The average theoretical impact is identical to the mean rule, however the variance is larger than zero as the impact range is* $[0, 50]$.

Contrary to the mean rule, the impact range under the median rule is determined by the distribution of votes and may be zero or anything greater than zero. Therefore, the variance of the theoretical impact is always greater than zero. Given the same average theoretical impact but uncertainty about the distribution of votes, individuals that are risk averse in theory will prefer the voting rule that comes with a lower variance of the impact, which is the mean rule. Since under this assumption, participation of risk averse subjects is lower under the median rule, a selection effect is expected, i.e. participants are on average more risk loving under median voting.

20 Experiment on Participation

In July 2017, a field experiment was conducted at the Karlsruhe Institute of Technology. We set up a vote over the allocation of a donation on two campus-projects using either the mean or the median rule. Our main focus areas are the participation rate and the role of impact under both voting rules. Subsequent to the vote, we implement a survey in order to elicit beliefs about the allocation result, about the participation rate and about the impact on the social outcome. Additionally, we ask for strategic voting behavior and elicit risk preferences.

20.1 General Setup and Design

The field experiment went on for one week in July 2017 at the Karlsruhe Institute of Technology. We invited 510 subjects to participate in a vote over the allocation of a donation on two campus-projects: the bike workshop (an assisted workshop, which provides tools, help and space for students that would like to repair their bike) and the campus-garden (the possibility to grow plants, cultivate and harvest fruits and vegetables). Both projects are on-campus and accessible free of charge for all students. We assess wide interest among the students for these projects and made sure that the implementation could be realized by the General Students' Committee (AStA) with every possible outcome.

We randomized subjects using the hroot subject pool (Bock et al., 2014) of the KD2Lab at the Karlsruhe Institute of Technology. Our randomly selected pool consists of 510 subjects, which were divided into 30 groups with 17 members. The subjects received an invitation to participate in the voting process via e-mail on July 11th, 2017. Each group votes over the allocation of 100 Euros on the two projects. We use a between-subjects design with the treatment variable voting rule: 255 subjects are randomly assessed to the mean rule and 255 to the median rule. In the e-mail, the subjects are informed that they belong to a group out of which 16 other persons are also invited to vote over the allocation of 100 Euros on the bike workshop and the campus-garden. We explain the voting rule (mean or median) and the conditions on a valid vote. Subjects are further informed that there exist other groups voting on the allocation of 100 Euros and that for an implementation of the group allocation at least one vote is necessary. Together with the remark that no individual payment will be made follows the link to submit the vote.

The vote for an allocation on the two projects affiliates with a short questionnaire. The questionnaire asks for information on the individual beliefs regarding the allocation result, the number of participants in each group and the degree of impact. We further query whether the voting rule was understood and if the participants revealed their true preferred allocation. Besides, we ask for demographic data and elicit risk preferences. In the appendix C.1, we provide the e-mail that subjects received in both treatments. Screenshots of the voting process and the questionnaire can also be found in the appendices C.2 and C.3.

20.2 Eliciting Impact Beliefs and Risk Preferences

When comparing the mean and the median rule for relatively small groups, we expect any difference in participation rates be driven by different beliefs about how participation affects the social outcome. We capture these beliefs in various ways.

We ask the participants directly to assess the impact of their vote. Therefore, we ask them to choose one out of six impact categories, ranging from 'my vote does not have any impact' to 'my vote is decisive for the outcome'. These categories are certainly open to individual interpretation but we find it a good way to elicit impact beliefs directly.

The indirect way of eliciting beliefs on the impact is to ask participants about their belief on the social outcome and the number of participants in their group. When a subject gives an estimation on the social outcome, we may calculate the distance between this allocation and the individual vote. However, it is not clear how to interpret the distance between the belief about the social outcome and the own vote. Consider a case in which both numbers are identical. It might mean that this voter believes his or her vote is decisive and thus the impact belief of his or her vote is maximal since the vote determines the outcome. In that sense the closer the outcome belief and the vote, the higher is a participant's belief on the impact this vote has on the outcome. Another and contrary interpretation of the equality of vote and outcome belief is that the subject believes the outcome would have been equal to the vote even without participation. In this case, the belief about the impact would be zero and minimal.

Another indirect way of eliciting impact beliefs is by asking for the belief about the number of participants in the group. For the mean rule, the impact decreases with the number of voters and also under the median rule the possibilities for effecting the outcome are higher if the number of voters is low.

As we expect that any difference in the impact beliefs affects subjects with the same class of risk preference, we elicit risk preferences. Charness et al. (2013) present and evaluate several methods for eliciting risk preferences in experiments. The authors argue that simple methods are especially suitable for capturing treatment effects, which is the aim of our study. We elicit risk preferences using the Eckel and Grossman (2002) method, where subjects have to choose one out of a series of lotteries. We adapt the values of the original gambles as suggested by Dave et al. (2010). Dave et al. (2010) let participants choose between six lotteries, each involving a high and a low payoff with equal probability of 50%. Table 20.1 presents the lottery choices. Lottery 1 represents a secure option as subjects receive a payoff of 28$ for sure.[15] Expected payoffs increase together with the risk level from lottery 1 to lottery 5. Lottery 5 represents risk-neutrality as it comes with the highest expected return combined with a lower standard deviation as compared to lottery 6, which implies risk-seeking behavior.

	Low Payoff	**High Payoff**	**Exp. Return**	**S.D.**
Lottery 1	28	28	28	0
Lottery 2	24	36	30	6
Lottery 3	20	44	32	12
Lottery 4	16	52	34	18
Lottery 5	12	60	36	24
Lottery 6	2	70	36	34

Table 20.1: Lottery choices from Dave et al. (2010)

Reynaud and Couture (2012) perform a field experiment and compare the Eckel and Grossman method to the elicitation method of Holt and Laury (2002). Therefore, they perform non-incentivized field experiment and find that while there exist differences among elicitation methods, the risk attitudes are significantly correlated across the different lottery tasks. The main advantage of the Eckel and Grossman method from our point of view is that by choosing only one lottery, we exclude inconsistent decisions like subjects switching lotteries in the Holt and Laury method. Moreover, the Eckel and Grossman task is

[15] We adapt the currency from $ to Euros in our experiment but stick to the same values.

simpler and the explanation can be done faster. One should keep in mind that subjects participated in the vote on a voluntary basis and we wanted to keep the questionnaire as short as possible.

20.3 Research Questions

Our theoretical analysis shows that there are many factors driving the impact on the social outcome. Nevertheless, we expect that the belief of subjects about their impact is higher under the mean rule.

Hypothesis (H1). ***The belief about the real impact is higher under the mean rule as compared to the median rule.***

Since strategic voting is possible under the mean rule and for at least $(k-1)$ subjects truth-telling is not a Nash strategy (assuming that the peaks are distinct), we expect that subjects believe they have a high impact under the mean rule. The median rule, by contrast, is strategy-proof. Since only the median vote (or the two median votes if the number of participants is even) is decisive, we expect that subjects underestimate their impact of participation.

We showed that the impact under the mean rule is small for a high number of participants; however, it is certain for any distribution of votes. Whereas the median outcome may not be affected by participation as soon as two or more votes are equal to the median outcome. The other extreme is a high impact in cases where votes are clustered around extreme allocations. Given an equal expected impact but a higher variance of impact under the median rule, we suppose that risk averse subjects prefer mean voting and therefore are less likely to participate in median voting. Since we expect that risk averse subjects do not participate under median voting, we hypothesize a selection effect.

Hypothesis (H2). ***Participants are more risk averse in mean voting as compared to median voting.***

Based on H1 and H2, together with standard assumptions about individual preferences, in particular about risk aversion (e.g. Holt and Laury, 2002), we expect that the voter turnout under the mean rule is higher as compared to the median rule.

Hypothesis (H3). ***The actual number of participants is higher under the mean rule as compared to the median rule.***

Our last hypothesis does not address to voter turnout but to the voting game. In the analysis about the Nash equilibria of the voting game, we already explained that under the mean rule at most one individual votes for an allocation that is not extreme. All other individuals vote for zero or Q. Contrary under the median rule truth-telling is a weakly dominant strategy for most of the voters. Regarding only the participants, we therefore that the votes are more extreme under the mean rule and thus hypothesize a greater variance of votes.

Hypothesis (H4). ***The variance of the participants' votes is higher under the mean rule as compared to the median rule.***

We design a field experiment that seeks to test all four hypotheses. H3 and H4 are tested using direct observations of the voter turnout and the votes, i.e. our field experiment measures participation or abstention of every subject that is invited to the vote and additionally the vote under participation. The data used for H1 and H2 are collected by a subsequent survey filled out by participants.

21 Results

Our main points of interest are the number of participants and the role of impact and risk preferences for participation. The major research question is whether there is a difference in the participation rates under both voting rules and if so, whether our data on impact beliefs and risk preferences is able to explain it. Regarding the voting game, we are interested in the voting behavior of participants under the mean and the median rule and whether subjects vote optimally regarding their beliefs.

21.1 Impact

As already explained in chapter 19, the theoretical impact under the median rule is highly sensitive on the distribution of votes. For the analysis of the experimental results, we calculate the *real impact* of a participant's vote by the difference of the actual social outcome and the hypothetical outcome without this participant. The individual real impact based on the actual voter turnout and the actual distribution of votes for each group is examined in the following chapter.

21.1.1 Real Impact

As we have the data on the vote of each participant and the social outcome of each group, we are able to calculate $x(q_{-i})$ and the real impact of q_i. The real impact of a vote is the absolute deviation between $x(q_{-i})$ and $x(q)$. For simplicity, most allocations are indicated only for the bike workshop. Given that $k > 1$ for each group, we do not have to deal with social outcomes $x(q_{-i})$ for $k = 0$. The real impact range over all individuals, i.e. the interval from the lowest to the highest real impact, is $[0, 30]$ under the median rule, the range under the mean rule is $[0.08, 18.33]$. The box plot in figure 21.1 represents the real impact for both voting rules and strengthens our theoretical implications: while the impact under the mean rule is small but certain (the smallest observation for the real impact is 0.08 Euros, not zero), the impact under the median rule may either be zero or high and has larger variance.[16] The average individual real impact is 4.48 Euros under the mean rule and the impact values are significantly lower as compared to the median rule with an average individual real impact of 7.90 Euros (Mann-Whitney U test, p=0.018).

Since the number of participants and the votes may differ among the voting rules, we control for these variations and consider additionally all 140 votes, i.e. the total distribution of votes under both voting rules. The mean of all 140 votes is an allocation of 65.06 Euros for the bike workshop. A hypothetical calculation of the real impact for each vote yields a maximal real impact value of 0.46 Euros under the mean rule and results from a vote for 0 Euros. If we consider only the 74 votes under the mean rule, the maximal real impact would be 0.87 Euros

The median of all 140 votes is 70 Euros and the maximal real impact would be hypothetically 0.00 Euros under the median rule. The occurrence of no real impact at all under the median rule is driven by the fact that 18 votes are identical to the median outcome and allocate 70 Euros on the bike workshop. The same is true when considering solely the 66 votes under the median rule.

[16] Note that the real impact is not identical to imp_i, since there are votes that did not change the social outcome but a change might have been possible. Thus, a real impact of zero is possible also under the mean rule.

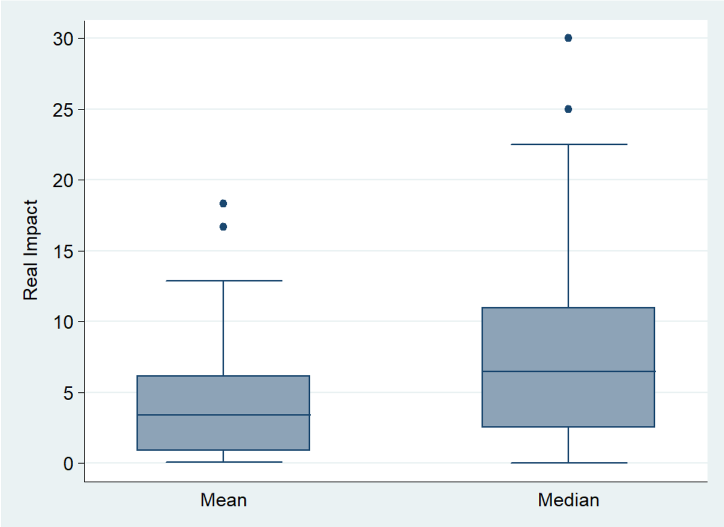


Figure 21.1: Real impact

Obviously, the real impact may be calculated only ex post but we observed significant differences among the voting rules. It is therefore interesting to regard whether these differences are also reflected in the participants' beliefs about the impact of their vote.

21.1.2 Belief about the Impact

We elicit the belief about the impact by letting the participants evaluate their impact in six categories. The categories range from 'my vote has no impact on the social outcome' to 'my vote is decisive'. The share of participants that chose the categories is displayed in figure 21.2.

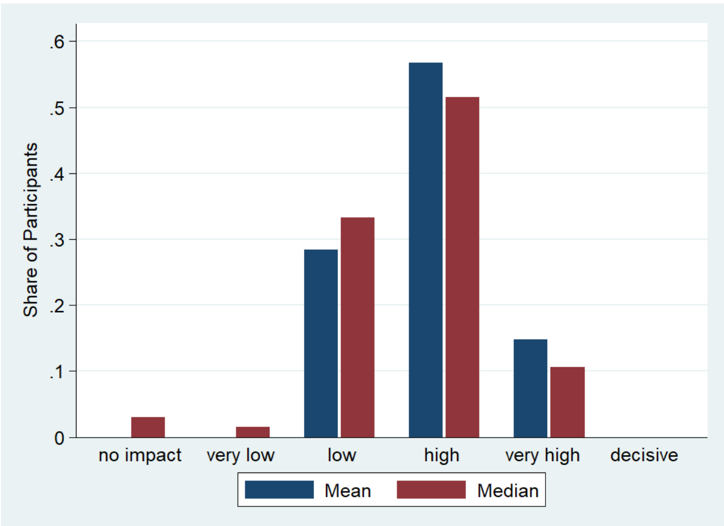


Figure 21.2: Belief about impact

As already pointed out, the real impact is significantly different for both voting rules. Nevertheless, this difference does not occur for the participants' beliefs about their impact. Figure 21.2 shows a slightly higher belief about the impact for mean rule participants, but the difference is not statistically significant, which implies our data does not support H1 (Mann-Whitney U test, p=0.159). Another interesting finding is that among those participants that have a high impact belief we do not observe more extreme votes.

Figure 21.3 illustrates the belief about the impact by voting rule together with the real impact for each participant. The bigger the bubbles, the more participants share the same belief and real impact. In order to assess the impact belief, we classify the real impact values into six categories, indicated by the blue frames in figure 21.3. Since the highest value of the real impact is 30, we classify the belief about the impact and the real impact in the following way. 'No impact' corresponds to an impact of 0 Euros, 'very low' is classified as an alteration of the outcome by more than 0 and up to 5 Euros and so on. The highest category is the belief that the vote is 'decisive', which we classify as a social outcome that varies from the outcome without this participant by more than 20 Euros. Participants with a belief that corresponds to the categories of the real impact are situated on or within the blue areas and make 12.16% of mean and 18.18% of median rule participants. Very few subjects underestimate their real impact under the mean rule (6.76%) and a higher share under the median rule (18.18%), displayed by the bubbles to the top-left of the blue sections. An overestimation occurs by 81.08% under mean and 63.64% under median rule, which is represented by the area below the blue areas. We find that the differences in proportions of over- and underestimation are significant (two-sample test of proportions, p=0.019 for underestimation, p=0.010 for overestimation). As already mentioned, the differences in the 'correct' estimation (according to our classification) of the real impact is not driven by the differences in the beliefs. It rather results from the variations in the real impact. We are aware that it is arguable whether our classification of the impact categories is reasonable and that impact beliefs differ for each individual. We cannot know if an individual classifies a vote as decisive but at the same time believes that the change of the social outcome is only 1 Euro. Obviously, the number of correct impact estimations may increase or decrease if we shift the blue areas. Nevertheless, the classification is identical for both voting rules and the large fraction of participants that believe to have a high or very high impact but whose real impact is below 5 Euros obtains validity without any classification.

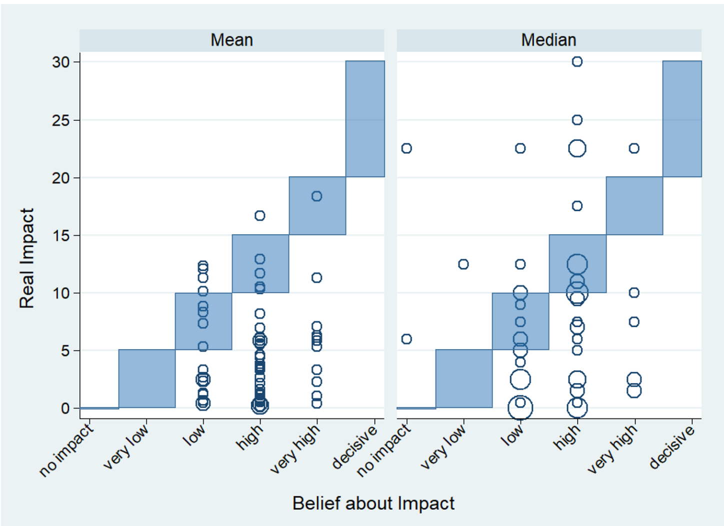


Figure 21.3: Belief about impact vs. real impact

We elicit beliefs about the impact also indirectly by asking the participants about their belief about the allocation result. As stated before, it is not clear how to interpret the distance between the vote and the belief about the result. Figure 21.4 plots the two different ways of eliciting the beliefs. The x-axis shows the categorization of the impact belief, whereas the y-axis depicts the distance between the vote and the belief about the result. A distance of zero indicates that a participant voted for the exact same allocation as the belief about the outcome. We additionally draw the regression line and find that there is no correlation between the two measures of impact belief. It is quite interesting that some subjects believe to have a very high impact when they are asked directly and at the same time indicate that they believe the group result will differ from their vote by more than 30 Euros.

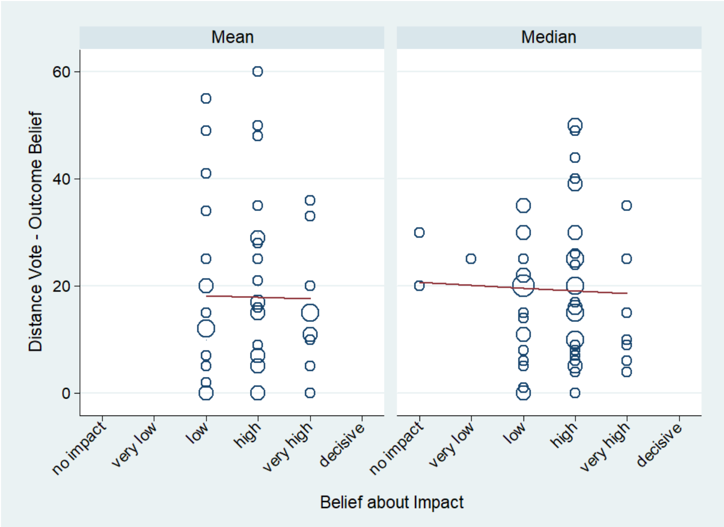


Figure 21.4: Measures of impact belief

We also examine gender differences in impact beliefs by performing a Mann-Whitney U test on the impact belief depending on gender.[17] We are able to reject the H_0 hypothesis that male and female participants have an equal belief about their impact with p=0.034 in support of the H_1 that the impact belief of men is on average higher as compared to women.

Since female participants have on average a lower belief about their impact, we test in a second step whether women also overestimate their impact less frequently. Out of the 102 participants that overestimate their impact, 63 are male and 37 are female. Therefore, the share of male participants that overestimate their impact is 68.48% (63 out of 92), where the share for women is 80.43% (37 out of 46). However, we do not find a significant correlation between overestimation and gender (chi-squared test, p=0.138).

When considering the votes separately for male and female participants, we find some interesting differences. While 24 male participants, i.e. 26.09% of all men, vote for an extreme allocation of either 0, 1, 99 or 100 Euros, the share of female participants that vote extreme is only 8.70% or four women. Extreme votes occur under both rules, as will be referred to later. Building on the differences in the voting behavior, we perform another Mann-Whitney U test to compare the real impact dependent on gender and find indeed that the real impact of male participants is on average significantly higher compared to the real impact of women (p=0.026).

21.1.3 Belief about Participation

When considering the high shares of overestimation of the real impact, the question arises whether the high beliefs are driven by an underestimation of the number of participants. The impact under the mean rule is negatively correlated to the number of participants and under the median rule, the expected impact is higher when there are less participants as the distance between the votes is increased in expectation. Similar in both treatments, subjects believe on average that the number of participants per group is 8.8 (no difference, two-sample t test, p=0.966), which means that the share of participants that overestimate the number of participants compared to the real number of participants is 84.85% in the median, where

[17] Two participants did not state their sex, therefore we exclude these observations.

the group average of the real participation rate is 4.4 and 74.32% in the mean groups, where the average amounts to 4.9.[18] We do not find gender differences in the belief about the number of participants (two-sample t test, p=0.394).

Since the number of participants is directly correlated to the impact, we use the belief about the voter turnout to get a second classification of the belief about the impact. If one measures the belief about impact indirectly by the belief about the number of participants, most of the participants underestimate this impact: as the belief about the number of participants is higher than the real number of participants, the indirect impact belief is lower as the indirect real impact. Therefore, the high beliefs about the impact measured by the six categories are even higher: participants overestimate their real impact directly and at the same time overestimate the number of participants. Figure 21.5 gives an insight of the indicated beliefs about the impact in combination with the beliefs about the number of the other participants ($k-1$) per group. The impact under the median rule depends on the distribution of votes and not only on the number of participants. Surprisingly, when drawing a regression line on the graphs, it has a positive slope for the mean treatment, where the correlation between the belief about the number of participants and the belief about the impact is 0.188 (Spearman correlation, p=0.109). As shown in the theoretical analysis in chapter 19, indeed the impact of participation under the mean rule is negatively correlated with the number of participants, which would predict a negative slope. We therefore find that subjects are inconsistent in their beliefs under the mean rule. The correlation under the median rule is slightly negative (-0.072) such that we do not find the same inconsistency in beliefs for the median rule (Spearman correlation, p=0.566). The difference in the correlations between the voting rules is not statistically significant (two-sample Fisher's z test, p=0.209).

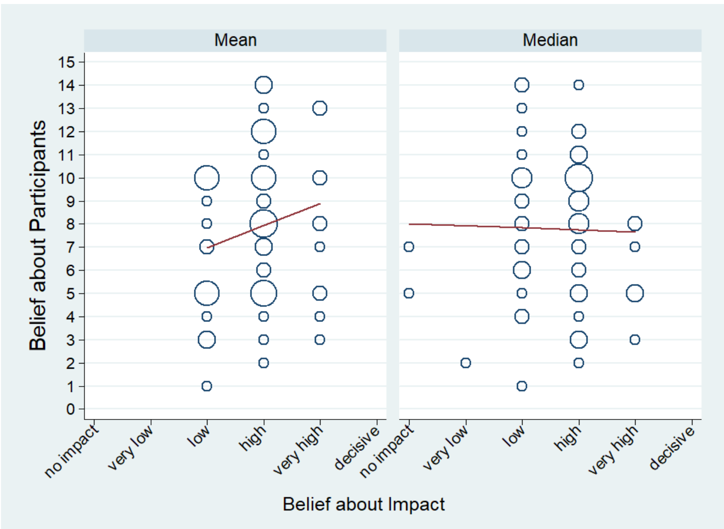


Figure 21.5: Belief about impact vs. belief about participants

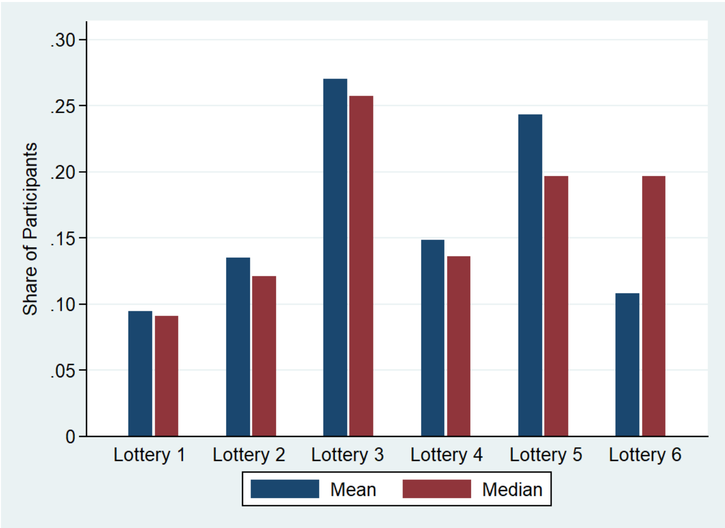
21.2 Risk Preferences

Since we only have the survey data of the subjects who completed the form, the following analysis will refer to these subjects solely. We elicit risk preferences using the Eckel and Grossman (2002) method. Following Dave et al. (2010), the participants were asked to choose one lottery out of six. Each lottery yields a low and a high payoff with the probability of 50%. The lotteries have different expected payoffs

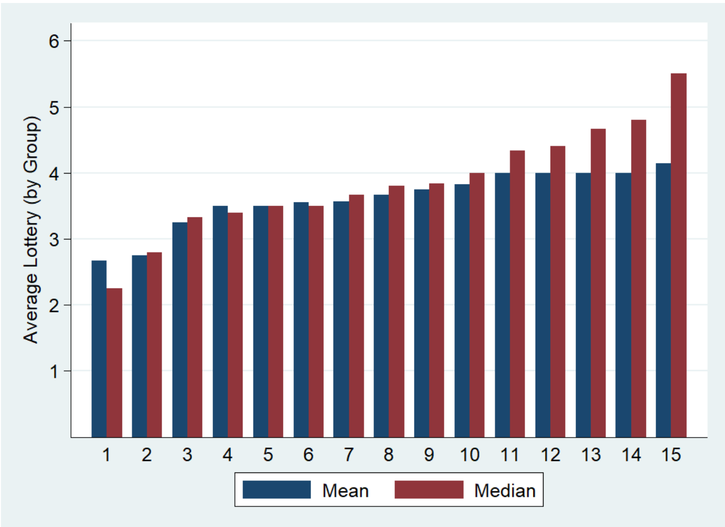
[18] In the field experiment, we ask subjects on their belief about the number of other participants, i.e. the belief about $k-1$. To avoid confusion, we add the one participant that answered the question and report the belief about k.

and variances, such that risk neutral participants choose the lottery with the highest expected payoff and the lowest standard deviation. In our experiment, lottery 5 indicates risk neutrality. Lottery 6 gives the same expected payoff as compared to 5 but with a higher standard deviation. Lottery 6 should therefore be chosen by risk-seeking individuals. The lotteries 1 to 4 have the lowest standard deviations combined with the lowest expected return. Therefore, these are chosen by risk-averse participants, with the highest degree of risk aversion for lottery 1. The detailed values of the lottery payoffs can be found in table 20.1 in chapter 20.2.

Figure 21.6a displays for each of the six lotteries the share of participants by the voting rule. The average lottery number per individual is 3.64 under the mean rule and slightly higher under the median rule, where the average lottery is 3.82. Both values indicate risk aversion on the individual level, since the average is below lottery 4, which is among the risk averse lotteries the one with the highest expected return and the highest standard deviation. On the individual level, we do not find a significant difference between the two rules (two-sample t test, p=0.485). Thus, the individual data does not support hypothesis H2.



(a) Shares by lottery



(b) Average lottery, by group (increasing)

Figure 21.6: Risk preferences

We label the six lotteries by corresponding values from 1 to 6 and run a regression of the chosen lottery on a set of independent variables. The results are displayed in table 21.1 and again we see that the voting rule (the relevant coefficient is 'rulemean', which takes the value 1 for the mean rule and zero for the median rule) is negative but not significant. Interestingly, we find a highly significant and positive coefficient for the dummy variable 'gendermale', which takes the value 1 if the participant is male and zero for female participants.

In order to analyze gender differences further, we perform a two-sample t test for the individual risk preferences indicated by male and female participants. We are able to reject the H_0 hypothesis that there is no difference in the risk preference between men and women (independent of the voting rule) at a p-value below 1% and find support for the H_1: the average risk preference level is lower under female compared to male participants (on average 2.98 vs. 4.09). Since we find a gender difference regarding the risk preferences, our next question is whether a different share of female participants occurs under the two voting rules. As indicated in chapter 19, risk averse subjects theoretically prefer the certain impact under the mean rule and the higher risk aversion of women would therefore predict higher female participation rates in mean voting as compared to the median rule. The overall share of female participants is 33.33% and splits up into 35.14% under the mean rule and 31.25% under the median rule. Our overall subject pool (participants and non-participants) consists of 91 women under the mean and 92 under the median rule. The adjusted share of female participants out of all female subjects is therefore 28.57% for the mean rule and 21.74% for the median, which makes an even higher difference. However, we do not find that the difference in participation rates by gender are statistically significant (chi-squared test, p=0.381).

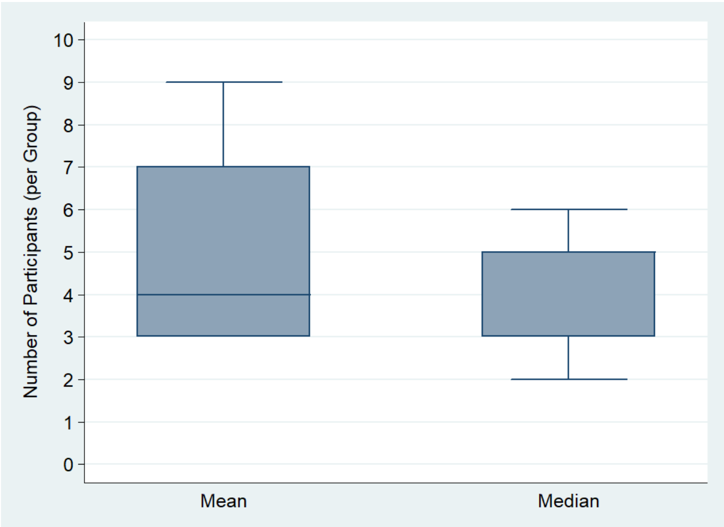
VARIABLES	lottery
rulemean	-0.0985
	(0.256)
extremevote	0.0138
	(0.336)
partbelief	0.0135
	(0.0403)
man	0.574
	(0.494)
impactbelief	-0.127
	(0.176)
gendermale	1.113***
	(0.275)
Constant	3.230***
	(0.580)
Observations	138
R-squared	0.130

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

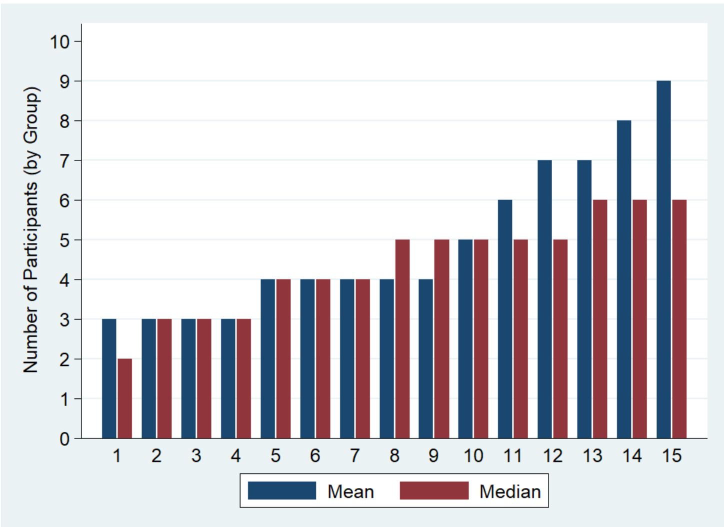
Table 21.1: Regression results risk preferences

21.3 Voter Turnout

Out of the $n = 510$ subjects that were invited, $k = 140$ participated and completed the entire voting process. Further 21 subjects visited the survey platform but did not complete the form. The 140 participants split up into the two treatments: 74 subjects participated under the mean rule and 66 under the median rule. The overall participation rate is rather high with 29.0% and 25.9%, which is 4.9 participants on average per group under the mean and 4.4 under the median rule. There were at least two participants per group and a maximum number of nine participants, which occurred under the mean rule. Figure 21.7a represents a boxplot containing the number of participants per group. The median number of participants per group is four under the mean and five under the median rule. The detailed number of participants by voting rule and groups is displayed in figure 21.7b, ordered by increasing number of participants. As one can see in the two figures, the spread of participation rates is higher under the mean rule. In six of the ordered groups, the number of participants under the mean rule exceeds the one under the median rule; the opposite is true for only two groups. We do find support for the hypothesis that the variance of mean rule participation is higher as compared to participation under the median rule groups (variance ratio test, p=0.046).



(a) Boxplot of participants



(b) Participants by group (increasing)

Figure 21.7: Average number of participants

In order to test our hypothesis H3, we run a regression of the dichotomous variable 'participation' on a dummy variable for the voting rule and the gender. These are the only independent variables that we have for participants and abstainers, as the subject pool contains information on gender and we assigned the voting rule. The regression results are displayed in table 21.2. The coefficient for 'rulemean', which takes the value 1 for the mean rule and 0 for the median rule, is $\beta_1 = 0.0370$ and the positive value indicates that the voter turnout is higher for the mean rule. However, we may not reject the null hypothesis that the deviation is significantly different from zero. The same is true for the coefficient of the gender dummy variable, indicating that male subjects have a higher participation rate, yet the difference is not significant.

VARIABLES	participation
rulemean	0.0370
	(0.0395)
gendermale	0.0314
	(0.0412)
Constant	0.233***
	(0.0384)
Observations	508
R-squared	0.003

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

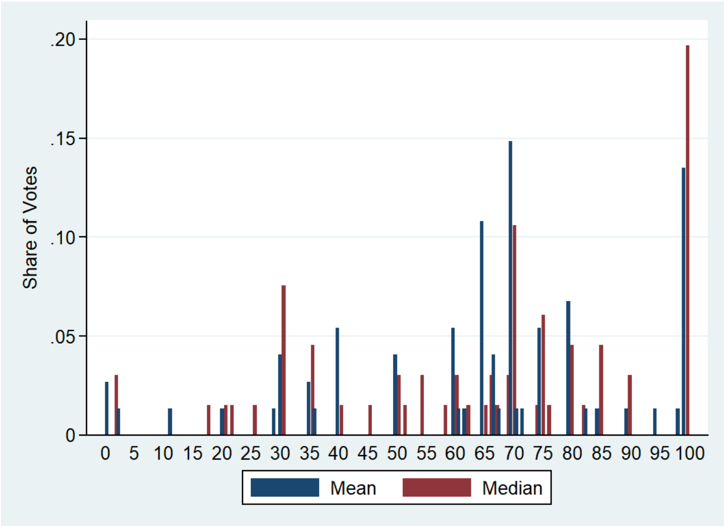
Table 21.2: Regression results participation

We additionally consider the aggregated group values for participation and perform a Mann-Whitney U test. The rank sum of the average group participation rate is higher under the mean rule, but the difference is also not statistically significant (p=0.735).

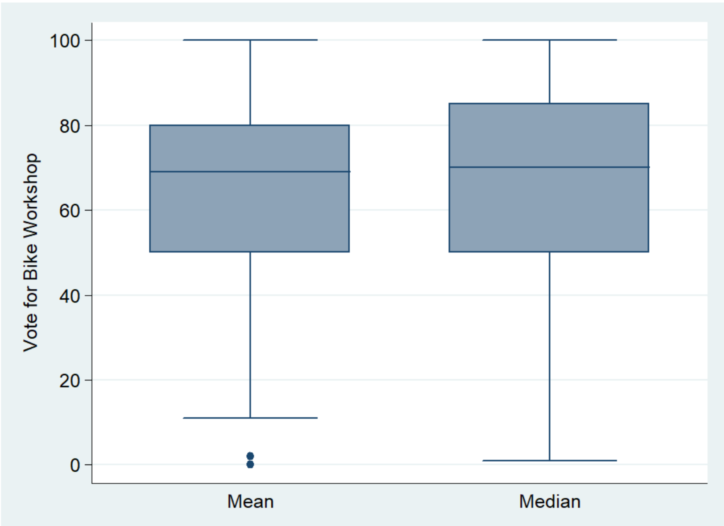
Our data does not support our hypothesis H3, as we do not find evidence for different numbers of participants among the rules. This result has to be considered cautiously, however. The number of groups that we compare is limited to fifteen and the average participation rates per group differ in only eight cases. It is open to future research to obtain more data on participation rates in the context of a simple resource allocation problem.

21.4 Distribution of Votes and Allocation Results

The real distribution of votes is displayed in figure 21.8a. The x-axis depicts the votes for the bike workshop in Euros, i.e. q_i^j for $j = \{\text{bike workshop}\}$, therefore, the distribution of votes for the garden project would yield a mirrored graph. On the y-axis, 'share of votes' describes the percentage of participants that vote for the respective amount. The overall distribution shows the highest percentage at 100 Euros, indicating the preference to allocate all the money on the bike workshop. Further interesting votes are a (70,30)-allocation and also the correspondent split of (30,70), giving a hint on some reference points besides the extreme allocations or the equal split, which only two participants under the median rule and three under the mean rule voted for. We cannot find a significant difference in the votes themselves (Mann-Whitney U test, p=0.680), in the distribution of votes (two-sample Kolmogorov-Smirnov test, p=0.830) nor in the variance of votes (variance ratio test, p=0.553). A boxplot of the votes for the bike workshop by rules is displayed in figure 21.8b. These results cause that our H4 may not be supported: we do not find that the variance of votes is higher under the mean rule. We find this quite surprising. In the first part of this thesis we found that in a controlled laboratory experiment, the shares of individual Nash play are high under both rules - even under no information on the peak distribution. Nash play under the mean rule implies most of the times strategic voting with votes at the extremes of the budget set. Under the median rule, truth-telling is a weakly dominant strategy for all non-pivotal voters and for pivotal voters if the number of participants is odd. Given that the distribution of votes does not differ significantly among the voting rules, we also conclude that the differences in the real impact are not driven by differences in the distribution of votes.



(a) Distribution of votes

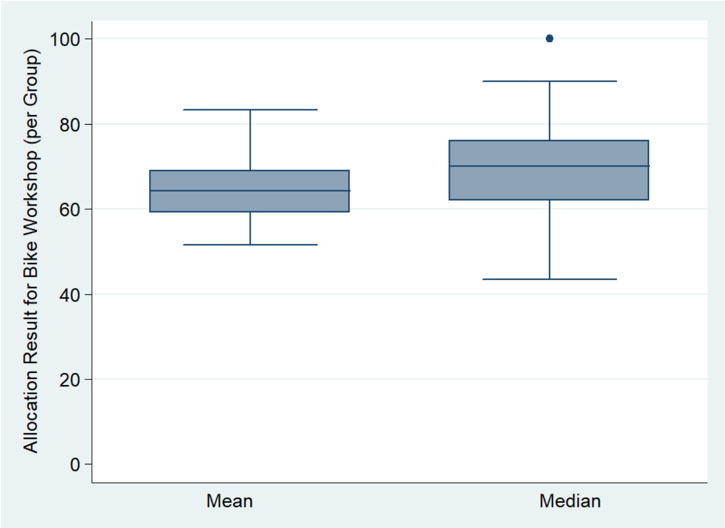


(b) Boxplot of votes

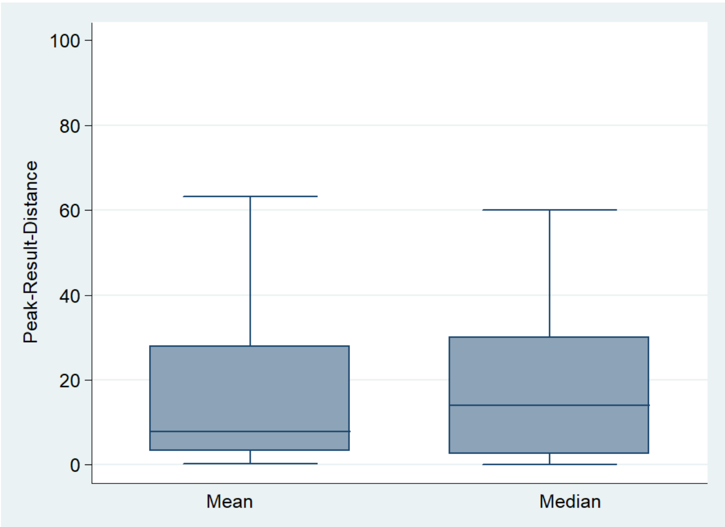
Figure 21.8: Votes for bike workshop

Assuming that the distribution of peaks does not differ between both treatment groups, we expect that votes are more extreme under the mean rule. We will later go into detail about how to interpret the equal distribution of votes when we consider Nash equilibria of the voting game in chapter 21.6.

We do also not find a significant difference in the group allocation results across the two voting rules (two-sample t test, p=0.227). The allocation result (bike,garden) is on average (64.97,35.03) under the mean rule and (70.47,29.53) under the median rule. Figure 21.9a shows a boxplot of the allocation results by rule. Though the group results are not significantly different, we do observe a greater spread in the group results under the median rule, where one group result was to donate the total budget of 100 Euros to the bike workshop project. In this group, three subjects participated and the vector of votes was $q = (85, 100, 100)$. Also solely under the median rule the social outcome was below 50 Euros for the bike workshop. The greater variance in the group allocation results under median voting is statistically significant (variance ratio test, p=0.021). The total donation for the bike workshop adds up to 2,031.60 Euros and for the campus garden to 968.40 Euros.



(a) Allocation results for bike workshop



(b) Distance between peak and result

Figure 21.9: Allocation results and distance to peaks

We asked for the belief about the allocation result, which is slightly more balanced as compared to the real result: (62.10,37.90) is the average belief under the mean and (62.87,37.13) under the median rule. We do not find a significant difference in the belief about the allocation result among voting rules (two-sample t test, p=0.786). Interestingly, we find a significant difference in the correct belief about the result (two-sample t test, p=0.019). Therefore, we calculate the difference between the belief about the result and the real result and find that the average difference under the mean rule is -2.08, as compared to -7.08 under the median rule. The negative sign indicates that participants under both rules believe that the result for the bike workshop is lower than it really is, i.e. they on average underestimate the share for the bike workshop, or in other words, they believe that the result is more balanced among the projects. We find that the average value for correct estimation is significantly higher under the mean rule, which means that the average of the participants' beliefs is only 2.08 Euros lower as the real result. Under the median rule, the average deviation from the real result is 7.08 Euros. The closeness to zero implies that participants estimate the allocation result on average more precisely under the mean rule. Figure 21.10 plots the belief about the result versus the real result. The correlation is positive for both rules (0.061 for the mean rule and 0.467 for the median rule), however the coefficient is significant only for the median rule (Spearman correlation, p=0.608 and p<0.001). We find that the difference between the rules is significant (two sample Fisher's z test, p=0.037).

Figure 21.10: Belief about result vs. real result

When evaluating the mean and the median rule in a single-dimensional resource allocation setting, we already described that while every Nash outcome under the mean rule is unique and efficient, under the median rule inefficient Nash outcomes may occur. However, the median outcome under the concepts partial honesty and coalition-proofness, as defined in part II of this thesis, has the desirable property that it is efficient and minimizes the total distance sum. As we define utility to be dependent on the distance between the peak and the social outcome, the median rule is considered to yield welfare optimal Nash outcomes under partial honesty and under coalition-proofness. In order to compare the welfare level under both voting rules, we calculate for each individual the distance between the peaks and the aggregated group results. The cost of participation is supposed to be identical for all individuals. Figure 21.9b displays the boxplots for the distance between peak and result for both voting rules. We do not find a significant difference in the average distance (two-sample t test, p=0.534) nor in the variance between both rules (variance ratio test, p=0.837). This is quite surprising as the occurrence of Nash equilibria under the median voting games is higher as under the mean rule, what we will refer to in chapter 21.6.

21.5 Non-Truthful and Strategic Voting

As already mentioned in chapter 14.2, there are experiments on budget allocation problems in the lab, which find that subjects try to manipulate the social outcome by not revealing their true preferences. In the controlled laboratory environment, strategic voting is observed especially under the mean rule. The median rule is either strategy-proof or very difficult to manipulate. However, results from the lab show that a high share of votes are untruthful either way. In a field experiment, where apart from the preferences of the other subjects also the votes and number of actual participants is unknown, strategic voting can only be belief-based.

Under the mean rule, a belief about the social outcome is sufficient for deciding on where to place the vote relative to the social outcome. If the belief about the social outcome is for example $Mean(q_{-i}) = (70, 30)$ and the most preferred outcome of i for $j = 1$ is $p_i^1 < 70$, submitting a vote $q_i^1 < p_i^1$ would be strategic. The distance between the optimal vote q_i^* and p_i depends on the (belief about the) number of voters and is limited by the set of feasible allocations. Table 21.3 lists all participants under the mean rule, that state to have voted non-truthfully. The share of all votes that are non-truthful (as per the statements of the participants) is 6.76% and rather low. Based on the true peak p_i^j for $j = \{\text{bike workshop}\}$ and the beliefs about the social outcome $b_i^{x(q)}$ and about the number of participants $b_i^{(k-1)}$, we calculate a theoretical best response $q_i^j(b_i^{x(q)}, b_i^{(k-1)})^{(*)}$ for each of the five individuals. After correcting for allocations that are feasible (only natural numbers between zero and 100), we are able to compare the theoretical belief-based best responses $q_i^j(b_i^{x(q)}, b_i^{(k-1)})^*$ to the actual votes q_i^j. Two individuals submit a vote that corresponds to the best response ($q_i^j = q_i^j(b_i^{x(q)}, b_i^{(k-1)})^*$), which implies that these votes are not only strategic but also optimal given the beliefs. Two further votes are very close to the best response and therefore strategic, as they deviate from the true peak and decrease the distance to the social outcome belief. The one individual that places a non-strategic vote according to the beliefs argues that he could not find a reason why his peak should be different from the equal split ("I don't find an argument why one project should be better for the community than the other"). Nevertheless, he votes for more budget on the project that seemed more useful personally ("I will probably never use the campus garden").[19] This argumentation puts the vote into perspective and makes it in some way strategic, as it seems that the indicated peak was based on the benefit for the community.

p_i^j	$b_i^{x(q)}$	$b_i^{(k-1)}$	$b_i^{x(q_{-i})}$	$q_i^j(b_i^{x(q)}, b_i^{(k-1)})^{(*)}$	$q_i^j(b_i^{x(q)}, b_i^{(k-1)})^*$	q_i^j
85	51	10	46.10	435.10	100	100
80	64	5	56.80	172.80	100	100
60	60	10	58.00	78.00	78	80
50	50	8	56.20	6.25	6	0
50	70	8	68.75	-81.25	0	80

Table 21.3: Non-truthful voting, mean rule

Table 21.4 summarizes the data for the six participants that state to have placed a non-truthful vote under the median rule. The share of non-truthful votes is 9.10% and, surprisingly, higher than under the mean rule. Strategic voting under the median may be possible only for an even number of participants and only if the regarding peak is one of the two in the middle. With the beliefs about the social outcome and the true peak, we get a hint on whether participants vote strategically according to their beliefs. As strategic voting is not possible for an odd number of voters, the best response given $b_i^{(k-1)}$ is even is a vote that is at least as high as the belief about the median outcome $b_i^{x(q)}$ if $p_i^j > b_i^{x(q)}$ or at most as high as $b_i^{x(q)}$ if $p_i^j < b_i^{x(q)}$. The three individuals that believe $(k-1)$ to be even play a best response given their beliefs, but the belief-based distance between their peak and the social outcome is not reduced. If a voter beliefs that the social outcome is identical to the peak, then the best response according to this belief is not necessarily distinct. It depends on the belief about the distribution of votes, more precisely, on whether the voter believes that he or she is the unique median voter. Voting for the true most preferred allocation is optimal given the belief to be the unique median voter. Otherwise, deviation from the true peak might not change the social outcome because the median vote is chosen also by other voters and thus the optimal vote regarding this belief might be any feasible allocation. Nevertheless, truth-telling is at least a weakly dominant strategy if the (belief about the) total number of voters is odd. As can be

[19] The original statements are in German.

observed in table 21.4, none of the voters that voted untruthfully and have the belief about $(k-1)$ is even additionally believes that the social outcome corresponds to the own peak.

p_i^j	$b_i^{x(q)})$	$b_i^{(k-1)}$	$q_i^j(b_i^{x(q)}, b_i^{(k-1)})^*$	q_i^j
85	75	10	≥ 75	100
78	61	11	?	62
70	60	8	≥ 60	85
50	50	3	?	35
30	40	14	≤ 40	1
26	66	9	?	17

Table 21.4: Non-truthful voting, median rule

If $(k-1)$ is odd, the median is determined by the mean of the two middle votes and strategic voting may be possible for these two voters. Consider the following example. Let $Q = 100$, $k = 4$, $q_1^1 = 10$, $p_2^1 = 20$, $p_3^1 = 50$, $q_4^1 = 75$. The median outcome under truth-telling of the two middle voters is 35. Given truth-telling of individual 2, a best response of voter 3 is $(q_3^1)^* = 2 \cdot p_3^1 - p_2^1 = 80$, which is a feasible allocation. However, any vote $q_3^1 \geq q_4^1 = 75$ would yield the same outcome, since individual 4 becomes a median voter. A best response of individual 2 given truth-telling of individual 3 is $(q_2^1)^* = 2 \cdot p_2^1 - p_3^1 = -10$, which corresponds to the feasible vote $(q_2^1)^* = 0$. Again, any $q_2^1 \leq q_1^1 = 10$ yields the same result. Therefore, determining a best response is only possible for a given distribution of votes. Since we did not elicit beliefs about the distribution, a closer look into the statements of the participants is interesting. One individual states that his vote should not be suppressed by the other votes, which hints on a willingness to give up the possibility of strategic voting and thereby casting the winning vote. By submitting a vote that allocates little money to the bike workshop, one participant argues that he wants to give the garden-supporters the possibility that their votes influence the social outcome: “Only few participants will vote for the extremes to make sure that the median is centered. My extreme position will give the garden-supporters the necessary room for a high median.” In order to evaluate the votes as strategic, a more detailed elicitation of the beliefs is necessary. One participant submits a non-truthful vote and believes that the social outcome is equal to her peak ($p_i^j = b_i^{x(q)} = 50$). One may assume that the submitted vote was strategic given her beliefs but we do not know for sure.

21.6 Equilibrium Analysis

In this section, we theoretically analyze the voting and participation behavior. Therefore, we evaluate whether the actual votes constitute a Nash equilibrium for the given distribution of votes and participation rates. The calculation of the Nash equilibria for the voting game is only possible ex post and under full information. One should keep in mind that the individuals in the experiment only had the information on the total group size but not on the number of participants or their peaks or votes.

Since the optimal voting behavior in the theoretical voting game is well-known and explained in chapter 15, we already see from the results in chapter 21.4, that the distribution of votes does not differ for both voting rules. Especially interesting is the fact that under the mean rule, Nash play occurs seldom, which is a direct contradiction to studies from the lab (Marchese and Montefiori, 2011; Block, 2014). We already saw in the last chapter that there is little strategic voting under the mean rule. However, to constitute a Nash equilibrium of the voting game, at most one voter may vote for the true most preferred allocation. Indeed, in none of the 15 groups, the votes constitute a Nash equilibrium in the mean rule voting game and only 12 voters (16.22%) vote for an extreme allocation of zero or 100.

Under the median rule in a single-dimensional allocation problem, truth-telling is an equilibrium for an odd number of voters, as well as a best response to truth-telling, where all non-pivotal voters stay within their rank. There exist eight groups in which the total number of voters is odd and seven of them constitute a Nash equilibrium in the voting game. In the one group where the votes do not comprise an equilibrium, two subjects do not state their true most preferred allocation and one of them thereby increases the distance between the outcome and his peak, as he is the pivotal median voter. In the seven groups where the number of participants is even (i.e. it is two, four or six, see figure 21.7b), no Nash equilibrium occurs. In five out of these, all participants, including the pivotal voters, tell the truth. In two groups, one of the pivotal voters votes strategically: the voters with peaks $p_i^j = 85$ and $p_i^j = 26$ from table 21.4. These subjects play a Nash strategy according to the real votes (and for the first one we know that he also plays a Nash strategy according to his beliefs), however since the opponent does not vote strategically, the group does not constitute a Nash equilibrium in the median rule voting game.

We will now consider individuals that state no to have voted strategically, i.e. that vote for their true most preferred allocation and evaluate whether these individuals play a best response according to their beliefs. For the mean rule, we calculate the theoretical best response using the individual peak, the belief about the allocation result and about the number of individuals. We find that 14 individuals, which is 18.92%, play a best response according to their beliefs and thereby not vote strategically, i.e. vote for their true peak. It is worth noticing that nine voter's most preferred allocations is either to vote for zero or for 100. There is no group in which all participants play a best response according to their beliefs. Hence, no belief-based Nash equilibrium exists in the mean rule voting game and thus also not in the participation game.

Our theoretical model of participation states that an individual participates in a vote if the utility gain from affecting the social outcome exceeds the cost of participation. For the mean rule, we are able to calculate the belief of the voters about the social outcome and giving the votes, we may calculate the belief about the outcome change. This belief provides the maximum cost for participation. In the model, participation costs are identical for all individuals. By calculating the upper cost limits individually based on beliefs, we observe different cost ranges. Figure 21.11 provides a boxplot of the upper cost values for the mean rule.

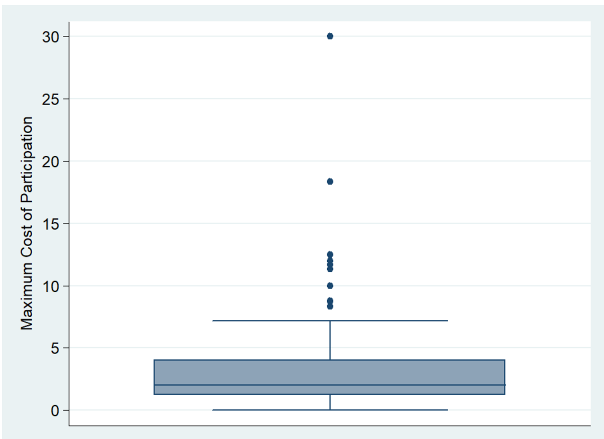


Figure 21.11: Maximum cost of participation under the mean rule

There are eight individuals, whose maximum cost is zero, meaning that they are indifferent in participation and abstention. Both, the mean and the median of the maximum cost is below four and there are only seven values greater than ten. The upper cost values state that the true individual costs must be equally high or lower; otherwise, the individuals would have abstained. Unfortunately, we do not know anything about the abstainers. According to the model, these individuals abstained because the cost exceeded the expected change of the social outcome. This might be the case when the peak is close to the expected outcome of the other votes or the costs of participation are evaluated high.

In the median voting game strategic voting is only possible for an even number of voters. Given the belief about the number of participants is odd, truth-telling is a best response. Thus, 33 subjects vote optimal as their belief about the number of participants is an odd number and they vote truthfully. There exist two groups in which all participants state the truth and believe that k is odd. Both groups thus constitute a Nash equilibrium in the voting game for the median rule.

For the other 27 subjects that voted truthfully, no statement about optimal voting behavior regarding the belief about an even number of participants can be made. These subjects only play a best response if their vote is semi-pivotal or non-pivotal. However, we do not know the belief about the distribution of votes and whether subjects believe that their vote is pivotal or not. For the median rule, the validity about optimal voting behavior and therefore Nash equilibria is limited.

22 Summary and Conclusion

The theory on the pivotal voter model predicts a correlation between the probability of casting a pivotal vote and the decision of participating in an election. In a budget allocation setting, we measure pivotality by the impact of a vote on the social outcome. We define the theoretical impact as the size of the option set, i.e. a larger theoretical impact increases the number of allocations that can be realized. We show that under the mean rule the theoretical impact is small for a large number of participants, however it is certain and always greater than zero as it only depends on the total number of participants. By contrast, the theoretical impact of participation under the median rule is highly sensitive on the distribution of the other votes and may be anything between high and zero. The expected impact under the median rule is larger as compared to the expected impact under the mean rule for bimodal distributions but smaller for normal distributions of votes. Due to the uncertainty of the impact under the median rule, our model predicts that risk averse subjects yield lower participation rates under the median rule, which is referred to as a selection effect.

In a field experiment, we evaluate impact beliefs, elicit risk preferences, and compare participation rates as well as voting behavior under the mean and the median rule. While the real impact of participation on the social outcome is significantly higher under the median rule, our data does not reveal the same differences in the participants' beliefs about the impact. We do observe that participants overestimate their impact, where the belief about the impact is higher under male subjects, who also vote more often for extreme allocations. By checking for risk preferences, we find no support for the selection effect, which would hypothesize a lower level of risk aversion under median voters as compared to mean voters. We also do not find support for the hypothesis of higher participation rates under the mean rule.

We did not ask participants for their motivation to vote but besides the impact, a possible reason why people vote is the 'D-term', formulated by Downs (1957) as a support of the democracy system and a prevention of the democratic collapse that would occur if nobody participated in elections. Riker and Ordeshook (1968) reinvent this term as a sense of civic duty and the resulting satisfaction from voting. If subjects feel a moral obligation to participate in elections, this driving force is given regardless of the voting rule. Therefore, the difference of participation rates among voting rules due to impact differences might be reduced and moral obligations may explain why subjects participate even if they believe that their impact is very low or not existent.

Future research is necessary to address participation rates for different voting rules. It is advisable to test participation under the mean and the median rule in a controlled laboratory environment, such that the cost of participation can be fixed and information on the abstainers is provided. Nevertheless, it should be kept in mind that measuring participation in the lab approaches a different question, since individuals are incentivized by monetary payoffs. Another important aspect of our field experiment is that participation and subsequently voting are both one-shot decisions. In the mean rule voting game of part I, we observe an increase in Nash play over time. After each vote, subjects received information on the social outcome and adapted their votes to Nash play. We also find that information on the peak distribution has a positive effect on Nash play under the mean rule and on the shares of best response to truth-telling of the other voters under the median rule. The experiment on the participation game in the lab conducted by Agranov et al. (2017) finds that participation rates among the majority increase when information on the electorate is provided. In the context of participatory budgeting, Niemeyer

(2017) finds that dynamic or continuous feedback stimulates the funding of projects more as compared to static feedback. It is also argued that with dynamic feedback, the participation process becomes more exiting and enjoyable, which can increase future participation rates. The role of information and feedback therefore seems to be an important factor not to be disregarded in participation games and should be covered in future research.

Even if the results are not as distinct as hypothesized, this field experiment provides a unique possibility in comparing impact beliefs and participation rates on ‘real’ voters.

23 Conclusion

This thesis covers the subject of voting over resource allocation. Resource allocations comprise several applications, whereas we focus on budget allocation on public projects via the mean and the median rule with single-peaked preferences. In single-dimensional allocation problems, we explain that the median rule is strategy-proof for an odd number of voters. However, multiple Nash equilibria exist and these equilibria include Pareto-inefficient outcomes. By contrast, using the mean rule yields a unique and efficient Nash equilibrium in which strategic voting is possible and at most one individual does not vote for an extreme allocation.

The first part of this thesis addresses to multi-dimensional allocation problems. We show that the median outcome might not satisfy the budget constraint such that an adaptation of the outcome is necessary. Implementing the normalized median engenders strategic voting and truth-telling is not always a Nash equilibrium. By construction, the mean outcome allocates the total resource for any set of votes. In a multi-dimensional allocation setting, we proof that even under the mean rule Pareto-inefficient Nash equilibria might occur. In a laboratory experiment, we empirically test voting over resource allocation on three public projects. We observe low shares of truth-telling under the mean rule and a strong tendency to Nash play. A large fraction of votes may also be explained by an optimal response to the result of the previous period. The normalized median rule yields contrary results. While most subjects play a best response to truth-telling of all other subjects, a large fraction does not vote truthfully themselves. Nevertheless, the shares of truth-telling are higher under the median rule as compared to the mean rule. Under both rules, we find low shares of Nash outcomes for the entire group; however, a large majority of the outcomes is classified as efficient.

Parts II and III approach the subject of costly voting. When participation in elections is accompanied by costs, abstention from the election might be optimal. In part II, we embed voting over single-resource allocation into the context of the pivotal voter model. To the best of our knowledge, we are the first to consider costly participation in budget allocation voting including strategic voting behavior. In our setting, pivotality is not only a yes-no question but votes affect the social outcome to a calculable extend. Therefore, the game-theoretic analysis of both the median and the mean rule becomes complex if voting is costly. In particular, the specific position of the median votes is necessary to determine Nash equilibria under the median rule. We develop refinement concepts and provide a complete classification of all Nash equilibria for single participation, for full participation as well as for any number of participants depending on participation costs. We show that the corresponding participation games in general have multiple Nash equilibria, frequently with different sets of participants.

Due to the existence of multiple equilibria and the complexity of the task to determine the equilibria, we hypothesize in part III that the individual participation decision is in practice not driven by equilibrium considerations but by other factors: the impact of a vote on the social outcome (or the belief about the impact) and risk attitude. While the impact of a vote on the social outcome under the mean rule is small for large electorates, it is certain and always greater than zero. By contrast, the impact under the median rule has large variance. In a field experiment, we test whether, and how, voter turnout varies with the voting rule. To this end, we conduct a vote using either the mean or the median rule to determine the allocation of a donation on two public projects. Our focus lies on the participation and voting behavior under either rule, and additionally we study the role of impact beliefs and risk attitude. While the real impact of participation on the social outcome is significantly higher under the median rule, our data does not reveal the same differences in the participants' beliefs about the impact. We observe that participants overestimate their impact, where the belief about the impact is higher under male subjects, who also vote more often for extreme allocations. We find no support for the selection effect, which hypothesizes a lower level of risk aversion under median voters as compared to mean voters. We also do not find support for the hypothesis of higher participation rates under the mean rule. Interestingly, we also do not find differences in the voting behavior when comparing both rules. Especially under the mean rule, where strategic voting is possible and often straightforward, only a handful of participants state that their vote deviates from their true most preferred allocation. This finding seems surprising when comparing it to the results from the voting game in the lab, where strategic voting and Nash play is prominent.

An important aspect with respect to the results of the field experiment is that the decision on participation and voting is one-shot. The data from part I of this thesis reveal changes in voting behavior over time and with different information levels. Information before and feedback after elections are factors that should be considered in participation games and are open to future research.

Another insight we gain is the crucial role of participation costs. We show that the theoretical model and equilibrium analysis become complex with the introduction of participation costs. Therefore, it seems unlikely for subjects to play a Nash equilibrium of the participation game, also in controlled lab experiments. This great effect of participation costs on equilibrium outcomes promotes the pursuit of making participation costless or at least reduce the costs to a minimum. Efforts of reducing the expected costs are already made in order to increase participation in various forms like implementing chances of winning a voucher that will be raffled off among all participants. In elections where lotteries are inappropriate, like voting for the next parliament, a good way to decrease participation costs is to provide information on the alternatives and on the election as simple and easy accessible as possible and to conduct elections online. Some countries already go as far as to make participation in elections compulsory.

To the best of our knowledge, the present thesis is the first study that covers theoretical and experimental analysis of voting over multi-dimensional budget allocation (part I) and costly participation in voting over simple resource allocations with strategic voting (parts II and III).

Appendix

A Instructions of the Laboratory Experiments

A.1 First Experiment

1 Vorbemerkungen

Willkommen zum Experiment und vielen Dank für Ihre Teilnahme. Zu Beginn möchten wir Sie bitten, Ihre Mobiltelefone auszuschalten und jegliche Kommunikation einzustellen. Wenn Sie Fragen haben, richten Sie diese bitte so leise wie möglich an die Experimentleitung und sprechen Sie nicht mit den anderen Teilnehmern.

In diesem Experiment verdienen Sie abhängig von Ihren Entscheidungen und den Entscheidungen der anderen Teilnehmer bares Geld. Während des Experiments wird Ihnen Ihr Kontostand in der Einheit *ECU* angezeigt. 100 *ECU* entsprechen 1,50 Euro. Am Ende des Experiments wird Ihnen Ihr letzter Kontostand ausbezahlt. Für Ihr pünktliches Erscheinen zum Experiment erhalten Sie zusätzlich 5,00 Euro.

2 Mathematische Grundlagen

Wir möchten Sie zunächst mit einigen mathematischen Grundlagen vertraut machen, die für das Experiment von Bedeutung sein werden.

2.1 Durchschnitt (arithmetisches Mittel)

Der Durchschnitt ist ein Mittelwert, der als Quotient aus der Summe aller Werte und der Anzahl der Werte definiert ist. Die Formel zur Berechnung lautet:

$$x = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{y_1 + \cdots + y_n}{n}$$

Beispiel: Gegeben seien die fünf Zahlen $3, 19, 58, 25, 80$. Als Durchschnitt ergibt sich $x = 37$, denn $\frac{1}{5} \cdot (3 + 19 + 58 + 25 + 80) = \frac{185}{5} = 37$.

2.2 Median

Der Median einer ungeraden Anzahl von Werten ist die Zahl, die nach Sortierung der Werte in aufsteigender Größe an der mittleren Stelle steht.

Beispiel: Gegeben seien die fünf Zahlen $3, 19, 58, 25, 80$. Als Median ergibt sich $x = 25$, nämlich die mittlere Zahl von $3, 19, \mathbf{25}, 58, 80$.

Falls mehrere Median-Werte ermittelt werden, die in Summe einen bestimmten Betrag erreichen sollen, muss gegebenenfalls eine Anpassung der Mediane vorgenommen werden. Diese kann durch Normalisierung erzielt werden.

Normalisierung

Bei der Anpassung der ermittelten Mediane durch Normalisierung wird das Verhältnis der Median-Werte zueinander beibehalten und die einzelnen Werte so lange gemeinsam erhöht oder verringert, bis deren Summe den erwünschten Betrag erreicht.

Beispiel 1: Es soll der Betrag von 100 auf drei Komponenten A, B und C aufgeteilt werden. Die zugrunde liegenden Werte sind wie folgt:

1. $A_1 = 80$, $B_1 = 0$, $C_1 = 20$
2. $A_2 = 20$, $B_2 = 70$, $C_2 = 10$
3. $A_3 = 10$, $B_3 = 50$, $C_3 = 40$

Daraus ergeben sich nach getrennter Betrachtung von A, B und C die Mediane $M_A = 20$, $M_B = 50$ und $M_C = 20$. Da $20 + 50 + 20 < 100$, muss eine Anpassung erfolgen. Das Verhältnis zwischen M_A, M_B und M_C ist $2 : 5 : 2$, dementsprechend folgt nach Normalisierung $M_A^* = 22\frac{2}{9}$, $M_B^* = 55\frac{5}{9}$ und $M_C^* = 22\frac{2}{9}$, sodass $M_A^* + M_B^* + M_C^* = 22\frac{2}{9} + 55\frac{5}{9} + 22\frac{2}{9} = 100$.

Beispiel 2: Es soll der Betrag von 100 auf drei Komponenten A, B und C aufgeteilt werden. Die zugrunde liegenden Werte sind wie folgt:

1. $A_1 = 50$, $B_1 = 50$, $C_1 = 0$
2. $A_2 = 0$, $B_2 = 90$, $C_2 = 10$
3. $A_3 = 70$, $B_3 = 10$, $C_3 = 20$

Daraus ergeben sich nach getrennter Betrachtung von A, B und C die Mediane $M_A = 50$, $M_B = 50$ und $M_C = 10$, sodass $50 + 50 + 10 > 100$ und eine Anpassung erfolgen muss. Das Verhältnis zwischen M_A, M_B und M_C ist $5 : 5 : 1$, woraus nach Normalisierung folgt: $M_A^* = 45\frac{5}{11}$, $M_B^* = 45\frac{5}{11}$ und $M_C^* = 9\frac{1}{11}$ mit $M_A^* + M_B^* + M_C^* = 45\frac{5}{11} + 45\frac{5}{11} + 9\frac{1}{11} = 100$.

2.3 Betragsfunktion

Es wird mit $|x|$ der Betrag von x bezeichnet. Dieser ist auch als Absolutwert oder abs(x) bekannt.

Beispiel: $|-15 + 12| = |-3| = 3$.

3 Aufbau des Experiments

Sie nehmen an einer Abstimmung teil, bei der die jeweilige Höhe der Förderung dreier Projekte bestimmt werden soll. An der Abstimmung nehmen noch vier weitere Personen teil. Jedem Teilnehmer werden individuelle Verteilungswünsche zugeteilt, die mit p_1, p_2 und p_3 bezeichnet sind. p_1 beschreibt die gewünschte Förderung für Projekt 1, p_2 und p_3 für die Projekte 2 und 3. Insgesamt steht ein Betrag von 100 Geldeinheiten zur Verfügung, der immer gänzlich ausgeschöpft werden muss. Somit liegen Ihre zugeteilten Werte stets zwischen 0 und 100 und ergeben in Summe 100. Ihre Verteilungswünsche bleiben mehrere Durchgänge gleich, ebenso die der anderen Teilnehmer. Sobald sich Ihr zugewiesener Verteilungswunsch ändert, ändern sich auch die der anderen Teilnehmer. Ihre Auszahlung ist abhängig vom Abstand Ihres Verteilungswunsches zum Abstimmungsergebnis aus allen fünf Vorschlägen. Die genaue Auszahlungsfunktion finden Sie in Abschnitt 3.3.

3.1 Abgabe der Vorschläge

Jeder der fünf Teilnehmer macht einen Vorschlag, der in die Abstimmung einfließt. Das heißt, alle fünf Teilnehmer nennen drei natürliche Zahlen zwischen 0 und 100, die in Summe wieder 100 ergeben. Sollte die Summe aus Ihren drei Werten ungleich 100 oder Ihre einzelnen Werte keine natürlichen Zahlen sein, erhalten Sie eine Fehlermeldung und müssen Ihre Vorschläge anpassen.

3.2 Abstimmung

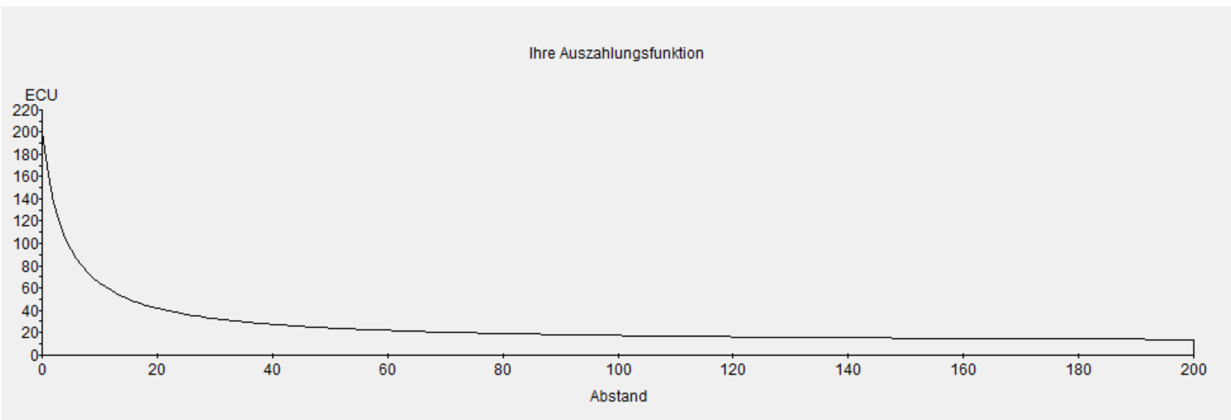
Anhand aller genannten Vorschläge wird die Höhe der Förderung für die drei Projekte bestimmt. Dazu werden die Vorschläge, also jeweils fünf Werte für jedes der drei Projekte, entweder der Größe nach sortiert und der drittgrößte Vorschlag wird gewählt (das entspricht dem Medianverfahren) oder aufsummiert und durch fünf geteilt (das entspricht dem Durchschnitt aller Vorschläge). Daraus ergibt sich das Abstimmungsergebnis x_1, x_2 und x_3. Falls die durch das Medianverfahren erhaltenen Werte x_1, x_2 und x_3 in Summe nicht 100 ergeben, erfolgt eine normalisierte Anpassung des Ergebnisses, wie in Abschnitt 2.2 weiter oben erklärt. Sie erfahren jeweils vor der Abstimmung, welches Verfahren benutzt wird. Erfolgt die Abstimmung anhand des Medians, so erfahren Sie vor der Abstimmung zusätzlich die zugeteilten Verteilungswünsche der anderen Teilnehmer.

3.3 Auszahlung

Je geringer die Differenz zwischen den aus der Abstimmung erhaltenen Werten x_1, x_2, x_3 und den Ihnen zugeteilten Werten p_1, p_2, p_3 ausfällt, desto höher ist Ihr Gewinn. Ihre individuelle Auszahlung f_i in der Einheit *ECU* berechnet sich wie folgt:

$$f_i(p^i, x) = 10 + \frac{760}{4 + \sum_{j=1}^{3} |p_j^i - x_j|} \ .$$

Die nachfolgende Abbildung stellt die Auszahlungsfunktion grafisch dar. Als Abstand wird die Summe aus den Beträgen der Differenzen von x_1, x_2, x_3 und p_1, p_2, p_3 bezeichnet.



Beispiel: Sollten sich aus der Abstimmung die Werte $x_1 = 15$, $x_2 = 50$ und $x_3 = 35$ ergeben und waren die Ihnen zugeteilten Verteilungswünsche $p_1 = 30$, $p_2 = 50$, $p_3 = 20$, dann ist Ihr Abstand vom Abstimmungsergebnis $|15 - 30| + |50 - 50| + |35 - 20| = 30$. Das würde eine Auszahlung von $f = 10 + \frac{760}{4+30} = 32,35\ ECU$, also $32,35 \cdot \frac{1,50}{100} = 0,4529e$ bedeuten.

4 Ablauf des Experiments

Die Abstimmung findet in mehreren Runden statt. Der Ablauf jeder Runde ist wie folgt:

1. Sie erfahren das Verfahren der Abstimmung, also Median oder Durchschnitt. Wenn das Abstimmungsverfahren der Median ist, erfolgt eine gegebenenfalls nötige Anpassung durch Normalisierung.
2. Sie erfahren Ihre Werte p_1, p_2 und p_3 und bei Abstimmung durch den Median zusätzlich die Werte der anderen Teilnehmer. Diese Werte bleiben mehrere Durchgänge gleich.
3. Sie machen einen Vorschlag für die Abstimmung.
4. Sie erfahren die aus den Vorschlägen aller Teilnehmer berechneten Werte x_1, x_2 und x_3 und Ihre Auszahlung.

5 Schlussbemerkungen

Bevor das Experiment beginnt, werden Ihnen auf dem Bildschirm einige Verständnisfragen gestellt. An Ihrem Platz finden Sie Papier und Stift. Wir bitten Sie, diese beim Verlassen des Raumes am Platz liegen zu lassen. Außerdem können Sie während der gesamten Studie den Taschenrechner verwenden.

Bitte bleiben Sie auch nach Ende des Experiments an Ihrem Platz und warten Sie auf weitere Anweisungen der Experimentleitung. Vielen Dank!

A.2 Pilot Session

As mentioned in 7.2.1, the conversion of ECU in Euros was different in the pilot session. Participants received 1.00 Euro per 100 ECU. Additionally to the first experiment, the pilot session included the sequential median rule. The instructions of the sequential median rule are provided in the following.

2.2.2 Sequentielle Anpassung

Falls die Anpassung sequentiell, d.h. nach einer festgelegten Reihenfolge, erfolgt, wird der Reihe nach geprüft, ob die Summe der bereits bestimmten Mediane den gewünschten Betrag erreicht. Sollte dies der Fall sein, erhalten alle nachfolgenden Mediane den Wert 0. Ist die Summe bis zum letzten Median nicht erreicht, so wird die Differenz zwischen dem gewünschten Gesamtbetrag und den bereits bestimmten Medianen als letzter Wert festgelegt.

Beispiel 1: Es soll der Betrag von 100 auf drei Komponenten A, B und C aufgeteilt werden. Die Aufteilungen sind wie folgt:

1. $A_1 = 80$, $B_1 = 0$, $C_1 = 20$
2. $A_2 = 20$, $B_2 = 70$, $C_2 = 10$

3. $A_3 = 10$, $B_3 = 50$, $C_3 = 40$

Daraus ergeben sich nach getrennter Betrachtung von A, B und C die Mediane $M_A = 20$, $M_B = 50$ und $M_C = 20$. Da $20+50+20 < 100$, muss eine Anpassung erfolgen. Da $M_A < 100$, ist auch $M_A^* = M_A = 20$, d.h. keine Anpassung ist nötig. Weiter ist $M_A^* + M_B < 100$, sodass $M_B^* = M_B = 50$. Schließlich gilt wegen $M_A^* + M_B^* < 100$ für den letzten Wert $M_C^* = 100 - (M_A^* + M_B^*) = 30$.

Beispiel 2: Es soll der Betrag von 100 auf drei Komponenten A, B und C aufgeteilt werden. Die Aufteilungen sind wie folgt:

1. $A_1 = 50$, $B_1 = 50$, $C_1 = 0$
2. $A_2 = 0$, $B_2 = 90$, $C_2 = 10$
3. $A_3 = 70$, $B_3 = 10$, $C_3 = 20$

Daraus ergeben sich nach getrennter Betrachtung von A, B und C die Mediane $M_A = 50$, $M_B = 50$ und $M_C = 10$, sodass $50+50+10 > 100$. Dementsprechend muss eine Anpassung erfolgen. Da $M_A < 100$, ist auch $M_A^* = M_A = 50$, d.h. keine Anpassung ist nötig. Weiter ist $M_A^*+M_B = 100$, sodass $M_B^* = M_B = 50$. Schließlich gilt wegen $M_A^* + M_B^* = 100$ für den letzten Wert $M_C^* = 0$.

A.3 Second Experiment

In the second experiment, we use a between-subject design, such that participants were confronted with solely one voting rule (the mean or the normalized median rule). Therefore, the handout was shortened as we only explained the relevant voting rule and the part in which we informed participants about the voting rule for the subsequent rounds could be dropped. Additionally, the payoff conversion is different from the first experiment: 100 ECU correspond to 1.00 Euro.

B Session Procedure of the Laboratory Experiments

Voting Rule	Information	Peak Distribution	Participant					Number of Periods
			1	**2**	**3**	**4**	**5**	
Mean	No Info	A	20	60	20	10	70	5
			70	20	20	80	20	
			10	20	60	10	10	
		B	20	25	25	10	60	
			20	50	65	70	15	
			60	25	10	20	25	
		C	60	20	0	20	30	
			20	20	80	80	40	
			20	60	20	0	30	
	Full Info	A	60	70	10	20	20	3
			20	20	80	70	20	
			20	10	10	10	60	
		B	25	25	20	60	10	
			50	65	20	15	70	
			25	10	60	25	20	
		C	20	0	30	20	60	
			80	80	40	20	20	
			0	20	30	60	20	
Normalized Median	No Info	A	70	20	60	20	10	5
			20	70	20	20	80	
			10	10	20	60	10	
		B	25	10	60	25	20	
			65	70	15	50	20	
			10	20	25	25	60	
		C	30	60	20	0	20	
			40	20	80	80	20	
			30	20	0	20	60	
	Full Info	A	10	20	20	70	60	3
			80	20	70	20	20	
			10	60	10	10	20	
		B	10	60	25	20	25	
			70	15	50	20	65	
			20	25	25	60	10	
		C	0	30	20	60	20	
			80	40	20	20	80	
			20	30	60	20	0	
Sequential Median	Full Info	C	20	20	60	30	0	3
			20	80	20	40	80	
			60	0	20	30	20	

Table B.1: Procedure of the pilot session

Voting Rule	**Information**	**Peak Distribution**	**Participant 1**	**2**	**3**	**4**	**5**	**Number of Periods**
Normalized Median	Full Info	D	25	25	60	20	10	3
			50	55	15	20	70	
			25	20	25	60	20	
		E	40	30	30	60	0	
			20	10	70	10	70	
			40	60	0	30	30	
		F	15	20	65	0	50	
			30	40	5	70	30	
			55	40	30	30	20	
Mean	No Info	G	50	40	20	10	20	5
			35	30	20	80	70	
			15	30	60	10	10	
		D	10	25	60	25	20	
			70	55	15	50	20	
			20	20	25	25	60	
		H	0	20	30	50	20	
			80	30	40	30	80	
			20	50	30	20	0	
		I	30	20	40	10	25	
			50	50	50	50	50	
			20	30	10	40	20	

Table B.2: Procedure of sessions one to ten

Session	Information	Peak Distribution	Participant					Number of Periods
			1	2	3	4	5	
1;5	No Info	J	10	5	70	20	10	5
			65	10	10	20	8	
			25	85	20	60	82	
		K	75	70	20	12	40	
			10	15	15	78	30	
			15	15	65	10	30	
		L	20	15	20	30	10	
			10	30	20	15	20	
			70	55	60	55	70	
	Full Info	J	20	10	10	70	5	3
			20	8	65	10	10	
			60	82	25	20	85	
		K	12	40	70	75	20	
			78	30	15	10	15	
			10	30	15	15	65	
		L	20	20	30	10	15	
			20	10	15	20	30	
			60	70	55	70	55	
2;6	No Info	J	5	10	20	10	70	5
			10	8	20	65	10	
			85	82	60	25	20	
		K	40	20	12	75	70	
			30	15	78	10	15	
			30	65	10	15	15	
		M	50	15	25	25	10	
			30	60	70	50	20	
			20	25	5	25	70	
	Full Info	J	10	20	5	70	10	3
			8	20	10	10	65	
			82	60	85	20	25	
		K	12	75	20	70	40	
			78	10	15	15	30	
			10	15	65	15	30	
		M	25	15	50	10	25	
			70	60	30	20	50	
			5	25	20	70	25	
3;7	No Info	M	15	25	25	50	10	5
			60	70	50	30	20	
			25	5	25	20	70	
		K	75	70	20	12	40	
			10	15	15	78	30	
			15	15	65	10	30	
		L	20	30	20	15	10	
			20	15	10	30	20	
			60	55	70	55	70	
	Full Info	M	25	15	10	25	50	3
			70	60	20	50	30	
			5	25	70	25	20	
		K	70	40	12	75	20	
			15	30	78	10	15	
			15	30	10	15	65	
		L	10	20	30	20	15	
			20	10	15	20	30	
			70	70	55	60	55	

4;8	No Info	J	5	70	10	20	10	5
			10	10	65	20	8	
			85	20	25	60	82	
		M	15	10	25	50	25	
			60	20	50	30	70	
			25	70	25	20	5	
		L	20	30	20	10	15	
			20	15	10	20	30	
			60	55	70	70	55	
	Full Info	J	10	20	70	10	5	3
			8	20	10	65	10	
			82	60	20	25	85	
		M	50	25	10	25	15	
			30	50	20	70	60	
			20	25	70	5	25	
		L	10	20	30	15	20	
			20	10	15	30	20	
			70	70	55	55	60	

Table B.3: Procedure of the second experiment

C Procedure of the Field Experiment

C.1 The Invitation

C.1.1 Mean Treatment

Abstimmung über Spendenprojekte am KIT

Guten Tag,

der Lehrstuhl für Wirtschaftstheorie des KIT möchte Sie an einer Abstimmung teilhaben lassen. Neben Ihnen wurden **16 weitere Personen** zu dieser Abstimmung eingeladen. Ihre Gruppe stimmt über die Aufteilung von **100 Euro** auf zwei Campus-Projekte ab. Dazu kann jedes Gruppenmitglied einen individuellen Aufteilungsvorschlag abgeben. Das Ergebnis der Abstimmung berechnet sich aus dem **Durchschnitt** aller Aufteilungsvorschläge, d.h. die Summe der Aufteilungsvorschläge geteilt durch die Anzahl der abgegebenen Stimmen.
Bei den beiden Projekten handelt es sich um eine **Fahrradwerkstatt** (betreute Selbsthilfewerkstatt) und den **Campus-Garten** (Möglichkeit zum Obst- und Gemüseanbau). Ihr Aufteilungsvorschlag besteht also aus zwei Geldbeträgen: der Betrag, der an die Fahrradwerkstatt gespendet werden soll und der Betrag, der an den Campus-Garten gespendet werden soll. Beide Beträge müssen in Summe 100 Euro ergeben.
Eine individuelle Vergütung erfolgt nicht. Der gesamte Betrag von 100 Euro wird gemäß der Abstimmungsergebnisse an die beiden Projekte gespendet. Die Abstimmung erfolgt online und dauert etwa 5 Minuten. Wenn Sie an der Abstimmung teilnehmen möchten, klicken Sie bitte hier:
`https://www.survio.com/survey/d/d1`
Neben Ihrer Gruppe existieren weitere Gruppen, die über die Aufteilung von 100 Euro für die beiden Projekte abstimmen. Jede Gruppe besteht aus 17 Teilnehmern und stimmt über die Verwendung von jeweils 100 Euro ab. Für die Umsetzung des Aufteilungsvorschlages jeder Gruppe ist mindestens eine abgegebene Stimme pro Gruppe nötig.
Die Abstimmung läuft bis zum 17.07.2017. Sollten Sie Interesse an dem Ergebnis der Abstimmung haben, geben Sie bitte nach der Abstimmung Ihre E-Mail-Adresse ein. Ihre Antworten werden anonym behandelt und sind der E-Mail-Adresse nicht zuzuordnen. Hier noch einmal der Link zur Abstimmung:
`https://www.survio.com/survey/d/d1`

Viele Grüße

Der Lehrstuhl für Wirtschaftstheorie des KIT

C.1.2 Median Treatment

Das Ergebnis der Abstimmung berechnet sich aus dem **Median** aller Aufteilungsvorschläge, d.h. der Aufteilungsvorschlag, der nach Sortierung aller Aufteilungsvorschläge in aufsteigender Reihenfolge an mittlerer Stelle steht, wird gewählt. Sollte die Anzahl der abgegebenen Stimmen gerade sein, berechnet sich der Median aus dem Mittelwert der beiden mittleren Aufteilungsvorschläge.

C.2 The Voting Process

Figure C.1: The vote

C.3 The Questionnaire

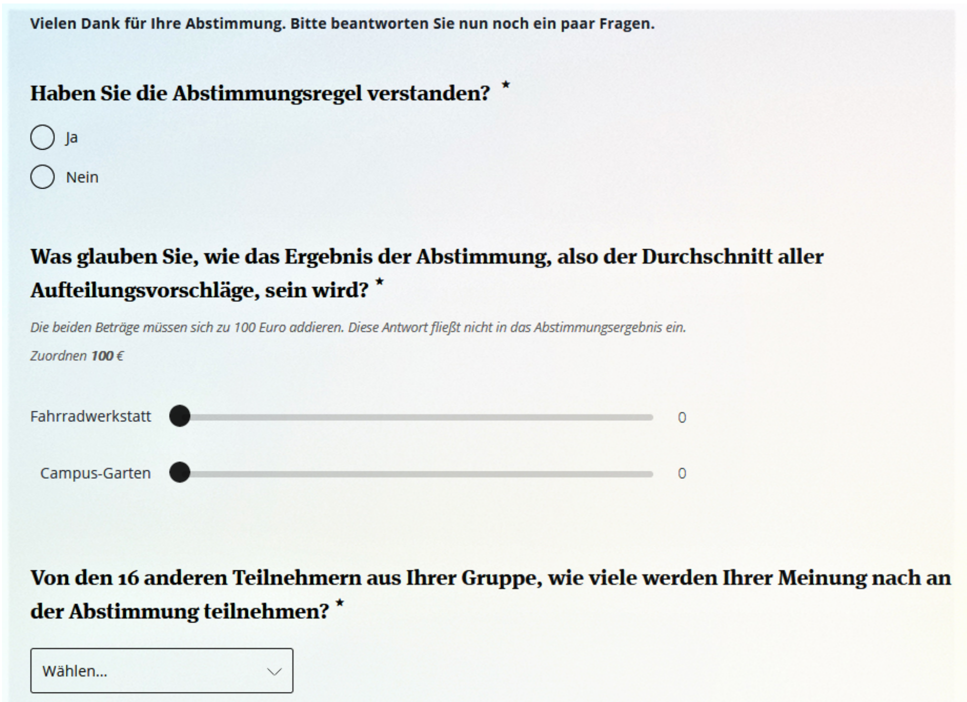
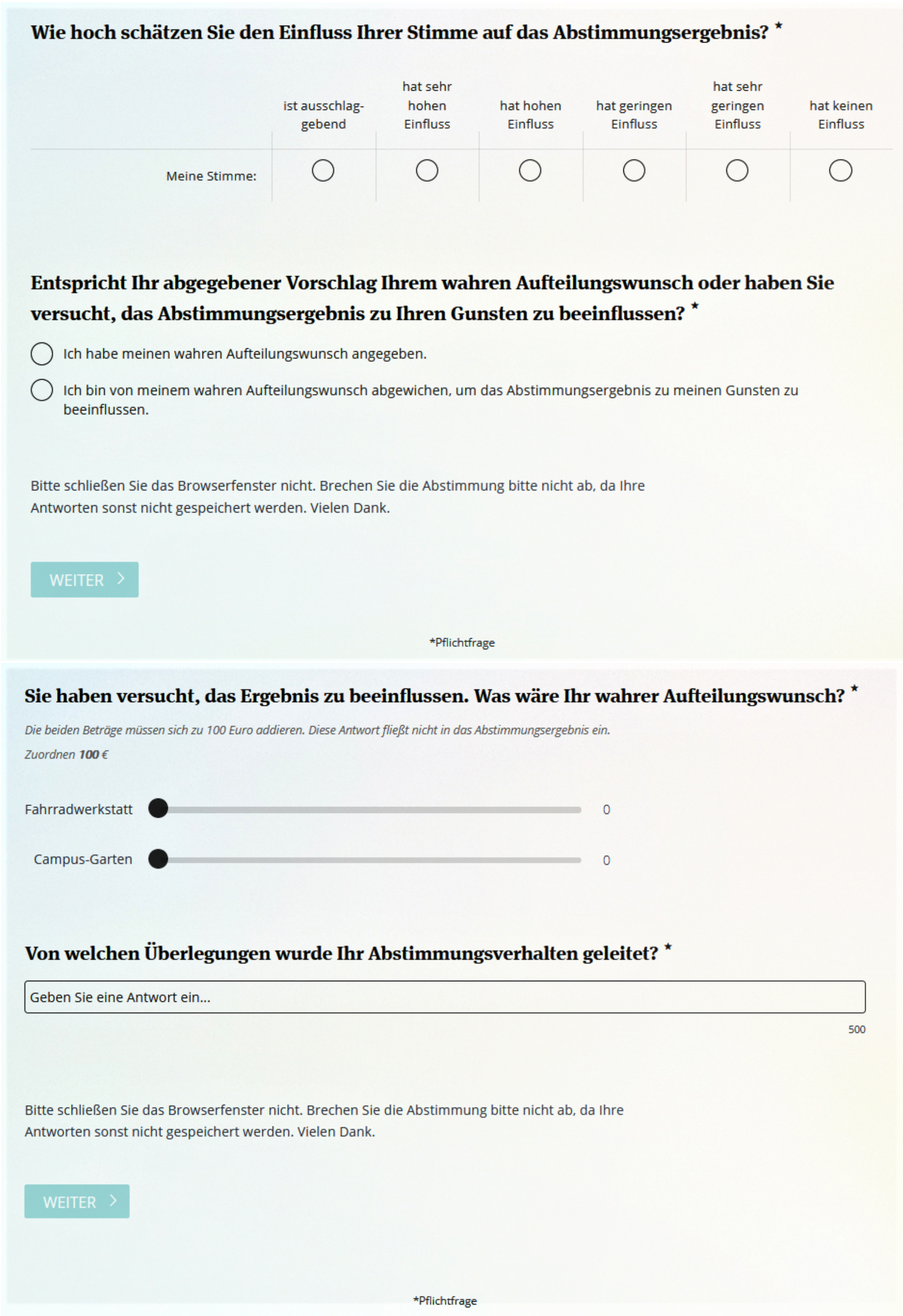


Figure C.2: Questions on the vote I



Wie hoch schätzen Sie den Einfluss Ihrer Stimme auf das Abstimmungsergebnis? *

	ist ausschlag-gebend	hat sehr hohen Einfluss	hat hohen Einfluss	hat geringen Einfluss	hat sehr geringen Einfluss	hat keinen Einfluss
Meine Stimme:	◯	◯	◯	◯	◯	◯

Entspricht Ihr abgegebener Vorschlag Ihrem wahren Aufteilungswunsch oder haben Sie versucht, das Abstimmungsergebnis zu Ihren Gunsten zu beeinflussen? *

◯ Ich habe meinen wahren Aufteilungswunsch angegeben.

◯ Ich bin von meinem wahren Aufteilungswunsch abgewichen, um das Abstimmungsergebnis zu meinen Gunsten zu beeinflussen.

Bitte schließen Sie das Browserfenster nicht. Brechen Sie die Abstimmung bitte nicht ab, da Ihre Antworten sonst nicht gespeichert werden. Vielen Dank.

WEITER >

*Pflichtfrage

Sie haben versucht, das Ergebnis zu beeinflussen. Was wäre Ihr wahrer Aufteilungswunsch? *

Die beiden Beträge müssen sich zu 100 Euro addieren. Diese Antwort fließt nicht in das Abstimmungsergebnis ein.

*Zuordnen **100 €***

Fahrradwerkstatt 0

Campus-Garten 0

Von welchen Überlegungen wurde Ihr Abstimmungsverhalten geleitet? *

Geben Sie eine Antwort ein...

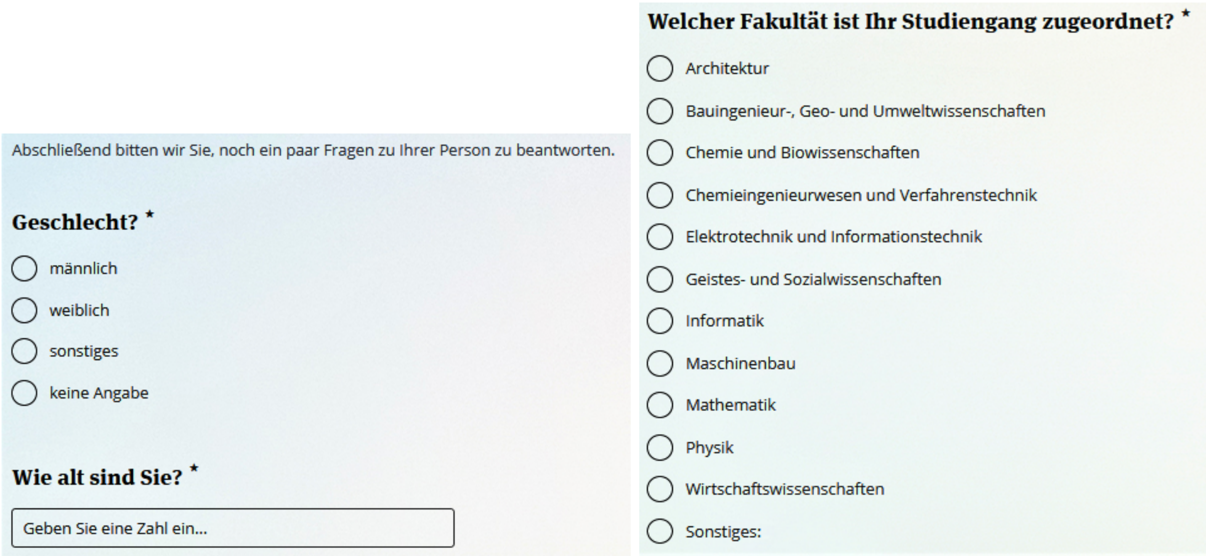
500

Bitte schließen Sie das Browserfenster nicht. Brechen Sie die Abstimmung bitte nicht ab, da Ihre Antworten sonst nicht gespeichert werden. Vielen Dank.

WEITER >

*Pflichtfrage

Figure C.3: Questions on the vote II



Abschließend bitten wir Sie, noch ein paar Fragen zu Ihrer Person zu beantworten.

Geschlecht? *

- ◯ männlich
- ◯ weiblich
- ◯ sonstiges
- ◯ keine Angabe

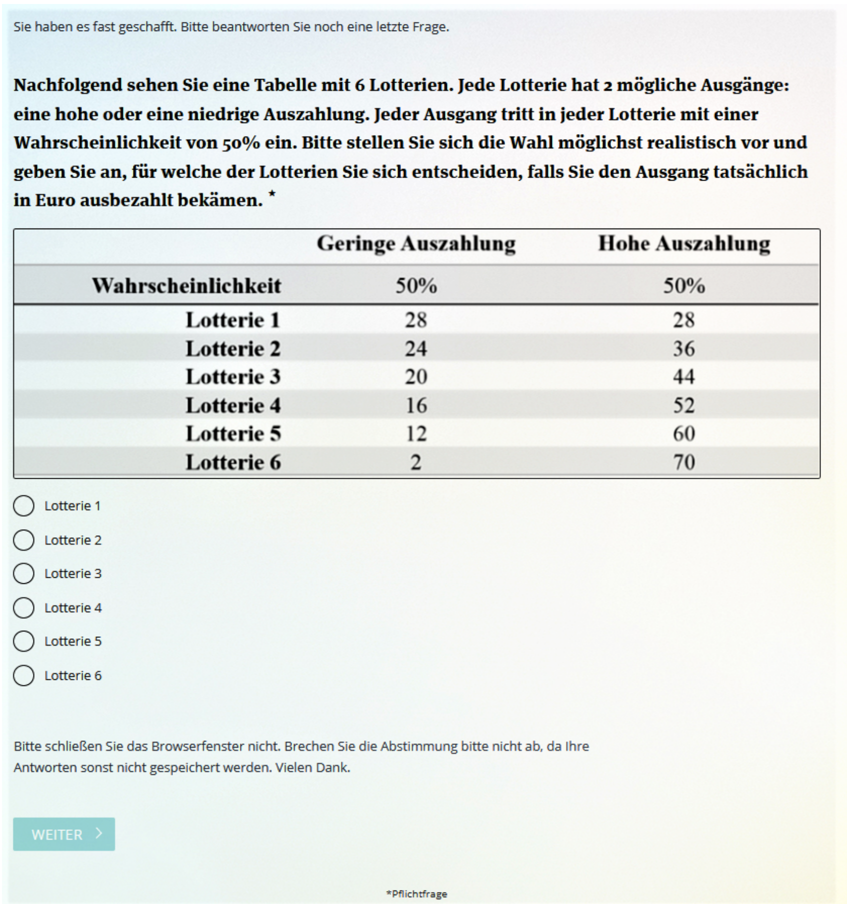
Wie alt sind Sie? *

Geben Sie eine Zahl ein...

Welcher Fakultät ist Ihr Studiengang zugeordnet? *

- ◯ Architektur
- ◯ Bauingenieur-, Geo- und Umweltwissenschaften
- ◯ Chemie und Biowissenschaften
- ◯ Chemieingenieurwesen und Verfahrenstechnik
- ◯ Elektrotechnik und Informationstechnik
- ◯ Geistes- und Sozialwissenschaften
- ◯ Informatik
- ◯ Maschinenbau
- ◯ Mathematik
- ◯ Physik
- ◯ Wirtschaftswissenschaften
- ◯ Sonstiges:

Figure C.4: Demographic questions



Sie haben es fast geschafft. Bitte beantworten Sie noch eine letzte Frage.

Nachfolgend sehen Sie eine Tabelle mit 6 Lotterien. Jede Lotterie hat 2 mögliche Ausgänge: eine hohe oder eine niedrige Auszahlung. Jeder Ausgang tritt in jeder Lotterie mit einer Wahrscheinlichkeit von 50% ein. Bitte stellen Sie sich die Wahl möglichst realistisch vor und geben Sie an, für welche der Lotterien Sie sich entscheiden, falls Sie den Ausgang tatsächlich in Euro ausbezahlt bekämen. *

	Geringe Auszahlung	Hohe Auszahlung
Wahrscheinlichkeit	50%	50%
Lotterie 1	28	28
Lotterie 2	24	36
Lotterie 3	20	44
Lotterie 4	16	52
Lotterie 5	12	60
Lotterie 6	2	70

- ◯ Lotterie 1
- ◯ Lotterie 2
- ◯ Lotterie 3
- ◯ Lotterie 4
- ◯ Lotterie 5
- ◯ Lotterie 6

Bitte schließen Sie das Browserfenster nicht. Brechen Sie die Abstimmung bitte nicht ab, da Ihre Antworten sonst nicht gespeichert werden. Vielen Dank.

WEITER >

*Pflichtfrage

Figure C.5: Risk preferences

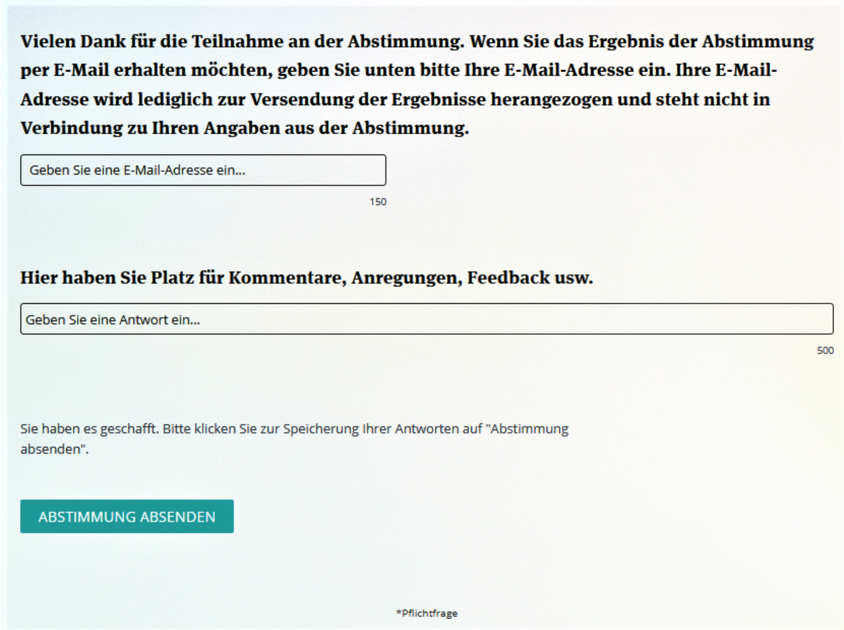


Figure C.6: Concluding remarks

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