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Numerical method to determine the inverse solution of two impacting rods of non-constant cross section

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Impacting rods are used in several scientific experiments and in everyday tools such as drilling machines. For deep hole drillings, the efficiency of the drilling process can be improved by shaping the longitudinal wave that is transmitted through the drill rod to the drill bit [1]. The approach implemented in this contribution adjusts the longitudinal wave shape by varying the geometry of the impacting piston.

1 Direct solution

Using rod theory, one dimensional impacting rods can mathematically be described by the partial differential equation

$$\frac{\partial}{\partial x}\left[E_i(x)A_i(x)\frac{\partial u_i}{\partial x}\right] = \rho_i(x)A_i(x)\frac{\partial^2 u_i}{\partial t^2},\ i = 1, 2 \tag{1}$$

for the displacements u_i of piston ($i = 1$) and rod ($i = 2$). For constant parameters Young's modulus E, density ρ and cross section A, the equation simplifies to the well known wave equation which can be solved analytically by the D'Alembert solution. For the numerical approach, the geometries of the rods are approximated by piecewise constant cross sections. The number of approximated cross sections equals the number of elements. The material properties can vary from element to element but are constant on each element. Therefore, the analytical solution can be applied on each element. At the interfaces between elements transition conditions have to be formulated, which can be derived by claiming force and displacement equilibrium. The method is described extensively in [2].

2 Inverse solution

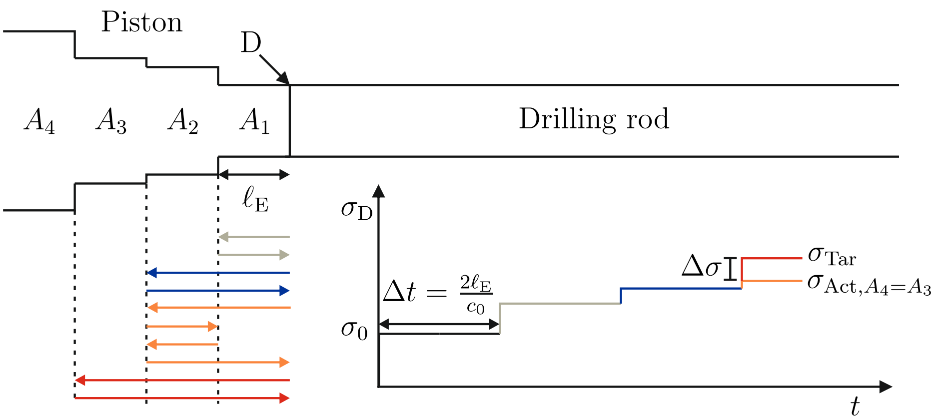

Fig. 1: Influence of the first cross sections of the piston on the stress wave shape at the interfaces between piston and rod at position D.

For the direct calculations, the initial conditions, the material properties and the geometries of the impacting rods are given. As a result of the direct solution the stress or strain of the rods can be determined. In contrast, the stress shape at an arbitrary position of the drilling rod is given over time if the inverse solution is calculated. The geometry of the piston is no longer given but adjusted in order to generate the desired target stress.

In Figure 1 the time evolution of the stress at the transition point D and the influence of the first cross sections of the piston on the stress shape is depicted. For a better understanding of the principle idea of the method, piston and rod are assumed to be homogeneous and of the same material. At the beginning, the piston which is traveling at constant initial velocity is hitting the rod which is at rest. This leads to a stress wave σ_0 that is both propagating into piston and rod. After the time Δt, which is the time it takes for the stress waves to travel twice the element length ℓ_E at wave propagation speed c_0, the influence of the second

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cross section A_2 can be observed on the graph. It is assumed that the target stress σ_{Tar} matches perfectly the actual stress σ_{Act} until $t = 3\Delta t$. The cross sections A_1 to A_3 are set accordingly. For the correct determination of the next cross section A_4, a first approach is to set $A_4 = A_3$. In general, the correct solution will not be achieved with that approach. However, the stress difference $\Delta\sigma$ between the actual stress and the target stress can be calculated. Since A_1, A_2, A_3 are already set to match the curve at the beginning, the only possibility to change the stress at this time is by adjusting cross section A_4. The corresponding formula for the difference of target stress and actual stress $\Delta\sigma$ with the known initial stress σ_0 calculates as follows

$$\Delta\sigma = \frac{2A_1}{A_1 + A_2}\frac{2A_2}{A_2 + A_3}\frac{A_4 - A_3}{A_4 + A_3}\frac{2A_3}{A_3 + A_2}\frac{2A_2}{A_2 + A_1}\sigma_0. \tag{2}$$

Therefore, the only unknown variable in equation (2) is A_4. For the next step, the cross sections A_1 to A_4 are given so that the only unknown variable will be A_5. Thus, this method enables to compute the cross sections iteratively.

3 Example and conclusion

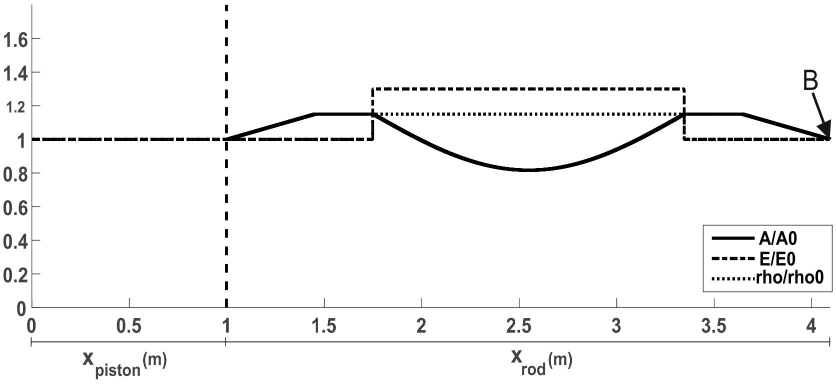

Fig. 2: Given parameters of the piston (on the left side of the vertical dashed line) and of the rod (on the right side of the vertical dashed line).

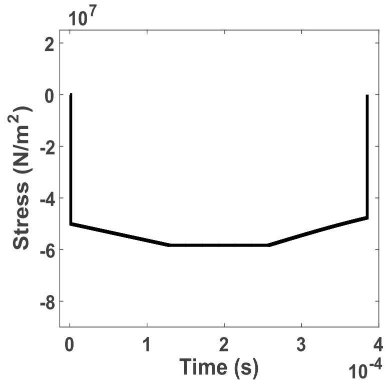

Fig. 3: Target stress at the end of the rod at position B.

In Figure 2 the parameters of piston and rod for the determination of the piston geometry are given. The piston is homogeneous whereas the material properties of the rod change in the middle of the rod. This can be interpreted as a rod part with different material. Moreover, the cross section of the rod varies as well. At the end of the rod at position B, the stress is predefined (Figure 3). The maximum time at which the stress can can be predefined is the time it takes for the wave front to travel twice the length of the piston. Afterwards, no cross section parts of the piston are left to fulfill the desired stress.
The rock is modeled as an infinite rod that has the same constant cross section and material properties as the rod at its end. Therefore, no waves will be reflected at the interface between rod and rock. The calculated cross section of the piston is depicted in Figure 4. The target stress can be determined exactly which can be proved by inserting the calculated cross section into the direct solution.

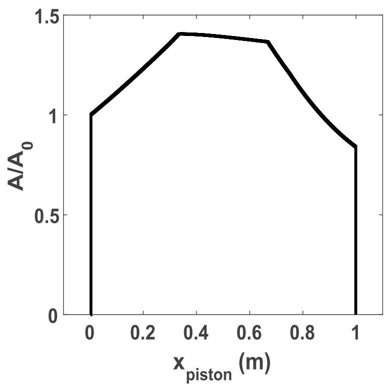

Fig. 4: Calculated cross section of the piston.

In this contribution, an iterative approach to determine the inverse solution of one dimensional impact problems with arbitrary cross sections and material parameters has been presented. The iterative approach enables to calculate the inverse solution exactly in a very short time ($\approx 2\,\mathrm{s}$ for 400 elements). For the application of rock drilling the modeling of the rock has to be revised since the modeling as an infinite rod does not describe sufficiently the interaction of the rod with the rock.

References

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