ROTA: Round Trip Times of Arrival for Localization with Unsynchronized Receivers

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***Abstract*—Multilateration systems reconstruct the location of a target that transmits electromagnetic or acoustic messages. The required measurement for localization is the time of arrival (TOA) of the transmitted signal, measured by a number of spatially distributed receivers. We present a novel multilateration algorithm that does not require a precise clock synchronization between the receivers. Furthermore, our method does not need any prior information about the transmitted signal. It just must be provided that accurate and identifiable TOA measurements can be derived. For example, the target could emit aperiodic pulses. Our method works with TOA differences between successive messages locally on each receiver. Therefore, our measurement equations do not depend on the receivers' clock offsets, and clock synchronization only needs to be accurate enough to identify each transmitted signal on all receivers. We derive a number of concrete maximum likelihood estimators for a single non-maneuvering target based on this method. In addition, we consider the clock drifts of the receivers.**

***Index Terms*—target tracking, multilateration, time of arrival, time difference of arrival, unsynchronized receivers.**

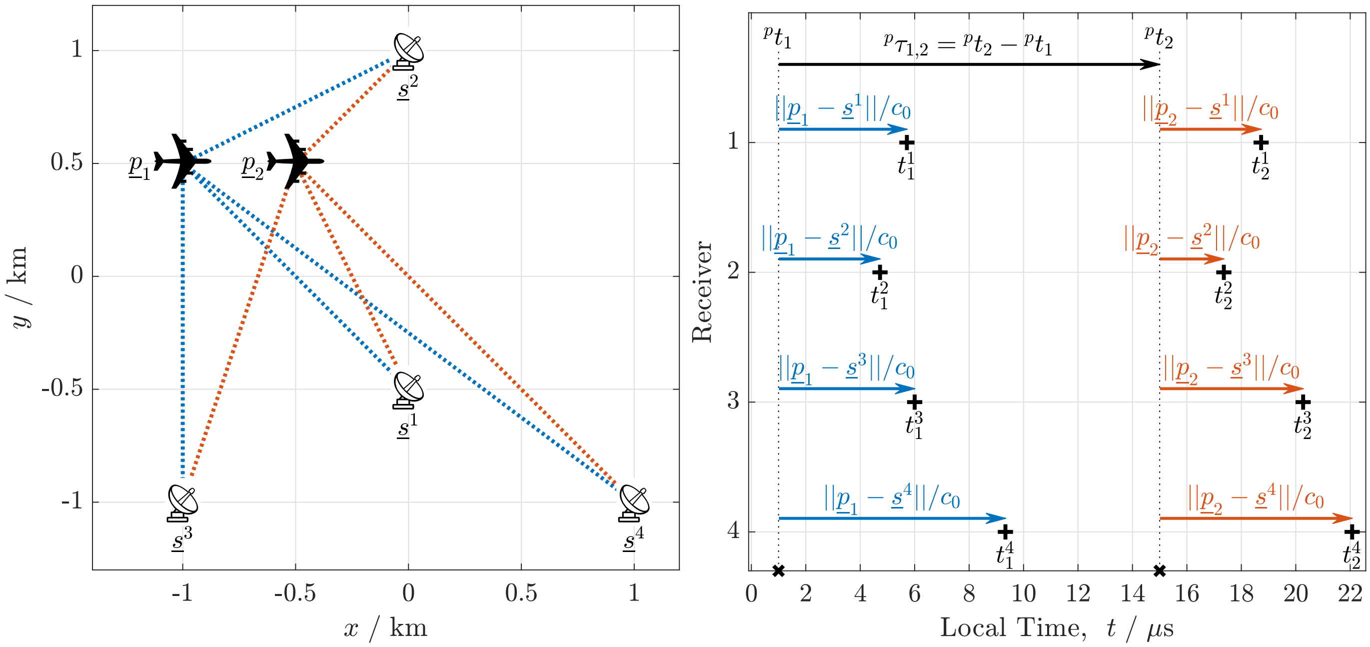
I. Introduction

a) Context: Electromagnetic and acoustic signals have many different properties that carry information and, when received at multiple sites, can be used to obtain the location of the signal transmitter. A wealth of algorithms have been proposed to locate transmitters based on one such property or a combination thereof. There is received signal strength (RSS)-based localization [1], [2], angle of arrival (AOA) or bearings localization [3], Doppler shift localization [4], [5], and time of arrival (TOA)-based localization.

Localization methods based on signal TOA are generally referred to as multilateration and have been widely studied [6]. For example, a target equipped with a transponder that sends messages traveling with a uniform propagation velocity can be localized when such messages are received at multiple sites. The same principle can also be used for navigation, i.e., an array of transmitters at known positions emits electromagnetic pulses simultaneously or with known time delays, and receivers in aircraft or ships measure the TOA and calculate the difference of times of arrival (DOTA) of the pulses at their position. This principle was first applied during the Second World War by the United States, called long range navigation system (LORAN) [7]. The corresponding "multilateration algorithm" for navigation involved looking into a map where many hyperbola were inscribed with their corresponding DOTA. Due to the simplicity and efficiency of the DOTA method, it is still used today [8]. Apart from the maximum likelihood estimation that has to be obtained iteratively, direct solutions have been proposed which are however suboptimal with noisy measurements [9]. Other methods directly use the TOA for localization [10], [11] or naviagation [12], without taking the difference of two TOA first. The target's messages can also be requested by an interrogation. This setup is called secondary surveillance radar (SSR) and can be used for localization, optionally together with DOTA [13]. Because the mathematically similar primary surveillance radar (PSR) is very expensive in acquisition and operation, and on top of that attains inferior accuracy, SSR is used instead wherever possible.

DOTA and TOA require the clocks in all receivers to be mutually synchronized, with an accuracy according to the propagation speed of the message. For electromagnetic waves, $1\,\mathrm{ns}$ corresponds to as much as $30\,\mathrm{cm}$, so the time synchronization necessary here can be costly [14], especially when receivers are hundreds of kilometers apart. One way to have everything synchronized is of course using GNSS [15], but this relies on external satellite infrastructure and an undisturbed radio reception of their signals, which should not be taken for granted in safety-critical systems [16].

b) State of Art: Methods for multilateration not requiring any accurate sensor synchronization reduce the overall complexity and communication overhead of distributed sensor networks. One basic idea to localize a moving target that inherently does not require synchronization was first introduced in [17]. Here the DOTA of two successive messages are measured locally at each receiver, so the constant offset of the receiver's clock is not relevant. This method was further elaborated in [18], where not only two successive messages but an entire transmission series in a time interval was considered. Furthermore, Kalman filter target tracking was implemented based on a linear constant velocity system model. However, this tracking was defined for two transmissions only, and the fact that successive measurements are actually correlated was noted but not considered. Clock drift was treated as general system noise and not explicitly modeled. For their method, time intervals between transmissions at the target must always be known.



(a) (b)

Fig. 1: Geometry plot (a) shows the position of an aircraft $\underline{p}_k$ at two time steps ${}^pt_k,\ \ k \in \{1, 2\}$. The aircraft transmits a message at ${}^pt_1$ and ${}^pt_2$, respectively. This message travels with a constant velocity in all directions and is detected by the receivers $\underline{s}^i,\ i \in \{1, 2, 3, 4\}$. In (b) you can see everything on the time axis. The first transmission takes place at ${}^pt_1 = 1\,\mu\text{s}$. From there, the signal travels through space (blue arrows) until it arrives at the individual receivers at times t_1^i (marked by fat plus signs). The second message is transmitted at ${}^pt_2 = 15\,\mu\text{s}$, i.e., ${}^p\tau_{1,2} = 14\,\mu\text{s}$ after the first transmission and arrives at the receivers at t_2^i. Signal traveling times (orange arrows) are different this time because the target has moved and hence the distances between receivers and aircraft have changed in the meantime. Each receiver now determines its local DOTA $\tau_{1,2}^i = t_2^i - t_1^i$. These are our synchronization-free measurements.

c) Contribution: We present a novel method to localize a target with asynchronous receivers that does not require the target transmission times to be known. Furthermore, we explicitly model the clock drift of the receivers and estimate it along with the target position.

II. Problem Formulation

We consider a single moving target at several successive positions $\underline{p}_k \in \mathbb{R}^D$ at time steps $k \in \{1, 2, \ldots, K\}$. The dimension D is typically two or three. The target transmits electromagnetic or acoustic messages at time steps ${}^pt_k$ traveling with uniform propagation speed c_0. The transmission times or transmission time intervals do not need to be known beforehand. Hence, the target's clock does not need to be synchronized with any other clock. Transmissions can be periodic or aperiodic.

Measurements are obtained by a sufficient number of receivers at locations $\underline{s}^i,\ i \in \{1, 2, \ldots, \mathcal{R}\}$. These receivers record the TOA t_k^i of the transmissions in terms of their own local clock. It is important to note that the $\mathcal{R}$ receivers are *not* time-synchronized with each other and also not synchronized with the target.

Our goal is to determine the target positions $\underline{p}_k$ at time steps ${}^pt_k$ based on the local time difference of arrival (L-TDOA) or local difference of times of arrival (L-DOTA) measured in the $\mathcal{R}$ receivers.

III. Key Idea

We will only use L-TDOA or L-DOTA. Hence, the receivers do not have to be synchronized. Time differences that have been obtained by taking the difference between two TOA from the same receiver we will call L-DOTA, and time differences that have been directly measured via cross correlation of two raw signals recorded on one receiver we will call L-TDOA. (In literature, differences of TOA are often called TDOA, too, but we use DOTA to emphasize their composite construction.) L-DOTA and L-TDOA differ in their measurement variance and covariance.

Now we formulate a "round-trip" in time as valid measurement equation. From Fig. 1 (b) we can see that it holds "black arrow length plus orange arrow length equals blue arrow length plus the measurement $\tau_{1,2}^i$." The equation states a relation between the L-DOTA or L-TDOA measurement $\tau_{1,2}^i = t_2^i - t_1^i$ of the receivers at position $\underline{s}^i \in \mathbb{R}^D$, and the unknown aircraft positions $\underline{p}_1$, $\underline{p}_2$, as well as the target's time difference of transmissions (TDOT) ${}^{p}\tau_{1,2} = {}^{p}t_2 - {}^{p}t_1$, and can be seen as a round trip through the times of arrival (ROTA)

$$ {}^{p}\tau_{1,2} + \frac{\left\| \underline{p}_2 - \underline{s}^i \right\|}{c_0} = \frac{\left\| \underline{p}_1 - \underline{s}^i \right\|}{c_0} + \tau^i_{1,2} \ , $$

which yields one measurement equation

$$ \tau^i_{1,2} = {}^{p}\tau_{1,2} + \frac{\left\| \underline{p}_2 - \underline{s}^i \right\| - \left\| \underline{p}_1 - \underline{s}^i \right\|}{c_0} \tag{1} $$

for every sensor $\underline{s}^i$, respectively,

$$ i \in \{1, 2, \ldots, \mathcal{R}\} \ . $$

IV. Measurement Equation

A. L-TDOA of Two Transmissions

For estimation and tracking, we deal with uncertain measurements. By treating unknowns and measurements in (1) as random variables and introducing additive Gaussian noise $\boldsymbol{v}^i_{k,l}$, we obtain our final measurement $\hat{\boldsymbol{\tau}}^i_{k,l}$ and the measurement equation of ROTA in its basic L-TDOA form

$$ \begin{aligned} \hat{\boldsymbol{\tau}}^i_{k,l} &= \boldsymbol{\tau}^i_{k,l} + \boldsymbol{v}^i_{k,l} \\ &= h^i(\underline{\boldsymbol{p}}_k, \underline{\boldsymbol{p}}_l, {}^{p}\boldsymbol{\tau}_{k,l}) + \boldsymbol{v}^i_{k,l} \ , \quad \text{with} \end{aligned} \tag{2} $$

$$ f^{\boldsymbol{v},i}_{k,l}(v) = \mathcal{N}(v;\ 0;\ C^{\boldsymbol{v},i}_{k,l}) = \frac{1}{\sqrt{2\pi\, C^{\boldsymbol{v},i}_{k,l} \exp\left\{ {}^{v^2}/_{C^{\boldsymbol{v},i}_{k,l}} \right\}}} $$

$$ \begin{aligned} \text{and} \quad & h^i(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) \\ &= {}^{p}\tau_{k,l} + \frac{\left\| \underline{p}_l - \underline{s}^i \right\| - \left\| \underline{p}_k - \underline{s}^i \right\|}{c_0} \ . \end{aligned} $$

We are going to formulate nonlinear optimization problems to obtain the maximum likelihood estimate. For efficient computation, we should provide the solver with analytic gradients of the nonlinear measurement function $h^i(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l})$, so we state these derivatives here, too.

$$ \frac{\partial}{\partial\, \underline{p}_k} h^i(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) = \frac{-\left(\underline{p}_k - \underline{s}^i \right)}{c_0 \left\| \underline{p}_k - \underline{s}^i \right\|} \ , $$

$$ \frac{\partial}{\partial\, \underline{p}_l} h^i(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) = \frac{\underline{p}_l - \underline{s}^i}{c_0 \left\| \underline{p}_l - \underline{s}^i \right\|} \ , $$

$$ \frac{\partial}{\partial\, {}^{p}\tau_{k,l}} h^i(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) = 1 \ . $$

Now we transform (2) into its probabilistic form

$$ \begin{aligned} & f^{\hat{\boldsymbol{\tau}},i}_{k,l}(\hat{\tau}^i_{k,l} \,|\, \underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}, v^i_{k,l}) \\ &\qquad = \delta\Big(h^i(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) + v^i_{k,l}) - \hat{\tau}^i_{k,l} \Big) \ , \end{aligned} $$

and by marginalizing over the joint probability distribution we obtain the likelihood given a single measurement $\hat{\tau}^i_{k,l}$,

$$ \begin{aligned} & f^{L,i}_{k,l}(\hat{\tau}^i_{k,l} \,|\, \underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) \\ &\quad = \int f^{\hat{\boldsymbol{\tau}},i}_{k,l}(\hat{\tau}^i_{k,l} \,|\, \underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}, v^i_{k,l})\, f^{\boldsymbol{v},i}_{k,l}(v^i_{k,l})\, \mathrm{d}v^i_{k,l} \\ &\quad = f^{\boldsymbol{v},i}_{k,l}\Big(\hat{\tau}^i_{k,l} - h^i(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) \Big) \\ &\quad = \mathcal{N}\Big(\hat{\tau}^i_{k,l};\ h^i(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l});\ C^{\boldsymbol{v},i}_{k,l} \Big) \ . \end{aligned} $$

Assuming the $\boldsymbol{v}^i_{k,l},\ i \in 1, 2, \ldots, \mathcal{R}$, to be mutually Gaussian distributed, the likelihood given $\mathcal{R}$ L-TDOA measurements $\hat{\tau}^i_{k,l}$ from $\mathcal{R}$ receivers reads

$$ \begin{aligned} & f^{L,1:\mathcal{R}}_{k,l}(\hat{\underline{\tau}}^{1:\mathcal{R}}_{k,l} \,|\, \underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) \\ &\qquad = \mathcal{N}\Big(\hat{\underline{\tau}}^{1:\mathcal{R}}_{k,l};\ \underline{h}^{1:\mathcal{R}}(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l});\ \mathbf{C}^{\boldsymbol{v},1:\mathcal{R}}_{k,l} \Big) \ , \\ & \hat{\underline{\tau}}^{1:\mathcal{R}}_{k,l} = \left[\hat{\tau}^1_{k,l}, \hat{\tau}^2_{k,l}, \ldots, \hat{\tau}^{\mathcal{R}}_{k,l} \right]^\top \ , \\ & \underline{h}^{1:\mathcal{R}}(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) = \begin{bmatrix} h^1(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) \\ h^2(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) \\ \vdots \\ h^{\mathcal{R}}(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) \end{bmatrix} \ . \end{aligned} $$

For $\mathcal{R}$ given L-TDOA measurements $\hat{\underline{\tau}}^{1:\mathcal{R}}_{k,l}$, we can now find the unknowns that maximize the likelihood. This involves nonlinear optimization and can be solved iteratively. The maximum likelihood estimate is equal to the Bayesian maximum a posteriori estimate without prior knowledge and can be used for initialization

$$ \begin{aligned} & \left(\hat{\underline{p}}^{ML}_k,\ \hat{\underline{p}}^{ML}_l,\ {}^{p}\hat{\tau}^{ML}_{k,l} \right) \\ &\quad = \underset{\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}}{\arg\max} \left\{ f^{L,1:\mathcal{R}}_{k,l}(\hat{\underline{\tau}}^{1:\mathcal{R}}_{k,l} \,|\, \underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) \right\} \\ &\quad = \underset{\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}}{\arg\min} \left\{ \left[\hat{\underline{\tau}}^{1:\mathcal{R}}_{k,l} - \underline{h}^{1:\mathcal{R}}(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) \right]^\top \right. \hookleftarrow \\ &\qquad \left. \left(\mathbf{C}^{\boldsymbol{v},1:\mathcal{R}}_{k,l} \right)^{-1} \left[\hat{\underline{\tau}}^{1:\mathcal{R}}_{k,l} - \underline{h}^{1:\mathcal{R}}(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) \right] \right\} \ . \end{aligned} \tag{3} $$

As the measurements errors of the L-TDOA from different receivers are mutually independent, the covariance matrix $\mathbf{C}^{\boldsymbol{v},1:\mathcal{R}}_{k,l}$ is diagonal

$$ \left[\mathbf{C}^{\boldsymbol{v},1:\mathcal{R}}_{k,l} \right]_{i,j} = \begin{cases} C^{\boldsymbol{v},i}_{k,l} = \left(\sigma^{\boldsymbol{v},i}_{k,l} \right)^2 \ , & i = j \\ 0 \ , & i \neq j \ . \end{cases} $$

This simplifies the maximum likelihood problem (3) a little, as it can be written as a weighted sum of squares of the deviations between the modeled measurements and the actual noisy measurements

$$ \begin{aligned} & \left(\hat{\underline{p}}^{ML}_k,\ \hat{\underline{p}}^{ML}_l,\ {}^{p}\hat{\tau}^{ML}_{k,l} \right) \\ & = \underset{\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}}{\arg\min} \left\{ \sum_{i=1}^{\mathcal{R}} \left(C^{\boldsymbol{v},i}_{k,l} \right)^{-1} \cdot \left(\hat{\tau}^i_{k,l} - h^i(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}) \right)^2 \right\} \ . \end{aligned} \tag{4} $$

This type of problem can be solved with the Levenberg-Marquardt algorithm [19].

Returning to the case of an exact model and exact measurements, we can see that there are $\mathcal{R}$ equations

$$\tau^i_{k,l} = h^i(\underline{p}_k, \underline{p}_l, {}^p\tau_{k,l})$$

confronted with five unknowns in 2D or seven unknowns in 3D. This way we can make up the balance and state that L-TDOA measurements of five or seven different receivers must be available in 2D or 3D, respectively, to find a solution. This is a lower bound, so there may be more measurements necessary in practice. More formally, we state the number of unknown variables V,

$$V = 2 \cdot D + 1 \ ,$$

and the number of equations E,

$$E = \mathcal{R} \ .$$

Now we demand having at least as many equations as unknowns

$$E \geq V \ \Rightarrow \ \mathcal{R} \geq 2 \cdot D + 1 \ . \tag{5}$$

B. L-DOTA of Two Transmissions

Cross correlations between pairs of signal wave forms are expensive in terms of storage and computational power. If just the TOA $\hat{\boldsymbol{t}}^i_k$ are measured, we can still subsequently subtract them and get a L-DOTA measurement $\hat{\boldsymbol{\tau}}^i_{k,l}$. We first define the noisy TOA measurements $\hat{\boldsymbol{t}}^i_k$ that include mutually independent additive zero-mean Gaussian noise $\boldsymbol{v}^i_k$

$$\hat{\boldsymbol{t}}^i_k = \boldsymbol{t}^i_k + \boldsymbol{v}^i_k \ .$$

From there, we come to L-DOTA measurements by subtraction

$$\begin{aligned} \hat{\boldsymbol{\tau}}^i_{k,l} &= \hat{\boldsymbol{t}}^i_l - \hat{\boldsymbol{t}}^i_k \\ &= \boldsymbol{t}^i_l - \boldsymbol{t}^i_k + \boldsymbol{v}^i_l - \boldsymbol{v}^i_k \\ &= \boldsymbol{\tau}^i_{k,l} + \boldsymbol{v}^i_{k,l} \ , \\ \text{with} \quad \boldsymbol{v}^i_{k,l} &= \boldsymbol{v}^i_l - \boldsymbol{v}^i_k \ . \end{aligned} \tag{6}$$

The L-DOTA measurement noise has a mean of zero

$$\mathrm{E}\{\boldsymbol{v}^i_{k,l}\} = \mathrm{E}\{\boldsymbol{v}^i_l - \boldsymbol{v}^i_k\} = \mathrm{E}\{\boldsymbol{v}^i_l\} - \mathrm{E}\{\boldsymbol{v}^i_k\} = 0 \ ,$$

and for the covariance, it follows

$$\begin{aligned} C^{\boldsymbol{v},i}_{k,l} &= \mathrm{Var}\{\boldsymbol{v}^i_{k,l}\} \\ &= \mathrm{Var}\{\boldsymbol{v}^i_l - \boldsymbol{v}^i_k\} \\ &= \mathrm{E}\Big\{\big(\boldsymbol{v}^i_l - \boldsymbol{v}^i_k\big)^2\Big\} \\ &= \mathrm{E}\Big\{\big(\boldsymbol{v}^i_l\big)^2 + \big(\boldsymbol{v}^i_k\big)^2 - 2\,\boldsymbol{v}^i_k \boldsymbol{v}^i_l\Big\} \\ &= \mathrm{E}\Big\{\big(\boldsymbol{v}^i_l\big)^2\Big\} + \mathrm{E}\Big\{\big(\boldsymbol{v}^i_k\big)^2\Big\} - 2\,\mathrm{E}\{\boldsymbol{v}^i_k \boldsymbol{v}^i_l\} \\ &= \mathrm{Var}\{\boldsymbol{v}^i_k\} + \mathrm{Var}\{\boldsymbol{v}^i_l\} - 2\,\mathrm{Cov}\{\boldsymbol{v}_k, \boldsymbol{v}_l\} \\ &= \begin{cases} \mathrm{Var}\{\boldsymbol{v}^i_k\} + \mathrm{Var}\{\boldsymbol{v}^i_l\} \ , & k \neq l \\ 0 \ , & k = l \ . \end{cases} \end{aligned}$$

Compared to L-TDOA, the L-DOTA covariance matrix has the same structure, only the variances are higher

$$\Big[\mathbf{C}^{\boldsymbol{v},1:\mathcal{R}}_{k,l}\Big]_{i,j} = \begin{cases} \mathrm{Var}\{\boldsymbol{v}^i_k\} + \mathrm{Var}\{\boldsymbol{v}^i_l\} \ , & i = j \\ 0 \ , & i \neq j \ , \end{cases} \quad k \neq l \ . \tag{7}$$

With these variances, the maximum likelihood estimator (3) can be also used for L-DOTA.

C. L-TDOA of Many Transmissions

We can increase the ratio of the number of equations to the number of unknowns by extending the model over multiple time steps. Let's say we consider $\mathcal{M}$ successive transmissions over time and independently determine the

$$\binom{\mathcal{M}}{2} = \frac{\mathcal{M} \cdot (\mathcal{M} - 1)}{2}$$

possible L-TDOA between all possible pairs of transmissions via multiple cross correlations of the raw signal wave forms. Formally, we first enumerate the L-TDOA between pairs of received signals

$$\hat{\boldsymbol{\tau}}^i_{k_m,l_m} \ , \quad m \in \left\{1, 2, \ldots, \binom{\mathcal{M}}{2}\right\}$$

and plug it into (2)

$$\hat{\boldsymbol{\tau}}^i_{k_m,l_m} = h^i(\underline{\boldsymbol{p}}_{k_m}, \underline{\boldsymbol{p}}_{l_m}, {}^p\boldsymbol{\tau}_{k_m,l_m}) + \boldsymbol{v}^i_{k_m,l_m} \ .$$

Then we obtain the iterative maximum likelihood estimator analog to (4)

$$\begin{aligned} &\left(\hat{\underline{p}}^{ML}_1, \hat{\underline{p}}^{ML}_2, \ldots, \hat{\underline{p}}^{ML}_{\mathcal{M}}, {}^p\hat{\tau}^{ML}_{1,2}, {}^p\hat{\tau}^{ML}_{2,3}, \ldots, {}^p\hat{\tau}^{ML}_{\mathcal{M}-1,\mathcal{M}}\right) \\ &\quad = \underset{(\ldots)}{\arg\min} \left\{ \sum_{i=1}^{\mathcal{R}} \sum_{m=1}^{\binom{\mathcal{M}}{2}} \left(C^{\boldsymbol{v},i}_{k_m,l_m}\right)^{-1} \right. \\ &\qquad \left. \cdot \left(\hat{\tau}^i_{k_m,l_m} - h^i(\underline{p}_{k_m}, \underline{p}_{l_m}, {}^p\tau_{k_m,l_m})\right)^2 \right\} \ . \end{aligned} \tag{8}$$

If $k_m - l_m > 1$, then ${}^p\tau_{k_m,l_m}$ is derived by summing up the target transmission intervals between k_m and l_m. Now we can make up the new balance between the number of unknowns V

$$V = \mathcal{M} \cdot D \ + \ \mathcal{M} - 1 \tag{9}$$

and the number of equations E

$$E = \binom{\mathcal{M}}{2} \cdot \mathcal{R} \ .$$

For the balance it follows

$$E \geq V \ \Rightarrow \ \mathcal{R} \geq 2 \cdot \frac{\mathcal{M} \cdot (D+1) - 1}{\mathcal{M} \cdot (\mathcal{M} - 1)} \ ,$$

or simpler,

$$E \geq V + 1 \ \Rightarrow \ \mathcal{R} \geq \frac{2D}{\mathcal{M} - 1} \ .$$

It may thus be possible to reduce the minimum number of required receivers $\mathcal{R}$ compared to (5).

D. L-DOTA of Many Transmissions

In this section we will transfer IV-C to L-DOTA measurements. That is, we directly measure two TOA $\hat{\boldsymbol{t}}_k^i, \hat{\boldsymbol{t}}_l^i$ at each receiver and obtain the L-DOTA $\hat{\boldsymbol{\tau}}_{k,l}^i$ by taking the difference between them

$$\hat{\boldsymbol{\tau}}_{k,l}^i = \hat{\boldsymbol{t}}_l^i - \hat{\boldsymbol{t}}_k^i \ .$$

Assuming that each TOA measurement $\hat{\boldsymbol{t}}_k^i$ contains additive white Gaussian noise $\boldsymbol{v}_k^i$

$$\hat{\underline{\boldsymbol{t}}}_k^i = \boldsymbol{t}_k^i + \boldsymbol{v}_k^i \ ,$$

when we set up the L-DOTA measurement as in (6),

$$\hat{\boldsymbol{\tau}}_{k,l}^i = \boldsymbol{\tau}_{k,l}^i + \boldsymbol{v}_{k,l}^i \ ,$$

we obtain again the variance of the measurement noise

$$\mathrm{Var}\{\boldsymbol{v}_{k,l}^i\} = \begin{cases} \mathrm{Var}\{\boldsymbol{v}_k^i\} + \mathrm{Var}\{\boldsymbol{v}_l^i\} \ , & k \neq l \\ 0 \ , & k = l \ . \end{cases}$$

L-DOTA measurements of more than two successive messages can be correlated, and the covariance matrix is not diagonal anymore

$$\begin{aligned}
&\mathrm{Cov}\{\boldsymbol{v}_{k_1,l_1}^i, \boldsymbol{v}_{k_2,l_2}^i\} \\
&= \mathrm{E}\{\boldsymbol{v}_{k_1,l_1}^i \cdot \boldsymbol{v}_{k_2,l_2}^i\} \\
&= \mathrm{E}\{(\boldsymbol{v}_{l_1}^i - \boldsymbol{v}_{k_1}^i) \cdot (\boldsymbol{v}_{l_2}^i - \boldsymbol{v}_{k_2}^i)\} \\
&= \mathrm{E}\{\boldsymbol{v}_{l_1}^i \boldsymbol{v}_{l_2}^i - \boldsymbol{v}_{l_1}^i \boldsymbol{v}_{k_2}^i - \boldsymbol{v}_{k_1}^i \boldsymbol{v}_{l_2}^i + \boldsymbol{v}_{k_1}^i \boldsymbol{v}_{k_2}^i\} \\
&= \mathrm{Cov}\{\boldsymbol{v}_{l_1}^i, \boldsymbol{v}_{l_2}^i\} - \mathrm{Cov}\{\boldsymbol{v}_{l_1}^i, \boldsymbol{v}_{k_2}^i\} \hookleftarrow \\
&\quad - \mathrm{Cov}\{\boldsymbol{v}_{k_1}^i, \boldsymbol{v}_{l_2}^i\} + \mathrm{Cov}\{\boldsymbol{v}_{k_1}^i, \boldsymbol{v}_{k_2}^i\}
\end{aligned}$$

$$\text{with } \mathrm{Cov}\{\boldsymbol{v}_k^i, \boldsymbol{v}_l^i\} = \begin{cases} \mathrm{Var}\{\boldsymbol{v}_k^i\} \ , & k = l \\ 0 \ , & k \neq l \ . \end{cases}$$

It does not make sense to use all $\binom{\mathcal{M}}{2}$ L-DOTA here, as the covariance matrix would become singular. Two typical cases are taking all differences between directly successive TOA

$$\begin{aligned}
&(k_1 < l_1) \ \wedge \ (l_1 = k_2) \ \wedge \ (k_2 < l_2) \\
&\Rightarrow \quad \mathrm{Cov}\{\boldsymbol{v}_{k_1,l_1}^i, \boldsymbol{v}_{k_2,l_2}^i\} = -\mathrm{Var}\{\boldsymbol{v}_{k_2}^i\} \ ,
\end{aligned} \tag{10}$$

and taking the differences between one specific TOA and all others

$$\begin{aligned}
&(k_1 = k_2) \ \wedge \ (l_1 > k_1) \ \wedge \ (l_2 > k_1) \ \wedge \ (l_1 \neq l_2) \\
&\Rightarrow \quad \mathrm{Cov}\{\boldsymbol{v}_{k_1,l_1}^i, \boldsymbol{v}_{k_2,l_2}^i\} = \mathrm{Var}\{\boldsymbol{v}_{k_1}^i\} \ .
\end{aligned} \tag{11}$$

As we only take local time differences, the covariance between L-DOTA of different receivers i, j is zero

$$\mathrm{Cov}\left\{\boldsymbol{v}_{k_1,l_1}^i, \boldsymbol{v}_{k_2,l_2}^j\right\} = 0 \,, \quad i \neq j \ .$$

At each receiver, from $\mathcal{M}$ TOA $\hat{t}_k^i$, we can get $\mathcal{M}-1$ L-DOTA measurements $\hat{\boldsymbol{\tau}}_{k_m,l_m}^i$, $m \in \{1, 2, \ldots, \mathcal{M}-1\}$,

$$\hat{\boldsymbol{\tau}}_{k_m,l_m}^i = h^i(\underline{\boldsymbol{p}}_{k_m}, \underline{\boldsymbol{p}}_{l_m}, {}^p\boldsymbol{\tau}_{k_m,l_m}) + \boldsymbol{v}_{k_m,l_m}^i \ .$$

Their noise is correlated but not linearly dependent, i.e., the covariance matrix $\mathbf{C}_{1:\mathcal{M}-1}^{\boldsymbol{v},i}$ is non-diagonal but still has full rank. Analog to (3), we get the iterative maximum likelihood estimator

$$\begin{aligned}
&\left(\hat{\underline{p}}_1^{ML}, \hat{\underline{p}}_2^{ML}, \ldots, \hat{\underline{p}}_{\mathcal{M}}^{ML}, {}^p\hat{\tau}_{1,2}^{ML}, {}^p\hat{\tau}_{2,3}^{ML}, \ldots, {}^p\hat{\tau}_{\mathcal{M}-1,\mathcal{M}}^{ML}\right) \\
&\quad = \arg\min_{(\ldots)} \left\{ \sum_{i=1}^{\mathcal{R}} \left[\hat{\underline{\tau}}_{1:\mathcal{M}-1}^i - \underline{h}_{1:\mathcal{M}-1}^i(\ldots)\right]^\top \hookleftarrow \right. \\
&\qquad \left. \left(\mathbf{C}_{1:\mathcal{M}-1}^{\boldsymbol{v},i}\right)^{-1} \left[\hat{\underline{\tau}}_{1:\mathcal{M}-1}^i - \underline{h}_{1:\mathcal{M}-1}^i(\ldots)\right] \right\} \ ,
\end{aligned} \tag{12}$$

with

$$\hat{\underline{\tau}}_{1:\mathcal{M}-1}^i = \begin{bmatrix} \hat{\tau}_{k_1,l_1}^i \\ \hat{\tau}_{k_2,l_2}^i \\ \vdots \\ \hat{\tau}_{k_{\mathcal{M}-1},l_{\mathcal{M}-1}}^i \end{bmatrix}$$

and

$$\underline{h}_{1:\mathcal{M}-1}^i(\ldots) = \begin{bmatrix} h^i(\underline{p}_{k_1}, \underline{p}_{l_1}, {}^p\tau_{k_1,l_1}) \\ h^i(\underline{p}_{k_2}, \underline{p}_{l_2}, {}^p\tau_{k_2,l_2}) \\ \vdots \\ h^i(\underline{p}_{k_{\mathcal{M}-1}}, \underline{p}_{l_{\mathcal{M}-1}}, {}^p\tau_{k_{\mathcal{M}-1},l_{\mathcal{M}-1}}) \end{bmatrix} \ .$$

The covariance matrix is

$$\left[\mathbf{C}_{1:\mathcal{M}-1}^{\boldsymbol{v},i}\right]_{m,n} = \begin{cases} \mathrm{Var}\left\{\boldsymbol{v}_{k_m,l_m}^i\right\} \ , & m = n \\ \mathrm{Cov}\left\{\boldsymbol{v}_{k_m,l_m}^i, \boldsymbol{v}_{k_n,l_n}^i\right\} \ , & m \neq n \ , \end{cases}$$

which for the typical choices of L-DOTA is

$$\left[\mathbf{C}_{1:\mathcal{M}-1}^{\boldsymbol{v},i}\right]_{m,n} = \begin{cases} \mathrm{Var}\{\boldsymbol{v}_{k_m}^i\} + \mathrm{Var}\{\boldsymbol{v}_{l_m}^i\} \ , & m = n \\ -\mathrm{Var}\{\boldsymbol{v}_{k_n}^i\} \ , & l_m = k_n \\ \mathrm{Var}\{\boldsymbol{v}_{k_n}^i\} \ , & k_m = k_n \\ 0 \ , & \text{elsewhere} \ . \end{cases}$$

We will write down the covariance matrix explicitly for the previously mentioned two special cases under the assumption that the measurement noise $\boldsymbol{v}^i$ is stationary. For differences between directly successive TOA (10),

$$\mathbf{C}_{1:\mathcal{M}-1}^{\boldsymbol{v},i} = \left(\sigma_{\boldsymbol{v}}^i\right)^2 \cdot \begin{bmatrix} 2 & -1 & & & 0 \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ 0 & & & -1 & 2 \end{bmatrix} \ ,$$

and when differences are taken between the first TOA and all others (11),

$$\mathbf{C}_{1:\mathcal{M}-1}^{\boldsymbol{v},i} = \left(\sigma_{\boldsymbol{v}}^i\right)^2 \cdot \begin{bmatrix} 2 & 1 & & & 1 \\ 1 & 2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 2 & 1 \\ 1 & & & 1 & 2 \end{bmatrix} \ .$$

As it turns out, the Levenberg-Marquardt nonlinear least squares (LS) method exhibits superior runtime and convergence to the global optimum from any starting point in the maximum likelihood problems discussed here. Thus, we are going to transform (12) into a pure sum of squares. For this purpose, we need to decompose the inverse covariance matrix. Since it is positive definite, we can use its Cholesky decomposition

$$\mathbf{C}^{-1} = \mathbf{R}^\top \mathbf{R} \ .$$

Then we can rewrite the quadratic form

$$\underline{\xi}^\top \mathbf{C}^{-1} \underline{\xi} = \underline{\xi}^\top \mathbf{R}^\top \mathbf{R} \, \underline{\xi} = \left(\mathbf{R}\, \underline{\xi}\right)^\top \left(\mathbf{R}\, \underline{\xi}\right)$$

into a sum of squares

$$\underline{\xi}^\top \mathbf{C}^{-1} \underline{\xi} = \sum_m \left(\left[\mathbf{R}\, \underline{\xi} \, \right]_m \right)^2 \ ,$$

and obtain the maximum likelihood estimate using a Levenberg-Marquardt optimizer. If analytic gradients are supplied, they must then of course also be multiplied accordingly with the Cholesky decomposition $\mathbf{R}$ of the inverse covariance matrix.

Again we count the number of equations E and unknowns V. In any case we need at least as many equations than unknowns to obtain a unique optimum. A theoretical lower bound for the number of receivers $\mathcal{R}$ is

$$E \geq V \ \ \Rightarrow \ \ \mathcal{R} \geq \frac{\mathcal{M}}{\mathcal{M}-1} \cdot D \, + \, 1 \ .$$

V. Measurement Equation With Clock Drift

A. Clock Drift

Even if precise clocks are used in the receivers, a certain drift is unavoidable [20]. The clocks may run at a very constant rate, but not at exactly the true speed. Clock drift typically is on the order of $100\,\mathrm{ns/s}$, which, multiplied by the speed of light, corresponds to $30\,\mathrm{m/s}$. As our basic measurement is the L-DOTA $\tau^i_{k,l} = t^i_l - t^i_k$ spanning over successive transmissions and hence is on the order of seconds to minutes, we cannot neglect the drift here. We do assume that the drift is stable, though. At receiver i, instead of the true time t^i_k, we measure a transformed value $\tilde{t}^i_k$, according to the clock offset b^i and the clock drift δ^i of the particular receiver

$$\tilde{t}^i_k = t^i_k \cdot \left(1 + \delta^i\right) + b^i \ .$$

This brings us to the L-DOTA measurement with clock drift

$$\begin{aligned} \tilde{\tau}^i_{k,l} &= \tilde{t}^i_l - \tilde{t}^i_k \\ &= t^i_l \cdot \left(1 + \delta^i\right) + b^i \ - \ \left(t^i_k \cdot \left(1 + \delta^i\right) + b^i\right) \\ &= \left(t^i_l - t^i_k\right) \cdot \left(1 + \delta^i\right) \\ &= \tau^i_{k,l} \cdot \left(1 + \delta^i\right) \ . \end{aligned}$$

Plugging this measurement into (1), we arrive at the more detailed ROTA measurement model that includes the drift

$$\tilde{\tau}^i_{k,l} = \frac{1+\delta^i}{c_0} \cdot \left(c_0 \, {}^p\tau_{k,l} + \left\| \underline{p}_l - \underline{s}^i \right\| - \left\| \underline{p}_k - \underline{s}^i \right\| \right) \ . \quad (13)$$

B. L-TDOA of Many Transmissions With Clock Drift

Especially with L-TDOA of many transmissions, we can have much more equations than unknowns, so we can think about including clock drift into our model. Here, we assume that every receiver has its own drift which is constant over the window of transmissions that we consider in the measurement equation. The stochastic measurement equation is derived by adding zero-mean white Gaussian noise to (13)

$$\begin{aligned} \hat{\boldsymbol{\tau}}^i_{k_m,l_m} &= h^i(\underline{\boldsymbol{p}}_{k_m}, \underline{\boldsymbol{p}}_{l_m}, \boldsymbol{\delta}^i, {}^p\boldsymbol{\tau}_{k_m,l_m}) + \boldsymbol{v}^i_{k_m,l_m} \ , \\ & m \in \left\{ 1, 2, \ldots, \binom{\mathcal{M}}{2} \right\} \ , \end{aligned}$$

with

$$\begin{aligned} & h^i(\underline{p}_{k_m}, \underline{p}_{l_m}, \delta^i, {}^p\tau_{k_m,l_m}) \qquad (14) \\ &= \frac{1+\delta^i}{c_0} \cdot \left(c_0 \, {}^p\tau_{k_m,l_m} + \left\| \underline{p}_{l_m} - \underline{s}^i \right\| - \left\| \underline{p}_{k_m} - \underline{s}^i \right\| \right) \ . \end{aligned}$$

Again we calculate the derivatives for improved numerical efficiency during optimization

$$\begin{aligned} \frac{\partial}{\partial \, \underline{p}_{k_m}} \ h^i(\ldots) &= \frac{-\left(1+\delta^i\right) \, \left(\underline{p}_{k_m} - \underline{s}^i\right)}{c_0 \left\| \underline{p}_{k_m} - \underline{s}^i \right\|} \ , \\ \frac{\partial}{\partial \, \underline{p}_{l_m}} \ h^i(\ldots) &= \frac{\left(1+\delta^i\right) \, \left(\underline{p}_{l_m} - \underline{s}^i\right)}{c_0 \left\| \underline{p}_{l_m} - \underline{s}^i \right\|} \ , \\ \frac{\partial}{\partial \, {}^p\tau_{k_m,l_m}} \ h^i(\ldots) &= \left(1+\delta^i\right) \ , \\ \frac{\partial}{\partial \, \delta^i} \ h^i(\ldots) &= \ \hookleftarrow \\ {}^p\tau_{k_m,l_m} &+ \tfrac{1}{c_0} \left(\left\| \underline{p}_{l_m} - \underline{s}^i \right\| - \left\| \underline{p}_{k_m} - \underline{s}^i \right\| \right) \ . \end{aligned}$$

We obtain the iterative maximum likelihood estimator analog to (8). The difference is basically that the nonlinear measurement function $h^i(\ldots)$ includes drift now. It is also a nonlinear LS problem

$$\begin{aligned} & \left(\underline{\hat{p}}^{ML}_1, \ldots, \ \underline{\hat{p}}^{ML}_{\mathcal{M}}, \ {}^p\hat{\tau}^{ML}_{1,2}, \ldots, \ {}^p\hat{\tau}^{ML}_{\mathcal{M}-1,\mathcal{M}}, \ \hat{\delta}^{1,ML}, \ldots, \ \hat{\delta}^{\mathcal{R},ML} \right) \\ &= \arg\min_{(\ldots)} \left\{ \sum_{i=1}^{\mathcal{R}} \sum_{m=1}^{\binom{\mathcal{M}}{2}} \left(C^{\boldsymbol{v},i}_{k_m,l_m} \right)^{-1} \right. \hookleftarrow \\ & \qquad \left. \cdot \left(\hat{\tau}^i_{k_m,l_m} - h^i(\underline{p}_{k_m}, \underline{p}_{l_m}, \delta^i, {}^p\tau_{k_m,l_m}) \right)^2 \right\} \ , \end{aligned}$$

and can be solved with the Levenberg-Marquardt nonlinear LS method. Compared to (9), we need $\mathcal{R}$ more equations $E = \binom{\mathcal{M}}{2}$ – one per receiver for its drift. Hence, the theoretical lower bound of the number of receivers is

$$E \geq V \ \ \Rightarrow \ \ \mathcal{R} \geq 2 \cdot \frac{\mathcal{M} \cdot (D+1) - 1}{(\mathcal{M}+1)\,(\mathcal{M}-2)} \ . \qquad (15)$$

C. L-DOTA of Many Transmissions With Clock Drift

Acquisition of $\binom{\mathcal{M}}{2}$ L-TDOA via individual cross-correlations can be tedious, so we see how far we can get with L-DOTA measurements that can be derived from simple TOA measurements. Note that we only get up to $\mathcal{M}-1$ L-DOTA measurements per receiver, and they are correlated. We obtain a maximum likelihood estimator similar to (12),

$$\hat{\boldsymbol{\tau}}^i_{k_m,l_m} = h^i(\underline{\boldsymbol{p}}_{k_m}, \underline{\boldsymbol{p}}_{l_m}, \boldsymbol{\delta}^i, {}^p\boldsymbol{\tau}_{k_m,l_m}) + \boldsymbol{v}^i_{k_m,l_m} \quad ,$$
$$m \in \{1, 2, \ldots, \mathcal{M}-1\} \quad ,$$

only this time with the the additional unknowns $\delta^1, \ldots, \delta^{\mathcal{R}}$,

$$\underline{h}^i_{1:\mathcal{M}-1}(\ldots) = \begin{bmatrix} h^i(\underline{p}_{k_1}, \underline{p}_{l_1}, \delta^i, {}^p\tau_{k_1,l_1}) \\ h^i(\underline{p}_{k_2}, \underline{p}_{l_2}, \delta^i, {}^p\tau_{k_2,l_2}) \\ \vdots \\ h^i(\underline{p}_{k_{\mathcal{M}-1}}, \underline{p}_{l_{\mathcal{M}-1}}, \delta^i, {}^p\tau_{k_{\mathcal{M}-1},l_{\mathcal{M}-1}}) \end{bmatrix} \quad ,$$

and with a different measurement function $h^i(\ldots)$ including drift (14). It can be written as a maximum likelihood problem

$$\left(\underline{\hat{p}}_1^{ML}, \ldots, \underline{\hat{p}}_{\mathcal{M}}^{ML}, {}^p\hat{\tau}^{ML}_{1,2}, \ldots, {}^p\hat{\tau}^{ML}_{\mathcal{M}-1,\mathcal{M}}, \hat{\delta}^{1,ML}, \ldots, \hat{\delta}^{\mathcal{R},ML}\right)$$
$$= \arg\min_{(\ldots)} \left\{ \sum_{i=1}^{\mathcal{R}} \left[\underline{\hat{\tau}}^i_{1:\mathcal{M}-1} - \underline{h}^i_{1:\mathcal{M}-1}(\ldots)\right]^\top \right. \hookleftarrow$$
$$\left. \left(\mathbf{C}^{\boldsymbol{v},i}_{1:\mathcal{M}-1}\right)^{-1} \left[\underline{\hat{\tau}}^i_{1:\mathcal{M}-1} - \underline{h}^i_{1:\mathcal{M}-1}(\ldots)\right] \right\} \quad ,$$

and with Cholesky decomposition

$$\left(\mathbf{C}^{\boldsymbol{v},i}_{1:\mathcal{M}-1}\right)^{-1} = \left(\mathbf{R}^{\boldsymbol{v},i}_{1:\mathcal{M}-1}\right)^\top \left(\mathbf{R}^{\boldsymbol{v},i}_{1:\mathcal{M}-1}\right) \quad ,$$

also as sum of squares

$$\arg\min_{(\ldots)} \left\{ \sum_{i=1}^{\mathcal{R}} \sum_{m=1}^{\mathcal{M}-1} \left[\mathbf{R}^{\boldsymbol{v},i}_{1:\mathcal{M}-1} \left[\underline{\hat{\tau}}^i_{1:\mathcal{M}-1} - \underline{h}^i_{1:\mathcal{M}-1}(\ldots)\right]\right]^2_m \right\} \quad ,$$

so it can be solved with the Levenberg-Marquardt nonlinear LS method.

Here we have less equations $E = (\mathcal{M}-1) \cdot \mathcal{R}$ than with the L-TDOA measurements. However, for all $\mathcal{M} \geq 3$, we can find a number of receivers $\mathcal{R}$ such that there are more equations than unknown variables,

$$\mathcal{R} \geq \frac{\mathcal{M} \cdot (D+1) - 1}{\mathcal{M} - 2} \quad .$$

Comparing this to (15), we see that $\frac{\mathcal{M}+1}{2}$ times as many L-DOTA receivers as L-TDOA receivers are necessary for a given time horizon $\mathcal{M}$ to obtain a balanced equation system.

VI. Evaluation

In this section, we evaluate the ROTA measurement model by employing a selection of measurement processing methods from Sec. IV on a specific arrangement of targets and receivers. A variable number of receivers is placed on a circle with a radius of $5\,\mathrm{km}$, see Fig. 2 (b) for five receivers. Inside this sensor array, successive positions of one moving target are placed. As we do not apply a system model herein, the concrete track is rather abitrary. First, all correct measurements are determined according to (1), and Gaussian noise is added via deterministic sampling. Standard deviation of the primary TOA measurements is set to $3\,\mathrm{ns}$ which, multiplied with the speed of light, corresponds to $0.9\,\mathrm{m}$. For the L-DOTA values consequently holds $\sigma_{\boldsymbol{v}} \approx 4.2\,\mathrm{ns}$ after subtraction (7). With these noisy measurements, the maximum likelihood problem (12) is solved using the Levenberg-Marquardt algorithm with analytic gradients. This yields estimated positions, of which root mean square error (RMSE) is determined. On the ordinate of Fig. 2 (a), the mean RMSE of the $\mathcal{M}$ target positions is shown respectively.

A common mistake in multilateration is to simply apply unweighted LS estimation. This corresponds to the dark blue and orange line in Fig. 2 (a), and unsurprisingly, they have the largest RMSE. Incorrectly assuming a diagonal covariance matrix is more problematic for successively chosen DOTA (10) than for the "star-shaped" model (11). When the LS estimator is correctly weighted however, both DOTA choices perform equally (yellow and purple line).

VII. Conclusion

The paper presents a systematic derivation of a new method for performing MLAT of a single target with multiple unsynchronized receivers. The method is very flexible as it does not require information about the transmission times at the target.

The measurement equations relating the target positions with time differences measured locally at the individual receivers are derived in detail. In addition, a maximum likelihood method is derived for estimating the target positions based on a standard nonlinear LS method (Levenberg-Marquardt).

Drifts of the local clocks in the individual receivers are explicitly considered during the estimation. They cannot be neglected as the time difference between target transmissions can be on the order of several seconds, where clock drift would lead to errors on the order of hundreds of meters.

VIII. Future Work

So far, we focused on the static measurement problem: The position of a target is calculated based on a finite number of transmissions of the target. The next step is to perform tracking of the target over time. In order to fully exploit the measurement information and to also provide an update with each new measurement, a sliding window of measurements will be used instead of consecutive, mutually exclusive measurement windows. Of course, this approach leads to correlations that have to be taken into account, which requires an appropriate estimation mechanism. In addition, a model for the target motion is required.

Two processing modes will be investigated. The first mode is batch processing, where measurements are collected over a certain time span and the target track is calculated all at once in batch mode. The second mode is recursive online processing, where every new measurement is assimilated in real time.

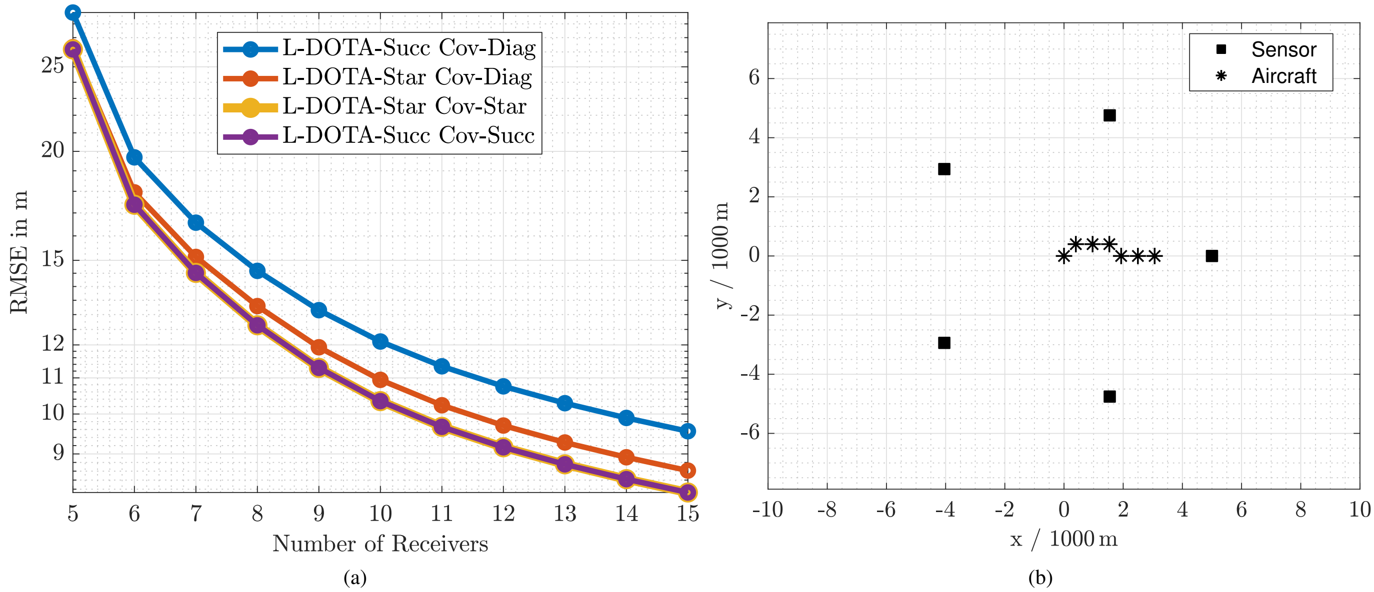


Fig. 2: Simulation with $\mathcal{M} = 7$ message transmissions according to Sec. IV-D. (a) RMSE plot for different numbers of receivers (abscissa) and various measurement processing methods. "L-DOTA-Succ Cov-Diag" means for example that differences between successive TOA have been used as in (10), and yet a diagonal covariance matrix was (incorrectly) assumed. (b) Geometry of the arrangement in this evaluation example, for $\mathcal{R} = 5$ receivers.

Finally, the proposed method will be extensively evaluated with real data. For this purpose, Frequentis Comsoft in Karlsruhe-Durlach (frequentis-comsoft.com) will provide data sets from an operational wide area MLAT system.

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