State Estimation in Networked Control Systems With Delayed And Lossy Acknowledgments

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Abstract— **In this paper, we consider state estimation in Networked Control Systems where both control inputs and measurements are transmitted via networks which are lossy and introduce random transmission delays. In contrast to the common notion of TCP-like communication, where successful transmissions are acknowledged instantaneously and without losses, we focus on the case where the acknowledgment packets provided by the actuator upon reception of applicable control inputs are also subject to delays and losses. Consequently, the estimator has only partial and belated knowledge on the actually applied control inputs, which results in additional uncertainty. We derive an estimator for the considered setup by generalizing an existing approach for UDP-like communication which integrates estimates of the applied control inputs into the overall state estimation. The presented estimator is assessed in terms of Monte Carlo simulations.**

I. INTRODUCTION

Networked Control Systems (NCSs) constitute a special class of control loops where the individual components, i.e., plant, sensor, and controller, use digital, packet-based, and often general-purpose networks such as Ethernet or WiFi for communication instead of dedicated point-to-point connections. Compared to traditional control loops, such systems usually benefit from enhanced flexibility and reliability as well as lesser costs for maintenance and installation due to, e.g., reduced wiring [1]. On the downside, they have to handle network-induced effects and constraints like random packet losses and delays and limited bandwidth which impact the overall performance and stability [2]–[4]. As communication and control should not be addressed independently from each other [5], several control methods have been proposed in recent years that explicitly factor in network effects. Among these, the approach of *sequence-based control* has gained much attention [6]–[10]. Here the underlying principle is to compute control inputs for the next, say N, time steps in addition to the one for the current time instant. By transmitting this sequence of control inputs in a single data packet which is buffered at the actuator upon reception, the problem of delayed or missing control inputs can be alleviated. Such controllers, often called predictive controllers, are usually adapted from nominal controllers which disregard the network [11], [12], or based on model predictive control approaches [7], [8]. Also, sequence-based controllers which minimize a quadratic cost function have been proposed [9], [10]. Since most of the derived control algorithms explicitly demand a state estimate or assume a perfectly known state or noise-free plants and measurements, state estimation is generally required in an NCS. Developing an estimator, based on the minimum mean squared error (MMSE) criterion, for a given predictive controller in an NCS scenario, where both the network connecting controller and actuator and the network between sensor and estimator are subject to random packet delays and losses, is the aim of this paper. Due to the presence of the networks, the estimator is confronted with the problem that measurements as well as control inputs can get lost or are subject to a delay so that out-of-sequence and burst arrivals are probable. In particular, the resulting uncertainty about the actual applied control inputs poses a major challenge.

Estimators which can cope with missing or delayed measurements have been proposed, for instance, in [13], [14], whereas the problem of estimation subject to missing control inputs due to a lossy network has been considered in [5], [15]. In [16], a filter for Networked Control Systems was presented which can handle both delayed and missing control inputs and measurements. Yet, here the probabilities of the applicable control inputs are assumed to be time-invariant and have to be known beforehand. As opposed to this, the filter developed in [17], where a setup similar to ours was considered, utilizes an estimate of the currently buffered sequence of control inputs for the state estimation. We consider the case where the actuator is able to acknowledge data packets which were successfully transmitted from the controller, which is in contrast to [17] where such acknowledgments are not provided. Additionally, we take into account that the acknowledgments sent from the actuator to the controller also suffer from random delays and losses. Consequently, the setup in [17] can be seen as a special case of the scenario we consider in that acknowledgments are provided by the actuator but always get lost.

Remark 1 *We want to mention that in the NCS literature the notion of UDP-like networks is used to describe transmissions where acknowledgments are not supplied by the receiver while the term TCP-like refers to idealized transmission schemes in which successful transmissions are acknowledged instantaneously and without losses [5]. A more realistic modeling of the TCP protocol, however, should factor in that acknowledgment packets also suffer from delays and losses. In particular in wireless environments these effects are inevitable due to channel contention or*

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interference [18]. In this regard, the setup we consider can be seen as a more realistic treatment of real networks using the TCP protocol.

Outline: The remainder of this paper is structured as follows. First, in Section II we give a detailed description of the considered scenario. Then, in Section III we design an estimator based on a formal model of the considered problem. The performance of the proposed estimator is then assessed in Section IV. Finally, Section V concludes this work.

Notation: Throughout this paper, vectors will be indicated by underlined letters ($\underline{x}$) while random vectors will be underlined and in bold ($\underline{\boldsymbol{x}}$). To denote matrices, we will employ boldface capital letters, e.g., $\mathbf{A}$. $\mathbf{0}$ and $\mathbf{I}$ are used to denote zero and identity matrix, respectively, and a subscript k indicates the time step. Finally, $\delta_{i,j}$ stands for the Kronecker delta, i.e., $\delta_{i,j} = 1$ if $i = j$ and 0 otherwise.

II. Problem Formulation

Consider an NCS where both plant and sensor are linear and described by

$$\underline{\boldsymbol{x}}_{k+1} = \mathbf{A}_k \underline{\boldsymbol{x}}_k + \mathbf{B}_k \underline{\boldsymbol{u}}_k + \underline{\boldsymbol{w}}_k \,, \tag{1}$$

$$\underline{\boldsymbol{y}}_k = \mathbf{C}_k \underline{\boldsymbol{x}}_k + \underline{\boldsymbol{v}}_k \,, \tag{2}$$

where $\underline{\boldsymbol{x}}_k \in \mathbb{R}^n$, $\underline{\boldsymbol{u}}_k \in \mathbb{R}^l$, and $\underline{\boldsymbol{y}}_k \in \mathbb{R}^m$ denote state, control input, and measurement, respectively. The control input is provided by a given controller. The zero mean, white noise sequences $\underline{\boldsymbol{w}}_k$ and $\underline{\boldsymbol{v}}_k$ are Gaussian and independent of each other with covariance matrices $\mathbf{C}_k^w$ and $\mathbf{C}_k^v$. The initial plant state $\underline{\boldsymbol{x}}_0$ is Gaussian with mean $\hat{\underline{x}}_0$ and covariance matrix $\mathbf{C}_0$ and is independent of $\underline{\boldsymbol{w}}_k$ and $\underline{\boldsymbol{v}}_k$. Furthermore, we assume that all components are synchronized and that the involved networks assign time stamps to data packets upon transmission.

Controller and actuator, which is collocated with the plant, are connected via a lossy network (CA-network) which means that each transmitted data packet can experience a (potentially unbounded) delay or even get lost. By interpreting losses as infinite delays, we can model the delay of a packet that is sent from the controller to the actuator at time instant k by the random variable $\boldsymbol{\tau}_k^{CA} \in \mathbb{N}_0$. We additionally assume that the $\boldsymbol{\tau}_k^{CA}$ are independent and identically distributed (i.i.d.) for all k with known probability mass function (PMF) f^{CA}. In order to account for these network-induced effects, the controller does not only transmit the current control input $\underline{u}_k$ at time k but also predicted control inputs for the next N time steps. Consequently, the data packet sent to the actuator consists of the *control sequence*

$$\underline{U}_k = \left[\underline{u}_{k|k}^{\mathrm{T}} \; \underline{u}_{k+1|k}^{\mathrm{T}} \; \cdots \; \underline{u}_{k+N|k}^{\mathrm{T}} \right]^{\mathrm{T}} ,$$

with $\underline{u}_{k+i|k}$, $i = 0, \ldots, N$ denoting the control input computed at time k and to be applied at time $k+i$. The buffer located at the actuator side employs the so called *past packets rejection logic* [1]: From the set of all received control sequences, only the most recent one, that is, the sequence with the largest time index, is maintained while all others are discarded. The control inputs provided by this sequence are then applied at the corresponding time steps until a newer sequence arrives at the actuator. However, in case of subsequent packet losses or large delays it may occur that the next control sequence arrives too late so that the control inputs from the buffered sequence are not applicable anymore. In such a case, the default input $\underline{u}_k^{df} = \underline{0}$ is applied.

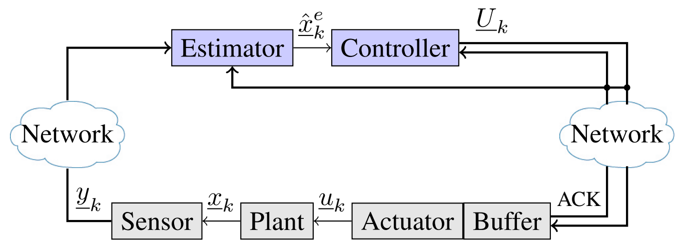


Fig. 1: Considered NCS Setup.

Remark 2 *Applying the default control input $\underline{u}_k^{df} = \underline{0}$ is known as zero-input strategy in the literature. Another common alternative is to apply the previous control input, i.e., $\underline{u}_k^{df} = \underline{u}_{k-1}$, which is known as the hold-input strategy. While the first one is mathematically more convenient, it has been shown in [19] that even for scalar systems and when when only packet dropouts are considered neither strategy can be deemed superior.*

Finally, each time the stored control sequence is replaced by a more recent one, an acknowledgment packet (ACK) is sent back to the controller to indicate a successful transmission of the corresponding sequence. ACKs can also be subject to delays and losses (infinite delays) which are modeled by the i.i.d. random variables $\boldsymbol{\tau}_k^{AC}$ with PMF f^{AC}. Hence, at each time step, the controller can receive none, one or multiple ACKs, meaning that none, one or multiple applied control inputs can be inferred.

Remark 3 *With the PMFs as defined above, it is possible to explicitly account for the additional delays caused by (unnecessary) retransmissions issued by real TCP implementations in case of delayed or missing acknowledgments and which often constitute a severe problem especially for relatively short transfers [18], [20]. On the other hand, this trading of packet losses for large delays is typically not desired in Networked Control Systems [3].*

At each time step, a sensor takes a noisy measurement of the state according to (2) which is sent to an estimator attached to the controller over another, yet UDP-like, network (SC-network). In this network, delays and losses also may happen according to the i.i.d. random variables $\boldsymbol{\tau}_k^{SC}$ with given PMF f^{SC}, so that at each time instant multiple measurements (or none) can arrive at the estimator. Note that in contrast to the CA-network, i) delayed packets do provide useful information about past states and hence should be incorporated by the estimator and ii) this network appears deterministic for the estimator because delays of the individual data packets are known due to the assigned time stamps [17]. However, as the estimator's buffer is finite, only up to $M \in \mathbb{N}$ measurements can be stored at the same time. As will be discussed in Section III, an appropriate approach to deal with burst and out-of-sequence arrivals of measurements is to maintain a fixed measurement history. Hence, we from now on assume that measurements with a delay larger than $M-1$ are discarded upon reception. The complete setup is depicted in Fig. 1.

Remark 4 *It is worth to mention that the above assumption results in a suboptimal estimator unless* $\boldsymbol{\tau}_k^{SC} \in \{0, \ldots, M-1, \infty\}$ *holds, that is, unless a measurement either gets lost or is at most* $M-1$ *time steps delayed [14].*

Our aim is now to design a state estimator which, at each time step k, supplies the given controller with an estimate $\hat{\underline{x}}_k^e$ of the plant state based on the MMSE criterion and is able to cope with missing, delayed and out-of-sequence measurements as well as with burst arrivals. We do so by extending the filter proposed in [17] for an UDP-like connection between controller and actuator.

III. Estimator Design

In order to design an estimator for the considered setup, it is essential to formulate a stochastic model to describe the CA-network and the actuator as a dynamical system. Chief ingredients of this model are a vector $\underline{\eta}_k \in \mathbb{R}^{lN(N+1)/2}$ which encompasses all control inputs from the sequences $\underline{U}_{k-N}, \ldots, \underline{U}_{k-1}$ that are still applicable at time k or later, and a scalar random variable $\boldsymbol{\theta}_k \in \{0, 1, \ldots, N+1\}$ denoting the state of a Markov chain. In the following we briefly summarize the resulting model, more detailed derivations can be found in [9], [17].

Formally, $\underline{\eta}_k$ is given by

$$\underline{\eta}_k = \begin{bmatrix} \begin{bmatrix} \underline{u}_{k|k-1}^{\mathrm{T}} & \underline{u}_{k+1|k-1}^{\mathrm{T}} & \cdots & \underline{u}_{k+N-1|k-1}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \\ \begin{bmatrix} \underline{u}_{k|k-2}^{\mathrm{T}} & \underline{u}_{k+1|k-2}^{\mathrm{T}} & \cdots & \underline{u}_{k+N-2|k-2}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \\ \vdots \\ \begin{bmatrix} \underline{u}_{k|k-N+1}^{\mathrm{T}} & \underline{u}_{k+1|k-N+1}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \\ \underline{u}_{k|k-N} \end{bmatrix}^{\mathrm{T}},$$

which is illustrated in Fig. 2 for the case $N = 2$.

The dynamics of $\underline{\eta}_k$ can then be expressed by

$$\underline{\eta}_{k+1} = \mathbf{F}\underline{\eta}_k + \mathbf{G}\underline{U}_k\,, \tag{3}$$

with

$$\mathbf{F} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \ldots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{I} & \mathbf{0} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

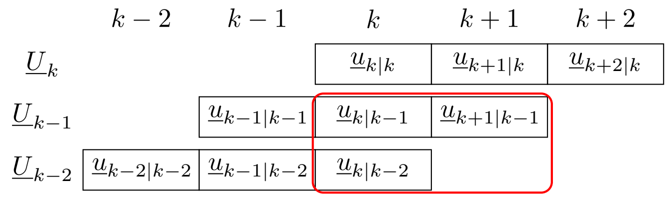


Fig. 2: Visualization of $\underline{\eta}_k$ (red rectangle) for $N = 2$. Applicable control inputs for the same time step are shown one below another.

By defining

$$\boldsymbol{\theta}_k = \min(k-t, N+1)\,,$$

where $\underline{U}_t$, $t \leq k$, is the control sequence currently buffered by the actuator and recognizing that when $\boldsymbol{\theta}_k = N+1$ holds the default input $\underline{u}_k^{df} = \underline{0}$ is applied, we get for the actual control input

$$\underline{\boldsymbol{u}}_k = \mathbf{H}_k\underline{\eta}_k + \mathbf{J}_k\underline{U}_k\,, \tag{4}$$

with

$$\begin{aligned} \mathbf{H}_k &= \begin{bmatrix} \delta_{\boldsymbol{\theta}_k,1}\mathbf{I} \ \mathbf{0} \ \delta_{\boldsymbol{\theta}_k,2}\mathbf{I} \ \mathbf{0} \ \ldots \ \delta_{\boldsymbol{\theta}_k,N}\mathbf{I} \end{bmatrix}, \\ \mathbf{J}_k &= \begin{bmatrix} \delta_{\boldsymbol{\theta}_k,0}\mathbf{I} \ \mathbf{0} \end{bmatrix}. \end{aligned}$$

We want to remark that i) $\underline{\boldsymbol{u}}_k$ is a random variable due to the stochasticity of $\mathbf{H}_k$ and $\mathbf{J}_k$ and ii) the transition matrix governing the Markov chain can be computed from the given PMF f^{CA}. Finally, combining (1), (3) and (4) yields the dynamics

$$\underline{\boldsymbol{\xi}}_{k+1} = \begin{bmatrix} \mathbf{A}_k & \mathbf{B}_k\mathbf{H}_k \\ \mathbf{0} & \mathbf{F} \end{bmatrix} \underline{\boldsymbol{\xi}}_k + \begin{bmatrix} \mathbf{B}_k\mathbf{J}_k \\ \mathbf{G} \end{bmatrix} \underline{U}_k + \begin{bmatrix} \underline{\boldsymbol{w}}_k \\ \underline{0} \end{bmatrix}, \tag{5}$$

of the augmented system with state $\underline{\boldsymbol{\xi}}_k = [\underline{\boldsymbol{x}}_k^{\mathrm{T}} \ \underline{\eta}_k^{\mathrm{T}}]^{\mathrm{T}}$. Eq. (5) represents a non-homogeneous Markov jump linear system (MJLS) [21] with parameter $\boldsymbol{\theta}_k$, which is usually referred to as the *mode* of the system. For this class of systems, it is well-known that the MMSE estimator is given by a time-varying Kalman filter in case the complete mode history, i.e., all mode realizations $\theta_0, \theta_1, \ldots, \theta_k$ up to time k, is available [21]. In the given setup, however, only a subset $\mathcal{I}_k$ of the mode realizations will be available to the estimator. Before we propose an estimator for this case, we first discuss the case of completely unknown mode history, that is, the special case $\mathcal{I}_k = \varnothing$, in the following Sections III-A and III-B.

A. Perfect Connection between Sensor and Estimator

In case of a perfect connection between sensor and estimator, i.e., measurements are received by the estimator immediately, it is well-known for a long time that the optimal estimator for system (5) in the MMSE sense is not only nonlinear but also intractable as its computational complexity grows exponentially in time [22], [23]. This is mainly due to the fact that an exponentially increasing number of possible mode realizations has to be tracked. Hence, a variety of approximations to the optimal solution have been proposed, ranging from LMMSE estimators [24]–[26] to approaches which at each time instant maintain only a fixed number of hypotheses about the mode history by applying some hypothesis reduction strategy [27]. Among these approaches, the *Interacting Multiple Model (IMM) filter* [22] is frequently applied since it exhibits a good trade-off between estimation quality and complexity. In particular, the IMM filter consists of a bank of Kalman filters, one for each mode, which are individually reinitialized at each time step by mixing all mode-conditioned estimates from the previous time step [27]. For the system given by (5) the IMM filter is hence composed of $N+2$ Kalman filters.

B. Missing, Delayed, and Out-of-Sequence Measurements

In [28], [29] estimators were presented for applications where measurements from a linear plant are transmitted to the estimator through communications channels which can be described by Markov chains so that the resulting combined systems can be modeled as MJLS. However, the used channel models do not account for packet delays and out-of-sequence arrivals but only for transmission errors which are considered as packet losses. Also, the proposed filters were derived with respect to $\mathcal{H}_2$ and $\mathcal{H}_\infty$ criteria, respectively, while we strive for an approximation of the MMSE filter. An MMSE estimator for MJLS which copes with delayed measurements has been derived in [30]. Here, a fixed delay is assumed for all measurements, so that losses and burst and out-of-sequence arrivals cannot be integrated. For the same reason, the LMMSE estimator proposed in [31] for randomly delayed measurements cannot be adapted to our scenario. Since MJLS are widely used in (multi-)target tracking scenarios where measurement delays and losses and out-of-sequence arrivals are common, work has been conducted in this community to handle these issues, yielding IMM filters where retrodiction techniques are used to incorporate delayed and out-of-sequence measurements [32]. Yet, applying retrodiction in our setup necessitates that the system matrix $\mathbf{A}_k$ in (1) is invertible which is not always given. The approach from [33] to maintain a history of past estimates and measurements which is recalculated upon the reception of a delayed measurement has been adapted to IMM filters for Networked Control Systems in [17]. Besides its simplicity, this approach has the advantage that is inherently suited for dealing with burst arrivals of measurements which are, for instance, processed one by one. On the downside, the computational complexity increases with the buffer length.

Finally, based on the observation that the Markov chain described by $\boldsymbol{\theta}_k$ possesses a stationary distribution [17], the Kalman filter approach from [16] can also be used for the given system (5). However, using this stationary distribution is clearly an approximation and it has been shown in [17] that for NCS scenarios this approach is inferior to an IMM-based approach with state and measurement history. Consequently, as indicated earlier, we will build upon this approach from [17] and equip it with the additional capability to handle delayed and out-of-sequence mode observations. This is detailed in the following section.

C. Delayed and Out-of-Sequence Mode Observations

At each time step, the IMM filter maintains the state estimate in terms of a mixture distribution which for the given setup consists of $N+2$ individual Gaussians, and each component is weighted according to the estimated mode probability distribution $\underline{\pi}_k$.[1] Hence, if at time k the estimator can infer a mode realization $\boldsymbol{\theta}_t = L$, $t \leq k$, $L \in \{0, \ldots, N\}$, due to a received acknowledgment packet, the probability distribution of the mode $\boldsymbol{\theta}_t$ reduces to

$$\underline{\pi}_t = \underline{e}_{L+1}\,, \tag{6}$$

where $\underline{e}_{L+1}$ is the $(N+2)$-dimensional unit vector with one at position $L+1$ and zero elsewhere. Please note that the mode realization $\boldsymbol{\theta}_t = N+1$ will never be available to the filter since this indicates that at time t the default input was applied. In such a case, no applicable control sequence would have been received in time by the actuator, and hence no ACK would have been sent back. By making use of the history kept by the filter and recognizing that $\boldsymbol{\theta}_t$ only affects the prediction step at $t+1$, integrating a delayed mode observation at time k consists of updating $\underline{\pi}_t$ according to (6) and then recomputing the estimates from $t+1$ to k. This procedure can also be utilized to handle burst arrivals of ACKs, which means that multiple mode realizations can be inferred at once. Then, starting with the oldest one, they are integrated into the recomputation of the history one after another.

Additionally, since we assumed that measurements with a delay larger than $M-1$ are discarded by the estimator, it is reasonable to keep a history of estimates that comprises the current estimate and the previous M ones so that a measurement with maximum delay $M-1$ can be processed belatedly. Hence, we discard all acknowledgment packets with a delay larger than M time steps. In total, the history kept by the estimator comprises the current and past M states $\underline{\boldsymbol{x}}_{k-M}, \ldots, \underline{\boldsymbol{x}}_k$, computed control sequences $\underline{U}_{k-M}, \ldots, \underline{U}_k$ and mode observations $\theta_{k-M}, \ldots, \theta_k$, and the last M measurements $\underline{y}_{k-M+1}, \ldots, \underline{y}_k$.

IV. Evaluation

In this section we assess the performance of the proposed estimator in an NCS scenario similar to the one used in [17], namely controlling an inverted pendulum on a cart which operates in a transient state. We compare the proposed estimation scheme with the original approach from [17] which does not have access to the partial mode history $\mathcal{I}_k$. Our aim is to evaluate whether the information advantage of the proposed filter results in improved estimates.

To that end, consider the state of the pendulum given by $\underline{\boldsymbol{x}}_k = \begin{bmatrix} \boldsymbol{s}_k & \dot{\boldsymbol{s}}_k & \boldsymbol{\phi}_k & \dot{\boldsymbol{\phi}}_k \end{bmatrix}^{\mathrm{T}}$, where $\boldsymbol{s}_k$ denotes the position of the

[1] The point estimate $\hat{\underline{x}}_k^e$ for the controller is then simply the mean of the mixture.

Mass of the cart	$0.5\,\mathrm{kg}$
Mass of the pendulum	$0.5\,\mathrm{kg}$
Coefficient of friction for the cart	$0.1\,\mathrm{N\,s/m}$
Length to pendulum center of mass	$0.3\,\mathrm{m}$
Moment of inertia of the pendulum	$0.006\,\mathrm{kg\,m^2}$
Gravitational acceleration	$9.81\,\mathrm{m/s^2}$

TABLE I: Parameters of the inverted pendulum used in the simulation.

cart (in m) and $\boldsymbol{\phi}_k$ is the deviation (in rad) of the pendulum from the upward equilibrium, and the model (1) and (2) with

$$\mathbf{A}_k = \begin{bmatrix} 1 & 0.0099911 & 0.0003871 & 0.0000013 \\ 0 & 0.9982114 & 0.0774447 & 0.0003872 \\ 0 & -0.0000263 & 1.0025820 & 0.0100086 \\ 0 & -0.0052630 & 0.5165563 & 1.0025820 \end{bmatrix},$$

$$\mathbf{B}_k = \begin{bmatrix} 0.0000894 \\ 0.0178855 \\ 0.0002631 \\ 0.0526298 \end{bmatrix}, \quad \mathbf{C}_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The model has been obtained by linearizing the nonlinear pendulum equations (cf., for instance, [34]) with parameters according to Table I around the upward equilibrium and a subsequent discretization with sampling time $t_A = 0.01\,\mathrm{s}$. The covariances of $\underline{\boldsymbol{w}}_k$ and $\underline{\boldsymbol{v}}_k$ are $\mathbf{C}_k^w = 0.01\mathbf{I}$ and $\mathbf{C}_k^v = 0.2\mathbf{I}$. As in [17], we employ a nominal predictive state feedback linear quadratic regulator [34] which computes the control inputs $\underline{u}_k$ based on the true state of the plant. State and input weighting matrix for the computation of the regulator gain $\mathbf{L}$ are given by

$$\mathbf{Q} = \begin{bmatrix} 5000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{R} = 100\,.$$

Overall, we carried out two Monte Carlo simulations with 2000 runs each where each run comprised 250 time steps. In each run, the initial plant state was randomly drawn from a Gaussian distribution with mean and covariance matrix

$$\hat{\underline{x}}_0 = \begin{bmatrix} 0 \; 0.2 \; 0.2 \; 0 \end{bmatrix}^{\mathrm{T}}, \quad \mathbf{C}_0 = 0.5\mathbf{I}\,.$$

The probability mass functions of the links between controller and actuator (f^{CA}), actuator and controller (f^{AC}) and sensor and controller (f^{SC}) are depicted in Fig. 3. We chose to set the length of the measurement history to $M = 6$, that is, ACKs with a delay larger than six time steps and measurements with a delay larger than five time steps were discarded. For the SC-link shown in Fig. 3a the measurement loss rate was thus $4.99\,\%$. Since ACKs are usually much smaller than regular data packets, the probabilities for large delays and losses are commonly considerably smaller as well [35]. Consequently, we decided to utilize a distribution for the AC-link (cf. Fig. 3b) where delays larger than three time steps were very unlikely. As mentioned above, the plant operates in a transient state, i.e., set point changes occur. Thus, in each simulation run, the initial set point of the pendulum was $[2\ 0\ 0\ 0]^{\mathrm{T}}$ which changed to $[-2\ 0\ 0\ 0]^{\mathrm{T}}$ after 100 time steps and then changed back after another 100 time steps. The length of the control sequences computed by the controller was $N + 1 = 7$, resulting in an MJLS with 8 modes. In each run, the mode-conditioned Kalman filters of both estimators were initialized with $\hat{\underline{x}}_0^e = \hat{\underline{x}}_0$ and $\mathbf{C}_0^e = \mathbf{C}_0$. The initial mode distribution was $\underline{\pi}_0 = \underline{e}_8 \in \mathbb{R}^8$. Note that neither estimator had impact on the computation of the control sequences.

In the first simulation, the true PMF f^{CA} was used to compute the transition matrix of the Markov chain $\boldsymbol{\theta}_k$, while in the second simulation we assumed that the filters were completely unaware of the behavior of the CA-link. Hence, we employed a uniform PMF instead of f^{CA} for the computation of the transition matrix. This decision was motivated by the time-varying nature of real networks, due to which model mismatches are likely in practical applications.

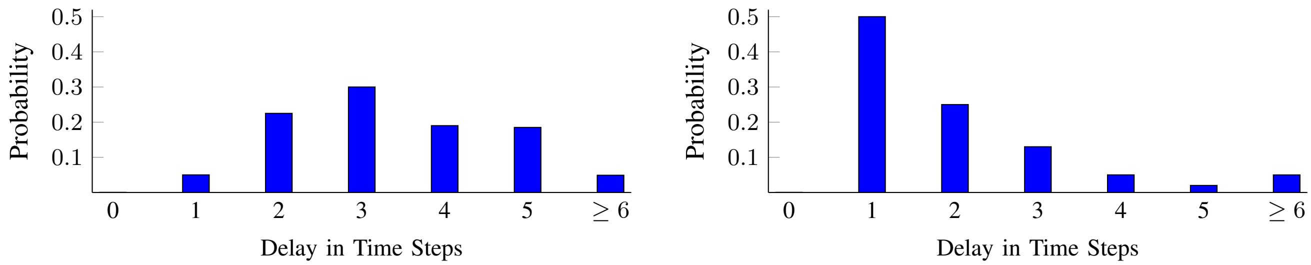
The simulation results in terms of the root mean squared error (RMSE) are shown in Fig. 4 for the directly accessible states $\boldsymbol{s}_k$ and $\boldsymbol{\phi}_k$ and in Fig. 5 for the non-accessible states $\dot{\boldsymbol{s}}_k$ and $\dot{\boldsymbol{\phi}}_k$. We can directly see from the results that the performance of both filters does not differ much most of the time. In particular with respect to the directly accessible states $\boldsymbol{s}_k$ and $\boldsymbol{\phi}_k$, the information advantage of the proposed filter only pays off at a single time step ($k = 190$) in both simulation scenarios. At that time step, the estimation error of the filter from [17] increases drastically while it remains at the same level for the proposed filter. An additional interesting observation is that the estimation quality of both filters is not affected by the model mismatch introduced in the second simulation.

On the contrary, the RMSE curves of the non-accessible states $\dot{\boldsymbol{s}}_k$ and $\dot{\boldsymbol{\phi}}_k$ exhibit that the overall estimation quality of both filters is corrupted by the wrong PMF assumed for $\boldsymbol{\tau}_k^{CA}$. However, although both filters achieve almost equal performance most of the time, the proposed approach can yield significantly lower estimation errors compared to the approach from [17] even in case of a model mismatch. In particular for the angular velocity component of the state ($\dot{\boldsymbol{\phi}}_k$), the original approach is not able to improve its estimates at times where the proposed approach achieves considerable improvements.

Overall, we can conclude that the additional available information in terms of the partial mode history $\mathcal{I}_k$ can result in an enhanced estimation quality and help increase the filter's robustness towards modeling errors with respect to the nature of the networks. Finally, it is worth to remark that the set point changes do not result in an increased estimation error, in contrast to what was reported in [17].

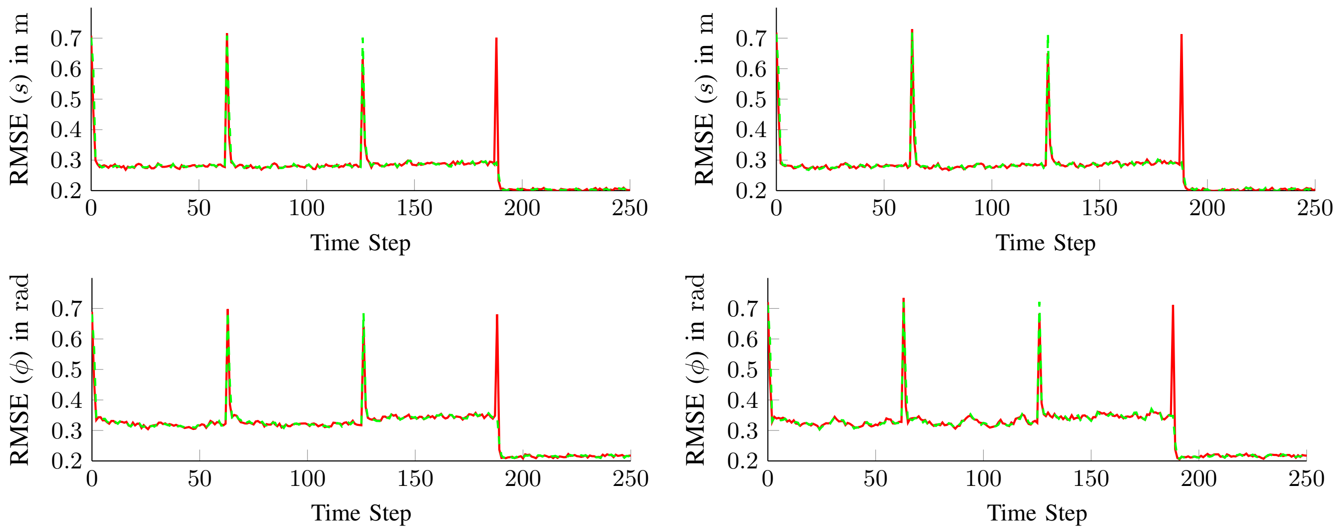
V. Conclusions

In this work, we investigated the issue of state estimation in Networked Control Systems, where, in contrast to the notion of TCP-like communication used in the literature, acknowledgment packets are also subject to random delays and dropouts which is a more realistic treatment of the real TCP protocol. We presented a state estimator based on an existing IMM approach which is able to incorporate the information on applied past control inputs retroactively into



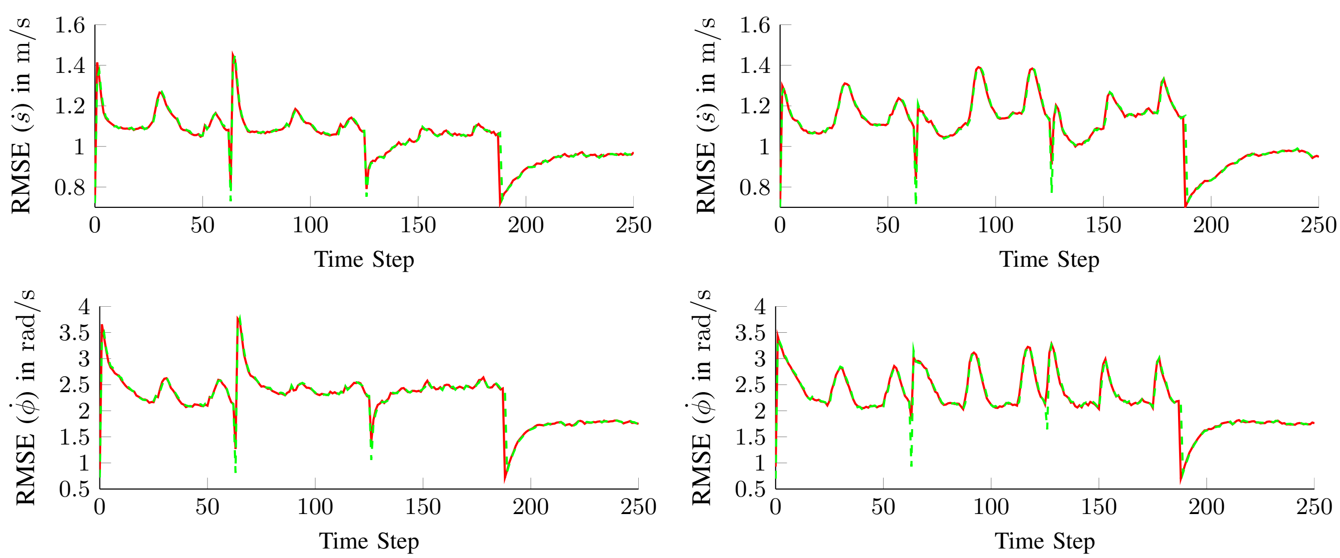
(a) f^{CA} and f^{SC} (b) f^{AC}

Fig. 3: PMFs of the packet delays. Delays larger than five time steps in the SC-link are treated as packet losses (infinite delay) by the estimators.



(a) True PMF f^{CA} used for τ_k^{CA} (b) Uniform PMF assumed for τ_k^{CA}

Fig. 4: Results of the proposed estimator and the original approach from [17]: Comparison of the RMSE for the directly accessible states $\boldsymbol{s}_k$ and $\boldsymbol{\phi}_k$.



(a) True PMF f^{CA} used for τ_k^{CA} (b) Uniform PMF assumed for τ_k^{CA}

Fig. 5: Results of the proposed estimator and the original approach from [17]: Comparison of the RMSE for the non-accessible states $\dot{\boldsymbol{s}}_k$ and $\dot{\boldsymbol{\phi}}_k$.

the current state estimate. We pointed out that UDP-like communication schemes are a special case of the considered problem and hence subsumed by the proposed approach. The evaluation results indicated that the integration of this belated information can be an appropriate means to make the filter more robust towards imperfect knowledge of the network characteristics, in particular for components of the state that are not directly accessible.

Future work in this context should examine the interdependence of control sequence length, nature of the network, and the size of the utilized history as well as their impact on the estimation quality in more detail. Moreover, it is worth to examine whether an additional, similar state augmentation which explicitly takes the network between sensor and estimator into account, can serve as a starting point for the derivation of an estimator for the given setup.

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