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A unified multiscale modelling approach for concrete fatigue with microscale damage-mechanism implementations

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A micro-scale-based approach for the numerical analysis of cement-based materials, subjected to low- and high-cycle fatigue actions, is presented in this paper. The constitutive model is aimed at describing the evolving microstructural changes caused by cyclic loading protocols. The plastic-damage-based model is formulated combining the concepts of fracture-energy theories and damage stiffness degradations, representing the key phenomena occurring in concrete under fatigue.

1 Introduction

Concrete structures like tall towers, bridges and modern buildings made of high strength concretes are highly sensible to suffer failures and damages under fatigue loads [1]. In this context, plenty of experimental tests are available in the scientific literature dealing with the fatigue behavior of cement-based composites [2, 3]. Many of these works showed a huge scatter in terms of fatigue performances of the composites which can be actually attributed to a high variability of the materials nature, stress levels, inner meso-structures, distribution of fibers, strain-hardening/softening mechanisms and specimen type/size [4, 5].

Current approaches, codes and design guidelines analyze the concrete fatigue performance through empirical rules, in most cases based on the well-known Wöhler concepts [6]. Fatigue degradation is represented in the so-called stress-life or simple S-N curves, where the Basquin relationship usually defines the behavior in a log-log straight line [7]. For example, the smeared crack approach for concrete by [8] can be mentioned, in which the S-N criterion is translated into the damage description and accounts for the consideration of the stress increments and number of cycles at each material point. Although, these kind of approaches have been valuably used in practical applications, they completely avoid deeper explanations of the mechanisms which drive the damage initiations and triggering of concrete-fatigue behavior.

In classical Wöhler curves, three different fatigue types can be differentiated: (i) Oligocyclic (O), (ii) Low (L) and (iii) High (H) Cycle Fatigue (CF). OCF is mainly due to very high stress levels, where the number of reversals needed to reach the failure is very limited. In this case, the material is hugely fractured and after very few cycles the crack propagation becomes instable and failure is quickly reached. On the other hand, LCF and HCF are characterized by stable crack propagation processes. In LCF, the nucleation and failure processes are driven by an increase in the intermediate-macro crack, which develops cycle by cycle. The number of failure cycles is commonly lower than $10t$. Constitutive models based on fatigue damage-based description mechanisms are commonly available in literature for this kind of fatigue description [9, 10].

In HCF, the level of stress reaching failure is much lower than that of LCF and the accumulation of damage is driven by the continuous accumulation of microscopic defects. Main differences between these two are the deformations and achieved strain levels, since HCF is characterized by remaining on the elastic range while LCF by repeated plastic deformations cycle by cycle. HCF represents a very complex phenomenon which deals with the generation of micro-cracks at lower scales under very high number of cycles and their coalescence into macro-cracks up to failure. Plenty of works demonstrated that non-linear constitutive models classically based on plasticity theory and damage mechanics, often suitable for capturing the non-linear responses of specimens under LCF, needs to be extended for HCF [11]. Stress levels, cycle times, regression indices, amplitudes, and frequencies are key variables are key variables in determining the loss of material load capacity. Available models took into the account the "number of cycle" as variable capable of modifying the internal plastic variables (i.e., plastic strains, work spent and damage parameters) and necessary to capture the strength loss occurring under HCF tests [12, 13].

In this context, the present research is mainly intended at proposing a unified model for the numerical analysis of concrete specimens possibly subjected to both LCF and HCF actions. This paper explores for the first time the potential of assessing fatigue microcracks formation and growth, and their influence on the macroscopic behavior which has not been proposed yet in the international scientific literature. The latter is representing the main significant contribution of this work to the current state-of-the-art.

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2 Unified fatigue damage-mechanisms model

A discontinuous-based approach for the numerical analysis of concrete specimens subjected to (both LCF and HCF) fatigue action is presented in a unified way.

Particularly, a coupled damage-elasto-plastic constitutive equation is proposed as follows

$$\dot{\mathbf{t}} = \mathbf{C_d}(\dot{\mathbf{u}} - \dot{\mathbf{u}}^{\mathbf{cr}}) \tag{1a}$$

$$\dot{\mathbf{u}} = \dot{\mathbf{u}}^{\mathbf{el}} + \dot{\mathbf{u}}^{\mathbf{cr}} \tag{1b}$$

where $\dot{\mathbf{t}} = [\dot{\sigma}_{\mathbf{N}}, \dot{\sigma}_{\mathbf{T}}]^{\mathbf{T}}$ is the interface stress rate vector, $\mathbf{C_d}$ the elastic-damage stiffness matrix, $\dot{\mathbf{u}} = [\dot{\mathbf{u}}, \dot{\mathbf{v}}]^{\mathbf{T}}$ is the vector of relative displacement rates across the interface, $\dot{\mathbf{u}}^{\mathbf{el}}$ and $\dot{\mathbf{u}}^{\mathbf{cr}}$ are the elastic and cracking (inelastic) components, respectively.

By providing the concept of effective interface stress, , the plastic (cracking) behavior can be independently integrated and isolated from the damage part. Thus, Eq. 1 can be thus reformulated as

$$\dot{\mathbf{t}} = (\mathbf{I} - \mathbf{D}) \cdot \mathbf{C} \cdot (\dot{\mathbf{u}} - \dot{\mathbf{u}}^{\mathbf{cr}}) \quad \rightarrow \quad \dot{\tilde{\mathbf{t}}} = \mathbf{C} \cdot \dot{\mathbf{u}}^{el} \tag{2}$$

being $\mathbf{I}$ the second order identity matrix and $\mathbf{C}$ the undamaged elastic matrix.

The unified procedure is thus based on a scalar-type approach which is employed to describe the isotropic damage. Particularly, the following global relationship is proposed to integrate the damage behavior under general cyclic events (i.e., dealing with either O-, L- or HCF)

$$\dot{\mathbf{D}} = \dot{\mathbf{D}}^{\mathbf{LCF}} + \dot{\mathbf{D}}^{\mathbf{HCF}} \tag{3}$$

where the timely accumulated damage can be evaluated as

$$\mathbf{D} = \int_{\mathbf{t_0}}^{\mathbf{t_f}} \dot{\mathbf{D}} \mathbf{dt} \in [\mathbf{0}, \mathbf{1}] \tag{4}$$

3 Low cycle fatigue description

The possibilities of modelling the fracture response, induced by LCF, can be approached through employing zero-thickness interface elements for discrete crack analysis. A cracking surface defines the (effective) stress level at which the post-elastic displacements starts [14]

$$f = \tilde{\sigma}_T^2 - (c - \tilde{\sigma}_N^2 \tan\phi)^2 + (c - \chi \tan\phi)^2 \tag{5}$$

where χ is the tensile strength, c the cohesion and ϕ the frictional angle.

The vector of crack displacement rates can be defined following a non-associated flow rule

$$\dot{\mathbf{u}}^{\mathbf{cr}} = \dot{\lambda} \mathbf{m} \tag{6}$$

where $\dot{\lambda}$ is the non-negative plastic multiplier, deriving from the classical Kuhn-Tucker ($\dot{\lambda} \geq 0, f \leq 0, f = 0$) and consistency conditions ($\dot{f} = 0$), $f = f[\tilde{\sigma}_N, \tilde{\sigma}_T]$ is the yield condition while $\mathbf{m}$ is the vector controlling the direction of the fracture displacements by means of a non-associated flow rule.

The softening post-cracking behavior is driven through considering scaling functions which are based on the ratio between the plastic work spent W_{cr} and the available fracture energies ($G_f^{\#}, \# = I, II$)

$$\xi_\star = \begin{cases} \frac{1}{2}\left[1 - \cos\left(\pi \frac{W_{cr}}{G_f^{\#}}\right)\right] & \text{if } W_{cr} \leq G_f^{\#} \\ 1 & \text{otherwise} \end{cases} \tag{7}$$

where $\star$ alternatively represents c, χ and tanϕ.

Finally, a scalar-type approach is employed to describe the isotropic damage for LCF

$$d^{\#} = \frac{e^{-\beta_d}\left(\frac{W_{cr}}{G_f^{\#}}\right)^{\alpha_d}}{1 + (e^{-\beta_d} - 1)\left(\frac{W_{cr}}{G_f^{\#}}\right)^{\alpha_d}}, \, \# = I, IIa \quad \wedge \quad \mathbf{D}^{\mathbf{LCF}} = \begin{pmatrix} d^I & 0 \\ 0 & d^{IIa} \end{pmatrix} \tag{8}$$

being $\alpha_d \geq 0$ and β_d parameters for controlling the decay shape of the damage induced by fracture.

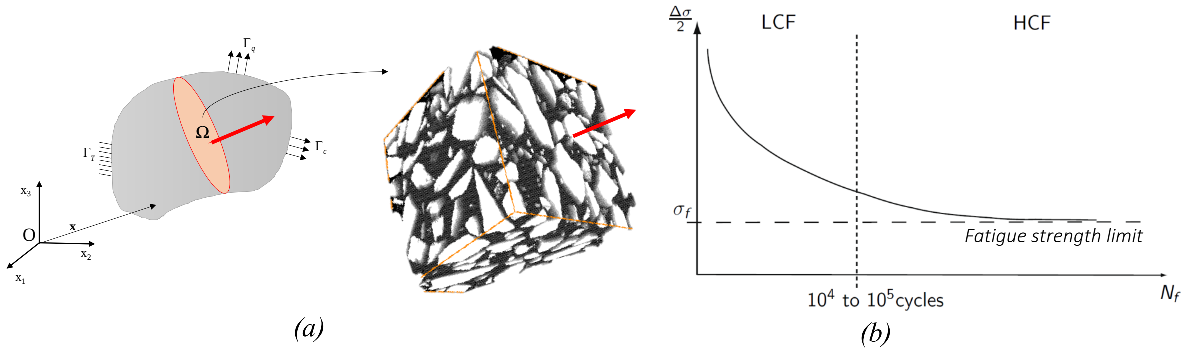

Fig. 1: **a** two scale (macro-micro) approach and **b** concept of fatigue stress limit.

4 High cycle fatigue description

Plastic strains (and/or cracks) are almost negligible in those tests which fail under HCF when compared with mechanisms which drive LCF. Therefore, classical approaches based on fracture energy, plasticity theory and damage approaches, are not able to capture HCF. Very small (micro-) plastic strains and microcracks are the main responsible of fatigue damage under HCF. After huge cycle reversals they mainly increase and coalesce into few macro-cracks [15].

Based on this idea, a two scale approach (Fig. 1a) is adopted where no plastic strains take place at the macro-scale while microplastic strains (cracks) are modelled at the micro-level. Therefore, assuming a Hollomon's type hardening model [16] at the microscale, it can be written that

$$\tilde{\sigma}_{eq} = K\delta^{M}_{micro}, \tilde{\sigma}_{eq} = \begin{cases} \sqrt{(\tilde{\sigma}_N)^2 + (\tilde{\sigma}_T)^2} & \tilde{\sigma}_N \geq 0 \\ \tilde{\sigma}_T - |\tilde{\sigma}_N| \tan\phi & \tilde{\sigma}_N < 0 \end{cases}, \delta_{micro} = \begin{cases} \sqrt{(u_{micro})^2 + (v_{micro})^2} & \tilde{\sigma}_N \geq 0 \\ v_{micro} & \tilde{\sigma}_N < 0 \end{cases} \tag{9}$$

being K a strength index (or strength coefficient) and M a crack-hardening exponent (which can be possible dependent on the frequency f); u^{cr}_{micro} and v^{cr}_{micro} are the cracking microscale relative displacements in the normal and tangential direction, respectively.

Following an isotropic damage approach, it can be stated that

$$\frac{\sigma_{eq}}{1-D} = K\delta^{M}_{micro} \Rightarrow \delta_{micro} = \frac{\sigma^{1/M}_{eq}}{(1-D)K^{1/M}} \Rightarrow \dot{\delta} = \frac{\sigma^{1/M}_{eq} - 1}{M(1-D)^{1/M}K^{1/M}}\dot{\sigma_{eq}} \tag{10}$$

Thus, the microscale damage description can be done by means of the following rule

$$\dot{D} = \begin{cases} 0 & \text{if } \frac{\sigma_{eq,M} - \sigma_{eq,m}}{2} \leq \sigma_f \\ L_{el^{-1}} \frac{Q(f)\sigma^{q(f)}_{eq}}{(1-D)^{q(f)+1}} \dot{\sigma}_{eq} & \text{if } \frac{\sigma_{eq,M} - \sigma_{eq,m}}{2} > \sigma_f \end{cases} \quad \wedge \quad \mathbf{D^{HCF}} = \begin{pmatrix} \dot{D} & 0 \\ 0 & \dot{D} \end{pmatrix} \tag{11}$$

being $q(f) = 1/M(f) - 1$, $Q(f) = \frac{K^{-1/M(f)}}{M(f)}$ and $L_{el^{-1}}$ (keeping in mind that $\dot{D} = L_{el^{-1}}\dot{\delta}_{micro}$) a characteristic length value. The latter three variables are mainly aiming at representing the link between the microscale and the macroscale; $\sigma_{eq,M}$ and $\sigma_{eq,m}$ are the "sup" and "inf" values of σ_{eq} under cyclic protocols, while σ_f is a fatigue stress limit to be calibrated and shown in Fig. 1b.

5 Model predictions and examples

This section aims at validating the proposed model by performing and comparing numerical examples against experimental results of literature on concrete specimens submitted to fatigue effects.

The numerical results tried to simulate flexural fatigue tests on concrete (having a target strength of about 40 MPa) proposed by Oh [17] who utilized concrete beams measuring 100 x 100 x 500 mm. The unnotched beam specimens were tested under four-point bending. The fatigue protocol was designed in consideration of the change in the bottom fiber stress from zero to a predetermined maximum stress. Therefore, the maximum cyclic stress ratio, namely f^{MAX}_R/f_{mon}, was analyzed in each series to reach the fatigue failure after a certain number of cycles/reversals, i.e., N_{FAIL}.

For the calibration purpose of the numerical examples, key geometric and material properties were chosen according to the experimental evidences, while the only two parameters which were fitted were $q(f) = 24.30$ and $\frac{q(f)+1}{2[q(f)+2]Q(f)L_{el^{-1}}} = 2.1$MPa. Furthermore, the following assumptions were considered for the model integration: $D = 0$ when the number of cycles is null and $D = 1$ in fatigue failure (i.e. $N = N_{FAIL}$).

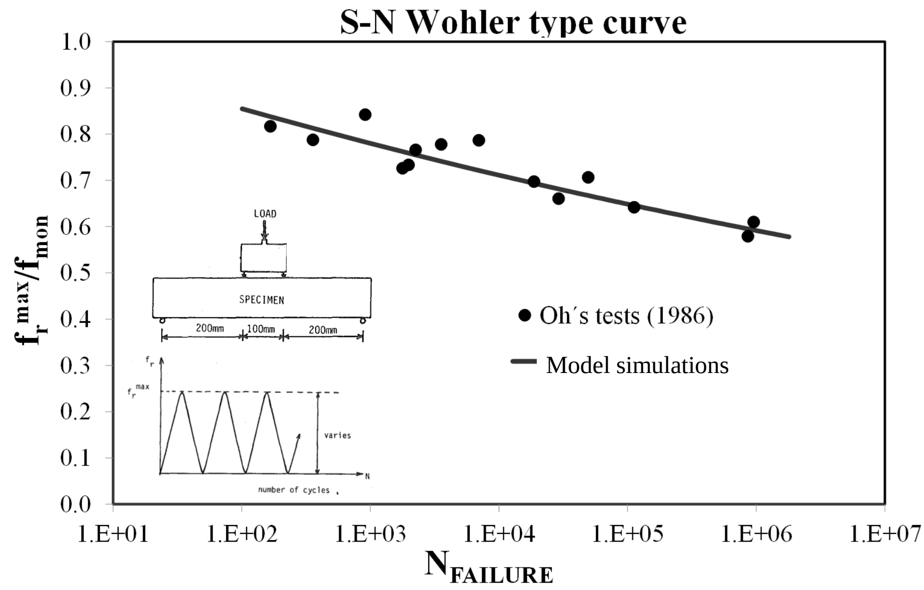

Fig. 2: Numerical examples vs. the experimental tests by Oh (1986) [17].

Numerical S-N curves can be generated from this approach and compared with experimental data. The numerical results were almost promising in predicting the experimental data and actually follow the well-known Wöhler expression for describing the S-N curve: i.e., $f_R^{MAX}/f_{mon} = a + b\log_{10}N_{FAIL}$.

Beyond the general soundness of the proposed unified model for fatigue failure predictions, the numerical results demonstrate the capabilities of the proposed formulation to reproduce the strong sensitivity of concrete mechanical behavior on the stress level affecting the HCF.

6 Conclusions

A unified numerical approach was proposed for analyzing low- and high-cycle fatigue actions in concrete. The discontinuous-based model is aimed at describing the evolving micro- and meso-structural changes caused by cyclic protocols by employing a damage accumulation rule. Concepts of fracture-energy theory, damage stiffness degradations and microscopic accumulated inelasticity were combined for predicting failure mechanisms occurring in concrete under cyclic responses.

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