Karlsruher Institut für Technologie

v-*K*-data for silica from lifetime measurements under step-shaped loading history

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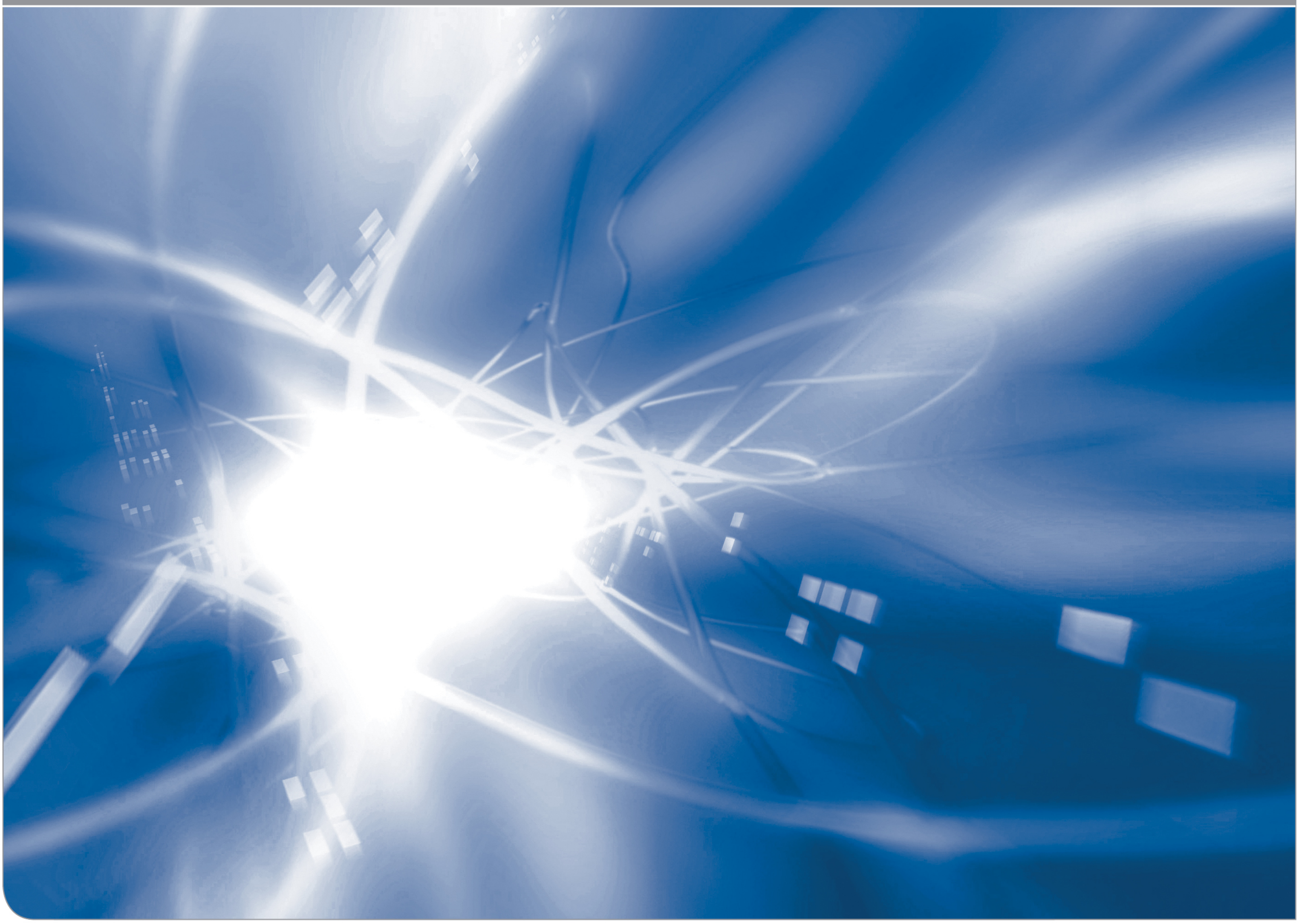
KIT SCIENTIFIC WORKING PAPERS 143



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Abstract

Different methods were applied so far in order to determine subcritical crack growth for silica. Mostly, fracture mechanics standard tests with *macro cracks* were used for this purpose. In this report, we evaluated the subcritical crack growth curves from lifetime tests on silica bending specimens. The survivors were then tested under increased stress. Crack growth rates down to $v=10^{-12}$m/s were reached in this way.

In the plot of $v=f(K/K_{Ic})$ slight material differences could be eliminated and suitable agreement with macro-crack results by Wiederhorn and Bolz [1] on DCB-specimens and Michalske et al. [2] on DCDC-specimens could be stated.

Contents

1. Introduction

Different types of test specimens were used in the past for the measurement of subcritical crack growth in silica. Figure 1a shows as the curves some subcritical crack growth results from literature for which the stress intensity factors were available in form of handbook solutions. In order to get results independent of the slightly different K_{Ic}-values reported in literature (K_{Ic}=0.72-0.80 MPa√m for data in Fig. 1), the abscissa in Fig. 1b is given in normalized form by K/K_{Ic}. The crack-growth data by Wiederhorn and Bolz [1] were measured with the Double-Cantilever Beam (DCB) method and Michalske *et al.* [2] used the Double Cleavage Drilled Compression (DCDC) specimen. Minimum crack-growth rates of 10^{-11}-10^{-10} m/s could be reached.

Lower rates are reachable by lifetime methods. This was demonstrated by evaluation of interrupted lifetime tests [3] based on measurements by Sglavo and Green [4]. In this report, we evaluate the v-K-curve from lifetime measurements performed by Braun et al. [5]. Methods were developed very early to determine v-K the dependence from lifetimes [6,7]. For our purpose, we use the procedure given by Fett and Munz [7]. The results of the computations outlined in this Report are included in Fig. 1. The applied procedure is described below.

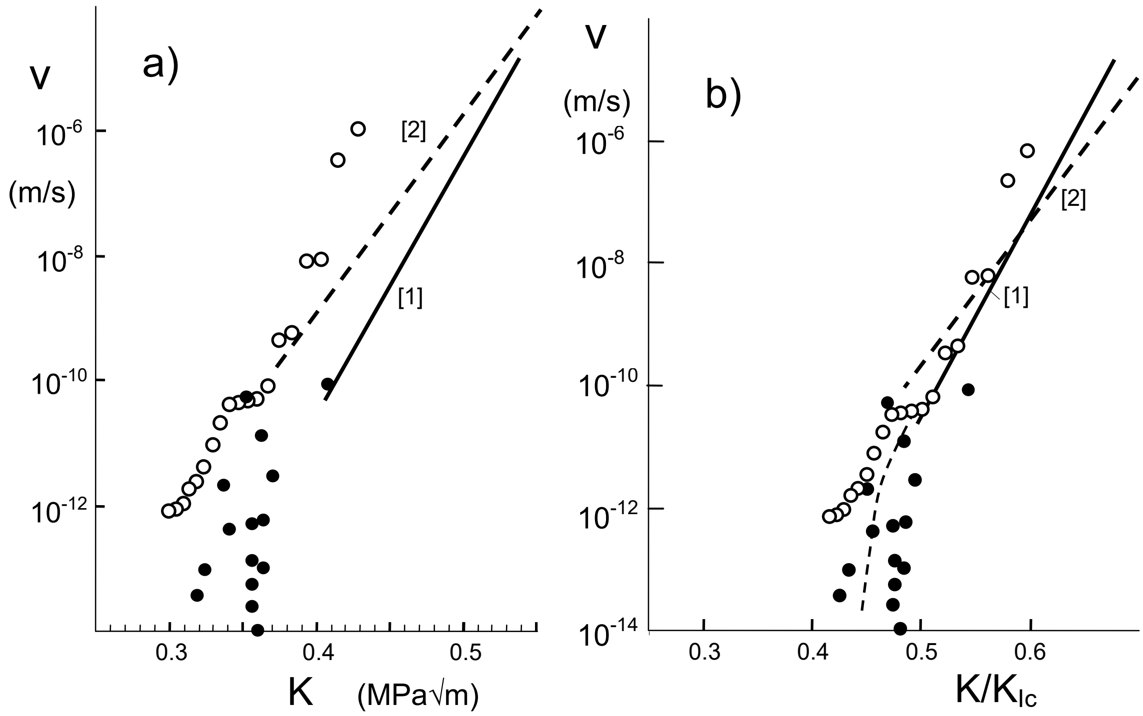

Fig. 1 Subcritical crack growth measurements on silica by Wiederhorn and Bolz [1] (DCB) and Michalske et al. [2] (DCDC), represented by the lines, open circles: results from Braun et al. [5], solid circles: data obtained from the ISF-procedure by Sglavo and Green [4].

2. Evaluation procedure for step-shaped loading

The evaluation of subcritical crack growth rates from lifetime measurements under sectional constant stresses is schematically explained in Fig. 2. A series of N test specimens (N=15 in Fig. 2) is loaded with a constant stress σ_1. During load application, a number of specimens may fail spontaneously as is indicated by the open circles. The specimens which fail on the constant stress level are represented by the solid circles (7 specimens for $\sigma=\sigma_1$). After a time span t_1, the load is increased to $\sigma_2>\sigma_1$. Also in this phase, specimens can fail spontaneously (one specimen in Fig. 2a). The remaining specimens may fail on the higher stress level (5 specimens in Fig. 2a).
Figure 2b shows the distribution of the inert strength that may be determined by N additional test specimens.
The lifetime $t_{f,1}$ for specimens fracture under constant stress σ_1 (Fig. 2a) is given by

$$t_{\mathrm{f},1} = \frac{2}{\sigma_1^2 Y^2} \int_{K_i}^{K_{\mathrm{Ic}}} \frac{K}{v(K)} \mathrm{d}K \,, \tag{1}$$

with the subcritical crack growth rate v, fracture toughness K_{Ic} and geometry factor $Y\cong1.3$ (see e.g. Fett and Munz [7]).
Taking the derivative of the integral in (1) with respect to the lower limit, K_i, yields

$$\frac{\mathrm{d}t_\mathrm{f}}{\mathrm{d}K_{\mathrm{Ii}}} = -\frac{2K_{\mathrm{Ii}}}{Y^2\sigma_1^2 v(K_{\mathrm{Ii}})} \tag{2}$$

For the initial K_I value, K_i, we make use of the relation

$$\frac{K_i}{K_{\mathrm{Ic}}} = \frac{\sigma_1}{\sigma_c} \tag{3}$$

(σ_c=inert strength). The representation of the inert strengths by a Weibull distribution reads

$$F = 1 - \exp\left[-\left(\frac{\sigma_c}{\sigma_0}\right)^m\right] \tag{4}$$

with the failure probability F and the two parameters σ_0 and m which can be determined by using the Maximum-Likelihood method [8].
Introducing logarithmic derivations, the crack-growth rate results in [7]:

$$v(K_\mathrm{i}) = -\frac{2}{t_{\mathrm{f},1}\sigma_1^2}\left(\frac{K_\mathrm{i}}{Y}\right)^2 \frac{\mathrm{d}[\log(K_\mathrm{i})]}{\mathrm{d}[\log(t_{\mathrm{f},1})]} = -\frac{2}{t_\mathrm{f}\sigma_c^2}\left(\frac{K_{\mathrm{Ic}}}{Y}\right)^2 \frac{\mathrm{d}[\log(K_\mathrm{i})]}{\mathrm{d}[\log(t_{\mathrm{f},1})]} \tag{5}$$

Samples that survived the limit time t_1 under load σ_1 are then reloaded to a higher stress $\sigma_2>\sigma_1$ which is again kept constant. Failure of a specimen may occur after the additional time t_2 on this stress level (Fig. 3a).

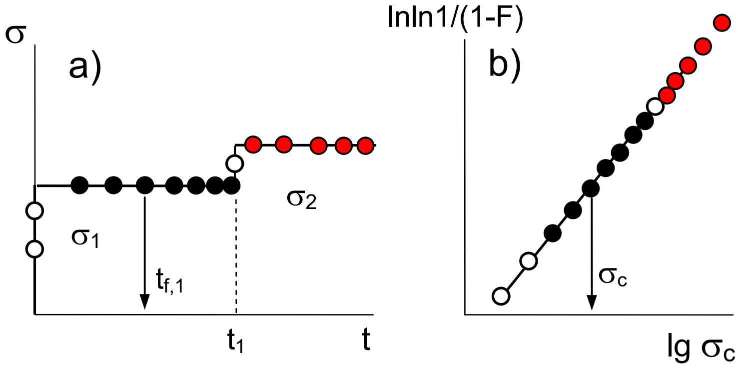

Fig. 2 Evaluation of lifetimes from the lower level of a test series carried out on two stress levels.

Figure 3b shows the increase of the stress intensity factor from the initial value $K=K_i$ up to fracture toughness $K=K_{Ic}$ at which lifetime is reached. When $K^{(1)}$ is the stress intensity factor reached at $t=t_1$, the stress intensity factor after load increase, $K^{(2)}$, is

$$K^{(2)} = K^{(1)} \frac{\sigma_2}{\sigma_1}, \tag{6}$$

For the time span t_2 from load increase to failure, it holds, similar to (1),

$$t_2 = \frac{2}{\sigma_2^2 Y^2} \int_{K^{(2)}}^{K_{Ic}} \frac{K}{v(K)} \mathrm{d}K\,, \tag{7}$$

Rewriting eq.(7) gives

$$t_2 \sigma_2^2 = \frac{2}{Y^2} \int_{K^{(2)}}^{K_{Ic}} \frac{K}{v(K)} \mathrm{d}K\,, \tag{7a}$$

Since the time t_2 and the stress σ_2 are known, the lower integral limit $K^{(2)}$ in eqs.(7, 7a) can be determined from the lifetime measurements obtained on level σ_1. The procedure is illustrated in Fig. 3c where the red symbols and coordinate labelling correspond to the stress σ_2. The black symbols and curve $t_{f1}=f(K_i)$ are from the evaluation of tests fractured on the lower stress level. At the measured lifetime part t_2 the initial stress intensity factor $K^{(2)}$ is obtained as indicated by the red arrows.

Figure 3d shows the values $K^{(1)}$ and $K^{(2)}$ in dependence of the related starting value K_i. For the first part of the lifetime test it holds

$$t_1\sigma_1^2 = \frac{2}{Y^2}\int_{K_i}^{K^{(1)}} \frac{K}{v(K)}\mathrm{d}K = \text{constant}\,, \tag{8}$$

If the stress intensity factor $K^{(1)}$ is known, the crack rate can be determined from eq.(8).

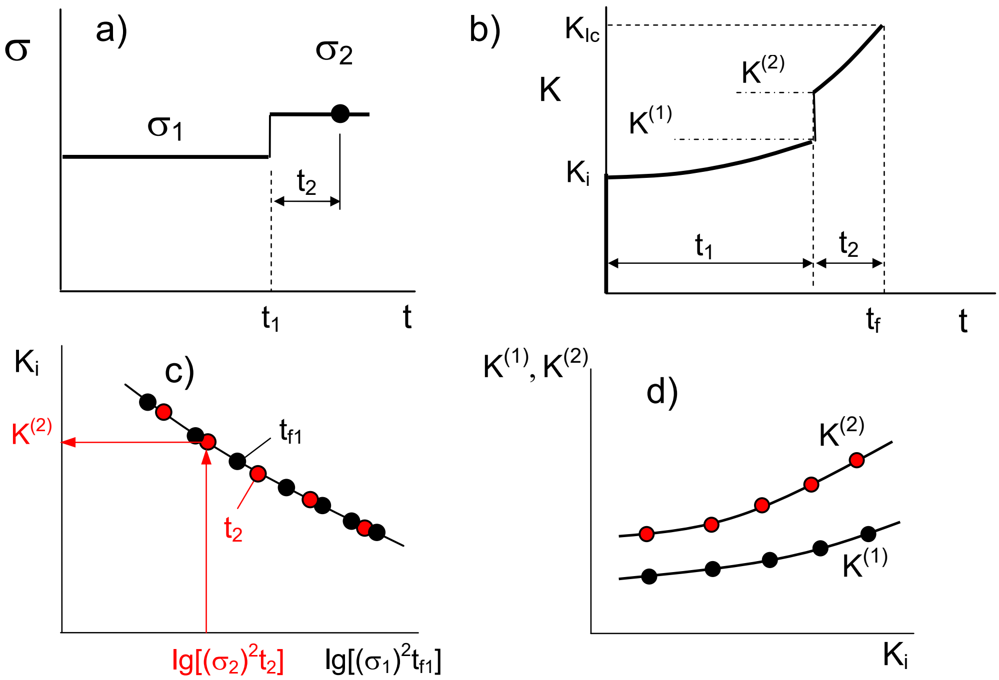

Fig. 3 Evaluation of lifetimes for specimens which failed on the increased stress level $\sigma_2 > \sigma_1$.

3. Evaluation of lifetime measurements from Braun et al. [5]

In order to show the procedure, the strength and lifetime results from Braun et al. [5] may be evaluated. Figure 4 represents the inert strength data in a Weibull plot. The Weibull parameters were reported in [5] as σ_0=156.5 MPa, m=4.44.

The lifetimes at the first stress level of σ=65.3 MPa are plotted in Fig. 5a as a function of the initial stress intensity factor K_i in the representation K_i=f($t_{f,1}$).This value was obtained via eq.(3) using fracture toughness K_{Ic}=0.72 MPa√m.

The v-K-results in Fig. 5b obtained by application of eq.(5) on the data points in Fig. 5a can be described by the straight-line relation

$$v(K) = v_0 \exp[bK] \tag{9}$$

with the parameters: v_0=1.5×10^{-26} m/s, b=102.3 (MPa√m)$^{-1}$. The results of Fig. 5b are introduced in Fig. 1 as the open circles.

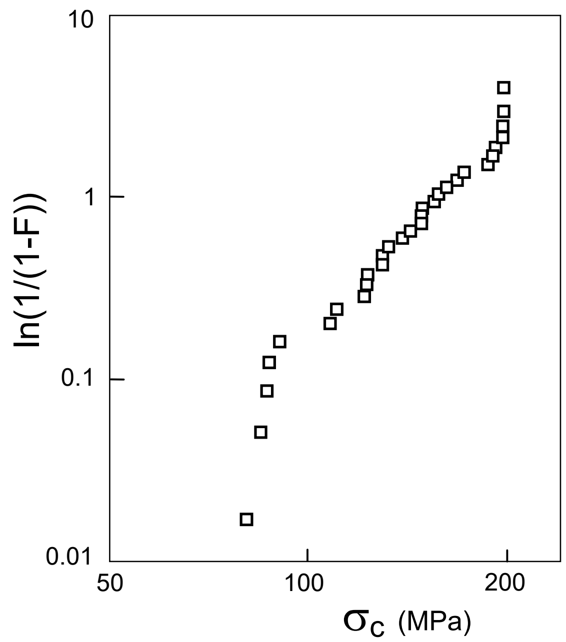

Fig. 4 Inert strength measured by Braun et al. [5].

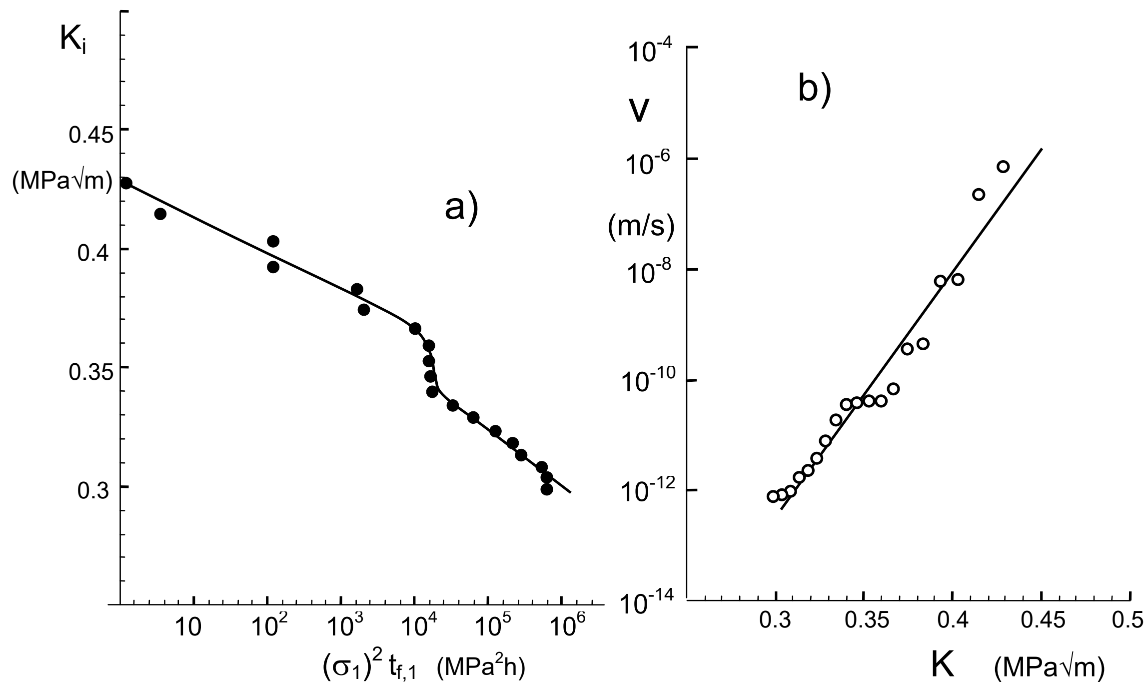

Fig. 5 a) Lifetimes at the lowest load level of σ=65.3 MPa as measured by Braun et al. [5] with fitting curve, b) *v*-*K*-data.

The lifetime results on the higher stress level are included in the normalized plot of Fig. 6 as the red circles. These data represent a stress intensity factor value given by $(\sigma_2/\sigma_1)K_\mathrm{i}$ that is of course not identical with $K^{(2)}$ directly after load increase, because it is computed for the initial crack length a_0. $K^{(2)}$ has to be determined according to Fig.3c so that for the same abscissa value the red circles agree with the black curve in Fig. 6. This is at least correct for the data points that fulfil $(\sigma_2)^2 t_2 < 10^6$ MPa^{2}h. For the three data points above this value a small extension of the curve in Fig. 6 is necessary as is indicated by the dotted line part. From these values, the stress intensity factor $K^{(1)}$ before load increase, $K^{(1)}$, is computed via eq.(6).

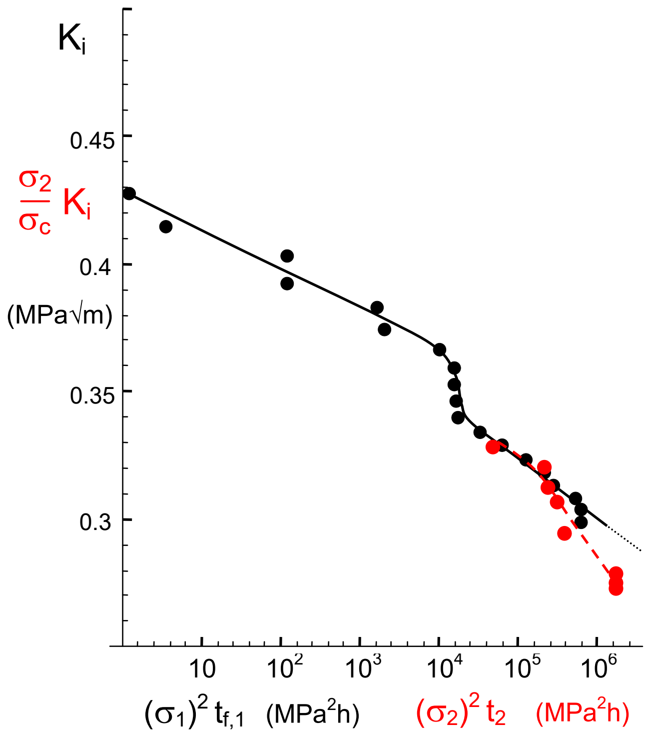

Fig. 6 Lifetimes at σ=65.3 MPa (black) and 72.1 MPa (red) plotted according to Fig. 3c.

In Fig. 7 the stress intensity factors $K^{(2)}$ and $K^{(1)}$ are plotted versus the initial stress intensity factor K_i. The blue circles show the individual results for $K^{(1)}$. Because of the fact that for K_i=0 also $K^{(1)}$ must disappear, a linear regression analysis yields

$$K^{(1)} = \alpha\, K_i \tag{10}$$

with the parameter α=1.002 [0.967, 1.036]. The numbers in brackets are the 95% confidence intervals. The solid line in Fig. 7 represents eq.(10) and the 95% confidence band is given by the dashed curves. It is a priori clear that the confidence band reflects pure scatter since the single values with $K^{(1)} < K_\mathrm{i}$ cannot exist.

From Fig. 7 and eq.(10) it becomes evident that the stress intensity factor $K^{(1)}$ is practically identical with the initial value K_i that is plotted by the blue straight line. Consequently, an exact determination of the crack rate via eq.(8) is not possible since the lower and upper integration boundaries are the same.

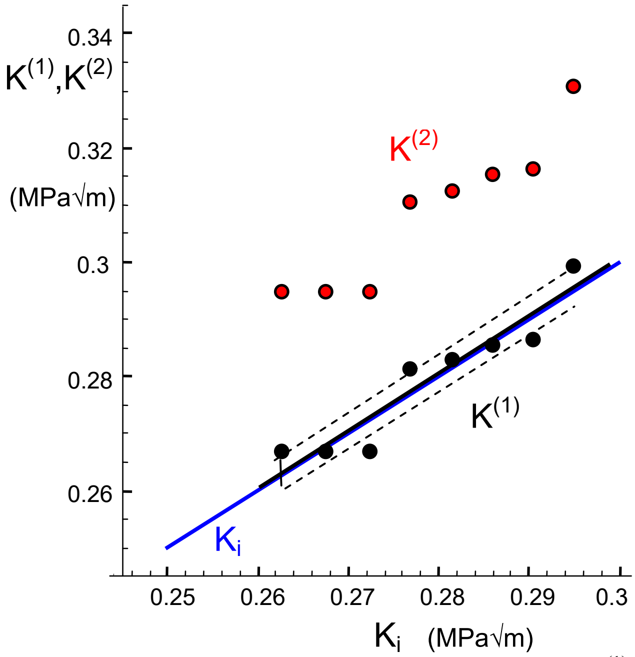

Fig. 7 Stress intensity factors at the end of the dwelling time (t_1=336 h), $K^{(1)}$, according to Fig. 3d.

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