Reconstruction of Cross-Correlations between Heterogeneous Trackers Using Deterministic Samples

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Abstract:
The exploitation of dependencies between state estimates from distributed trackers plays a vital role in so-called track-to-track fusion and has been extensively studied for state estimates with the same state space. In contrast, dependencies are often neglected when considering heterogeneous state estimates referring to different state spaces, since the necessary transformations make the analytic calculation complex or infeasible. This paper aims to develop an overarching framework for the reconstruction of cross-covariances between state estimates obtained in heterogeneous state spaces. The proposed method uses a set of deterministic samples to calculate dependent information. Thus, it allows for a distributed track-keeping of correlations that also encodes the transformation into the local subsystems. To highlight the algorithm, we use a linear problem with heterogeneous trackers only and discuss the correlation problem in detail. The results show superior performance compared to neglecting the correlations.

Keywords: Distributed Estimation, Heterogeneous Track-to-Track Fusion, Cross-covariance Reconstruction, Deterministic Samples

1. INTRODUCTION

Modern tracking systems benefit from a variety of different sensors that are often spatially distributed. Collecting all measurements from these sensors in a central node to process them simultaneously is expensive in terms of processing resources and often infeasible in terms of communication bandwidth and reliability. Therefore, the distributed processing of sensor data directly at their source is often preferred. This more robust approach comes with the need to account for the correlations between state estimates when fusing them. This problem of track-to-track fusion has extensively been studied in recent decades, e.g., by Chang et al. (1997) or Mori et al. (2012).

A special case is the fusion of state estimates that originate from different state spaces and use different system models and that we will call heterogeneous state estimates. Often the transformation into the subspaces is subject to a nonlinear transformation. Bar-Shalom and Chen (2007) first introduced this problem for the track association problem. Yuan et al. (2011) describe a linear minimum mean square error approach (LMMSE) and a maximum likelihood approach (ML) to handle the heterogeneous fusion. Further, they use an approximation technique for the cross-correlations, but state that their application has little impact on the fused result. Another often described approach introduced by Yang et al. (2019a) is information matrix fusion, which is also used by Mallick et al. (2019).

Yang et al. (2019b) introduces an approach to calculate the cross-covariance matrix between heterogeneous estimates analytically using the jacobian to cope with the nonlinear transformation. Their results show that using the cross-covariance matrix is improving the fused estimates. Overall, cross-covariances are often either neglected or only approximated. Using linearization to approximate the nonlinear transformation can lead to unreliable results when nonlinearities are severe. As it has already been done for Kalman filtering, e.g., the UKF as described by Julier and Uhlmann (1997), sampling of distributions is a useful tool for coping with nonlinearities. Samples can also be used to reconstruct correlations between state estimates of different trackers. Reinhardt et al. (2014) use a set of random samples to reconstruct the cross-covariances approximately. Steinbring et al. (2016) firstly introduce deterministic samples to allow for optimal fusion. Here, every sensor node processes a set of samples in parallel to its state estimate and covariance matrix using the local Kalman filter. When the fusion step is executed, the cross-covariances between the state estimates are reconstructed using the sample sets. Since the number of samples grows over time, Radtke et al. (2018) propose a method to keep the number of samples constant. Radtke et al. (2019) extend the method for the fusion of overlapping state estimates.

This paper proposes two key contributions to the fields of heterogeneous track-to-track fusion:

(1) We introduce a heterogeneous track-to-track fusion problem where none of the trackers has full knowledge

about the complete system state. Further, we show that trackers are correlated due to common process noise even though they use different subspaces of the state space for tracking.

(2) We propose a method to keep track of the occurring cross-correlations between state estimates in a distributed fashion using deterministic samples.

In Section 3, this paper gives a brief overview of the considered setup and the problem of distributed estimation and the emergence of correlations between state estimates. Section 4 addresses the dependencies between heterogeneous trackers and introduces the proposed approach using deterministic samples. Section 5 presents numerical results and comparisons with other methods. Finally, we conclude our findings and discuss further research opportunities.

2. PRELIMINARIES

Underlined variables $\underline{x}$ denote vector-valued functions, and lowercase boldface letters $\underline{\boldsymbol{x}}$ are used for random quantities. Matrices are written in uppercase boldface letters $\mathbf{P}$. The notation $\hat{\underline{x}}$ is used for the mean of a random variable, an estimate of uncertain quality or an observation. The matrix $\mathbf{I}$ is the identity matrix of the appropriate dimension. By $\{\underline{p}^{(m)}\}_{m=1}^{M}$, we denote a sample set with a number of M sample vectors.

3. PROBLEM FORMULATION

We assume a target moving in Euclidean space $\mathbb{R}^N$. Its motion can be described by a linear time-invariant discrete system model $\mathbf{A}$ and is subject to additive white Gaussian process noise $\underline{\boldsymbol{w}}_k$ with covariance matrix $\mathbf{Q}$ according to

$$\underline{\boldsymbol{x}}_{k+1} = \mathbf{A}\underline{\boldsymbol{x}}_k + \underline{\boldsymbol{w}}_k \,, \text{with } \underline{\boldsymbol{w}}_k \sim \mathcal{N}(\underline{0}, \mathbf{Q}) \,. \quad (1)$$

The target is observed by a number of L sensor nodes with measurement models of nodes $i = 1, \ldots, L$ according to

$$\underline{\boldsymbol{z}}_k^i = \mathbf{C}_g^i \cdot (\underline{\boldsymbol{x}}_k + \underline{t}_g^i) + \underline{\boldsymbol{v}}_k^i \,, \text{with } \underline{\boldsymbol{v}}_k^i \sim \mathcal{N}(\underline{0}, \mathbf{R}^i) \,, \quad (2)$$

where every sensor node i has a measurement model in the global state space with measurement matrix $\mathbf{C}_g^i$ and is subject to white Gaussian measurement noise $\underline{\boldsymbol{v}}_k^i$ with covariance matrix $\mathbf{R}^i$. Further, $\underline{t}_g^i$ is a sensor-specific known offset from the origin of the global coordinate system to the origin O^i of the local coordinate system. Although fusing all measurements in a single global Kalman filter usually yields the most accurate results, communicating measurements is not always possible or reasonable. Therefore, every sensor will be equipped with a Kalman filter to process the measurements into state estimates locally. An example for such a setup can be seen in Figure 3. We will assume that the local trackers are estimating the target in a lower-dimensional state space $\mathbb{R}^{N_i}$ that is a subspace of $\mathbb{R}^N$ so that $N_i < N$. The transformation from the global state space to the local state space is given by

$$\underline{\boldsymbol{x}}_k^i = \mathbf{G}^i \cdot (\underline{\boldsymbol{x}}_k + \underline{t}_g^i) = \mathbf{G}^i \underline{\boldsymbol{x}}_k + \underline{t}^i \,, \quad (3)$$

with transformation $\mathbf{G}^i$ accounting for the linear transformation from $\mathbb{R}^N$ into the linear Euclidean subspace $\mathbb{R}^{N_i}$. The local measurement model with local measurement matrix $\mathbf{C}^i$ is now defined by

$$\underline{\boldsymbol{z}}_k^i = \mathbf{C}^i \underline{\boldsymbol{x}}_k^i + \underline{\boldsymbol{v}}_k^i \,, \text{with } \underline{\boldsymbol{v}}_k^i \sim \mathcal{N}(\underline{0}, \mathbf{R}^i) \,. \quad (4)$$

An example of such a transformation $\mathbf{G}^i$ into a lower subspace is shown in Figure 1(a).

3.1 Distributed Estimation Problem

While the local processing of state estimates lowers the necessary communication between sensor nodes, it creates a need to address the dependencies between estimates when executing the fusion step. We define an unbiased state estimate as

$$\hat{\underline{m}}_k^i = \hat{\underline{x}}_k^i - \underline{t}^i = \mathbf{G}^i \hat{\underline{x}}_k \,,$$

where the subtraction of $\underline{t}^i$ removes the offset between the different coordinate systems introduced by equation (3). We can then formulate the fusion as a weighted least squares problem as described by Li et al. (2003) according to

$$\hat{\underline{x}}_k^{\mathrm{WLS}} = \arg\min \left[\hat{\underline{m}}_k - \mathbf{G}\underline{\boldsymbol{x}}_k\right]^{\mathrm{T}} \mathbf{J}_k^{-1} \left[\hat{\underline{m}}_k - \mathbf{G}\underline{\boldsymbol{x}}_k\right] \,, \quad (5)$$

with the common state estimate $\hat{\underline{m}}_k = \left[(\hat{\underline{m}}_k^i)^{\mathrm{T}} \ldots (\hat{\underline{m}}_k^L)^{\mathrm{T}}\right]^{\mathrm{T}}$ treated as a measurement. The matrix $\mathbf{G}$ accounts for the known transformation of the global state into all local subspaces according to

$$\mathbf{G} = \left[(\mathbf{G}^1)^{\mathrm{T}} \ (\mathbf{G}^2)^{\mathrm{T}} \ \ldots \ (\mathbf{G}^L)^{\mathrm{T}}\right]^{\mathrm{T}} \,.$$

The state estimates from the distributed trackers include common process noise and common prior information and are thus correlated by

$$\mathbf{J}_k = E\left[(\hat{\underline{m}}_k - \mathbf{G}\underline{\boldsymbol{x}}_k)(\hat{\underline{m}}_k - \mathbf{G}\underline{\boldsymbol{x}}_k)^{\mathrm{T}}\right] \,,$$

which is called the joint error covariance matrix and accounts for the correlation between state estimates

$$\mathbf{J}_k = \begin{bmatrix} \mathbf{P}_k^1 & \mathbf{P}_k^{1,2} & \cdots & \mathbf{P}_k^{1,L} \\ \mathbf{P}_k^{2,1} & \mathbf{P}_k^2 & & \vdots \\ \vdots & & \ddots & \\ \mathbf{P}_k^{L,1} & \cdots & & \mathbf{P}_k^L \end{bmatrix} \,. \quad (6)$$

The solution of (5) can be computed as a gain matrix according to

$$\mathbf{K}_k = \left(\mathbf{G}^{\mathrm{T}}\mathbf{J}_k^{-1}\mathbf{G}\right)^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{J}^{-1}$$

from which we can finally formulate the fusion rule for the fusion of multiple state estimates similar to the work in Sun and Deng (2004) as

$$\hat{\underline{x}}_k^{\mathrm{f}} = \mathbf{K}_k \, \hat{\underline{m}}_k = \mathbf{P}_k^{\mathrm{f}}\mathbf{G}^{\top}\mathbf{J}_k^{-1}\hat{\underline{m}}_k \,, \quad (7)$$

$$\mathbf{P}_k^{\mathrm{f}} = \left(\mathbf{G}^{\top}\mathbf{J}_k^{-1}\mathbf{G}\right)^{-1}. \quad (8)$$

While elements on the main diagonal of $\mathbf{J}_k$ are known since they include the covariance matrices of the local Kalman filters, the cross-covariance matrices on the off diagonals are usually unknown. Since neglecting these dependencies can lead to inconsistent fusion results that underestimate the uncertainty, they can be either approximated which results in a suboptimal fusion rule or reconstructed enabling optimal fusion.

3.2 Calculation of Correlation

We consider two sensor nodes i and j that both employ a Kalman filter or one of its derivatives and estimate the state of the same target of interest. Both trackers share the prior common information $\mathbf{P}_0$ that originates from initialization with the same covariance and that causes them to correlate fully after the initialization step. Further, both sensors are using the same process model, according to (1), and incorporate the same process noise $\underline{w}_k$. Considering this,
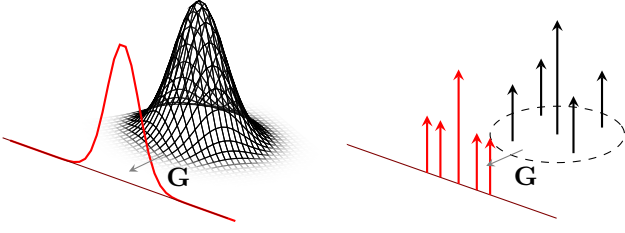
(a) Transformation of two-dimensional Gaussian into one-dimensional Gaussian.

(b) Transformation of samples in two-dimensional space into one-dimensional space.

Fig. 1. Transformation from a two-dimensional to a one-dimensional state space

we can observe how the trackers are correlated after the prediction step

$$\mathbf{P}_{k|k-1}^{i,j} = E[(\hat{\underline{x}}_{k+1|k}^i - \underline{x}_k)(\hat{\underline{x}}_{k|k-1}^j - \underline{x}_k)^{\mathrm{T}}]$$
$$= \mathbf{A}\mathbf{P}_{k-1|k-1}\mathbf{A}^{\mathrm{T}} + \mathbf{Q}_k \,. \qquad (9)$$

This correlation is further altered during the update step by multiplication with the gain matrix $\mathbf{L}_k^i = \mathbf{I} - \mathbf{K}_k^i\mathbf{C}_k^i$ containing the Kalman gain $\mathbf{K}_k^i$

$$\mathbf{P}_{k|k}^{i,j} = E[(\hat{\underline{x}}_{k|k}^i - \underline{x}_k)(\hat{\underline{x}}_{k|k}^j - \underline{x}_k)^{\mathrm{T}}]$$
$$= \mathbf{L}_k^i \mathbf{P}_{k+1|k+1}^{i,j}(\mathbf{L}_k^j)^{\mathrm{T}} \,. \qquad (10)$$

These formulas are recursive and allow the complete calculation of the cross-covariance matrix. Yet, they require full communication of all model parameters at every time step, which is infeasible in large sensor networks.

3.3 Deterministic Sample-based Approach for Reconstruction of Cross-Correlations

Steinbring et al. (2016) introduce a technique to keep track of the cross-covariance matrix in a distributed fashion by deterministically sampling the correlated process noise distributions. While all sample sets are identical after their creation, they are updated by the Kalman filter steps of every sensor node individually and afterwards allow the reconstruction of cross-covariances based on their modifications. The following section briefly reviews the key elements of the method.

The first step is to create a sample set $\{\underline{p}\}_{m=1}^M$ that contains a number of M samples with sample weights ω having the following characteristics

$$\sum_{m=1}^M \omega^{(m)}\underline{p}^{(m)} = \underline{0} \,, \quad \sum_{m=1}^M \omega^{(m)}\underline{p}^{(m)}\big(\underline{p}^{(m)}\big)^{\mathrm{T}} = \mathbf{I}_{D\times D} \,.$$

The dimension of the created sample set is given by $M = D + 1 = N \times (\mathcal{T} + 1) + 1$, where N is the dimension of the state space and $\mathcal{T}$, called the time horizon, is a user-defined number of processing steps that can be included in the sample set. Steinbring et al. (2016) suggest to use the spherical simplex sampling method as proposed by Julier (2003), but other methods can also be employed as long as they share the same characteristics. This sample set is weighed with the following matrix containing the prior common information and common process noise terms

$$\mathbf{\Sigma}_k = \mathrm{diag}\left(\sqrt{\mathbf{P}_{k|k}^{(i,j)}}, \sqrt{\mathbf{Q}_{k+1}}, \dots, \sqrt{\mathbf{Q}_{k+\mathcal{T}}}\right). \quad (11)$$

By weighting the sample set, we obtain a sample set $\{\underline{d}_k\}_{m=1}^M$ according to

$$\underline{d}_k^{(m)} = \mathbf{\Sigma}_k \underline{p}^{(m)} \,, \quad m = 1, \dots, M$$
$$= \left[(\underline{s}_{k|k}^{(i,m)})^{\mathrm{T}}, (\underline{w}_{k+1}^{(m)})^{\mathrm{T}}, \dots, (\underline{w}_{k+\mathcal{T}}^{(m)})^{\mathrm{T}}\right]^{\mathrm{T}} \,, \quad (12)$$

which includes one sample set $\{\underline{s}_{k|k}^{(i,m)}\}_{m=1}^M$ accounting for the common prior information and a sample set $\{\underline{w}_{k+\tau}^{(m)}\}_{m=1}^M$ for every processing step $k + \tau$ accounting for the common process noise until a user defined time horizon $\mathcal{T}$. Multiplying sample sets that account for the same common information yields the underlying cross-covariance

$$\mathbf{P}_{k|k}^{(i,j)} = \sum_{m=1}^M \omega^{(m)}\underline{s}_{k|k}^{(i,m)}\big(\underline{s}_{k|k}^{(i,m)}\big)^{\mathrm{T}} \,,$$
$$\mathbf{Q}_{k+\tau'} = \sum_{m=1}^M \omega^{(m)}\underline{w}_{k+\tau'}^{(i,m)}\big(\underline{w}_{k+\tau'}^{(i,m)}\big)^{\mathrm{T}} \,,$$

while multiplying other terms that are uncorrelated to each other yields zero

$$\sum_{m=1}^M \omega^{(m)}\underline{s}_{k|k}^{(i,m)}\big(\underline{w}_{k+\tau'}^{(m)}\big)^{\mathrm{T}} = \sum_{m=1}^M \omega^{(m)}\underline{w}_{k+\tau'}^{(m)}\big(\underline{w}_{k+\tau''}^{(m)}\big)^{\mathrm{T}} = \mathbf{0} \,,$$

with τ' and τ'' being arbitrary time steps and $\tau' \neq \tau''$. To incorporate the dependencies between state estimates, the samples are propagated through the Kalman filter. Based on the recursive formula (9), we take the sample set accounting for the current common information $\{\underline{s}_{k|k}^{(i,m)}\}_{m=1}^M$ and include the process noise referring to the current time step

$$\underline{s}_{k|k-1}^{(i,m)} = \mathbf{A}\underline{s}_{k-1|k-1}^{(i,m)} + \underline{w}_k^{(m)} \,, \quad m = 1, \dots, M \,. \qquad (13)$$

The sample set is then modified by the update step of the Kalman filter gain according to

$$\underline{s}_{k|k}^{(i,m)} = \mathbf{L}_k^i \mathbf{s}_{k|k-1}^{(i,m)} \,, \quad m = 1, \dots, M \,. \qquad (14)$$

The sample set $\{\underline{s}_{k|k}^{(i,m)}\}_{m=1}^M$ is further updated recursively until the time horizon is reached or the fusion step should be executed. The cross-covariance matrix $\mathbf{P}^{(i,j)}$ can be reconstructed by multiplying the correlation samples of node i and j

$$\mathbf{P}_k^{(i,j)} = \sum_{m=1}^M \omega^{(m)}\big(\underline{s}_{k|k}^{(i,m)} - \bar{\underline{s}}_{k|k}^{(i)}\big)\big(\underline{s}_{k|k}^{(j,m)} - \bar{\underline{s}}_{k|k}^{(j)}\big)^{\mathrm{T}} \,, \quad (15)$$

where $\bar{\underline{s}}$ is the mean of the sample set $\{\underline{s}_{k|k}^{(i,m)}\}_{m=1}^M$ since the processing of the samples, e.g., during the update step, can lead to a nonzero sample mean.

4. RECONSTRUCTION OF CROSS-CORRELATIONS BETWEEN HETEROGENEOUS TRACKERS

While the reconstruction of cross-covariances between homogeneous trackers with state estimates in the same coordinate system is an already solved problem, there is yet no convincing solution for the state estimates from heterogeneous trackers. In the following section, we will, therefore, introduce an analytic formulation for the cross-covariance. Our results are similar to the ones obtained by Yang et al. (2019b), but the extended explanation helps to motivate the proposed method. Based on this, we will show
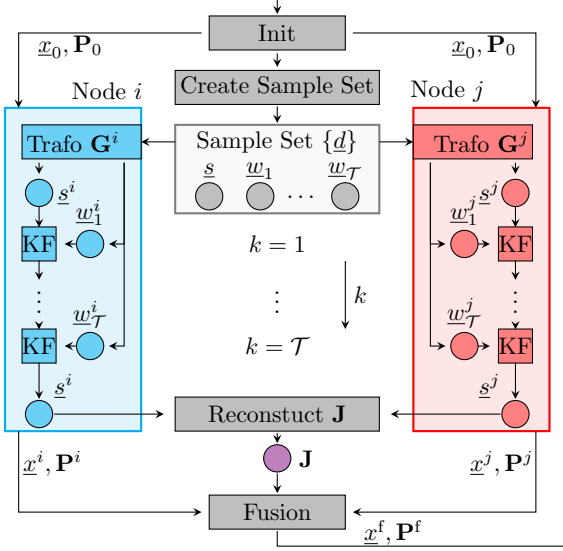
Fig. 2. Flow diagram for the sample-based reconstruction of the cross-covariance matrix between heterogeneous trackers.

why deterministic samples are useful for reconstructing correlations between different state spaces.

4.1 Calculation of Correlations between Subspaces

We consider two sensor nodes i and j estimating the state of a target in different state spaces. Depending on their respective local state space, the information they include is only partially correlated. Therefore the formula (9) needs to be adapted to include the transformation into the local subsystems. The estimation error of node i is defined as $\hat{\underline{x}}^i_{k|k} - \mathbf{G}^i \underline{x}_k$ and the transformation into the sub space is given according to (3) with transformation matrix $\mathbf{G}^i$. Then, we can calculate the cross-covariance matrix after the prediction step as

$$\mathbf{P}^{(i,j)}_{k+1|k} = E\left[\left(\tilde{\underline{x}}^i_{k+1|k}\right)\left(\tilde{\underline{x}}^j_{k+1|k}\right)^{\mathrm{T}}\right]$$
$$= E\left[\left(\mathbf{A}^i \hat{\underline{x}}^i_{k|k} - \mathbf{G}^i \cdot \left(\mathbf{A}\underline{x}_k + \underline{w}_k\right)\right)\left(\dots\right)^{\mathrm{T}}\right],$$

where the local system model can be calculated by $\mathbf{A}^i = \mathbf{G}^i \mathbf{A}\left(\mathbf{G}^i\right)^{\mathrm{T}}$ (see Appendix A). When substituting this into our formula, we receive

$$\mathbf{P}^{(i,j)}_{k+1|k} = E\left[\left(\mathbf{A}^i \hat{\underline{x}}^i_{k|k} - \mathbf{A}^i \mathbf{G}^i \underline{x}_k - \mathbf{G}^i \underline{w}_k\right)\left(\dots\right)^{\mathrm{T}}\right]$$
$$= E\left[\mathbf{A}^i \tilde{\underline{x}}^i_{k|k}\left(\tilde{\underline{x}}^j_{k|k}\right)^{\mathrm{T}}\left(\mathbf{A}^j\right)^{\mathrm{T}}\right]$$
$$+ E\left[\mathbf{G}^i \underline{w}_k\left(\underline{w}_k\right)^{\mathrm{T}}\left(\mathbf{G}^j\right)^{\mathrm{T}}\right]$$
$$= \mathbf{A}^i \mathbf{P}^{i,j}_{k|k}\left(\mathbf{A}^j\right)^{\mathrm{T}} + \mathbf{G}^i \mathbf{Q}\left(\mathbf{G}^j\right)^{\mathrm{T}}.$$

From this formula, we can identify $\mathbf{G}^i \mathbf{Q}\left(\mathbf{G}^j\right)^{\mathrm{T}}$ as the common process noise between i and j. The update during the filter step is similar to (10).

4.2 Calculation of Correlations between Subspaces using Deterministic Samples

In the following section, we will extend the sample-based reconstruction of the cross-covariance matrix to

heterogeneous trackers. The flow chart of the proposed approach is shown in Figure 2, where the processing of the state estimate and covariance matrix, as well as the time index k are neglected for simplicity. Since the process noise is a phenomenon from the global state space, the samples have to be drawn from the full state space to include the complete correlation. Therefore the initial steps of weighting the identity sample set with the common information according to (11) and (12) remain identical. Afterwards, the sample sets are communicated with the local trackers and transformed into the local state space by the transformation matrix $\mathbf{G}^i$ to allow the processing by the local Kalman filter

$$\underline{d}^{(i,m)}_k = \left[\left(\mathbf{G}^i \underline{s}^{(i,m)}_{k|k}\right)^{\mathrm{T}}, \left(\mathbf{G}^i \underline{w}^{(m)}_{k+1}\right)^{\mathrm{T}}, \dots, \left(\mathbf{G}^i \underline{w}^{(m)}_{k+\mathcal{T}}\right)^{\mathrm{T}}\right]^{\mathrm{T}}.$$

A representation of this transformation of samples into a lower dimensional subspace is shown in Figure 1(b). To incorporate the correlation, the samples are updated by the Kalman filter steps according to equations (13) and (14). When the time horizon $\mathcal{T}$ is reached or the fusion step has to be executed, the samples are transferred back to the fusion center and used to reconstruct the joint cross-covariance matrix $\mathbf{J}$ using equations (6) and (15). Afterwards the fusion is carried out using (7) and (8). Lastly, the fused state estimate and covariance are used to reinitialize the local trackers and a new set of samples is created with $\mathbf{P}^{\mathrm{f}}$ as their common prior information.

5. EVALUATION

In this section, we will discuss our numerical results. We assume a target moving on a two-dimensional plane where the motion can be described by a constant velocity model with time constant $\Delta T = 0.1$

$$\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{v}_x \\ \boldsymbol{v}_y \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0 & \Delta T & 0 \\ 0 & 1 & 0 & \Delta T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{v}_x \\ \boldsymbol{v}_y \end{bmatrix}_k + \underline{w}_k,$$

and that is affected by additive white Gaussian noise $\underline{w}_k$ with noise power $q_1 = 2$, $q_2 = 0.5$ according to

$$\mathbf{Q} = \begin{bmatrix} q_1 \frac{1}{3}\Delta T^3 & 0 & q_1 \frac{1}{2}\Delta T^2 & 0 \\ 0 & q_2 \frac{1}{3}\Delta T^3 & 0 & q_2 \frac{1}{2}\Delta T^2 \\ q_1 \frac{1}{2}\Delta T^2 & 0 & q_1 \Delta T & 0 \\ 0 & q_2 \frac{1}{2}\Delta T^2 & 0 & q_2 \Delta T \end{bmatrix}. \quad (16)$$

The target will be observed by three trackers arranged in an isosceles triangle. This setup is sketched out in Figure 3. The subsystems are two dimensional and only observe the position and the velocity along a single axis that is the x-axis of the original system rotated by an angle ϕ_i

$$\phi_1 = -\pi/3, \ \phi_2 = \pi/3, \ \phi_3 = -\pi.$$

Further, the origins O_i of the new coordinate systems have an offset $\underline{t}^i_g$ to the original euclidean coordinate system according to

$$\underline{t}^1_g = \begin{bmatrix} -5 \\ -5 \\ 0 \\ 0 \end{bmatrix}, \ \underline{t}^2_g = \begin{bmatrix} 5 \\ -5 \\ 0 \\ 0 \end{bmatrix}, \ \underline{t}^3_g = \begin{bmatrix} 0 \\ 5 \\ 0 \\ 0 \end{bmatrix}.$$

The transformation from the full state space to the sub state spaces can be described by equation (3), where the linear transformation is given by
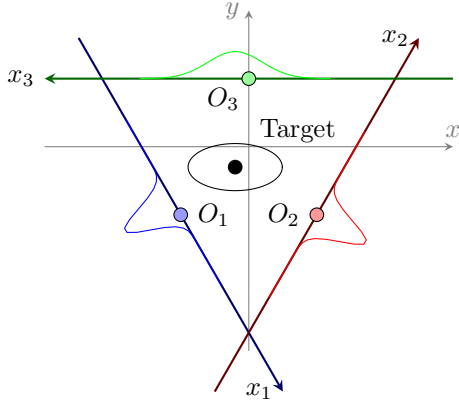
Fig. 3. Problem sketch of the linear evaluation example.

$$\mathbf{G}^i = \begin{bmatrix} \cos(\phi_i) & -\sin(\phi_i) & 0 & 0 \\ 0 & 0 & \cos(\phi_i) & -\sin(\phi_i) \end{bmatrix}.$$

The local measurement model is given by (4), where the measurements are drawn from the global state space by equation (2). The local measurement model matrix in the global state space (see (2)) and the local measurement model matrix (see (4)) are given by

$$\mathbf{C}_g^i = [\cos(\phi_i) \; -\sin(\phi_i) \; 0 \; 0] \;,\; \mathbf{C}^i = [1 \; 0] \;,$$

with the local measurement noise matrices

$$\mathbf{R}^1 = 1 \;,\; \mathbf{R}^2 = 0.5 \;,\; \mathbf{R}^3 = 0.25 \,.$$

Every local tracker uses a constant velocity model describing the motion on a one-dimensional plane

$$\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{v}_x \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{v}_x \end{bmatrix}_k + \underline{\boldsymbol{w}}_k \,.$$

The proof that this is true for every of the considered trackers is given in Appendix A. The process noise matrices of the local trackers on the other hand are different from each other and depend on the transformation as can be seen in Appendix B. The samples are created by using the method used in the unscented Kalman filter as described by Julier and Uhlmann (2004), but where the weight of the sample in the middle is double the weights of the other samples. We want to point out that the sampling technique might depend on the transformation carried out to obtain the local subspaces. The fusion step is carried out every 5th time step. Therefore, our proposed method will be initialized with a sample set including process noise samples till a time horizon of $\mathcal{T} = 5$. Note that the number of samples included in the sample set grows linearly with the number of processing steps until the fusion step is executed. We compare the proposed method (SbF) with a naive fusion approach, where the correlations are neglected, with the optimal reconstruction (Opt) of the correlation using the recursive formulas and with a global Kalman filter as a reference which uses all measurements directly.

Figure 4 shows the mean squared error (MSE) for all compared methods. It can be seen that the global Kalman filter yields the best results. This was excepted since it uses the measurements more efficiently. The optimal and the sample-based reconstruction yield the same results by design. The results also show that neglecting the correlations leads to a higher error. The consistency of the different algorithms can be seen in Figure 5. Here, we compared the estimated error of the tracker, using the trace
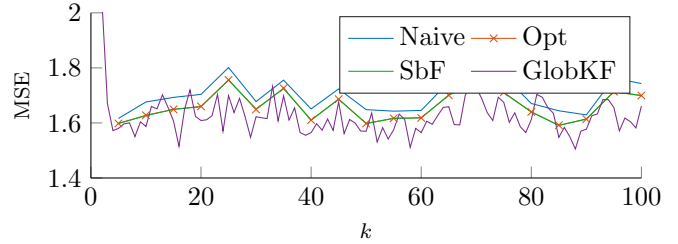

Fig. 4. Mean squared error (MSE) for linear evaluation example for 1000 MCR.

(a) Naive Fusion

(b) Optimal Fusion

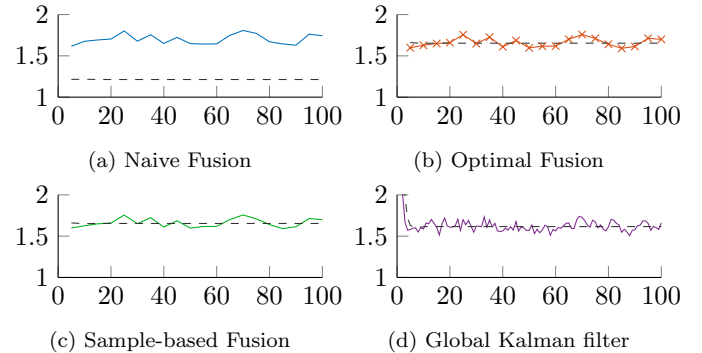(c) Sample-based Fusion

(d) Global Kalman filter

Fig. 5. Comparison of the trace of the fused covariance matrix (dashed) and actual mean squared error for 1000 MCR.

of the covariance matrix, with the actual mean squared error. It can be seen that for the Global Kalman filter (Figure 5(d)), the optimal fusion (Figure 5(b)) and the proposed approach (Figure 5(c)) the actual error matches the estimated error. As expected, the naive fusion (see Figure 5(a)) results in an overconfident covariance matrix that does not match the real error and therefore leads to an inconsistent tracker.

6. CONCLUSION

This paper introduced a sample-based approach to reconstruct the cross-covariance between distributed heterogeneous trackers. Further, it introduced an example where every tracker only observes a subspace of the targets state space and the nodes need to cooperate to obtain the full state of the target. While we used the particular case of a linear transformation to highlight the algorithm itself, the method can be adapted to a nonlinear transformation. Using deterministic samples can be especially useful since it renders the linearization of the transformation unnecessary. Because of the nonlinear dependencies between heterogeneous state estimates, the joint covariance matrix is insufficient to model the joint probability distribution and further research has to be undertaken to solve this problem. Yet, deterministic samples could provide a missing puzzle piece for modeling dependencies needed in many heterogeneous track-to-track fusion problems.

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Appendix A. TRANSFORMATION OF TWO-DIM. CONSTANT VELOCITY MODEL (CVM) INTO A LOWER-DIM. STATE SPACE

Let $\underline{x}$ be the state vector in the global state space and further let $\underline{x}^i$ be the state vector of node i in the local lower-dimensional subspace. Then the transformation from the global to the local state space is defined by

$$\underline{x}_k^i = \mathbf{G}^i \cdot \left(\underline{x}_k + \underline{t}_g^i \right),$$

where $\mathbf{G}^i$ is a linear transformation matrix and $\underline{t}_g^i$ is a linear translation between the two coordinate systems. The propagation of the global state can be described by a linear transition matrix $\mathbf{A}$. We now want to obtain the local transition matrix $\mathbf{A}^i$ which is the global transition matrix transformed into the local subspace. The prediction step from time step k to time step $k+1$ can be described by

$$\underline{x}_{k+1}^i = \mathbf{G}^i \mathbf{A} \left(\underline{x}_k + \underline{t}_g^i \right) = \mathbf{G}^i \mathbf{A} \underline{x}_k + \mathbf{G}^i \mathbf{A} \underline{t}_g^i$$
$$= \mathbf{A}^i \mathbf{G}^i \underline{x}_k + \mathbf{A}^i \mathbf{G}^i \underline{t}_g^i,$$

We can set the equations equal, resulting in $\mathbf{G}^i \mathbf{A} \underline{t}_g^i \overset{!}{=} \mathbf{A}^i \mathbf{G}^i \underline{t}_g^i$ and

$$\mathbf{A}^i \mathbf{G}^i \underline{x}_k \overset{!}{=} \mathbf{G}^i \mathbf{A} \underline{x}_k^i$$
$$\mathbf{A}^i \mathbf{G}^i \overset{!}{=} \mathbf{G}^i \mathbf{A}$$
$$\mathbf{A}^i = \mathbf{G}^i \mathbf{A} \mathbf{G}^{i*},$$

where $\mathbf{G}^{i*}$ is the pseudo inverse of $\mathbf{G}^i$. The transformation matrix $\mathbf{G}^i$ is a basis change matrix where the new basis also has a lower dimension than the original state space. Revisiting the evaluation example from Section 5, the new basis is the span of the vectors $\{\underline{w}_1, \underline{w}_2\}$. We choose a new transformation matrix $\mathbf{G}^i$ of the following form

$$\mathbf{G}^i = \begin{bmatrix} w_1^{\mathrm{T}} \\ w_2^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & 0 & 0 \\ 0 & 0 & c_1 & c_2 \end{bmatrix},$$

where c_1 and c_2 are arbitrary constants for which $c_1^2 + c_2^2 = 1$ holds. This normalization creates a new state space that has the same dimensions as the original state space without stretching or compression. We define a pseudo inverse $\mathbf{G}^{i*} = (\mathbf{G}^i)^{\mathrm{T}}$ that fulfills $\mathbf{G}^i (\mathbf{G}^i)^{\mathrm{T}} = \mathbf{I}$, with $\mathbf{I}$ in the lower-dimensional subspace. We finally obtain the local transition matrix as

$$\mathbf{A}^i = \mathbf{G}^i \mathbf{A} (\mathbf{G}^i)^{\mathrm{T}} = \begin{bmatrix} c_1^2 + c_2^2 & \Delta T c_1^2 + \Delta T c_2^2 \\ 0 & c_1^2 + c_2^2 \end{bmatrix} = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix}.$$

In the considered evaluation example, the offset to the local coordinate system $\underline{t}_g^i$ does not include a velocity component which leads to a constant component $\mathbf{A} \underline{t}_g^i = \underline{t}_g^i$.

Appendix B. TRANSFORMATION OF THE PROCESS NOISE MATRIX INTO A LOWER-DIM. STATE SPACE

Analogously to Appendix A, the local process noise $\mathbf{Q}^i$ is obtained by $\mathbf{Q}^i = \mathbf{G}^i \mathbf{Q} (\mathbf{G}^i)^{\mathrm{T}}$. Compared to the transition matrix $\mathbf{A}$, the process noise is not symmetric. Considering a global process noise matrix $\mathbf{Q}$ as defined in (16)), the local process noise matrix can be calculated by

$$\mathbf{Q}^i = \begin{bmatrix} c_1 q_1 \frac{1}{3} \Delta T^3 + c_2 q_2 \frac{1}{3} \Delta T^3 & c_1 q_1 \frac{1}{2} \Delta T^2 + c_2 q_2 \frac{1}{2} \Delta T^2 \\ c_1 q_1 \frac{1}{2} \Delta T^2 + c_2 q_2 \frac{1}{2} \Delta T^2 & c_1 q_1 \Delta T + c_2 q_2 \Delta T \end{bmatrix}.$$

While the transformation of the system model is independent of the local state space, the transformation of $\mathbf{Q}^i$ depends on the new coordinate system if $q_1 \neq q_2$.