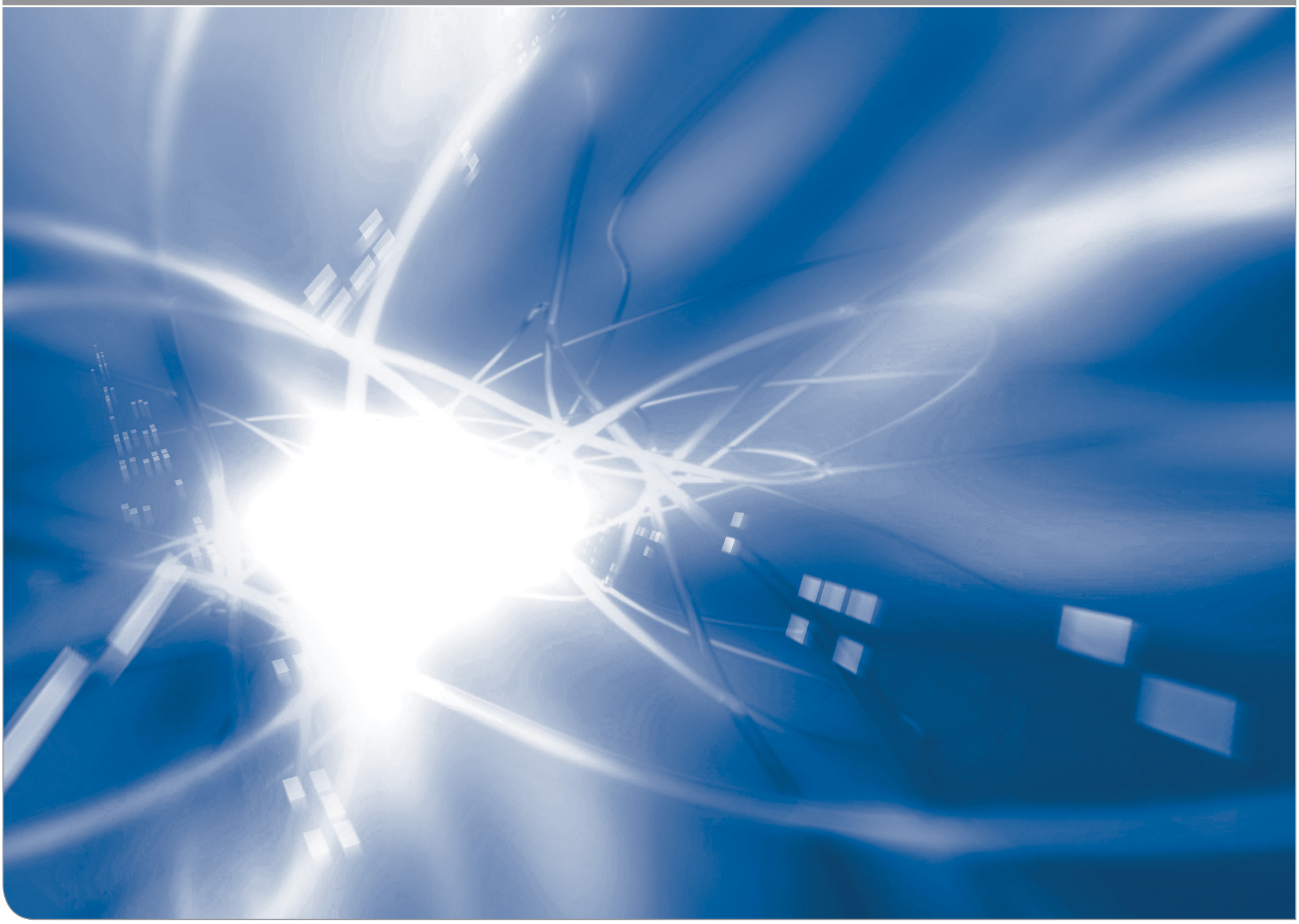
Karlsruher Institut für Technologie

# Crack-flank displacements for cracks with zones of reduced Young's modulus

G. Rizzi, T. Fett, K.G. Schell, C. Bucharsky

KIT SCIENTIFIC WORKING PAPERS **150**



KIT – Universität des Landes Baden-Württemberg und nationales Forschungszentrum in der Helmholtz-Gemeinschaft

IAM Institute for Applied Materials

**Impressum**

Karlsruher Institut für Technologie (KIT)
www.kit.edu



2020

ISSN: 2194-1629

## Abstract

In this report, the influence of different elastic modules ahead of a crack tip and in the bulk on the crack-flank displacements is discussed.
From our FE-computations on crack-flank displacements it was found that the effect of a reduced Young's modulus is visible only in a relative crack-tip distance of about $r/\omega<1.5$ ($\omega$=height of the zone showing a reduced Young's modulus).

The slopes of the asymptotes, reached for distances $r/\omega>1.5$, are within about ±5% independent of the modulus.

A fictive crack length increased by an amount of $\Delta a/\omega$ into the bulk material is in the order of $\Delta a/\omega\approx1$.

The steeply rising displacements near $r/\omega\approx0$ may suggest the occurrence of "crack-tip blunting" as dealt in elastic-plastic fracture mechanics, although purely linear-elastic material behaviour is present.

# Contents

**1 Introduction** 1

**2 Crack-Flank Displacements** 2

2.1 CFD for heart-shaped zones 3

2.2 CFD for zones extended in the crack wake 4

**3 Comparison of parameters for the two zone shapes** 5

**Summary** 6

**References** 7

# 1. Introduction

In [1] and [2] we reported stress intensity factor results from a study on heart-shaped zones ahead of a crack tip for arrested and extended zones. In these FE-studies the Young's modules $E$ in the zones were chosen to be different from that of the bulk material, $E_0$. Figure 1a illustrates the heart-shaped zone and Fig. 1b shows the zone extended in the crack wake.

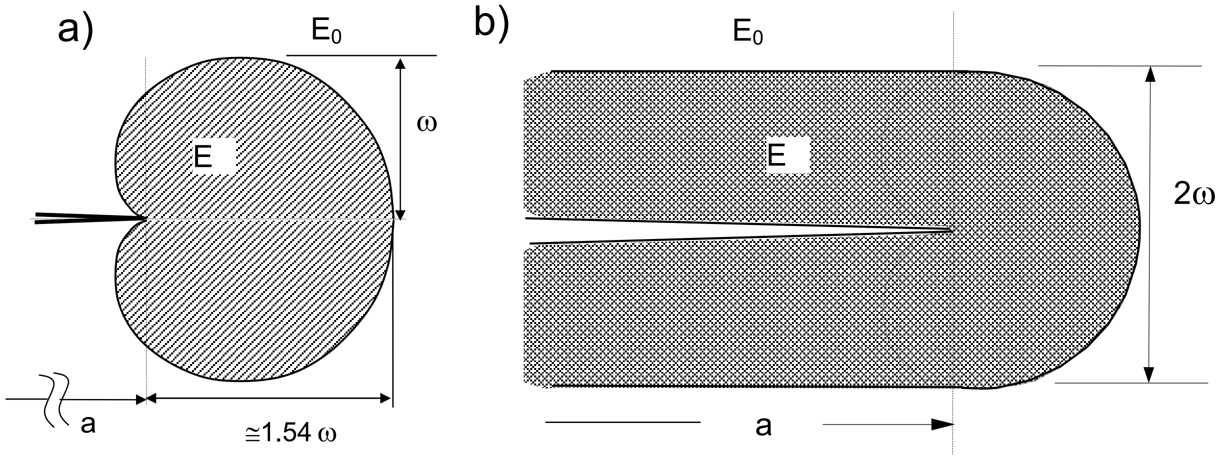



**Fig. 1** a) Heart-shaped crack-tip zone with a Young's modulus $E$ deviating from the bulk modulus $E_0$, $E<E_0$, b) zone extending over the whole crack length $a$.

The stress intensity factors can be determined via the J-Integral by Rice [3], the stress distribution ahead the crack tip and the crack-flank displacements in the crack wake. Very good agreement of all the methods was stated. Figure 2 shows $K$-results obtained by using the J-Integral.

In the case of the heart-shaped zone, the results could be expressed by

$$K = K_{appl}\sqrt{\frac{E}{E_0}} \tag{1}$$

indicated by the dashed straight line in Fig.2. The solid curve representing the results for the extended zone could be fitted by

$$\frac{K(E)}{K(E_0)} \cong \frac{2E}{E+E_0} \tag{2}$$

The stress intensity factor $K(E_0)$ is the value obtained for $E$=$E_0$, i.e. for homogeneous material.

The FE results confirm eq.(1) for $0.1<E/E_0<1$ as had to be expected e.g. from the theoretical analysis by Merkle [4] on slender notches embedded in the zone of reduced modulus at the notch-root. From the agreement between the theoretical solution by

Merkle [4] and the FE-results for $0.1 \le E/E_0 \le 1$, it can be concluded that our crack-zone modelling and the accuracy of our FE-mesh is sufficient at least for this range.

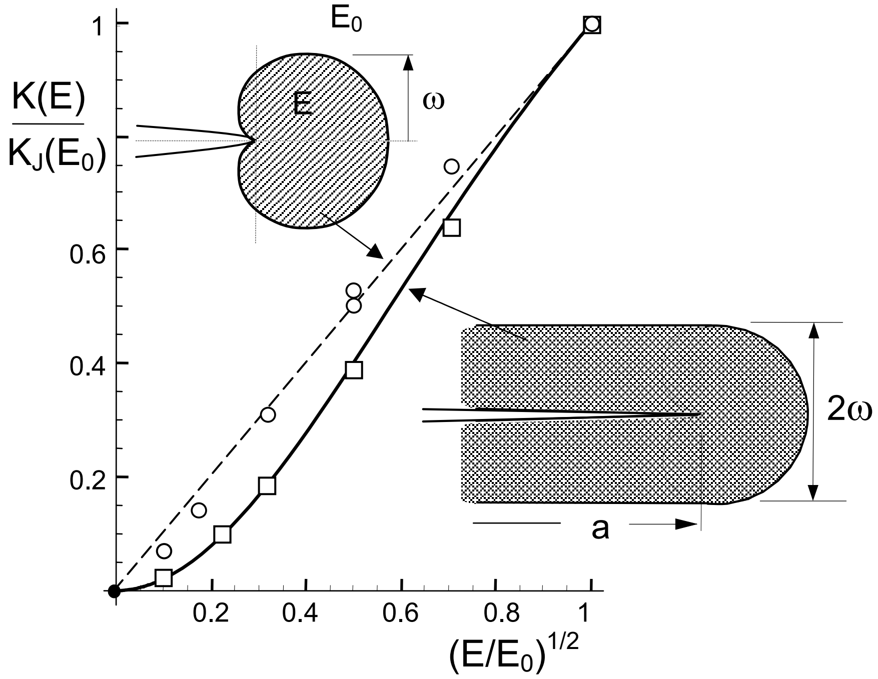



**Fig. 2** Stress intensity factors from J-Integral, $K_J$ (normalized on the value for $E=E_0$) for heart-shaped (circles) and extended zones (squares).

## 2. Crack-Flank Displacements

The results reported below are only intended to complete the FE data. We evaluated crack-opening displacement results in [1] in order to compute the stress intensity factors for different Young's moduli and obtained the near-tip profiles. In this context, it has to be noted that crack opening *displacements* COD are defined as the distance from the symmetry line to one of the surfaces. Unfortunately, the (total) crack opening, i.e. the distance between the crack flanks, is sometimes also referred to as COD. In order to avoid misinterpretations, we denote the displacements of the crack flanks from the symmetry line as "Crack-Flank Displacement" CFD. Stress intensity factors $K_{CFD}$ were computed from the Irwin-parabola

$$v = \sqrt{\frac{8}{\pi}} \frac{K_{CFD}}{E/(1-\nu^2)} \sqrt{r} \tag{3}$$

where $v$ is the crack-flank displacement in the distance $r$ from the crack tip and ν is Poisson's ratio (0.17 for silica). Normalized displacements $v'$ represented in Fig. 3 are defined by

$$v' = v \frac{E_0}{K(E_0)\sqrt{\omega}(1-\nu^2)} \tag{4}$$

where $K(E_0)$ denotes the stress intensity factor for the case of homogeneous material.

## 2.1 CFD for heart-shaped zones

Crack-flank displacements for the crack with a heart-shaped zone ahead of the tip are shown in Fig. 3a in the form $v'$=f($r$/ω) and in Fig. 3b in the form of $(v')^2$=f($r$/ω). From the diagram of $(v')^2$=f($r$/ω) it becomes obvious that the curves tend to straight lines for about $r$/ω>1.5. The straight lines for $E/E_0$<1 show fictive crack origins as indicated by the intersections of the dashed extensions of the crack profiles with the abscissa. In the case of very short crack-tip distance the CFD is similar to crack-tip blunting in elastic-plastic fracture mechanics although the material behaviour is purely linear-elastic. In larger distances of about $r$/ω>1, the straight-line for $E/E_0$=0.01 shows nearly the same slope as obtained for $E/E_0$=1 and seems to be shifted by the constant amount $v_T$/2.

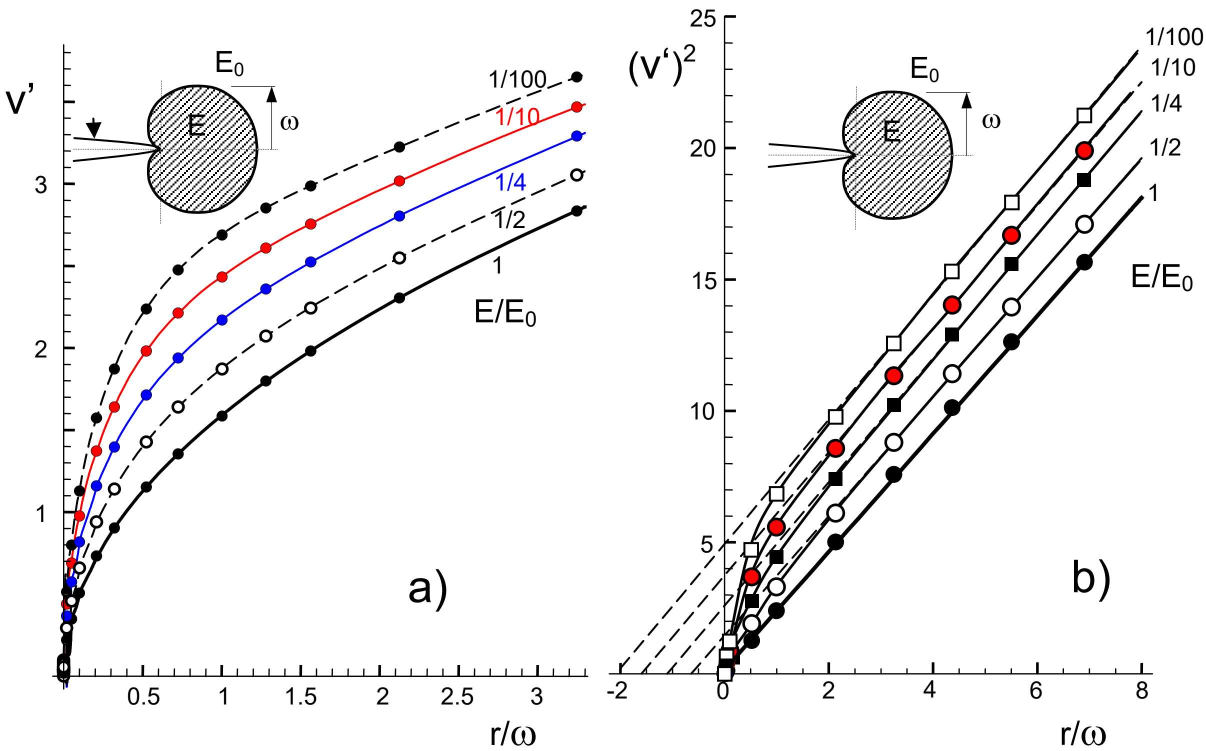



**Fig. 3** Influence of reduced Young's modulus in the heart-shaped crack-tip zone, a) linear representation of the normalized crack-flank displacements CFD according to eq.(4), b) square of the crack-flank displacements as the ordinate.

The value of $2v'$ at $r$=0 is the normalized Crack-Tip-Opening Displacement (CTOD) $v_T$ as is defined in elastic-ideal plastic fracture mechanics. Figure 4 again shows the flank displacements for the extremely different Young's moduli ratios $E/E_0$ =1 and

1/100. The apparent crack-tip blunting is illustrated by the inset in Fig. 4, when the CFD is simplified by the thick red contours, resulting in an apparent crack-tip opening $v_T=2v(r\rightarrow 0)$ and an apparent increase $\Delta a$ of the fictive crack length.
By least-squares fitting of the data points for $r/\omega>3$, we obtained the dashed straight lines characterized by the slope and the offset from the origin.
The two parameters $v'$ and the slopes of the straight lines are compiled in Table 1 together with the CTOD (column 4). From simple geometry, also the apparent extension of the crack $\Delta a/\omega$, i.e. the shift of the fictive crack tip into the bulk material is obtained. This quantity is given in the 5th column of Table 1.

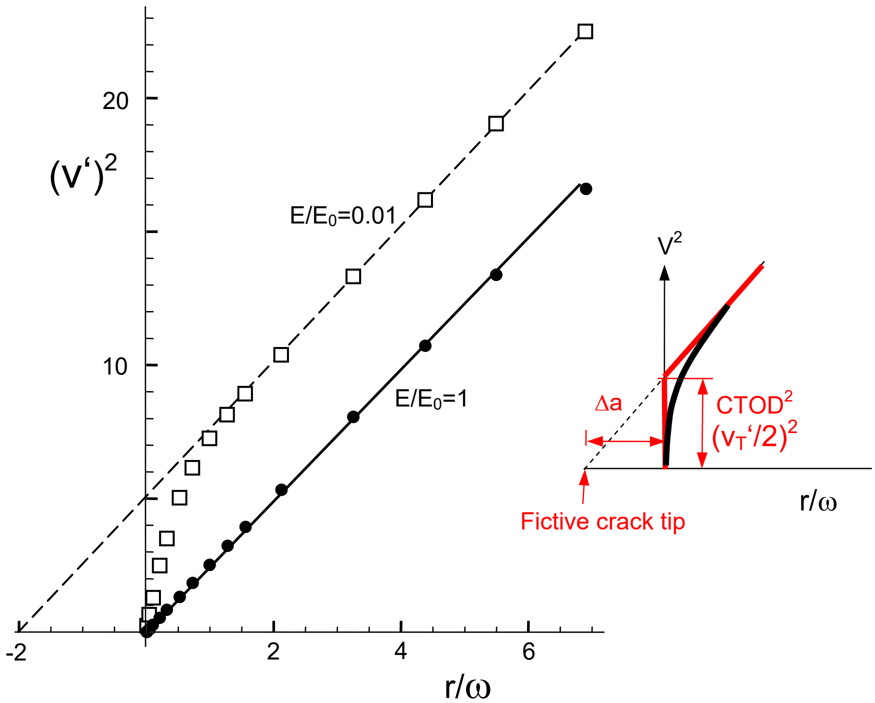



**Fig. 4** Near-tip CFD for $E/E_0$=0.01 and 1, inset: Definition of crack-tip opening displacement (CTOD).

| $E/E_0$ | $d(v')^2/d(r/\omega)$ | $(v'(r=0))^2$ | $v_T'$ | $\Delta a/\omega$ |
|---|---|---|---|---|
| 1 | 2.436 | 0 | 0 | 0 |
| 0.5 | 2.433 | 1.396 | 2.363 | 0.573 |
| 0.25 | 2.494 | 2.746 | 3.314 | 1.101 |
| 0.1 | 2.488 | 3.966 | 3.983 | 1.594 |
| 0.01 | 2.511 | 5.197 | 4.559 | 2.067 |

**Table 1** Parameter of straight lines in Fig. 3b: Slopes of straight lines, crack tip displacements $(v')^2$, and $v_T'$, fictive crack-length increase $\Delta a$ as a function of Young's modulus ratio $E/E_0$ ($\nu$=0.17).

## 2.2 CFD for zones extended in the crack wake

For the zone extending over the whole crack wake, Fig. 1b, we performed the same evaluations as done for the heart-shaped zone. Figure 5 and Table 2 show the results.

| $E/E_0$ | $d(v')^2/d(r/\omega)$ | $(v'(r=0))^2$ | $v_T'$ | $\Delta a/\omega$ |
|---|---|---|---|---|
| 1 | 2.58 | 0 | 0 | 0 |
| 0.5 | 2.69 | 0.47 | 1.37 | 0.17 |
| 0.25 | 2.76 | 1.32 | 2.30 | 0.48 |
| 0.1 | 2.81 | 2.25 | 3.00 | 0.80 |
| 0.05 | 2.83 | 2.74 | 3.31 | 0.97 |
| 0.01 | 2.86 | 3.19 | 3.57 | 1.12 |

**Table 2** Parameters characterizing the dashed straight lines in Fig. 5.

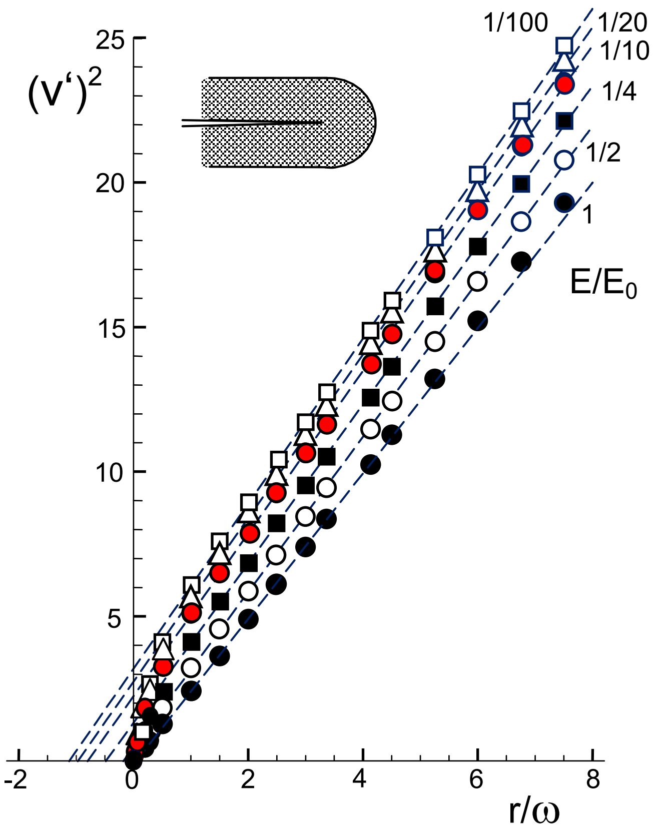



**Fig. 5** Influence of a reduced Young's modulus in the extended zone on normalized displacements, square of the CFD as the ordinate.

## 3. Comparison of parameters for the two zone shapes

Figure 6 shows again the parameters from Tables 1 and 2. In Fig. 6a the slopes are given and in Fig. 6b the CTOD, i.e. the offsets of the straight line asymptotes at $r/\omega=0$. It is obvious that the slopes hardly vary noticeably with the modulus of elasticity. Only slight increase with increasing modulus can be stated.

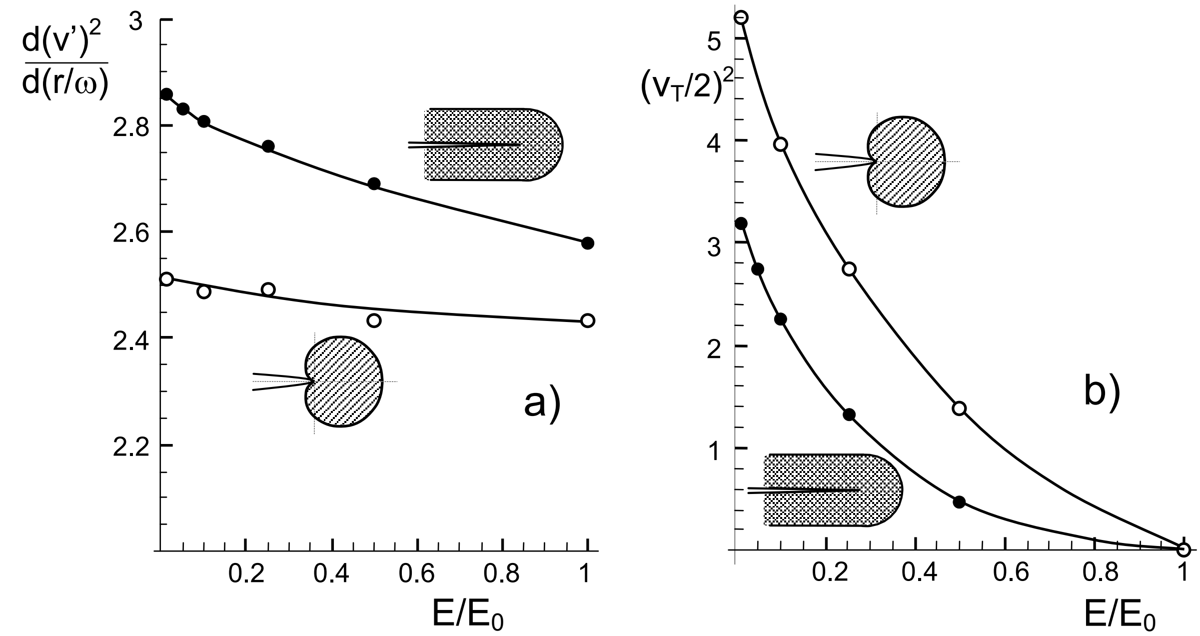



**Fig. 6** a) Comparison of slopes and b) CTODs from Tables 1 and 2.

## 4. Summary

We studied the crack-flank displacements CFD for cracks in a bi-material. The following points may be emphasized:

- From our FE-computations on crack-flank displacements it can be stated that the effect of a reduced Young's modulus is visible only in a crack-tip distance of $r/\omega<1.5$.
- The slopes of the asymptotes, reached for distances $r/\omega>1.5$, are within about ±5% independent of the modulus.
- The fictive crack length defined by the intersection of the straight asymptotes with the abscissa is in the order of $\Delta a/\omega\approx 1$.
- The steeply rising displacements at $r/\omega\approx 0$ may suggest to the observer an occurrence of "crack-tip blunting", although purely linear-elastic material behaviour is present.

## References

1 G. Rizzi, K.G. Schell, C. Bucharsky, T. Fett, FE-Study on heart-shaped crack-tip zones, **142**, 2020, ISSN: 2194-1629, Karlsruhe, KIT

2 K.G. Schell, C. Bucharsky, G. Rizzi, T. Fett, Heart-shaped crack-tip zones – Effect of the zone length, **148**, 2020, ISSN: 2194-1629, Karlsruhe, KIT

3 Rice, J.R., A path independent integral and the approximate analysis of strain concentration by notches and cracks, Trans. ASME, J. Appl. Mech. (1986), 379-386.

4 J. G. Merkle, An application of the J-integral to an incremental analysis of blunt crack behavior, Mechanical Engineering. Publications, London, 1991, 319-332.

KIT Scientific Working Papers
ISSN 2194-1629