

Probabilistic relationships between strain range, stress range and loading cycles. Application on ASTM A969 steel

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ABSTRACT

Keywords:

Stress-life
Strain-life
Cyclic loading
Weibull distribution
Stüssi model

Fatigue life estimations can be made by considering the $\Delta\sigma$ - N or $\Delta\epsilon$ - N curves. Reliable estimations allow engineers designing save stress levels in structures or machines. In this paper, the Weibull distribution is combined with the Stüssi function to model the quantiles of the $\Delta\sigma$ - N and $\Delta\epsilon$ - N curves and the Ramberg-Osgood relationship. The nonlinear Stüssi function offers a good geometric approach to model the relationship $\Delta\sigma$ - N or $\Delta\epsilon$ - N and the Weibull distribution is the most adequate to handle lifetime magnitudes as load cycles N . To show the application of the proposed model, simulated and experimental data of ASTM A969 steel are evaluated.

1. Introduction

In the behaviour of materials under cyclic loading three magnitudes are mainly considered: the strain range, the stress range and the number of load cycles. Several attempts have been done to describe the relationships between these variables, either in a deterministic or a probabilistic way.

On the one hand, several models have been proposed in [1–11] to describe the relationship between the stress range and the number of load cycles, see Table 1.

On the other hand, to describe the relationship between the strain range and loading cycles, the models proposed by Coffin-Manson [12], Smith-Watson-Topper [13], Walker [14] and Castillo-Fernández-Canteli [15] are usually applied, see Table 2. However, in few cases a probabilistic model has been proposed.

In [16] the author defines a general Cumulative Distribution Function (CDF) for the lifetime. This CDF depends on a two-parameter Weibull distribution whose parameters should be estimated. Afterwards, in the loglog scale, linear confidence intervals for the strain are obtained. Because of its linearity, this model is not suitable to perform estimations in Low Cycle Fatigue (LCF) or High Cycle Fatigue (HCF) regime.

In [17,15] the authors proposed a model based on the Ramberg-Osgood relationship and a three-parameter Weibull distribution. An application of this model is presented in [18,19]. Unfortunately, this model does not offer a suitable geometrical description of the strain in LCF regime.

In [20], the authors present a three-dimensional model to describe

the relationship between stress, strain and load cycles. Since this deterministic model is based on the linear model of Basquin, its results are not suitable for estimations neither in LCF or HCF regime.

In [21] a linear regression model for statistical analysis of strain-life fatigue data is proposed. This model considers the LCF and the HCF separately and it defines a transition fatigue life as the point where the two linear regressions meet.

In [22] a strain-life curve based on the combination of HCF and LCF damage is presented. In [11] the authors propose a general fatigue model based on the Kohout-Věchet function. This model considers several fatigue damage parameters and one of its applications allows to depict strain-life curves.

At the present time, modelling the strain-life and stress-life curves is still a very important research topic in steel structures. For instance, several applications on riveted steel structures have been recently done by considering the models mentioned above, see [23–26].

In this paper the authors propose a model which describes the probabilistic relationships between strain range, stress range and loading cycles. This model is built by considering the following components:

- The nonlinear function proposed by Stüssi in [27,28] and applied by the authors to model the Wöhler curves in [29]
- The three-parameter Weibull distribution $W(a, b, c)$, see [30]
- The Ramberg-Osgood relationship considered in [31]

This paper is organized as follows. In Section 2 the Ramberg-Osgood relationship is presented and its inverse is calculated in order to express

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Table 1
Some models to represent $S-N$ curves.

Model	$S-N$ curves equation
Basquin (1910)	$\log N = A - B \log \Delta \sigma; \Delta \sigma \geq \Delta \sigma_{\infty}$
Stromeyer (1914)	$\log N = A - B \log(\Delta \sigma - \Delta \sigma_{\infty})$
Palmgreen (1924)	$\Delta \sigma = b(N + B)^{-a} + \Delta \sigma_{\infty}$
Bastenaire (1972)	$N = \frac{A}{\Delta \sigma - E} \exp[-C(\Delta \sigma - E)] - B$
Ling & Pan (1997)	$F = \sum_{i=1}^n \left\{ \ln \sigma(S_i) + \frac{[\log N_i - \mu(S_i)]^2}{2\sigma^2(S_i)} \right\}$
Kohout & Věchet (2001)	$\log \left(\frac{\Delta \sigma}{\Delta \sigma_{\infty}} \right) = \log \left(\frac{N + N_1}{N + N_2} \right)^b$
Castillo et al. (2009)	$p = 1 - \exp \left\{ - \left[\frac{(\log N - B)(\log \Delta \sigma - C) - a}{b} \right]^c \right\}$

Table 2
Some models to represent $\Delta \epsilon-N$ curves.

Model	$\Delta \epsilon-N$ curves equation
Coffin-Manson (1954)	$\Delta \epsilon = 2 \left[\left(\frac{S_e}{E} \right) (2N)^{b_e} + S_p (2N)^{b_p} \right]$
Smith-Watson-Topper (1970)	$\sigma_{max} \epsilon_a = A_1 N^{a_1} + A_2 N^{a_2}$
Walker (1970)	$\epsilon_a = \frac{\sigma_f}{E} \left(\frac{1-R}{2} \right)^{(1-\gamma)} (2N)^b + \epsilon'_f \left(\frac{1-R}{2} \right)^{c(1-\gamma)/b} (2N)^c$
Castillo et al. (2009)	$p = 1 - \exp \left\{ - \left[\frac{(\log N - B)(\log \Delta \epsilon - C) - a}{b} \right]^c \right\}$

the stress range as function of the strain range. In Section 3 a function type Stüssi to express the strain range as function of the loading cycles is presented. Afterwards, a probabilistic model to express the quantile of this function is defined. In Section 4 a probabilistic function of the Ramberg–Osgood relationship is defined. In Section 5 the analytical results from the previous sections are applied on experimental data obtained from specimens made of ASTM A969 steel. Finally, in Section 6 the conclusions of this work are presented.

2. The Ramberg–Osgood relationship and its inversion

Before establishing the probabilistic relationship between strain range and the number of load cycles up to failure, it is necessary to find the inverse of the Ramberg–Osgood relationship.

According to Mostaghel [31], the Ramberg–Osgood relationship for

the elastic and plastic regions can be denoted by

$$\sigma_a = E \epsilon_a - \alpha \Delta \epsilon^\beta \quad \text{elastic,} \quad (1)$$

$$\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'} \right)^{\frac{1}{n'}} \quad \text{plastic,} \quad (2)$$

where

- σ_a : stress cyclic amplitude
- ϵ_a : strain cyclic amplitude.

In the case of considering the stress range and strain range, the Ramberg–Osgood relationship can be described by

$$\Delta \sigma = E \Delta \epsilon - 2\alpha \left(\frac{\Delta \epsilon}{2} \right)^\beta = RO_{el}(\Delta \epsilon) \quad \text{if } 0 \leq \Delta \epsilon \leq \Delta \epsilon_o, \quad (3)$$

$$\Delta \epsilon = \frac{\Delta \sigma}{E} + \left(\frac{\Delta \sigma}{2^{(1-n')} K'} \right)^{\frac{1}{n'}} = RO_{pl}^{-1}(\Delta \sigma) \quad \text{if } \Delta \epsilon \geq \Delta \epsilon_o, \quad (4)$$

where

- E : modulus of elasticity
 - K' : cyclic strength coefficient
 - n' : cyclic strain hardening exponent
 - $\Delta \epsilon_o$: interface strain range
 - α, β : geometrical parameters
 - RO: Ramberg–Osgood relationship
- Fig. 1 shows the Ramberg–Osgood relationship according to Eqs. (3) and (4).

Manipulating the Eq. (4) leads to

$$\Delta \sigma = 2^{(1-n')} K' \Delta \epsilon^{n'} \left(1 - \frac{\Delta \sigma}{E \Delta \epsilon} \right)^{n'}. \quad (5)$$

The power series of the last part in Eq. (5) is given by

$$\left(1 - \frac{\Delta \sigma}{E \Delta \epsilon} \right)^{n'} = 1 - \frac{n' \Delta \sigma}{E \Delta \epsilon} + \frac{(n' - 1)n' \Delta \sigma^2}{2E^2 \Delta \epsilon^2} - \frac{(n' - 2)(n' - 1)n' \Delta \sigma^3}{6E^3 \Delta \epsilon^3} + O(\Delta \sigma^4). \quad (6)$$

Taking only the terms up to 3rd order and replacing them in Eq. (5) leads to the 3rd degree polynomial

$$P^3(\Delta \sigma) = A - B \Delta \sigma + C \Delta \sigma^2 - D \Delta \sigma^3, \quad (7)$$

where

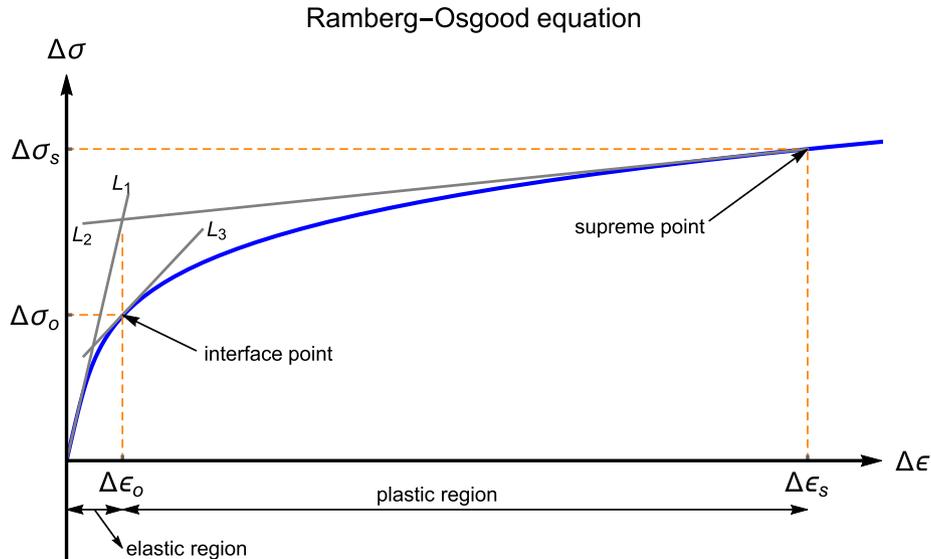


Fig. 1. Ramberg–Osgood relationship. The transition between the elastic and the plastic regions is given by the interface point $(\Delta \epsilon_o, \Delta \sigma_o)$. The upper limit of the plastic region is given by the supreme point $(\Delta \epsilon_s, \Delta \sigma_s)$.

$$\begin{aligned}
A &= 2^{(1-n')} K' \Delta \epsilon^{n'}, \\
B &= A \frac{n'}{E \Delta \epsilon}, \\
C &= A \frac{(n'-1)n'}{2E^2 \Delta \epsilon^2}, \\
D &= A \frac{(n'-2)(n'-1)n'}{6E^3 \Delta \epsilon^3}.
\end{aligned}$$

Solving analytically the polynomial given in Eq. (7) leads to the Ramberg–Osgood relationship $RO_{pl}(\Delta \epsilon)$ given by

$$\begin{aligned}
\Delta \sigma &= \frac{C}{3D} \\
&+ \frac{\frac{1}{23}(-C^2 + 3BD)}{3D(-2C^3 + 9BCD - 27AD^2 + \sqrt{4(-C^2 + 3BD)^3 + (-2C^3 + 9BCD - 27AD^2)^2})^{\frac{1}{3}}} \\
&- \frac{(-2C^3 + 9BCD - 27AD^2 + \sqrt{4(-C^2 + 3BD)^3 + (-2C^3 + 9BCD - 27AD^2)^2})^{\frac{1}{3}}}{3 \cdot 23D}.
\end{aligned}$$

For simplicity, this solution will be denoted by

$$\Delta \sigma = RO_{pl}(\Delta \epsilon) \text{ if } \Delta \epsilon \geq \Delta \epsilon_o. \quad (8)$$

Thus, the Ramberg–Osgood relationship is defined explicitly by Eqs. (3) and (8).

From now on, if no difference between the elastic and plastic region should be remarked, the general and explicit Ramberg–Osgood relationship will be denoted as

$$\Delta \sigma = RO(\Delta \epsilon). \quad (9)$$

Within this paper, the authors consider the 3rd order approximation is enough for the purpose of this work. A 4th order approximation is presented in [31], and an alternative method to invert the Ramberg–Osgood relationship can be found in [32].

2.1. Finding the interface point ($\Delta \epsilon_o, \Delta \sigma_o$)

In order to establish the strain range domains of Eqs. (3) and (8), it is necessary to find the interface point ($\Delta \epsilon_o, \Delta \sigma_o$), which separates the elastic and the plastic regions, see Fig. 1.

Since the interface point lies in the intersection of lines L_1 and L_2 , the task is done if the equations of the lines are established, see Fig. 1.

It is well known that the equation of line L_1 is given by

$$\Delta \sigma = E \Delta \epsilon. \quad (10)$$

From now on, it is assumed that a large enough strain range $\Delta \epsilon_s$ known as supreme strain range is given. Then, by applying Eq. (8) its corresponding supreme stress range $\Delta \sigma_s$ can be calculated by

$$\Delta \sigma_s = RO_{pl}(\Delta \epsilon_s). \quad (11)$$

Afterwards, it is necessary to calculate the slope E_s of line L_2 . For this purpose, the derivative of Eq. (4)

$$\frac{d\Delta \epsilon}{d\Delta \sigma} = \frac{1}{E} + \frac{1}{n'} \frac{\Delta \sigma (\frac{1}{n'} - 1)}{2(\frac{1}{n'} - 1) K' \frac{1}{n'}}, \quad (12)$$

has to be considered.

From elementary calculus, the condition $\frac{d\Delta \epsilon}{d\Delta \sigma}(\Delta \sigma_s) = \frac{1}{E_s}$ has to be satisfied. Then, the slope E_s of line L_2 is given by

$$E_s = \frac{E}{1 + \frac{E \Delta \sigma_s (\frac{1}{n'} - 1)}{n' 2 (\frac{1}{n'} - 1) K' \frac{1}{n'}}}. \quad (13)$$

Thus, equation of line L_2 is defined by

$$\Delta \sigma = E_s \Delta \epsilon - E_s \Delta \epsilon_s + \Delta \sigma_s. \quad (14)$$

As mentioned before, considering the intersection of the two lines given by Eqs. (10) and (14) leads to the interface strain range

$$\Delta \epsilon_o = \frac{\Delta \sigma_s - E_s \Delta \epsilon_s}{E - E_s}, \quad (15)$$

and then by applying the equation RO_{pl} , its corresponding interface stress range is obtained by

$$\Delta \sigma_o = RO_{pl}(\Delta \epsilon_o). \quad (16)$$

2.2. Calculating the geometrical parameters α and β

The parameters α and β define the Ramberg–Osgood relationship in the elastic region, see Eq. (3). These parameters can be calculated by the following considerations.

First, it is necessary to calculate the slope E_o of line L_3 . In other words, the condition $\frac{d\Delta \epsilon}{d\Delta \sigma}(\Delta \sigma_o) = \frac{1}{E_o}$, has to be satisfied. Then the slope of line L_3 is given by

$$E_o = \frac{E}{1 + \frac{E \Delta \sigma_o (\frac{1}{n'} - 1)}{n' 2 (\frac{1}{n'} - 1) K' \frac{1}{n'}}}. \quad (17)$$

Since, the interface point ($\Delta \epsilon_o, \Delta \sigma_o$) belongs also to the curve RO_{el} given by Eq. (3), it leads to

$$\alpha = \frac{2^{(\beta-1)} E \Delta \epsilon_o - \Delta \sigma_o}{\Delta \epsilon_o^\beta}. \quad (18)$$

Now consider the derivative of Eq. (3) which is given by

$$\frac{d\Delta \sigma}{d\Delta \epsilon} = E - \alpha \beta \left(\frac{\Delta \epsilon}{2} \right)^{\beta-1}. \quad (19)$$

Evaluating this derivative in the interface strain range $\Delta \epsilon_o$ leads to the slope E_o of line L_3 . It means, that the condition $\frac{d\Delta \sigma}{d\Delta \epsilon}(\Delta \epsilon_o) = E_o$, has to be satisfied.

Then

$$\alpha = \frac{2^{\beta-1} (E - E_o)}{\beta \Delta \epsilon_o^{\beta-1}}. \quad (20)$$

Afterwards, equalizing Eqs. (18) and (20) leads to the parameter β by

$$\beta = \frac{\Delta \epsilon_o (E - E_o)}{E \Delta \epsilon_o - \Delta \sigma_o}. \quad (21)$$

2.3. Application on simulated data

In order to prove the suitability of the inversion method of the Ramberg–Osgood relationship, simulated data were generated.

The simulation was performed according to Eq. (2) and by considering a stress ratio $R = -1$ and the properties of ASTM A969 hot dipped galvanized sheet steel given in Table 3.

ASTM A969 is a cold-rolled, low carbon, extra deep drawing steel (EDDS), which usually is used in the automotive industry, see [16,33].

The calculations corresponding to the simulation were performed with Mathematica 12.0.

The results corresponding to the coordinates of the interface point

Table 3
Material properties.

Material properties - ASTM A969			
Modulus of elasticity	E	206824	[MPa]
Cyclic strength coefficient	K'	800,57	
Cyclic strain hardening exponent	n'	0,2261	
Yield strength	R_{el}	164	[MPa]
Tensile strength	R_m	297	[MPa]
Elastic coefficient	S_e	669,32	
Elastic exponent	b_e	-0,1174	
Plastic coefficient	S_p	0,299	
Plastic exponent	b_p	-0,4844	
Fatigue limit	$\Delta \sigma_{\infty}$	122,84	[MPa]

Table 4
Parameters of the Ramberg–Osgood relationship.

Interface Point		
$\Delta \epsilon_0$	0,00267525	[-]
$\Delta \sigma_0$	302,033	[MPa]
Geometrical parameters		
α	1, 27516·10 ⁷	
β	1,74218	

and to the values of the geometrical parameters of Eq. (3) are shown in Table 4.

A plot of a simulated Ramberg–Osgood relationship and its corresponding data are shown in Fig. 2.

3. Consideration of the strain range from the Ramberg–Osgood relationship in the Stüssi model

The interest in fatigue design is obtaining a reliable probabilistic model to estimate the fatigue behaviour of a structure. For this purpose, a suitable model has to be developed. This model has to consider: a) a geometrical function which describes properly the relation between the stress range and the applicable load cycles or between the strain range and the load cycles and b) a statistical distribution for the lifetime. In this paper, the model proposed by the authors in [29], which is based on the Stüssi function [27,28] and a three-parameter Weibull distribution $W(a, b, c)$ is considered.

3.1. The Stüssi function for the strain range

Consider the Stüssi function for the stress range given by

$$\Delta \sigma = \frac{R_m(1 - R) + \alpha N^\beta \Delta \sigma_\infty}{1 + \alpha N^\beta} = S(N), \quad (22)$$

where

- $\Delta \sigma$: stress range during the fatigue test
- N : number of load cycles up to failure or up to end of the test
- R_m : ultimate tensile strength
- $\Delta \sigma_\infty$: fatigue limit¹
- α, β : geometrical parameters
- R : stress ratio
- S : Stüssi function.

The method to estimate the geometrical parameters is described in [29]. Be aware that the geometrical parameters α and β from Eq. (22) are different than the parameters from Eq. (3).

Combining the Stüssi function given by Eq. (22) with the inverse of the Ramberg–Osgood relationship given by Eqs. (3) and (8), a new Stüssi function for the strain range as function of the loading cycles given by

$$\Delta \epsilon = \text{RO}^{-1} \left(\frac{R_m(1 - R) + \alpha N^\beta \Delta \sigma_\infty}{1 + \alpha N^\beta} \right) = \text{RO}^{-1}(S(N)) \quad (23)$$

is obtained.

Moreover, this function describes clearly the asymptotic behaviour regarding the ultimate tensile strength R_m in the plastic region and the fatigue limit $\Delta \sigma_\infty$ in the elastic region by.

- $\lim_{N \rightarrow 0} \Delta \epsilon = \text{RO}^{-1}(R_m(1 - R))$
- $\lim_{N \rightarrow \infty} \Delta \epsilon = \text{RO}^{-1}(\Delta \sigma_\infty)$.

¹ The existence of the fatigue limit is still an open debate, see for example [34,35].

Figs. 3 shows the $\Delta \epsilon - N$ curve given by the Stüssi function in the log–log scale.

3.2. Probabilistic model

In [29] a probabilistic model based on the Stüssi function and the Weibull distribution was proposed. For this model it was considered that $\Delta \sigma - S(N)$ is a random variable which follows a Weibull distribution $W(a, b, c)$. Under these considerations, and using the notation of Eq. (23) the probabilistic model is defined by

$$p = 1 - \exp \left\{ - \left[\frac{\Delta \sigma - S(N) - a}{b} \right]^c \right\}, \quad (24)$$

where

- p : probability of failure
- $a \in \mathbb{R}$: Weibull location parameter
- $b > 0$: Weibull scale parameter
- $c > 0$: Weibull shape parameter

Replacing the general Ramberg–Osgood relationship given by Eq. (9) in Eq. (24), leads to a Stüssi probabilistic function to model the $\Delta \epsilon - N$ curves in the elastic and plastic regions given by

$$p = 1 - \exp \left\{ - \left[\frac{\text{RO}(\Delta \epsilon) - S(N) - a}{b} \right]^c \right\}. \quad (25)$$

The model given by Eq. (25) depends on two geometrical parameters α, β of Eq. (22) and on three Weibull parameters a, b and c . The geometrical parameters can be estimated by a linear regression, see [29] and the Weibull parameters can be estimated by applying the PWM method, see [36].

Once all parameters have been estimated, the $\Delta \epsilon - N$ curves can be depicted.

3.3. Application on simulated data

As in Section 2.3, simulated data corresponding to ASTM A969 hot dipped galvanized sheet steel are considered. Moreover, the Stüssi quantile will be compared with the well known Coffin–Manson equation (CM), which relates the strain range and the loading cycles and is given by

$$\Delta \epsilon = 2 \left[\left(\frac{S_e}{E} \right) (2N)^{b_e} + S_p (2N)^{b_p} \right], \quad (26)$$

where

- S_e : elastic coefficient
- E : modulus of elasticity
- b_e : elastic exponent
- S_p : plastic coefficient
- b_p : plastic exponent.

The geometrical parameters of the Stüssi function given by Eq. (22) are estimated by applying the method proposed in [29], see Table 5.

The Weibull parameters estimated by applying the method proposed in [36] are shown in Table 6.

A graphical representation of the $\Delta \epsilon - N$ curves given by Eq. (25) is shown in Fig. 4. In this Figure, it can be seen that the 50% quantile of the $\Delta \epsilon - N$ curve and the curve given by the Coffin–Manson equation have a very similar geometry between 10⁴ and 10⁷ load cycles. The quantiles given by Eq. (25) seem to be a reliable probabilistic alternative to the Coffin–Manson equation.

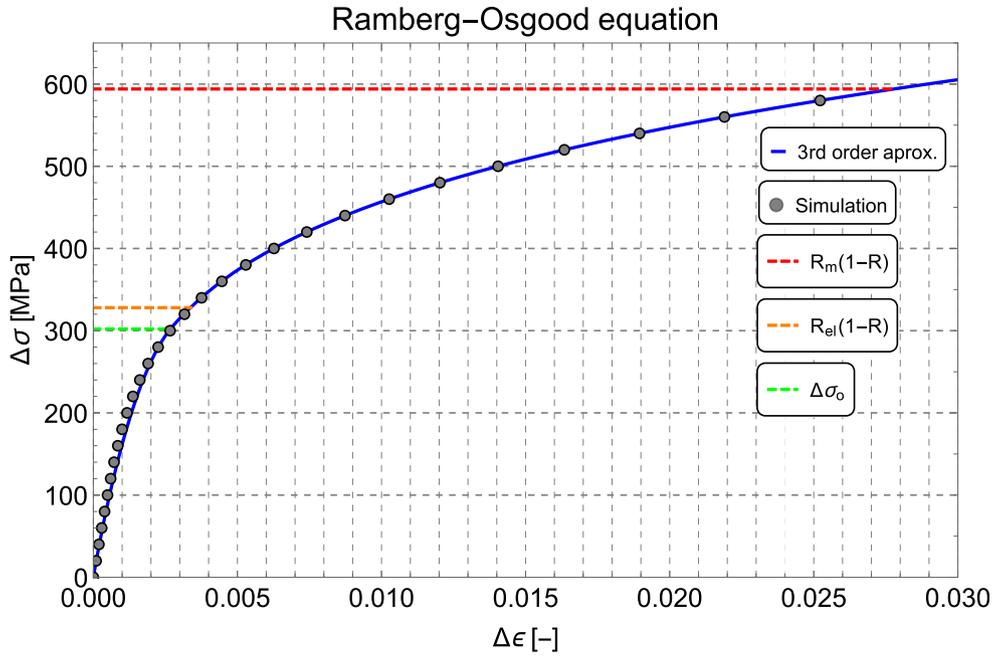


Fig. 2. Ramberg–Osgood relationship and its corresponding simulated data. Supreme strain $\Delta \epsilon_s = 0,06$. The stress ratio of the fatigue tests is $R = -1$.

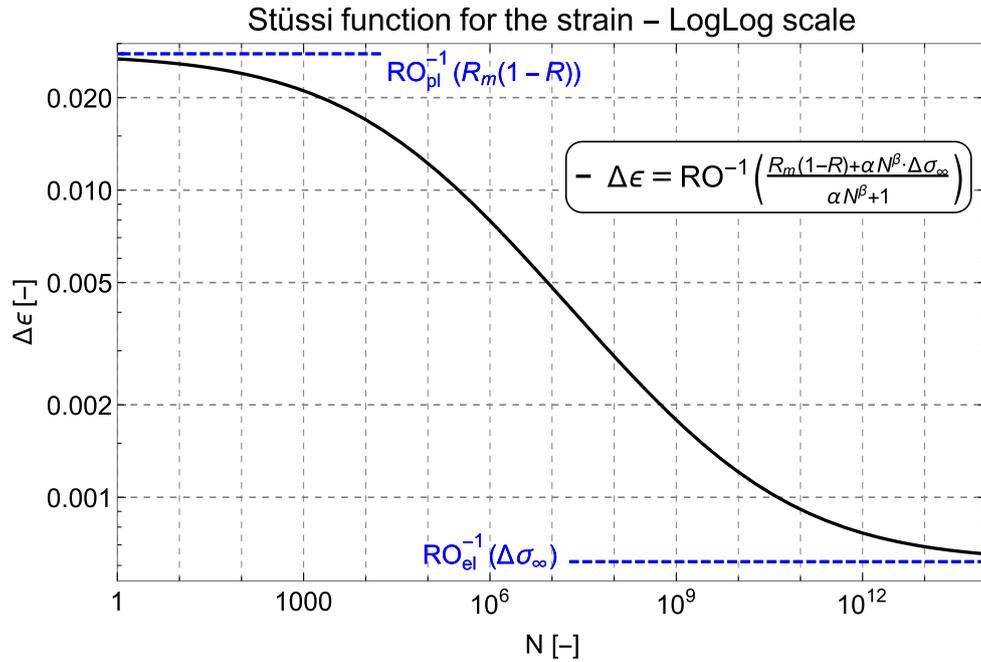


Fig. 3. Stüssi function for the strain range.

4. Probabilistic Ramberg–Osgood relationship

Based on the Stüssi probabilistic functions proposed in the previous section, a probabilistic formulation for the Ramberg–Osgood relationship can be defined.

On the one hand, in the probabilistic Stüssi model given by the Eq. (24), the considered random variable was $\Delta \sigma = S(N)$.

On the other hand, from the Ramberg–Osgood relationship and Stüssi function, given by Eq. (9) and Eq. (22) respectively, it can be established that $S(N) = RO(\Delta \epsilon)$.

Table 5
Parameters estimation from the Stüssi function.
Simulated data.

Geometrical parameters	
α	0,03274
β	0,32457

Table 6
Weibull parameters estimation of the probabilistic Stüssi function. Simulated data.

Weibull parameters	
a	-25,7662
b	28,8251
c	3,2157

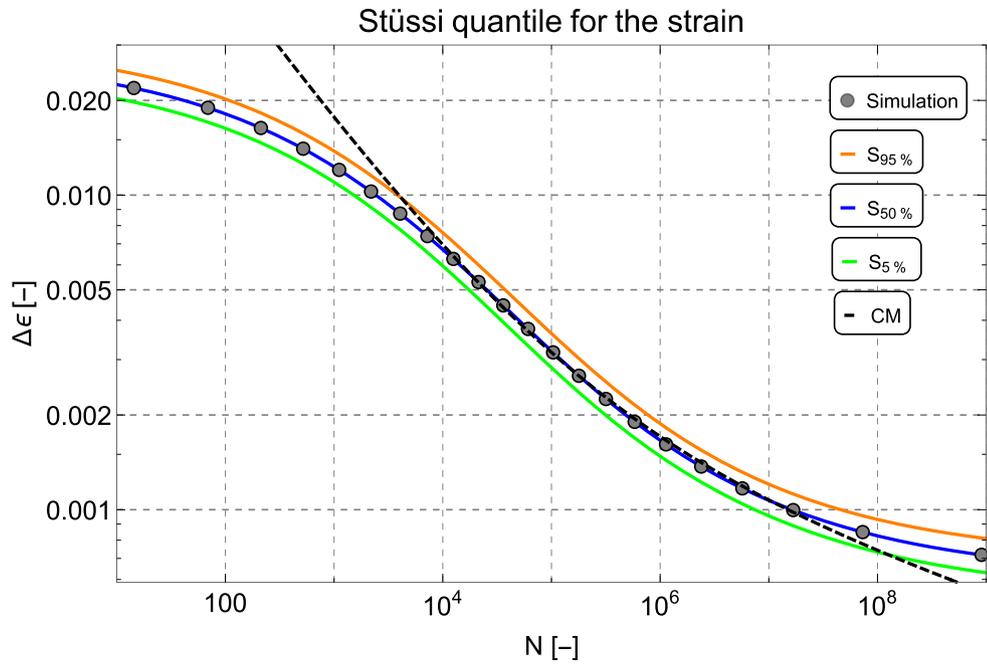


Fig. 4. $\Delta \epsilon$ - N curves based on the Stüssi model. The plotted curves correspond to a probability of 5%, 50% and 95%.

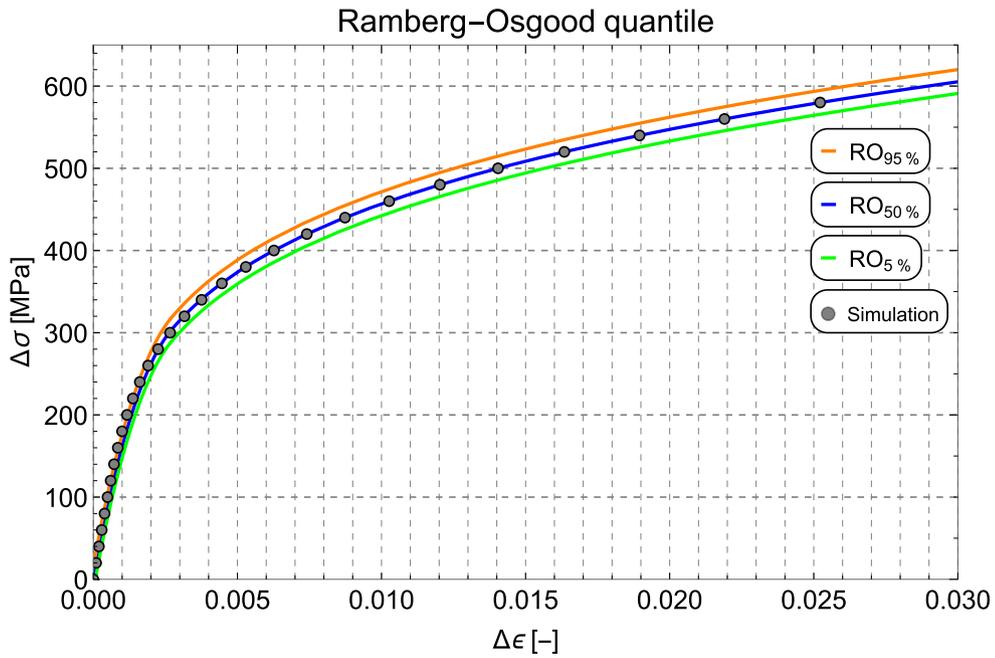


Fig. 5. Quantile of the Ramberg-Osgood relationship based on the Stüssi model. The plotted curves correspond to a probability of 5%, 50% and 95%.

Table 7

Parameters estimation from the Stüssi function. Experimental data.

Geometrical parameters	
α	0,02262
β	0,35543

Table 8

Weibull parameters estimation of the probabilistic Stüssi function. Experimental data.

Weibull parameters	
a	-40,9495
b	45,3065
c	2,52388

For these reasons, it seems plausible to consider an equivalent random variable given by $\Delta \sigma - RO(\Delta \epsilon)$. Since this variable is only obtained by replacing equations, it should also follow the same three-parameter Weibull distribution $W(a, b, c)$.

Under these considerations a probabilistic function for the Ramberg-Osgood relationship can be defined as follows.

$$p = 1 - \exp \left\{ - \left[\frac{\Delta \sigma - RO(\Delta \epsilon) - a}{b} \right]^c \right\}. \quad (27)$$

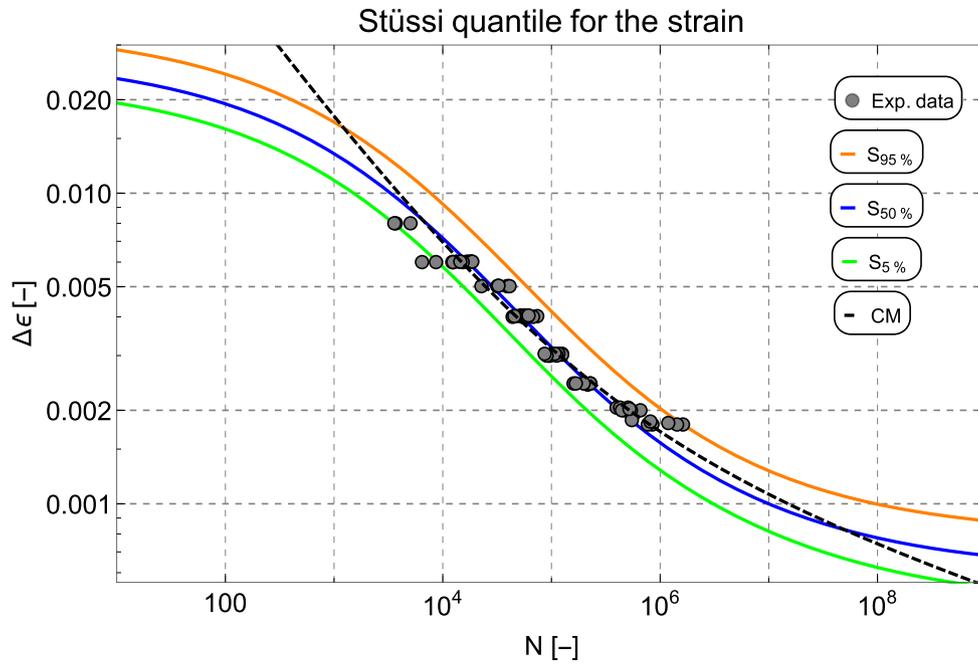


Fig. 6. Stüssi quantile for the strain range. The plotted curves correspond to a probability of 5%, 50% and 95%.

4.1. Application on simulated data

As in the Sections 2.3 and 3.3, the same simulated data are considered to evaluate the model.

A graphical representation of the quantile of the Ramberg–Osgood relationship given by Eq. (27) is shown in Fig. 5.

5. Application on specimens made of ASTM A969 steel

The application presented in this section is based on the experimental data considered in [16]. These data were provided by the Steel Market Development Institute (SMDI)².

5.1. Material and specimens

Sixty-nine specimens made of hot dipped galvanized sheet steel ASTM A969 were tested. The main properties of the material are described in Table 3.

5.2. Fatigue tests

The specimens were tested under a constant stress range with a triangular wave form and a stress ratio $R = -1$. The tests were terminated when the tensile load was dropped by 50% from the maximum load or the test was a runout up to 10^7 loading cycles. The corresponding experimental data provide the stress range, strain range and loading cycles.

5.3. The Stüssi model for the strain range

In order to plot the corresponding $\Delta \epsilon$ - N quantile, some steps should be done. First, it is necessary to find the inverse of the Ramberg–Osgood relationship. This step is accomplished by estimating of the geometrical parameters α and β from Eq. (3). Since the inversion of the Ramberg–Osgood relationship does not depend on the experimental data but

on the material properties shown in Table 3, this step is already done, see Table 4.

Afterwards, the geometrical parameters of Eq. (22) are estimated by applying the method proposed in [29]. The corresponding estimations are shown in Table 7.

Once the geometrical parameters have been estimated, it is possible to estimate the Weibull parameters of Eq. (25). The method of the PWMs has been applied for this task, see [36]. For this case the estimations of the Weibull parameters are shown in Table 8.

The $\Delta \epsilon$ - N curves corresponding to a probability of 5%, 50% and 95% are shown in Fig. 6.

5.4. The Stüssi model for the Ramberg–Osgood relationship

As it was explained in Section 4, based on the experimental data and applying Eq. (27), it is possible also to depict quantile of the Ramberg–Osgood relationship.

The quantiles of the Ramberg–Osgood relationship corresponding to a probability of 5%, 50% and 95% are shown in Fig. 7.

5.5. The Stüssi model for the stress range

Finally, by applying the Stüssi model for the stress range given by Eq. (24) the corresponding $\Delta \sigma$ - N quantile can be depicted.

The quantiles of the $\Delta \sigma$ - N curve, corresponding to a probability of 5%, 50% and 95% are shown in Fig. 8.

6. Conclusions and recommendations

As it has been shown in the previous sections, a new statistical proposal for modelling the $\Delta \sigma$ - N , $\Delta \epsilon$ - N curves and the Ramberg–Osgood relationship has been made by combining the Stüssi function and the three-parameter Weibull distribution $W(a, b, c)$. Moreover, this proposal allows to obtain estimations of the fatigue lifetime of a structure either in LCF and HCF. This fact represents an advantage compared to the Basquin model which is considered in the official standards.

On the one hand, the Stüssi function offers a good geometric

² IF_DDQ_HDG70G_Strain_Life_Fatigue.xls

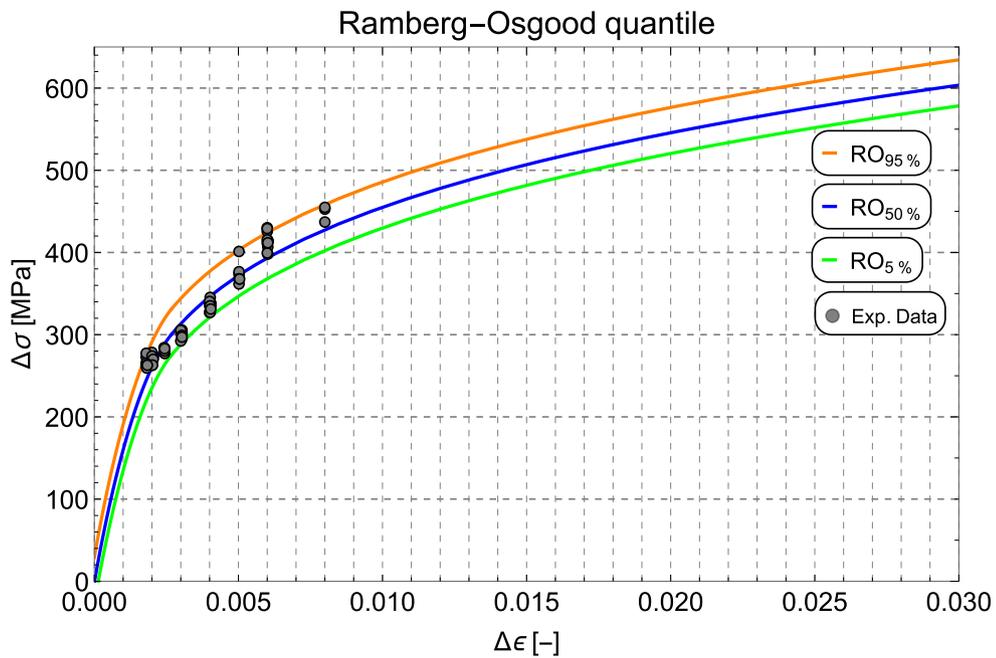


Fig. 7. Quantile of the Ramberg–Osgood relationship. The plotted curves correspond to a probability of 5%, 50% and 95%.

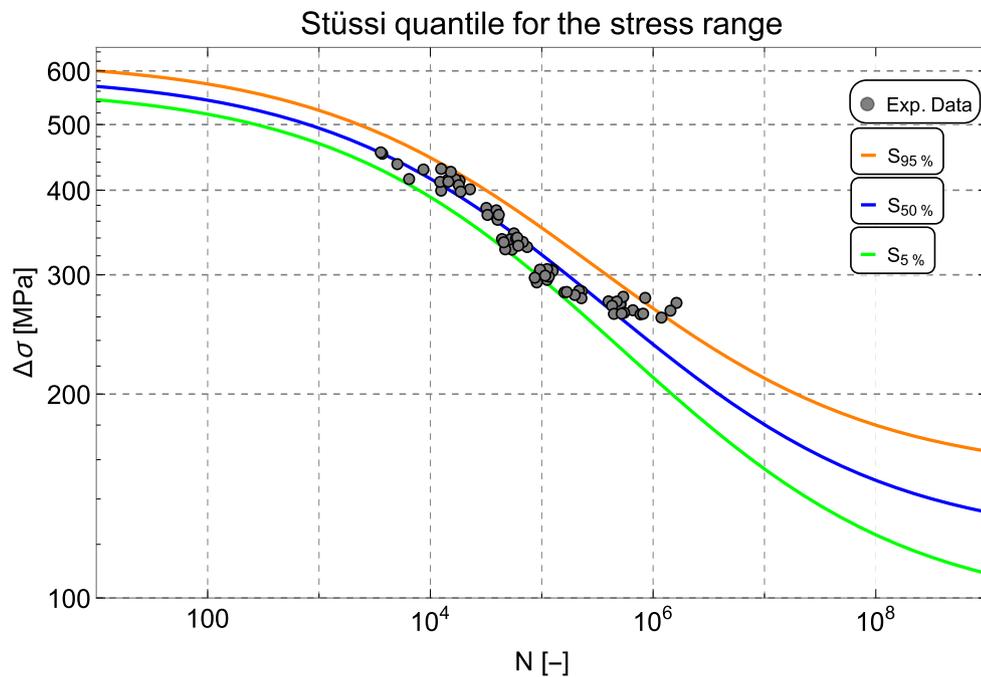


Fig. 8. Stüssi quantile for the stress range. The plotted curves correspond to a probability of 5%, 50% and 95%.

description of the $\Delta\sigma$ - N and $\Delta\epsilon$ - N curves. In fact, it describes properly their asymptotic characteristics given by the ultimate tensile strength and by the fatigue limit.

In LCF this function is suitable to estimate the fatigue lifetime, since it converges smoothly to the ultimate tensile strength, while in HCF it converges to the fatigue limit.

On the other hand, since the number of load cycles can be considered as random lifetime variable, the three-parameter Weibull distribution $W(a, b, c)$ is the adequate one to describe this magnitude.

In order to obtain the best results applying the method proposed in this work, the suggestions made in [29] can be taken into account for

the planning of fatigue tests.

Regarding the lifetime estimation there are still a lot of questions and concerns, which have to be considered in future investigations. One of them is making estimations in the ultra low cycle fatigue (ULCF) regime where the plastic behaviour prevails.

The authors want to express their gratitude to the German Research Foundation DFG³ for the funds granted to perform the research in fatigue of steel structures.

³ Deutsche Forschungsgemeinschaft.

Declaration of Competing Interest

None.

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Dies ist ein Postprint/begutachtetes Manuskript.

Empfohlene Zitierung:

Toasa Caiza, P. D.; Ummenhofer, T.
[Probabilistic relationships between strain range, stress range and loading cycles. Application on ASTM A969 steel.](#)
2020. International journal of fatigue, 137.
doi: [10.5445/IR/1000123484](#)

Zitierung der Originalveröffentlichung:

Toasa Caiza, P. D.; Ummenhofer, T.
[Probabilistic relationships between strain range, stress range and loading cycles. Application on ASTM A969 steel.](#)
2020. International journal of fatigue, 137, 105626.
doi: [10.1016/j.ijfatigue.2020.105626](#)