Applying the Weibull and Stüssi Methods that Derive Reliable Wöhler Curves to Historical German Bridges

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Abstract: Preserving the structural integrity of historical bridges made of wrought iron is a mandatory technical requirement in several countries in Europe. A reliable fatigue lifetime estimation of theses bridges allows for proposal of adequate maintenance policies to keep these historical structures as part of the historical legacy for the next generations. In this paper, fatigue data of German bridges built between 1850 and 1900 are considered. The Weibull model of Castilllo and the model of Stüssi to depict the Wöhler curves and estimate the fatigue limit are applied. Their results are compared with those from the linear model of Basquin, which is used in international standards.

Author keywords: Fatigue; Lifetime; Estimation; Weibull; Stüssi.

Introduction

Modeling the fatigue of materials has been a technical challenge for engineers, phyisicians, and mathematicians since the late nine teenth century. The interest on this research topic increased during the Industrial Revolution. [The period of time covered by the In dustrial Revolution varies with different historians. Hobsbawm (1999) held that it "broke out" in Britain in 1789 and was not fully felt until 1848, whereas Ashton (1998) held that it occurred roughly between 1760 and 1830.]

The research of August Wöhler (Soltau, June 22, 1819 to Hannover, March 21, 1914, German engineer, Royal *Obermaschi nenmeister* of the *Niederschlesisch Mährische* Railways in Frank furt an der Oder) on railway axles established the fundaments of fatigue tests whose results currently are represented as *S N* curves. To carry out these experiments, Wöhler designed a rotating bending machine, which ran at a very low frequency. Wöhler also designed machines that performed axial bending and torsion tests on different notched and unnotched specimens. In 1870, Wöhler published the results, which contained his conclusions (Wöhler 1870) and often are called Wöhler's laws.

One alternative for predicting the fatigue life of steel struc tures is based on the Wöhler curves, which are obtained from experimental fatigue data. Considering the stress based approach, several models have been proposed to represent the Wöhler curves (Toasa Caiza 2018).

The estimation of the fatigue life of historical steel bridges has been studied by several researchers (e.g., Helmerich 2005; Mang and Bucak 1991, 1993; Pedrosa et al. 2019; Mayorga et al. 2017).

In this paper, two applications for modeling the Wöhler curves are presented. The experimental fatigue data were obtained by test ing the main girders of antique bridges built between 1850 and 1900. Two methods to depict the Wöhler curves are applied, and their results are compared with those from the linear model of Basquin (1910), which is based on the Wöhler results and used in international standards.

Basquin Model of 1910

The linear model of Basquin (1910) given by the equation

$$\log N = A - B \log \Delta\sigma, \qquad \Delta\sigma \geq \Delta\sigma_\infty \qquad (1)$$

has been applied to depict the Wöhler curves (Schijve 2009) on the finite life fatigue zone by international standards such as Eurocode 3 (Sedlacek et al. 2005; CMC 2010), ISO 12107 (ISO 2008), and ASTM E739 10 (ASTM 2015).

The estimation of the parameters A and B can be done by ap plying standard least squares methods (e.g., Toasa Caiza 2008b). It is assumed that the lifetime N_i and the stress range $\Delta\sigma_i$ follow a normal or t student distribution, so that the stress range and its con fidence intervals can be estimated on the high cycle fatigue (HCF) zone (Spindel and Haibach 1979). In this way the percentile curves of the Basquin model can be determined in order to facilitate its comparison with respect to the Weibull and Stüssi models.

Unfortunately, this model presents some limitations. One the one hand, because of the assumed linear relationship between the stress range and the number of load cycles, this model is neither suitable to extrapolate the Wöhler curves from the finite life region to the HCF region nor able to provide an estimation of the fatigue limit. A linear model allows one to make estimations or extrapo lations only within the experimental frame and range because the geometry of the mentioned relationship can change beyond this re gion. And in fact, this is the case of the fatigue life.

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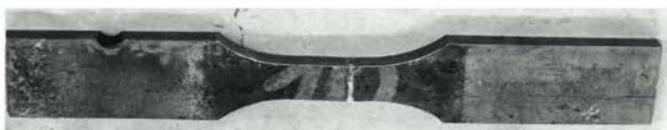
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Fig. 1. Main girders of the Dingolfing Bridge. (Reproduced with permission from Bähre and Kosteas 1979.)

Fig. 2. Main girders of the Loiching Bridge. (Reproduced with permission from Bähre and Kosteas 1979.)

Fig. 3. Geometry of the samples for the fatigue tests. (Reproduced with permission from Bähre and Kosteas 1979.)

Table 1. German historical bridge data on fatigue failures (R 0.1)

$\Delta\sigma$ (MPa)	N	N	N	N
288	124,900	156,700	197,000	
261	137,000	194,000	252,400	263,800
	297,300	437,400		
243	373,200			
225	599,200			
216	246,600	689,800	779,500	872,200
	1,031,000			
207	1,205,800	1,268,120	1,602,200	1,886,800
	2,049,500	2,139,900		
198	1,193,400	1,596,300	2,179,700	

Table 2. German historical bridge data of runouts (R 0.1)

$\Delta\sigma$ (MPa)	N	N	N
216	3,287,420		
207	2,278,100	2,451,600	2,956,700
198	3,348,400	3,640,420	12,335,620
189	2,000,000	8,119,200	8,292,100

Table 3. German historical bridge data of runouts and their retests (R 0.1)

$\Delta\sigma_1$ (MPa)	N_1	$\Delta\sigma_2$ (MPa)	N_2	N_2	N_2
261	100,000	189	715,800	2,052,400	1,660,400
			1,088,100	2,519,700	2,407,900
		225	249,000	938,700	573,000
			919,700	539,300	573,000
189	2,000,000	261	134,100	156,500	196,000
			217,400	254,500	395,600
		225	543	237,900	427,300
			1,080,600	1,396,900	1,650,500

Table 4. German historical bridge estimations of the fatigue life (N 5×10^6)

Model	Probability (%)	$\Delta\sigma$ (MPa)
EC3	5	66.3
Greiner	5	74.9
Basquin	5	148.9
Weibull	5	173.9
Stüssi	5	127.9

For this reason, in order to overcome this limitation, an arbitrary knee point on the linear regression is suggested where the slope changes (Hobbacher 2009, 2010; CMC 2010). This point represents the classical transition from the endurance line to the fatigue limit, and its existence is assumed and based on empirical knowledge (Hobbacher 2010). On the other hand, from the statistical point of view, it has been proved that the arbitrary assumption of a normal or log normal distribution in this model is not suitable to model fatigue data (Hobbacher 2010; Castillo and Fernández Canteli 2009).

Weibull Model of Castillo et al.

An alternative model based on a Weibull distribution $W(a, b, c)$ is given by

$$p = 1 - \exp\left\{-\left[\frac{(\log N - B)(\log \Delta\sigma - C) - a}{b}\right]^c\right\};$$
$$(\log N - B)(\log \Delta\sigma - C) \geq a \tag{2}$$

where p = probability of fatigue failure (Castillo and Fernández Canteli 2009). This model depends on two geometrical parameters, namely B and C, and three Weibull parameters, namely a, b, and c. The geometrical parameters are estimated by solving a nonlinear problem (Castillo et al. 2008), and the Weibull parameters can be estimated by the methods suggested by Toasa Caiza and Ummenhofer (2011) or Gupta and Panchang (1989).

The model allows one to consider the runouts and their subsequent retests in the statistical evaluation of the experimental data (Toasa Caiza and Ummenhofer 2018b). Moreover, it describes properly the asymptotic behavior of the fatigue life in VHCF. However, in LCF the application of this model is not suitable.
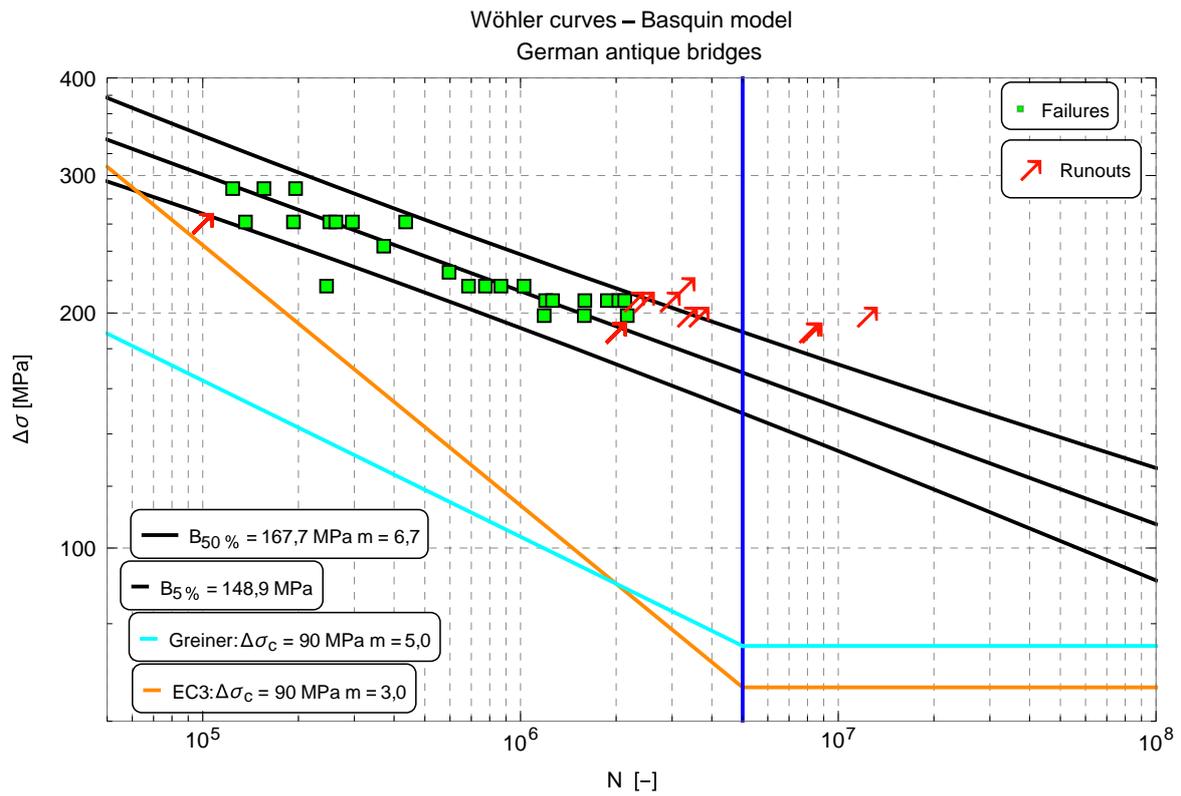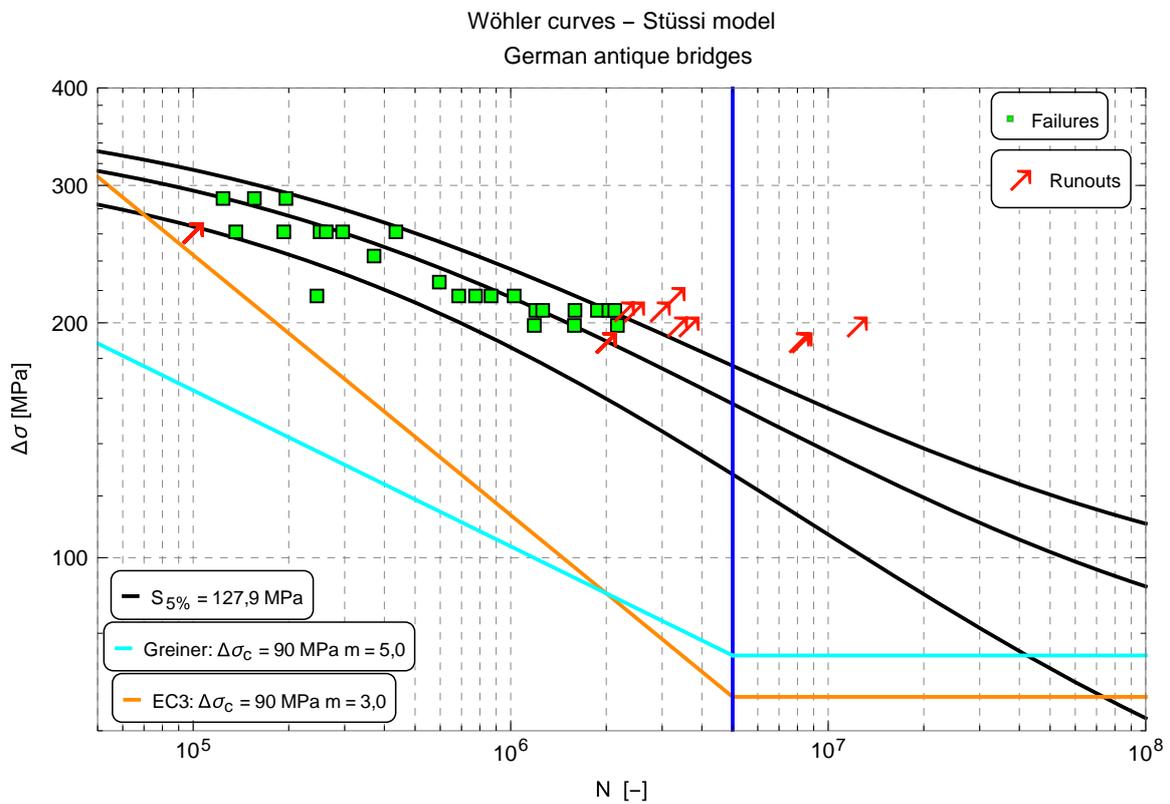
Fig. 4. German historical bridge Wöhler curves for a fatigue failure probability of 5%, 50%, and 95%: Basquin model.
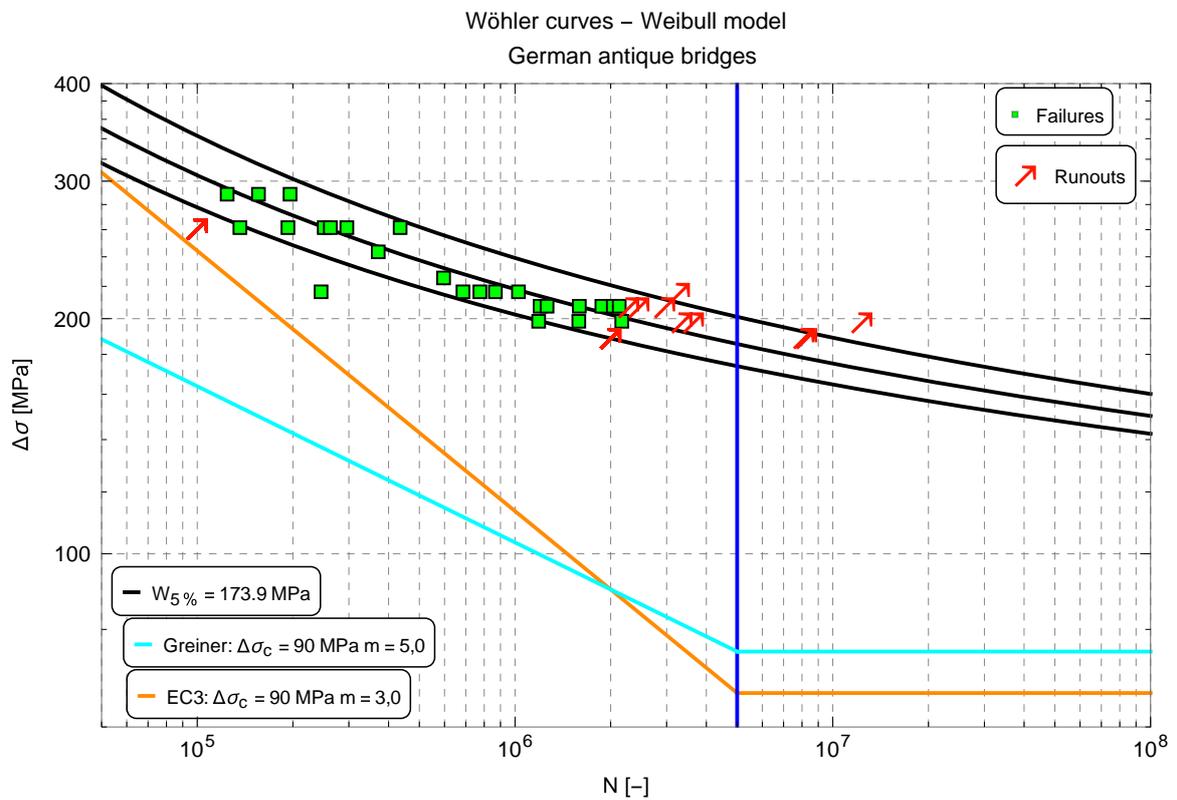
Fig. 5. German historical bridge Wöhler curves for a failure probability of 5%, 50%, and 95%: Weibull model.
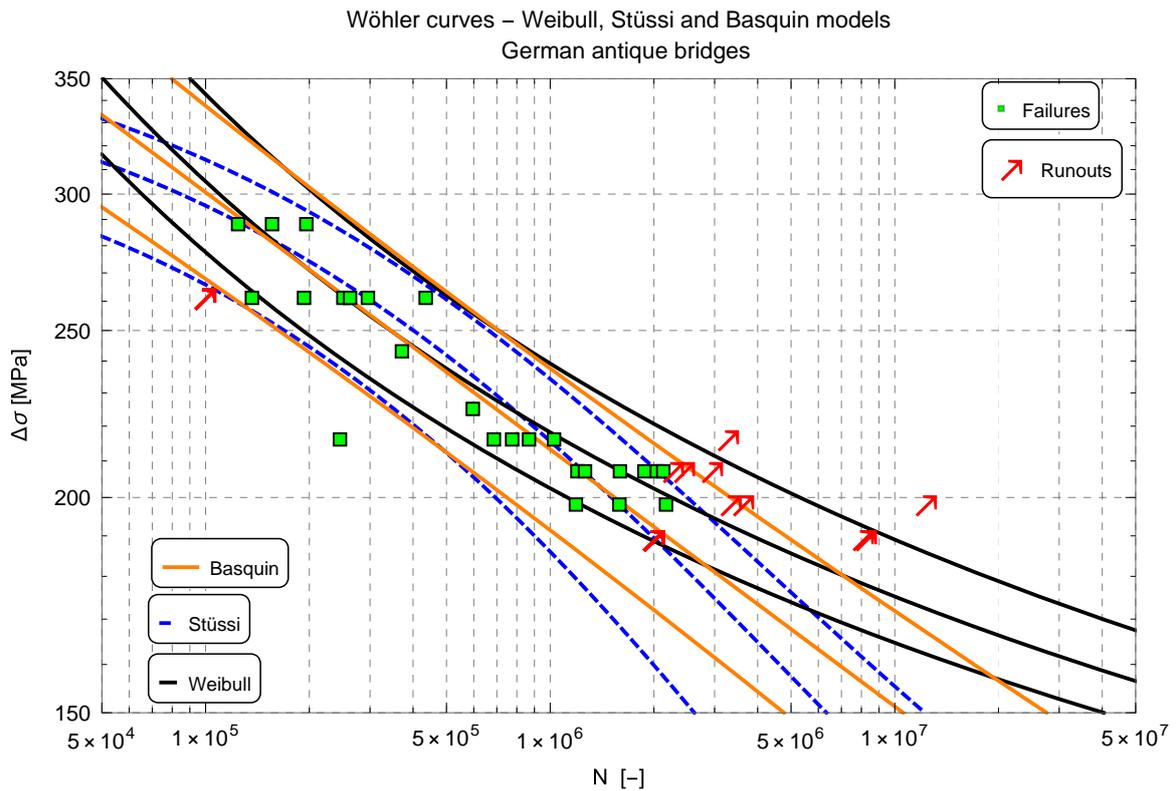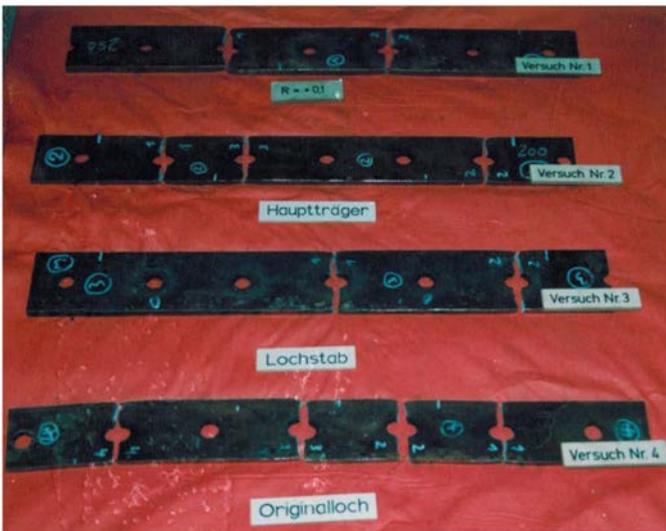
Fig. 6. German historical bridge Wöhler curves for a failure probability of 5%, 50%, and 95%: Basquin, Stüssi, and Weibull models.

Fig. 7. German historical bridge Wöhler curves for a failure probability of 5%, 50%, and 95%: Stüssi model.

Fig. 8. Main girder taken from the Stahringer Bridge. (Reproduced with permission from Bähre and Kosteas 1979.)

Fig. 9. Specimens taken from the Stahringer Bridge. (Reproduced with permission from Bähre and Kosteas 1979.)

Table 5. Stahringer Bridge (1895) Fatigue data of failures and runouts ($R \approx 0,1$)

$\Delta\sigma$ (MPa)	N	N	N	N	N
225	38,927	46,474			
180	112,302	138,322	142,491	133,710	127,800
	235,782	302,732	133,500	133,840	289,800
	330,300				
144	523,200	398,319	554,726	496,652	647,726
	897,183	885,600	1,200,640	661,550	683,264
	739,415	827,473	989,000	456,020	456,480
	292,680	309,570	355,100		
126	1,333,748	1,904,087	1,964,597	766,126	1,236,622
	1,514,865	1,272,776	1,313,096	1,689,579	1,882,968
	775,688	1,084,235	1,334,498	1,401,002	
108	2,980,233	3,504,216	5,484,220	17,804820[a]	

[a]The experiment for this entry was a runout.

Table 6. Stahringer Bridge (1895) estimations of the fatigue life ($N \approx 5 \times 10^6$)

Model	Probability (%)	$\Delta\sigma$ (MPa)
EC3	5	66.3
Greiner	5	74.9
Basquin	5	90.6
Weibull	5	100.8
Stüssi	5	81.3

Stüssi Model

The following model is based on the Stüssi equation (Stüssi 1955), and it can be identified as a Weibull model in which a correction is introduced to account for the low cycle fatigue (LCF) effect:

$$p = 1 - \exp\left\{-\left[\frac{\Delta\sigma - \frac{R_m + \alpha N^\beta \Delta\sigma_\infty}{1 + \alpha N^\beta} - a}{b}\right]^c\right\} \qquad (3)$$

The ultimate tensile strength R_m and the fatigue limit $\Delta\sigma_\infty$ are assumed to be known or already estimated, the parameters α and β are estimated by applying a linear regression, and the Weibull parameters a, b, and c can be estimated as in the previous model (Toasa Caiza and Ummenhofer 2018a). This model describes properly the fatigue life in LCF, but in very high cycle fatigue (VHCF) it is much more conservative than the models of Basquin (1910) and Castillo and Fernandez Canteli (2009).

Application to German Historical Bridges

The aforementioned models are applied on the experimental data obtained from seven German antique bridges made of wrought iron and built between 1850 and 1900. The obtained results are compared with the fatigue strength curve of Eurocode 3 ($m = 3$), the curve proposed by Greiner et al. (2007) ($m = 5$), and the model of Basquin applied in ISO 12107 and ASTM E739 10.

German Bridges 1850–1900

In a research project financed by the German Federal Railway [*Deutsche Bundesbahn* (DB)], 59 identical samples taken from the main girders of six antique bridges: Loiching, Dingolfing, and Pilsting Plattning (42.93 km), Landshut Plattning (6.086 and 7.745 km), and Weiden Hof were tested (Bähre and Kosteas 1979). The main girders of the Dingolfing and Loiching Bridges are shown in Figs. 1 and 2. The geometry of the samples is shown in Fig. 3.

The experimental data are given in Tables 1 3.

At 5 million load cycles, the estimations of the 5% quantile of the fatigue lifetime obtained by applying the Basquin, Weibull, and Stüssi models are higher than the proposals from Eurocode 3 and Greiner [Table 4]. The slope given by the Basquin model ($m = 6.7$) differs from the slope given in Eurocode 3 and the suggested one by Greiner (Fig. 4).

Because the Weibull model considers the runouts and their retests, the asymptotic behavior of the Wöhler curves in HCF and VHCF regions is properly described (Fig. 5). Moreover, the estimation of the fatigue lifetime and fatigue limit ($\Delta\sigma_\infty = 61.4$ MPa) are the highest (Fig. 6). By applying the Stüssi model, the Wöhler curves in LCF region, where the plastic behavior prevails, are properly described (Fig. 7).

Stahringer Bridge Built in 1895

For the second application, 49 experiments performed on specimens cut out from the main girders with original perforations of the Stahringer bridge are considered (Toasa Caiza 2008a, 2018). One main girder and the corresponding taken specimens are shown in Figs. 8 and 9. The experimental data are given in Table 5.

In this case, the estimations of the fatigue lifetime at 5 million load cycles obtained by applying the Basquin, Weibull, and Stüssi models are higher than the proposals from Eurocode 3 and from Greiner as well [Table 6].
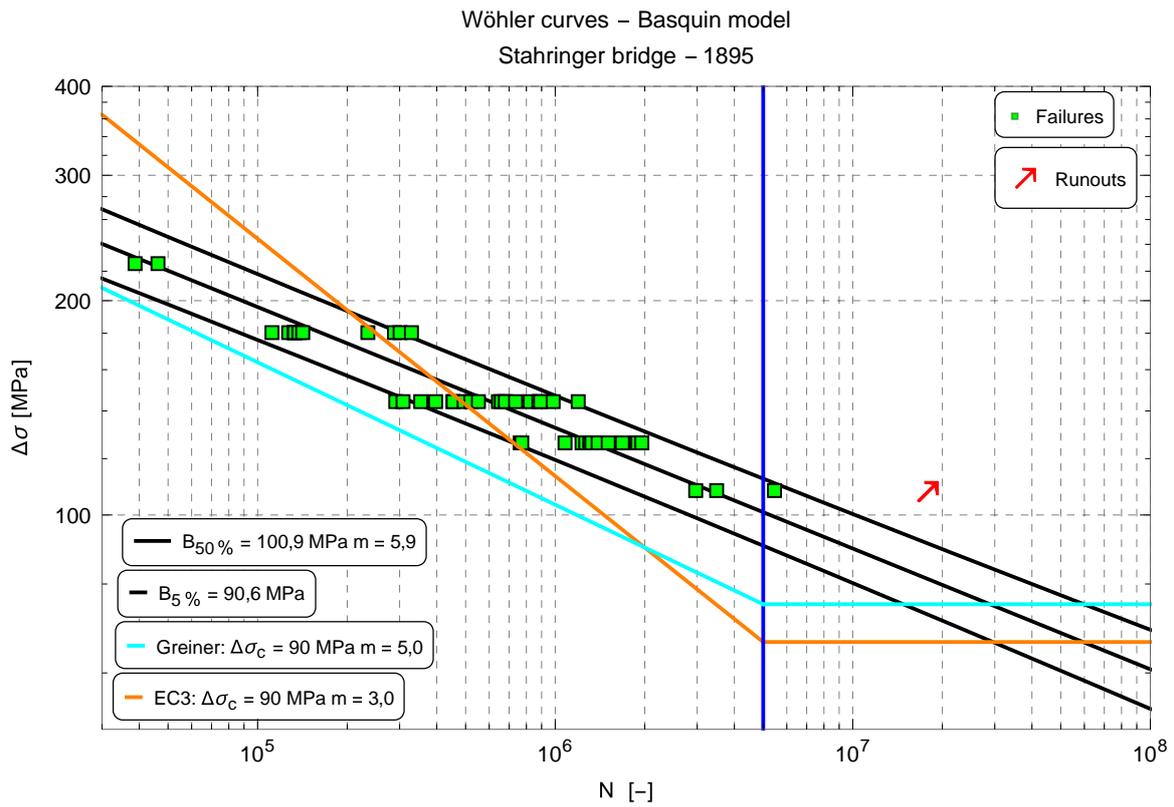
Fig. 10. Stahringer Bridge (1895) Wöhler curves for a failure probability of 5%, 50%, and 95%: Basquin model.
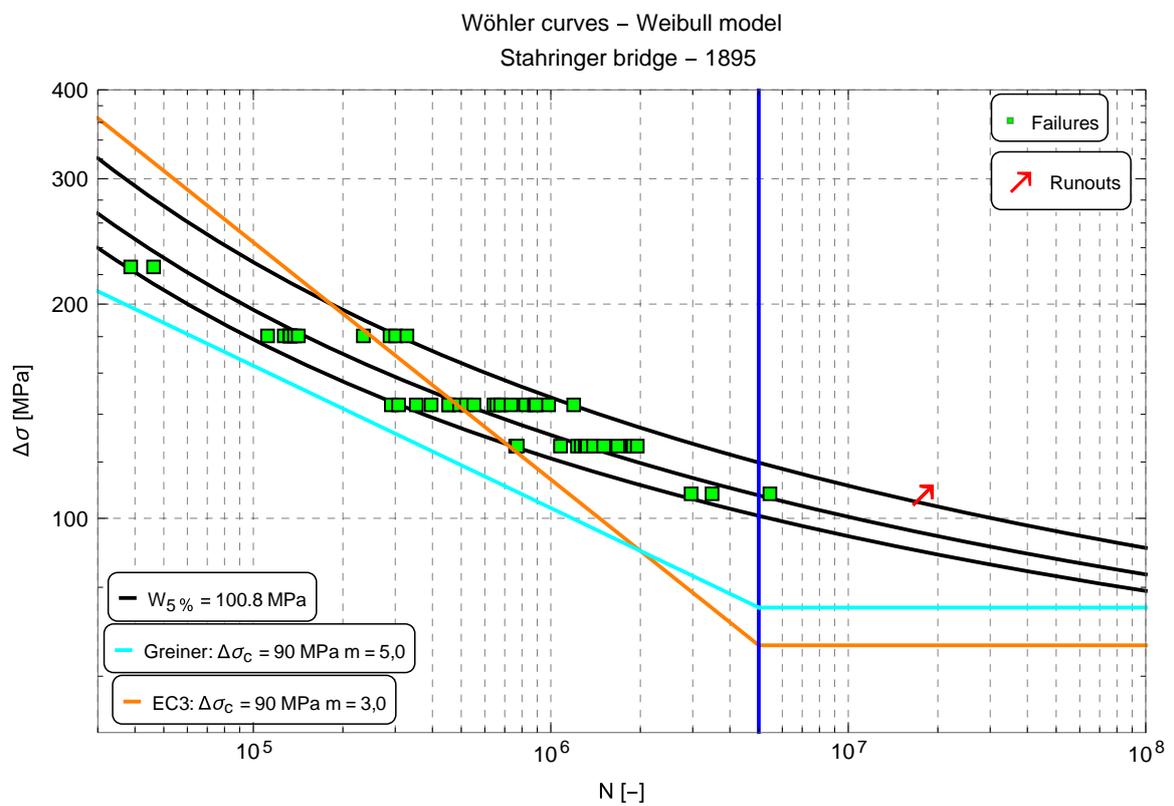
Fig. 11. Stahringer Bridge (1895) Wöhler curves for a failure probability of 5%, 50%, and 95%: Weibull model.
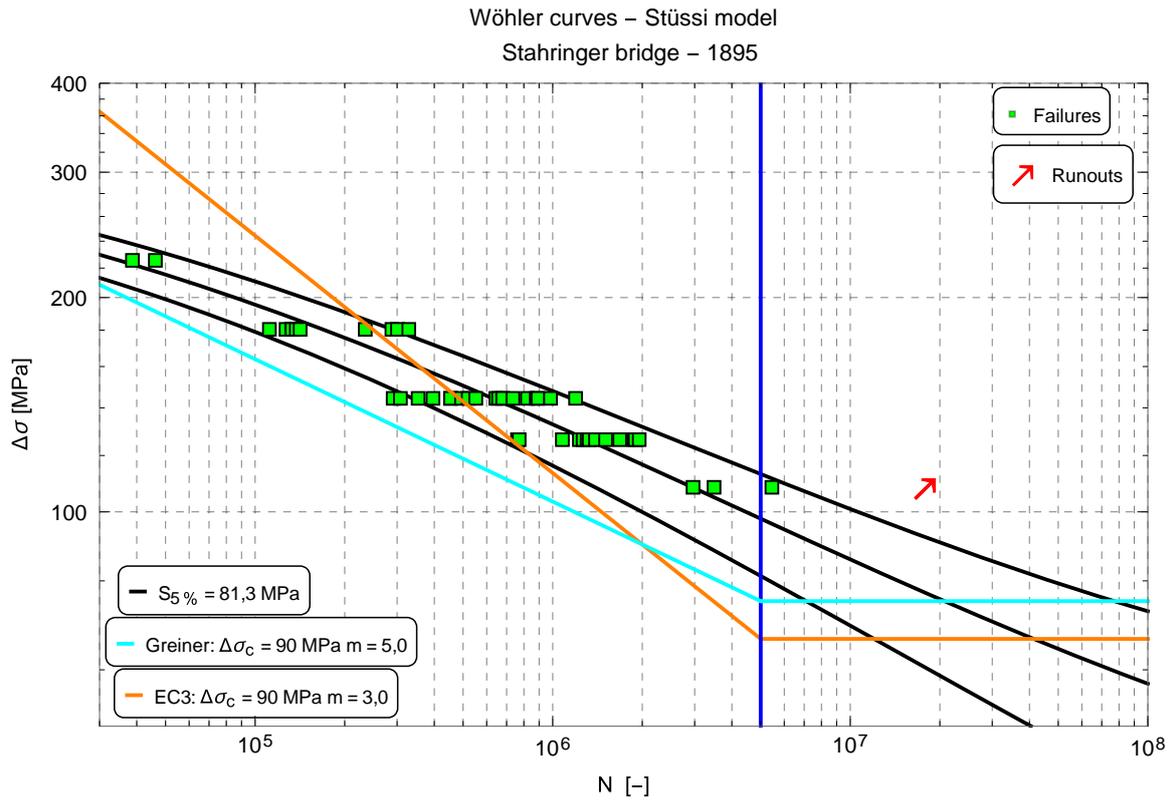
Fig. 12. Stahringer Bridge (1895) Wöhler curves for a failure probability of 5%, 50%, and 95%: Stüssi model.
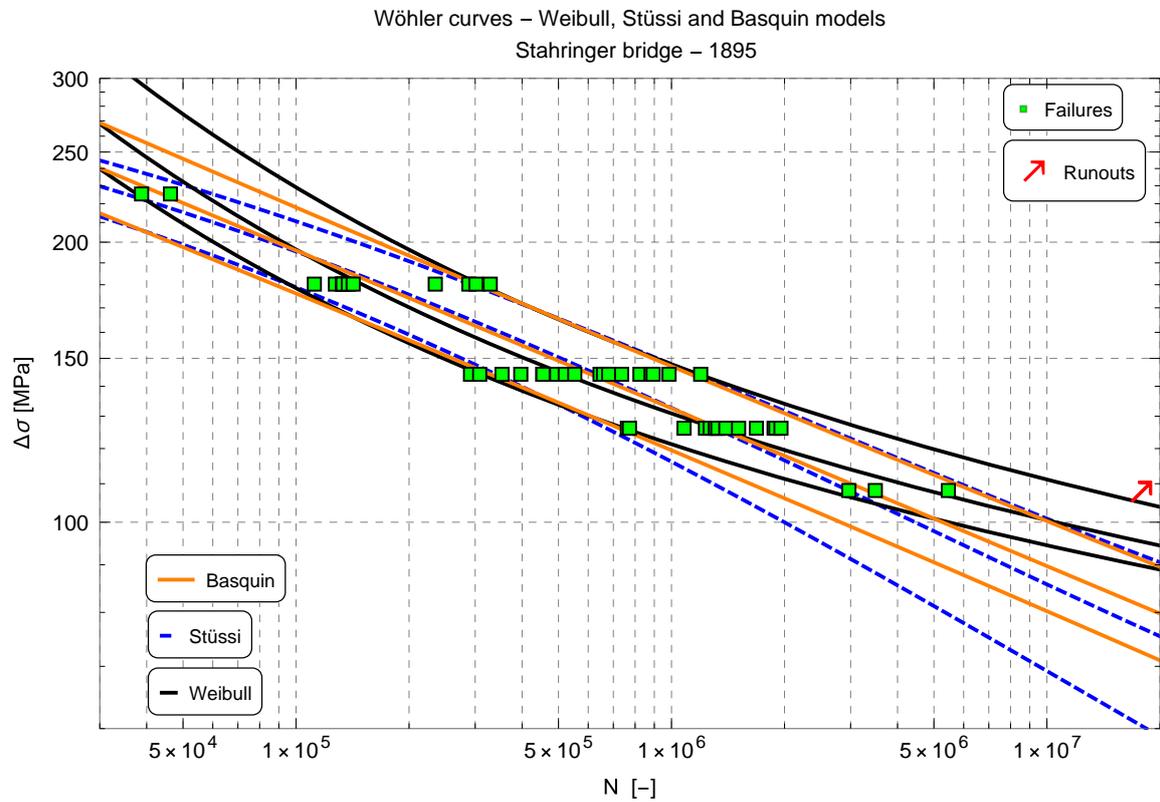

Fig. 13. Stahringer Bridge (1895) Wöhler curves for a failure probability of 5%, 50%, and 95%: Basquin, Stüssi, and Weibull models.

On the one hand, the slope given by the Basquin model ($m = 5.9$) is revealed to be similar to the slope ($m = 6.0$) obtained for single shear riveted connections of Pedrosa et al. (2019). This similarity does not depend on the stress ratio R of the experiments. In fact, in the Basquin model, R affects only the vertical displacement of the regression. On the other hand, it differs from the slope given in Eurocode 3 ($m = 3.0$) and the suggested one by Greiner ($m = 5.0$) (Fig. 10).

Even though there is one runout, the Weibull model describes properly the asymptotic behavior of the Wöhler curves in HCF and VHCF regions (Fig. 11). The estimated fatigue limit $\Delta\sigma_\infty$ is 29.07 MPa when only the fatigue failures are considered, and it is 28.59 MPa when the fatigue failures and the runout are considered (Toasa Caiza and Ummenhofer 2018b). These estimations of the fatigue limit are similar to the estimated fatigue limit of 30.14 MPa obtained from other antique European bridges such as the Eiffel Bridge (Portugal), Fäo Bridge (Portugal), and the bridge over the Adour River (France) (Mayorga et al. 2017). This particular fact requires attention because the corresponding stress ratios R are different. For this reason, additional investigations are in order to establish the effect of the stress ratio. Moreover, the estimation of the fatigue lifetime and fatigue limit are the highest (Fig. 13).

In the LCF region, the Stüssi model describes properly the geometry of the Wöhler curves (Fig. 12).

Conclusions

The two applications presented in this paper show that the application of the linear Basquin model or assigning a predetermined slope to the Wöhler curves is not appropriate to estimate fatigue life of an historical riveted structure.

On the one hand, the Weibull model describes properly the asymptotic behavior of the fatigue lifetime in the VHCF regime; however, this model does not help to make an estimation in the LCF regime.

On the other hand, because the Stüssi model considers the ultimate tensile strength, it allows one to describe the fatigue lifetime in the LCF regime; however, in the HCF and VHCF regimes, this model is more conservative than the Weibull model.

Based on the experimental results and aforementioned comparisons, it can be concluded that a suitable estimation of the fatigue life can be made by applying the Weibull model in both the HCF and VHCF regimes. The same applies for the Stüssi model but only in the LCF regime.

Data Availability Statement

The fatigue experimental data of the historical bridges considered in this paper are available from the corresponding author upon reasonable request.

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