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A Simple Stick-Slip Model for the Overturning Stability of unanchored Containers

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ABSTRACT:

Unanchored containers, e.g. for toxic waste, are required to be checked against overturning under seismic actions. Usually this is done by calculating static equilibrium, therewith ensuring stiction and avoiding overturning.

When concepting seismic actions as displacement controlled base excitation with reversals, it becomes obvious that neither the beginning of an overturning motion nor the loss of stiction necessarily causes the structure to fall over.

Based on a simplified rigid body assumption for the unanchored container, analytical methods are used to describe the states of no-motion, sliding, rocking, overturning and restoring of the overturning motion by reversion of the base displacement. Of course, if this is extended into a numerical simulation, the rigid body assumption ("rocking block") can be dropped and deformations of the container, e.g. shell modes, can be considered.

Depending on the aspect ratio and a variation of friction coefficients parameter confidence boundaries are identified, within which the structure is safe against overturning.

Keywords: earthquake design; stiction; base excitation; rigid body;

1 Introduction

Motivation of the present paper is an industrial project, where the first author investigated the tilting stability of stacked steel boxes as given in Figure 1 under earthquake and wind in Germany [1]. It was obvious, that the Lateral Force Method ([2], see worked example in [3]) is very conservative, when it comes to reversed base motion as with earthquake. In a quasi-static approach, loss of stiction indicates loss of bearing capacity; really this is not the case, there is just a displacement. As well, in a quasi-static approach tilting indicates loss of bearing capacity; really, if tilting starts short before reversal of the ground motion, some bumping will happen, but the stack will not fall over.

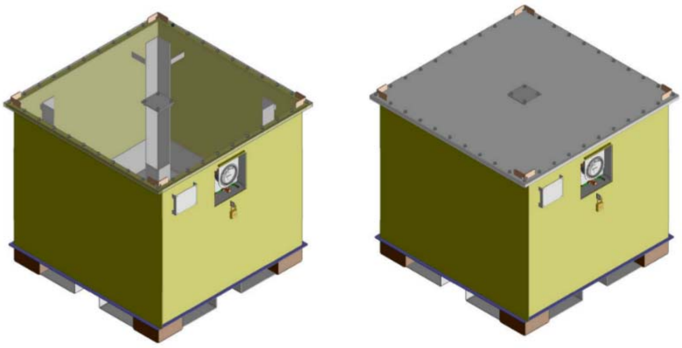

Figure 1: Outer steel box of a double wall mercury container [4]

In the present paper we develop a less conservative approach, but still using quasi-static formulations for limit conditions instead of employing a strict dynamic solution. Thus, "simple" means the model is wrong in a mechanical sense, because we are describing dynamic phenomena with a quasi-static approach. However, from a designer's point of view, the model given in Figure 2 is not worse than the Lateral Force Method proposed in EC8 [2].

2 Terms, Definitions and Abbreviations

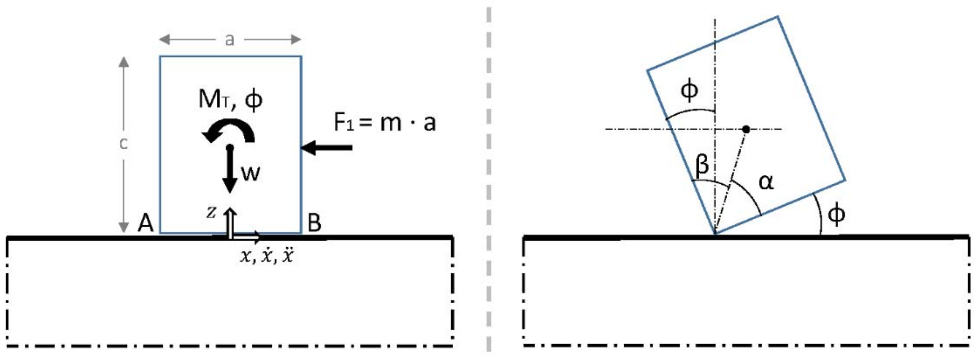

Figure 2: Mechanical model (elevation); tilting/sliding body on a rigid surface

a, b, c [m]	dimensions of the body in direction of the x-, y- and z-coordinate
a [m/s^2]	acceleration
A [m]	amplitude of the ground motion

aspect ratio c/a – a big aspect ratio is describing a slender body

coordinates: origin: on the surface between body and ground; right hand rule
x: horizontal coordinate; z: vertical (upward) coordinate

DAF dynamic amplification factor

DOF degree(s) of freedom

Lyapunov Stability engineering definition used in this paper:
a) long time amplitudes do not diverge; i.e. existence of a limit cycle (attracting orbit);
b) without steady state drive the amplitudes are converging to zero motion;
Note, that Klotter used the term semi-stable for the limit cycle described under a), because perturbations to the outside are divergent while perturbations to the inside are stable [5].

m [kg] mass

non-smooth dynamical system system with discontinuity in the restoring force

PGA peak ground acceleration

$r = c/2 / \sin \alpha$ radius of the trajectory of the center of mass when rotating around a bottom corner

T (s) oscillating period; (see ω)

$\alpha = \arctan (c/a)$ [rad] angle against the horizontal; pointing from a bottom corner of the body towards the center of gravity

$\beta = \pi/2 - \alpha$ angle against the vertical; pointing from a bottom corner of the body towards the center of gravity

$-\frac{\pi}{2} \leq \varphi \leq +\frac{\pi}{2}$ [rad] rotational DOF of the body

$\eta = \frac{\Omega}{\omega}$ normalized driving frequency

μ_s; μ_k; (constant) coefficients of static and kinetic friction

ρ [kg/m^3] density

Ω [rad/s] driving frequency

ω [rad/s] pseudo-eigenfrequency of the oscillator; The term pseudo-eigenfrequency was used by Nagel [6] to make clear, that a nonlinear oscillator exhibits a periodic motion, but the latter cannot be described by means of classical (linear) dynamics as $T = 2\pi \sqrt{(m/c)}$. However, using T in a less rigorous definition, we can refer to the period as describing the time along three zero-crossings, regardless of the motion in between being harmonic or periodic non-harmonic.

3 Assumptions

We investigate a prismatic body standing on a horizontal surface; body and surface are rigid; for stacked boxes this implies, that either gapping between the boxes is counterbalanced by the self-weight or the boxes are bolted; the stack remains rigid, "whipping" is excluded.

The body has a continuous density, so that the mass of the body is given by

$$m = a \cdot b \cdot c \cdot \rho \qquad (1)$$

In this first study we investigate a 2D problem in the x-z-plane. Thus, the findings do also hold for a vertical cylindrical body instead of a prism.

For simplicity, in this study the seismic action is assumed to be a horizontal harmonic ground motion given by $x(t) = A \cdot \sin \Omega t$. Restrictions and further discussion of this simplification is given by Knoedel/Hrabowski [7].

For simplification, we assume the asperity of both contact surfaces being represented by constant coefficients of static friction and kinetic friction.

Due to the harmonic ground motion we have decaying speeds towards the extreme points, so that the relative motion between body and ground gradually comes to a halt. This may lead to stick-slip phenomena towards the reversal of the motion ("rattling"), which have been investigated e.g. by Vielsack [8]. However, these effects near the reversal do not affect the findings of this study.

4 Dynamic Behavior

4.1 General

Nonlinear dynamics of rocking (tilting) oscillators have been described by e.g. Klotter [5], DeJong/Dimitrakopoulos [9] and Vassiliou/Markis [10].

Nonlinear dynamics of sliding oscillators have been described by e.g. den Hartog [11], Klotter [5], Leine et al. [12], Hong/Liu (including a review of literature [13]), Vielsack [8] and Gaus [14]. Typically, the model of a mass on a belt is used, which is derived from technical applications such as screeching of a disc brake.

To the authors knowledge, there is no simple engineering solution published, which includes friction with "belt-reversals" and which allows to estimate he behavior of an oscillator rocking and sliding.

4.2 Properties of Rocking Oscillators

In Klotter 5.43 [5] a linearized solution (small rotation; requires small amplitudes) for a quarter fundamental period is given, which is here extended to a full period for convenience and retransformed into physical units.

$$T = 8\sqrt{\frac{\varphi_0 \cdot r^2}{a \cdot g}} \qquad (2)$$

with φ_0 being the initial rotation. Remark: As Klotter includes only a translatory point-mass in the center of gravity of the block, this describes a mathematical pendulum rather than a physical pendulum.

In DeJong/Dimitrakopoulos [9] a "frequency parameter" is given, which is here rearranged as fundamental period for convenience.

$$T = 2\pi\sqrt{\frac{4 \cdot r}{3 \cdot g}} \qquad (3)$$

This solution coincides with a physical bar pendulum of L = 2r including rotational inertia. Further on, we refer to the value given by Eq. 3 because this seems to be a better representation for our engineering problem.

As will be seen in the example below, the rocking period of bodies with practical dimensions is $T \geq 2$ s. Thus, the period of the body is by far larger than the control period T_C given in EC8-1 Tab. 3.2 and 3.3 [2]. The effective acceleration is at least by a factor of

$$k = \frac{T_C}{T} = \frac{0.8\,s}{2.0\,s} = 0.4 \qquad (4)$$

smaller than the plateau value. This means in turn, that the big amplitudes in the response spectrum do not really affect the oscillator. With a prescribed base displacement and $\eta > 1$, even for a steady state drive the DAF for the absolute amplitude of the oscillator remains < 1: the displacement amplitudes of the center of gravity of the oscillator are smaller than the driving amplitudes at the base.

4.3 A Note on Friction Coefficients

In EN 12812 informative Annex B [15] values are given for steel on concrete:

$$0{,}3 \leq \mu_s \leq 0{,}4 \qquad (5)$$

In this study we will be using (arbitrary)

$$\mu_s = 0{,}4 \qquad (6)$$
$$\mu_k = 0{,}2$$

Note, that also the relation of μ_k being app. 50 % of μ_s is arbitrary:

Vatansever/Yardımcı [16] reported a slip coefficient being 92 % of the stiction. Afzali et al. [17] reported a slip coefficient of nearly 100 % of the stiction, compare also EN 1090-2:2018 Fig. G.2 [18].

It seems, that big differences result from higher displacement rates, while in the test setup according to EN 1090-2:2018 the load increase up to slip is very slow. Furthermore, the procedure in EN 1090-2 is for qualifying coating, and thus might have a different behavior than a loose block on a surface. However, due to space restrictions, this cannot be discussed in detail in the present paper.

5 Identification of Modes

5.1 Quasi-Static Mode (sticking)

Assessment of the quasi-static mode is possible only, if the center of gravity of the body is accelerated instead of the base.

Limit conditions
Base shear does not exceed friction
Tilting moment does not exceed restoring moment from self weight
(φ = 0)

Inertia of the mass

$$F_I(t) = -m \cdot a(t) \quad (7)$$

Static friction

$$F_F^s(t) = +m \cdot a(t) \;\; \leq \;\; G \cdot \mu_s = F_{F,lim}^s \quad (8)$$

Tilting moment

$$M_T(t) = \; +m \cdot a(t) \cdot \frac{c}{2} \quad (9)$$

Restoring Moment

$$M_R = -G \cdot \frac{a}{2} \quad (10)$$

5.2 Slipping Mode

Limit conditions
Base shear does exceed friction
Tilting moment does not exceed restoring moment from self weight
(φ = 0)

Kinetic friction

$$F_F^k(t) = +m \cdot a(t) \;\; \leq \;\; G \cdot \mu_k \; = \; F_{F,lim}^k \quad (11)$$

Compared to stiction, the accelerating base shear is reduced during slipping. Thus, the body is less accelerated, leading to a smaller rotation φ if tilting. With the same displacement amplitude A, the motion of the body becomes less critical.

5.3 Rocking Mode

Limit conditions
Base shear does not exceed friction (more complex terms for slipping)
Tilting moment does exceed restoring moment from self weight
($\varphi \neq 0$)

Tilting moment

$$M_T(t) = +m \cdot a(t) \cdot \frac{c}{2 \cdot \sin\alpha} \sin(\varphi + \alpha) \qquad (12)$$

Restoring Moment

$$M_R = -G \cdot \frac{a}{2 \cdot \cos\alpha} \cos(\varphi + \alpha) \qquad (13)$$

Note, that the restoring moments is negative for $\varphi > \beta$.

Note that the condition $A \leq a/2$ is only a conservative description of the falling over limit. It neglects the acceleration of the center of mass during the first quarter period, leading to $\varphi < \beta$ for $A = a/2$.

5.4 Overturning Mode

Limit conditions
Base shear does not exceed friction (more complex terms for slipping)
Tilting moment does exceed restoring moment from self weight
Steady state amplitude $A > a/2$ (quasi-static, weak condition)

6 Numerical Analysis

In parallel to the analytical solutions presented here, a FEA demonstrator model is prepared. This enables to perform a transient analysis and to check the plausibility of the above assumptions. When trying to drive a highly non-linear system to maximum response amplitudes, the tuning of the driving frequency can be difficult, as described in [19]. In order to overcome this difficulty, a wide range of driving frequencies are investigated, and the response of the container is mapped. However, due to space restrictions, the results such as phase portraits etc. will not be included in this paper.

7 Example

In the example we use the features of the container shown in Fig. 1.

Steel box L/W/H = app. 1.100/1.100/1.000 mm; center of gravity at app. 500 mm elevation; self weight plus fill app. 25 kN (data taken from [1]), resulting in a virtual density of 2.066 kg/m^3.

Stiction limit load

$$F^{s}_{F,lim} = G \cdot \mu_s = 25\,kN \cdot 0{,}4 = 10\,kN \quad (14)$$

Restoring moment

$$M_R = -G \cdot \frac{a}{2} = -25\,kN \cdot \frac{1.100\,mm}{2} = -13{,}8\,kNm \quad (15)$$

Sliding limit load

$$F^{k}_{F,lim} = G \cdot \mu_k = 25\,kN \cdot 0{,}2 = 5\,kN \quad (16)$$

We assume a PGA of 1,6 m/s^2 as used in the example in [7] with a soil factor S = 1,5, a behavior factor of q = 1,5 and an importance factor of γ_I = 1,0. Thus, according to EC8-1 4.3.3.1 with Eqs. 4.5 and 3.14 [2], we obtain a quasi-static plateau base shear of

$$F_b = m \cdot \gamma_I \cdot a_{gR} \cdot S \cdot 2{,}5/q \quad (17)$$
$$F_b = 2.500\,kg \cdot 1{,}0 \cdot 1{,}6\frac{m}{s^2} \cdot 1{,}5 \cdot \frac{2{,}5}{1{,}5} = 2.500\,kg \cdot 4.0\frac{m}{s^2} = 10\,kN$$

This corresponds to a harmonic base acceleration of 2,4 m/s^2 corresponding to a harmonic base displacement with an amplitude of app. 50 mm [7].

The tilting moment amounts to

$$M_T(t) = +10\,kN \cdot 500\,mm = 5\,kNm \quad (18)$$

In this example, the body will just remain in the sticking mode and will not tilt.

Fundamental period

$$T = 2\pi\sqrt{\frac{4 \cdot 0.74m}{3 \cdot 10\frac{m}{s^2}}} = 1.98\,s \quad (19)$$

In the same way we obtain the numbers when stacking up 4 boxes:
$F_{F,lim}$ = 40 kN; M_R = –55,0 kNm; F_b = 40 kN; M_T = 40 kN * 2,0 m = 80 kNm;
T = 3,30 s

In this example, the body will just remain in the sticking mode and will tilt.

A driving displacement amplitude of 50 mm is << a/2 = 600 mm. Regardless of sticking or slipping the tilting of the body will remain well within the limit cycle. Thus, Lyapunov stability is given.

8 Summary

By extending the analysis also to dynamic modes, it was possible, to come to less conservative results than before.

– Slipping is not affected by the number of boxes in the stack, only by the base acceleration and the coefficient of friction.

- According to the above findings slipping leads to increased overall stability.

- We assume that the stack has been designed for a maximum quasi-static load coming from the plateau value of the response spectrum. Then, by dynamic effects, there is no amplification that leads to higher horizontal loads.
- Employing dynamic criteria enables a less conservative statement on global rocking stability.
- Even more precise findings can be expected by FE time history analysis.

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