Next-to-Leading Order QCD Corrections to Higgs Boson Production and Mixed QCD-Electroweak Corrections to Z Boson Production at High Transverse Momentum

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Kirill Kudashkin

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Referent: Prof. Dr. Kirill Melnikov
Korreferent: Prof. Dr. Matthias Steinhauser

Abstract

In this thesis we study two processes: the Higgs boson production when the Higgs transverse momentum larger than the top-quark mass $p_{H,T} \gg m_t$ and the Z boson production at high transverse momentum. The key point of our study is the evaluation of master integrals. Both processes share a similar signature at the high transverse momentum: a hierarchy of scales. This signature is exploited to construct asymptotic expansions of master integrals which allows us to simplify their evaluation. Master integrals are then evaluated using differential equations. The analytical results are expressed in terms of Goncharov polylogarithms. In the case of the Higgs boson production, the evaluation of relevant master integrals allows us to compute two-loop amplitudes for partonic processes $gg \to Hg$, $q\bar{q} \to Hg$ and $qg \to Hg$. These two-loop amplitudes are the long-missing part of next-to-leading order QCD corrections to the Higgs+jet production at high-$p_{H,T}$. The evaluation of next-to-leading order QCD corrections increases the exiting leading-order results by almost a factor of two. This result removes one of the largest theoretical uncertainties in the description of the Higgs+jet production at large-$p_{H,T}$.

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Overview

We present the results of two projects: the Higgs boson production and the Z boson production at high transverse momentum. The first project is often referred to as $H + j$ production, while the second is referred to as $Z + j$ production.

For $H + j$ project consists of two parts. One part is devoted to calculations of two-loop amplitudes for processes $gg \to Hg$, $q\bar{q} \to Hg$ and $qg \to Hq$ that are relevant for next-to-leading order QCD corrections to $H + j$ production at high Higgs transverse momenta $p_{H,T}$. They were originally calculated in Ref. [1]. Another part is devoted to the evaluation of next-to-leading order QCD corrections to $H + j$ production at high-$p_{H,T}$ [2].

The second project is devoted to calculations of all master integrals that are relevant to two-loop mixed QCD&EW corrections to the production of Z bosons accompanied by a high transverse momentum jet. This project is a matter of ongoing research. We refer to this process as $Z + j$ production.

In Chapter 1 we discuss how differential observables can be used in searches of physics beyond Standard Model (BSM). In particular, we motivate studies of two cases: $H + j$ production at high transverse momenta and $Z + j$ production at high transverse momenta.

We discuss the perturbative structure of $H + j$ production in Chapter 2. In particular, we present the main ingredients which are used to derive the two-loop scattering amplitudes: form factors. We discuss how to renormalize UV and IR divergences appearing in these form factors. We show how to derive helicity amplitudes for $H + j$ case.

Chapter 3 is devoted to a general discussion of Feynman integrals. We discuss some of their basic properties including a singular structure of Feynman integrals and how to regularize it.

In Chapter 4, the differential equation method is reviewed. The idea of high energy asymptotic expansions is reviewed. It is shown how these expansions can be used to solve differential equations. We analyze differential equations of both cases of interest. On several examples, we show how boundary constants relevant to $H + j$ and $Z + j$ are calculated.

In Chapter 5, we discuss how to perform analytic continuation of two-loop amplitudes to the relevant kinematic regions. We then show leading order (LO) results for $H + j$ production expanded in the mass of the Higgs boson and the top quark. These results are compared with existing LO results. In the end, the next-to-leading order predictions for $H + j$ production at high transverse momenta are presented.

The notation and Feynman rules which were used in calculations of $H + j$ production are listed in Appendix A.1. Some analytical techniques and formulas are given in Appendix B. We present all integral families, top-sectors and some Feynman diagrams of master integrals relevant to both cases of interest in Appendix C. In Appendix D, selected solutions of master integrals for the cases of $H + j$ and $Z + j$ are shown. Finally, Appendix E contains helicity amplitudes for the $H + j$ case.

We used LaTeX to write this thesis. All Feynman diagrams were drawn with the help of JaxoDraw [3] and *Mathematica*-based package `LiteRed` [4, 5].

Chapter 1

Precision vs Sensitivity

High-energy particles collisions at the Large Hadron Collider (LHC) are one of the main tools to test different theories of fundamental physics. At present, the Standard Model (SM) is the most adequate theory giving accurate predictions for the outcome of these collisions. All the predicted particles of the SM have been discovered. However, there are several experimental phenomena which cannot be described within the SM. Prominent examples are neutrino oscillations, the hierarchy problem, dark energy, baryon asymmetry. Moreover, gravity, one of the four fundamental forces of Nature, is not incorporated in the SM. It is then possible to conclude that there must be physics beyond the Standard Model (BSM) to account for the aforementioned phenomena.

The hope was that already during the first Run of the LHC, BSM physics would be observed. This hope was fuelled by promising theories, in particular various supersymmetric extensions of the SM which could solve, for instance, the hierarchy problem. These theories were able to predict the existence of new particles at scales reachable at the LHC. However, aside from the discovery of the Higgs boson [6], which is the major achievement of particle physics in the past two decades, no new particles were observed. This taught us a good lesson: new physics will probably not be discovered straightforwardly, rather it will be hidden somewhere among the SM background processes.

Therefore, to find any inconsistencies in current description of collision processes, one needs to increase the *precision* of both experiments and theoretical calculations. At the experimental end, this would mean increasing the center-of-mass energy of colliding particles, at present 14 TeV, and luminosity of $10^{34}\text{cm}^{-1}\text{s}$. Improving both factors increases the number of events per year the LHC can deliver, consequently, reducing the statistical error of measured observables at higher rates. Increasing the precision of experimental measurements will challenge the theory. Indeed, for some observables, the experimental precision is already of $O(1\%)$. A prominent example is the Z boson p_T spectrum [7, 8] which we address in this thesis. This means that to match the experimental precision, very accurate theoretical predictions will be required reaching next-to-next-to-next-to-leading order (N^3LO) accuracy for some inclusive observables.

Reaching such high accuracy is a tremendous work which requires the combined efforts of many physicists. Indeed, different studies must be carried out simultaneously at a sufficient level of precision. Such as fixed-order calculations, various resummations, construction of optimal subtraction schemes that take care of infrared divergences of gauge theories, etc. Moreover, it becomes more and more pressing issue that Parton Distribution Functions (PDFs) are the main source of theoretical uncertainties for many observables [9]. While many ongoing projects are pursuing N^3LO accuracy for different processes, we might in the mean time study LHC data with means that we have at our disposal from a different perspective, both theoretically and experimentally. To this end, one way to proceed is to study differential observables which are sensitive to the dynamics of production processes. Below, we are going to motivate this statement on a rather rudimentary, qualitative basis. However, it does demonstrate the philosophy which inspired us to write the thesis.

Consider the leading-order effect of new physics on an SM observable which comes from higher dimensional effective operators [10][1]

$$\mathrm{O} = \mathrm{O}_{\mathrm{SM}} \left(1 + \mathcal{O}\left(\frac{\mu^2}{\Lambda^2} \right) + \ldots \right), \tag{1.1}$$

where Λ is the scale of new physics and μ is a typical scale of the SM observable O_{SM}. Considering the inclusive Higgs production cross-sections or the Higgs decay rates, we take μ to be the Higgs vacuum expectation value v. Then, an effect of new physics for this kind of Higgs observable at a scale $\Lambda = 2\,\mathrm{TeV}$ is of $\mathcal{O}(1\%)$ [11]. On other hand, consider a dynamical observable with large momentum transfer Q, as the Higgs p_T distribution with the scale $\mu \sim Q$. It is straightforward to see that for the same Λ as before, we now get an effect of $\mathcal{O}(20\%)$ at $Q \sim 1\,\mathrm{TeV}$ [12, 13]. What this example shows is that studying dynamical observables at a higher energy scale may be beneficial since they are often more *sensitive* to the effects of BSM physics. They offer higher kinematic reach to probe the larger scale Λ of BSM physics. We note that this discussion applies only to a certain type of models [11]. Moreover, we revisit this discussion in a more detailed manner later in the text. Finally, this idea is not new and was considered before in many publications (cf. [12–14] and references therein). These studies might be complicated experimentally due to low statistics, and, for instance, lower signal to background ratio [14, 15] at higher transverse momenta. In the light of the ongoing High Luminosity (HL) upgrade at the LHC, this and many other experimental issues will become less severe [16]. Additionally, if the Future Circular Collider (FCC) is approved, the low-statistics issue will be pushed to even higher energy scales [17].

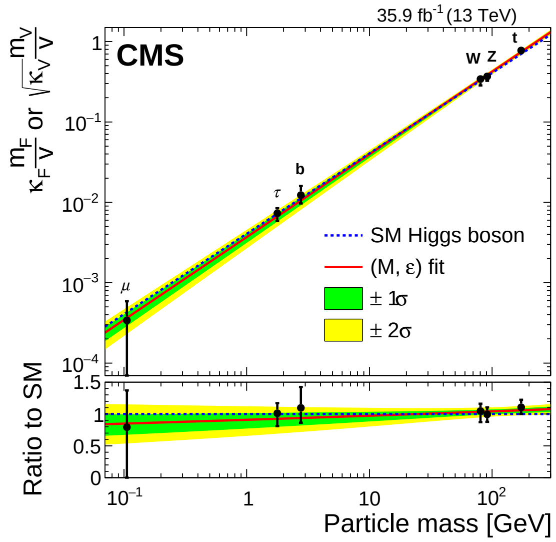
What kind of observables may exhibit substantial BSM effects?

1.1. Phenomenology of the Higgs production at the LHC

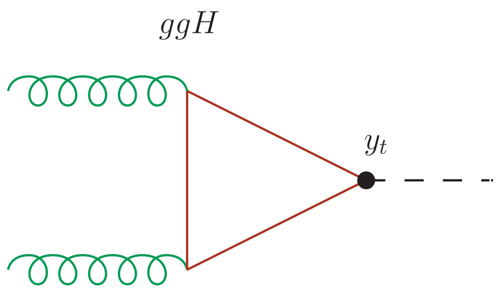
The discovery of the Higgs boson opens up a new chapter in searches for physics beyond the SM (BSM). Indeed, the Higgs boson is the least studied and the most peculiar particle of the SM. It is then likely that it couples to unobserved particles that appear in many extensions of the SM. Such interactions will affect the production of the Higgs boson and its decay rates. Up to now such deviations have not been observed which implies that the energy scale of New Physics is higher than what has been expected previously. If so, effects of BSM particles on Higgs boson production and decays processes may be smaller than radiative corrections in the SM. Hence, on the one hand, we have to bring the predictions within SM to high precision in order to have a chance to observe a particular BSM signal, and on the other hand, we need to develop an intuition as to where to look for it.

To this end, it is natural to explore four main production channels of the Higgs bosons at the LHC: the gluon fusion process $gg \to H$, the vector boson fusion $q\bar{q} \to q\bar{q}H$, the Higgs-strahlung process $q\bar{q} \to HV$ and the associated production process $gg \to t\bar{t}H$. These channels allow us to

[1]Here, we follow M. L. Mangano's discussion from the workshop “Higgs couplings 2016”.



(a) Values for Higgs SM couplings within κ-framework [15].



(b) Gluon fusion process.

Figure 1.1.: Gluon fusion is mostly facilitated by the top quark, since it has the strongest coupling to the Higgs boson.

access different Higgs couplings, so that we can probe a variety of BSM scenarios. In this thesis, we will focus on the gluon fusion process $gg \longrightarrow H + X$.

There are at least two reasons to consider the gluon fusion channel. First, we can use this channel to indirectly probe the top-Yukawa coupling. Second, Higgs production in the gluon fusion process is the largest contribution to the Higgs boson cross-section, as can be seen from Tab. 1.1. Hence, given the importance of this process, it is essential to have good theoretical understanding of the $gg \longrightarrow H$ production, if we are to probe extensions of the SM through this channel at the LHC.

Interactions of Higgs bosons with gluons are facilitated by quark loops, cf. Fig. 1.1b. From the SM Lagrangian we can extract the term which is relevant for the ggH coupling

$$\mathcal{L}_{SM}^{Yukawa} = \sum_i \frac{m_i}{v} q_i \bar{q}_i h, \tag{1.2}$$

where q_i denotes the field and m_i denotes the mass of a quark of a flavor i, $i = \{u, d, s, c, b, t\}$, h is the Higgs boson field and v is the vacuum expectation value.

Since $m_t \gg m_b \gg m_c, \ldots$ the top quark couples strongly to Higgs bosons and provides the largest contribution to the gluon fusion cross-section.[2] Hence, the top-Yukawa coupling plays an essential role in the production of the Higgs boson. At present, it is known experimentally with the precision of about $20 - 30\%$ from the $t\bar{t}H$ production channel [18]. Hence, it is still possible to have additional, point-like contributions to Hgg interaction vertex that compensate deviations caused by changes in the Yukawa coupling. We can describe such contributions by modifying the top-Yukawa coupling

[2]The next relevant contribution comes from the bottom quark, but the top mass and the bottom mass is separated by two orders of magnitude cf. Fig. 1.1a

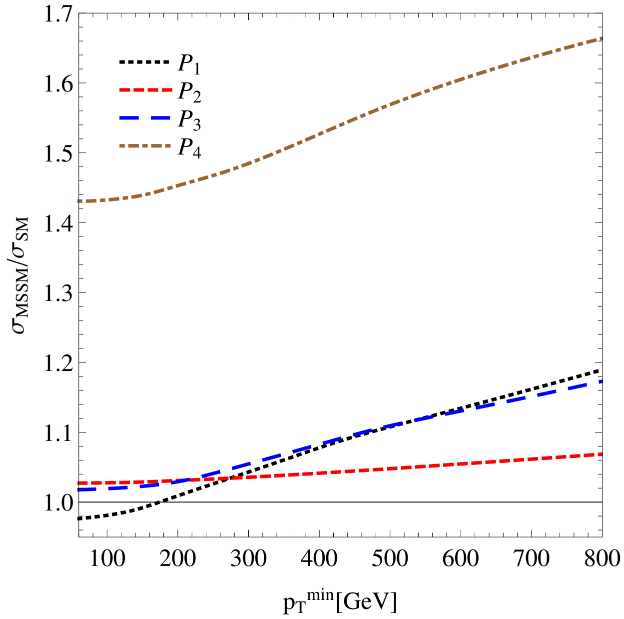


Figure 1.2.: Cross section for boosted Higgs boson in the MSSM, normalized to the SM value, as a function of the transverse momentum cut p_T^{min}. Different lines correspond to different stop masses and the stop trilinear terms that are given in Ref. [12].

and adding the point-like ggH interaction term directly to the SM Lagrangian

$$\frac{m_t}{v}\bar{t}tH \to -\kappa_g \frac{\alpha_s}{12\pi v} G^a_{\mu\nu} G^{\mu\nu,a} H + \kappa_t \frac{m_t}{v}\bar{t}tH. \tag{1.3}$$

We note that the first term on the r.h.s. in Eq. (1.3) is the point-like contribution to the Higgs-gluon coupling and the second term is the modified top Yukawa coupling. Quantities κ_t, κ_g represent anomalous couplings.[3]

	ggH	VVH	WH	ZH	$t\bar{t}H$
σ [pb]	$54.7^{+5\%}_{-5\%}$	$4.28^{+2\%}_{-2\%}$	$1.51^{+2\%}_{-2\%}$	$0.99^{+5\%}_{-5\%}$	$0.60^{+9\%}_{-9\%}$

Table 1.1.: The SM Higgs boson production cross sections for various channels with $m_H = 125$ GeV in pp collisions at the center of mass energy $\sqrt{s} = 14$ TeV, [19].

1.1.1. Production of the Higgs boson in the gluon fusion

We will leave aside a discussion about possible extensions of the SM that lead to modifications of the SM Lagrangian as in Eq. (1.3). Many of them are based on the idea that the Higgs boson is a pseudo-Goldstone boson [20]. Two prominent examples are the Little Higgs model [21] and the composite Higgs model [22]. What is relevant to us is that these models generically contain new heavy particles that couple to the Higgs boson. These particles are usually referred to as top-partners:

[3] Taking $\kappa_t = 1, \kappa_g = 0$ leads to the SM Lagrangian Eq. (1.2).

they are fermions with same quantum numbers as the top quark but with larger masses [12, 13]. Upon integrating them out, one obtains the Lagrangian shown in Eq. (1.3). The modified Higgs Lagrangian Eq. (1.3) changes the Higgs production cross-section. It is straightforward to derive the $gg \to H$ production cross-section under the assumption $m_H \ll 2m_t$ which is an excellent approximation. The result reads

$$\sigma_{gg\to H} \sim \frac{\alpha_s^2}{v^2}\left(\kappa_g + \kappa_t\right)^2, \tag{1.4}$$

where α_s is the strong coupling constant and v is the Higgs field vacuum expectation value. It follows that by measuring the gluon fusion cross section we can only constrain the sum of the two anomalous couplings Eq. (1.3) and not both of them separately.

A possible way to access κ_g and κ_t separately is to “resolve” the ggH vertex. This can be achieved by studying the production of Higgs bosons with high transverse momenta $|p_T| \gg 2m_t$. In this case the top-quark contribution cannot be treated as point-line any more and we obtain

$$\frac{d\sigma_H}{dp_T^2} \sim \frac{\sigma_0}{p_T^2}\left(\kappa_g + \kappa_t \frac{4m_t^2}{p_T^2}\right)^2. \tag{1.5}$$

Eq. (1.5) suggests that measurements of the Higgs boson production with high p_T may allow one to determinate the anomalous coupling $\kappa_g \neq 0$. This general consideration is supported by the analysis within a specific model of Refs. [12, 23]. As can be seen from Fig. 1.2, effects of heavy BSM particle strongly increase with p_T and can reach $O(100\%)$.

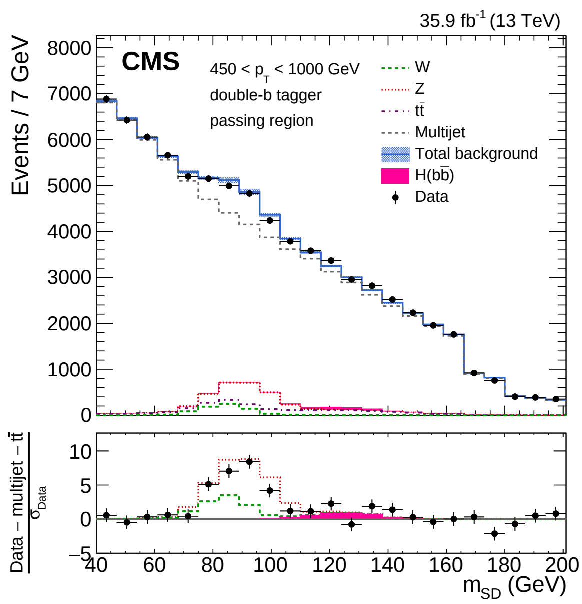


Figure 1.3.: The distribution of the jet mass (m_{SD}). In each bin, experimental data is collected at high p_T-region. The lower panel shows the signal strength. The strength of the Higgs signal is 1.5σ [24].

1.1.2. The experimental status of the the Higgs plus jet production at high p_T region

Up to now, we have discussed the production of the Higgs boson from a pure theoretical point of view. We pointed out that it is important to study the Higgs transverse momentum distribution at high p_T. Yet, as it turns out, it is very difficult to do in practice. The most significant issue is that the cross section of Higgs boson production at high p_T is small. Indeed, it is easy to estimate that $O(10000)$ Higgs bosons have been produced in $p_T > 400$ GeV region at the LHC so far. Despite the low statistics, it was possible provide evidence of the high-p_T Higgs production [24]. The main result of Ref. [24] is presented below.

To observe the Higgs decaying into $b\bar{b}$-pair in the gluon fusion, one needs to distinguish between a large multijet QCD backgrounds and b-jets that are produced in the Higgs decays. Considering very boosted Higgs bosons is a way to do so. Indeed, very boosted Higgs boson $p_T \gg 450$ GeV decaying into $b\bar{b}$-pair has a small decaying radius which can be used as a discriminating feature. To this end, $H \to b\bar{b}$ is considered to be a single jet and tagged using b-jet tagging techniques [24]. For the first time, the single-jet topology was used to observe $Z \to b\bar{b}$ decays Fig. 1.3. In the case of the boosted Higgs bosons production, the measured cross-section times the branching ratio $H \to b\bar{b}$ is $74 \pm 48(\text{stat})^{+17}_{-10}(\text{syst})$ fb is in agreement with the SM predictions given huge uncertainties of the measurement [24].

1.1.3. Theoretical status of the Higgs plus jet production at high transverse momenta

The production of Higgs bosons at high transverse momenta is an interesting observable. The study of this observable is challenging both experimentally and theoretically. Indeed, until recently, the leading order (LO) calculations in the strong coupling constant [1, 2, 24, 25] were the most accurate predictions to include top quark mass effects exactly for the high-p_T region [26, 27]. These calculations were done thirty years ago. The NLO analytic calculations which include the full top mass dependence is still a matter of ongoing research [28].

It is useful to separate the physical scales of the problem under consideration in order to understand the dynamics of the Higgs bosons production better. Let us consider the case of inclusive Higgs boson production $pp \to H + X$. It appears that a natural scale to separate different production regimes of Higgs bosons is the top mass m_t. Indeed, taking the center of mass energy $\sqrt{s}$ to be much smaller than m_t allows us to calculate higher order QCD corrections to $pp \to H + X$. In such a regime, as it was discussed previously, a top quark loop which facilitates the production of Higgs bosons is contracted to a point-like interaction ggH. In other words, the top quark is integrated out from the SM. Such theory without the top quark is referred as Higgs Effective Field Theory (HEFT). Within HEFT, it was possible to achieve quite remarkable results: the inclusive cross section for Higgs bosons production was first calculated to next-to-leading (NLO) order in α_s in Ref. [29], then to next-to-next-to leading order (N^{2}LO) in Ref. [30, 31] and finally to ultimate next-to-next-to-next leading order (N^{3}LO) in Ref. [32, 33]. Additionally, the $H + j$ production cross section was calculated to N^{2}LO [34–36].

Top mass effects were estimated for the inclusive Higgs boson production at NLO QCD by S. Dawson and R. Kauffman in Ref. [37]. By explicitly calculating top mass corrections of the type $\mathcal{O}(\alpha_s^3 m_H^2/m_t^2)$, i.e. going beyond the heavy-top mass approximation, they observed that relative effect of the top mass on the total cross-section at NLO does not exceed 1%. For quite some time, there was no quantitative study that the same order effect is taking place at higher perturbative

QCD orders. To resolve this issue, explicit calculations of the same type, i.e. $O(\alpha_s^4 m_H^2/m_t^2)$, but at NNLO QCD were performed in Refs. [38–42]. These studies showed that the effect of the top mass on the inclusive cross-section remains at the level of 1%. In the same manner, the $H + j$ production cross-section was studied in Ref. [43]. It was shown that the effect of the top mass on the differential K-factor was $2 - 3\%$ for $p_T \lesssim 150$ GeV.

Since we are interested in estimating m_t effects in high transverse momentum distribution of the Higgs boson, using HEFT will not be possible. Indeed, in the kinematic region $p_T > 2m_t$, the top quark loop can no longer be considered point-like as discussed previously. Despite this observation, people estimated these effects in case of $H + j$ production for large-p_T in a similar manner to the aforementioned works in Ref. [44]. LO results and NLO real emission corrections to the $H + j$ production were known exactly including exact top mass dependence, while the virtual two-loop correction was calculated by expanding corresponding Feynman amplitudes up to the fourth order in $1/m_t^2$. Such an approximation is justified as long as the virtual two-loop corrections are much smaller than the ones coming from real radiation and $p_T \leq 300$ GeV [44]. Given the fact that we are interested in even higher transverse momentum of Higgs bosons and that such calculations are performed with an arguable assumption, quantitative analysis of theoretical uncertainties of $H + j$ production for large-p_T is required. To this end, it is mandatory to explicitly calculate the virtual two-loop corrections.

In the main part of this thesis we discuss a solution to this problem based on the m_t/p_T expansion. In particular, we explain how the two-loop virtual amplitudes for $H + j$ production were computed using the small mass expansion. We note that a similar expansion, but in the bottom quark mass, was used in Refs. [45, 46]. It is also worth mentioning that similar high energy expansions were applied to double-Higgs production in the high energy limit in Refs. [47–49].

1.2. Missing transverse energy at the LHC

Missing transverse energy (MET) signature refers to a phenomenon when an object is produced at a hadron collider but there appears to be nothing that it recoils against. In the SM this happens, for instance, when Z or W bosons are produced and decay into $\nu\bar{\nu}$ or $l\nu$ and neutrinos leave the detectors unobserved. Interestingly, a presumptive production of Dark Matter (DM) particles at the LHC leads to similar signature. Indeed, assuming that non-gravitational interactions of DM particles with SM particles do exist, there are quite a few scenarios how it can occur [50]. One way to observe DM at the LHC detector is to look for an associate production of SM particles $X = \{Z, W, \gamma, g, q, \text{Higgs}\}$ with missing energy. Since, SM and DM observables may have similar missing energy signatures, there will be a discrepancy in the number of missing energy events. To observe it, the precise predictions of SM backgrounds for experimentally controlled regions is needed.

The leading SM background at the LHC is generated by $pp \to Z(\to \nu\bar{\nu}) + j$ process. Other processes such as $W(\to l\nu) + j$ and $Z/\gamma(\to l\bar{l}) + j$ also play a role . By carefully measuring $V + j$ production processes in control regions, one can put constrains on $pp \to Z(\to \nu\bar{\nu}) + j$ at higher MET values. To do so, one requires to preform an extrapolation of observed data form control regions to a signal region $Z \to \nu\bar{\nu}$. Such an extrapolation is highly nontrivial and relies on a theoretical input. In Ref. [9] all up-to-date predictions for aforementioned production processes were combined. In particular, they used NNLO QCD and NLO EW corrections for all $V + j$ production processes.

p_T^Z, GeV	$K_{\text{NLO}}^{\text{QCD}}$	$K_{\text{NNLO}}^{\text{QCD}}$	$K_{\text{nNLO}}^{\text{EW}}$	PDFs , %
650	$1.45 \pm 7\%$	$1.06 \pm 2\%$	$0.85\pm < 1\%$	1
1000	$1.5 \pm 8\%$	$1.07 \pm 2\%$	$0.8 \pm 1\%$	3
2000	$1.6 \pm 10\%$	$1.08 \pm 4\%$	$0.75 \pm 2\%$	>5

Table 1.2.: A breakdown of radiative corrections and uncertainties for $pp \to Z/\gamma(\to l\bar{l})+j$ production channel from Ref. [2]. K-factors of QCD, EW radiative corrections and PDFs uncertainties for different values of Z boson transverse momentum. $K_{\text{NLO}}^{\text{QCD}}$ is a ratio between NLO and LO predictions, while $K_{\text{NNLO}}^{\text{QCD}}$ is a ratio between NNLO and NLO predictions. Finally, $K_{\text{nNLO}}^{\text{EW}}$ is ratio between LO and NLO+NLL EW corrections.

An important point of these calculations is inclusion of EW NNLO Sudakov logarithms [51–54]. These logarithms were firstly studied in Ref. [55] for Abelian and in Ref. [56] for Non-Abelian field theories. They stem from the infrared structure of an underline theory under consideration. They appear when invariant energy and all transverse momenta become larger than the mass scale which is running inside a loop. In the case of the EW theory, considering energy scales far larger than the mass of the Z (W) boson, corrections due to Sudakov logarithms are proportional to a power of $g^2/(16\pi^2)\log^2(s/m^2_{Z(W)})$ [51]. These logarithms must be under the theoretical control, if we want to reach a percent precision, since their effect become of the same magnitude of NNLO QCD corrections at high energy limit $p_T \sim 1$ TeV for $V + j$ productions [2].

However, not all relevant calculations for $V + j$ processes are available. In particular, mixed QCD&EW correction to the $Z + j$ process is still absent. It lead to relative theoretical uncertainty for full mixed corrections that varies between 10% and 20% while their relative contribution (wrt. $\sigma_{\text{NLO QCD}}$) range from $8 - 15\%$ depending on $p_{T,Z}$ and a decay channel [9].

Part of this thesis is devoted to the first step towards the calculation of QCD&EW corrections of $Z + j$ process. Namely, we compute all Feynman integrals relevant to this process in the m_Z/p_T expansion.

We note here, that both cases of interest, i.e. $H + j$ and $Z + j$ have a similar hierarchy of scales. Namely, in the case of $H + j$, we have $m_H < m_T \ll p_{T,H}$ while in the case of $Z + j$, we have $m_Z, m_W \ll p_{T,Z}$. As we will explain later, there exist small parameters in scattering processes which can be used to simplify the evaluation of relevant Feynman integrals and eventually corresponding scattering amplitudes.

Chapter 2

Production of Higgs boson with non-vanishing transverse momentum

In this chapter, we consider the Higgs+jet production. Our goal is to introduce two-loop form factors. To this end, we consider a differential cross section for the production of Higgs bosons and study its pertubative structure. In particular, we overview the next-to-leading order QCD contributions. Its consists of elastic and inelastic processes. Computing two-loop scattering amplitudes for the former processes is of main interest. We derive projector operators that are needed to extract unrenormalized two-loop form factors. Next, it is shown how regularize these form factors. Finally, we introduce helicity amplitudes.

2.1. The physical cross-section of the $H + j$ production

We will start this section by writing a differential cross section for the production of Higgs bosons in the collisions of protons $pp \to H + jet$. Thanks to *factorization theorem* [57], we can write

$$\frac{d\sigma_{pp}}{dp_T} = \sum_{i,j} \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 f_i(x_1, \mu_F^2) f_j(x_2, \mu_F^2) \frac{d\sigma_{ij\to H+j}}{dp_T}, \tag{2.1}$$

where $\sigma_{ij\to H+j}$ denotes a partonic cross-section for the process $ij \to H + X$. Indices $\{i, j\}$ denote colliding partons, i.e. quarks and antiquars of flavors $\{u,\ d,\ s,\ c,\ b\}$ and a gluon g. Functions $f_i(x, \mu_F^2)$ and $f_j(x, \mu_F^2)$ are parton distribution functions (PDFs), x_i is a fraction of a proton momentum P_i carried by a parton “i” into a hard process and μ_F^2 is a factorization scale.

The partonic cross-sections $\sigma_{ij\to H+j}$ is a function of Mandelstam variables $\{\hat{s},\ \hat{t},\ \hat{u}\}$, where we use hats to indicate that these variables are related to partonic processes. They can be defined as

$$\hat{s} = (p_i + p_j)^2,\ \hat{t} = (p_i - p_H)^2,\ \hat{u} = (p_j - p_H)^2. \tag{2.2}$$

Mandelstam variables can be expressed in the following way

$$\begin{aligned} \hat{s} &= x_1 x_2 s, \\ \hat{t} &= \frac{m_H^2 - \hat{s}}{2}(1 - \cos\theta), \\ \hat{u} &= \frac{m_H^2 - \hat{s}}{2}(1 + \cos\theta), \end{aligned} \tag{2.3}$$

where θ is the angle between the collision axis and the Higgs boson momentum in the partonic center-of-mass frame, and m_H is the mass of the Higgs boson.

We note that the following partonic processes contribute to $H+j$ production cross-section Eq. (2.1)

$$\begin{aligned} q\bar{q} &\to Hg, \quad \bar{q}q \to Hg, \\ qg &\to Hq, \quad gq \to Hq, \\ g\bar{q} &\to H\bar{q}, \quad \bar{q}g \to H\bar{q}, \\ gg &\to Hg. \end{aligned} \tag{2.4}$$

In the center-of-mass frame of the colliding partons we can write a fully differential partonic cross section as

$$\frac{d\sigma_{ij}}{d\Omega} = \frac{\hat{s} - m_H^2}{64\pi^2 \hat{s}^2 N_{ij}} \overline{\sum} \left|M_{ij}\right|^2 \Theta\left(\sqrt{\hat{s}} - \left(p_T + \sqrt{p_T^2 + m_H^2}\right)\right), \tag{2.5}$$

where Θ is the Heaviside function, p_T is the Higgs transverse momentum, M_{ij} is a scattering amplitude for a particle partonic process and N_{ij} is the colour-average factors. They read $N_{q\bar{q}} = N_{\bar{q}q} = N_c^2$, $N_{qg} = N_{gq} = N_{\bar{q}g} = N_{g\bar{q}} = N_c(N_c^2 - 1)$ and, finally, $N_{gg} = (N_c^2 - 1)^2$. The sum goes over polarizations and colours of the initial and final state particles

$$\overline{\sum} = \frac{1}{4} \sum_{pol} \sum_{col} . \tag{2.6}$$

The prefactor $1/4$ is the average factor for the initial-state polarizations.

Taking into account that the scattering angle can be expressed as

$$\cos\theta = \sqrt{1 - 4p_T^2 \hat{s}/(\hat{s} - m_H^2)^2}, \tag{2.7}$$

with p_T being the transverse momentum of the Higgs, we can write the (partonic) transverse momentum distribution

$$\frac{d\sigma_{ij}}{dp_T} = \frac{p_T}{8\pi\hat{s}\left|\hat{t} - \hat{u}\right| N_{ij}} \overline{\sum} (|M_{ij}|^2 + (\hat{t} \leftrightharpoons \hat{u}))\Theta\left(\sqrt{\hat{s}} - \left(p_T + \sqrt{p_T^2 + m_H^2}\right)\right), \tag{2.8}$$

where we took into account the fact that $\hat{t} - \hat{u}$ can take negative values.

2.2. Perturbative QCD for the H+j production

Partonic cross-sections described in the previous Section admit perturbation expansion in the strong coupling constant

$$\mathrm{d}\sigma_{ij} = \mathrm{d}\sigma_{ij}^{\mathrm{LO}} + \mathrm{d}\sigma_{ij}^{\mathrm{NLO}} + \ldots, \tag{2.9}$$

where $\sigma_{ij}^{\mathrm{LO}}$ is the leading order (LO) and $\sigma_{ij}^{\mathrm{NLO}}$ is the next-to-leading order contribution (NLO), respectively.

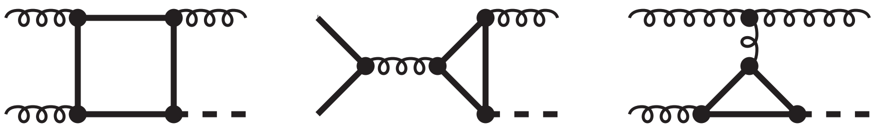


Figure 2.1.: Examples of Feynman diagrams at LO.

2.2.1. The $H + j$ production at leading order α_s

The production of the Higgs boson in association with a jet at leading order is a one-loop process. The production is facilitated by three partonic processes $gg \to Hg$, $qg \to Hq$ and $q\bar{q} \to Hg$. Some Feynman diagrams are shown in Fig. 2.1. The production cross section at LO was originally computed in Ref. [26] and later in Ref. [27]. Since, the Higgs boson and the top quark were not yet discovered, these papers focused on the dependence of the production cross section on the Higgs and the top mass. It was noticed there that one can simplify the analytic expressions either in the $m_t \to \infty$ or the $m_t \to 0$ limit.

2.2.2. The $H + j$ production at the next-to-leading order α_s

There are two contributions to the partonic cross sections for $H + j$ at the NLO QCD

$$\mathrm{d}\sigma_{ij}^{\mathrm{NLO}} = \mathrm{d}\sigma_{ij}^{\mathrm{real}} + \mathrm{d}\sigma_{ij}^{\mathrm{virtual}}, \tag{2.10}$$

where $\sigma_{ij}^{\mathrm{real}}$ is the real emission and $\sigma_{ij}^{\mathrm{virtual}}$ the virtual NLO corrections, respectively.

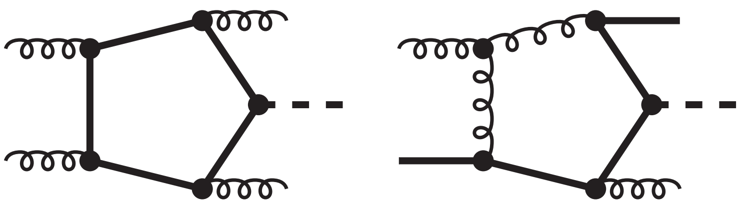


Figure 2.2.: Examples of Feynman diagrams for the real emission corrections.

Real emission corrections are described by one-loop inelastic processes $gg \to Hg+g$, $qg \to Hq+g$, and etc. Examples of Feynman diagrams that contribute to $\mathrm{d}\sigma_{ij}^{\mathrm{real}}$ are shown in Fig. 2.2. Computation of these processes requires evaluations of three-, four- and five-point one-loop Feynman integrals. The real emission corrections were computed in Ref. [58] and recomputed later in Ref. [44].

Elastic partonic processes $q\bar{q} \to Hg$, $gg \to Hg$, etc. receive the virtual corrections. Some examples of their Feynman diagrams are shown in Fig. 2.3 and Fig. 2.4. The analytical computations of these processes with full top mass dependence are not available. In Ref. [25], form factors for virtual processes were computed numerically using computer program *pySecDec* [59] that allowed to compute the Higgs p_T-distribution.

To evaluate NLO corrections to the Higgs production at high transverse momenta analytically, virtual corrections need to be computed. They can be calculated in a m_t/p_T expansion. Below, we explain how this can be done .

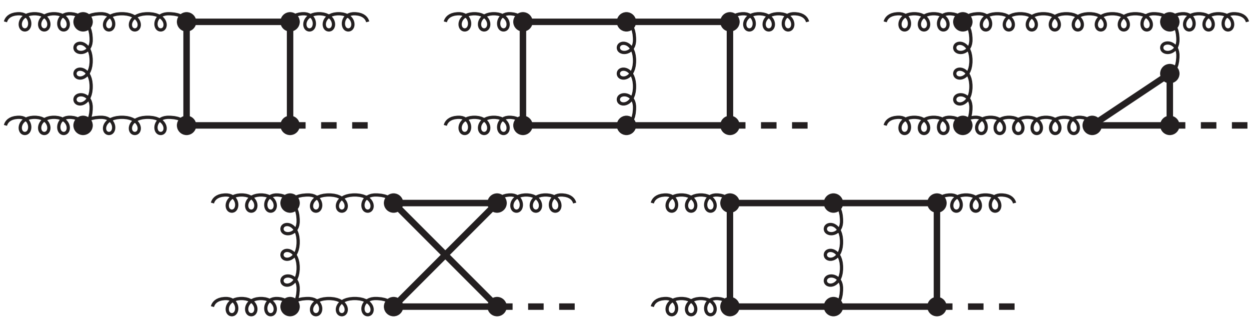


Figure 2.3.: Examples of two-loop Feynman diagrams for $gg \to Hg$

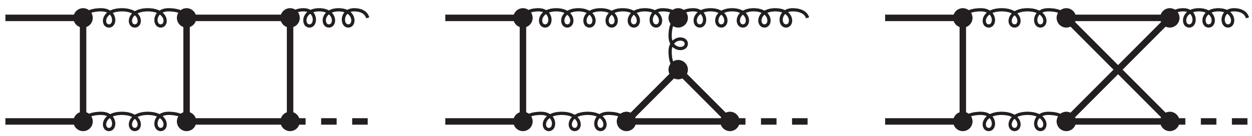


Figure 2.4.: Examples of two-loop Feynman diagrams for $qq \to Hg$

2.3. Form factors

In this section, we show how one can express scattering amplitudes M_{ij} as the Lorentz-scalar functions of Mandelstam variables that are known as form factors. For the process $pp \to H + jet$ this has already been done in Ref. [60]. However, we will follow closely a slightly different approach described in Refs. [45, 46].

Only two partonic amplitudes from Equation 2.4 need to be computed; the remaining ones can be obtained by crossings. We have chosen to consider the following processes

$$H(p_4) \to g_1(p_1) + g_2(p_2) + g_3(p_3), \tag{2.11}$$

$$H(p_4) \to q(p_1) + \bar{q}(p_2) + g(p_3), \tag{2.12}$$

to compute the two independent amplitudes. Later we analytically continue them to the required kinematic region. We will discuss such continuation in the Section where we introduce helicity amplitudes.

2.3.1. $H \to ggg$

We begin by considering the amplitude $H(p_4) \to g(p_1) + g(p_2) + g(p_3)$. We write it as

$$A(p_1^{a_1}, p_2^{a_2}, p_3^{a_3}) = f^{a_1 a_2 a_3} \epsilon_1^{\mu} \epsilon_2^{\nu} \epsilon_3^{\rho} \mathcal{A}_{\mu\nu\rho}(p_1, p_2, p_3, m_t), \tag{2.13}$$

where $f^{a_1 a_2 a_3}$ are structure constants of the $\mathfrak{su}(3)$ color algebra, $\epsilon_{1,2,3}$ are polarization vectors of gluons with momenta $p_{1,2,3}$, respectively, and μ, ν, ρ are Lorentz indices.

Since the amplitude $A(p_1^{a_1}, p_2^{a_2}, p_3^{a_3})$ is Lorentz invariant, the rank-3 tensor $\mathcal{A}_{\mu\nu\rho}(p_1, p_2, p_3, m_t)$ must contain only Lorentz covariant objects. Since we work in QCD, the amplitudes should be parity-conserving. Therefore, we can use Minkowski metric tensor $\eta^{\mu\nu}$ and four-vectors $p_1^{\mu}, p_2^{\nu}, p_3^{\rho}$

to construct a rank-3 tensor. We write

$$\mathcal{A}_{\mu\nu\rho}(s,t,u,m_t) = \sum_{\sigma} \sum_{\{i,j,k\}=1}^{3} F_{\sigma}^{ijk} p_{i,\sigma(\mu)} p_{j,\sigma(\nu)} p_{k,\sigma(\rho)} + \frac{1}{2} \sum_{\sigma} \sum_{i=1}^{3} F_{\sigma}^{i} \eta_{\sigma(\mu)\sigma(\nu)} p_{i,\sigma(\rho)}, \tag{2.14}$$

where σ denotes a set of all possible permutations of Lorentz indices and F_{σ}^{ijk} and F_{σ}^{i} are Lorentz scalars. These functions are called form factors.

The number of terms in Eq. (2.14) can be reduced by using the transversality conditions

$$\epsilon \cdot p_i, \quad i = 1, 2, 3. \tag{2.15}$$

This set of conditions is insufficient to ensure that a gluon has only two polarizations. In fact, it is necessary to impose an additional gauge-fixing condition for each gluon that we choose to be in the following cyclic form

$$\epsilon_1.p_2 = 0, \quad \epsilon_2.p_3 = 0, \quad \epsilon_3.p_1 = 0. \tag{2.16}$$

A discussion of gauge-fixing conditions can be found in Appendix B.1 and in many textbooks (see, for instance, section 25.4.3 in Ref. [61]).

As the result, only four terms in Eq. (2.14) provide non-vanishing contributions to $\mathcal{A}$. We write

$$\mathcal{A}^{\mu\nu\rho}(s,t,u,m_t) = F_1 \eta^{\mu\nu} p_2^{\rho} + F_2 \eta^{\mu\rho} p_1^{\nu} + F_3 \eta^{\rho\nu} p_3^{\mu} + F_4 p_3^{\mu} p_1^{\nu} p_2^{\rho}. \tag{2.17}$$

In spite of the relative simplicity of Eq. (2.14), a direct computation of the scattering amplitude will still be cumbersome due to gluon polarization vectors ϵ_i. For this reason, we would like to compute the form factors directly. To this end, we construct operators that project the amplitude on a particular form factor

$$F_j(s,t,u,m_t) = \sum_{\lambda_1,\lambda_2,\lambda_3} P_{j,\mu\nu\rho} (\epsilon^{\mu}_{1,\lambda_1})^* (\epsilon^{\nu}_{2,\lambda_2})^* (\epsilon^{\rho}_{3,\lambda_3})^* \epsilon^{\mu_1}_{1,\lambda_1} \epsilon^{\nu_1}_{2,\lambda_2} \epsilon^{\rho_1}_{3,\lambda_3} \mathcal{A}_{\mu_1\nu_1\rho_1}(s,t,u,m_t). \tag{2.18}$$

We note that in Eq. (2.18) the sum goes over polarizations of external gluons.

The polarization sums, consistent with gauge-fixing conditions shown in Eqs. (2.15, 2.16), read

$$\begin{aligned}
\sum_{\lambda_1} (\epsilon^{\mu}_{1,\lambda_1})^* \epsilon^{\mu_1}_{1,\lambda_1} &= -\eta^{\mu\mu_1} + \frac{p_1^{\mu} p_2^{\mu_1} + p_1^{\mu_1} p_2^{\mu}}{p_1 . p_2}, \\
\sum_{\lambda_2} (\epsilon^{\nu}_{1,\lambda_1})^* \epsilon^{\nu_1}_{2,\lambda_2} &= -\eta^{\nu\nu_1} + \frac{p_2^{\nu} p_3^{\nu_1} + p_2^{\nu_1} p_3^{\nu}}{p_2 . p_3}, \\
\sum_{\lambda_3} (\epsilon^{\rho}_{3,\lambda_3})^* \epsilon^{\rho_1}_{3,\lambda_3} &= -\eta^{\rho\rho_1} + \frac{p_3^{\rho} p_1^{\rho_1} + p_3^{\rho_1} p_1^{\rho}}{p_3 . p_1}.
\end{aligned} \tag{2.19}$$

Up to this point, we have not mentioned the dimensionality of Minkowski space. To use dimensional regularization, it must be treated as d-dimensional space. With this in mind, we can make an Ansatz for the projectors P_j using tensors that enter the definition of the amplitude Eq. (2.17). We write

$$P_j^{\mu\nu\rho} = \frac{1}{d-3} \Big(c_{1,j} \eta^{\mu\nu} p_2^{\rho} + c_{2,j} \eta^{\mu\rho} p_1^{\nu} + c_{3,j} \eta^{\rho\nu} p_3^{\mu} + c_{4,j} p_3^{\mu} p_1^{\nu} p_2^{\rho} \Big), \quad j = 1,\ 2,\ 3,\ 4. \tag{2.20}$$

In Eq. (2.20), $c_{i,j}$ are unknown functions of Mandelstam variables. Upon inserting the Ansatz Eq. (2.20) into Eq. (2.18) we obtain a system of linear equations for $c_{j,\,i}$. We solve it to find

$$\begin{array}{llll} c_{1,1} = \frac{t}{su}, & c_{2,1} = 0, & c_{3,1} = 0, & c_{4,1} = -\frac{1}{su}, \\ c_{1,2} = 0, & c_{2,2} = \frac{u}{st}, & c_{3,2} = 0, & c_{4,2} = -\frac{1}{st}, \\ c_{1,3} = 0, & c_{2,3} = 0, & c_{3,3} = \frac{s}{tu}, & c_{4,3} = -\frac{1}{tu}, \\ c_{1,4} = -\frac{1}{su}, & c_{2,4} = -\frac{1}{st}, & c_{3,4} = -\frac{1}{tu}, & c_{4,4} = \frac{d}{stu}. \end{array} \tag{2.21}$$

There are other symmetries which will help us to simplify the structure of the form factors; we discuss them later on in this chapter when we talk about helicity amplitudes. To finalize this section we note that form factors admit perturbative expansion

$$F_j^{\mathrm{un}}(s,t,u,m_{t,bare}) = \sqrt{\frac{\alpha_0}{\pi}}\Big(F_j^{(1),\,\mathrm{un}} + (\frac{\alpha_0}{2\pi})F_j^{(2),\,\mathrm{un}} + O(\alpha^2)\Big), \tag{2.22}$$

where F_j^{un} denotes the unrenormalized form factors, α_0 is a bare QCD coupling constant and $m_{t,bare}$ is a bare top quark mass.

2.3.2. $H \longrightarrow q\bar{q}g$

We study the partonic process $H(p_4) \longrightarrow q(p_1) + \bar{q}(p_2) + g(p_3)$ in a way that is similar to the discussion in the previous Section. We write the scattering amplitude

$$B(p_1^j, p_2^i, p_3^a) = iT_{ij}^a \epsilon^\mu \bar{u}(p_1)\mathcal{B}_\mu(s,t,u,m_t)v(p_2), \tag{2.23}$$

where i, j are quark color indices, a is the color index of a gluon, μ is Lorentz index, ϵ is the gluon polarization vector, $\bar{u}$, v are quark spinors and $\mathcal{B}_\mu$ is a combination of Dirac matrices γ^μ. We will use a short-hand notation for the matrix element $\bar{u}(p_1)\mathcal{B}_\mu(s,t,u,m_t)v(p_2) = \langle \mathcal{B}_\mu(s,t,u,m_t)\rangle$.

The amplitude $B(p_1^j, p_2^i, p_3^a)$ is Lorentz-invariant and obeys the Ward identity

$$p_3^\mu \langle \mathcal{B}_\mu(s,t,u,m_t)\rangle = 0. \tag{2.24}$$

Spinors $\bar{u}$, v satisfy massless Dirac equations $\not{p}_1 u(p_1) = 0$, $\not{p}_2 v(p_2) = 0$. Additionally, we impose parity and helicity conservation on the amplitude. Then, the most general ansatz for $\mathcal{B}_\mu(s,t,u,m_t)$ reads

$$\langle \mathcal{B}_\mu\rangle = F_1\langle \not{p}_3\rangle p_1^\mu + F_2\langle \not{p}_3\rangle p_2^\mu + F_3\langle \gamma^\mu\rangle, \tag{2.25}$$

where $\not{p}_i = p_i^\mu \gamma_\mu$ and F_1, F_2, F_3 are the form factors.

We use the Ward identity Eq. (2.24) to obtain the following relation among form factors

$$F_3 = -p_1 \cdot p_3 F_1 - p_2 \cdot p_3 F_2. \tag{2.26}$$

Additionally, imposing the gauge conditions for the gluon polarization vector $\epsilon \cdot p_3$, $\epsilon \cdot p_1$ leads to the following expression

$$\langle \mathcal{B}_\mu\rangle = -p_1 \cdot p_3 F_1 \langle \gamma^\mu\rangle + F_2(\langle \not{p}_3\rangle p_2^\mu - p_2 \cdot p_3\langle \gamma^\mu\rangle). \tag{2.27}$$

The polarization sum consistent with chosen gauge fixing conditions reads

$$\sum_{\lambda}(\epsilon_{\lambda}^{\mu})^{*}\epsilon_{\lambda}^{\mu_1}) = -\eta^{\mu\mu_1} + \frac{p_1^{\mu}p_3^{\mu_1} + p_1^{\mu_1}p_3^{\mu}}{p_1{\cdot}p_3}. \tag{2.28}$$

We also compute spinor sums as

$$\sum_{s=1}^{2} u_s(p_1)\bar{u}_s(p1) = \not{p}_1, \quad \sum_{s=1}^{2} v_s(p_1)\bar{v}_s(p1) = \not{p}_2. \tag{2.29}$$

To construct projection operators, we use tensors from Eq. (2.27) and write[1]

$$\langle P_{j,\mu}\rangle = -p_1 \cdot p_3 c_{1,j}\langle\gamma_{\mu}\rangle + c_{2,j}(\langle\not{p}_3\rangle p_{2,\mu} - p_2 \cdot p_3\langle\gamma_{\mu}\rangle). \tag{2.30}$$

Projectors from Eq. (2.30) are used to extract form factors using the following equation

$$F_j(s,t,u,m_t) = \sum_{\lambda}(\epsilon_{\lambda}^{\mu})^{*}\epsilon_{\lambda}^{\mu_1}\,\mathrm{Tr}(P_{j,\mu}\not{p}_1\mathcal{B}_{\mu_1}\not{p}_2) \tag{2.31}$$

where the sum goes over polarization and we have used Eq. (2.29).

Solving Eq. (2.31) for $c_{i,j}$ yields

$$\begin{aligned} c_{1,1} &= \tfrac{d-2}{2(d-3)st^2}, \quad c_{2,1} = \tfrac{4-d}{2(d-3)stu}, \\ c_{1,2} &= \tfrac{4-d}{2(d-3)stu}, \quad c_{2,2} = \tfrac{d-2}{2(d-3)su^2}. \end{aligned} \tag{2.32}$$

The pertubative expansion of form factors for $H \to q\bar{q}g$ partonic process has the same form as Eq. (2.22)

$$F_j^{\mathrm{un}}(s,t,u,m_{t,bare}) = \sqrt{\frac{\alpha_0}{\pi}}\Big(F_j^{(1),\,\mathrm{un}} + (\frac{\alpha_0}{2\pi})F_j^{(2),\,\mathrm{un}} + O(\alpha^2)\Big). \tag{2.33}$$

2.4. Renormalization and regularization

The form factors discussed in sections Section 2.3.2 and Section 2.3.1 are unrenormalized. To renormalize them, we follow Refs. [1, 60]. We use dimensional regularization to regularize divergences which are present in the original form factors. Such divergences appear as poles in the dimensional regularization parameter ϵ and correspond to ultraviolet (UV) and/or infra-red (IR) divergences as will be discussed later in this thesis. Renormalization of form factors is performed in two steps. As the first step, we renormalize UV divergences in Eq. (2.33) and Eq. (2.22)

$$F_j^{\mathrm{UV}}(s,t,u,m_t) = \sqrt{\frac{\alpha_s^3}{\pi S_\epsilon^3}}\left[F_j^{(1),\mathrm{UV}} + \left(\frac{\alpha_s}{2\pi}\right)F_j^{(2),\mathrm{UV}} + O\left(\alpha_s^3\right)\right]. \tag{2.34}$$

The bare coupling constant and the top quark mass are expressed in terms of renormalized parameters. Additionally, we include for each external gluon the wave-function renormalization

[1] We have transpose dirac brackets, i.e. $\langle P_{i,\mu}\rangle = \bar{v}(p_2)P_{i,\mu}u_{p_1}$.

factor. The strong coupling constant is renormalized in the mixed scheme, i.e. contributions of N_f massless quarks are renormalized in the $\overline{\text{MS}}$-scheme, while top quark contributions are subtracted at zero momentum. The top quark mass is renormalized in an on-shell scheme. The relations between bare and renormalized coupling constant and the top quark mass read

$$\alpha_0 \mu_0^{2\epsilon} S_\epsilon = \alpha_s \mu_R^{2\epsilon} \left[1 - \frac{1}{\epsilon} \left(\beta_0 + \delta_w \right) \left(\frac{\alpha_s}{2\pi} \right) + O\left(\alpha_s^2 \right) \right], \tag{2.35}$$

$$m_{t,0} = m_t \left[1 + \left(\frac{\alpha_s}{2\pi} \right) \delta_m + O\left(\alpha_s^2 \right) \right], \tag{2.36}$$

where $S_\epsilon = (4\pi)^\epsilon e^{-\epsilon\gamma_E}$ is the typical phase-space volume, γ_E is the Euler constant, $\beta_0 = 11/6 C_A - 2/3 T_R N_f$ with $T_R = 1/2$ and $C_A = N_c$ is the number of colors. The gluon wave-function and mass renormalization constants are

$$\begin{aligned} \sqrt{Z_A} &= 1 + \frac{1}{2} \left(\frac{\alpha_s}{2\pi} \right) \delta_w + O\left(\alpha_s^2 \right), \\ \delta_w &= -2/3 T_R \left(m_t^2 / \mu_R^2 \right)^{-\epsilon}, \\ \delta_m &= C_F \left(\frac{m_t^2}{\mu_R^2} \right)^{-\epsilon} \left(-\frac{3}{2\epsilon} - 2 + O(\epsilon) \right). \end{aligned} \tag{2.37}$$

Following the described procedure, we express the UV-renormalized form factors in terms of bare ones as

$$\left(F^i \right)_j^{(1),\text{UV}} = \left(F^i \right)_j^{(1),\text{un}}, \tag{2.38}$$

$$\left(F^i \right)_j^{(2),\text{UV}} = S_\epsilon^{-1} \left(F^i \right)_j^{(2),\text{cun}} - \left(\frac{3\beta_0}{2\epsilon} + \delta_{i,q} \frac{\delta_w}{\epsilon} \right) \left(F^i \right)_j^{(1),\text{un}} + m_t \frac{d \left(F^i \right)_j^{(1),\text{un}}}{d m_t} \delta_m, \tag{2.39}$$

where $i = g, q$ denotes the $H \to ggg$ and $H \to q\bar{q}g$ form factors, respectively. Note that the LO contributions $(F_j^i)^{(1),\text{un}}$ has no poles in ϵ, even though it is an one-loop process.

The UV-renormalized F_j^{UV} form factors still contain IR divergences. These are infrared and collinear poles that appear in the virtual amplitude. They cancel once elastic (i.e. $gg \to Hg$) and inelastic partonic (i.e. $gg \to Hg + g$) processes are combined to compute physical cross-sections. The structure of IR singularities is universal [62]. As the result, it is possible to separate the virtual amplitude into IR-divergent and finite parts. We write[2]

$$\left(F^i \right)_j^{(1),\text{UV}} = \left(F^i \right)_j^{(1),\text{fin}}, \quad \left(F^i \right)_j^{(2),\text{UV}} = \mathrm{I}_1^i(\epsilon) \left(F^i \right)_j^{(1),\text{UV}} + \left(F^i \right)_j^{(2),\text{fin}}, \tag{2.40}$$

where again $i = q, g$ and $\mathrm{I}_1^{q,g}(\epsilon)$ are the so-called Catani operators. Here, we derive these operators for two processes $H \to g(p_1)g(p_2)g(p_3)$ and $H \to q(p_1)\bar{q}(p_2)g(p_3)$. First, we write the explicit form of Catani operators [62][3]

$$\mathrm{I}_1^i = \frac{1}{2} \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \sum_j \frac{1}{\mathbf{T}_j^2} \mathcal{V}_j^{\text{sing}} \sum_{j \neq k} \mathbf{T}_j \cdot \mathbf{T}_k \left(\frac{2 p_j \cdot p_k}{\mu_R^2} e^{i\pi\lambda_{jk}} \right)^{-\epsilon}, \tag{2.41}$$

[2] We note again that the $H + j$ at LO is a one-loop process. Hence, we do not consider the two-loop insertion operators I_2^i.

[3] We follow closely the notation presented in S. Catani's paper [62]. We redirect a reader to this paper for definitions of all quantities that we use here.

where $\mathbf{T}$ is a color charge, $\mathcal{V}_j^{\text{sing}}$ is a singular function, $i = g, q$ and $j, k = g, q, \bar{q}$; a unitary phase factor $e^{i\pi\lambda_{jk}}$ is -1 if all partons are outgoing/incoming, and 0 otherwise. We need to calculate various products of $\mathbf{T}$. To do this, we use the color conservation and commutation relations

$$\sum_j^m \mathbf{T}_j \left|M_m(p_1, p_2, \ldots, p_m)\right\rangle = 0, \tag{2.42}$$

$$\mathbf{T}_j \cdot \mathbf{T}_k = \mathbf{T}_k \cdot \mathbf{T}_j, \quad \text{if } j \neq k, \tag{2.43}$$

$$\mathbf{T}_j^2 = C_j, \tag{2.44}$$

where $|M\rangle_m$ is a color singlet amplitude for a production of m partons; $C_j = C_A$ if j is a gluon and $C_j = C_F = (N_c^2 - 1)/2N_c$ if j is a quark or an antiquark. Now, it is straightforward to see that for $m = 3$

$$2\mathbf{T}_1 \cdot \mathbf{T}_2 = -\mathbf{T}_1^2 - \mathbf{T}_2^2 - \mathbf{T}_3^2, \tag{2.45}$$

and all possible permutations of partons in Eq. (2.45). It follows then

$$\mathbf{T}_j \cdot \mathbf{T}_k = -\frac{C_A}{2},\ \mathbf{T}_q \cdot \mathbf{T}_{\bar{q}} = \frac{1}{2}(C_A - 2C_F) = \frac{1}{2C_A}, \tag{2.46}$$

for all possible combinations of $g, q, \bar{q}$ except when $j = q$ and $k = \bar{q}$.

What remains to calculate is singular functions. They are given by [62]

$$\mathcal{V}_g = C_A \frac{1}{\epsilon^2} + \beta_0 \frac{1}{\epsilon}, \tag{2.47}$$

$$\mathcal{V}_{q,\bar{q}} = C_F\left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon}\right). \tag{2.48}$$

We use Eq. (2.41) to obtain Catani operators for two cases of interest ($H \longrightarrow ggg$ and $H \longrightarrow q\bar{q}g$)

$$\mathrm{I}_1^g = -\frac{e^{\epsilon\gamma_E}}{2\Gamma(1-\epsilon)} \mathcal{V}_g^{\text{sing}} \left(\left(-\frac{s}{\mu_R^2}\right)^{-\epsilon} + \left(-\frac{t}{\mu_R^2}\right)^{-\epsilon} + \left(-\frac{u}{\mu_R^2}\right)^{-\epsilon} \right), \tag{2.49}$$

$$\begin{aligned} \mathrm{I}_1^q = \frac{e^{\epsilon\gamma_E}}{2\Gamma(1-\epsilon)} &\left(\left(\frac{\mathcal{V}_q^{\text{sing}}}{\mathbf{T}_q^2} + \frac{\mathcal{V}_g^{\text{sing}}}{\mathbf{T}_g^2} \right) \mathbf{T}_q \cdot \mathbf{T}_g \left(\left(-\frac{t}{\mu_R^2}\right)^{-\epsilon} + \left(-\frac{u}{\mu_R^2}\right)^{-\epsilon} \right) \right. \\ &\left. 2\mathbf{T}_q \cdot \mathbf{T}_{\bar{q}} \frac{\mathcal{V}_q^{\text{sing}}}{\mathbf{T}_q^2} \left(-\frac{s}{\mu_R^2}\right)^{-\epsilon} \right). \end{aligned} \tag{2.50}$$

We insert Eqs. (2.46, 2.47) into Eq. (2.49) and obtain

$$\mathrm{I}_1^g(\epsilon) = -\frac{C_A e^{\epsilon\gamma_E}}{2\Gamma(1-\epsilon)} \left(\frac{1}{\epsilon^2} + \frac{\beta_0}{C_A} \frac{1}{\epsilon} \right) \left(\left(-\frac{s}{\mu_R^2}\right)^{-\epsilon} + \left(-\frac{t}{\mu_R^2}\right)^{-\epsilon} + \left(-\frac{u}{\mu_R^2}\right)^{-\epsilon} \right), \tag{2.51}$$

$$\begin{aligned} \mathrm{I}_1^q(\epsilon) = -\frac{e^{\epsilon\gamma_E}}{2\Gamma(1-\epsilon)} &\left(C_A \left(\frac{1}{\epsilon^2} + \frac{3}{4\epsilon} + \frac{\beta_0}{2C_A\epsilon} \right) \left(\left(-\frac{t}{\mu_R^2}\right)^{-\epsilon} + \left(-\frac{u}{\mu_R^2}\right)^{-\epsilon} \right) \right. \\ &\left. - \frac{1}{C_A} \left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} \right) \left(-\frac{s}{\mu_R^2}\right)^{-\epsilon} \right). \end{aligned} \tag{2.52}$$

Since, in Eq. (2.40), one loop amplitudes are multiplied by singular Catani operators, we require the one-loop amplitudes to order $O(\epsilon^2)$ in dimensional regularization parameter. The infrared parts of Eq. (2.39) are listed in Appendix E which we denoted there as $F^{\rm IR}_{q,g}$.

2.5. Helicity amplitudes

In the section we rewrite scattering amplitudes Eqs. (2.17, 2.25) using the helicity basis described in Appendix B.1. To do this, we use the helicity basis for polarization vectors and quark spinors to write helicity amplitudes

$$\begin{aligned}\mathcal{A}^g_{\lambda_1\lambda_2\lambda_3}(s,t,u,m_t) &= \epsilon^{\mu}_{1,\lambda_1}(p_1)\,\epsilon^{\nu}_{2,\lambda_2}(p_2)\,\epsilon^{\rho}_{3,\lambda_3}(p_3)\,\mathcal{A}^g_{\mu\nu\rho}(s,t,u,m_t)\,,\\ \mathcal{A}^q_{\lambda_1\lambda_2\lambda_3}(s,t,u,m_t) &= \epsilon^{\mu}_{3,\lambda_3}(p_3)\,\overline{u}_{\lambda_1}(p_1)\,\mathcal{A}^q_{\mu}(s,t,u,m_t)\,v_{\lambda_2}(p_2)\,,\end{aligned} \tag{2.53}$$

where λ_1, λ_2 and λ_3 take two values $\{+,-\}$.[4] It is readily seen that in order to compute form factors for $H \to ggg$, we need 8 helicity amplitudes in total. However in practise, we need only two, since other helicity amplitudes can be evaluated using charge and parity conjugation. In the case of $H \to q\bar{q}g$, there are only four helicity configurations which have to be computed since QCD interactions conserve quark helicity. In other words, helicity of an outgoing quark has to be opposite to helicity of an outgoing anti-quark. In the case of $H \to q\bar{q}g$, there is only one independent helicity configuration due to parity and charge conjugation. We choose to compute the following amplitudes

$$\begin{aligned}\mathcal{A}^g_{+++}(s,t,u,m_t) &= \frac{s}{\sqrt{2}\langle 12\rangle\langle 23\rangle\langle 31\rangle}\Omega^g_{+++}(s,t,u,m_t)\,,\\ \mathcal{A}^g_{+-+}(s,t,u,m_t)\,, &= \frac{[13]^3}{\sqrt{2}[12][32]s}\Omega^g_{+-+}(s,t,u,m_t)\,,\\ \mathcal{A}^q_{-+}(s,t,u,m_t) &= \frac{1}{\sqrt{2}}\frac{[23]^2}{[12]s}\Omega^q_{-++}(s,t,u,m_t)\,.\end{aligned} \tag{2.54}$$

Spinor products are defined in B.1. Functions $\Omega_{\lambda_1\lambda_2\lambda_3}$ are helicity coefficients given by linear combinations of form factors Section 2.3

$$\Omega^g_{+++} = u\left(F^g_1 + \frac{t}{u}F^g_2 + \frac{t}{s}F^g_3 + \frac{t}{2}F^g_4\right), \quad \Omega^g_{+-+} = \frac{-s^2}{t}\left(F^g_2 + \frac{u}{2}F^g_4\right), \quad \Omega^q_{-++} = s^2F^q_1. \tag{2.55}$$

All other helicity amplitudes are obtained by permuting external legs and complex conjugation

$$\begin{aligned}\mathcal{A}^g_{++-}(p_1,p_2,p_3) &= \mathcal{A}^g_{+-+}(p_1,p_3,p_2)\,,\\ \mathcal{A}^g_{+--}(p_1,p_2,p_3) &= \left[\mathcal{A}^g_{+-+}(p_2,p_1,p_3)\right]^*,\\ \mathcal{A}^q_{+-+}(p_1,p_2,p_3) &= \mathcal{A}^q_{-++}(p_2,p_1,p_3)\,,\\ \mathcal{A}^i_{\lambda_1\lambda_2\lambda_3}(p_1,p_2,p_3) &= \left[\mathcal{A}^i_{(-\lambda_1)(-\lambda_2)(-\lambda_3)}(p_1,p_2,p_3)\right]^*.\end{aligned} \tag{2.56}$$

We note here that only spinor products must be complex-conjugated but not the form factors $F^{g,q}_i$.

[4] Usually, $\{+,-\}$ is used for gauge particles, while $\{R, L\}$ for fermion particles. We use $\{+,-\}$ for both kinds of particles.

Functions Ω^i admit pertubative expansion in the strong coupling constant α_s

$$\Omega^i = \frac{m_t^2}{v}\sqrt{\frac{\alpha_s^3}{\pi}}\left[\Omega^{i,(1l)} + \frac{\alpha_s}{2\pi}\Omega^{i,(2l)} + \mathcal{O}\left(\alpha_s^2\right)\right], \tag{2.57}$$

where we extract a common prefactor m_t^2/v such that we have dimensionless one-loop and two-loop helicity coefficients.

To proceed further, we need to compute scalar Feynman integrals which appear in form factors defined in Section 2.3. In the next chapter, we discuss how relevant Feynman integrals can be computed.

Chapter 3

Feynman integrals

In this section, we consider Feynman integrals. We discuss their properties and, in particular, focus on their singularity structure. We show how one can efficiently regularize such integrals using dimensional regularization. We describe the Feynman-parameter technique and explain how to use it to integrate over loop momenta. We provide the definition of Symanzik polynomials and outline their properties. We consider linear relations among Feynman integrals that follow from the so-called integration-by-parts identities (IBPs) and discuss the concept of basis (master) integrals.

As we discussed in Section 2.3, we need to compute form factors. To obtain them, we proceed as follows. First, we generate Feynman diagrams using computer algebra programs such as *QGRAF* [63] and *FeynArts* [64]. Second, we use Feynman rules to write amplitudes in *FORM*- or *Mathematica*-readable form [65, 66]. Third, as we discussed in Sections 2.3.1 and 2.3.2, we apply projectors to extract the form factors. We compute traces of Dirac matrices, contract Lorentz indices and perform algebraic manipulations using *FORM* and *Mathematica*.

Once all this is accomplished, the form factors are written as linear combinations of products of rational functions $C_{i,j}$ of the Mandelstam variables and integrals I_j over loop momenta

$$F_i = \sum C_{ij} I_j. \tag{3.1}$$

In the next section we discuss how these integrals are computed.

3.1. Feynman integrals: definition

Feynman integrals have been known since long ago, and their definition and properties are discussed in many textbooks [61, 67, 68]. Since the focus of this thesis is the evaluation of Feynman integrals $H + j$ and $Z + j$ processes, we describe general properties of Feynman integrals.

A L-loop scalar Feynman integral in d dimensions with $E + 1$ external legs is defined as

$$I\left(p_1, \ldots, p_{E+1}; a_1, \ldots, a_N; d\right) = \int \prod_{i=1}^{L} \frac{\mathrm{d}^d k_i}{(2\pi)^d} \frac{1}{D_1^{a_1} \cdots D_N^{a_N}} \tag{3.2}$$

where $N = L(L+1)/2 + LE$; $p_1, \ldots p_{E+1}$ are external momenta, $k_1 \ldots k_L$ are loop momenta, $D_j = (q_j^2 - m_j^2 + i0)$ are inverse propagators and the $\{q_j\}$ are a linear combinations of loop and external

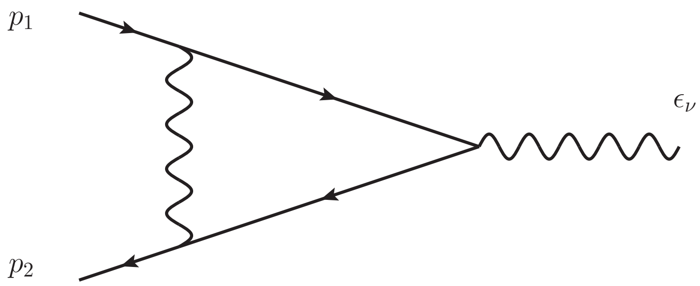


Figure 3.1.: One loop vertex correction

momenta that flows along a line j. We denote masses of internal particles as m_j. Indices $a_1 \dots a_N$ can take arbitrary integer values. It is usually said that, Eq. (3.2) defines a *family* of Feynman integrals.

We need Feynman integrals in four space-time dimensions. However, it is easy to see that in $d = 4$ many of such integrals diverge and are not well-defined. There are two main types of divergences which arise in the computation of Feynman integrals: ultraviolet (UV) and infrared (IR) ones. To explain what they are, it is instructive to consider one-loop electron vertex function $\Gamma(p_1, p_2, s)$ in quantum electrodynamics, c.f. Fig. 3.1. The external electrons are taken to be on-shell, i.e. $p_1^2 = m^2, p_2^2 = m^2$. We write down the corresponding expression for this diagram using QED Feynman rules that can be found e.g. in Ref. [61]. We use Feynman gauge for the internal photon. The expression reads

$$\Gamma^\mu(p_1, p_2, s) = -e^3 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{\bar{v}(p_2)\gamma_\mu(\not{k} - \not{p}_2)\gamma^\nu(\not{k} + \not{p}_1)\gamma^\mu u(p_1)}{(k^2 + i0)((k - p_2)^2 - m^2 + i0)((k + p_1)^2 - m^2 + i0)}. \tag{3.3}$$

First, we discuss the UV divergence. This divergence arises when all components of the loop momentum become very large. Studying this limit, we first neglect all external momenta relative to the loop momentum and then Wick-rotate the time component of the loop momentum, $k^0 \to ik^0$. In the resulting Euclidean space, we introduce the four-dimensional spherical coordinates $\mathrm{d^4k = r^3 dr dS_3}$ with $\mathrm{d}S_3$ being a surface element of the unit sphere in four dimensions. The loop momentum reads $\mathrm{k = r\hat{k}}$ where $\mathrm{\hat{k}}$ is a unit vector and $\mathrm{r = |k|}$ is the length of the loop momentum. The integral reduces to

$$\Gamma^\mu_{\mathrm{UV}}(s) \approx -e^3 \int \frac{\mathrm{d}S_3}{(2\pi)^4} \frac{\bar{v}(p_2)\gamma_\mu\hat{\not{k}}\gamma^\nu\hat{\not{k}}\gamma^\mu u(p_1)}{(\hat{k}^2)^3} \int_\Lambda^\infty \frac{\mathrm{d}r}{r}, \tag{3.4}$$

where the integration with respect to r diverges logarithmically in the $r \to \infty$ limit and Λ is a cut-off.

IR divergences arise when all components of the loop momentum become small. In this $k \to 0$ limit, the integral becomes

$$\Gamma^\mu_{\mathrm{IR}}(p_1, p_2, s) \approx -e^3 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{\bar{v}(p_2)\gamma_\mu(\not{p}_2)\gamma^\nu(\not{p}_1)\gamma^\mu u(p_1)}{(k^2 + i0)(k \cdot p_2 + i0)(k \cdot p_1 + i0)}. \tag{3.5}$$

A power-counting argument implies that the integral is logarithmically divergent.

The collinear divergences are IR divergences of a different type. They appear when a loop momentum is collinear to an external *light-like* momentum. To see now it happens, we consider the integral Eq. (3.6) in case when $m^2 = 0$, $p_{1,2} = 0$. For the sake of clarity, we keep only denominators and omit numerators and prefactors. The expression reads

$$\Gamma_{\mathrm{IR}}(p_1, p_2, s) \sim \int \frac{\mathrm{d}^4 k}{(k^2 + i0)(k^2 + 2p_1 \cdot k + i0)(k^2 - 2p_2 \cdot k + i0)}. \tag{3.6}$$

Following [69], we compute the integral using Cauchy theorem. The residue at $k^0 = |\vec{k}| + i0$ reads

$$\mathrm{Res}_{k^0=|\vec{k}|+i0}\left(\frac{1}{k^2(k^2 + 2p_1 \cdot k)(k^2 - 2p_2 \cdot k)}\right) \propto \frac{1}{|\vec{k}|(p_1 \cdot k)(-p_2 \cdot k)}. \tag{3.7}$$

The scalar products at $k^0 = |\vec{k}|$ read $p_i \cdot k = p_i^0 |\vec{k}|(1 - \cos\theta_i)$ where θ_i is the angle between the loop momentum and the corresponding external momentum p_i. For small values of θ the integrand behaves as $\mathrm{d}\cos\theta/(1 - \cos\theta) \longrightarrow \mathrm{d}\theta/\theta$ which implies that the integral diverges logarithmically at $\theta = 0$

3.2. Dimensional regularization

We have shown on a simple example that Feynman integrals have a non-trivial singularity structure. Since physical results have to be free of singularities, we have to regularize and extract singularities that appear at intermediate stages of calculations. We turn to a discussion of a possible way to do so.

The dimensional regularization scheme [70] is based on the observation that Feynman integrals can be defined to be analytical functions of the space-time dimension parameter d . It has been proven [71] that Feynman integrals in complex space-time dimensions are mathematically well-defined. While the general discussion of this proof goes beyond the scope of this thesis, we list only requirements which must be imposed on dimensional regularized integrals to make the scheme self-consistent [67]. They are

- Linearity: For any two complex numbers a and b

$$\int \mathrm{d}^d\mathrm{k}(af(\mathrm{k}) + bg(\mathrm{k})) = a\int \mathrm{d}^d\mathrm{k}f(\mathrm{k}) + b\int \mathrm{d}^d\mathrm{k}g(\mathrm{k}). \tag{3.8}$$

- Scaling: For any α

$$\int \mathrm{d}^d\mathrm{k}f(\alpha\mathrm{k}) = \alpha^{-d}\int \mathrm{d}^d\mathrm{k}f(\mathrm{k}). \tag{3.9}$$

- Translation invariance: for any vector p

$$\int \mathrm{d}^d\mathrm{k}f(\mathrm{k} + p) = \int \mathrm{d}^d\mathrm{k}f(\mathrm{k}). \tag{3.10}$$

- Differentiation and integration commute

$$\frac{\partial}{\partial p_\mu}\int \mathrm{d}^d\mathrm{k}f(\mathrm{k}, p, \ldots) = \int d^\mathrm{d}\mathrm{k}\frac{\partial}{\partial p_\mu}f(\mathrm{k}, p, \ldots), \tag{3.11}$$

The immediate consequences of imposing such requirements are the following expressions

- Any scaleless integral vanishes
$$\int \mathrm{d}^d\mathrm{k}(\mathrm{k}^2)^\alpha = 0. \tag{3.12}$$
- Convergence at infinity
$$\int \mathrm{d}^d\mathrm{k}\frac{\partial}{\partial \mathrm{k}^\mu} f(\mathrm{k}) = 0. \tag{3.13}$$

We will show now that divergences discussed in the previous Section emerge as $(d-4)$ poles in the dimensional regularization scheme. In the case of UV divergence, the integration measure of generalized spherical coordinates becomes $d^dk = r^{d-1}drd\Omega_{d-1}$. Then, in the $r \to \infty$ limit, the integral Eq. (3.3) reduces to

$$\Gamma^\mu_{UV}(s;d) = -e^3 \int \frac{\mathrm{d}S_{d-1}}{(2\pi)^4} \frac{\bar{v}(p_2)\gamma_\mu(\hat{\not{k}})\gamma^\nu(\hat{\not{k}})\gamma^\mu u(p_1)}{(\hat{k}^2)^3} \int_\Lambda^\infty r^{d-5}\mathrm{d}r. \tag{3.14}$$

It follows that the integration over r is now regularized and it can be easily performed (by choosing $\mathrm{Re}(d) < 4$)
$$I = \int_\Lambda^\infty r^{d-5}\mathrm{d}r = -\frac{\Lambda^{d-4}}{d-4}. \tag{3.15}$$

Since, our goal is to evaluate physical observables in four dimensions, we have to expand the results of the integration around $d = 4$. We write $d = 4 - 2\epsilon$ and expand Eq. (3.15) in ϵ. The result reads
$$I = \frac{1}{2\epsilon} + \log(\Lambda) + O(\epsilon), \tag{3.16}$$
which is divergent in the $\epsilon \to 0$ limit . This divergence is exactly the UV divergence we have observed before, but now regulated dimensionally.

3.3. Parametric Representation

In this section, we introduce the technique of Feynman parameters, which turns out to be very useful in combination with dimensional regularization. The idea is to combine denominators in the following way

$$\prod_{j=1}^N \frac{1}{(q_j - m_j^2)^{a_j}} = \frac{\Gamma(a}{\Gamma(a_1)\Gamma(a_2)\ldots\Gamma(a_N)} \int_{x_j\geq 0} \mathrm{d}^N x\delta\left(1 - \sum_{j=1}^N x_j\right) \frac{\prod_{j=1}^N x_j^{a_j-1}}{\left(\sum_{j=1}^N x_j(q_j - m_j^2)\right)^a}, \tag{3.17}$$

where, $a = \sum_{j=1}^N a_j$, Γ is the Euler gamma function and x_j are the Feynman parameters. The integration boundaries of the Feynman parameters are fixed by the delta function. Other quantities have been already defined in Section 3.1. The Feynman prescription $p^2 - m^2 \to p^2 - m^2 + i0$ is omitted for brevity.

Eq. (3.17) can be modified. Indeed, according to Cheng-Wu theorem [72] the integration with respect to Feynman parameters can be performed in various ways. We adopt the definition of this

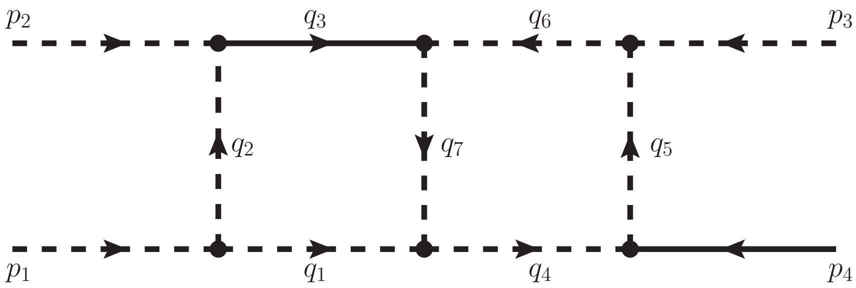


Figure 3.2.: One of the master integral for $Z + j$ production. In this chapter, we use the following notation for the Feynman diagrams: a dashed line corresponds to a massless particle while a solid line corresponds to a massive particle. Different masses correspond to different thickness of solid lines.

theorem from Ref. [73]. It states that a projective[1] (Feynman) integral over a domain Δ has the same value when the integration domain is deformed as

$$\Delta_S = \{\vec{x}, x_i \geq 0, \sum_{i \in S} x_i = 1\}, \tag{3.18}$$

where $S \subseteq \{1, \ldots, N\}$ is a non-empty domain. Projectivity implies invariance with respect to rescaling of an integrand $x_i \rightarrow \lambda x_i$, $dx_i \rightarrow \lambda dx_i$. If a Feynman integral does not have such an invariance, it can be restored via the following projective transformation

$$x_i \longrightarrow \frac{x_i'}{\sum_i^N x_i'}. \tag{3.19}$$

We insert Eq. (3.17) into Eq. (3.2) and integrate over loop momenta.[2] We obtain the following expression [74]

$$I = i \frac{\Gamma(a - LD/2)}{(4\pi)^{d/2} \prod_{j=1}^N \Gamma\left(a_j\right)} \int_{x_j \geq 0} \left(\prod_{j=1}^N \mathrm{d}x_j x_j^{a_j - 1} \right) \delta \left(1 - \sum_{\Delta_S} x_i \right) \frac{\mathcal{U}^{a-(L+1)D/2}}{\mathcal{F}^{a-LD/2}}, \tag{3.20}$$

where we have introduced the Symanzik polynomials $\mathcal{U}$ and $\mathcal{F}$.

To calculate them, we rewrite the inverse propagators from Eq. (3.17) as

$$\sum_{j=1}^N x_j \left(-q_j^2 + m_j^2 \right) = - \sum_{i=1}^L \sum_{j=1}^L k_i k_j M_{ij} + \sum_{i=1}^L 2k_i \cdot Q_i + J, \tag{3.21}$$

where M_{ij} is a $(L \times L)$ matrix which depends on Feynman parameters, Q_i is a vector which depends on external momenta as well as Feynman parameters and J is a scalar function which depends on

[1] Choosing a particular domain corresponds to a choice of a particular hyperplane spanned by Feynman parameters in a parametric space. Hence, Feynman integrals are projective integrals.

[2] We give examples of such integration in the Section where we discuss boundary conditions of Feynman integrals. Other examples can be found in many textbooks, see e.g. Refs. [61, 67]

the Mandelstam variables and Feynman parameters. The Symanzik polynomials are computed as follows [75]

$$\mathcal{U} = \det(M), \quad \mathcal{F} = \det(M)\left(J + QM^{-1}Q\right). \tag{3.22}$$

We will show now how to calculate the Symanzik polynomials $\mathfrak{U}$ and $\mathfrak{F}$ considering a two-loop planar box diagram, Fig. 3.2. We consider all momenta to be incoming and require

$$p_1^2 = 0,\ p_2^2 = 0,\ p_3^2 = 0,\ p_4^2 = m^2. \tag{3.23}$$

Also, we define Mandelstam variables as

$$s = (p_1+p_2)^2,\ t = (p_1+p_3)^2,\ u = (p_2+p_3)^2, \tag{3.24}$$

First, we calculate the quantities M, Q, J. To do this, we use Eq. (3.21) and write

$$\begin{aligned}\sum_{j=1}^{7} x_j\left(-q_j^2+m_j^2\right) =& k_1^2(-(x_1+x_2+x_3+x_7))- \\ & 2k_1(k_2x_7+x_3(p_1+p_2)+p_1x_2) - k_2^2(x_4+x_5+x_6+x_7)- \\ & 2k_2(x_6(p_3+p_4)+p_4x_5) - m^2(x_5+x_6) - s(x_3+x_6).\end{aligned} \tag{3.25}$$

From this expression, we find

$$\begin{aligned} M &= \begin{bmatrix} x_1+x_2+x_3+x_7 & x_7 \\ x_7 & x_1+x_2+x_3+x_7 \end{bmatrix}, \\ Q &= \begin{bmatrix} -p_1x_2-p_1x_3-p_2x_3 \\ -p_4x_5-p_3x_6-p_4x_6 \end{bmatrix}, \\ J &= s(-x_3-x_6)+m^2(-x_5-x_6). \end{aligned} \tag{3.26}$$

Using Eq. (3.22), we arrive at the following Symanzik polynomials

$$\begin{aligned}\mathcal{U} =& x_1x_4+x_2x_4+x_3x_4+x_1x_5+x_2x_5+x_3x_5+x_1x_6+x_2x_6+x_3x_6 \\ & +x_1x_7+x_2x_7+x_3x_7+x_4x_7+x_5x_7+x_6x_7, \\ \mathcal{F} =& m^2(-x_3x_4x_5-2x_3x_7x_5-x_4x_7x_5-x_3x_4x_6-3x_3x_6x_7-x_4x_6x_7- \\ & x_1(x_5+x_6)(x_4+x_7)-x_2(x_5+x_6)(x_4+2x_7))+s(x_2(x_5x_7-x_4x_6)- \\ & x_4(x_6x_7+x_3(x_6+x_7))-x_1(x_6(x_4+x_7)+x_3(x_4+x_5+x_6+x_7)))+tx_2x_5x_7.\end{aligned} \tag{3.27}$$

3.4. Integration-by-parts identities and master integrals

From the axioms of dimensional regularization follows that integrals of total derivatives have to vanish Eq. (3.13). This property has important consequences, because it allows us to construct various linear relations among Feynman integrals. Consider the following equations

$$\int \prod_{i=1}^{L} \frac{\mathrm{d}^d k_i}{(2\pi)^d} \frac{\partial}{\partial k_n^\mu}\left(q_m^\mu \frac{1}{D_1^{a_1}\dots D_N^{a_N}}\right) = 0. \tag{3.28}$$

Computing derivatives and expressing scalar products of loop and external momenta through inverse propagators in Eq. (3.28), one arrives[3] at the following relation between Feynman integrals

$$\sum C_j I_j(a_1 + b_1, a_2 + b_2, \ldots, a_N + b_N) = 0. \tag{3.29}$$

These relations are known as integration-by-parts (IBP) identity [76]. In Eq. (3.29), C_j are rational functions of Mandelstam variables and the space-time dimension d and b_j are shifts of exponents a_j caused by taking derivatives in Eq. (3.28).

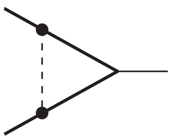
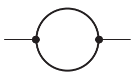
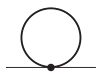
Since exponents a_j can be arbitrary, IBPs provide infinitely many linear relations among scalar integrals. These relations are linearly depended. Solving such a system is difficult and, unfortunately, there is no natural way to do so. However, there exist a finite set of integrals [69, 77] such that other integrals can be written as a linear combination of these integrals using IBP relations. We are left with a problem of finding a set of basis integrals which are traditionally called *master integrals* (MI).

This problem was solved by S. Laporta in Ref. [78], where he introduced a general reduction algorithm that is guaranteed to converge. The Laporta's algorithm has been implemented in many computer-algebra programs such as *REDUZE2* [79], *FIRE6* [80], *KIRA* [81], *LiteRed* [4].

To finish this section, we give an example of an IBP relation. We return to the one-loop electron vertex function $\Gamma(p_1, p_2, s)$. The top topology is read off from the diagram (Fig. 3.1), i.e. we simply discard the tensorial structure of Eq. (3.6) and raise propagators to arbitrary powers. We write

$$I(a_1, a_2, a_3; d) = \int \frac{\mathfrak{D}k}{(k^2)^{a_1}((k+p_1)^2 - m^2)^{a_2}((k-p_2)^2 - m^2)^{a_3}}, \tag{3.30}$$

where $\mathfrak{D}k = \mathrm{d}^{\mathrm{d}}/(2\pi)^{\mathrm{d}}$ is an integration measure and $p_i^2 = m^2$. Performing the reduction for this family, we get two master integrals: a bubble  and a tadpole . For the triangle diagram $I(1, 1, 1; d)$, the IBP identity reads

 $= \dfrac{2(d-3)}{(d-4)\,(4m^2 - s)} \cdot$  $+ \dfrac{d-2}{(d-4)m^2\,(s-4m^2)} \cdot$  $.$ (3.31)

It follows that to compute the tree-point functions, we need to calculate two master integrals that are two-point function $I(0, 1, 1; d)$ and the tadpole. In this particular case, it is straightforward to do that while in general it can be quite demanding. We will discuss computations of master integrals in Chapter 4 where we discuss the differential equation method.

[3]Only if algebra of momenta closes

Chapter 4

Evaluating master integrals

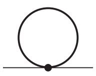
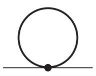
In this chapter we discuss the evaluation of master integrals. We have used the method of differential equations to compute them both in the case of Higgs production with high p_T and for mixed QCD&EW correction to $q\bar{q} \to Zg$. In both cases, a small parameter appears - m_t/p_T in the $H + j$ case and m_V/p_T in the $Z + j$ case - that we make use of. Hence, we discuss the concept of a small parameter expansion and how to use it to construct solutions for the differential equations. Finally, we show how boundary conditions of master integrals are calculated using several selected examples.

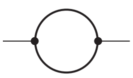
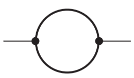
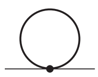
4.1. Method of differential equations

Direct computation of Feynman integrals is often complicated. Method of differential equations (DEs) provides an important alternative and sometimes works in cases when other methods fail.

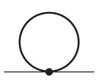
The method of differential equations is based on the following observation. Differentiating master integrals with respect to kinematic variables and masses leads to scalar integrals that, although different from the original one, belong to the same family. However, we have seen in Section 3.4 that IBP identities allow us to express these scalar integrals in terms of master integrals. By doing so, we obtain a system of differential equations which is closed for master integrals.

To illustrate this point, we return to the example of the one-loop electron vertex function. As we noted in Section 3.4, the three-point function Eq. (3.31) can be expressed as a linear combination of two master integrals. To generate differential equations, we use *REDUZE2*. The master integrals are functions of $\mu = -4m^2/s$ and s, which leads to two differential equations per integral. However, the dependence on s is trivial and can be restored through dimensional analysis. We set s to be -1 and substitute $d = 4 - 2\epsilon$. The differential equations for the variable μ read

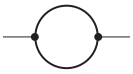
$\frac{\partial}{\partial \mu}$  $= \frac{1-\epsilon}{\mu} \cdot$  ,

$\frac{\partial}{\partial \mu}$  $= -\frac{2\epsilon - 1}{2(\mu+1)} \cdot$  $+ \frac{2(\epsilon-1)}{\mu(\mu+1)} \cdot$  . (4.1)

The first differential equation can be straightforwardly integrated. Taking into account the normalization Eq. (C.2), the result reads [61]

 $= -\frac{\Gamma(\epsilon-1)}{\Gamma(1+\epsilon)}(\mu)^{1-\epsilon}.$ (4.2)

Integrating the second differential equation gives



$$= C(2\mu+2)^{\frac{1}{2}(1-2\epsilon)} + \frac{2^{\epsilon+\frac{1}{2}}\mu^{1-\epsilon}(2\mu+2)^{\frac{1}{2}(1-2\epsilon)}\Gamma(\epsilon-1)\,{}_2F_1\left(\frac{1}{2}(3-2\epsilon),1-\epsilon;2-\epsilon;-\mu\right)}{\Gamma(\epsilon+1)}, \quad (4.3)$$

where ${}_2F_1$ is the ordinary hypergeometric function and C is an integration constant. This integration constant is fixed by considering the limit $\mu \longrightarrow 0$. On the right hand side of Eq. (4.3), the part containing $\mu^{1-\epsilon}$ vanishes for $\epsilon < 1$. On the left side, setting $\mu \longrightarrow 0$, we obtain a diagram which corresponds to a massless bubble. Hence, we find

$$C = 2^{\frac{1}{2}(-1+2\epsilon)} \quad (4.4)$$

The massless bubble  can be easily calculated using Feynman parameters. The results reads

$$= \frac{\Gamma(1-\epsilon)^2\Gamma(\epsilon)}{\Gamma(2-2\epsilon)\Gamma(\epsilon+1)}. \quad (4.5)$$

In general, a system of differential equations of master integrals can be written as

$$\frac{\partial}{\partial\vec{x}}\vec{I}(x;\epsilon) = A(x;\epsilon)\cdot\vec{I}(x;\epsilon), \quad (4.6)$$

where the matrix A contains rational functions of kinematic variables $\vec{x}$ and the dimensional regularization parameter ϵ. The vector $\vec{I}$ is a vector of all master integrals. The master integrals in $\vec{I}$ can be rearranged according to their topologies in the order of increasing complexity. It forces A to assume a lower-triangular form. In such a case, a generic system of differential equations can be solved by first finding solution for simpler integrals and then using them as input for differential equations of more complex integrals.

Integrating Eq. (4.6) can become difficult, if one wants to do it exactly. Indeed, modern multiloop calculations involve hundreds of master integrals. Such integrals usually have a complicated analytic structure. In most cases studied up to now, the properties of master integrals are known (see iterated integrals in Ref. [82]), it is understood how to integrate DE's by putting them in so-called canonical form of DEs [83] and what classes of functions appear [84]. There are, however, master integrals that are different. For instance, there are integrals that have elliptic kernels [85]. Such integrals appear, for instance, in the $H+j$ case (an example of elliptic integrals is shown in Fig. 4.1). It is then unclear how to define a canonical form of differential equations in such cases and whether these integrals can be iterated. For some of such integrals the solution is known (see iterated integral on elliptic curves [86]). Yet, this is still insufficient for an exact computation of the most complicated master integrals (see hyperelliptic sectors in the two-loop master integrals for Higgs+jet production with full top-mass dependence in Refs. [28, 87]).

We present a different approach below. It allows one to obtain physical results if a small parameter exists.

Figure 4.1.: An example of an elliptic sector in H + j case [28].In this chapter, we use the following notation for the Feynman diagrams: a dashed line corresponds to a massless particle while a solid line corresponds to a massive particle. Different masses correspond to different thickness of solid lines.

4.1.1. Small parameter expansion

Solving differential equations can be simplified in a case when one is interested in obtaining physical results in a certain kinematic approximation. For instance, as we discussed in Section 1.1 and Section 1.2, we are interested in the production the Higgs boson and the Z boson, with high transverse momenta. This requirement leads to an existence of the small parameter that can be used to simplify computations. To put it in a formal language, imagine that we have a kinematic parameter Q that exceeds all other kinematic and mass parameters X

$$Q \gg X, \tag{4.7}$$

We can define a small parameter μ as

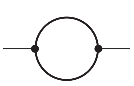
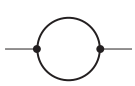
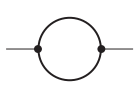
$$\mu = \frac{X}{Q}. \tag{4.8}$$

A master integral is then expanded asymptotically in the following way

$$\mathbf{I}(Q, X, x_1, x_2, \ldots; \epsilon) = \sum_i^{N_1} \sum_j^{N_2} \sum_k^{N_3} \tilde{\mathbf{I}}_{ijk}(Q, x_1, x_2, \ldots; \epsilon)\mu^{i-j\epsilon} \log^k(\mu) + \ldots \quad \text{as} \quad \mu \longrightarrow 0, \tag{4.9}$$

where numbers $N_1, N_2 \in \mathbb{Q}$ (rational numbers) and $N_3 \in \mathbb{Z}$ (integer numbers) are finite and $\tilde{\mathbf{I}}_{ijk}$ are unknown functions that need to be determined. We see immediate consequences of such an expansion: the dependence on μ is fixed in the limit $\mu \longrightarrow 0$. To arrive to this form of asymptotic expansion, one needs to analyze the differential equations of a master integral $\mathbf{I}(Q, X, x_1, x_2, \ldots; \epsilon)$.[1]

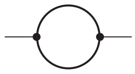
To find unknown functions from Eq. (4.9), one need to use the differential equations. Indeed, a system of differential equations provide full information about corresponding master integrals, including their behaviour in various limits. To show how this is done in practice, we return to master integrals from Eq. (4.1). We are interested in the kinematic limit where $|s| \gg 4m^2$. Hence, a small parameter is defined as $\mu = -4m^2/s$. For simplicity, s is taken again to be -1. Rewriting and expanding the differential equation of the bubble Eq. (4.1) with respect to μ leads to the following differential equation

$\frac{\partial}{\partial\mu}$  $= -\frac{1}{2}(2\epsilon - 1)$  $+ \frac{1}{2}\mu(2\epsilon - 1)$ 

$$+ 2(\epsilon - 1)\mu^{-\epsilon} - 2(\epsilon - 1)\mu^{1-\epsilon} + \mathcal{O}(\mu^2), \tag{4.10}$$

[1]The convergence of an asymptotic expansion is studied on a case-by-case basis. However, the uniqueness of an asymptotic expansion is given by differential equations and boundary conditions.

where we have inserted the solution for the tadpole Eq. (4.2) (we omitted the prefactor $\Gamma(\epsilon-1)/\Gamma(1+\epsilon)$). It follows from the differential equation that the following *Ansatz* is valid



$$= A(\epsilon) + \mu^{-\epsilon}(B(\epsilon) + C(\epsilon)\mu) + D(\epsilon)\mu + \mathcal{O}(\mu^2). \tag{4.11}$$

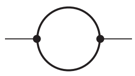
Quantities A, B, C, D are unknown functions of ϵ. To find them, we insert the Ansatz Eq. (4.11) into the differential equation. Matching powers in μ on both sides of the differential equation gives three linear equations for A, B, C, D

$$\begin{aligned} &2(B+2)\epsilon - B - 2C(\epsilon-1) - 4 = 0, \\ &A\left(\frac{1}{2} - \epsilon\right) - D = 0, \\ &B = 0. \end{aligned} \tag{4.12}$$

The solution reads

$$B = 0, \quad D = \frac{1}{2}A(1-2\epsilon), \quad C = 2. \tag{4.13}$$

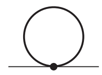
We arrive at the following solution



$=$

$+ \dfrac{\mu(1-2\epsilon)}{2} \cdot$

$+ 2 \cdot$



$+ \mathcal{O}(\mu^2), \quad (4.14)$

where we set the function A to be a massless bubble.[2] This results agrees with the full result obtained in the previous section upon its expansion in the $\mu \to 0$ limit.

Finally, we want to summarize some of the properties of the approach presented in this section. It is based on the example of the differential equation of the bubble diagram and our own experience applying this method to more complicated problems discussed in the next Sections.

- Asymptotic expansion of a master integral Eq. (4.9) is evaluated using differential equations that follow from IBPs.
- Using such an expansion reduces partially the problem of evaluation the master integral to solving linear equations.
- Systems of linear equations which appear in calculations are underdetermined which can be seen even from Eq. (4.12).
- We refer to terms that scale as $\mu^{-c\epsilon}$ as *branches*. Calculations of different branches can be done separately.
- Different branches correspond to different scaling of loop momenta of a master integral (see method of regions in Ref. [88]).
- As a drawback, we must compute boundary conditions that respect an asymptotic expansion of an integral. Such boundary conditions are sometimes hard to calculate.

[2] It was calculated in Eq. (4.5). However, we will return to this point in the section on boundary conditions.

4.1.2. Evaluating master integrals of two-loop amplitudes for the H-production at high transverse momenta

In this section, we discuss how the master integrals for the two-loop contribution to the scattering amplitudes $gg \to Hg$, $qg \to Hq$ and $q\bar{q} \to Hg$ are computed. Originally, these master integrals have been calculated in Ref. [1]. We follow closely the procedure described in that paper. We use the integration measure as in Appendix C. Top-sectors and topologies that appear in calculations are presented in Appendix C. Several selected solutions for master integrals are presented in Appendix D.1. There are 160 master integrals in total (their Feynman diagrams are listed in Appendix C).

$H + j$ amplitudes depend on 3 Mandelstam variables as well as the Higgs and the top mass

$$\hat{s} = (p_1 + p_2)^2, \quad \hat{t} = (p_1 + p_3)^2, \quad \hat{u} = (p_2 + p_3)^2, \quad \hat{s} + \hat{t} + \hat{u} = m_H^2. \tag{4.15}$$

It is convenient to use the following dimensionless variables

$$\eta = -\frac{m_H^2}{4m_t^2}, \quad \kappa = -\frac{m_t^2}{\hat{s}}, \quad z = \frac{\hat{u}}{\hat{s}}. \tag{4.16}$$

We take the Higgs and the top mass to be $m_H \sim 125\,\text{GeV}$ and $m_t \sim 173\,\text{GeV}$, respectively. We consider $|\eta| \sim 0.13$ to be a small parameter.

Computations of master integrals are usually performed in an Euclidean region where all Mandelstam variables $\hat{s}, \hat{t}$ and $\hat{u}$ are negative. It is not possible to perform computations in an Euclidian region in our case, since we are interested in the kinematical region $\left|m_H^2\right| = |\hat{s} + \hat{t} + \hat{u}| \ll |\hat{s}|, |\hat{t}|, |\hat{u}|$. Instead, we choose Minkowski region $\hat{t} > 0$, $\hat{s} < 0$, $\hat{u} < 0$. We also take $m_H^2 < 0$, $m_t^2 > 0$ such that parameters defined in Eq. (4.16) are positive in the chosen Minkowski region

$$0 < \eta \ll 1, \quad 0 < \kappa \ll 1, \quad 0 < z, \quad s < 0. \tag{4.17}$$

The scattering amplitude in the Minkowski region has imaginary part. To produce physical results, the Higgs mass is analytically continued to the region $m_H^2 > 0$. To calculate the cross-section Eq. (2.1), one needs to calculate virtual amplitudes in kinematic regions described in Section 2.1. It is done by an analytic continuation of the computed scattering amplitudes from the region Eq. (4.17) to another regions, say $\hat{s} > 0, \hat{t} < 0, \hat{u} < 0$. We discuss this procedure in later on the thesis.

To derive differential equations, we follow the procedure described in Section 4.1. In practise, we use routines provided by *REDUZE2* [79] to generate differential equations for a given set of master integrals. To work with differential equations written with dimensionless variables, we derive the following relations

$$\partial_\eta = 4\hat{s}\kappa\partial_{\hat{t}}, \quad \partial_\kappa = \hat{s}\left(4\eta\partial_{\hat{t}} - \partial_{m_t^2}\right), \quad \partial_z = \hat{s}\left(\partial_{\hat{u}} - \partial_{\hat{t}}\right). \tag{4.18}$$

After setting $\hat{s}$ to -1, the differential equations read

$$\partial_x \mathbf{I}_i(\kappa, \eta, z, \epsilon) = \sum_j A_{ij}^x(\kappa, \eta, z, \epsilon)\mathbf{I}_j(\kappa, \eta, z, \epsilon), \quad x \in \{\kappa, \eta, z\}, \tag{4.19}$$

where ∂_x denotes a partial derivative with respect to a variable x. The $\hat{s}$-dependence of the scattering amplitudes is later restored through dimensional analysis. The three matrices A^x are rational functions of η, κ, z and ϵ. We have chosen master integrals in such a way which makes the differential equation assume the low-triangular form

$$\partial_k \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} a_{11}^k & 0 & \dots & 0 \\ a_{21}^k & a_{22}^k & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1}^k & a_{N2}^k & \dots & a_{NN}^k \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}, \tag{4.20}$$

where we assume that the integrals are arranged according to their complexity (see the previous Section).

Analysis of the matrices A^x reveals singularities at $\eta = 0, -1/2, -1$. They correspond to $m_H^2 = 0, m_H^2 = 2m_t^2$ and $m_H^2 = 4m_t^2$ respectively. We find that the singularity at $m_H^2 = 2m_t^2$ is spurious. By taking combinations of master integrals as in Ref. [28] this singularity disappears. At the singular point $\eta = m_H^2 = 0$, we find singularities at $\kappa = 0, -1/4, -(1+z)/4, -z/4$. Taking into account the structure of matrices A^x, we expand them in the Laurent series in η and κ.

To solve the differential equations for the master integrals, we use the method described in the previous Section. Namely, we consider an asymptotic expansion of the master integrals at the singular points $\eta = 0$ and $\kappa = 0$. Because these points are singular, we expect that series expansions of master integrals differ from Taylor series. Indeed, by considering differential equations for master integrals, we find that they admit the following Ansatz for master integrals

$$\mathbf{I}_i(\kappa, \eta, z, \epsilon) = \sum_{j\geq 0, l\geq -3} \sum_{k=0}^{1} \sum_{m=0}^{2} \sum_{n=0}^{2} c_{i,j,k,l,m,n}(z, \epsilon) \eta^{j-k\epsilon} \kappa^{l/2-m\epsilon} \log^n(\kappa). \tag{4.21}$$

The maximal power of expansion of master integrals in κ and η are chosen for each master integral individually such that the two-loop form factors Eq. (2.22) and Eq. (2.33) can be computed to leading order in η and subleading order in κ. We note that for the majority of integrals, the expansion with respect to η turns out to be just a Taylor expansion. However, for a few master integrals this is not the case. Examples of integrals that have singularities in η are presented in Fig. 4.2. Indeed, as we already implied in Eq. (4.21), we include in the ansatz terms that scale as $\eta^{-\epsilon}$ to account for singularities that arise when Higgs couples to massless particles is induced. We note that in the final result such singularities have to disappear since, as it is seen from the analysis of the corresponding Feynman diagrams, a Higgs boson *only* couples to massive particles. We have confirmed that the cancellation of such singularities takes place.

Figure 4.2.: Examples where an external massive leg is connected to internal massless lines. Such diagrams appear only at the master integral level. One can get these master integrals from the planar topology 2 by pinching propagator's lines (see PL2 in Appendix C.1).

Additionally, we note that some of the master integrals contain non-analytic terms proportional to $\kappa^{-1/2}$, $\kappa^{1/2}$, $\kappa^{3/2} \dots$, etc.

As we explained above, after inserting the Ansatz Eq. (4.21) into differential equations, we require that the coefficients of $\eta^{j-k\epsilon}\kappa^{l/2-m\epsilon}\log^n(\kappa)$ vanish in order to fulfill the DEs. Resulting linear equations with respect $c_{i,j,k,l,m,n}(z,\epsilon)$ are solved in a standard way. By doing so, we "integrate" differential equations for η and κ.

Coefficients $c_{i,j,k,l,m,n}(z,\epsilon)$ that cannot be fixed in previous step have yet to be integrated with respect to the variable z. We insert Ansatz Eq. (4.21) into z-differential equations with fixed κ- and η-dependence from the previous step and we match powers in μ on both sides of the differential equations to get the following differential equations for $c(z;\epsilon)$

$$\partial_z c_i(z,\epsilon) = \sum_j \tilde{A}^z_{ij}(z;\epsilon)c_j(z,\epsilon) + f_i(z,\epsilon). \tag{4.22}$$

Since all indices $\{i,j,k,l,m,n\}$ are fixed to certain values by κ- and η-differential equations, we label unknown functions $c(z;\epsilon)$ with only one index i, since there is only one unknown function per master integral is to integrate. Function $f_i(z,\epsilon)$ is the inhomogeneous part of a differential equation which consists of terms generated by κ- and η-integration and their z-derivatives.

Integration with respect to one variable in many cases can be performed using algorithm of Ref. [77] by putting the differential equations into canonical form, and there are many programs which implemented this algorithm [89, 90]. We observe a significant simplification of analytic structure of the master integrals compared to full analytic results of Refs. [28, 87] that compute master integrals keeping exact the top mass dependence. Indeed, all our integrals can be written in terms of Harmonic Polylogarithms (HPL) with simpler arguments (see their definition in Appendix B.2), their properties are well-known. The last step in the integration of differential equations is determination of boundary constants. This step is discussed later in this thesis, after reviewing differential equations for the $Z + j$ production.

4.1.3. Evaluating master integrals of two-loop amplitudes for the $Z + j$ production at high transverse momenta

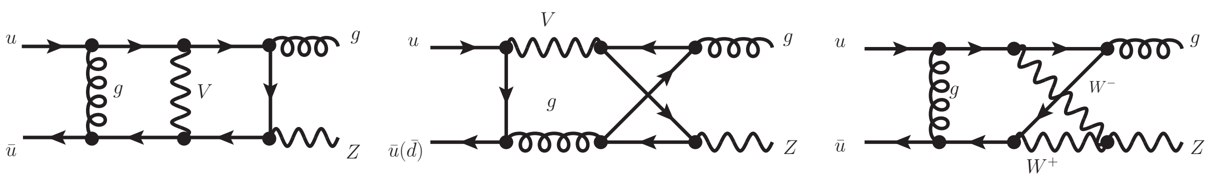


Figure 4.3.: Examples of Feynman diagrams that are contributing to mixed QCD&EW corrections of $Z + j$ production. The initial states (quarks) are chosen to be u-quarks. Vector boson V can be either a photon, Z or $W^{+/-}$ boson.

In this section, we review how master integrals for the $Z + j$ production are computed. Computations are quite similar to the $H + j$ case, so we will structure the discussion along similar lines.

To get the set of master integrals, we analyse the Feynman diagrams that are contributing to mixed QCD&EW corrections of $Z + j$ production. We consider only four quark-flavors running in the loop (u, d, c, s) which taken to be massless. Gauge particles running in loops are gluons g and vector bosons $V = \gamma,\ Z,\ W^{+/-}$ photons and massive vector bosons, respectively. We note that we work in the equal-mass approximation $m_Z = m_W = m_V$. Upon restricting the structure of Feynman diagrams, we identified 18 Feynman integral families (see Appendix C). In these 18 families we choose 481 master integrals (their Feynman diagrams are listed in Appendix C). We give selected solutions of master integrals in Appendix D.2.

To estimate the error of the equal-mass approximation, we consider corrections due to Sudakov logarithms and write

$$\frac{g^2}{16\pi^2}\log^2\left(\frac{s}{m_Z^2(1-\delta)}\right) = \frac{g^2}{16\pi^2}\log^2\left(\frac{s}{m_Z^2}\right) + \delta\frac{g^2}{8\pi^2}\log\left(\frac{s}{m_Z^2}\right) + O(\delta^2), \tag{4.23}$$

where $\delta = 1 - m_W^2/m_Z^2$. It is now straightforward to see that such an approximation $\delta \longrightarrow 0$ allows us to compute mixed QCD&EW corrections at high p_T with $O(10\%)$ precision.

Additionally, we neglect all diagrams with top quarks and treat all quarks as massless. Quark-mass corrections of $O(m_q^2/m_V^2)$ can be neglected for all quarks except the top quark. Top-mass corrections are indeed sizeable at the partonic level, however, not at the hadronic level due to suppression from b-quark's PDFs [52, 91].[3] Subsequently, we neglect all diagrams involving the Higgs boson.

The Mandelstam variables are defined as follows

$$\hat{s} = (p_1+p_2)^2\,, \quad \hat{t} = (p_1+p_3)^2\,, \quad \hat{u} = (p_2+p_3)^2\,, \quad \hat{s}+\hat{t}+\hat{u} = m_V^2, \tag{4.24}$$

with m_V being the Z boson mass.

For the production of the Z boson with a high transverse momentum $p_{Z,T} \sim 1$ TeV, we use the following dimensionless variables

$$\chi = \frac{\hat{t}}{\hat{s}}, \quad \mu = -\frac{m_V^2}{\hat{s}}. \tag{4.25}$$

Since we assume that $\{|\hat{s}|,\ |\hat{t}|,\ |\hat{u}|\} \gg m_V^2$, the condition $\hat{s}+\hat{t}+\hat{u} = m_V^2$ is only fulfilled if some of the Mandelstam variables are negative. As a starting region for our computation, we take $\hat{u} > 0$ and $\hat{s},\ \hat{t} < 0$. In this region, the dimensionless variables defined above take the following values

$$0 < \mu \ll 1, \quad 0 < \chi. \tag{4.26}$$

Amplitudes in other kinematic regions are obtained by analytic continuation. We note that in Minkowski region master integrals have imaginary parts.

To derive differential equations in dimensionless variables Eq. (4.25), we use the following relations

$$\partial_\chi = -\hat{s}(\partial_{\hat{u}} + \partial_{m_V^2}), \quad \partial_\mu = \hat{s}(\partial_{\hat{t}} - \partial_{\hat{u}}). \tag{4.27}$$

Differential equations in these variables take the following form

$$\partial_x \mathbf{I}_i(\chi,\mu;\epsilon) = \sum_j A^x_{ij}(\chi,\mu;\epsilon)\mathbf{I}_j(\chi,\mu;\epsilon), \quad \text{with} \quad x \in \{\chi,\mu\}. \tag{4.28}$$

To solve these differential equations, we apply the very same method that was used for the H + jet calculations. Namely, we expand matrices $A^x(\chi,\mu;\epsilon)$ in the small parameter μ and analyse resulting

[3] This statement is true for EW corrections to $Z+j$, but for mixed QCD&EW corrections it is not, since there could be such diagrams that include top-loops. Including the top mass corrections may be the subject of a future research.

Figure 4.4.: Two examples of integrals which have non-analytic terms in the μ expansion.

differential equations. Since, $\mu = 0$ is a singular point of matrices A^k, we expect that master integrals will contain terms which differ from Taylor series. We take the following Ansatz

$$\mathbf{I}_i = \sum_{j,k,l} c_{i,j,k,l}(\chi;\epsilon)\mu^{j/2-k\epsilon}\log^l(\mu), \tag{4.29}$$

where, for the most general case, the indices take the following values $j/2 \geq -4$, $k = 0, 1, 2, 3, 4$ and $l = 0, 1, 2, 3, 4$. we note that master integrals have non-analytical terms such as $\mu^{-3/2}$, $\mu^{-1/2}$, $\ldots$, etc. Examples of Feynman diagrams where such terms appear are given in Fig. 4.4.

After the μ-integration, there is one unknown coefficient per master integral is left to compute. To calculate these terms, we insert the Ansatz Eq. (4.29) into χ-differential equations.

$$\partial_\chi c_i(z;\epsilon) = \sum_{ij} A^\chi_{ij} c_j(z;\epsilon) + f_i(\chi,\epsilon). \tag{4.30}$$

In Eq. (4.30), as before, function $f_i(\chi,\epsilon)$ is the inhomogeneous part of the differential equation that arises due to the μ-integration. We note that unknown coefficients are again relabeled as $\{j,k,l\} \longrightarrow i$. Integration of such a system of differential equations is performed in the standard way, explained in the previous Section.

We get the following "z" alphabet (see App. B.2)

$$a_i \in \{0, -1, 1, \frac{-1+i\sqrt{3}}{2}, \frac{1-i\sqrt{3}}{2}\}, \tag{4.31}$$

where the latter two letters are in fact roots of a *cyclotomic* polynomial $\Phi_n(x)$ for $n = 3$ (see, for instance, [92, 93]). Because of this, we get *Clausen* functions in solutions of the master integrals. Clausen functions correspond to imaginary and real parts of a polylogarithm on a unit circle. We note that intermediate results also contain letters (see App. B.2) $\{-1/2, -2\}$. However, they disappear after applying an appropriate change of variables and using identities which relate GPLs of different arguments (see [94]). We used function `ToFibrationBasis` from *Mathematica*-based package *PolyLogTools* [95] which implements these identities.

We have shown how to integrate differential equations using expansions in small parameters. However, we still need to evaluate boundary conditions. We now turn to the discussion of determining boundary conditions.

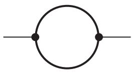
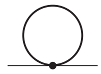
4.2. Computing boundary constants

Computing Feynman integrals using asymptotic expansions provides required information about original integrals. However, it also forces one to obtain boundary conditions from the behavior of integrals in these limits.

In this section, we explain how boundary constants are computed. To this end, we employ several methods, including regularity conditions, Mellin-Barnes representation or a direct computation using method of regions [88].

4.2.1. Massless branches

Since we are solving an underdetermined system of linear equations as explained in Section 4.1.1, we have the freedom to choose which unknown coefficients to solve this system for. It is very convenient, but not always possible, to choose the unknown coefficient of the massless branch. To demonstrate how it is done, we return to the example of the bubble diagram. Its solution in the $\mu \to 0$ limit reads

 $=$  $+ \dfrac{\mu(1-2\epsilon)}{2} \cdot$  $+ 2 \cdot$  $+ O(\mu^2)$.

We have solved Eq. (4.10) for the massless branch of the bubble and identified it with the massless diagram . The evaluation of a massless integral is usually easier than in the massive case.

We note that this idea can be applied to more general calculations that we have discussed in previous sections. Fortunately for us, we do not need to calculate the massless two-loop integrals, since they have already been calculated for planar topologies and non-planar ones in Refs. [96–98]. However, it is not always possible to fix all boundary constants using this trick. In these cases, we employ different ways to fix boundary constants.

4.2.2. Regularity conditions

By studying discontinuities of a given Feynman diagram, it is possible to infer which type of singularities in terms of Mandelstam variables one can encounter in the corresponding differential equations. To evaluate a discontinuity in the external channels of a Feynman integral, we make use of the famous Cutkosky rules [99]. These rules amounts to the following replacement of a propagator

$$\frac{1}{p^2 - m^2 + i0} \to 2\pi i \delta(p^2 - m^2)\theta(p^0), \tag{4.32}$$

which enforces an intermediate state to be on its mass shell. Consider a massless one-loop two-point function with incoming momentum $p^2 = s > 0$. We write

 $$= \int \frac{\mathrm{d}^d k}{(k^2 + i0)((k-p)^2 + i0)} \propto (-s)^{-\epsilon}. \tag{4.33}$$

Upon applying Cutkosky rules to evaluate the discontinuity in the s channel, we obtained

$\mathrm{Cut}_s \Big($  $\Big)$ $$= \int \mathrm{d}^d k\, \delta(k^2)\theta(k^0)\delta((k-p)^2)\theta(k^0 - p^0) \propto s^{-\epsilon}\Theta(s), \tag{4.34}$$

where θ is the Heavyside function. We see that $s = 0$ is the branch point which separates $s > 0$ and $s < 0$ regions in a complex plane. Eq. (4.34) is the imaginary part of Eq. (4.33).

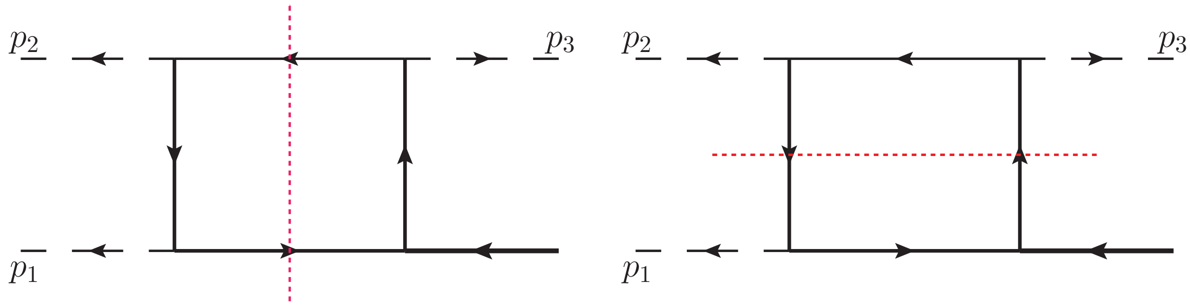


Figure 4.5.: Cuts in s-channel (left) and u-channel (right). The red line crosses the black ones that are set to be on-shell. The t-channel for the chosen momenta is not existent.

To get logarithmic functions in the solution, a singularity at $s = 0$ in the DE must manifest. Indeed, the differential equation of the massless bubble reads

$$\frac{\mathrm{d}}{\mathrm{d}s}$$  $$= \frac{-\epsilon}{s}$$  . (4.35)

We see that the branch point $s = 0$ becomes a singularity at the differential equation level. This is general property, i.e. a discontinuity in a particular channel gives rise to a singularity at the differential equation level [97].

The massless bubble diagram is a quite trivial example, since it only has one kinematic channel s. In practice though, we must work with functions which have many kinematic channels. At the differential equation level, as we saw, discontinuities appears as singularities in corresponding the Mandelstam variables in the DE [83]. Note that not only singularities in external channels appear at the differential equations [100]. It happens quite often that some of this singularities do not correspond to any discontinuity of corresponding Feynman diagram [83]. To demonstrate this point, we consider for simplicity a one-loop box diagram for the $H + j$ production. The corresponding Feynman integral reads

$$\mathbf{BOX}(\eta, \kappa, z, s) = \int \frac{\mathrm{d}^d k}{(k^2 - m_t^2)((k - p_1)^2 - m_t^2)((k - p_{12})^2 - m_t^2)((k - p_{123})^2 - m_t^2)}, \tag{4.36}$$

where $p_{12} = p_1 + p_2$ and $p_{123} = p_1 + p_2 + p_3$ and we used the same kinematic setup as in Section 4.1.2. It has two non-vanishing cuts in external channels: s and u (cf. Fig. 4.5). However, a cut in t-channel is absent

$$\mathrm{Cut}_t\left(\mathbf{BOX}(\eta, \kappa, z, s)\right) = 0, \tag{4.37}$$

in accordance with the definition of discontinuity and Cutkosky rules [100]. This means that the solutions of the DE must be independent of $t = -s - u \equiv 1 + z = 0$ discontinuity. Consider the homogeneous part of a differential equation in z for $\mathbf{BOX}(m_H^2, m_t^2, u, s)$. It reads

$$\frac{\partial}{\partial z}\mathbf{BOX}(\eta, \kappa, z, s) = -\frac{1 + z + \epsilon}{z(1 + z)}\mathbf{BOX}(\eta, \kappa, z, s) + O(\eta, \kappa), \tag{4.38}$$

which can be easily integrated. We obtain

$$\mathbf{BOX}(\eta, \kappa, z, s) = f(\eta, \kappa) z^{-1-\epsilon}(1 + z)^{\epsilon} = \frac{f(\eta, \kappa)}{z}(1 + \epsilon(-\log(z) + \log(1 + z)) + O(\epsilon^2)), \tag{4.39}$$

where $f(\eta, \kappa)$ is a boundary function which irrelevant for our discussion. We see that the solutions of the homogeneous part of the differential equations includes logarithmic functions which do not satisfy the condition of Eq. (4.37). Such a solution stems from the term $1/(1+z)$. To satisfy the condition Eq. (4.37), we must require that the singularity $1/(1+z)$ does not appear at the differential equation level. To this end, we collect all terms that multiply $1/(1+z)$ and set the corresponding coefficient to be zero in the $z \to -1$ limit. Remembering that we use the ansatz of Section 4.1.1 to solve differential equation, we get the following expression (schematically)

$$\partial_z \mathbf{BOX}(0,0,z,s) = \frac{1}{z}\left(\sum a_i c_i\right) + \frac{1}{(1+z)}\left(\sum b_i c_i\right) + O(\eta, \kappa), \tag{4.40}$$

where a_i, b_i are just coefficients that depends on kinematic invariants and the dimension regularization parameter and c_i are boundary constants. Upon requiring that the coefficient of $1/(1+z)$ must vanish in the $z \to -1$ limit, we obtained the additional linear relations

$$\sum b_i c_i \Big|_{z=-1} = 0, \tag{4.41}$$

which can be used to fix the yet unknown boundary constants.

This method of obtaining boundary constants was used in both $H + j$ and $Z + j$ cases. It appears that it is best used in the case of planar topologies since it is always possible to ensure that one (or more) channels (discontinuities) are missing. For non-planar top sectors, this is not the case. In order to fix the boundary constants for non-planar diagrams, we use different methods, discussed in the next sections.

4.2.3. Fixing boundary conditions with the Mellin-Barnes representation

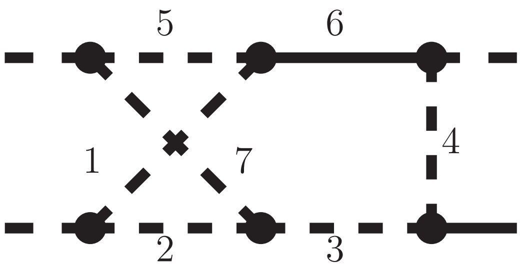


Figure 4.6.: A non-planar master integral for $Z+j$ production. We use it as an example to demonstrate MB techniques.

Mellin-Barnes (MB) representation (described in Appendix B.3) is used to evaluate Feynman integrals and their asymptotic expansions. To illustrate how this can be done, we consider one of the integrals required for the computations of the $Z + j$ production.[4] Consider the family of integrals

$$\mathbf{I} = \int \frac{\mathfrak{D}k_1\, \mathfrak{D}k_2}{[1]^{a_1}[2]^{a_2}[3]^{a_4}[4]^{a_4}[5]^{a_5}[6]^{a_6}[7]^{a_7}[8]^{a_8}[9]^{a_9}} \tag{4.42}$$

[4] Multiple examples of derivations of similar boundary conditions can be found in the literature [1, 46, 101]. In Ref. [101], the authors rely on the method of regions and on a geometric approach to the expansion by regions [88, 102, 103], while we opt for the approach developed in Refs. [96, 104].

with inverse propagators defined as

$$
\begin{aligned}
&[1] = -k_1^2, && [2] = -(k_1+p_1)^2, && [3] = -(k_2+p_1+p_2)^2, \\
&[4] = -(k_2-p_3)^2, && [5] = -(k_2-p_3)^2, && [6] = -(k_1-k_2)^2, \\
&[7] = -(k_1-k_2)^2, && [8] = -(k_1+p_1+p_2)^2, && [9] = -(k_1-k_2+p_3)^2 .
\end{aligned}
\tag{4.43}
$$

Having defined propagators, we calculate corresponding Symanzik polynomials following the discussion in Section 3.3

$$
\begin{aligned}
\mathfrak{U} &= x_1x_3+x_2x_3+x_5x_3+x_7x_3+x_1x_4+x_2x_4+x_1x_5+x_2x_5 \\
&\quad + x_4x_5+x_1x_6+x_2x_6+x_5x_6+x_1x_7+x_2x_7+x_4x_7+x_6x_7, \\
\mathfrak{F} &= m_V^2\, x_6((x_3+x_4+x_6)(x_5+x_7)+x_1(x_3+x_4+x_5+x_6+x_7) \\
&\quad + x_2(x_3+x_4+x_5+x_6+x_7)) \\
&\quad + s\,(-x_1x_3(x_4+x_5+x_6)-(x_4+x_6)(x_2(x_3+x_7)+x_3(x_5+x_7))) \\
&\quad - t\,x_4(x_1x_3+(x_5+x_7)x_3+x_2(x_3+x_5+x_7)) \\
&\quad - u\,x_4(x_1(x_3+x_7)+x_2(x_3+x_7)+x_3(x_5+x_7)),
\end{aligned}
\tag{4.44}
$$

where we set a_8 and a_9 to zero, since we are interested in the $\{111111100\}$ sector of Eq. (4.43) (its Feynman diagram is given in Fig. 4.6).

We would like to simplify $\mathfrak{F}$-polynomial. To do so, we redefine this it in the following way

$$
\mathfrak{F} \to \mathfrak{F} + (x_1x_3x_4+x_2x_3x_4+x_3x_5x_4+x_2x_7x_4+x_3x_7x_4)\left(\tilde{\sigma}m_V^2+s+t+u\right), \tag{4.45}
$$

with $\tilde{\sigma} = -1-i0$. Since we add zero ($m_V^2 = s+t+u$), we do not change the integrals. The second Symanzik polynomial then reads

$$
\begin{aligned}
\mathfrak{F} = &m_V^2(\tilde{\sigma}x_4(x_1(x_3+x_7)+x_2(x_3+x_7)+x_3(x_5+x_7))+x_6((x_3+x_4+x_6)(x_5+x_7)+ \\
&x_1(x_3+x_4+x_5+x_6+x_7)+x_2(x_3+x_4+x_5+x_6+x_7)))+ \\
&\tilde{s}(x_1x_3(x_5+x_6)+x_6(x_2(x_3+x_7)+x_3(x_5+x_7)))+\Delta x_1x_4x_7+\tilde{t}x_2x_4x_5.
\end{aligned}
\tag{4.46}
$$

To arrive at this form, we used identity $u = -\Delta - \tilde{\sigma}m_V^2$ with $\Delta = s+t$ and changed variables $s \to -\tilde{s}$, $t \to -\tilde{t}$ after that. We note that Δ is negative, while quantities $\tilde{s}$, $\tilde{t}$, m_V^2 are positive. We treat $\Delta, \tilde{s}, \tilde{t}$ and m_V^2 as independent variables, otherwise the sign of $\mathcal{F}$ is indefinite. After all these manipulations, the integral takes the form

$$
\mathbf{I}_{a_1a_2a_3a_4a_6a_700} = \frac{\Gamma(d+a)}{\Gamma(1+\epsilon)^2\prod_{i=1}^{7}\Gamma(a_i)}\int_0^\infty\left(\prod_{j=1}^{7}\mathrm{d}x_jx_j^{a_j-1}\right)\delta(1-x_1)\,\frac{\mathfrak{U}^{a-\frac{3d}{2}}}{\mathfrak{F}^{d-a}}, \tag{4.47}
$$

where $a = \sum_{j=1}^{7} a_j$, and $d = 4-2\epsilon$.[5] We note that boundaries in this integral are chosen according to Cheng-Wu theorem as explained in Section 3.3. Namely, we choose the following hyperplane in the parametric space

$$
1-\sum_{i=1}^{7}x_i \equiv 1-x_1, \tag{4.48}
$$

[5]In what follows, a variable x with a subscript is Feynman parameter while a variable z with a subscript is MB variable.

which is true if and only if the rest of Feynman parameters are integrated from 0 to ∞. We do not use δ-function until the end.

We are interested particularly in a master integral $\mathbf{I}_{111111100}$. We set powers of propagators to be

$$\begin{aligned} &a_1 \to 1-\delta,\ a_2 \to \delta+1,\ a_3 \to \delta+1,\ a_4 \to 1-\delta, \\ &a_5 \to 1-\delta,\ a_6 \to \delta+1,\ a_7 \to 1, \end{aligned} \tag{4.49}$$

where we have introduced an additional analytical regulator δ which is needed to ensure that poles of Γ's with positive arguments and poles of Γ's negative arguments are separated upon introducing MB representation. Otherwise, the integration contour may end up on a pole of a Γ - function [105].

The sector $\{111111100\}$ contains three master integrals. Upon solving their differential equations, we need to calculate three boundary constants. Two out of three constants are fixed to massless branches in the way we explained in Section 4.2.1 while the last one corresponds to a branch $(m_V^2)^{-2\epsilon}$ of the master $\mathbf{I}_{111111100}$. Hence, we need to extract this branch from Eq. (4.47).

We introduce the MB representation for the integral Eq. (4.47). As we explained in Appendix B.3, to introduce a MB variable we split a polynomial into two pieces. It is clear, that it can be done in various ways that, however, lead to different number of MB integrals. Although derivation of MB representation for Feynman integrals is automatized in the public version of *AMBRE* [106], one can often do better by carefully analyzing the integrand. For example, we managed to get a MB representation with only 7 MB integrals where AMBRE produces 14 MB integrals.[6] We show now how this is done.

Our first steps is to separate the dependence on kinematic and mass parameters. The $\mathfrak{F}$-polynomial contains all these parameters. Hence, we split $\mathfrak{F}$ in five terms by introducing four MB parameters

$$\begin{aligned} \mathfrak{U}^{1+3\epsilon}\mathfrak{F}^{-3-2\epsilon} = &\iiiint_{-i\infty}^{+\infty} \mathrm{d}z_1\mathrm{d}z_2\mathrm{d}z_3\mathrm{d}z_4 \sigma^{z_4} s^{z_1-z_2-z_3} t^{z_2} \left(m_V^2\right)^{-z_1-2\epsilon-3} \Delta^{z_3} \\ &\times \mathfrak{U}^{-z_1-z_4+\epsilon-2} x_1^{z_3} x_2^{z_2} x_4^{z_2+z_3+z_4} x_5^{z_2} x_6^{-z_1-z_4-2\epsilon-3} x_7^{z_3} P_1^{z_4} P_2^{z_1-z_2-z_3} \\ &\times \frac{\Gamma\left(-z_2\right)\Gamma\left(-z_3\right)\Gamma\left(-z_1+z_2+z_3\right)\Gamma\left(-z_4\right)\Gamma\left(2\epsilon+z_1+z_4+3\right)}{\Gamma(2\epsilon+3)}, \end{aligned} \tag{4.50}$$

where

$$\begin{aligned} P_1 &= \left(x_1\left(x_3+x_7\right)+x_2\left(x_3+x_7\right)+x_3\left(x_5+x_7\right)\right), \\ P_2 &= \left(x_1x_3\left(x_5+x_6\right)+x_6\left(x_2\left(x_3+x_7\right)+x_3\left(x_5+x_7\right)\right)\right). \end{aligned} \tag{4.51}$$

In Eq. (4.50), after applying MB splitting, it is just happened that one of polynomials is the same as the $\mathfrak{U}$-polynomial. We notice that the $\mathfrak{U}$ is the only polynomial which depends on variable x_4. To perform integration over this variable we use one of definitions of Euler B-function Eq. (B.31). We write

$$\begin{aligned} \int_0^\infty \mathrm{d}x_4 x_4^{z_2+z_3+z_4-\delta}\mathfrak{U}^{-2-z_1-z_4+\epsilon} = &\frac{\Gamma(z_2+z_3+z_4+1-\delta)\Gamma(z_2+z_3+z_4+1+\delta)}{\Gamma(-\epsilon+z_1+z_4+2)} \\ &P_3^{-1-z_1+z_2+z_3+\epsilon-\delta} P_4^{-z_2-z_3-z_4-1+\delta}, \end{aligned} \tag{4.52}$$

[6]We used *AMBRE* 1.2. while newer versions of *AMBRE* seem to perform better.

with P_3 being

$$
\begin{aligned}
&P_3 = (x_3 + x_6)(x_5 + x_7) + x_1 \left(x_3 + x_5 + x_6 + x_7\right) + x_2 \left(x_3 + x_5 + x_6 + x_7\right), \\
&P_4 = (x_1 + x_2 + x_5 + x_7).
\end{aligned}
\tag{4.53}
$$

If we split P_2 - polynomial in a nice way, we can perform integration over the variable x_6, since the only other polynomial that depends on the variable x_6 is P_3. We split P_2 and integrate over x_6 in the following equation

$$
\begin{aligned}
&\int_0^\infty \mathrm{d}x_6 x_6^{\delta-z_1-z_4-2\epsilon-3} P_2^{z_1-z_2-z_3} P_3^{-1-z_1+z_2+z_3+\epsilon-\delta} = \int_{-\infty}^{+\infty} \mathrm{d}z_5 \int_0^\infty \mathrm{d}x_6 \\
&\times x_1^{z_5} x_3^{z_5} x_5^{z_5} x_6^{\delta-z_2-z_3-z_4-z_5-2\epsilon-3} \\
&\times \frac{\Gamma\left(-z_5\right) \Gamma\left(-z_1+z_2+z_3+z_5\right)}{\Gamma\left(-z_1+z_2+z_3\right)} P_3^{-1-z_1+z_2+z_3+\epsilon-\delta} P_4^{z_1-z_2-z_3-z_5} = \int_{-\infty}^{+\infty} \mathrm{d}z_5 \\
&\times x_1^{z_5} x_3^{z_5} x_5^{z_5} P_4^{-\delta+z_2+z_3+z_4+z_5+2\epsilon+2} P_5^{z_1-z_2-z_3-z_5} P_6^{-z_1-z_4-z_5-\epsilon-3} \\
&\times \frac{\Gamma\left(-z_5\right) \Gamma\left(-z_1+z_2+z_3+z_5\right)}{\Gamma\left(-z_1+z_2+z_3\right) \Gamma\left(\delta-\epsilon+z_1-z_2-z_3+1\right)} \\
&\times \Gamma\left(\epsilon+z_1+z_4+z_5+3\right) \Gamma\left(\delta-2\epsilon-z_2-z_3-z_4-z_5-2\right),
\end{aligned}
\tag{4.54}
$$

where we have used

$$
\begin{aligned}
&P_5 = x_1x_3 + x_2x_3 + x_5x_3 + x_7x_3 + x_2x_7, \\
&P_6 = x_1x_3 + x_2x_3 + x_5x_3 + x_7x_3 + x_1x_5 + x_2x_5 + x_1x_7 + x_2x_7.
\end{aligned}
\tag{4.55}
$$

Following the same strategy, we use Eq. (B.27) to split P_6 into tree terms

$$
\begin{aligned}
P_6^{-z_1-z_4-z_5-\epsilon-3} = &\iint_{-\infty}^{+\infty} \mathrm{d}z_6 \mathrm{d}z_7 \left(x_1 + x_2\right)^{z_6+z_7} \\
&x_5^{z_6} x_7^{z_7} x_3^{-z_1-z_4-z_5-z_6-z_7-\epsilon-3} P_4^{-z_1-z_4-z_5-z_6-z_7-\epsilon-3} \\
&\frac{\Gamma\left(-z_6\right) \Gamma\left(-z_7\right) \Gamma\left(\epsilon+z_1+z_4+z_5+z_6+z_7+3\right)}{\Gamma\left(\epsilon+z_1+z_4+z_5+3\right)}.
\end{aligned}
\tag{4.56}
$$

The next and the last step is to integrate over remaining Feynman parameters. We rewrite Eq. (4.47) applying all above manipulations

$$
\begin{aligned}
\mathbf{I}_{111111100} = &\int_{-i\infty}^{+i\infty} \prod_{k=1}^{7} \mathrm{d}z_k \int_0^\infty \prod_{k'=1, k'\neq\{4,6\}}^{7} \mathrm{d}x_{k'} \\
&\times \tilde{\sigma}^{z_4} \tilde{s}^{z_1-z_2-z_3} \tilde{t}^{z_2} \Delta^{z_3} \left(x_1 + x_2\right)^{z_6+z_7} \left(m_V^2\right)^{-z_1-2\epsilon-3} P_4^{-z_1-z_6-z_7+\epsilon-2} P_5^{z_1-z_2-z_3-z_5} \\
&\times x_1^{-\delta+z_3+z_5} x_2^{\delta+z_2} x_3^{\delta-z_1-z_6-z_7-\epsilon-3} x_5^{-\delta+z_2+z_5+z_6} x_7^{z_3+z_7} \\
&\times \frac{\Gamma\left(-z_2\right) \Gamma\left(-z_3\right) \Gamma\left(-z_4\right) \Gamma\left(-z_5\right) \Gamma\left(-z_1+z_2+z_3+z_5\right)}{\Gamma(1-\delta)^3 \Gamma(\delta+1)^3 \Gamma(\epsilon+1)^2} \\
&\times \frac{\Gamma\left(-z_6\right) \Gamma\left(-z_7\right) \Gamma\left(\delta-2\epsilon-z_2-z_3-z_4-z_5-2\right)}{\Gamma\left(-\epsilon+z_1+z_4+2\right) \Gamma\left(\epsilon+z_1+z_5+z_6+3\right)}.
\end{aligned}
\tag{4.57}
$$

Integrating over Feynman parameters is now straightforward. We use the δ-function to integrate over x_1. The MB representation of $\mathbf{I}_{111111100}$ then reads

$$
\begin{aligned}
\mathbf{I}_{111111100} = & \int_{-i\infty}^{+i\infty} \prod_{k=1}^{7} \mathrm{d}z_k \, \left(m_V^2\right)^{-\delta-z_1-2\epsilon-3} \tilde{\sigma}^{z_4} \tilde{s}^{z_1-z_2-z_3} \Delta^{z_3} \tilde{t}^{z_2} \Gamma\left(-z_2\right) \\
& \times \frac{\Gamma\left(-z_5\right)\Gamma\left(-z_6\right)\Gamma\left(-z_7\right)\Gamma(\delta-\epsilon)\Gamma\left(-\delta+z_2+z_3+z_4+1\right)\Gamma\left(-z_3\right)\Gamma\left(-z_4\right)}{\Gamma(1-\delta)^3\Gamma(\delta+1)^2\Gamma(2\delta+1)\Gamma(\epsilon+1)^2} \\
& \times \frac{\Gamma\left(-\delta+z_3+z_5+1\right)\Gamma\left(-\delta+z_2+z_5+z_6+1\right)\Gamma\left(\epsilon+z_1+z_4+z_5+z_6+3\right)}{\Gamma(\delta-2\epsilon)\Gamma\left(-\epsilon+z_1+z_4+2\right)\Gamma\left(\epsilon+z_1+z_5+z_6+3\right)\Gamma\left(\delta-\epsilon-z_6-z_7\right)} \\
& \times \Gamma\left(\epsilon+z_1+z_5+z_6+z_7+3\right)\Gamma\left(\delta+2\epsilon+z_1+z_4+3\right) \\
& \times \Gamma\left(\delta-\epsilon-z_2-z_5-z_6-1\right)\Gamma\left(\delta-\epsilon-z_1-z_6-z_7-2\right) \\
& \times \Gamma\left(\delta-2\epsilon-z_2-z_3-z_4-z_5-2\right)\Gamma\left(2\delta-\epsilon-z_3-z_5-z_6-z_7-1\right) \\
& \times \Gamma\left(-\delta+\epsilon+z_2+z_3+z_5+z_6+z_7+2\right).
\end{aligned}
\tag{4.58}
$$

This integral is well-defined for finite values ϵ and δ. We choose $\epsilon = -\frac{17}{32}$ and $\delta = \frac{1}{16}$. However, we need results in $d = 4$ dimensions. To evaluate the integral Eq. (4.58) in these dimensions, we need to analytically continue it in such a way that we can safely take $\delta \to 0$ and $\epsilon \to 0$ limits. How it can be done, we explain below.

MB integrals are performed along the contours that run parallel to the imaginary axis and are chosen such that the $\Gamma(z+\ldots)$ and $\Gamma(-z+\ldots)$ poles are to the left and to the right respectively of the contours. We shift integration contours, by taking the $\delta \to 0$ limit and keeping ϵ fixed. Each time a pole of a gamma function crosses an integration contour, we simply add the residue of an integral at corresponding pole. Residue integrals may also contain poles so that the procedure is iterated. After that, Eq. (4.58) reads (schematically)

$$
\mathbf{I}_{111111100} \to \tilde{\mathbf{I}} + \sum R_i, \tag{4.59}
$$

where $\tilde{\mathbf{I}}$ is an analytic continuation of $\mathbf{I}_{111111100}$ in the neighborhood of the point $\delta = 0$; functions R_i are residues of $\mathbf{I}_{111111100}$. We apply the same procedure for the analytic continuation $\epsilon \to 0$. This algorithm was first discussed in Ref. [96].

In practise, an analytic continuation of the integral is performed using the *Mathematica*-based package *MB.m* [104], where above algorithm is implemented. In our case, the analytic continuation in ϵ led to 18 distinct integrals. To select a particular m_V^2 branch, we perform an asymptotic expansion $m_V^2 \to 0$ of these 18 integrals using the *Mathematica*-based package *MBasymptotics.m* written by M. Czakon. We take one of these 18 integral to show the procedure of extracting a branch. The integral reads

$$
\begin{aligned}
\tilde{I} = & \oint_{\Gamma} \mathrm{d}z_1 \, \Delta^{2\epsilon} t^{2\epsilon} \left(m_V^2\right)^{-z_1-2\epsilon-3} \tilde{\sigma}^{-4\epsilon-1} s^{z_1-4\epsilon} \\
& \times \frac{\Gamma(-2\epsilon)^2\Gamma(-\epsilon)^2\Gamma(2\epsilon+1)\Gamma(3\epsilon+1)}{\Gamma(\epsilon+1)^3} \\
& \times \Gamma(4\epsilon+1)\Gamma\left(2\epsilon-z_1-1\right)\Gamma\left(-2\epsilon+z_1+2\right),
\end{aligned}
\tag{4.60}
$$

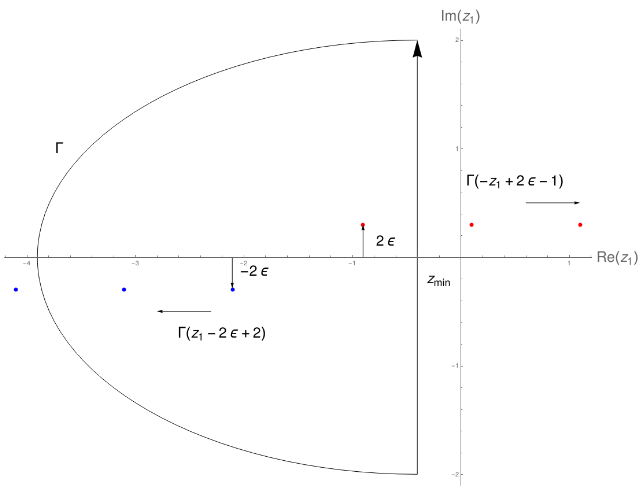


Figure 4.7.: The integration contour from Eq. (4.60).

where the integral over z_1 is performed along the contour Γ shown in Fig. 4.7. The contour Γ encircles all relevant poles of Γ-functions from Eq. (4.60) in such a way that the integration along this contour gives a desired order of expansion in m_V^2. In our case, we require the expansion up to the leading term in m_V^2.

From Eq. (4.60) it follows that different poles in z_1 give different powers of m_V^2 in the asymptotic expansion. If we require terms of asymptotic expansion up to $(m_V^2)^0$, we fixed the integration contour by imposing the following constraint

$$z_1^{\max} = -3 - 2\epsilon. \tag{4.61}$$

This constraint is used to close correctly the contour (Fig. 4.7), i.e. to take only those poles of $\Gamma(2\epsilon - z_1 - 1)$ and $\Gamma(-2\epsilon + z_1 + 2)$ which gives the requested power of m_V^2. We close contour to the left $\text{Re}(z_1) < 0$ and encircle all poles including the one at Eq. (4.61). We get three terms of the asymptotic expansion

$$\tilde{I} = (m_V^2)^{-4\epsilon}\left(\frac{R_1}{(m_V^2)^2} + \frac{R_2}{m_V^2} + R_3 + O(m_V^2)\right), \tag{4.62}$$

where R_1, R_2, R_3 are residues in $z_1 = -1, -2, -3$, respectively.

As we showed in Eq. (4.62), we need to perform an asymptotic expansion of $\mathbf{I}_{111111100}$ in the $m_V^2 \to 0$ limit to extract the branch $(m_V^2)^{-2\epsilon}$. Such an asymptotic expansion results in hundreds of integrals contributing in this branch. Most of these integrals can be computed by means of Barnes' lemmas which are shown in Appendix B.3. We use a *Mathematica*-based package called *barnesroutines.m* [107] which systematically apply Barnes' lemmas.

However, there are often few MB integrations left which one cannot calculate using Barnes' lemmas. In our case, there are two such integrals that contribute to the branch $(m_V^2)^{-2\epsilon}$. They read

$$I_1 = \int_{0-i\infty}^{0+i\infty} \mathrm{d}z_3 \frac{2\tilde{t}^{-z_3-2}\Delta^{z_3}\Gamma\left(-z_3-1\right)^2\Gamma\left(-z_3\right)\Gamma\left(z_3+1\right)^2\Gamma\left(z_3+2\right)}{\tilde{s}\epsilon}, \tag{4.63}$$

$$I_2 = \int_{0-i\infty}^{0+i\infty} \mathrm{d}z_5 \frac{3\Gamma\left(-z_5-1\right)\Gamma\left(-z_5\right)\Gamma\left(z_5+1\right)^2\psi^{(0)}\left(z_5\right)}{4\tilde{s}^2\tilde{t}\epsilon}, \tag{4.64}$$

where $\psi^{(0)}(z_5) = d/dz_5 \log\Gamma(z_5)$ is a polygamma function. To evaluate these integrals, we close contour to the left of the real part of the integration variable, i.e $\mathrm{Re}(z) < 0$.[7] Using Cauchy's theorem, we get an infinite sum of residues at poles of Γ functions. We used *MBsums.m* [108] to close the contour and obtained the following

$$
\begin{aligned}
I_1 \sim \sum_{n=1}^{\infty} & \frac{(-1)^{-2n}}{6\Delta(n-1)^2 n\tilde{\sigma}} \left(\frac{6\sigma}{n-1} + \frac{3\sigma}{n} + 3(n-1)((n-1)\sigma - 3n)\psi^{(2)}(n-1) + \right.\\
& \left(3n - \frac{3(n-1)^2\sigma}{n}\right)\psi^{(1)}(n-1) - \frac{51n}{(n-1)^2} - 8\pi^2 n - 3n\psi^{(0)}(n)^2 + \\
& \left. \frac{6n((n-1)^2\psi^{(1)}(n-1)+1)\psi^{(0)}(n)}{n-1} \right), \\
I_2 \sim \sum_{n=1}^{\infty} & \frac{(-1)^{-2n}(n^2(n+1)\psi^{(1)}(n+1) - (3n+2)(n\psi^{(0)}(n)+1))}{\Delta n^4 (n+1)^2},
\end{aligned} \tag{4.65}
$$

where we have used $\sim$ to emphasis that we skip some terms which are already resummed. To do the summation, we use the program *Xsummer* [109].

Finally, we sum up all terms together to get the following expression for the branch $(m_V^2)^{-2\epsilon}$

$$
\begin{aligned}
& \mathbf{I}_{111111100} \longrightarrow c(\chi;\epsilon)(m_V^2)^{-2\epsilon}, \quad \text{with} \\
& c(\chi;\epsilon) = -\frac{61}{8(\chi+1)\epsilon^4} + \frac{11G(-1,\chi) - 48i\pi - 10}{4(\chi+1)\epsilon^3} \\
& \quad + \frac{96i\pi G(-1,\chi) + 12G(-1,\chi) + 305\pi^2 - 96i\pi + 96}{24(\chi+1)\epsilon^2} \\
& \quad + \frac{1}{12(\chi+1)\epsilon}(-57\pi^2 G(-1,\chi) - 24G(-1,-1,-1,\chi) + 381\zeta(3) \\
& \quad + 120i\pi^3 + 46\pi^2 + 72i\pi - 72) \\
& \quad + \frac{1}{48(\chi+1)}(-224i\pi^3 G(-1,\chi) + 8\pi^2 G(-1,\chi) + 48\pi^2 G(-1,-1,\chi) \\
& \quad - 192i\pi G(-1,-1,-1,\chi) + 192G(-1,-1,-1,-1,\chi) - 360\zeta(3)G(-1,\chi) \\
& \quad + 2496i\pi\zeta(3) + 912\zeta(3) - 287\pi^4 + 128i\pi^3 - 320\pi^2 - 576i\pi + 576) + O(\epsilon),
\end{aligned} \tag{4.66}
$$

where $\chi = t/s$ and G-functions are GPLs Appendix B.2.

To summarize, the MB representation is powerful method which allows us to compute boundary conditions required in $H + j$ and $Z + j$ calculations. Moreover, it can been seen from the definition of an integral family Eq. (4.42), finding a MB representation with the lowest number of MB variables allows one to compute all boundary conditions for lower sectors. It is done by pinching propagators, i.e. setting a_i to zero. However, it is hard to come up with a such representation for some cases. For these cases, we get two- and three-fold MB integrals which are difficult to calculate. For such difficult sectors, we use a different method which is reviewed next.

[7] In the very same way as shown in Fig. 4.7, but now the contour must include *all* residues to the left of $\mathrm{Re}(z) < 0$.

4.2.4. Expansion by regions

The asymptotic behaviour of a Feynman integral can be evaluated using the *strategy of regions* also known as the *expansion by regions.* The idea of the "strategy of regions" stems from a paper by M. Beneke and V. Smirnov [88], where it was demonstrated on a number of one-loop examples how one can, in principle, derive an asymptotic behaviour for a Feynman integral in a particular kinematic limit.[8] Over the years, this original idea was reconsidered and refined in many publications [102, 103, 110, 111], and it was shown to work in many multiloop examples (cf. Refs [46, 112]), however, it is still holds the status of experimental mathematics, as it is stated in Ref. [68].

The strategy of regions is a rather heuristic approach, i.e. it consists of several rules, by following which, one can evaluate a Feynman integral in a kinematic region of interest. These rules are listed below [88, 103].

1. *Divide the integration domain into several subdomains (regions) which are distinct from each other with respect to their scaling properties of a given integration variable.*[9]

Formally, this step can be cast in the following form [103]. Given an integration domain $\mathcal{D}_x$, we split it as

$$\mathcal{D}_x = \bigcup_{j=J} \mathcal{D}_j, \quad \mathcal{D}_{j_1} \bigcap \mathcal{D}_{j_2} = \emptyset, \tag{4.67}$$

where J is a set of regions. These subdomains are defined as follows

$$\mathcal{D}_j = \{x \in (\Lambda_{j-1}, \Lambda_j) : \tilde{x} \sim z_j\}, \tag{4.68}$$

with Λ_i being an integration cut-off which separates regions. An integration cut-off Λ_i has a direct relation to the scaling properties of adjacent regions. For each region, the scaling must be specified, i.e. the integration variables x are set to be of the same order as z_j, which is usually considered to be a small or large number (scale). In practice, it means that one needs to find a certain change of variables where a new variable $\tilde{x}$ is set to be of a certain order z_j.

2. *Taylor expand the integrand in each subdomain according to its scaling properties.*

Once *all* relevant regions are identified, the integrand will be expanded with respect to $\tilde{x}$. Formally, it amounts to defining an expansion operator for each subdomain of the form

$$T_j = \sum_{n=0}^{\infty} \frac{(\tilde{x} - z_j)^n}{n!} \frac{\partial^n}{\partial \tilde{x}^n}. \tag{4.69}$$

Here we assumed that these operators commute with each other. For non-commuting cases see Ref. [111].

3. *Set each region to be the whole integration domain.*

This step is the most crucial for the method to make sense and requires some elaboration. Working with covariant gauges in the dimensional regularization scheme (see Section 3.2), allows to regularize Feynman integrals which otherwise would diverge. One of the essential advantages

[8] Originally, it was considered by M. Beneke for some toy examples. However, we do not know if these results have been published. The only reference to M. Beneke original results can be found in Ref. [103].

[9] We often refer to regions as branches though, practically they correspond to slightly different features of a Feynman integral.

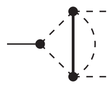
of using such a scheme is that any scaleless integral vanishes (Eq. (3.12)). In particular, once the boundaries are set to be the original integration domain, the surplus of the integration region has to be subtracted to avoid what naively looks like double counting, i.e. schematically

$$\int_{\mathcal{D}_j} = \int_{\mathcal{D}_x} - \int_{\mathcal{D}_x/\mathcal{D}_j} . \tag{4.70}$$

However, once that all these extra terms added together, they *must* vanish. More precisely, they add up to scaleless integrals which subsequently vanish in dimensional regularization scheme. This is a necessary condition for the whole scheme to work. Yet, it is not sufficient to assure, that the final result is correct, i.e., it does not tell whether all the relevant regions were taken into account.

4. *Integrate.*
At this point, the complexity of an integral for a given region is reduced and usually allows a straightforward evaluation.

To demonstrate all this steps in practice, consider the following master integral that appears in the Z+jet amplitudes. The integral reads



$$= \int \frac{\mathfrak{D}k\mathfrak{D}l}{[1][2][3][4]^2} \tag{4.71}$$

where we use the integration measure from Appendix C and

$$[1] = -(k+p_1+p_2)^2,\ [2] = -k^2,\ [3] = -(p_3-l)^2,\ [4] = m_V^2 - (l-k)^2. \tag{4.72}$$

This master integral corresponds to the sector $\{1,0,1,0,1,0,1,0,0\}$ of the integral family *FamPlanar7* (Tab. C.6). It is a two-by-two sector, and hence, upon integrating the corresponding differential equations, the sector required two boundary constants. One of them was matched with a massless branch as discussed previously, while the other one appears in the $(m_V^2)^{-2\epsilon}$ branch.

We start with the integration over the loop momentum l. To this end, we parametrize the integral in the following way

$$I_l = \int \frac{\mathfrak{D}l}{[3][4]^2} \overset{\text{F.P.}}{=} 2\int \mathfrak{D}l \int_0^1 \mathrm{d}\omega\, \frac{1-\omega}{(\omega[3]+(1-\omega)[4])^3}, \tag{4.73}$$

with ω being a Feynman parameter. In this form, the integration over the loop momentum l is straightforward and it reads

$$I_l = \int_0^1 \mathrm{d}\,\omega \frac{(1-\omega)^{-\epsilon}\omega^{-1-\epsilon}}{(m_V^2/\omega - (k-p_3)^2)^{1+\epsilon}}. \tag{4.74}$$

The next step is to integrate over the loop momentum k. We write

$$I_k = \int \frac{\mathfrak{D}k}{[1][2][5]^{1+\epsilon}} \overset{\text{F.P.}}{=}$$

$$\frac{\Gamma(3+\epsilon)}{\Gamma(1+\epsilon)} \int \mathfrak{D}k \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y\, \frac{y^{\epsilon}}{(-[1]x-(1-x-y)[2]+y[5]))^{3+\epsilon}}, \tag{4.75}$$

where we have introduced the new Feynman parameters x, y and the inverse propagator $[5] = m_V^2/\omega - (k-p_3)^2$. To perform the integration over k, we bring the denominator of Eq. (4.75) into a $\Delta - k^2$ form, where Δ is a function of the external momenta and Feynman parameters. Upon doing that, we integrated over k and obtained

$$I_k = \frac{\Gamma(1+2\epsilon)}{\Gamma(1+\epsilon)^2} \int_0^1 \mathrm{d}x\,\mathrm{d}y\, \frac{(1-x)^{-\epsilon} y^{\epsilon} \omega^{1+2\epsilon}}{(\omega x(1-y) + m_V^2 y(1-\omega x))^{1+2\epsilon}}, \tag{4.76}$$

where we have additionally rescaled the variable y as $y \to y(1-x)$ and set s to be -1. By doing so, the parameter m_V^2 becomes scaleless, though we keep the notation unchanged.

We multiply the two integrals and obtain

$$I = \frac{\Gamma(1+2\epsilon)}{\Gamma(1+\epsilon)^2} \int_0^1 \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\omega\, \frac{(1-x)^{-\epsilon} y^{\epsilon} \omega^{\epsilon}(1-\omega)^{-\epsilon}}{(\omega x(1-y) + m_V^2 y(1-\omega x))^{1+2\epsilon}}. \tag{4.77}$$

After the change of variables $u = (1-(1+m_V^2)y)/m_V^2 y$, the integral takes the final form

$$I = (m_V^2)^{-2\epsilon} \frac{\Gamma(1+2\epsilon)}{\Gamma(1+\epsilon)^2} \int_{-1}^{\infty} \mathrm{d}u\,(1+m_V^2(u+1))^{\epsilon-1} \int_0^1 \mathrm{d}x\,\mathrm{d}\omega\, \frac{(1-\omega)^{-\epsilon}\omega^{\epsilon}(1-x)^{-\epsilon}}{(\omega x u+1)^{1+2\epsilon}}. \tag{4.78}$$

We notice that the integration over the Feynman parameters x and ω can be easily performed. We obtain

$$I_\delta = \frac{\Gamma(1+2\epsilon)B(1+\epsilon,1-\epsilon)}{2\epsilon^2\Gamma(1+\epsilon)^2}(m_V^2)^{-2\epsilon} \int_{-1}^{\infty} \mathrm{d}u\,(1+m_V^2(u+1))^{\epsilon-1}\, \frac{1 - {}_2F_1(\epsilon,2\epsilon,1-\epsilon;-u)}{u^{1+\delta}}, \tag{4.79}$$

where ${}_2F_1$ is an ordinary hypergeometric function. We note that the integral of Eq. (4.79) is well-defined in the integration region $u \in (-1,\infty)$ with $\epsilon \in (-1,1)$. However, we split the integral into several integrals, and individually, these do not converge in the newly defined integration regions. To avoid this issue, we introduce an analytic parameter δ which governs the singular behaviour of the new integrals.

We begin by noticing that the required factor of $(m_V^2)^{-2\epsilon}$ is already factorized in Eq. (4.79). It follows that only the polynomial behaviour with respect to m_V^2 of the I_δ integral is required. We then write

$$I_\delta = \underbrace{\int_{-1}^{0} \ldots}_{I_0} + \underbrace{\int_{0}^{\alpha} \ldots}_{I_s} + \underbrace{\int_{\alpha}^{\infty} \ldots}_{I_h}, \tag{4.80}$$

where meaning of each region is explained below.

Consider first the integral

$$I_0 = \int_{-1}^{0} \mathrm{d}u\,(1+m_V^2(u+1))^{\epsilon-1}\, \frac{1 - {}_2F_1(\epsilon,2\epsilon,1-\epsilon;-u)}{u^{1+\delta}}, \tag{4.81}$$

where the normalization prefactor is omitted everywhere for brevity (it will be restored in the end). In Eq. (4.81), we can safely put δ to zero, since a divergence of the integrand at $u=0$ is integrable.

It is readily seen that due to boundaries of I_0, the term $(m_V^2(u+1))$ is always much smaller than 1. Therefore, we simply expand the integrand in m_V^2 to leading order

$$I_0 \approx \int_{-1}^{0} \mathrm{d}u \, (1 + O(m_V^2)) \, \frac{1 - {}_2F_1(\epsilon, 2\epsilon, 1-\epsilon; -u)}{u}. \tag{4.82}$$

To integrate Eq. (4.82), we expand the integrand in ϵ. We are allowed to do so since there is no unregulated divergences. To preform the expansion in ϵ of ${}_2F_1$, we use the *Mathematica*-based package `HypExp` [113]. Upon expanding the integrand in ϵ, we obtained

$$I_0 = \int_{-1}^{0} \mathrm{d}u \, \frac{2G(0,-1,u)\epsilon^2 - 2(4G(0,-1,-1,u) - G(0,0,-1,u))\epsilon^3 + O(\epsilon^4)}{u}, \tag{4.83}$$

which can be easily integrated following the prescription in Appendix B.2, and where it is also taken into account that GPLs at fixed points can be evaluated to Riemann zeta functions (see lectures of Ref. [92] for details). After the evaluation integral reads

$$I_0 \approx 2\zeta(3)\epsilon^2 + \frac{2\pi^4\epsilon^3}{45} + O(\epsilon^4) + O(m_V^2). \tag{4.84}$$

We proceed with the I_s and I_h integrals, which are regulated by a cut-off parameter α. As it was mentioned above, a cut-off parameter has a straightforward relation to the scaling properties of the adjacent regions. In the case under consideration, the scaling can be read off from $(1 + m_V^2(u+1))^{\epsilon-1}$. Indeed, it requires that $m_V^2(u+1) + 1 \sim 1$ which is possible if $m_V^2(\alpha+1) \ll 1$ in order to extract the desired branch. Then, $\alpha \ll \frac{1}{m_V^2}$, where we took into account the fact that m_V^2 is by definition small. However, we can constrain α from below, i.e. $\alpha \gg 1$, since it does not contradicts our original assumption. Finally, we arrive at the following condition

$$1 \ll \alpha \ll \frac{1}{m_V^2}. \tag{4.85}$$

Following the outlined procedure, we identify two regions which are called, following Ref. [103], *soft* and *hard*, respectively. They read

$$\begin{aligned} \mathcal{D}_s &= \{u \in (0,\alpha) : u \approx 1\}, \\ \mathcal{D}_h &= \{u \in (\alpha,\infty) : u \approx 1/m_v^2\}. \end{aligned} \tag{4.86}$$

In principle we can define the expansion operators at this point. However, for our needs, only the leading term of the expansion is needed. Yet, it may be always assumed that these operators are implied when we perform an expansion.

The integral I_s corresponds to the soft region. This integrand is expanded accordingly to its domain $\mathcal{D}_s$. We write

$$\begin{aligned} I_s = \int_{\mathcal{D}_s} \mathrm{d}u \, (1 + m_V^2(u+1))^{\epsilon-1} \, \frac{1 - F(\epsilon, 2\epsilon; 1-\epsilon; -u)}{u^{1+\delta}} \approx \\ \int_0^\infty \mathrm{d}u \, (1 + O(m_V^2)) \, \frac{1 - F(\epsilon, 2\epsilon; 1-\epsilon; -u)}{u^{1+\delta}} - \tilde{I}_s(\alpha,\epsilon), \end{aligned} \tag{4.87}$$

where we have expanded the integrand in m_V^2 and in the small parameter $m_V^2 u \ll 1$ and set the integration region to infinity. To avoid double-counting, we subtracted the resulting surplus integral, i.e., $\tilde{I}_s(\alpha, \epsilon)$. This integral is evaluated at the end of this section.

The integration over u can be performed using the well-known formulas for the integration of hypergeometric functions from Ref. [114]. The result reads

$$I_s = \frac{1}{\delta} + (-\psi^{(0)}(1-\epsilon) + \psi^{(0)}(\epsilon) + \psi^{(0)}(2\epsilon) + \gamma_E) + O(\delta) + +O\left(m_V^2\right), \tag{4.88}$$

where we have expanded the final answer in δ; ψ^0 is a polygamma function of order zero and γ_E is the Euler–Mascheroni constant.

The I_h integral requires a bit more care to be evaluated. Indeed, in $\mathcal{D}_h$ the ${}_2F_1$-function is not analytical. To analytically continue it to the integration region $\mathcal{D}_h$, we use the well-known relations for hypergeometric functions with arguments of forms x and $1/x$. In particular, we use the formula (9.132.2) for the analytic continuation of ${}_2F_1$ from Ref. [114]. It reads

$$\begin{aligned} {}_2F_1(\epsilon, 2\epsilon; 1-\epsilon; -u) = \frac{\Gamma(1-\epsilon)\Gamma(\epsilon)}{\Gamma(2\epsilon)\Gamma(1-2\epsilon)} u^{-\epsilon}\, {}_2F_1(\epsilon, 2\epsilon; 1-\epsilon; -1/u) \\ + \frac{\Gamma(1-\epsilon)\Gamma(-\epsilon)}{\Gamma(\epsilon)\Gamma(1-3\epsilon)} u^{-2\epsilon}\, {}_2F_1(2\epsilon, 3\epsilon; 1+\epsilon; -1/u). \end{aligned} \tag{4.89}$$

Upon applying Eq. (4.89) and expanding the integrand from Eq. (4.80) in accordance with its predefined region $\mathcal{D}_h$, we obtained the following integral

$$I_h = \int_0^\infty \mathrm{d}u\, (1 + m_V^2 u)^{\epsilon-1} u^{-1-\delta} - \tilde{I}_h(\alpha, m_V^2, \epsilon), \tag{4.90}$$

where $\tilde{I}_h(\alpha, m_V^2, \epsilon)$ is calculated below. Such drastic simplification is given by the scaling of terms that are generated by analytic continuation, i.e. $u^{-\epsilon}$ and $u^{-2\epsilon}$. These terms are omitted, since they contribute to different regions. However, we have to keep them in the $\tilde{I}_h(\alpha, m_V^2, \epsilon)$ integral since they are necessary ingredients to show that the surplus integrals vanish. The integration of Eq. (4.90) is now straightforward and amounts to using a definition of Euler B-function Eq. (B.31). We write

$$I_h \approx \int_0^\infty \mathrm{d}u\, (1 + m_V^2 u)^{\epsilon-1} u^{-1-\delta} = -\frac{1}{\delta} + (-\psi^{(0)}(1-\epsilon) + \log(m_V^2) - \gamma_E) + O(\delta), \tag{4.91}$$

We note that we have acquired a $\log(m_V^2)$ term which is not needed in the final answer.

The prefactor that we have omitted so far reads

$$F = \frac{\Gamma(2\epsilon+1) B(\epsilon+1, 1-\epsilon)}{2\epsilon^2 \Gamma(\epsilon+1)^2}. \tag{4.92}$$

Combining it with Eqs. (4.91, 4.88, 4.84) and expanding in ϵ, we obtain the final answer

$$I \propto (m_V^2)^{-2\epsilon} \left(\frac{3}{4\epsilon^3} - \frac{\pi^2}{6\epsilon} - \zeta(3) - \frac{53\pi^4 \epsilon}{360} + O\left(\epsilon^2\right) \right). \tag{4.93}$$

It is still necessary to evaluate the surplus integrals that appeared during the computation. First, we write the soft integral

$$\tilde{I}_s(\alpha, \epsilon) = \int_\alpha^\infty \mathrm{d}u\, u^{-1-\delta}(1 - {}_2F_1(\epsilon, 2\epsilon, 1-\epsilon; -u)) \approx \int_\alpha^\infty u^{-1-\delta}(1 - C_1 u^{-\epsilon} - C_2 u^{-2\epsilon}), \tag{4.94}$$

where the constants C_i can be read off from Eq. (4.89). We have taken into account the boundaries of this integral to expand the $\tilde{I}_s$ integral. Practically, we expanded the integrand in the *hard* region [103]. Note that we have omitted power suppressed terms. This is justified since these terms have the same structure as the leading ones and what changes is their exponents. Similarly, we proceed with the hard surplus integral

$$\tilde{I}_h(\alpha, m_V^2, \epsilon) = \int_0^\alpha \mathrm{d}u\, (1 + m_V^2 u)^{\epsilon-1} u^{-1-\delta}(1 - C_1 u^{-\epsilon} - C_2 u^{-2\epsilon}) \approx$$
$$\int_0^\alpha \mathrm{d}u\, (1 + O(m_V^2))\, u^{-1-\delta}(1 - C_1 u^{-\epsilon} - C_2 u^{-2\epsilon}), \tag{4.95}$$

We add up these integrals obtaining

$$\tilde{I}_h(\alpha, m_V^2, \epsilon) + \tilde{I}_s(\alpha, \epsilon) = \int_0^\infty \mathrm{d}u\, u^{-1-\delta}(1 - C_1 u^{-\epsilon} - C_2 u^{-2\epsilon}) = 0, \tag{4.96}$$

since in the dimensional regularization scheme scaleless integrals vanish. As it was pointed out at the beginning of this section, this is necessary condition, but it does not guarantee that all relevant regions are included. To this end, we cross-checked our results against the MB method discussed in the previous section as well as against a numerical evaluation with *SecDec* [59], finding the perfect agreement.

In conclusion, despite technical challenges and its open status [68], the method of regions offers a powerful alternative to compute boundary constants. It works well together with the advocated method of small parameter expansion. Indeed, by looking at the asymptotic solutions of differential equations, it is easy to see what kind of scaling of a region to expect. Then, the extraction of the required branch from a Feynman integral is just a technical problem. To this end, the tool `asy2.m` is able, in principle, to identify all the relevant regions [103]. Once this is done, it is possible to proceed with the algorithm described above.

Chapter 5

Results for the Higgs transverse momentum distribution

In this chapter we review how the physical cross section discussed in Chapter 2 is calculated. First, we discuss how helicity amplitudes are analytically continued to relevant kinematic regions. Second, we discuss the computational setup. Finally, we present the results for the production of the Higgs boson at leading order and next-to-leading order in the strong coupling constant α_s.

5.1. Analytic continuation of helicity amplitudes

We have showed in Section 2.5 how to calculate helicity amplitudes in the kinematic region $t > 0$, $s < 0$, $u < 0$ with $m_H^2 < 0$ for processes $H \to ggg$ and $H \to gq\bar{q}$ from Sections 2.11 and 2.12, respectively. We list the helicity coefficients of Section 2.5, that were computed in this kinematic region in Appendices (E.2, E.3). However, in order to calculate two-loop corrections to the $H + j$ production processes, i.e. $gg \to Hg$ and $q\bar{q} \to Hg$, we require other kinematic regions with positive invariant mass $m_H^2 > 0$.

We define the relevant kinematic regions by

$$2a_+ : \quad s > 0, t, u < 0, \tag{5.1}$$

$$3a_+ : \quad t > 0, s, u < 0, \tag{5.2}$$

$$4a_+ : \quad u > 0, s, t < 0. \tag{5.3}$$

These regions correspond to the production processes

$$2a_+ : g(-p_1) + g(-p_2) \to H(-p_4) + g(p_3),\ q(-p_2) + \bar{q}(-p_1) \to H(-p_4) + g(p_3), \tag{5.4}$$

$$3a_+ : g(-p_1) + g(-p_3) \to H(-p_4) + g(p_2),\ \bar{q}(-p_1) + g(-p_3) \to H(-p_4) + \bar{q}(p_2), \tag{5.5}$$

$$4a_+ : g(-p_2) + g(-p_3) \to H(-p_4) + g(p_1),\ q(-p_2) + g(-p_3) \to H(-p_4) + q(p_1). \tag{5.6}$$

In each region, we must follow the Feynman prescription. Hence, positive Mandelstam variables receive a small imaginary part in each corresponding region

$$2a_+ : s \to s + i0, \tag{5.7}$$

$$3a_+ : t \to t + i0, \tag{5.8}$$

$$4a_+ : u \to u + i0. \tag{5.9}$$

To acquire results in the $2a_+$ and $4a_+$ regions, we analytically continue the helicity amplitudes from the $3a_+$ region. To do so, we followed the procedure explained in Ref. [97], and, we refer to this paper for details. Here, we just state that in order to calculate the analytic continuation of the helicity amplitudes, one must follow a specific path in the complex plane spanned by the Mandelstam variables. This path crosses the branch cuts at $u = 0$, $t = 0$ or $s = 0$ and thus, affects the GPLs: they receive an imaginary part. For instance, a path from the region $3a_+$ to $2a_+$ crosses two branch cuts: $t = 0$ and $s = 0$. Logarithms $\log(-t/s)$ and $\log(u/s)$ changes according to

$$\begin{aligned} &\log\left(-\frac{t}{s}\right) \underbrace{\longrightarrow}_{t<0} \log\left(\frac{t}{s}\right) + \pi i \underbrace{\longrightarrow}_{s>0} \log\left(-\frac{t}{s}\right) + 2\pi i, \\ &\log\left(\frac{u}{s}\right) \underbrace{\longrightarrow}_{t<0} \log\left(\frac{u}{s}\right) \underbrace{\longrightarrow}_{s>0} \log\left(-\frac{u}{s}\right) + \pi i, \end{aligned} \tag{5.10}$$

while GPLs changes as in

$$G\left(a_1, \ldots, a_n; \frac{u}{s} \equiv z\right) \to \sum_i c_i G\left(b_{1,i}, \ldots, b_{n,i}; -\frac{1}{z}\right). \tag{5.11}$$

Such transformations are described in Ref. [94]. To do such continuation in practice, we have used the *Mathematica*-based package `HPL.m` from Ref. [115] where all relevant transformations (e.g. Eq. (5.11)) are implemented.

To present our results for the two-loop helicity amplitudes, we have performed analytic continuation to all relevant regions. GPLs that appear in our results are all real-valued functions and their imaginary parts are accurately extracted upon an analytic continuation. In each region, we have defined a new set of variables

$$2a_+ : \ u_{2a} = -\frac{u}{s} = -z, \tag{5.12}$$

$$3a_+ : \ u_{3a} = -\frac{s}{t} = \frac{1}{1+z}, \tag{5.13}$$

$$4a_+ : \ u_{4a} = -\frac{u}{s} = -\frac{1}{z}, \tag{5.14}$$

with $0 \leq u_i \leq 1$. All these transformations are checked against the *CHAPLIN* code from Ref. [116].

5.2. The Higgs transverse momentum distribution at leading order

We have calculated virtual corrections in the high transverse momentum limit up to leading order in the Higgs boson mass and subleading order in the top mass.

To see that such approximation is appropriate, we first check the leading order cross section (Fig. 5.1). This figure depicts three high energy expansions of the LO cross section which are compared to the exact cross section at LO: $O((m_H^2)^0, (m_t^2)^0)$, $O((m_H^2)^0, m_t^2)$ and $O(m_H^2, m_t^2)$. In the first case, we see a deviation of $O(30\%)$ at 800 GeV decreasing at higher values of p_T (dotted line). Including the subleading order in the top mass, the deviation decreases to the order of few percent (dashed line). Finally, expanding the exact cross section to $O(m_t^2, m_H^2)$ we get a per mill accuracy

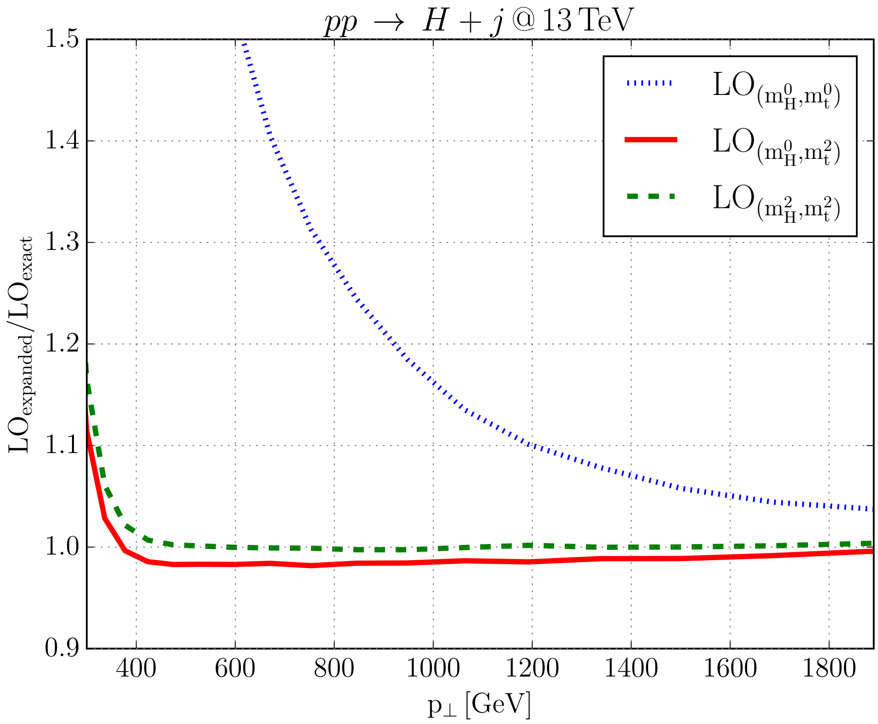


Figure 5.1.: We compare different expansions of LO cross section for $H + j$ against exact LO cross section. Expanding to the leading order in the Higgs and subleading order in the top mass leads to the difference of $O(1\%)$ above 400 GeV.

above 500 GeV (solid line). These results indicate that m_t/p_T and $m_H/2m_t$ are indeed good small parameters at LO and may be applied to the NLO calculations.

We note here that LO form factors are required to produce the NLO cross section. We keep them exact in the mass of the Higgs boson and in the top quark throughout NLO calculations.

5.3. Computation setup for $H + j$ production

The partonic cross sections $\sigma_{gg\to Hg}$ and $\sigma_{q\bar{q}\to Hg}$ consist of two parts: virtual and real emission corrections as we explained in Section 2.2.

While there exist analytical results with exact top-mass dependence for real emission processes $gg \to Hg + g$, $q\bar{q} \to Hg + g$, and etc. [58], we used their numerical implementation in the program *OpenLoops* [117, 118] which uses following codes: *COLLIER* [119], *CutTools* [120] and *OneLOop* [121]. The technical details of these programs can be found in the mentioned papers.

We note here that the numerical calculations of real emission corrections are highly non-trivial. Indeed, one must, e.g., deal with the singularities arising from the infrared structure of the underlain gauge theory. *OpenLoops* had been used in previous (N)NLO computations and is capable of dealing with the present calculations.

All virtual and real amplitudes have been inserted into the program *POWHEG-BOX* [122–124]. Using this program has two important advantages: it regularizes the infrared divergences using FKS subtraction [125] and it allows one to match fixed NLO calculation with parton showers.

5.4. The Higgs transverse momentum distribution at next-to-leading order

In this section, we review the results of Ref. [2], where the two-loop corrections [1] computed in the small parameter expansion have been combined with the real emission corrections to produce the p_T-distribution of the Higgs boson in the high transverse momentum region.

In our paper [2], we study proton-proton collisions at the LHC with a center-of-mass energy of $\sqrt{s} = 13\,\text{TeV}$. We use the pole mass scheme to renormalize the top mass $m_t = 173.2\,\text{GeV}$. The Higgs boson mass is taken to be $m_H = 125\,\text{GeV}$. We use a Fixed-Flavor Number Scheme (FFNS) with the number of flavors $n_f = 5$ and consider the bottom quark to be a massless parton. We choose to use the *NNPDF3.0* set of parton distribution functions [91, 126] and their provided implementation of the running of the strong coupling constant α_s. The renormalization and factorization scales are taken to be equal $\mu_R = \mu_F = \mu$. We choose the central value to be

$$\mu_0 = \frac{H_T}{2}, \quad H_T = \sqrt{m_H^2 + p_T^2} + \sum_j p_{T,j}, \tag{5.15}$$

where j indicates the parton in the final state, $p_{T,j}$ is the transverse momenta of the respective parton and p_T is the transverse momentum of the Higgs boson. To estimate theoretical uncertainties, we vary the factorization and the renormalization scales μ by a factor of two around the central value μ_0.

In Tab. 5.1, we present the inclusive cross sections at LO and NLO along with corresponding K-factor for different values of the minimal Higgs transverse momentum (1 column). In the 2 column, we include the inclusive results in the infinite top mass approximation. In the 3 column, we show our results obtained in the small parameter expansion. We refer to the former case as the Higgs Effective Field Theory (HEFT), and to the latter case as the Standard Model (SM). We observer that K_{SM} is close to 1.90 and it is almost not affected by changing p_T^{min}. This indicates that the QCD radiation corrections are almost insensitive to the effects of the top mass. We compare K_{SM} and K_{HEFT} at the central value to estimate the effect of the top mass. We observe a 4% difference at 400 GeV which increases with p_T^{min} reaching an effect of 6% at 1 TeV.

In Fig. 5.2, we present the transverse momentum distribution of the Higgs bosons. In the upper panel, the HEFT (dashed lines) and SM (solid lines) p_T-distribution are shown. The NLO QCD corrections increase the LO predictions by $\mathcal{O}(100\%)$ for both cases. We see that the HEFT results overestimate the production of the Higgs bosons at the high-p_T region as expected, since they do not include the top mass effects. In the lower panel, we present the theoretical uncertainties due to scale variation. We estimated theoretical uncertainty of our predictions to be at the level of 20%, which is a factor of two lower than LO theoretical uncertainties. We note that these uncertainties are almost insensitive to p_T^{min}.

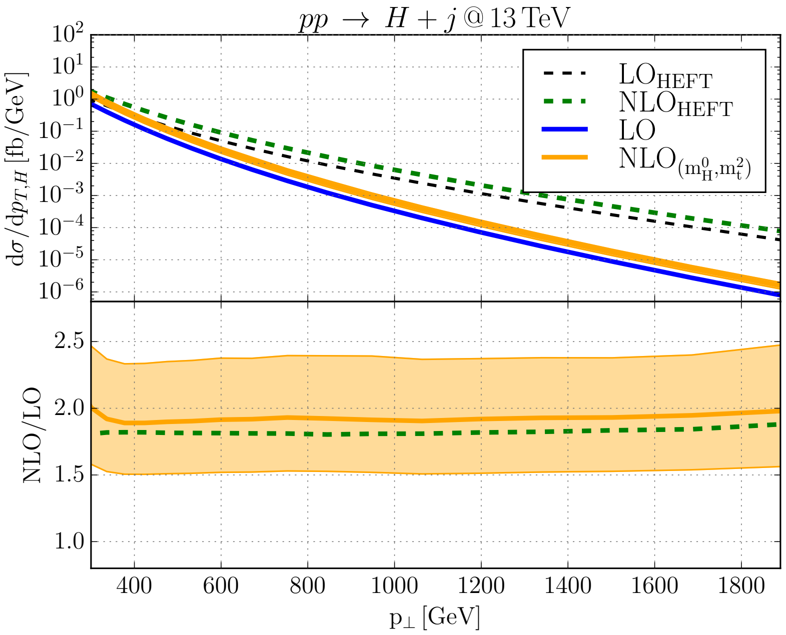


Figure 5.2.: In the upper panel, we show transverse momentum distribution of Higgs bosons at LO and NLO for HEFT (dashed lines) and the SM (solid lines). In the lower panel, ratio NLO/LO is shown. The band shows the theoretical uncertainty of our results due to scale variation.

To conclude this Section, results of Ref. [25] and Refs. [1, 2] in Fig. 5.3 are compared.[1] In particular, we compare partonic contributions of the type $\sum \mathrm{Re}(M_i^{\mathrm{LO}} M_i^{\mathrm{NLO}})$ where $i = g, q$ and $M_i^{\mathrm{LO}}, M_i^{\mathrm{NLO}}$ are matrix elements at LO and NLO, respectively. The sum goes over helicity configurations discussed in Section 2.5. In Fig. 5.3, we see the convergence of expanded results to exact ones. The major error is introduced by the partonic process $gg \to Hg$ and it is of $\mathcal{O}(10\%)$. This is the expected error since we consider the $m_H^2/(4m_t^2)$-expansion of the form factors from Section 2.3 only to leading order. Its value is ~ 0.1 for our choice of the Higgs and the top mass. Such large error is suppressed at the cross section, since the virtual contribution is of $\mathcal{O}(10\%)$ in the Higgs p_T-distribution at NLO. In Ref. [25], the effect of the top mass was calculated to be of the order 9%. We report that our results converge to this value and get very close to it at $p_T \sim \mathcal{O}(2\,\mathrm{TeV})$.

[1] Courtesy of Dr. M. Kerner and Dr. C. Wever.

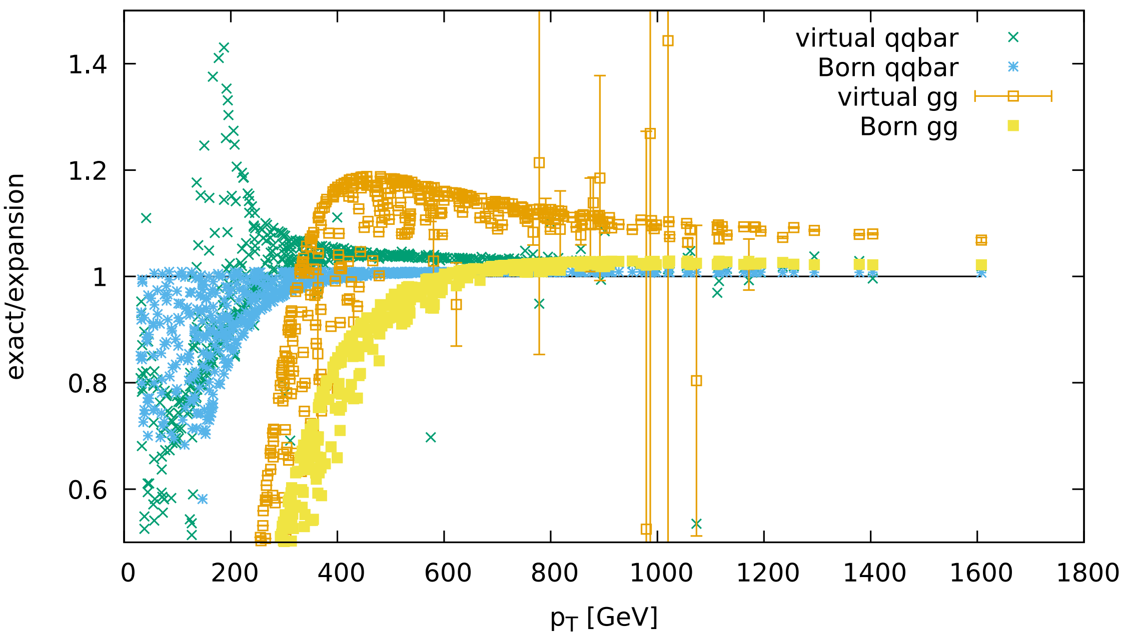


Figure 5.3.: Comparison of results for the Higgs boson production at high-p_T presented in Refs. [1, 2] with results of Ref. [25]. "Virtual" denotes virtual contributions, "Born" denotes Born contributions [27].

	LO$_{HEFT}$ [fb]	NLO$_{HEFT}$ [fb]	K_{HEFT}	LO [fb]	NLO [fb]	K_{SM}
$p_T^{\min}>$ 400 GeV	$33.8^{+44\%}_{-29\%}$	$61.4^{+20\%}_{-19\%}$	1.82	$12.4^{+44\%}_{-29\%}$	$23.6^{+24\%}_{-21\%}$	1.90
$p_T^{\min}>$ 450 GeV	$22.0^{+45\%}_{-29\%}$	$39.9^{+20\%}_{-19\%}$	1.81	$6.75^{+45\%}_{-29\%}$	$12.9^{+24\%}_{-21\%}$	1.91
$p_T^{\min}>$ 500 GeV	$14.7^{+44\%}_{-28\%}$	$26.7^{+20\%}_{-19\%}$	1.81	$3.80^{+45\%}_{-29\%}$	$7.28^{+24\%}_{-21\%}$	1.91
$p_T^{\min}>$ 1000 GeV	$0.628^{+46\%}_{-30\%}$	$1.14^{+21\%}_{-19\%}$	1.81	$0.0417^{+47\%}_{-30\%}$	$0.0797^{+24\%}_{-21\%}$	1.91

Table 5.1.: Inclusive cross sections and K-factors for $H + j$ production at different values of the minimal Higgs transverse momentum. The second column contains results obtained within the infinite top-mass approximation while the third one contains our results which includes the top-mass effects. The theoretical uncertainties are estimated by varying renormalization and factorization scales by a factor of two around the central value (Eq. (5.15)). K-factors are defined as an ration σ_{NLO}/σ_{LO}.

Chapter 6

Conclusions

In this thesis, we have studied the two production processes $pp \to H + j$ and $pp \to Z + j$. In the high-p_T region, these processes may be sensitive to the effects of physics beyond the Standard Model. To observe these effects, one needs to give accurate theoretical predictions for the Higgs boson and Z boson p_T-distributions. Unfortunately, higher-order corrections to differential observables are usually hard to calculate. In particular, calculations of multiloop Feynman integrals pose still a difficult problem in spite of remarkable advancements of computational techniques in the past few decades. Being interested mainly in the phenomenology of $H + j$ and $Z + j$ processes, we advocated a different approach to this problem. This approach stems from an observation that these processes have a similar signature in the high transverse momentum region: a hierarchy of scales. In the $H + j$ case, the hierarchy is given by $m_H < m_t \ll p_T$ which leads to two small parameters, $m_H^2/(4m_t^2)$ and m_t^2/p_T^2, while in the case of $Z + j$ production there is only one small parameter m_V^2/p_T^2. Using the expansions of Feynman integrals in these small parameters we were able to extend the applicability of the differential equation method to both processes of interest.

In the case of $H + j$ production, we have computed all Feynman integrals in the small parameter expansions that are relevant for the two-loop processes $gg \to Hg$, $q\bar{q} \to Hg$ and $qg \to Hq$ in the high-p_T region. It allowed us to compute two-loop helicity amplitudes to the aforementioned processes in the high-p_T region. Upon calculating two-loop contributions, we combined the existing real emission corrections with virtual contributions to produce NLO QCD corrections to the production of Higgs bosons at large transverse momenta. These NLO corrections increased the exiting LO predictions by $\mathcal{O}(100\%)$. The top mass effects grows with p_T-cut converging to 9% at $p_T \sim \mathcal{O}(2\,\text{TeV})$ as predicted by numerical computations [25]. We showed that the radiative corrections are almost independent of the top mass effects. The comparison between [2] and [25] is presented in this thesis.

We have made the first step towards the calculations of mixed QCD&EW corrections to the $Z + j$ production. Namely, we have computed master integrals that are relevant to the $q\bar{q} \to Zg$ process at large transverse momentum. Additionally, we employed the equal-mass approximation $m_W \approx m_Z = m_V$. To compute the master integrals, we first analysed relevant Feynman amplitudes for $Z+j$ and identified 18 Feynman integral families. In our analysis, we neglected contributions from the top quark. In these 18 integral families, we have selected the master integrals, and subsequently, we preformed reductions for these integrals. This allowed us to obtain differential equations and then to solve them using the expansion in a small parameter m_V/p_T. As the last step, we computed all boundaries conditions for the master integrals.

Since expanding Feynman integrals in small parameters tends to decrease the difficulty of multiloop calculations, one can think of applying this method to different observables that may be out of reach of the current multiloop computational techniques. In particular, including the top quark contributions will further improve QCD&EW corrections to $Z + j$ production.

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Appendix A

Notation

A.1. Feynman rules for QCD

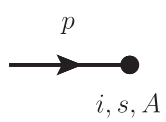
Here we present Feynman rules that we used to draw Feynman diagrams for the scattering amplitudes $q\bar{q} \rightarrow Hg$, $qg \rightarrow Hq$ and $gg \rightarrow Hg$. Following conventions are assumed, when we draw Feynman diagrams

- For color algebra SU(N), we use Latin letters $a, b, c, \ldots$ to denote the adjoint representation, while $i, j, k, \ldots$ to denote the fundamental representation.
- Greek letters $\mu, \nu, \rho, \sigma, \ldots$ are Lorenz indices. There are two exceptions: η is Minkowski metric tensor and λ is used to denote polarization states of gluons.
- Capital Latin letters $A, B, \ldots$ denote Dirac indices.
- We reserve Latin letters p, q, k for momenta of particles. Also, s is reserved to indicate spin states.

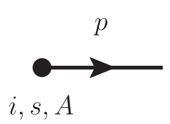
Additionally, $g_s = \sqrt{4\pi\alpha_S}$ is the strong charge with α_S being the strong coupling constant. We used [61, 127] to prepare this section.

External legs

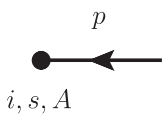
- Fermion external legs



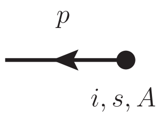
$$= u^s_A(p), \tag{A.1}$$



$$= \bar{u}^s_A(p), \tag{A.2}$$

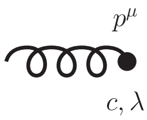


$$= v^s_A(p), \tag{A.3}$$

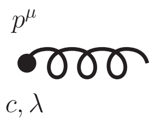


$$= \bar{v}^s_A(p). \tag{A.4}$$

- Gluon external legs



$$= \epsilon_{\mu,\lambda}, \tag{A.5}$$



$$= \epsilon^\star_{\mu,\lambda}. \tag{A.6}$$

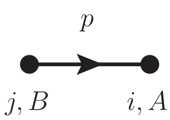
- Higgs external legs

$=$

$$= 1. \tag{A.7}$$

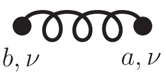
Propagators

- Top quark propagator



$$= \frac{i\delta_{ij}}{(\not{p} - m_t + i0)_{AB}} \tag{A.8}$$

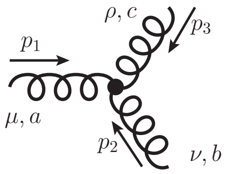
- Gluon propagator



$$= \frac{i\delta_{ij}\eta_{\mu\nu}}{p^2 + i0} \tag{A.9}$$

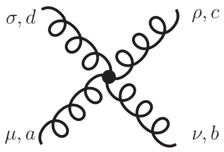
Vertices

- Three gluon vertex (all momenta is incoming)



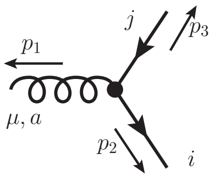
$$= -g_s f^{abc}[\eta^{\mu\nu}(p_1 - p_2)^\rho + \eta^{\nu\rho}(p_1 - p_2)^\mu + \eta^{\mu\rho}(p_1 - p_2)^\nu] \tag{A.10}$$

- Four gluon vertex



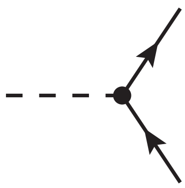
$$= -g_s^2[f_{eab}f_{ecd}(\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho}) + f_{eac}f_{edb}(\eta_{\mu\sigma}\eta_{\rho\nu} - \eta_{\mu\nu}\eta_{\sigma\rho}) + f_{ead}f_{ebc}(\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma})] \tag{A.11}$$

- Fermion gauge interaction (all momenta is outgoing)



$$= -ig_s\gamma^\mu T^a_{ij} \tag{A.12}$$

- Top-Higgs vertex



$$= -i\frac{g}{2}\frac{m_t}{m_W} \tag{A.13}$$

Appendix B

Analytical techniques

B.1. Spinor-helicity formalism

In this appendix, we describe the spinor-helicity formalism.

Since it is a known procedure [61, 128], we will briefly outline the main idea. Then, we will cast the characters which appears when one is performing computations in the spinor-helicity formalism and show how one should handle spinors.

When computing a scattering amplitude, one always face the problem of having huge expressions. As we have already seen in Section 2.3, there are many ways to facilitate computations. Most of them (if not all of them) come from symmetries. However, some symmetries are more obvious than others. This way of thinking leads to a question: is there a more suitable set of variables, functions, etc. for a particular scattering amplitude? To answer this question, we should consider two observations which are relevant for the type of computations performed in this thesis. First, it is often the case that external fermions are considered to be massless. Second, almost all particles have non-zero spin. Then, we can reformulate our question: which is the most convenient way of treating spinors? In Section 2.3 we did not assume anything special except for the fact fact that we are working with regular infinite-dimensional spinors. The answer is then: instead of using an infinite-dimensional representation of spinors, i.e. working with spinor fields, it is more convenient to choose a "smaller" representation: the left and/or right Weyl representations.

Since we are going to work with the Weyl representations, we need to define Pauli matrices which appear quite often

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{B.1}$$

Out of these matrices we can compose two 4-vectors $\sigma^\mu = (\mathbf{1}, \vec{\sigma})$ and $\bar{\sigma}^\mu = (\mathbf{1}, -\vec{\sigma})$, where the bold unity represents the 2×2 identity matrix. With these definitions we can easily define the Dirac matrices in the Weyl basis

$$\gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{bmatrix} \tag{B.2}$$

Now, we are going to identify Weyl spinors with the 4-momentum of a fermion

$$k_i^\mu \quad \to \quad u_R(k_i) \equiv |i_R\rangle \equiv (\tilde{\lambda}_i)_{\dot{\alpha}}, \quad u_L(k_i) \equiv |i_L\rangle \equiv \lambda_i^\alpha . \tag{B.3}$$

Here k_i^μ is the momentum of the fermion under consideration, $u_L(k_i)$ and $u_R(k_i)$ are left-handed and right-handed bispinors in Dirac notation, λ_i^α and $(\lambda_i)_{\dot\alpha}$ are left-handed and right-handed spinors, respectively, in Weyl representation, $\alpha, \dot\alpha = 1, 2$. In order to construct full bracket products we also need a notation for conjugated spinors

$$k_i^\mu \quad \rightarrow \quad \bar u_R(k_i) \equiv \langle i_R| \equiv \lambda_i^\alpha, \quad \bar u_L(k_i) \equiv \langle i_L| \equiv (\tilde\lambda_i)_{\dot\alpha}. \tag{B.4}$$

The dot denotes a different representation of the Lorenz group.

These spinors satisfy the Dirac equations for massless particles

$$\hat k_i u_{L/R} = 0, \tag{B.5}$$

where $\hat k = \gamma^\mu k_\mu$. At this points, it should be noted that the positive and negative energy solutions are the same up to a normalization factor. In other words, we only need one of them; the other solution can be acquired automatically.

To construct a scalar product we need an additional tensor which will act as a metric tensor in the spinor space. In Weyl basis this tensor is anti-symmetric[1]

$$i\sigma_2^{\alpha\beta} \equiv \varepsilon^{\alpha\beta} = -\varepsilon_{\alpha\beta} = \varepsilon^{\dot\alpha\dot\beta} = -\varepsilon_{\dot\alpha\dot\beta} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{B.6}$$

To rise/lower spinor indices we will use the following rules

$$\begin{aligned} (\lambda_i)_\beta \varepsilon^{\alpha\beta} = \lambda_i^\alpha, \quad \lambda_i^\beta \varepsilon_{\alpha\beta} = (\lambda_i)_\alpha \\ \tilde\lambda_i^{\dot\beta} \varepsilon_{\dot\alpha\dot\beta} = (\tilde\lambda_i)_{\dot\alpha}, \quad (\tilde\lambda_i)_{\dot\beta} \varepsilon^{\dot\alpha\dot\beta} = (\tilde\lambda_i)^{\dot\alpha}. \end{aligned} \tag{B.7}$$

It is necessary since ε is an anti-symmetric tensor. Thus, we can easily demonstrate how spinor products computed in this formalism

$$\begin{aligned} \langle ij\rangle \equiv \langle i_R i_L\rangle = \varepsilon^{\alpha\beta}(\lambda_i)_\alpha(\lambda_j)_\beta = (\lambda_i)_\alpha \lambda_j^\alpha, \\ [ij] \equiv \langle i_L i_R\rangle = \varepsilon^{\dot\alpha\dot\beta}(\tilde\lambda_i)_{\dot\beta}(\tilde\lambda_j)_{\dot\alpha} = \tilde\lambda_i^{\dot\alpha}(\tilde\lambda_j)_{\dot\alpha} \end{aligned} \tag{B.8}$$

We have omitted R/L subscripts in the leftmost part of the relations. Indeed, square and angle brackets unambiguously indicate the spinor representation. Note that it is exactly the order we need to be consistent with four dimensional spinors. In such a way, we can easily derive that

$$[ij] = \varepsilon^{\dot\alpha\dot\beta}(\tilde\lambda_i)_{\dot\beta}(\tilde\lambda_j)_{\dot\alpha} = \tilde\lambda_i^{\dot\alpha}(\tilde\lambda_j)_{\dot\alpha} = -(\tilde\lambda_i)_{\dot\alpha}\tilde\lambda_j^{\dot\alpha} = -[ji], \tag{B.9}$$

we can also show that $\langle ij\rangle = -\langle ji\rangle$. In a similar way, it is trivial to show that $[ii] = \langle ii\rangle = 0$ due to anti-symmetry of the $\varepsilon_{\alpha\beta}$ tensor. Also, it is easy to show that

$$\langle ij\rangle^* = \varepsilon^{\alpha\beta}(\tilde\lambda_i)_{\dot\alpha}(\tilde\lambda_j)_{\dot\beta} = \tilde\lambda_j^{\dot\alpha}(\tilde\lambda_i)_{\dot\alpha} = [ji] \tag{B.10}$$

Now we turn our attention to 4-momenta. As said, 4-momenta transform under the vector representation of $SO(3,1)$. However, we can also defined a 4-momentum in the spinor basis

$$k^{\alpha\dot\alpha} \equiv k^\mu \sigma_\mu^{\alpha\dot\alpha}, \quad k_{\dot\alpha\alpha} = k_\mu \bar\sigma^\mu_{\dot\alpha\alpha}, \quad k^\mu = \frac{1}{2}(\sigma^\mu)^{\alpha\dot\alpha}k_{\alpha\dot\alpha} \quad k^\mu = \frac{1}{2}\bar\sigma^\mu_{\dot\alpha\alpha}k^{\dot\alpha\alpha}, \tag{B.11}$$

[1]There are actually two tensors which are defined in two different spinor bases.

where we have used the following equation to derive the rest of relations

$$\sigma_{\mu}^{\alpha\dot{\alpha}}(\sigma^{\mu})^{\beta\dot{\beta}} = 2\varepsilon^{\alpha\beta}\varepsilon^{\dot{\alpha}\dot{\beta}}. \tag{B.12}$$

Then, a 4-momentum in spinor representation takes the following matrix form

$$k^{\alpha\dot{\alpha}} = \begin{bmatrix} k^0 - k^3 & -k^1 + ik^2 \\ -k^1 - ik^2 & k^0 + k^3 \end{bmatrix}. \tag{B.13}$$

For massless momenta, the determinant of this matrix is zero

$$\det(k^{\alpha\dot{\alpha}}) = 0. \tag{B.14}$$

It follows from linear algebra that any 2×2 matrix with zero determinant can be decomposed into a product of two vectors. In the case of 4-momenta, it turns out that these vectors are exactly the spinors from Eq. (B.3)

$$k^{\alpha\dot{\alpha}} = \lambda^{\alpha}\tilde{\lambda}^{\dot{\alpha}} = i\rangle[i, \quad k_{\dot{\alpha}\alpha} = \tilde{\lambda}_{\dot{\alpha}}\lambda_{\alpha} = i]\langle i. \tag{B.15}$$

Then, due to Eq. (B.12) and Eq. (B.8) the scalar product becomes

$$k_i \cdot k_j = k_i^{\mu}(k_j)_{\mu} = \frac{1}{2}\langle\lambda_i\lambda_j\rangle\,[\lambda_j\lambda_i]. \tag{B.16}$$

Finally, we arrive to the point when we can discuss how we can incorporate vector bosons into this formalism. It is well known that polarisations of a vector boson satisfy $\epsilon \cdot \epsilon = 0$. In a spinor basis, it can be represented by an outer product of two spinors. Yet, this representation should be fixed with an extra condition since a complex 4-vector has, in general, three degrees of freedom while a polarisation of a massless gauge boson only two. It is done by introducing an a lightlike 4-momentum r^{μ} called the reference momentum. It can be any lightlike vector which is not aligned with a 4-momentum of a corresponding gauge boson. In practice, it is usefully to use other external momenta as a reference vector. Then, the expression for a polarization vector in the spinor basis is

$$\epsilon_R^{\alpha,\dot{\alpha}}(i,j) = \sqrt{2}\frac{i\rangle[j}{[ij]}, \quad \epsilon_L^{\dot{\alpha},\alpha} = \sqrt{2}\frac{j]\langle i}{\langle ji\rangle}, \tag{B.17}$$

where $i \longrightarrow k_i$ the 4-momentum of the gauge boson, $j \longrightarrow k_j, \forall j : k_j \not\parallel k_i$; k_i denotes the momentum of an external particle. It is readily seen that all formulae involving polarizations still hold.

There is a very useful property of polarizations which we want to mention here. Since spinors are two-dimensional objects and $k_i \cdot k_j \neq 0$, we are confined in a linear space spanned by these two vectors. Then, the reference vector k_j admits the following relation

$$k_{j'} \longrightarrow k_j + k_i : \epsilon_R(i,j') = \sqrt{2}\frac{k_i\rangle[k_{j'}}{[k_i k_{j'}]} = \sqrt{2}\frac{k_i\rangle[(k_j + k_i)}{[k_i(k_j + k_i)]} = \sqrt{2}\frac{k_i\rangle[k_j}{[k_i k_j]} + \sqrt{2}\frac{k_i\rangle[k_i}{[k_i k_j]}. \tag{B.18}$$

We have chosen an arbitrary reference vector, hence any scattering amplitude must fulfil this condition $M_{\mu}k^{\mu} = 0$ which nothing else but a Ward identity. The translation $k_{j'} \longrightarrow k_j + k_i$ is a gauge transformation.

B.2. Goncharov polylogarithms

Higher order corrections involves a computation of multi-loop integrals in the dimensional regularisation scheme. Such integrals have complicated analytical structure which encapsulated in logarithms and polylogarithms $Li_n(z)$ at one loop level. However, it appears that even more a general extension of logarithms is needed when one interested in even higher corrections. Such a generalisation is given by Goncharov polylogarithms (GPLs) [84] (also known as generalized polylogarithms, or hyperlogarithms [129]). They play an essential role in contemporary computations. We will define them recursively

$$\begin{aligned} G(a_1, \ldots, a_n; x) &= \int_0^x \frac{dt}{t-a_1} G(a_2, \ldots, a_n; t) \quad \text{with} \\ G(a; x) &:= \int_0^x \frac{dt}{t-a} \end{aligned} \tag{B.19}$$

where $\vec{a} = \{a_1 \ldots a_n\}$ is called the *weight vector* and its length is called the *weight*. There are two special cases

$$\begin{aligned} G(\vec{a}_n; x) &= \frac{1}{n!} \log^n \left(1 - \frac{x}{a}\right), \\ G(\vec{0}_n; x) &= \frac{1}{n!} \log^n(x), \end{aligned} \tag{B.20}$$

when all entries are the same or zero respectively. When the recursion reaches the last element, i.e. the rightmost entry, it stops: $G(;x) = 1$. These functions are analytic whenever $a_n \neq 0$ at $x = 0$. We refer to elements of a vector a as *letters*. The set of all letters that appear in the computation of master integrals is called *alphabet*

GPLs present the *shuffle* algebra, with the shuffle product

$$G(a_1, \ldots, a_m; x)\, G(a_{m+1}, \ldots, a_n; x) = \sum_\sigma G\left(a_{\sigma(1)}, \ldots, a_{\sigma(n)}; x\right), \tag{B.21}$$

where the sum goes over all permutations σ which preserves the original order in sets $\{a_1 \ldots a_m\}$ and $\{a_{m+1} \ldots a_n\}$. The shuffle product preserves the combined weight of GPLs, i.e. if W is a function which maps GPLs to the weight, then $W[G(\vec{a}; x)G(\vec{b}; x)] = n + m$. Hence, this algebra is graded with respect to the weight.

Let us discuss some of the basic properties of GPLs. If the left most entry in a GPL is not zero, then this GPL is invariant under the following rescaling

$$G(\lambda\vec{a}; \lambda x) = G(\vec{a}; x), \tag{B.22}$$

where λ is a complex number. GPLs diverge whenever $x = a_1$ due to end-point singularity (see Eq. (B.19)). Also, we mentioned that they are analytic at $x = 0$ whenever $a_n \neq 0$, but if it is happened that the last entry is zero, thanks to the shuffle algebra, we can rewrite GPLs. For example,

$$G(a, 0; x) = G(a; x)G(0; x) - G(0, a; x) \tag{B.23}$$

GPLs admit *functional equations*. One of them is *Hölder convolution*

$$G(a_1,\ldots,a_n;1)=\sum_{k=0}^{n}(-1)^k G(1-a_k,\ldots,1-a_1;1-\frac{1}{p})G(a_{k+1},\ldots,a_n;\frac{1}{p}). \tag{B.24}$$

This expression is valid for any $p\in\mathbb{C}$, if $a_1\neq 0$ and $a_n\neq 0$. Then, if $p\longrightarrow\infty$ we see that all entries are shifted by 1

$$G(a_1,\ldots,a_n;1)=(-1)^n G(1-a_n,\ldots,1-a_1;1). \tag{B.25}$$

There is plenty of these relations which were derived by means of Hopf algebra [92]. We will not review them here.

Goncharov polylogarithms can be related to different instances of polylogarithms. One of this instances is called Harmonic polylogarithms (HPLs). HPLs are a special type of GPLs, since all entries of HPLs are from the set $\{-1,0,1\}$. They are related via

$$H(a_1,\ldots,a_n;z)=(-1)^p G(a_1,\ldots,a_n;x) \tag{B.26}$$

where p denots the number of element in $\vec{a}$ which equal to $+1$.

B.3. Mellin-Barnes representation

In this section, we present Millin-Barnes (MB) representation.

A denominator $D=A+B$, raised to arbitrary power p can be written as a contour integral in a complex domain

$$D^{-p}\equiv(A+B)^{-p}=\lim_{\delta\to 0}\frac{1}{2\pi i\Gamma(p)}\int_{-i\infty}^{+i\infty}\mathrm{d}z A^{-p+z+\delta}B^{z+\delta}\Gamma(p+z+\delta)\Gamma(-p-\delta), \tag{B.27}$$

where δ is a real number defined in such a way, that poles of gamma functions $\Gamma(p+z)\,\Gamma(-p)$ are separated.

Sometimes the integration of MB integrals can be performed exactly by means of two Barnes' lemmas. The first Barnes lemma reads

$$\int_{-i\infty}^{+i\infty}\mathrm{d}z\Gamma(a_1+z)\Gamma(a_2+z)\Gamma(a_3+z)\Gamma(a_4+z)=\frac{\Gamma(a_1+a_3)\Gamma(a_1+a_4)\Gamma(a_2+a_3)\Gamma(a_2+a_4)}{\Gamma(a_1+a_2+a_3+a_4)}. \tag{B.28}$$

The second Barnes lemma reads

$$\int_{-i\infty}^{+i\infty}\mathrm{d}z\frac{\Gamma(a_1+z)\Gamma(a_2+z)\Gamma(a_3+z)\Gamma(a_4-z)\Gamma(a_5-z)}{\Gamma(a_1+a_2+a_3+a_4+a_5+z)}= \\ \frac{\Gamma(a_1+a_4)\Gamma(a_1+a_5)\Gamma(a_2+a_4)\Gamma(a_2+a_5)\Gamma(a_3+a_4)\Gamma(a_3+a_5)}{\Gamma(a_1+a_2+a_4+a_5)\Gamma(a_1+a_3+a_4+a_5)\Gamma(a_2+a_3+a_4+a_5)}. \tag{B.29}$$

Another useful formula is the MB representation of the ${}_2F_1$ hypergeometric function

$$\frac{\Gamma(a_1)\Gamma(a_2)}{\Gamma(a_3)}\,{}_2F_1(a_1,a_2;a_3;a_4)=\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\mathrm{d}z(-z)^{a_4}\frac{\Gamma(a_1+z)\Gamma(a_2+z)\Gamma(-z)}{\Gamma(a_3+z)}, \tag{B.30}$$

where $|\arg(-z)| < \pi$. The path of the contour integral is chosen in a such a way that separates poles of $\Gamma(a_1 + z)\Gamma(a_2 + z)$ from the poles of $\Gamma(-z)$.

We also give definition of Euler B-function, since we used it quite often. It reads

$$B(x, y) = \int_0^\infty \mathrm{d}t \frac{t^{x-1}}{(1+t)^{x+y}}, \tag{B.31}$$

with $\mathrm{Re}(x) > 0$ and $\mathrm{Re}(y) > 0$.

Appendix C

Master integrals

Here we list propagators and solutions of master integrals which appear in calculations of the virtual amplitude for the processes $gg \to Hg$, $q\bar{q} \to Hg$ and $q\bar{q} \to Zg$.

We denote the two-loop four-points scalar integral as

$$\mathcal{I}_m(a_1, a_2, \ldots, a_8, a_9) = \int \frac{\mathfrak{D}^d k \mathfrak{D}^d l}{[1]^{a_1}[2]^{a_2}[3]^{a_3}[4]^{a_4}[5]^{a_5}[6]^{a_6}[7]^{a_7}[8]^{a_8}[9]^{a_9}}. \tag{C.1}$$

The index m indicates a particle in the loop. For $H + j$ it is the top quark, for $Z + j$ is the massive vector boson particle.

For both processes, we use the same integration measure. It reads

$$\mathfrak{D}^d k = (-s)^{(4-d)/2} \frac{(4\pi)^{d/2}}{i\Gamma(1+\epsilon)} \int \frac{\mathrm{d}^d k}{(2\pi)^d}. \tag{C.2}$$

C.1. Topologies and top sectors for H + jet two-loop amplitudes

Prop.	PL1	PL2	NPL
[1]	k^2	$k^2 - m_t^2$	$k^2 - m_t^2$
[2]	$(k-p_1)^2$	$(k-p_1)^2 - m_t^2$	$(k+p_1)^2 - m_t^2$
[3]	$(k-p_1-p_2)^2$	$(k-p_1-p_2)^2 - m_t^2$	$(k-p_2-p_3)^2 - m_t^2$
[4]	$(k-p_1-p_2-p_3)^2$	$(k-p_1-p_2-p_3)^2 - m_t^2$	$l^2 - m_t^2$
[5]	$l^2 - m_t^2$	$l^2 - m_t^2$	$(l+p_1)^2 - m_t^2$
[6]	$(l-p_1)^2 - m_t^2$	$(l-p_1)^2 - m_t^2$	$(l-p_3)^2 - m_t^2$
[7]	$(l-p_1-p_2)^2 - m_t^2$	$(l-p_1-p_2)^2 - m_t^2$	$(k-l)^2$
[8]	$(l-p_1-p_2-p_3)^2 - m_t^2$	$(l-p_1-p_2-p_3)^2 - m_t^2$	$(k-l-p_2)^2$
[9]	$(k-l)^2 - m_t^2$	$(k-l)^2$	$(k-l-p_2-p_3)^2$

Table C.1.: Feynman propagators of the three integral families for the processes $gg \to Hg$, $q\bar{q} \to Hg$.

Planar PL1 master integrals

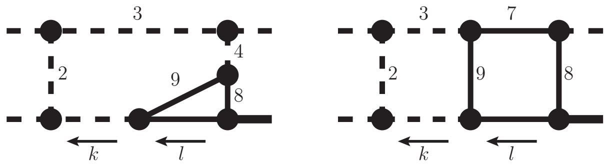
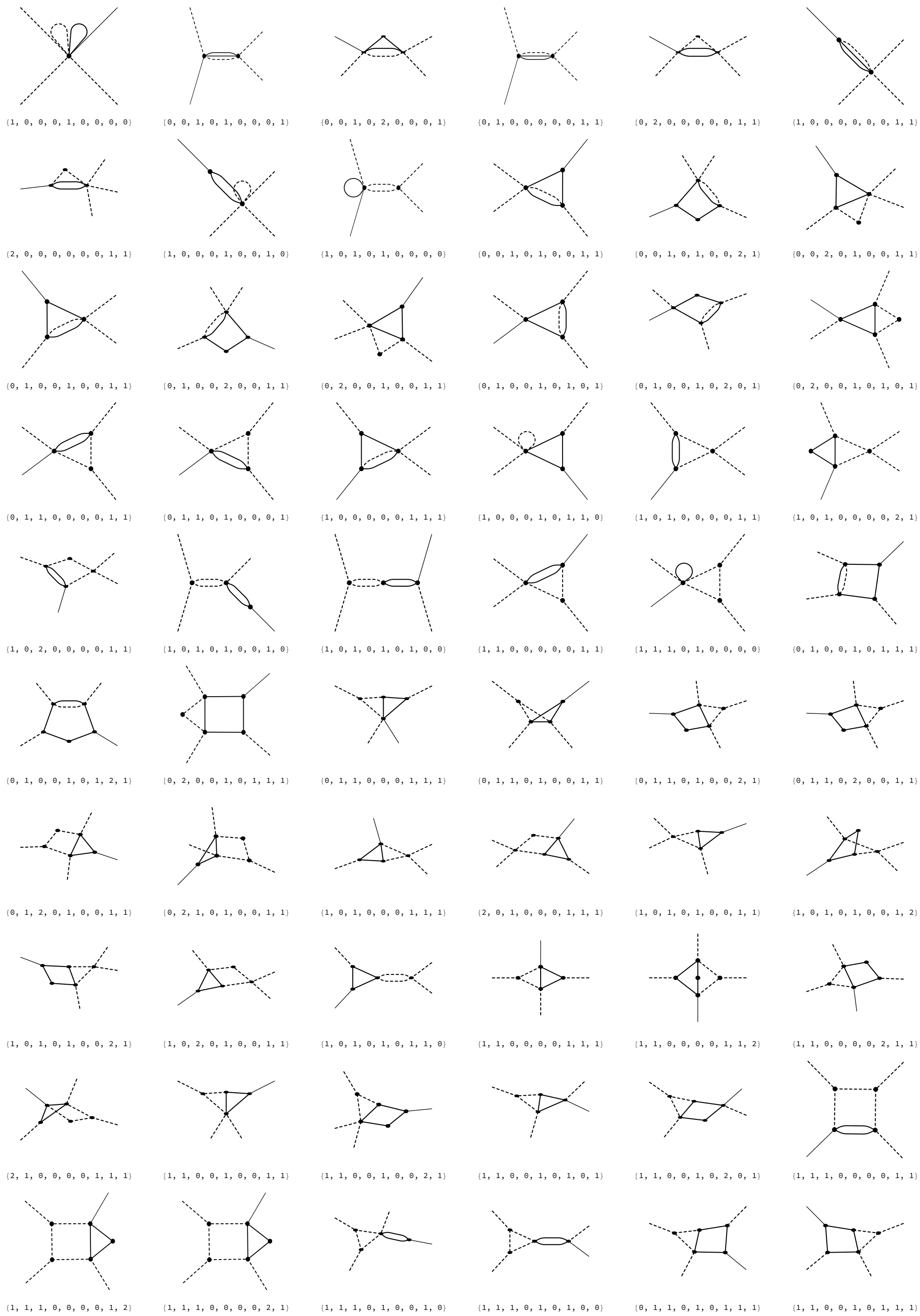
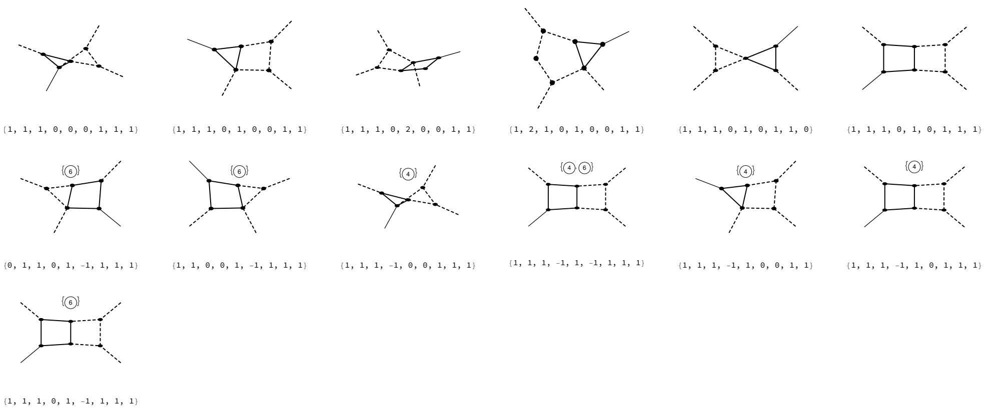
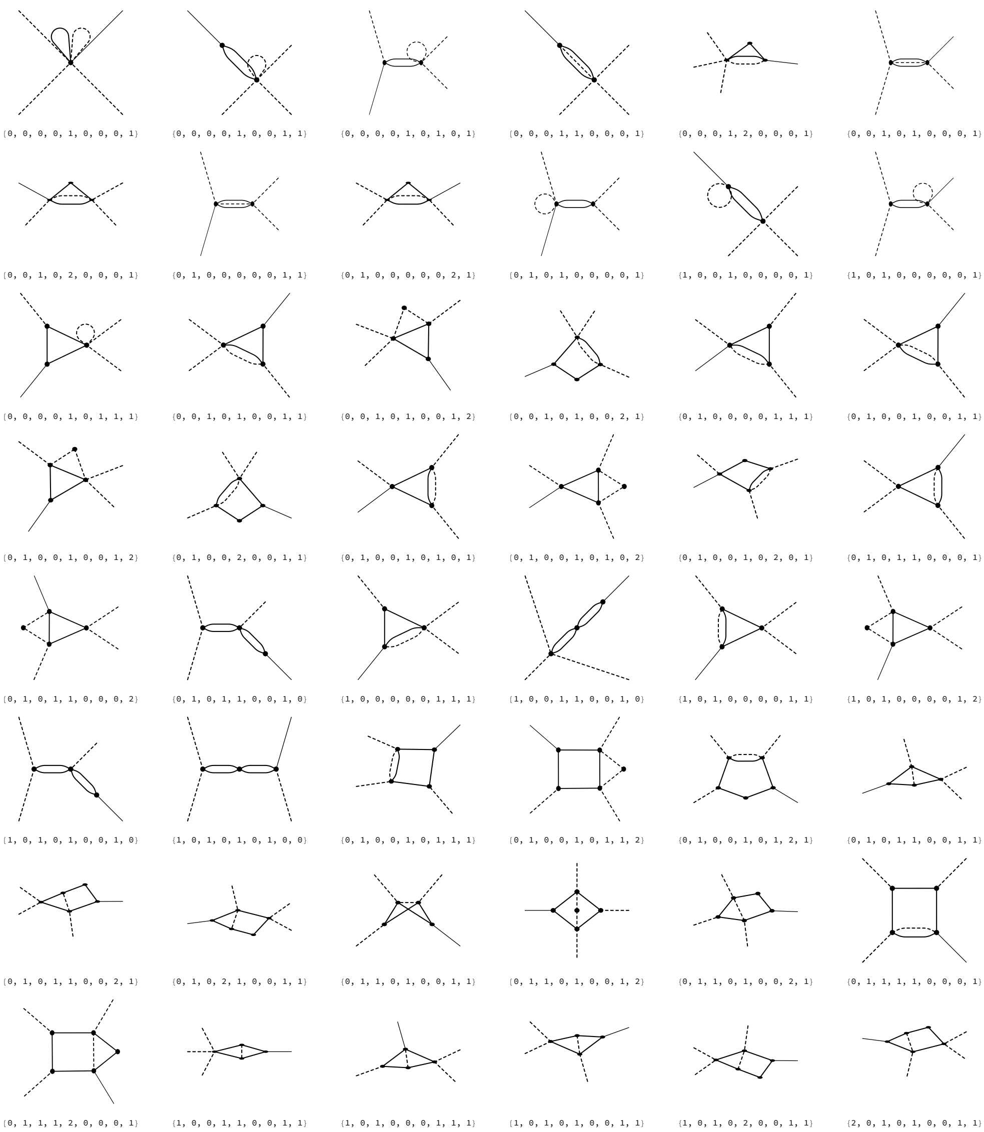


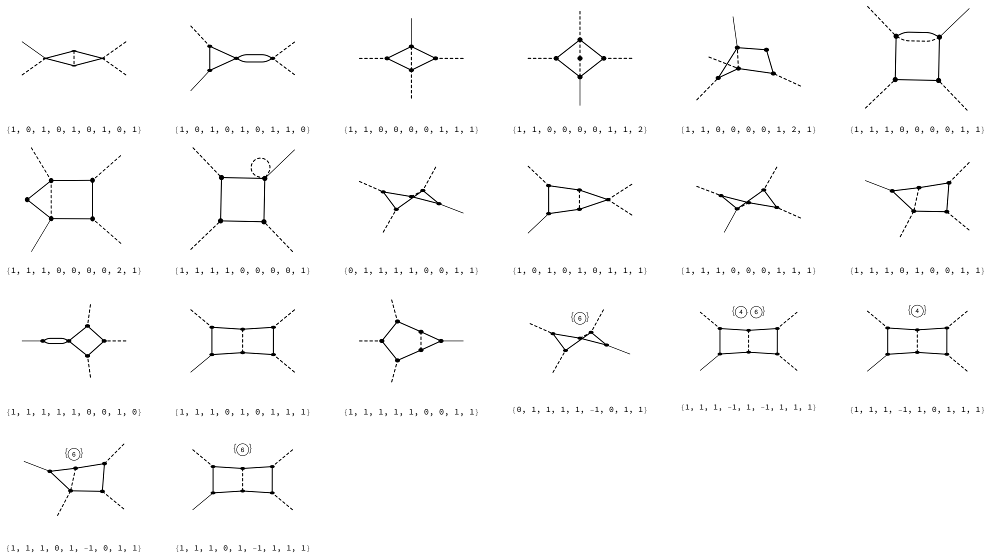
Figure C.1.: Top-sectors of the topology *PL1*.





Planar PL2 master integrals





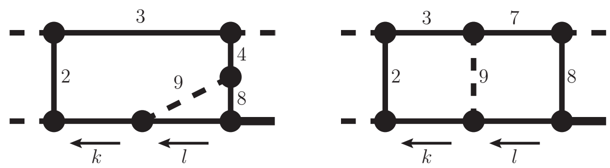
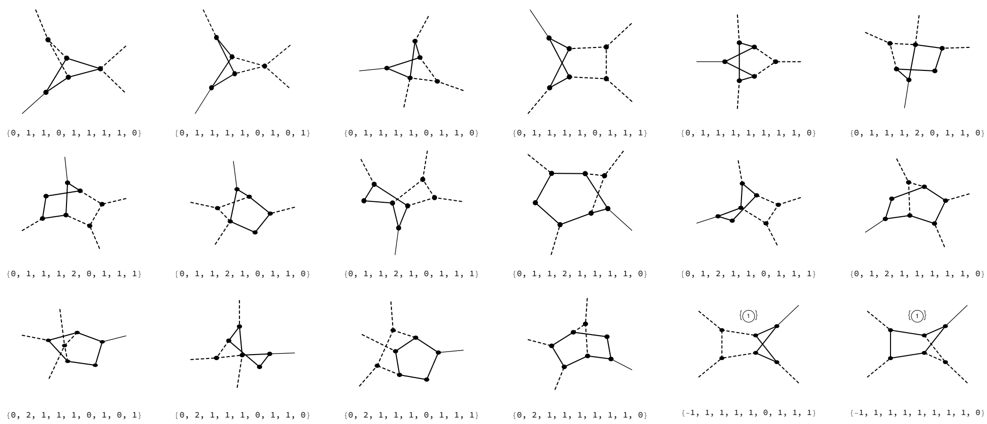


Figure C.2.: Top-sectors of the topology *PL2*.

Non-Planar NPL master integrals



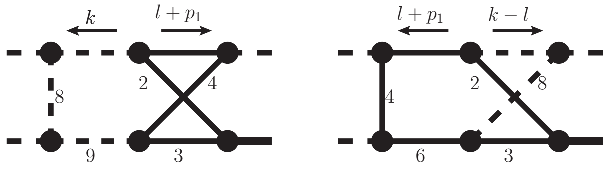


Figure C.3.: Top-sectors of the topology *NPL*.

C.2. Topologies and top sectors for $Z + j$

Prop.	FamPlanar0	FamPlanar1	FamPlanar2
[1]	k_1^2	$k_1^2 - m_V^2$	k_1^2
[2]	$(k_1 + p_1)^2$	$(k_1 + p_1)^2$	$(k_1 + p_1)^2 - m_V^2$
[3]	$(k_1 + p_1 + p_2)^2$	$(k_1 + p_1 + p_2)^2$	$(k_1 + p_1 + p_2)^2$
[4]	$(k_2 + p_1 + p_2)^2$	$(k_2 + p_1 + p_2)^2$	$(k_2 + p_1 + p_2)^2$
[5]	$(k_2 - p_3)^2$	$(k_2 - p_3)^2$	$(k_2 - p_3)^2$
[6]	k_2^2	k_2^2	k_2^2
[7]	$(k_1 - k_2)^2$	$(k_1 - k_2)^2$	$(k_1 - k_2)^2$
[8]	$(k_1 - p_3)^2$	$(k_1 - p_3)^2$	$(k_1 - p_3)^2$
[9]	$(k_2 + p_1)^2$	$(k_2 + p_1)^2$	$(k_2 + p_1)^2$

Table C.5.: Feynman propagators of the three integral families for the process $q\bar{q} \to Zg$

Prop.	FamPlanar3	FamPlanar6	FamPlanar7
[1]	k_1^2	k_1^2	k_1^2
[2]	$(k_1+p_1)^2$	$(k_1+p_1)^2$	$(k_1+p_1)^2$
[3]	$(k_1+p_1+p_2)^2-m_V^2$	$(k_1+p_1+p_2)^2$	$(k_1+p_1+p_2)^2$
[4]	$(k_2+p_1+p_2)^2$	$(k_2+p_1+p_2)^2$	$(k_2+p_1+p_2)^2$
[5]	$(k_2-p_3)^2$	$(k_2-p_3)^2$	$(k_2-p_3)^2$
[6]	k_2^2	$k_2^2-m_V^2$	k_2^2
[7]	$(k_1-k_2)^2$	$(k_1-k_2)^2$	$(k_1-k_2)^2-m_V^2$
[8]	$(k_1-p_3)^2$	$(k_1-p_3)^2$	$(k_1-p_3)^2$
[9]	$(k_2+p_1)^2$	$(k_2+p_1)^2$	$(k_2+p_1)^2$

Table C.6.: Feynman propagators of the three integral families for the process $q\bar{q} \to Zg$

Prop.	FamPlanar8	FamPlanar9	FamPlanarM2
[1]	k_1^2	k_1^2	k_1^2
[2]	$(k_1+p_1)^2$	$(k_1+p_1)^2$	$(k_1+p_1)^2$
[3]	$(k_1+p_1+p_2)^2$	$(k_1+p_1+p_2)^2$	$(k_1+p_1+p_2)^2$
[4]	$(k_2+p_1+p_2)^2$	$(k_2+p_1+p_2)^2$	$(k_2+p_1+p_2)^2-m_V^2$
[5]	$(k_2-p_3)^2$	$(k_2-p_3)^2$	$(k_2-p_3)^2-m_V^2$
[6]	k_2^2	k_2^2	k_2^2
[7]	$(k_1-k_2)^2$	$(k_1-k_2)^2$	$(k_1-k_2)^2$
[8]	$(k_1-p_3)^2-m_V^2$	$(k_1-p_3)^2$	$(k_1-p_3)^2$
[9]	$(k_2+p_1)^2$	$(k_2+p_1)^2-m_V^2$	$(k_2+p_1)^2$

Table C.7.: Feynman propagators of the three integral families for the process $q\bar{q} \to Zg$

Prop.	FamNonPlanar0	FamNonPlanar1	FamNonPlanar2
[1]	k_1^2	$k_1^2-m_V^2$	k_1^2
[2]	$(k_1+p_1)^2$	$(k_1+p_1)^2$	$(k_1+p_1)^2-m_V^2$
[3]	$(k_1+p_1+p_2)^2$	$(k_1+p_1+p_2)^2$	$(k_1+p_1+p_2)^2$
[4]	$(k_2+p_1+p_2)^2$	$(k_2+p_1+p_2)^2$	$(k_2+p_1+p_2)^2$
[5]	$(k_2-p_3)^2$	$(k_2-p_3)^2$	$(k_2-p_3)^2$
[6]	$(k_1-k_2+p_3)^2$	$(k_1-k_2+p_3)^2$	$(k_1-k_2+p_3)^2$
[7]	$(k_1-k_2)^2$	$(k_1-k_2)^2$	$(k_1-k_2)^2$
[8]	k_2^2	k_2^2	k_2^2
[9]	$(k_1-k_2-p_2)^2$	$(k_1-k_2-p_2)^2$	$(k_1-k_2-p_2)^2$

Table C.8.: Feynman propagators of the three integral families for the process $q\bar{q} \to Zg$

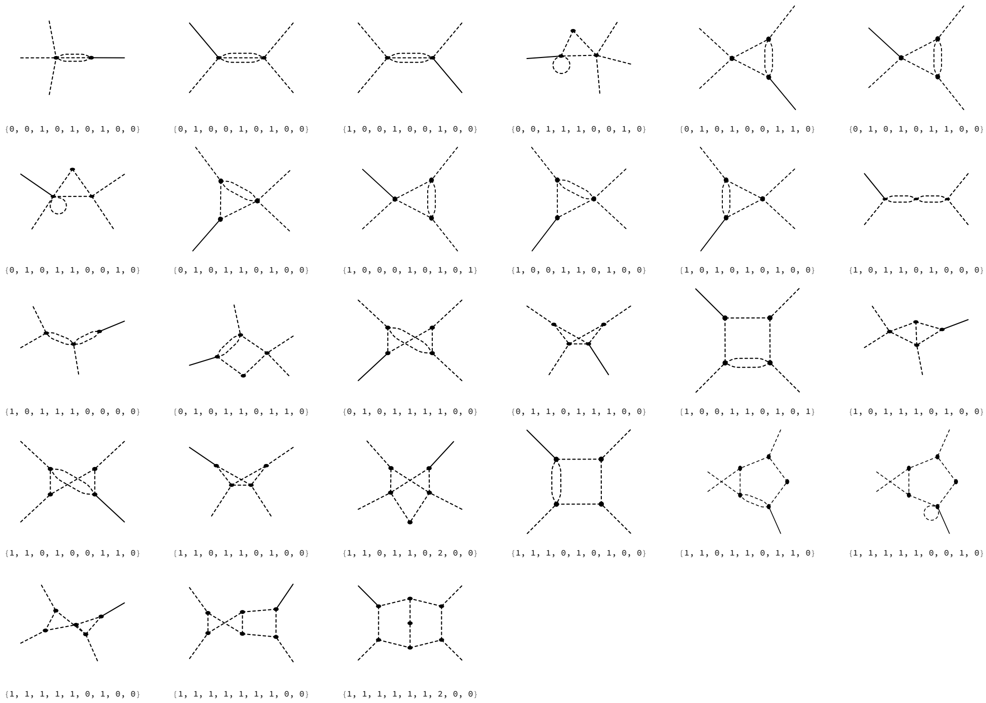
Prop.	FamNonPlanar3	FamNonPlanar6	FamNonPlanar7
[1]	k_1^2	k_1^2	k_1^2
[2]	$(k_1 + p_1)^2$	$(k_1 + p_1)^2$	$(k_1 + p_1)^2$
[3]	$(k_1 + p_1 + p_2)^2 - m_V^2$	$(k_1 + p_1 + p_2)^2$	$(k_1 + p_1 + p_2)^2$
[4]	$(k_2 + p_1 + p_2)^2$	$(k_2 + p_1 + p_2)^2$	$(k_2 + p_1 + p_2)^2$
[5]	$(k_2 - p_3)^2$	$(k_2 - p_3)^2$	$(k_2 - p_3)^2$
[6]	$(k_1 - k_2 + p_3)^2$	$(k_1 - k_2 + p_3)^2 - m_V^2$	$(k_1 - k_2 + p_3)^2$
[7]	$(k_1 - k_2)^2$	$(k_1 - k_2)^2$	$(k_1 - k_2)^2 - m_V^2$
[8]	k_2^2	k_2^2	k_2^2
[9]	$(k_1 - k_2 - p_2)^2$	$(k_1 - k_2 - p_2)^2$	$(k_1 - k_2 - p_2)^2$

Table C.9.: Feynman propagators of the three integral families for the process $q\bar{q} \to Zg$

Prop.	FamNonPlanar8	FamNonPlanar9	FamNonPlanarM2
[1]	k_1^2	k_1^2	k_1^2
[2]	$(k_1 + p_1)^2$	$(k_1 + p_1)^2$	$(k_1 + p_1)^2$
[3]	$(k_1 + p_1 + p_2)^2$	$(k_1 + p_1 + p_2)^2$	$(k_1 + p_1 + p_2)^2$
[4]	$(k_2 + p_1 + p_2)^2$	$(k_2 + p_1 + p_2)^2$	$(k_2 + p_1 + p_2)^2 - m_V^2$
[5]	$(k_2 - p_3)^2$	$(k_2 - p_3)^2$	$(k_2 - p_3)^2 - m_V^2$
[6]	$(k_1 - k_2 + p_3)^2$	$(k_1 - k_2 + p_3)^2$	$(k_1 - k_2 + p_3)^2$
[7]	$(k_1 - k_2)^2$	$(k_1 - k_2)^2$	$(k_1 - k_2)^2$
[8]	$k_2^2 - m_V^2$	k_2^2	k_2^2
[9]	$(k_1 - k_2 - p_2)^2$	$(k_1 - k_2 - p_2)^2 - m_V^2$	$(k_1 - k_2 - p_2)^2$

Table C.10.: Feynman propagators of the three integral families for the process $q\bar{q} \to Zg$

FamPlanar0 master integrals



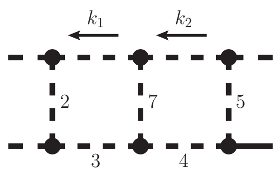
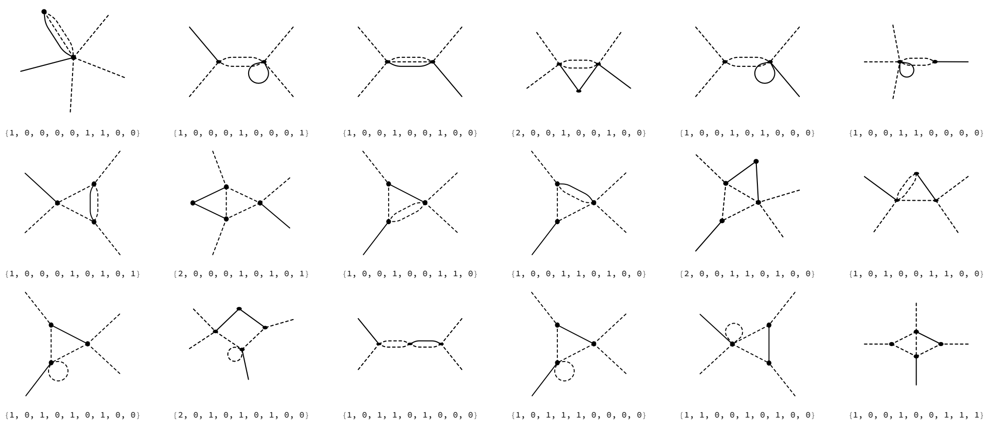
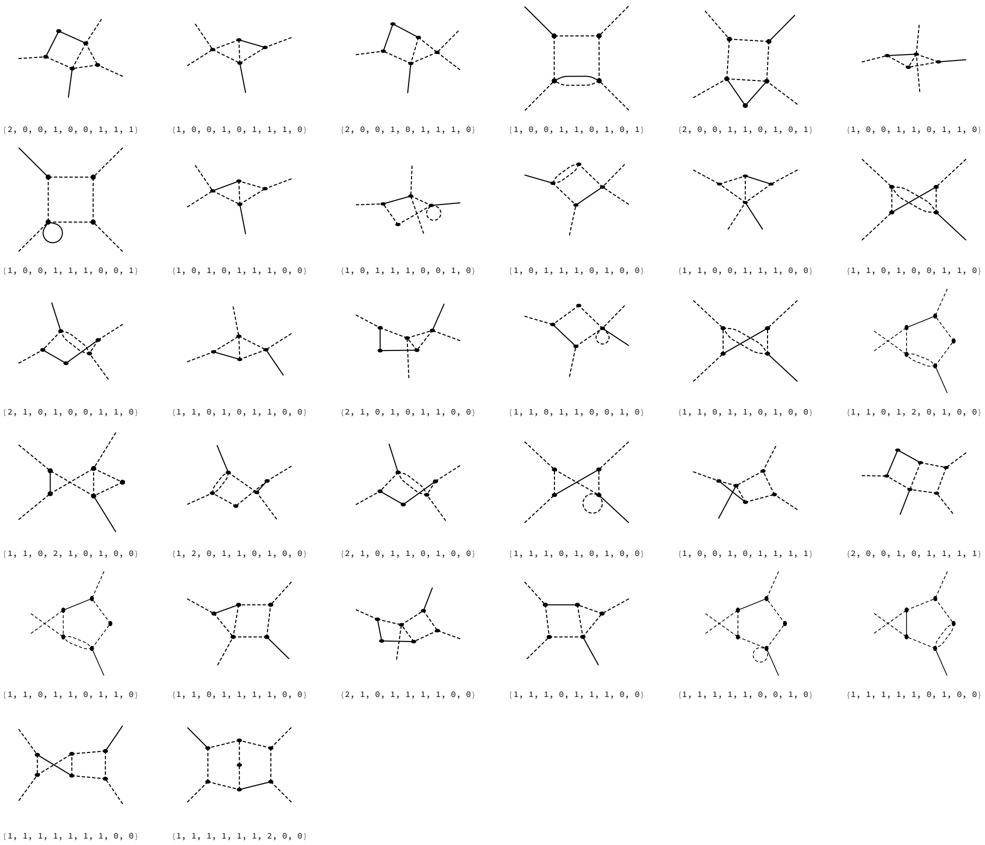


Figure C.4.: Top-sector of *FamPlanar0*.

FamPlanar1 master integrals





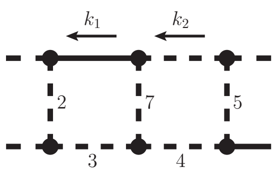
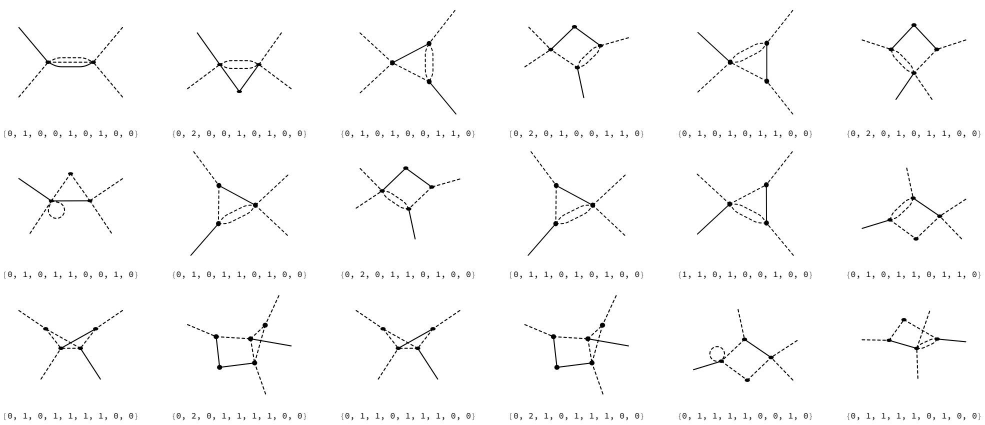
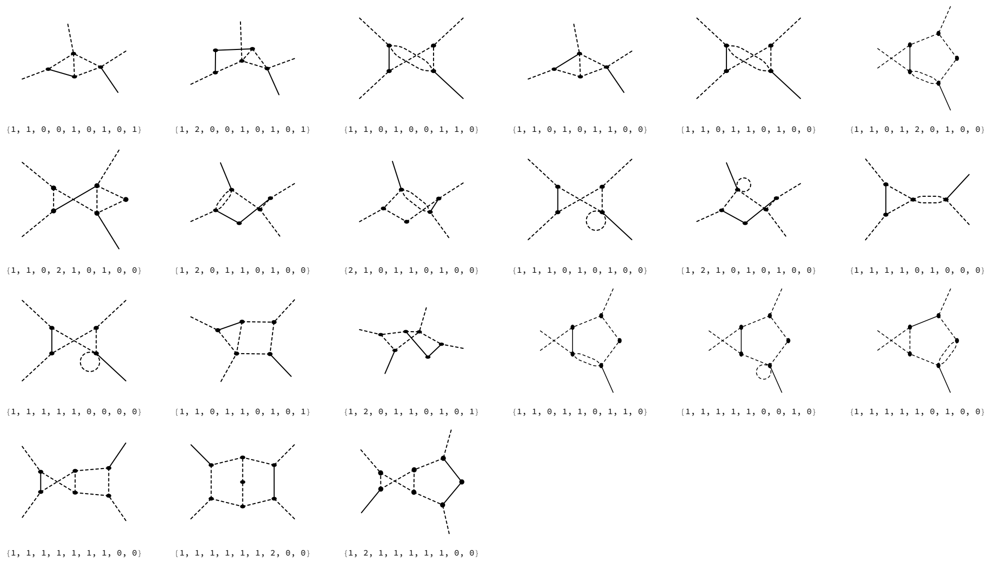


Figure C.5.: Top-sector of *FamPlanar1*.

FamPlanar2 master integrals





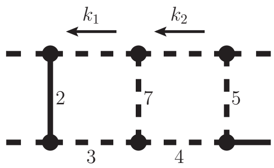
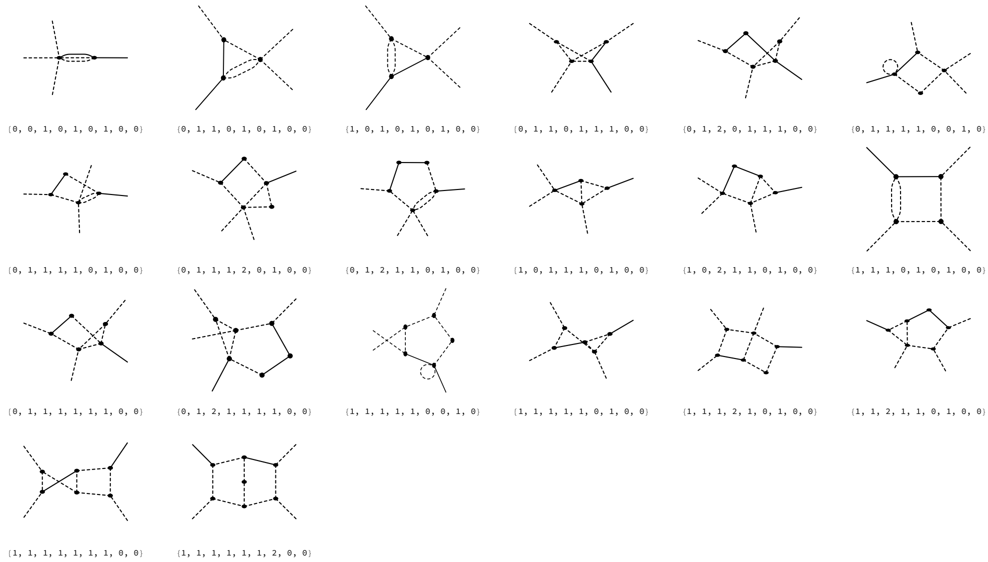


Figure C.6.: Top-sector of *FamPlanar2*.

FamPlanar3 master integrals



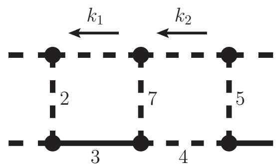
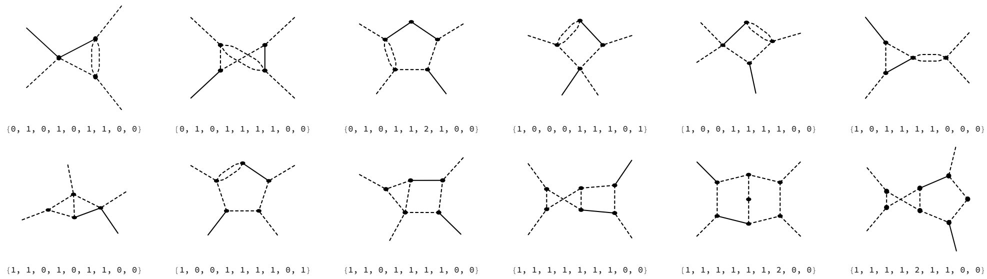


Figure C.7.: Top-sector of *FamPlanar3*.

FamPlanar6 master integrals



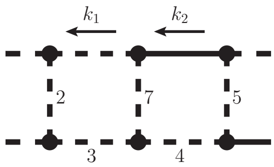
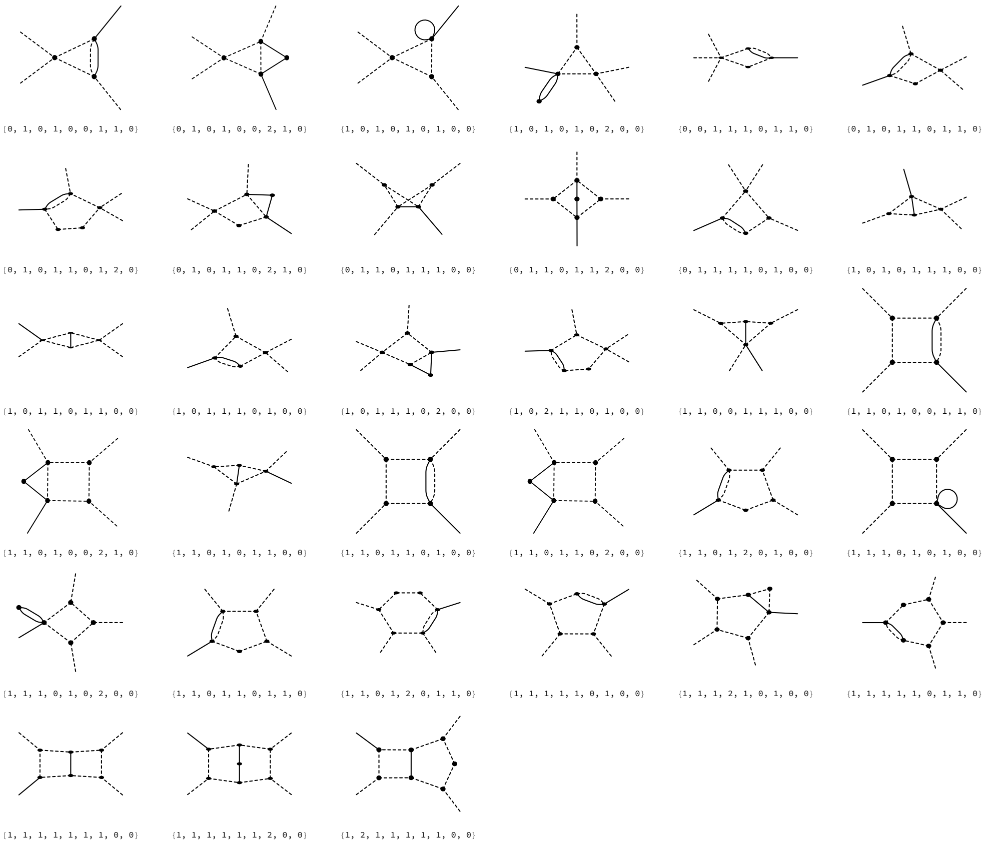


Figure C.8.: Top-sector of *FamPlanar6*.

FamPlanar7 master integrals



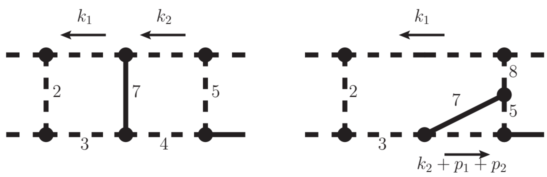
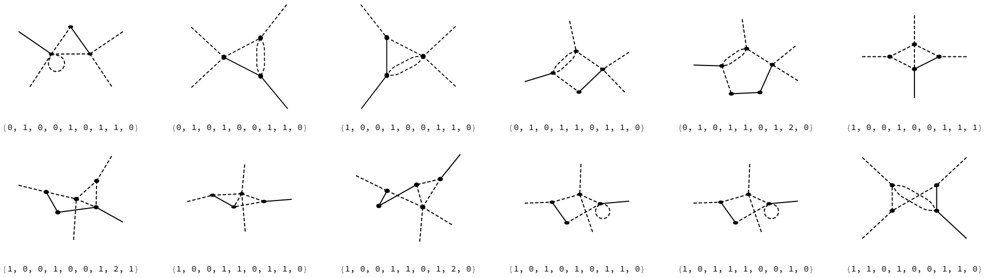
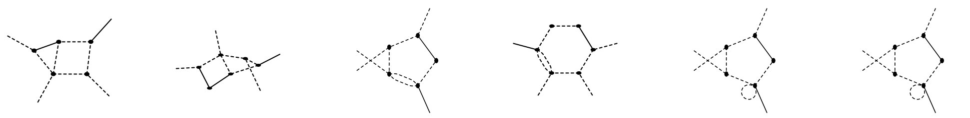


Figure C.9.: Top-sectors of the *FamPlanar7* topology.

FamPlanar8 master integrals





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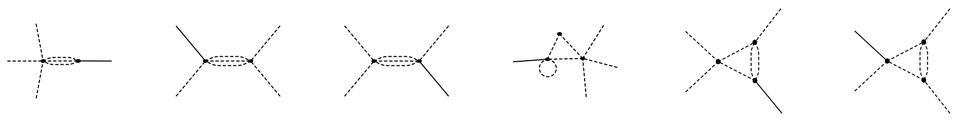
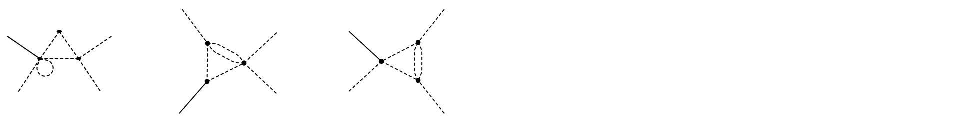
FamPlanar9 master integrals



{0, 0, 1, 0, 1, 0, 1, 0, 0} {0, 1, 0, 0, 1, 0, 1, 0, 0} {1, 0, 0, 1, 0, 0, 1, 0, 0} {0, 0, 1, 1, 1, 0, 0, 1, 0} {0, 1, 0, 1, 0, 0, 1, 1, 0} {0, 1, 0, 1, 0, 1, 1, 0, 0}



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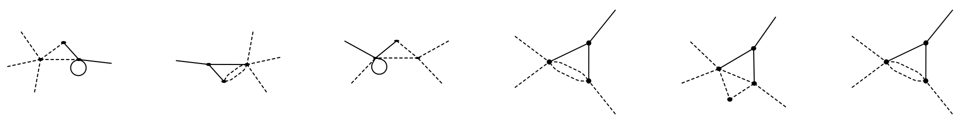
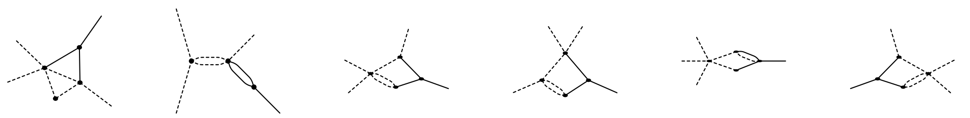
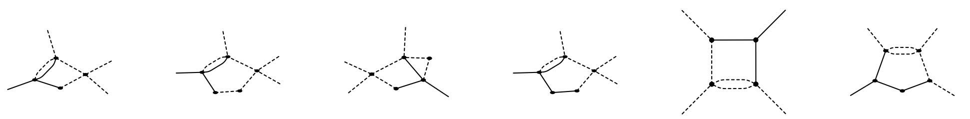
FamPlanarM2 master integrals



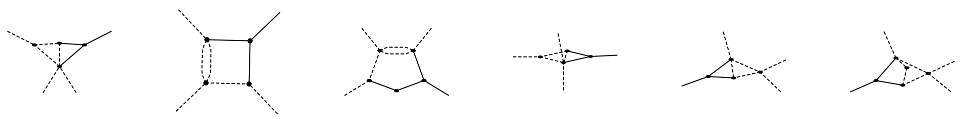
{0, 0, 1, 1, 1, 0, 0, 1, 0} {0, 0, 1, 1, 1, 0, 1, 0, 0} {0, 1, 0, 1, 1, 0, 0, 1, 0} {0, 1, 0, 1, 1, 0, 1, 0, 0} {0, 2, 0, 1, 1, 0, 1, 0, 0} {1, 0, 0, 1, 1, 0, 1, 0, 0}



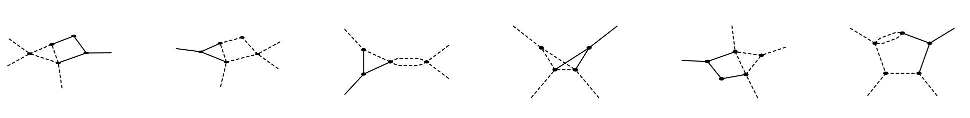
{2, 0, 0, 1, 1, 0, 1, 0, 0} {1, 0, 1, 1, 1, 0, 0, 0, 0} {0, 0, 0, 1, 1, 0, 1, 1, 1} {0, 0, 0, 1, 1, 1, 1, 1, 0} {0, 0, 1, 1, 1, 0, 1, 1, 0} {0, 0, 1, 1, 1, 1, 1, 0, 0}



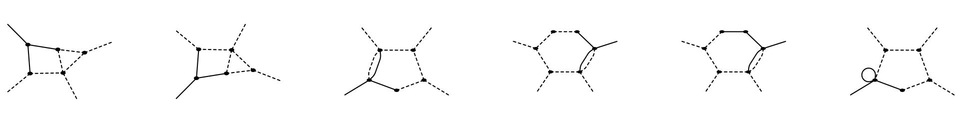
{0, 1, 0, 1, 1, 0, 1, 1, 0} {0, 1, 0, 1, 1, 0, 1, 2, 0} {0, 1, 0, 1, 1, 0, 2, 1, 0} {0, 1, 0, 1, 2, 0, 1, 1, 0} {0, 1, 0, 1, 1, 1, 1, 0, 0} {0, 1, 0, 1, 2, 1, 1, 0, 0}



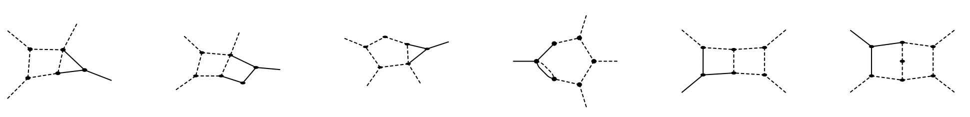
{0, 1, 1, 1, 1, 0, 1, 0, 0} {1, 0, 0, 1, 1, 0, 1, 0, 1} {1, 0, 0, 2, 1, 0, 1, 0, 1} {1, 0, 0, 1, 1, 0, 1, 1, 0} {1, 0, 1, 1, 1, 0, 1, 0, 0} {1, 0, 1, 1, 1, 0, 2, 0, 0}



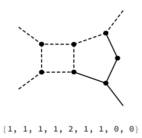
{1, 0, 1, 2, 1, 0, 1, 0, 0} {1, 0, 2, 1, 1, 0, 1, 0, 0} {1, 0, 1, 1, 1, 1, 0, 0, 0} {1, 1, 0, 1, 1, 0, 1, 0, 0} {1, 1, 0, 1, 2, 0, 1, 0, 0} {0, 0, 0, 1, 1, 1, 1, 1, 1}



{0, 1, 1, 1, 1, 1, 1, 0, 0} {1, 0, 0, 1, 1, 0, 1, 1, 1} {1, 1, 0, 1, 1, 0, 1, 1, 0} {1, 1, 0, 1, 1, 0, 1, 2, 0} {1, 1, 0, 1, 2, 0, 1, 1, 0} {1, 1, 1, 1, 1, 0, 0, 1, 0}



{1, 1, 1, 1, 1, 0, 1, 0, 0} {1, 1, 1, 2, 1, 0, 1, 0, 0} {1, 1, 2, 1, 1, 0, 1, 0, 0} {1, 1, 1, 1, 1, 0, 1, 1, 0} {1, 1, 1, 1, 1, 1, 1, 0, 0} {1, 1, 1, 1, 1, 1, 2, 0, 0}



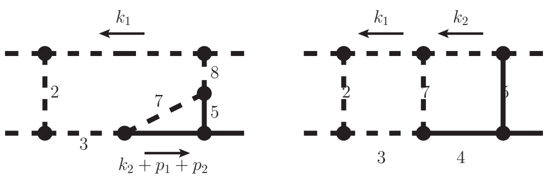
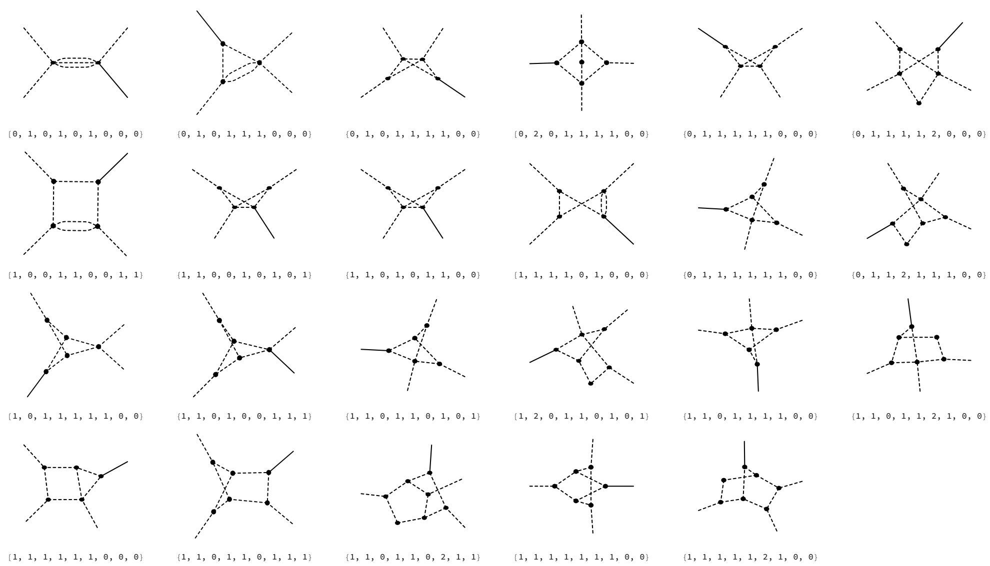


Figure C.10.: Top-sectors of the *FamPlanarM2* topology.

FamNonPlanar0 master integrals



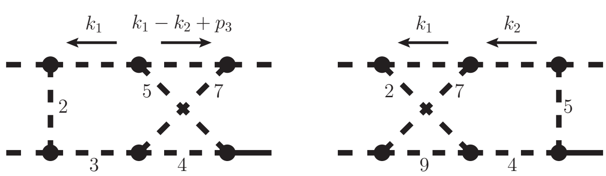
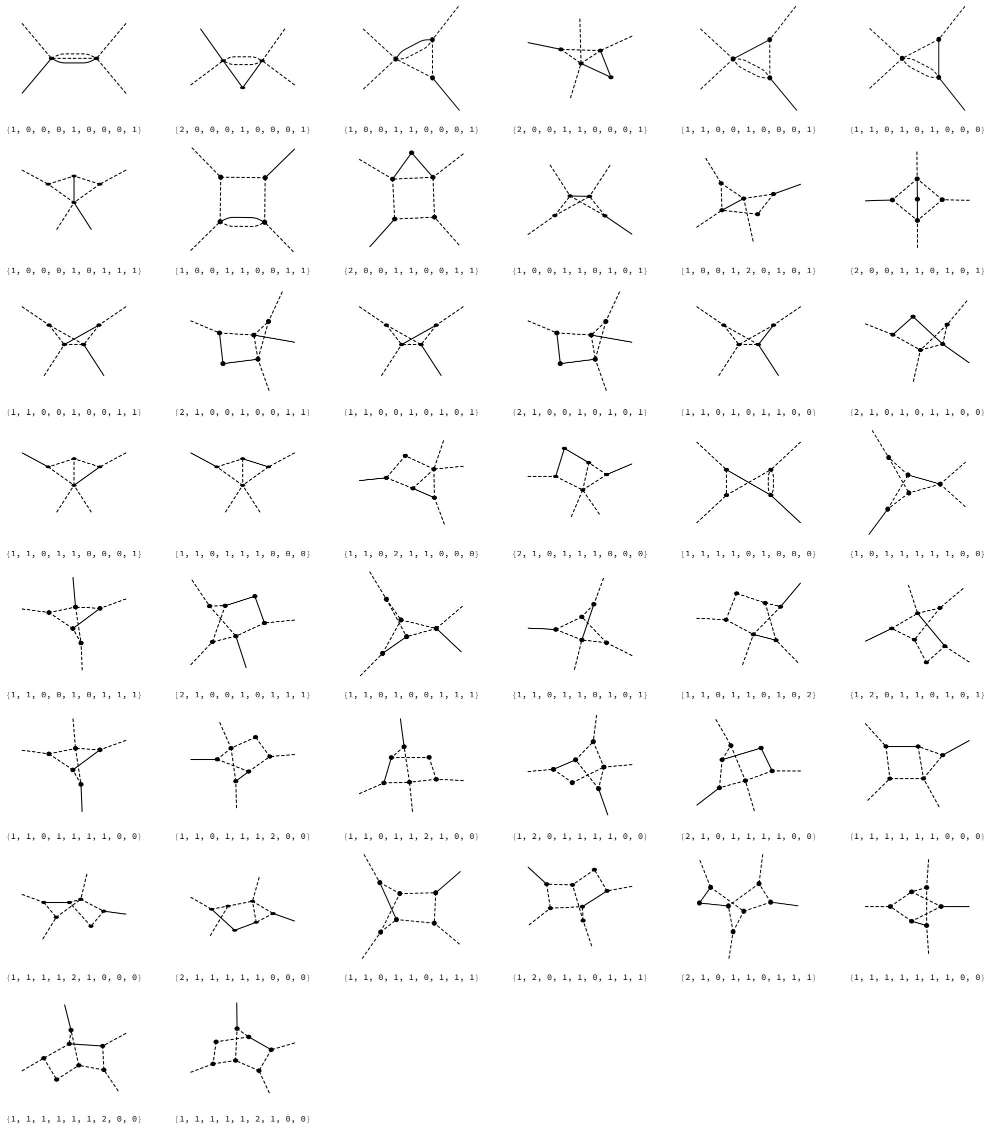


Figure C.11.: Top-sectors of *FamNonPlanar0* topology.

FamNonPlanar1 master integrals



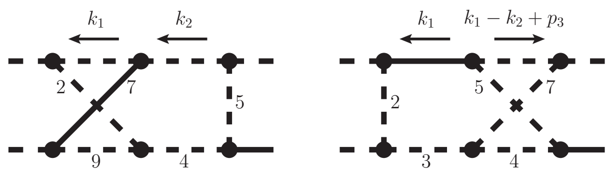
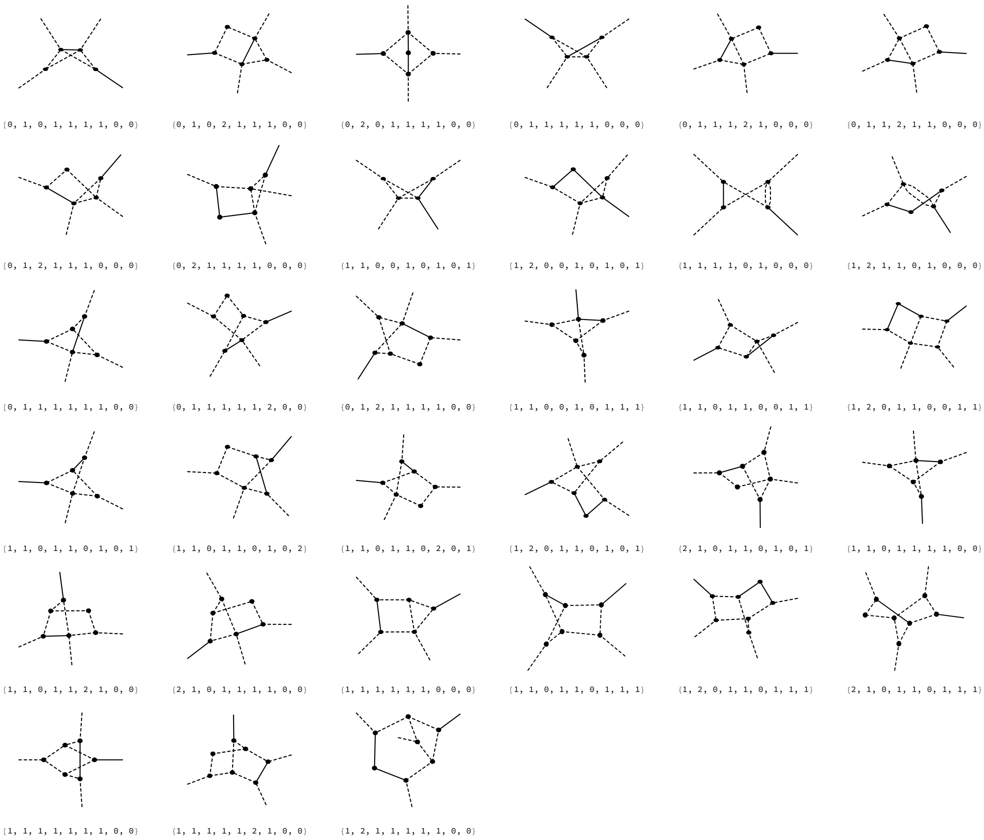


Figure C.12.: Top-sectors of *FamNonPlanar1* topology.

FamNonPlanar2 master integrals



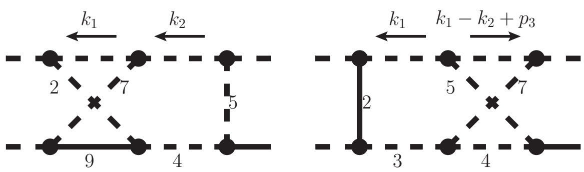
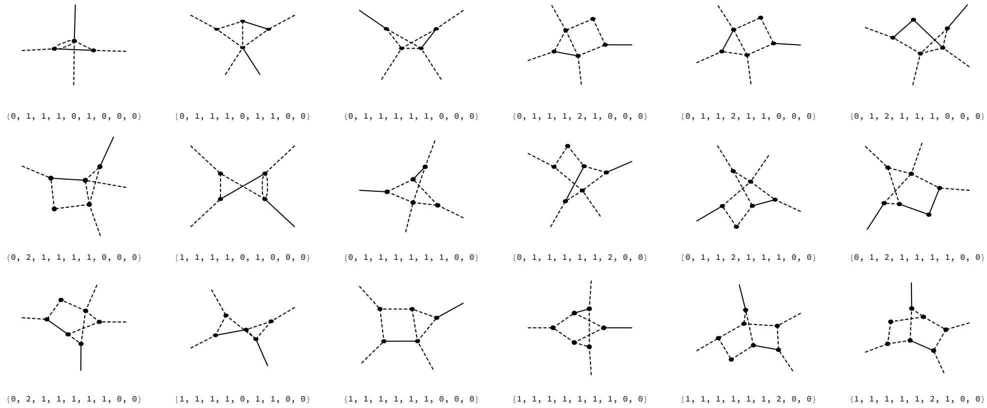


Figure C.13.: Top-sectors of *FamNonPlanar2* topology.

FamNonPlanar3 master integrals



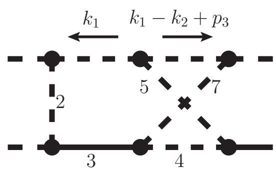
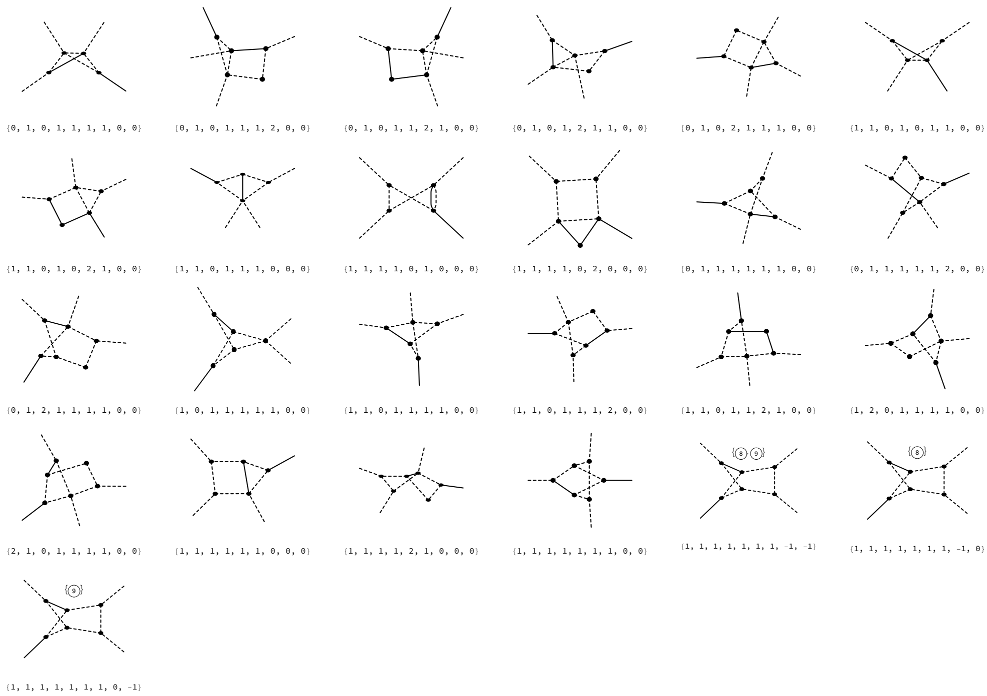


Figure C.14.: Top-sector of *FamNonPlanar3*.

FamNonPlanar6 master integrals



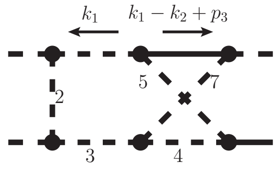
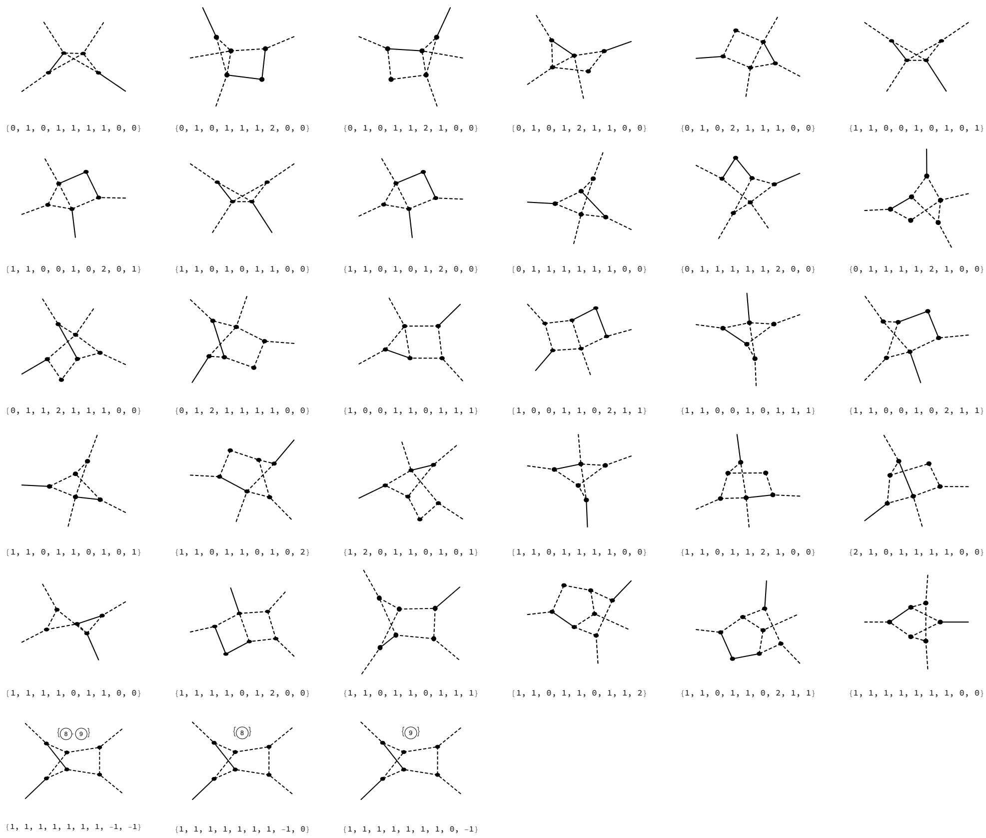


Figure C.15.: Top-sector of *FamNonPlanar6*.

FamNonPlanar7 master integrals



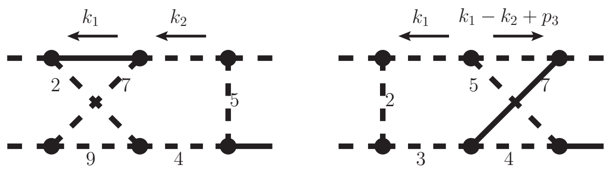
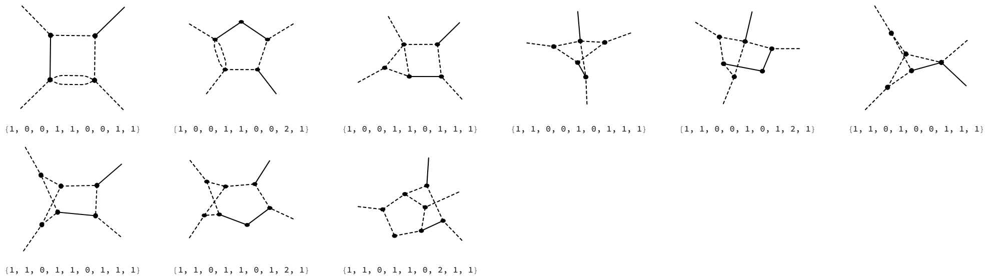


Figure C.16.: Top-sectors of *FamNonPlanar7* topology.

FamNonPlanar8 master integrals



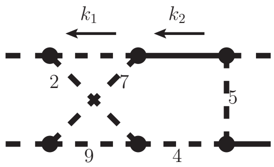
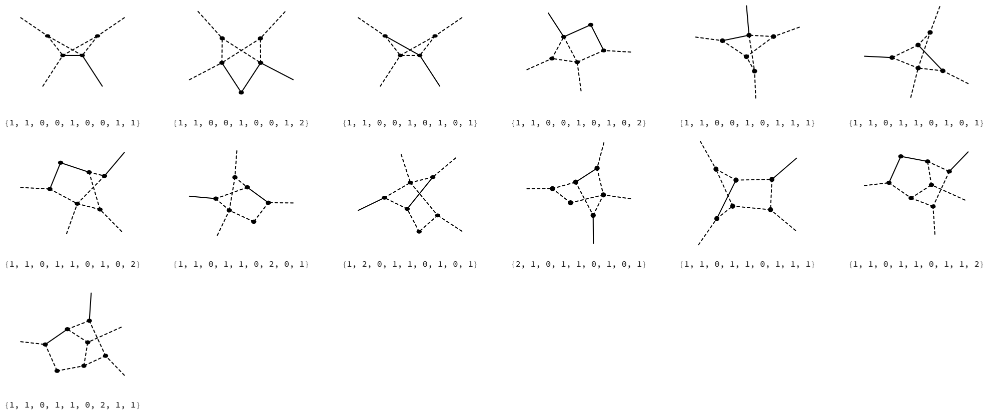


Figure C.17.: Top-sector of *FamNonPlanar8*.

FamNonPlanar9 master integrals



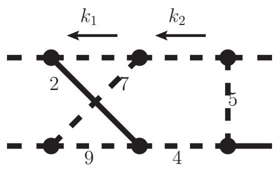
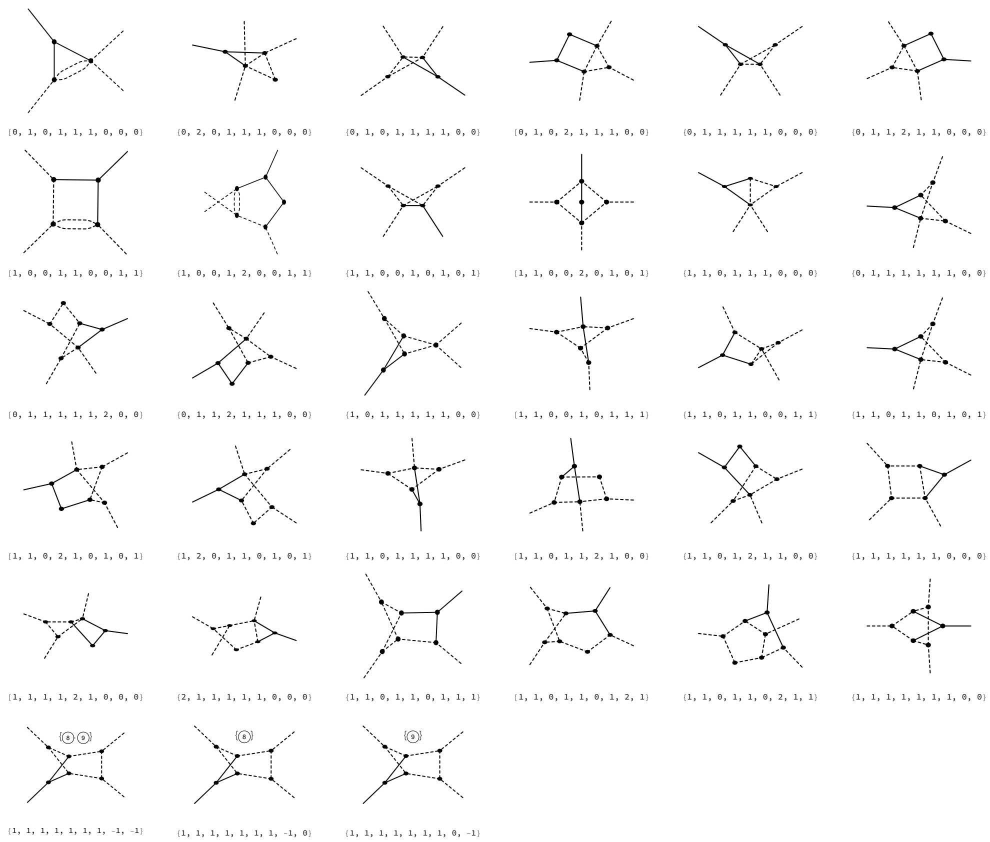


Figure C.18.: Top-sector of *FamNonPlanar9*.

FamNonPlanarM2 master integrals



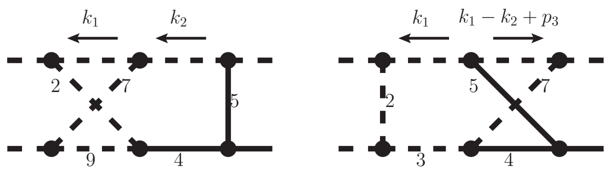


Figure C.19.: Top-sectors of *FamNonPlanarM2* topology.

Appendix D

Solutions of master integrals

In this appendix, we show master integral solutions for both cases of interest: $H+j$ and $Z+j$. For $Z+j$ master integrals we use following constants

$$r_{61}=\frac{1+i\sqrt{3}}{2},\ r_{62}=\frac{-1+i\sqrt{3}}{2},\ r_{64}=-\frac{1+i\sqrt{3}}{2},\ r_{65}=\frac{1-i\sqrt{3}}{2}. \tag{D.1}$$

Additionally, we use the following shorthand notation for polylogarithms at fixed points

$$\begin{aligned}
&K2\to\Im\left(\mathrm{Li}_2\left(\tfrac{1}{2}+\tfrac{i\sqrt{3}}{2}\right)\right), && K31\to\Im\left(\mathrm{Li}_3\left(\tfrac{i}{\sqrt{3}}\right)\right), && K32\to\Im\left(\mathrm{Li}_3\left(\tfrac{1}{4}+\tfrac{i\sqrt{3}}{4}\right)\right),\\
&K33\to\Re\left(\mathrm{Li}_3\left(\tfrac{i}{\sqrt{3}}\right)\right), && K34\to\Re\left(\mathrm{Li}_3\left(\tfrac{1}{4}+\tfrac{i\sqrt{3}}{4}\right)\right), && K41\to\Im\left(\mathrm{Li}_4\left(\tfrac{1}{2}+\tfrac{i\sqrt{3}}{2}\right)\right),\\
&K42\to\Im\left(\mathrm{Li}_4\left(\tfrac{i}{\sqrt{3}}\right)\right), && K43\to\Re\left(\mathrm{Li}_4\left(\frac{e^{\frac{i\pi}{6}}}{\sqrt{3}}\right)\right), && K44\to\Im\left(\mathrm{Li}_4\left(\tfrac{1}{4}+\tfrac{i\sqrt{3}}{4}\right)\right),\\
&K45\to\Im\left(\mathrm{Li}_4\left(\tfrac{3}{4}+\tfrac{i\sqrt{3}}{4}\right)\right), && K46\to\Im\left(\mathrm{Li}_{22}\left(\tfrac{1}{2}+\tfrac{i\sqrt{3}}{2},2\right)\right), && K47\to\Re\left(\mathrm{Li}_4\left(\tfrac{i}{\sqrt{3}}\right)\right).
\end{aligned} \tag{D.2}$$

D.1. The $H+j$ master integrals

I[NPL, 0, 1, 1, 1, 1, 1, 1, 1, 0] =

$$\begin{aligned}
&-\frac{3}{4}\frac{32z\zeta_3+111\zeta_4+379z\zeta_4}{z(z+1)^2}+\frac{1}{\epsilon^4}2\frac{1}{(z+1)z}-3\frac{-32+28\zeta_2+9\zeta_3}{(z+1)z}G_{-1}(z)+\left(2\frac{1}{(z+1)z}G_{-1}(z)-\frac{1}{2}\frac{7z+5}{z(z+1)^2}G_0(z)\right)\frac{1}{\epsilon^3}\\
&+\frac{-96z+84z\zeta_2+13\zeta_3+51z\zeta_3}{z(z+1)^2}G_0(z)-26\frac{\zeta_2}{(z+1)z}G_{-1,-1}(z)+4\frac{12+19\zeta_2+2z\zeta_2}{z(z+1)^2}G_{-1,0}(z)+4\frac{12+4\zeta_2+17z\zeta_2}{z(z+1)^2}G_{0,-1}(z)\\
&+\left(\frac{5}{2}\frac{\zeta_2}{(z+1)z}+6\frac{1}{(z+1)z}G_{-1}(z)-6\frac{1}{(z+1)^2}G_0(z)-4\frac{1}{(z+1)z}G_{-1,-1}(z)+\frac{z-1}{z(z+1)^2}G_{-1,0}(z)+\frac{z-1}{z(z+1)^2}G_{0,-1}(z)\right.\\
&\left.+2\frac{1}{(z+1)z}G_{0,0}(z)\right)\frac{1}{\epsilon^2}-10\frac{(5z-1)\zeta_2}{z(z+1)^2}G_{0,0}(z)-24\frac{1}{(z+1)z}G_{-1,-1,-1}(z)+24\frac{1}{z(z+1)^2}G_{-1,-1,0}(z)+24\frac{1}{z(z+1)^2}G_{-1,0,-1}(z)\\
&-24\frac{1}{(z+1)z}G_{-1,0,0}(z)+24\frac{1}{(z+1)^2}G_{0,-1,-1}(z)+24\frac{1}{z(z+1)^2}G_{0,-1,0}(z)+24\frac{1}{z(z+1)^2}G_{0,0,-1}(z)+\left(-\frac{19}{2}\frac{\zeta_3}{(z+1)z}\right.\\
&-\frac{29\zeta_2+24}{(z+1)z}G_{-1}(z)+\frac{24z-5\zeta_2+29z\zeta_2}{z(z+1)^2}G_0(z)-12\frac{1}{z(z+1)^2}G_{-1,0}(z)-12\frac{1}{z(z+1)^2}G_{0,-1}(z)-4\frac{1}{(z+1)z}G_{-1,-1,-1}(z)\\
&+4\frac{z+2}{z(z+1)^2}G_{-1,-1,0}(z)+4\frac{z+2}{z(z+1)^2}G_{-1,0,-1}(z)-6\frac{1}{(z+1)z}G_{-1,0,0}(z)+4\frac{z+2}{z(z+1)^2}G_{0,-1,-1}(z)-4\frac{1}{(z+1)z}G_{0,-1,0}(z)\\
&\left.-4\frac{1}{(z+1)z}G_{0,0,-1}(z)+2\frac{3z+1}{z(z+1)^2}G_{0,0,0}(z)\right)\frac{1}{\epsilon}+24\frac{1}{(z+1)^2}G_{0,0,0}(z)+44\frac{1}{(z+1)z}G_{-1,-1,-1,-1}(z)-2\frac{7z+11}{z(z+1)^2}G_{-1,-1,-1,0}(z)\\
&-2\frac{7z+11}{z(z+1)^2}G_{-1,-1,0,-1}(z)+6\frac{1}{(z+1)z}G_{-1,-1,0,0}(z)-2\frac{7z+11}{z(z+1)^2}G_{-1,0,-1,-1}(z)-12\frac{1}{(z+1)z}G_{-1,0,-1,0}(z)\\
&-12\frac{1}{(z+1)z}G_{-1,0,0,-1}(z)+4\frac{5z+7}{z(z+1)^2}G_{-1,0,0,0}(z)-2\frac{13z+5}{z(z+1)^2}G_{0,-1,-1,-1}(z)+2\frac{7z+1}{z(z+1)^2}G_{0,-1,-1,0}(z)
\end{aligned}$$

$$
\begin{aligned}
&+2\frac{7z+1}{z(z+1)^2}G_{0,-1,0,-1}(z)+6\frac{z-1}{z(z+1)^2}G_{0,-1,0,0}(z)-4\frac{z+4}{z(z+1)^2}G_{0,0,-1,-1}(z)+4\frac{3z+5}{z(z+1)^2}G_{0,0,-1,0}(z)+4\frac{3z+5}{z(z+1)^2}G_{0,0,0,-1}(z)\\
&-16\frac{2z+1}{z(z+1)^2}G_{0,0,0,0}(z)+\Bigg(\frac{-96-96z+12\zeta_2+36z\zeta_2+89\zeta_3+77z\zeta_3}{z(z+1)^2}-\frac{1}{\epsilon^3}2\frac{1}{(z+1)z}-8\frac{\zeta_2}{(z+1)z}G_{-1}(z)+\Bigg(-6\frac{1}{(z+1)z}\\
&+4\frac{1}{(z+1)z}G_{-1}(z)-\frac{z-1}{z(z+1)^2}G_0(z)\Bigg)\frac{1}{\epsilon^2}-8\frac{6+7\zeta_2+2z\zeta_2}{z(z+1)^2}G_0(z)+24\frac{1}{(z+1)z}G_{-1,-1}(z)-24\frac{1}{z(z+1)^2}G_{-1,0}(z)\\
&-24\frac{1}{(z+1)^2}G_{0,-1}(z)+\Bigg(\frac{31\zeta_2+24}{(z+1)z}+12\frac{1}{z(z+1)^2}G_0(z)+4\frac{1}{(z+1)z}G_{-1,-1}(z)-4\frac{z+2}{z(z+1)^2}G_{-1,0}(z)-4\frac{z+2}{z(z+1)^2}G_{0,-1}(z)\\
&+4\frac{1}{(z+1)z}G_{0,0}(z)\Bigg)\frac{1}{\epsilon}-24\frac{1}{z(z+1)^2}G_{0,0}(z)-44\frac{1}{(z+1)z}G_{-1,-1,-1}(z)+2\frac{7z+11}{z(z+1)^2}G_{-1,-1,0}(z)+2\frac{7z+11}{z(z+1)^2}G_{-1,0,-1}(z)\\
&+12\frac{1}{(z+1)z}G_{-1,0,0}(z)+2\frac{13z+5}{z(z+1)^2}G_{0,-1,-1}(z)-2\frac{7z+1}{z(z+1)^2}G_{0,-1,0}(z)+4\frac{z+4}{z(z+1)^2}G_{0,0,-1}(z)-4\frac{3z+5}{z(z+1)^2}G_{0,0,0}(z)\Bigg)i\pi\\
&+\Bigg(\frac{1}{4}\frac{32z\zeta_3+127\zeta_4+327z\zeta_4}{z(z+1)^2}-\frac{1}{\epsilon^4}3\frac{1}{(z+1)z}+2\frac{-16+16\zeta_2+5\zeta_3}{(z+1)z}G_{-1}(z)+\Bigg(-5\frac{1}{(z+1)z}G_{-1}(z)+\frac{7z+5}{z(z+1)^2}G_0(z)\Bigg)\frac{1}{\epsilon^3}\\
&-4\frac{-8z+8z\zeta_2+3\zeta_3+5z\zeta_3}{z(z+1)^2}G_0(z)+11\frac{\zeta_2}{(z+1)z}G_{-1,-1}(z)-\frac{16+29\zeta_2+11z\zeta_2}{z(z+1)^2}G_{-1,0}(z)-\frac{16+13\zeta_2+27z\zeta_2}{z(z+1)^2}G_{0,-1}(z)\\
&+\frac{(27z+11)\zeta_2}{z(z+1)^2}G_{0,0}(z)+\Bigg(-\frac{\zeta_2}{(z+1)z}-8\frac{1}{(z+1)z}G_{-1}(z)+8\frac{1}{(z+1)^2}G_0(z)+5\frac{1}{(z+1)z}G_{-1,-1}(z)-\frac{z-1}{z(z+1)^2}G_{-1,0}(z)\\
&-\frac{z-1}{z(z+1)^2}G_{0,-1}(z)-3\frac{1}{(z+1)z}G_{0,0}(z)\Bigg)\frac{1}{\epsilon^2}+8\frac{1}{(z+1)z}G_{-1,-1,-1}(z)-8\frac{1}{z(z+1)^2}G_{-1,-1,0}(z)-8\frac{1}{z(z+1)^2}G_{-1,0,-1}(z)\\
&+8\frac{1}{(z+1)z}G_{-1,0,0}(z)-8\frac{1}{(z+1)^2}G_{0,-1,-1}(z)-8\frac{1}{z(z+1)^2}G_{0,-1,0}(z)-8\frac{1}{z(z+1)^2}G_{0,0,-1}(z)+\Bigg(6\frac{\zeta_3}{(z+1)z}+\frac{23\zeta_2+16}{(z+1)z}G_{-1}(z)\\
&-\frac{16z+7\zeta_2+25z\zeta_2}{z(z+1)^2}G_0(z)+8\frac{1}{z(z+1)^2}G_{-1,0}(z)+8\frac{1}{z(z+1)^2}G_{0,-1}(z)+3\frac{1}{(z+1)z}G_{-1,-1,-1}(z)-\frac{3z+5}{z(z+1)^2}G_{-1,-1,0}(z)\\
&-\frac{3z+5}{z(z+1)^2}G_{-1,0,-1}(z)+5\frac{1}{(z+1)z}G_{-1,0,0}(z)-\frac{3z+5}{z(z+1)^2}G_{0,-1,-1}(z)+3\frac{1}{(z+1)z}G_{0,-1,0}(z)+3\frac{1}{(z+1)z}G_{0,0,-1}(z)\\
&-\frac{5z+3}{z(z+1)^2}G_{0,0,0}(z)\Bigg)\frac{1}{\epsilon}-8\frac{1}{(z+1)^2}G_{0,0,0}(z)-15\frac{1}{(z+1)z}G_{-1,-1,-1,-1}(z)+\frac{5z+7}{z(z+1)^2}G_{-1,-1,-1,0}(z)+\frac{5z+7}{z(z+1)^2}G_{-1,-1,0,-1}(z)\\
&-\frac{1}{(z+1)z}G_{-1,-1,0,0}(z)+\frac{5z+7}{z(z+1)^2}G_{-1,0,-1,-1}(z)+5\frac{1}{(z+1)z}G_{-1,0,-1,0}(z)+5\frac{1}{(z+1)z}G_{-1,0,0,-1}(z)-\frac{13z+15}{z(z+1)^2}G_{-1,0,0,0}(z)\\
&+3\frac{3z+1}{z(z+1)^2}G_{0,-1,-1,-1}(z)-\frac{5z+1}{z(z+1)^2}G_{0,-1,-1,0}(z)-\frac{5z+1}{z(z+1)^2}G_{0,-1,0,-1}(z)-\frac{3z+1}{z(z+1)^2}G_{0,-1,0,0}(z)+\frac{z+5}{z(z+1)^2}G_{0,0,-1,-1}(z)\\
&-\frac{5z+7}{z(z+1)^2}G_{0,0,-1,0}(z)-\frac{5z+7}{z(z+1)^2}G_{0,0,0,-1}(z)+\frac{17z+13}{z(z+1)^2}G_{0,0,0,0}(z)+\Bigg(-4\frac{-8-8z+2\zeta_2+4z\zeta_2+8\zeta_3+7z\zeta_3}{z(z+1)^2}+\frac{1}{\epsilon^3}5\frac{1}{(z+1)z}\\
&+\frac{\zeta_2}{(z+1)z}G_{-1}(z)+\Bigg(8\frac{1}{(z+1)z}-5\frac{1}{(z+1)z}G_{-1}(z)+\frac{z-1}{z(z+1)^2}G_0(z)\Bigg)\frac{1}{\epsilon^2}+\frac{16+19\zeta_2+9z\zeta_2}{z(z+1)^2}G_0(z)-8\frac{1}{(z+1)z}G_{-1,-1}(z)\\
&+8\frac{1}{z(z+1)^2}G_{-1,0}(z)+8\frac{1}{(z+1)^2}G_{0,-1}(z)+\Bigg(-\frac{23\zeta_2+16}{(z+1)z}-8\frac{1}{z(z+1)^2}G_0(z)-3\frac{1}{(z+1)z}G_{-1,-1}(z)+\frac{3z+5}{z(z+1)^2}G_{-1,0}(z)\\
&+\frac{3z+5}{z(z+1)^2}G_{0,-1}(z)-3\frac{1}{(z+1)z}G_{0,0}(z)\Bigg)\frac{1}{\epsilon}+8\frac{1}{z(z+1)^2}G_{0,0}(z)+15\frac{1}{(z+1)z}G_{-1,-1,-1}(z)-\frac{5z+7}{z(z+1)^2}G_{-1,-1,0}(z)\\
&-\frac{5z+7}{z(z+1)^2}G_{-1,0,-1}(z)-5\frac{1}{(z+1)z}G_{-1,0,0}(z)-3\frac{3z+1}{z(z+1)^2}G_{0,-1,-1}(z)+\frac{5z+1}{z(z+1)^2}G_{0,-1,0}(z)-\frac{z+5}{z(z+1)^2}G_{0,0,-1}(z)\\
&+\frac{5z+7}{z(z+1)^2}G_{0,0,0}(z)\Bigg)i\pi\Bigg)\kappa^{-\epsilon}+\Bigg(-\frac{1}{\epsilon^3}4\frac{1}{(z+1)z}+\frac{1}{\epsilon}10\frac{\zeta_2}{(z+1)z}+10\frac{\zeta_3}{(z+1)z}+6\frac{\zeta_2}{(z+1)z}G_{-1}(z)+\frac{1}{\epsilon^2}2\frac{1}{(z+1)z}G_0(z)\\
&-8\frac{\zeta_2}{(z+1)z}G_0(z)+2\frac{1}{(z+1)z}G_{-1,0,0}(z)-2\frac{1}{(z+1)z}G_{0,0,0}(z)+\Bigg(2\frac{\zeta_3}{(z+1)z}G_{-1}(z)-6\frac{\zeta_3}{(z+1)z}G_0(z)+6\frac{\zeta_2}{(z+1)z}G_{-1,-1}(z)\\
&-6\frac{\zeta_2}{(z+1)z}G_{-1,0}(z)-6\frac{\zeta_2}{(z+1)z}G_{0,-1}(z)+6\frac{\zeta_2}{(z+1)z}G_{0,0}(z)+2\frac{1}{(z+1)z}G_{-1,-1,0,0}(z)-4\frac{1}{(z+1)z}G_{-1,0,0,0}(z)\\
&-2\frac{1}{(z+1)z}G_{0,-1,0,0}(z)+4\frac{1}{(z+1)z}G_{0,0,0,0}(z)\Bigg)\epsilon\Bigg)\log(\kappa)\kappa^{-\epsilon}+\Bigg(\frac{1}{\epsilon^4}\frac{1}{(z+1)z}+\frac{3}{2}\frac{\zeta_4}{(z+1)z}-4\frac{\zeta_3}{(z+1)z}G_{-1}(z)+\Bigg(3\frac{1}{(z+1)z}G_{-1}(z)\\
&-\frac{1}{2}\frac{7z+5}{z(z+1)^2}G_0(z)\Bigg)\frac{1}{\epsilon^3}+\Bigg(\frac{7}{2}\frac{\zeta_3}{(z+1)z}+2\frac{\zeta_2}{(z+1)z}G_0(z)\Bigg)\frac{1}{\epsilon}+2\frac{\zeta_3}{(z+1)z}G_0(z)+\Bigg(-\frac{3}{2}\frac{\zeta_2}{(z+1)z}+2\frac{1}{(z+1)z}G_{-1}(z)-2\frac{1}{(z+1)^2}G_0(z)\\
&-\frac{1}{(z+1)z}G_{-1,-1}(z)+\frac{1}{(z+1)z}G_{0,0}(z)\Bigg)\frac{1}{\epsilon^2}+\Bigg(-\frac{1}{\epsilon^3}3\frac{1}{(z+1)z}+4\frac{\zeta_3}{(z+1)z}+\Bigg(-2\frac{1}{(z+1)z}+\frac{1}{(z+1)z}G_{-1}(z)\Bigg)\frac{1}{\epsilon^2}\Bigg)i\pi\Bigg)\kappa^{-2\epsilon}\\
&+\Bigg(-\epsilon\frac{5}{6}\frac{(60\log(2)^2+\pi^2)\pi^2}{z^{\frac{1}{2}}(z+1)^{\frac{3}{2}}}-\frac{1}{\epsilon}6\frac{\zeta_2}{z^{\frac{1}{2}}(z+1)^{\frac{3}{2}}}+60\frac{\log(2)\zeta_2}{z^{\frac{1}{2}}(z+1)^{\frac{3}{2}}}\Bigg)i\pi\kappa^{-\frac{1}{2}}\kappa^{-2\epsilon}+\Bigg(-\epsilon\frac{3}{2}\frac{\zeta_4}{(z+1)z}+\frac{1}{\epsilon^3}3\frac{1}{(z+1)z}-\frac{1}{\epsilon}4\frac{\zeta_2}{(z+1)z}\\
&+\Bigg(\frac{1}{(z+1)z}G_{-1}(z)-2\frac{1}{(z+1)z}G_0(z)\Bigg)\frac{1}{\epsilon^2}-\frac{1}{\epsilon^2}\frac{1}{(z+1)z}i\pi\Bigg)\log(\kappa)\kappa^{-2\epsilon}+\frac{1}{\epsilon^2}\frac{3}{2}\frac{1}{(z+1)z}\log^2(\kappa)\kappa^{-2\epsilon}+O(\eta)+O(\kappa)
\end{aligned}
\tag{D.3}
$$

I[NPL, 0,1, 1, 1, 1, 1, 1, 1, 0] =

$$- \frac{3}{4} \frac{32z\zeta_3 + 111\zeta_4 + 379z\zeta_4}{z(z+1)^2} + \frac{1}{\epsilon^4} 2 \frac{1}{(z+1)z} - 3 \frac{-32 + 28\zeta_2 + 9\zeta_3}{(z+1)z} G_{-1}(z) + \Bigg(2 \frac{1}{(z+1)z} G_{-1}(z) - \frac{1}{2} \frac{7z+5}{z(z+1)^2} G_0(z) \Bigg) \frac{1}{\epsilon^3}$$

$$+ \frac{-96z + 84z\zeta_2 + 13\zeta_3 + 51z\zeta_3}{z(z+1)^2} G_0(z) - 26 \frac{\zeta_2}{(z+1)z} G_{-1,-1}(z) + 4 \frac{12 + 19\zeta_2 + 2z\zeta_2}{z(z+1)^2} G_{-1,0}(z) + 4 \frac{12 + 4\zeta_2 + 17z\zeta_2}{z(z+1)^2} G_{0,-1}(z)$$

$$+ \Bigg(\frac{5}{2} \frac{\zeta_2}{(z+1)z} + 6 \frac{1}{(z+1)z} G_{-1}(z) - 6 \frac{1}{(z+1)^2} G_0(z) - 4 \frac{1}{(z+1)z} G_{-1,-1}(z) + \frac{z-1}{z(z+1)^2} G_{-1,0}(z) + \frac{z-1}{z(z+1)^2} G_{0,-1}(z)$$

$$+ 2 \frac{1}{(z+1)z} G_{0,0}(z) \Bigg) \frac{1}{\epsilon^2} - 10 \frac{(5z-1)\zeta_2}{z(z+1)^2} G_{0,0}(z) - 24 \frac{1}{(z+1)z} G_{-1,-1,-1}(z) + 24 \frac{1}{z(z+1)^2} G_{-1,-1,0}(z) + 24 \frac{1}{z(z+1)^2} G_{-1,0,-1}(z)$$

$$- 24 \frac{1}{(z+1)z} G_{-1,0,0}(z) + 24 \frac{1}{(z+1)^2} G_{0,-1,-1}(z) + 24 \frac{1}{z(z+1)^2} G_{0,-1,0}(z) + 24 \frac{1}{z(z+1)^2} G_{0,0,-1}(z) + \Bigg(- \frac{19}{2} \frac{\zeta_3}{(z+1)z}$$

$$- \frac{29\zeta_2 + 24}{(z+1)z} G_{-1}(z) + \frac{24z - 5\zeta_2 + 29z\zeta_2}{z(z+1)^2} G_0(z) - 12 \frac{1}{z(z+1)^2} G_{-1,0}(z) - 12 \frac{1}{z(z+1)^2} G_{0,-1}(z) - 4 \frac{1}{(z+1)z} G_{-1,-1,-1}(z)$$

$$+ 4 \frac{z+2}{z(z+1)^2} G_{-1,-1,0}(z) + 4 \frac{z+2}{z(z+1)^2} G_{-1,0,-1}(z) - 6 \frac{1}{(z+1)z} G_{-1,0,0}(z) + 4 \frac{z+2}{z(z+1)^2} G_{0,-1,-1}(z) - 4 \frac{1}{(z+1)z} G_{0,-1,0}(z)$$

$$- 4 \frac{1}{(z+1)z} G_{0,0,-1}(z) + 2 \frac{3z+1}{z(z+1)^2} G_{0,0,0}(z) \Bigg) \frac{1}{\epsilon} + 24 \frac{1}{(z+1)^2} G_{0,0,0}(z) + 44 \frac{1}{(z+1)z} G_{-1,-1,-1,-1}(z) - 2 \frac{7z+11}{z(z+1)^2} G_{-1,-1,-1,0}(z)$$

$$- 2 \frac{7z+11}{z(z+1)^2} G_{-1,-1,0,-1}(z) + 6 \frac{1}{(z+1)z} G_{-1,-1,0,0}(z) - 2 \frac{7z+11}{z(z+1)^2} G_{-1,0,-1,-1}(z) - 12 \frac{1}{(z+1)z} G_{-1,0,-1,0}(z)$$

$$- 12 \frac{1}{(z+1)z} G_{-1,0,0,-1}(z) + 4 \frac{5z+7}{z(z+1)^2} G_{-1,0,0,0}(z) - 2 \frac{13z+5}{z(z+1)^2} G_{0,-1,-1,-1}(z) + 2 \frac{7z+1}{z(z+1)^2} G_{0,-1,-1,0}(z)$$

$$+ 2 \frac{7z+1}{z(z+1)^2} G_{0,-1,0,-1}(z) + 6 \frac{z-1}{z(z+1)^2} G_{0,-1,0,0}(z) - 4 \frac{z+4}{z(z+1)^2} G_{0,0,-1,-1}(z) + 4 \frac{3z+5}{z(z+1)^2} G_{0,0,-1,0}(z) + 4 \frac{3z+5}{z(z+1)^2} G_{0,0,0,-1}(z)$$

$$- 16 \frac{2z+1}{z(z+1)^2} G_{0,0,0,0}(z) + \Bigg(\frac{-96 - 96z + 12\zeta_2 + 36z\zeta_2 + 89\zeta_3 + 77z\zeta_3}{z(z+1)^2} - \frac{1}{\epsilon^3} 2 \frac{1}{(z+1)z} - 8 \frac{\zeta_2}{(z+1)z} G_{-1}(z) + \Bigg(-6 \frac{1}{(z+1)z}$$

$$+ 4 \frac{1}{(z+1)z} G_{-1}(z) - \frac{z-1}{z(z+1)^2} G_0(z) \Bigg) \frac{1}{\epsilon^2} - 8 \frac{6 + 7\zeta_2 + 2z\zeta_2}{z(z+1)^2} G_0(z) + 24 \frac{1}{(z+1)z} G_{-1,-1}(z) - 24 \frac{1}{z(z+1)^2} G_{-1,0}(z)$$

$$- 24 \frac{1}{(z+1)^2} G_{0,-1}(z) + \Bigg(\frac{31\zeta_2 + 24}{(z+1)z} + 12 \frac{1}{z(z+1)^2} G_0(z) + 4 \frac{1}{(z+1)z} G_{-1,-1}(z) - 4 \frac{z+2}{z(z+1)^2} G_{-1,0}(z) - 4 \frac{z+2}{z(z+1)^2} G_{0,-1}(z)$$

$$+ 4 \frac{1}{(z+1)z} G_{0,0}(z) \Bigg) \frac{1}{\epsilon} - 24 \frac{1}{z(z+1)^2} G_{0,0}(z) - 44 \frac{1}{(z+1)z} G_{-1,-1,-1}(z) + 2 \frac{7z+11}{z(z+1)^2} G_{-1,-1,0}(z) + 2 \frac{7z+11}{z(z+1)^2} G_{-1,0,-1}(z)$$

$$+ 12 \frac{1}{(z+1)z} G_{-1,0,0}(z) + 2 \frac{13z+5}{z(z+1)^2} G_{0,-1,-1}(z) - 2 \frac{7z+1}{z(z+1)^2} G_{0,-1,0}(z) + 4 \frac{z+4}{z(z+1)^2} G_{0,0,-1}(z) - 4 \frac{3z+5}{z(z+1)^2} G_{0,0,0}(z) \Bigg) i\pi$$

$$+ \Bigg(\frac{1}{4} \frac{32z\zeta_3 + 127\zeta_4 + 327z\zeta_4}{z(z+1)^2} - \frac{1}{\epsilon^4} 3 \frac{1}{(z+1)z} + 2 \frac{-16 + 16\zeta_2 + 5\zeta_3}{(z+1)z} G_{-1}(z) + \Bigg(-5 \frac{1}{(z+1)z} G_{-1}(z) + \frac{7z+5}{z(z+1)^2} G_0(z) \Bigg) \frac{1}{\epsilon^3}$$

$$- 4 \frac{-8z + 8z\zeta_2 + 3\zeta_3 + 5z\zeta_3}{z(z+1)^2} G_0(z) + 11 \frac{\zeta_2}{(z+1)z} G_{-1,-1}(z) - \frac{16 + 29\zeta_2 + 11z\zeta_2}{z(z+1)^2} G_{-1,0}(z) - \frac{16 + 13\zeta_2 + 27z\zeta_2}{z(z+1)^2} G_{0,-1}(z)$$

$$+ \frac{(27z+11)\zeta_2}{z(z+1)^2} G_{0,0}(z) + \Bigg(- \frac{\zeta_2}{(z+1)z} - 8 \frac{1}{(z+1)z} G_{-1}(z) + 8 \frac{1}{(z+1)^2} G_0(z) + 5 \frac{1}{(z+1)z} G_{-1,-1}(z) - \frac{z-1}{z(z+1)^2} G_{-1,0}(z)$$

$$- \frac{z-1}{z(z+1)^2} G_{0,-1}(z) - 3 \frac{1}{(z+1)z} G_{0,0}(z) \Bigg) \frac{1}{\epsilon^2} + 8 \frac{1}{(z+1)z} G_{-1,-1,-1}(z) - 8 \frac{1}{z(z+1)^2} G_{-1,-1,0}(z) - 8 \frac{1}{z(z+1)^2} G_{-1,0,-1}(z)$$

$$+ 8 \frac{1}{(z+1)z} G_{-1,0,0}(z) - 8 \frac{1}{(z+1)^2} G_{0,-1,-1}(z) - 8 \frac{1}{z(z+1)^2} G_{0,-1,0}(z) - 8 \frac{1}{z(z+1)^2} G_{0,0,-1}(z) + \Bigg(6 \frac{\zeta_3}{(z+1)z} + \frac{23\zeta_2 + 16}{(z+1)z} G_{-1}(z)$$

$$- \frac{16z + 7\zeta_2 + 25z\zeta_2}{z(z+1)^2} G_0(z) + 8 \frac{1}{z(z+1)^2} G_{-1,0}(z) + 8 \frac{1}{z(z+1)^2} G_{0,-1}(z) + 3 \frac{1}{(z+1)z} G_{-1,-1,-1}(z) - \frac{3z+5}{z(z+1)^2} G_{-1,-1,0}(z)$$

$$- \frac{3z+5}{z(z+1)^2} G_{-1,0,-1}(z) + 5 \frac{1}{(z+1)z} G_{-1,0,0}(z) - \frac{3z+5}{z(z+1)^2} G_{0,-1,-1}(z) + 3 \frac{1}{(z+1)z} G_{0,-1,0}(z) + 3 \frac{1}{(z+1)z} G_{0,0,-1}(z)$$

$$- \frac{5z+3}{z(z+1)^2} G_{0,0,0}(z) \Bigg) \frac{1}{\epsilon} - 8 \frac{1}{(z+1)^2} G_{0,0,0}(z) - 15 \frac{1}{(z+1)z} G_{-1,-1,-1,-1}(z) + \frac{5z+7}{z(z+1)^2} G_{-1,-1,-1,0}(z) + \frac{5z+7}{z(z+1)^2} G_{-1,-1,0,-1}(z)$$

$$- \frac{1}{(z+1)z} G_{-1,-1,0,0}(z) + \frac{5z+7}{z(z+1)^2} G_{-1,0,-1,-1}(z) + 5 \frac{1}{(z+1)z} G_{-1,0,-1,0}(z) + 5 \frac{1}{(z+1)z} G_{-1,0,0,-1}(z) - \frac{13z+15}{z(z+1)^2} G_{-1,0,0,0}(z)$$

$$+ 3 \frac{3z+1}{z(z+1)^2} G_{0,-1,-1,-1}(z) - \frac{5z+1}{z(z+1)^2} G_{0,-1,-1,0}(z) - \frac{5z+1}{z(z+1)^2} G_{0,-1,0,-1}(z) - \frac{3z+1}{z(z+1)^2} G_{0,-1,0,0}(z) + \frac{z+5}{z(z+1)^2} G_{0,0,-1,-1}(z)$$

$$- \frac{5z+7}{z(z+1)^2} G_{0,0,-1,0}(z) - \frac{5z+7}{z(z+1)^2} G_{0,0,0,-1}(z) + \frac{17z+13}{z(z+1)^2} G_{0,0,0,0}(z) + \Bigg(-4 \frac{-8 - 8z + 2\zeta_2 + 4z\zeta_2 + 8\zeta_3 + 7z\zeta_3}{z(z+1)^2} + \frac{1}{\epsilon^3} 5 \frac{1}{(z+1)z}$$

$$+ \frac{\zeta_2}{(z+1)z} G_{-1}(z) + \Bigg(8 \frac{1}{(z+1)z} - 5 \frac{1}{(z+1)z} G_{-1}(z) + \frac{z-1}{z(z+1)^2} G_0(z) \Bigg) \frac{1}{\epsilon^2} + \frac{16 + 19\zeta_2 + 9z\zeta_2}{z(z+1)^2} G_0(z) - 8 \frac{1}{(z+1)z} G_{-1,-1}(z)$$

$$+ 8 \frac{1}{z(z+1)^2} G_{-1,0}(z) + 8 \frac{1}{(z+1)^2} G_{0,-1}(z) + \Bigg(- \frac{23\zeta_2 + 16}{(z+1)z} - 8 \frac{1}{z(z+1)^2} G_0(z) - 3 \frac{1}{(z+1)z} G_{-1,-1}(z) + \frac{3z+5}{z(z+1)^2} G_{-1,0}(z)$$

$$+ \frac{3z+5}{z(z+1)^2} G_{0,-1}(z) - 3 \frac{1}{(z+1)z} G_{0,0}(z) \Bigg) \frac{1}{\epsilon} + 8 \frac{1}{z(z+1)^2} G_{0,0}(z) + 15 \frac{1}{(z+1)z} G_{-1,-1,-1}(z) - \frac{5z+7}{z(z+1)^2} G_{-1,-1,0}(z)$$

$$- \frac{5z+7}{z(z+1)^2} G_{-1,0,-1}(z) - 5\frac{1}{(z+1)z} G_{-1,0,0}(z) - 3\frac{3z+1}{z(z+1)^2} G_{0,-1,-1}(z) + \frac{5z+1}{z(z+1)^2} G_{0,-1,0}(z) - \frac{z+5}{z(z+1)^2} G_{0,0,-1}(z)$$
$$+ \frac{5z+7}{z(z+1)^2} G_{0,0,0}(z)\Big) i\pi\Big)\kappa^{-\epsilon} + \Big(-\frac{1}{\epsilon^3} 4\frac{1}{(z+1)z} + \frac{1}{\epsilon} 10\frac{\zeta_2}{(z+1)z} + 10\frac{\zeta_3}{(z+1)z} + 6\frac{\zeta_2}{(z+1)z} G_{-1}(z) + \frac{1}{\epsilon^2} 2\frac{1}{(z+1)z} G_0(z)$$
$$- 8\frac{\zeta_2}{(z+1)z} G_0(z) + 2\frac{1}{(z+1)z} G_{-1,0,0}(z) - 2\frac{1}{(z+1)z} G_{0,0,0}(z) + \Big(2\frac{\zeta_3}{(z+1)z} G_{-1}(z) - 6\frac{\zeta_3}{(z+1)z} G_0(z) + 6\frac{\zeta_2}{(z+1)z} G_{-1,-1}(z)$$
$$- 6\frac{\zeta_2}{(z+1)z} G_{-1,0}(z) - 6\frac{\zeta_2}{(z+1)z} G_{0,-1}(z) + 6\frac{\zeta_2}{(z+1)z} G_{0,0}(z) + 2\frac{1}{(z+1)z} G_{-1,-1,0,0}(z) - 4\frac{1}{(z+1)z} G_{-1,0,0,0}(z)$$
$$- 2\frac{1}{(z+1)z} G_{0,-1,0,0}(z) + 4\frac{1}{(z+1)z} G_{0,0,0,0}(z)\Big)\epsilon\Big)\log(\kappa)\kappa^{-\epsilon} + \Big(\frac{1}{\epsilon^4}\frac{1}{(z+1)z} + \frac{3}{2}\frac{\zeta_4}{(z+1)z} - 4\frac{\zeta_3}{(z+1)z} G_{-1}(z) + \Big(3\frac{1}{(z+1)z} G_{-1}(z)$$
$$- \frac{1}{2}\frac{7z+5}{z(z+1)^2} G_0(z)\Big)\frac{1}{\epsilon^3} + \Big(\frac{7}{2}\frac{\zeta_3}{(z+1)z} + 2\frac{\zeta_2}{(z+1)z} G_0(z)\Big)\frac{1}{\epsilon} + 2\frac{\zeta_3}{(z+1)z} G_0(z) + \Big(-\frac{3}{2}\frac{\zeta_2}{(z+1)z} + 2\frac{1}{(z+1)z} G_{-1}(z) - 2\frac{1}{(z+1)^2} G_0(z)$$
$$- \frac{1}{(z+1)z} G_{-1,-1}(z) + \frac{1}{(z+1)z} G_{0,0}(z)\Big)\frac{1}{\epsilon^2} + \Big(-\frac{1}{\epsilon^3} 3\frac{1}{(z+1)z} + 4\frac{\zeta_3}{(z+1)z} + \Big(-2\frac{1}{(z+1)z} + \frac{1}{(z+1)z} G_{-1}(z)\Big)\frac{1}{\epsilon^2}\Big) i\pi\Big)\kappa^{-2\epsilon}$$
$$+ \Big(-\epsilon\frac{5}{6}\frac{(60\log(2)^2 + \pi^2)\pi^2}{z^{\frac{1}{2}}(z+1)^{\frac{3}{2}}} - \frac{1}{\epsilon} 6\frac{\zeta_2}{z^{\frac{1}{2}}(z+1)^{\frac{3}{2}}} + 60\frac{\log(2)\zeta_2}{z^{\frac{1}{2}}(z+1)^{\frac{3}{2}}}\Big) i\pi\kappa^{-\frac{1}{2}}\kappa^{-2\epsilon} + \Big(-\epsilon\frac{3}{2}\frac{\zeta_4}{(z+1)z} + \frac{1}{\epsilon^3} 3\frac{1}{(z+1)z} - \frac{1}{\epsilon} 4\frac{\zeta_2}{(z+1)z}$$
$$+ \Big(\frac{1}{(z+1)z} G_{-1}(z) - 2\frac{1}{(z+1)z} G_0(z)\Big)\frac{1}{\epsilon^2} - \frac{1}{\epsilon^2}\frac{1}{(z+1)z} i\pi\Big)\log(\kappa)\kappa^{-2\epsilon} + \frac{1}{\epsilon^2}\frac{3}{2}\frac{1}{(z+1)z}\log^2(\kappa)\kappa^{-2\epsilon} + O(\eta) + O(\kappa) \tag{D.4}$$

I[PL2, 1,1, 1, 1, 1, 0, 0, 1, 1] =

$$- \frac{1}{\epsilon^4}\frac{3}{4}\frac{z+1}{z^2} - \frac{3}{4}\frac{-8 - 8z + 4\zeta_3 + 4z\zeta_3 - 7\zeta_4 + 10z\zeta_4}{z^2} - 9\frac{(z+1)(2\zeta_2+\zeta_3)}{z^2} G_{-1}(z) + \Big(-\frac{3}{4}\frac{z+1}{z^2} + \frac{3}{2}\frac{1}{z^2} G_0(z)\Big)\frac{1}{\epsilon^3}$$
$$+ 3\frac{2 + 2\zeta_2 + 6z\zeta_2 - 2\zeta_3 + 3z\zeta_3}{z^2} G_0(z) - 27\frac{(z+1)\zeta_2}{z^2} G_{-1,-1}(z) + 3\frac{(8z+7)\zeta_2}{z^2} G_{-1,0}(z) + 9\frac{(3z+2)\zeta_2}{z^2} G_{0,-1}(z)$$
$$+ \Big(\frac{3}{2}\frac{z+1}{z^2} + \frac{3}{2}\frac{1}{z^2} G_0(z) - 3\frac{1}{z^2} G_{0,0}(z)\Big)\frac{1}{\epsilon^2} - 6\frac{-1+4z\zeta_2}{z^2} G_{0,0}(z) - 6\frac{z+1}{z^2} G_{-1,0,0}(z) + \Big(-3\frac{(z+1)(1+\zeta_2-\zeta_3)}{z^2} - 9\frac{(z+1)\zeta_2}{z^2} G_{-1}(z)$$
$$+ 3\frac{-1+3z\zeta_2}{z^2} G_0(z) - 3\frac{1}{z^2} G_{0,0}(z) - 3\frac{z+1}{z^2} G_{-1,0,0}(z) + 3\frac{z+2}{z^2} G_{0,0,0}(z)\Big)\frac{1}{\epsilon} + 6\frac{z+1}{z^2} G_{0,0,0}(z) - 9\frac{z+1}{z^2} G_{-1,-1,0,0}(z)$$
$$+ 3\frac{5z+4}{z^2} G_{-1,0,0,0}(z) + 3\frac{3z+2}{z^2} G_{0,-1,0,0}(z) - 3\frac{5z+4}{z^2} G_{0,0,0,0}(z) + \Big(\frac{3}{4}\frac{9 + 36z + 9z^2 + 12\zeta_2 + 272z\zeta_2 + 12z^2\zeta_2 - 8\zeta_3 + 112z\zeta_3 - 8z^2\zeta_3}{z^3}$$
$$+ \frac{1}{\epsilon^3}\frac{3}{2}\frac{z^2-4z+1}{z^3} + 18\frac{(z^2+4z+1)\zeta_2}{z^3} G_{-1}(z) - \frac{3}{2}\frac{-13 - 10z + 56z\zeta_2 + 12z^2\zeta_2}{z^3} G_0(z) + \Big(\frac{3}{4}\frac{z^2-36z+1}{z^3} + 3\frac{2z-1}{z^3} G_0(z)\Big)\frac{1}{\epsilon^2}$$
$$- 3\frac{2z-1}{z^3} G_{0,0}(z) + \Big(-\frac{3}{4}\frac{13 + 20z + 13z^2 - 64z\zeta_2}{z^3} + \frac{3}{2}\frac{18z-1}{z^3} G_0(z) + 6\frac{1}{z^3} G_{0,0}(z)\Big)\frac{1}{\epsilon} + 6\frac{z^2+4z+1}{z^3} G_{-1,0,0}(z) - 6\frac{z^2+4z+2}{z^3} G_{0,0,0}(z)$$
$$+ \Big(\frac{3}{4}\frac{1 - 32z + z^2 + 28\zeta_2 + 128z\zeta_2 + 28z^2\zeta_2 + 36\zeta_3 + 496z\zeta_3 + 36z^2\zeta_3 - 14\zeta_4 + 40z\zeta_4 + 20z^2\zeta_4}{z^3}$$
$$+ 6\frac{12\zeta_2 + 48z\zeta_2 + 12z^2\zeta_2 + 3\zeta_3 + 8z\zeta_3 + 3z^2\zeta_3}{z^3} G_{-1}(z) - \frac{3}{2}\frac{9 + 18z + 12\zeta_2 + 232z\zeta_2 + 48z^2\zeta_2 - 8\zeta_3 + 72z\zeta_3 + 12z^2\zeta_3}{z^3} G_0(z)$$
$$+ 18\frac{(3z^2+8z+3)\zeta_2}{z^3} G_{-1,-1}(z) - 6\frac{(8z^2+24z+7)\zeta_2}{z^3} G_{-1,0}(z) - 18\frac{(3z^2+8z+2)\zeta_2}{z^3} G_{0,-1}(z) + 3\frac{-13 - 4z + 48z\zeta_2 + 16z^2\zeta_2}{z^3} G_{0,0}(z)$$
$$+ 24\frac{z^2+4z+1}{z^3} G_{-1,0,0}(z) - 6\frac{4z^2+14z+1}{z^3} G_{0,0,0}(z) + 6\frac{3z^2+8z+3}{z^3} G_{-1,-1,0,0}(z) - 6\frac{5z^2+16z+4}{z^3} G_{-1,0,0,0}(z)$$
$$- 6\frac{3z^2+8z+2}{z^3} G_{0,-1,0,0}(z) + 6\frac{5z^2+16z+4}{z^3} G_{0,0,0,0}(z)\Big)\epsilon\Big)\kappa + \Big(\frac{1}{2}\frac{-4 - 4z + 8\zeta_2 + 8z\zeta_2 + 12\zeta_3 + 12z\zeta_3 - 7\zeta_4 + 10z\zeta_4}{z^2} + \frac{1}{\epsilon^4} 2\frac{z+1}{z^2}$$
$$+ 4\frac{(z+1)(3\zeta_2+\zeta_3)}{z^2} G_{-1}(z) + \Big(2\frac{z+1}{z^2} - 2\frac{1}{z^2} G_0(z)\Big)\frac{1}{\epsilon^3} - 2\frac{1 + 2\zeta_2 + 6z\zeta_2 + 2z\zeta_3}{z^2} G_0(z) + 12\frac{(z+1)\zeta_2}{z^2} G_{-1,-1}(z) - 2\frac{(5z+4)\zeta_2}{z^2} G_{-1,0}$$
$$- 6\frac{(2z+1)\zeta_2}{z^2} G_{0,-1}(z) + \Big(-2\frac{z+1}{z^2} - 2\frac{1}{z^2} G_0(z) + 2\frac{1}{z^2} G_{0,0}(z)\Big)\frac{1}{\epsilon^2} + 2\frac{-1+5z\zeta_2}{z^2} G_{0,0}(z) + 4\frac{z+1}{z^2} G_{-1,0,0}(z) + \Big(2\frac{(z+1)(2\zeta_2+1)}{z^2}$$
$$+ 6\frac{(z+1)\zeta_2}{z^2} G_{-1}(z) - 2\frac{-1+3z\zeta_2}{z^2} G_0(z) + 2\frac{1}{z^2} G_{0,0}(z) + 2\frac{z+1}{z^2} G_{-1,0,0}(z) - 2\frac{z+1}{z^2} G_{0,0,0}(z)\Big)\frac{1}{\epsilon} - 2\frac{2z+1}{z^2} G_{0,0,0}(z)$$
$$+ 4\frac{z+1}{z^2} G_{-1,-1,0,0}(z) - 2\frac{3z+2}{z^2} G_{-1,0,0,0}(z) - 2\frac{2z+1}{z^2} G_{0,-1,0,0}(z) + 2\frac{3z+1}{z^2} G_{0,0,0,0}(z)\Big)\kappa^{-\epsilon}$$
$$+ \Big(-8\frac{-3 - 12z - 3z^2 + \zeta_2 + 20z\zeta_2 + z^2\zeta_2 + 5z\zeta_3}{z^3} + \frac{1}{\epsilon^3} 16\frac{1}{z^2} - 24\frac{\zeta_2}{z^2} G_{-1}(z) + \Big(8\frac{z^2+8z+1}{z^3} - 8\frac{1}{z^2} G_0(z)\Big)\frac{1}{\epsilon^2}$$
$$+ \Big(8\frac{3 + 12z + 3z^2 - 5z\zeta_2}{z^3} - 8\frac{4z+1}{z^3} G_0(z)\Big)\frac{1}{\epsilon} + 8\frac{-3 - 6z + 4z\zeta_2}{z^3} G_0(z) + 8\frac{1}{z^3} G_{0,0}(z) - 8\frac{1}{z^2} G_{-1,0,0}(z) + 8\frac{1}{z^2} G_{0,0,0}(z)$$
$$+ \Big(-8\frac{-3 - 12z - 3z^2 + 3\zeta_2 + 30z\zeta_2 + 3z^2\zeta_2 + 2\zeta_3 + 20z\zeta_3 + 2z^2\zeta_3}{z^3} - 8\frac{12\zeta_2+\zeta_3}{z^2} G_{-1}(z) + 8\frac{-3 - 6z + \zeta_2 + 16z\zeta_2 + 3z\zeta_3}{z^3} G_0(z)$$
$$- 24\frac{\zeta_2}{z^2} G_{-1,-1}(z) + 24\frac{\zeta_2}{z^2} G_{-1,0}(z) + 24\frac{\zeta_2}{z^2} G_{0,-1}(z) - 24\frac{-1+z\zeta_2}{z^3} G_{0,0}(z) - 32\frac{1}{z^2} G_{-1,0,0}(z) + 8\frac{4z-1}{z^3} G_{0,0,0}(z) - 8\frac{1}{z^2} G_{-1,-1,0,0}(z)$$
$$+ 16\frac{1}{z^2} G_{-1,0,0,0}(z) + 8\frac{1}{z^2} G_{0,-1,0,0}(z) - 16\frac{1}{z^2} G_{0,0,0,0}(z)\Big)\epsilon\Big)\kappa\kappa^{-\epsilon} + \Big(-\frac{2}{5}\frac{(z+1)(3\zeta_2+5)\zeta_2}{z^2} - \frac{1}{\epsilon}\frac{(z+1)(\zeta_2-\zeta_3)}{z^2} - \frac{1}{\epsilon^4}\frac{5}{4}\frac{z+1}{z^2}$$
$$+ \Big(\frac{1}{2}\frac{z+1}{z^2} + \frac{1}{2}\frac{1}{z^2} G_0(z)\Big)\frac{1}{\epsilon^2} + \Big(-\frac{5}{4}\frac{z+1}{z^2} + \frac{1}{2}\frac{1}{z^2} G_0(z)\Big)\frac{1}{\epsilon^3}\Big)\kappa^{-2\epsilon} + \Big(-\frac{1}{\epsilon^2}\frac{1}{z} + \frac{\zeta_2-7}{z} - \epsilon\frac{-15+3\zeta_2-\zeta_3}{z} + \epsilon^2\frac{-31+7\zeta_2-3\zeta_3+\zeta_4}{z}$$

$$
\begin{aligned}
&+\frac{1}{\epsilon}3\frac{1}{z}\Bigg)\kappa^{-1}\kappa^{-2\epsilon}+\Bigg(-\frac{1}{\epsilon^3}\frac{1}{2}\frac{3z^2+20z+3}{z^3}+\frac{-35-111z-35z^2+\zeta_2+12z\zeta_2+z^2\zeta_2+2\zeta_3+12z\zeta_3+2z^2\zeta_3}{z^3}\\
&+\Bigg(-\frac{1}{4}\frac{83+276z+83z^2-16z\zeta_2}{z^3}+\frac{1}{2}\frac{14z+9}{z^3}G_0(z)\Bigg)\frac{1}{\epsilon}+\Bigg(-\frac{1}{4}\frac{35z^2+148z+35}{z^3}+\frac{2z+1}{z^3}G_0(z)\Bigg)\frac{1}{\epsilon^2}+\frac{13z+10}{z^3}G_0(z)\\
&+\Bigg(\frac{1}{2}\frac{-121-384z-121z^2+6\zeta_2+36z\zeta_2+6z^2\zeta_2+20\zeta_3+80z\zeta_3+20z^2\zeta_3-12\zeta_4-40z\zeta_4-12z^2\zeta_4}{z^3}+\frac{19z+16}{z^3}G_0(z)\Bigg)\epsilon\Bigg)\kappa\kappa^{-2\epsilon}\\
&+\Bigg(\epsilon^2\frac{1}{12}\frac{8\log(2)^4-24\log(2)^2\zeta_2+48\log(2)\zeta_3-27\zeta_4}{z}-\epsilon\frac{2}{3}\frac{2\log(2)^3-3\log(2)\zeta_2+3\zeta_3}{z}+\frac{1}{\epsilon^2}\frac{1}{z}+\frac{2\log(2)^2-\zeta_2}{z}-\frac{1}{\epsilon}2\frac{\log(2)}{z}\Bigg)\kappa^{-1}\eta^{-\epsilon}\kappa^{-2\epsilon}\\
&+\Bigg(-\frac{1}{\epsilon^3}\frac{1}{2}\frac{z+1}{z^2}-\frac{1}{\epsilon^2}\frac{1}{2}\frac{z+1}{z^2}\Bigg)\log(\kappa)\kappa^{-2\epsilon}+\Bigg(-\frac{1}{\epsilon}\frac{1}{2}\frac{9z^2+28z+9}{z^3}-\frac{1}{\epsilon^2}\frac{z^2+4z+1}{z^3}-2\frac{5z^2+13z+5}{z^3}-\epsilon 2\frac{8z^2+19z+8}{z^3}\\
&-\epsilon^2 2\frac{11z^2+25z+11}{z^3}\Bigg)\kappa\log(\kappa)\kappa^{-2\epsilon}+O(\eta)+O(\kappa^2)
\end{aligned}
\tag{D.5}
$$

I[PL2, 1,1, 1, 0, 1, 0, 1, 1, 1] =

$$
\begin{aligned}
&-\frac{1}{\epsilon^4}4\frac{1}{z}+\frac{93}{2}\frac{\zeta_4}{z}+4\frac{\zeta_3}{z}G_{-1}(z)+\frac{1}{\epsilon^3}5\frac{1}{z}G_0(z)-26\frac{\zeta_3}{z}G_0(z)+12\frac{\zeta_2}{z}G_{-1,-1}(z)-20\frac{\zeta_2}{z}G_{-1,0}(z)-60\frac{\zeta_2}{z}G_{0,-1}(z)+\Bigg(19\frac{\zeta_2}{z}-4\frac{1}{z}G_{0,0}(z)\Bigg)\frac{1}{\epsilon^2}\\
&+76\frac{\zeta_2}{z}G_{0,0}(z)+\Bigg(19\frac{\zeta_3}{z}+12\frac{\zeta_2}{z}G_{-1}(z)-38\frac{\zeta_2}{z}G_0(z)+4\frac{1}{z}G_{-1,0,0}(z)-4\frac{1}{z}G_{0,0,0}(z)\Bigg)\frac{1}{\epsilon}+4\frac{1}{z}G_{-1,-1,0,0}(z)-16\frac{1}{z}G_{-1,0,0,0}(z)\\
&-20\frac{1}{z}G_{0,-1,0,0}(z)+32\frac{1}{z}G_{0,0,0,0}(z)+\Bigg(\frac{1}{2}\frac{17+129z+40\zeta_2+536z\zeta_2-112\zeta_3+96z\zeta_3-186\zeta_4-186z\zeta_4}{z^2}+\frac{1}{\epsilon^4}8\frac{z+1}{z^2}\\
&-4\frac{15\zeta_2+2\zeta_3+2z\zeta_3}{z^2}G_{-1}(z)+2\frac{7-2z+110\zeta_2+48z\zeta_2+26\zeta_3+26z\zeta_3}{z^2}G_0(z)+\Bigg(\frac{14z+23}{z^2}-10\frac{z+1}{z^2}G_0(z)\Bigg)\frac{1}{\epsilon^3}-24\frac{(z+1)\zeta_2}{z^2}G_{-1,-1}(z)\\
&+40\frac{(z+1)\zeta_2}{z^2}G_{-1,0}(z)+120\frac{(z+1)\zeta_2}{z^2}G_{0,-1}(z)-2\frac{1+76\zeta_2+76z\zeta_2}{z^2}G_{0,0}(z)+\Bigg(-\frac{1}{2}\frac{-5+66z+76\zeta_2+76z\zeta_2}{z^2}-4\frac{4z+7}{z^2}G_0(z)\\
&+8\frac{z+1}{z^2}G_{0,0}(z)\Bigg)\frac{1}{\epsilon^2}-20\frac{1}{z^2}G_{-1,0,0}(z)+\Bigg(-\frac{1}{2}\frac{11+21z+208\zeta_2+64z\zeta_2+76\zeta_3+76z\zeta_3}{z^2}-24\frac{(z+1)\zeta_2}{z^2}G_{-1}(z)\\
&+2\frac{-1+14z+38\zeta_2+38z\zeta_2}{z^2}G_0(z)+4\frac{4z+5}{z^2}G_{0,0}(z)-8\frac{z+1}{z^2}G_{-1,0,0}(z)+8\frac{z+1}{z^2}G_{0,0,0}(z)\Bigg)\frac{1}{\epsilon}+32\frac{1}{z^2}G_{0,0,0}(z)-8\frac{z+1}{z^2}G_{-1,-1,0,0}(z)\\
&+32\frac{z+1}{z^2}G_{-1,0,0,0}(z)+40\frac{z+1}{z^2}G_{0,-1,0,0}(z)-64\frac{z+1}{z^2}G_{0,0,0,0}(z)\Bigg)\kappa+\Bigg(\frac{1}{\epsilon^4}4\frac{1}{z}+4\frac{\zeta_4}{z}-2\frac{\zeta_3}{z}G_{-1}(z)-\frac{1}{\epsilon^3}6\frac{1}{z}G_0(z)+12\frac{\zeta_3}{z}G_0(z)\\
&-6\frac{\zeta_2}{z}G_{-1,-1}(z)+10\frac{\zeta_2}{z}G_{-1,0}(z)+18\frac{\zeta_2}{z}G_{0,-1}(z)+\Bigg(-14\frac{\zeta_2}{z}+4\frac{1}{z}G_{0,0}(z)\Bigg)\frac{1}{\epsilon^2}-26\frac{\zeta_2}{z}G_{0,0}(z)+\Bigg(-6\frac{\zeta_3}{z}-6\frac{\zeta_2}{z}G_{-1}(z)+22\frac{\zeta_2}{z}G_0(z)\\
&-2\frac{1}{z}G_{-1,0,0}(z)+2\frac{1}{z}G_{0,0,0}(z)\Bigg)\frac{1}{\epsilon}-2\frac{1}{z}G_{-1,-1,0,0}(z)+8\frac{1}{z}G_{-1,0,0,0}(z)+6\frac{1}{z}G_{0,-1,0,0}(z)-12\frac{1}{z}G_{0,0,0,0}(z)\Bigg)\kappa^{-\epsilon}\\
&+\Bigg(-4\frac{-18-32z+8\zeta_2+53z\zeta_2+7z\zeta_3+2\zeta_4+2z\zeta_4}{z^2}-\frac{1}{\epsilon^4}8\frac{z+1}{z^2}+4\frac{12\zeta_2+\zeta_3+z\zeta_3}{z^2}G_{-1}(z)-2\frac{17+22z+50\zeta_2+14z\zeta_2+12\zeta_3+12z\zeta_3}{z^2}G_0(z)\\
&+\Bigg(4\frac{z-2}{z^2}+12\frac{z+1}{z^2}G_0(z)\Bigg)\frac{1}{\epsilon^3}+12\frac{(z+1)\zeta_2}{z^2}G_{-1,-1}(z)-20\frac{(z+1)\zeta_2}{z^2}G_{-1,0}(z)-36\frac{(z+1)\zeta_2}{z^2}G_{0,-1}(z)+2\frac{11-4z+26\zeta_2+26z\zeta_2}{z^2}G_{0,0}(z)\\
&+\Bigg(2\frac{19+46z+14\zeta_2+14z\zeta_2}{z^2}+8\frac{z+2}{z^2}G_0(z)-8\frac{z+1}{z^2}G_{0,0}(z)\Bigg)\frac{1}{\epsilon^2}+16\frac{1}{z^2}G_{-1,0,0}(z)+\Bigg(2\frac{33+64z+28\zeta_2+6\zeta_3+6z\zeta_3}{z^2}+12\frac{(z+1)\zeta_2}{z^2}G_{-1}(z)\\
&-2\frac{9+16z+22\zeta_2+22z\zeta_2}{z^2}G_0(z)-8\frac{1}{z}G_{0,0}(z)+4\frac{z+1}{z^2}G_{-1,0,0}(z)-4\frac{z+1}{z^2}G_{0,0,0}(z)\Bigg)\frac{1}{\epsilon}-40\frac{1}{z^2}G_{0,0,0}(z)+4\frac{z+1}{z^2}G_{-1,-1,0,0}(z)\\
&-16\frac{z+1}{z^2}G_{-1,0,0,0}(z)-12\frac{z+1}{z^2}G_{0,-1,0,0}(z)+24\frac{z+1}{z^2}G_{0,0,0,0}(z)\Bigg)\kappa\kappa^{-\epsilon}+\Bigg(\frac{1}{\epsilon^3}4\frac{1}{z}-\frac{1}{\epsilon}10\frac{\zeta_2}{z}-10\frac{\zeta_3}{z}-6\frac{\zeta_2}{z}G_{-1}(z)-\frac{1}{\epsilon^2}2\frac{1}{z}G_0(z)\\
&+8\frac{\zeta_2}{z}G_0(z)-2\frac{1}{z}G_{-1,0,0}(z)+2\frac{1}{z}G_{0,0,0}(z)+\Bigg(-2\frac{\zeta_3}{z}G_{-1}(z)+6\frac{\zeta_3}{z}G_0(z)-6\frac{\zeta_2}{z}G_{-1,-1}(z)+6\frac{\zeta_2}{z}G_{-1,0}(z)+6\frac{\zeta_2}{z}G_{0,-1}(z)-6\frac{\zeta_2}{z}G_{0,0}(z)\\
&-2\frac{1}{z}G_{-1,-1,0,0}(z)+4\frac{1}{z}G_{-1,0,0,0}(z)+2\frac{1}{z}G_{0,-1,0,0}(z)-4\frac{1}{z}G_{0,0,0,0}(z)\Bigg)\epsilon\Bigg)\log(\kappa)\kappa^{-\epsilon}+\Bigg(2\frac{-7-6z+26\zeta_2+12z\zeta_2+10\zeta_3+10z\zeta_3}{z^2}\\
&-\frac{1}{\epsilon^3}8\frac{z+1}{z^2}+12\frac{(z+1)\zeta_2}{z^2}G_{-1}(z)-2\frac{-1-2z+8\zeta_2+8z\zeta_2}{z^2}G_0(z)+\Bigg(2\frac{-7-8z+10\zeta_2+10z\zeta_2}{z^2}+4\frac{z+1}{z^2}G_0(z)\Bigg)\frac{1}{\epsilon}+\Bigg(-4\frac{3z+4}{z^2}\\
&+4\frac{z+1}{z^2}G_0(z)\Bigg)\frac{1}{\epsilon^2}+8\frac{1}{z^2}G_{0,0}(z)+4\frac{z+1}{z^2}G_{-1,0,0}(z)-4\frac{z+1}{z^2}G_{0,0,0}(z)+\Bigg(2\frac{-7-8z+25\zeta_2+14z\zeta_2+22\zeta_3+14z\zeta_3}{z^2}\\
&+4\frac{9\zeta_2+3z\zeta_2+\zeta_3+z\zeta_3}{z^2}G_{-1}(z)-2\frac{-1-2z+20\zeta_2+8z\zeta_2+6\zeta_3+6z\zeta_3}{z^2}G_0(z)+12\frac{(z+1)\zeta_2}{z^2}G_{-1,-1}(z)-12\frac{(z+1)\zeta_2}{z^2}G_{-1,0}(z)\\
&-12\frac{(z+1)\zeta_2}{z^2}G_{0,-1}(z)+2\frac{5+6\zeta_2+6z\zeta_2}{z^2}G_{0,0}(z)+4\frac{z+3}{z^2}G_{-1,0,0}(z)-4\frac{z+5}{z^2}G_{0,0,0}(z)+4\frac{z+1}{z^2}G_{-1,-1,0,0}(z)-8\frac{z+1}{z^2}G_{-1,0,0,0}(z)\\
&-4\frac{z+1}{z^2}G_{0,-1,0,0}(z)+8\frac{z+1}{z^2}G_{0,0,0,0}(z)\Bigg)\epsilon\Bigg)\kappa\log(\kappa)\kappa^{-\epsilon}+\Bigg(\frac{1}{\epsilon^2}\frac{\zeta_2}{z}+\frac{1}{\epsilon}\frac{\zeta_3}{z}-15\frac{\zeta_4}{z}+\frac{1}{\epsilon^3}\frac{1}{z}G_0(z)\Bigg)\kappa^{-2\epsilon}\\
&+\Bigg(\frac{1}{2}\frac{-209-353z+32\zeta_2+40z\zeta_2+24\zeta_3+64z\zeta_3+60\zeta_4+60z\zeta_4}{z^2}+\Bigg(-3\frac{6z+5}{z^2}-2\frac{z+1}{z^2}G_0(z)\Bigg)\frac{1}{\epsilon^3}+\Bigg(-\frac{1}{2}\frac{97+134z+4\zeta_2+4z\zeta_2}{z^2}\\
&+4\frac{z+1}{z^2}G_0(z)\Bigg)\frac{1}{\epsilon^2}+\Bigg(\frac{1}{2}\frac{-169-219z+8\zeta_2+8z\zeta_2-4\zeta_3-4z\zeta_3}{z^2}+4\frac{4z+5}{z^2}G_0(z)\Bigg)\frac{1}{\epsilon}+12\frac{3z+4}{z^2}G_0(z)\Bigg)\kappa\kappa^{-2\epsilon}\\
&+\Bigg(\frac{1}{\epsilon}2\frac{\zeta_2}{z}-4\frac{\zeta_3}{z}+\epsilon\frac{23}{2}\frac{\zeta_4}{z}\Bigg)\log(\kappa)\kappa^{-2\epsilon}+\Bigg(-\frac{1}{\epsilon}\frac{33+30z+4\zeta_2+4z\zeta_2}{z^2}+\frac{-77-59z-8\zeta_2+8\zeta_3+8z\zeta_3}{z^2}\\
&-\epsilon\frac{163+117z+14\zeta_2+8z\zeta_2-16\zeta_3+23\zeta_4+23z\zeta_4}{z^2}-\frac{1}{\epsilon^2}2\frac{4z+5}{z^2}\Bigg)\kappa\log(\kappa)\kappa^{-2\epsilon}+O(\eta)+O(\kappa^2)
\end{aligned}
\tag{D.6}
$$

I[PL1, 1,1, 1, 0, 1, 0, 1, 1, 1] =

$$
\begin{aligned}
&-\frac{1}{\epsilon^4}4\frac{1}{z}+\frac{93}{2}\frac{\zeta_4}{z}+4\frac{\zeta_3}{z}G_{-1}(z)+\frac{1}{\epsilon^3}5\frac{1}{z}G_0(z)-26\frac{\zeta_3}{z}G_0(z)+12\frac{\zeta_2}{z}G_{-1,-1}(z)-20\frac{\zeta_2}{z}G_{-1,0}(z)-60\frac{\zeta_2}{z}G_{0,-1}(z)+\bigg(19\frac{\zeta_2}{z}-4\frac{1}{z}G_{0,0}(z)\bigg)\frac{1}{\epsilon^2}\\
&+76\frac{\zeta_2}{z}G_{0,0}(z)+\bigg(19\frac{\zeta_3}{z}+12\frac{\zeta_2}{z}G_{-1}(z)-38\frac{\zeta_2}{z}G_0(z)+4\frac{1}{z}G_{-1,0,0}(z)-4\frac{1}{z}G_{0,0,0}(z)\bigg)\frac{1}{\epsilon}+4\frac{1}{z}G_{-1,-1,0,0}(z)-16\frac{1}{z}G_{-1,0,0,0}(z)\\
&-20\frac{1}{z}G_{0,-1,0,0}(z)+32\frac{1}{z}G_{0,0,0,0}(z)+\bigg(\frac{-23+69z-124\zeta_2+52z\zeta_2-128\zeta_3-196z\zeta_3-93\zeta_4-186z\zeta_4}{z^2}+\frac{1}{\epsilon^4}8\frac{2z+1}{z^2}\\
&-8\frac{21\zeta_2+24z\zeta_2+\zeta_3+2z\zeta_3}{z^2}G_{-1}(z)+4\frac{2+5z+76\zeta_2+104z\zeta_2+13\zeta_3+26z\zeta_3}{z^2}G_0(z)+\bigg(2\frac{21z+13}{z^2}-10\frac{2z+1}{z^2}G_0(z)\bigg)\frac{1}{\epsilon^3}\\
&-24\frac{(2z+1)\zeta_2}{z^2}G_{-1,-1}(z)+40\frac{(2z+1)\zeta_2}{z^2}G_{-1,0}(z)+120\frac{(2z+1)\zeta_2}{z^2}G_{0,-1}(z)-4\frac{-7+38\zeta_2+76z\zeta_2}{z^2}G_{0,0}(z)+\bigg(-\frac{-25+38\zeta_2+76z\zeta_2}{z^2}\\
&-4\frac{12z+7}{z^2}G_0(z)+8\frac{2z+1}{z^2}G_{0,0}(z)\bigg)\frac{1}{\epsilon^2}-8\frac{8z+7}{z^2}G_{-1,0,0}(z)+\bigg(-\frac{7+33z+152\zeta_2+204z\zeta_2+38\zeta_3+76z\zeta_3}{z^2}-24\frac{(2z+1)\zeta_2}{z^2}G_{-1}(z)\\
&+4\frac{-8+z+19\zeta_2+38z\zeta_2}{z^2}G_0(z)+8\frac{4z+1}{z^2}G_{0,0}(z)-8\frac{2z+1}{z^2}G_{-1,0,0}(z)+8\frac{2z+1}{z^2}G_{0,0,0}(z)\bigg)\frac{1}{\epsilon}+16\frac{4z+5}{z^2}G_{0,0,0}(z)\\
&-8\frac{2z+1}{z^2}G_{-1,-1,0,0}(z)+32\frac{2z+1}{z^2}G_{-1,0,0,0}(z)+40\frac{2z+1}{z^2}G_{0,-1,0,0}(z)-64\frac{2z+1}{z^2}G_{0,0,0,0}(z)\bigg)\kappa+\bigg(\frac{1}{\epsilon^4}8\frac{1}{z}+8\frac{\zeta_4}{z}-4\frac{\zeta_3}{z}G_{-1}(z)\\
&-\frac{1}{\epsilon^3}12\frac{1}{z}G_0(z)+24\frac{\zeta_3}{z}G_0(z)-12\frac{\zeta_2}{z}G_{-1,-1}(z)+20\frac{\zeta_2}{z}G_{-1,0}(z)+36\frac{\zeta_2}{z}G_{0,-1}(z)+\bigg(-28\frac{\zeta_2}{z}+8\frac{1}{z}G_{0,0}(z)\bigg)\frac{1}{\epsilon^2}-52\frac{\zeta_2}{z}G_{0,0}(z)\\
&+\bigg(-12\frac{\zeta_3}{z}-12\frac{\zeta_2}{z}G_{-1}(z)+44\frac{\zeta_2}{z}G_0(z)-4\frac{1}{z}G_{-1,0,0}(z)+4\frac{1}{z}G_{0,0,0}(z)\bigg)\frac{1}{\epsilon}-4\frac{1}{z}G_{-1,-1,0,0}(z)+16\frac{1}{z}G_{-1,0,0,0}(z)+12\frac{1}{z}G_{0,-1,0,0}(z)\\
&-24\frac{1}{z}G_{0,0,0,0}(z)\bigg)\kappa^{-\epsilon}+\bigg(8\frac{9+13z+19\zeta_2+3z\zeta_2+12\zeta_3+19z\zeta_3-2\zeta_4-4z\zeta_4}{z^2}-\frac{1}{\epsilon^4}16\frac{2z+1}{z^2}+8\frac{15\zeta_2+18z\zeta_2+\zeta_3+2z\zeta_3}{z^2}G_{-1}(z)\\
&-4\frac{3+4z+70\zeta_2+90z\zeta_2+12\zeta_3+24z\zeta_3}{z^2}G_0(z)+\bigg(-8\frac{11z+7}{z^2}+24\frac{2z+1}{z^2}G_0(z)\bigg)\frac{1}{\epsilon^3}+24\frac{(2z+1)\zeta_2}{z^2}G_{-1,-1}(z)-40\frac{(2z+1)\zeta_2}{z^2}G_{-1,0}(z)\\
&-72\frac{(2z+1)\zeta_2}{z^2}G_{0,-1}(z)+4\frac{-7-4z+26\zeta_2+52z\zeta_2}{z^2}G_{0,0}(z)+\bigg(4\frac{-11+14\zeta_2+28z\zeta_2}{z^2}+32\frac{3z+2}{z^2}G_0(z)-16\frac{2z+1}{z^2}G_{0,0}(z)\bigg)\frac{1}{\epsilon^2}\\
&+8\frac{6z+5}{z^2}G_{-1,0,0}(z)+\bigg(4\frac{13+24z+50\zeta_2+68z\zeta_2+6\zeta_3+12z\zeta_3}{z^2}+24\frac{(2z+1)\zeta_2}{z^2}G_{-1}(z)-4\frac{-15-6z+22\zeta_2+44z\zeta_2}{z^2}G_0(z)\\
&-24\frac{2z+1}{z^2}G_{0,0}(z)+8\frac{2z+1}{z^2}G_{-1,0,0}(z)-8\frac{2z+1}{z^2}G_{0,0,0}(z)\bigg)\frac{1}{\epsilon}-16\frac{3z+4}{z^2}G_{0,0,0}(z)+8\frac{2z+1}{z^2}G_{-1,-1,0,0}(z)-32\frac{2z+1}{z^2}G_{-1,0,0,0}(z)\\
&-24\frac{2z+1}{z^2}G_{0,-1,0,0}(z)+48\frac{2z+1}{z^2}G_{0,0,0,0}(z)\bigg)\kappa\kappa^{-\epsilon}+\bigg(\frac{1}{\epsilon^3}8\frac{1}{z}-\frac{1}{\epsilon}20\frac{\zeta_2}{z}-20\frac{\zeta_3}{z}-12\frac{\zeta_2}{z}G_{-1}(z)-\frac{1}{\epsilon^2}4\frac{1}{z}G_0(z)+16\frac{\zeta_2}{z}G_0(z)\\
&-4\frac{1}{z}G_{-1,0,0}(z)+4\frac{1}{z}G_{0,0,0}(z)+\bigg(-4\frac{\zeta_3}{z}G_{-1}(z)+12\frac{\zeta_3}{z}G_0(z)-12\frac{\zeta_2}{z}G_{-1,-1}(z)+12\frac{\zeta_2}{z}G_{-1,0}(z)+12\frac{\zeta_2}{z}G_{0,-1}(z)-12\frac{\zeta_2}{z}G_{0,0}(z)\\
&-4\frac{1}{z}G_{-1,-1,0,0}(z)+8\frac{1}{z}G_{-1,0,0,0}(z)+4\frac{1}{z}G_{0,-1,0,0}(z)-8\frac{1}{z}G_{0,0,0,0}(z)\bigg)\epsilon\bigg)\log(\kappa)\kappa^{-\epsilon}+\bigg(4\frac{-5-6z+28\zeta_2+32z\zeta_2+10\zeta_3+20z\zeta_3}{z^2}\\
&-\frac{1}{\epsilon^3}16\frac{2z+1}{z^2}+24\frac{(2z+1)\zeta_2}{z^2}G_{-1}(z)-4\frac{-3-2z+8\zeta_2+16z\zeta_2}{z^2}G_0(z)+\bigg(-8\frac{7z+5}{z^2}+8\frac{2z+1}{z^2}G_0(z)\bigg)\frac{1}{\epsilon^2}+\bigg(4\frac{-9-8z+10\zeta_2+20z\zeta_2}{z^2}\\
&+8\frac{3z+2}{z^2}G_0(z)\bigg)\frac{1}{\epsilon}+8\frac{1}{z^2}G_{0,0}(z)+8\frac{2z+1}{z^2}G_{-1,0,0}(z)-8\frac{2z+1}{z^2}G_{0,0,0}(z)+\bigg(4\frac{-9-8z+27\zeta_2+14z\zeta_2+26\zeta_3+34z\zeta_3}{z^2}\\
&+8\frac{9\zeta_2+9z\zeta_2+\zeta_3+2z\zeta_3}{z^2}G_{-1}(z)-4\frac{1-2z+22\zeta_2+24z\zeta_2+6\zeta_3+12z\zeta_3}{z^2}G_0(z)+24\frac{(2z+1)\zeta_2}{z^2}G_{-1,-1}(z)-24\frac{(2z+1)\zeta_2}{z^2}G_{-1,0}(z)\\
&-24\frac{(2z+1)\zeta_2}{z^2}G_{0,-1}(z)+12\frac{1+2\zeta_2+4z\zeta_2}{z^2}G_{0,0}(z)+24\frac{z+1}{z^2}G_{-1,0,0}(z)-8\frac{3z+4}{z^2}G_{0,0,0}(z)+8\frac{2z+1}{z^2}G_{-1,-1,0,0}(z)\\
&-16\frac{2z+1}{z^2}G_{-1,0,0,0}(z)-8\frac{2z+1}{z^2}G_{0,-1,0,0}(z)+16\frac{2z+1}{z^2}G_{0,0,0,0}(z)\bigg)\epsilon\bigg)\kappa\log(\kappa)\kappa^{-\epsilon}+\bigg(-\frac{1}{\epsilon^4}4\frac{1}{z}-\frac{25}{2}\frac{\zeta_4}{z}-2\frac{\zeta_3}{z}G_0(z)+\bigg(-7\frac{\zeta_3}{z}\\
&-6\frac{\zeta_2}{z}G_0(z)\bigg)\frac{1}{\epsilon}+\frac{1}{\epsilon^3}7\frac{1}{z}G_0(z)+\bigg(9\frac{\zeta_2}{z}-4\frac{1}{z}G_{0,0}(z)\bigg)\frac{1}{\epsilon^2}\bigg)\kappa^{-2\epsilon}+\bigg(-\frac{97+141z+28\zeta_2+28z\zeta_2-24\zeta_3-28z\zeta_3-25\zeta_4-50z\zeta_4}{z^2}\\
&+\frac{1}{\epsilon^4}8\frac{2z+1}{z^2}+4\frac{1-z+6\zeta_2+6z\zeta_2+\zeta_3+2z\zeta_3}{z^2}G_0(z)+\bigg(2\frac{23z+15}{z^2}-14\frac{2z+1}{z^2}G_0(z)\bigg)\frac{1}{\epsilon^3}+\bigg(-\frac{-19+18\zeta_2+36z\zeta_2}{z^2}-12\frac{4z+3}{z^2}G_0(z)\\
&+8\frac{2z+1}{z^2}G_{0,0}(z)\bigg)\frac{1}{\epsilon^2}+16\frac{z+1}{z^2}G_{0,0}(z)+\bigg(-\frac{45+63z+48\zeta_2+68z\zeta_2-14\zeta_3-28z\zeta_3}{z^2}+4\frac{-7-7z+3\zeta_2+6z\zeta_2}{z^2}G_0(z)\\
&+16\frac{z+1}{z^2}G_{0,0}(z)\bigg)\frac{1}{\epsilon}\bigg)\kappa\kappa^{-2\epsilon}+\bigg(-2\frac{\zeta_3}{z}+\epsilon 5\frac{\zeta_4}{z}-\frac{1}{\epsilon^3}8\frac{1}{z}+\frac{1}{\epsilon}10\frac{\zeta_2}{z}+\frac{1}{\epsilon^2}6\frac{1}{z}G_0(z)\bigg)\log(\kappa)\kappa^{-2\epsilon}+\bigg(-2\frac{9+3z+20\zeta_2+20z\zeta_2-2\zeta_3-4z\zeta_3}{z^2}\\
&+\frac{1}{\epsilon^3}16\frac{2z+1}{z^2}+\bigg(4\frac{15z+11}{z^2}-12\frac{2z+1}{z^2}G_0(z)\bigg)\frac{1}{\epsilon^2}+\bigg(-2\frac{-15-16z+10\zeta_2+20z\zeta_2}{z^2}-24\frac{z+1}{z^2}G_0(z)\bigg)\frac{1}{\epsilon}\\
&+\bigg(-2\frac{51+33z+26\zeta_2+20z\zeta_2-4\zeta_3-4z\zeta_3+5\zeta_4+10z\zeta_4}{z^2}-24\frac{z+1}{z^2}G_0(z)\bigg)\epsilon-24\frac{z+1}{z^2}G_0(z)\bigg)\kappa\log(\kappa)\kappa^{-2\epsilon}-\frac{1}{\epsilon^2}4\frac{1}{z}\log^2(\kappa)\kappa^{-2\epsilon}\\
&+\bigg(\frac{1}{\epsilon^2}8\frac{2z+1}{z^2}+16\frac{z+1}{z^2}+\frac{1}{\epsilon}16\frac{z+1}{z^2}+\epsilon 16\frac{z+1}{z^2}+\epsilon^2 16\frac{z+1}{z^2}\bigg)\kappa\log^2(\kappa)\kappa^{-2\epsilon}+O(\eta)+O(\kappa^2)
\end{aligned}
\tag{D.7}
$$

D.2. The $Z + j$ master integrals

I[FP0, 1, 1, 1, 1, 1, 1, 1, 0, 0] =

$$
\begin{aligned}
&-\frac{1}{\epsilon^4}4\frac{1}{\chi}i+\frac{1}{\epsilon^3}5\frac{1}{\chi}G_0(\chi)i+\frac{1}{\epsilon^2}\bigg(15\frac{\zeta_2}{\chi}-4\frac{1}{\chi}G_{0,0}(\chi)\bigg)i+\frac{1}{\epsilon}\bigg(\frac{65}{3}\frac{\zeta_3}{\chi}+12\frac{\zeta_2}{\chi}G_{-1}(\chi)-33\frac{\zeta_2}{\chi}G_0(\chi)+4\frac{1}{\chi}G_{-1,0,0}(\chi)-4\frac{1}{\chi}G_{0,0,0}(\chi)\bigg)i\\
&+\bigg(87\frac{\zeta_4}{\chi}+4\frac{\zeta_3}{\chi}G_{-1}(\chi)-\frac{88}{3}\frac{\zeta_3}{\chi}G_0(\chi)+12\frac{\zeta_2}{\chi}G_{-1,-1}(\chi)-20\frac{\zeta_2}{\chi}G_{-1,0}(\chi)-60\frac{\zeta_2}{\chi}G_{0,-1}(\chi)+72\frac{\zeta_2}{\chi}G_{0,0}(\chi)+4\frac{1}{\chi}G_{-1,-1,0,0}(\chi)\\
&-16\frac{1}{\chi}G_{-1,0,0,0}(\chi)-20\frac{1}{\chi}G_{0,-1,0,0}(\chi)+32\frac{1}{\chi}G_{0,0,0,0}(\chi)\bigg)i+\bigg(\frac{1}{\epsilon^4}4\frac{1}{\chi}i+\bigg(44\frac{\zeta_4}{\chi}-2\frac{\zeta_3}{\chi}G_{-1}(\chi)+18\frac{\zeta_3}{\chi}G_0(\chi)-6\frac{\zeta_2}{\chi}G_{-1,-1}(\chi)\\
&+8\frac{\zeta_2}{\chi}G_{-1,0}(\chi)+24\frac{\zeta_2}{\chi}G_{0,-1}(\chi)-25\frac{\zeta_2}{\chi}G_{0,0}(\chi)+6\frac{1}{\chi}G_{-1,0}(\chi)G_{0,0}(\chi)+6\frac{1}{\chi}G_{0,-1}(\chi)G_{0,0}(\chi)-2\frac{1}{\chi}G_{-1,-1,0,0}(\chi)-12\frac{1}{\chi}G_{-1,0,0,0}(\chi)\\
&-10\frac{1}{\chi}G_{0,-1,0,0}(\chi)-18\frac{1}{\chi}G_{0,0,-1,0}(\chi)-18\frac{1}{\chi}G_{0,0,0,-1}(\chi)-11\frac{1}{\chi}G_{0,0,0,0}(\chi)\bigg)i+\bigg(-3\frac{1}{\chi}G_0(\chi)i+4\frac{\pi}{\chi}\bigg)\frac{1}{\epsilon^3}-\frac{62}{3}\frac{\pi\zeta_3}{\chi}-\frac{\pi^3}{\chi}G_{-1}(\chi)\\
&+\frac{17}{6}\frac{\pi^3}{\chi}G_0(\chi)+\bigg(\bigg(-22\frac{\zeta_2}{\chi}+\frac{1}{\chi}G_{0,0}(\chi)\bigg)i-3\frac{\pi}{\chi}G_0(\chi)\bigg)\frac{1}{\epsilon^2}+\bigg(\bigg(-\frac{62}{3}\frac{\zeta_3}{\chi}-6\frac{\zeta_2}{\chi}G_{-1}(\chi)+23\frac{\zeta_2}{\chi}G_0(\chi)-2\frac{1}{\chi}G_{-1,0,0}(\chi)+3\frac{1}{\chi}G_{0,0,0}(\chi)\bigg)i\\
&-\frac{7}{3}\frac{\pi^3}{\chi}+\frac{\pi}{\chi}G_{0,0}(\chi)\bigg)\frac{1}{\epsilon}-2\frac{\pi}{\chi}G_{-1,0,0}(\chi)+3\frac{\pi}{\chi}G_{0,0,0}(\chi)\bigg)\mu^{-\epsilon}+\bigg(-\frac{1}{\epsilon^4}\frac{1}{\chi}i+\frac{1}{\epsilon^2}\frac{25}{2}\frac{\zeta_2}{\chi}i-\frac{149}{2}\frac{\zeta_4}{\chi}i\\
&+\bigg(-\frac{1}{96}\frac{-117579+134940\zeta_2+29440\zeta_3-47268\zeta_4}{\chi}i-\frac{1}{24}\frac{(3328\zeta_3+1150\pi^2-17913)\pi}{\chi}\bigg)\epsilon\\
&+\bigg(\frac{3}{32}\frac{-43359+45708\zeta_2+8960\zeta_3-10908\zeta_4}{\chi}i+\frac{3}{8}\frac{(768\zeta_3+350\pi^2-6405)\pi}{\chi}\bigg)\epsilon^2\\
&+\bigg(-\frac{3}{16}\frac{-17913+17940\zeta_2+3328\zeta_3-3636\zeta_4}{\chi}i-\frac{3}{4}\frac{(256\zeta_3+130\pi^2-2595)\pi}{\chi}\bigg)\epsilon^3+\bigg(\frac{43}{6}\frac{\zeta_3}{\chi}i+\frac{3}{2}\frac{\pi^3}{\chi}\bigg)\frac{1}{\epsilon}-\frac{1}{\epsilon^3}2\frac{\pi}{\chi}+\frac{43}{3}\frac{\pi\zeta_3}{\chi}\bigg)\mu^{-2\epsilon}+O(\mu)
\end{aligned}
\tag{D.8}
$$

I[FP1, 1, 1, 1, 1, 1, 1, 1, 0, 0] =

$$
\begin{aligned}
&-\frac{1}{\epsilon^4}4\frac{1}{\chi}i+\frac{1}{\epsilon^3}5\frac{1}{\chi}G_0(\chi)i+\frac{1}{\epsilon^2}\bigg(15\frac{\zeta_2}{\chi}-4\frac{1}{\chi}G_{0,0}(\chi)\bigg)i+\frac{1}{\epsilon}\bigg(\frac{65}{3}\frac{\zeta_3}{\chi}+12\frac{\zeta_2}{\chi}G_{-1}(\chi)-33\frac{\zeta_2}{\chi}G_0(\chi)+4\frac{1}{\chi}G_{-1,0,0}(\chi)-4\frac{1}{\chi}G_{0,0,0}(\chi)\bigg)i\\
&+\bigg(87\frac{\zeta_4}{\chi}+4\frac{\zeta_3}{\chi}G_{-1}(\chi)-\frac{88}{3}\frac{\zeta_3}{\chi}G_0(\chi)+12\frac{\zeta_2}{\chi}G_{-1,-1}(\chi)-20\frac{\zeta_2}{\chi}G_{-1,0}(\chi)-60\frac{\zeta_2}{\chi}G_{0,-1}(\chi)+72\frac{\zeta_2}{\chi}G_{0,0}(\chi)+4\frac{1}{\chi}G_{-1,-1,0,0}(\chi)\\
&-16\frac{1}{\chi}G_{-1,0,0,0}(\chi)-20\frac{1}{\chi}G_{0,-1,0,0}(\chi)+32\frac{1}{\chi}G_{0,0,0,0}(\chi)\bigg)i+\bigg(\frac{1}{\epsilon^4}9\frac{1}{\chi}i+\bigg(3\frac{\zeta_4}{\chi}-4\frac{\zeta_3}{\chi}G_{-1}(\chi)+\frac{98}{3}\frac{\zeta_3}{\chi}G_0(\chi)+\frac{23}{4}\frac{\zeta_2}{\chi}G_0(\chi)^2\\
&-\frac{11}{48}\frac{1}{\chi}G_0(\chi)^4-12\frac{\zeta_2}{\chi}G_{-1,-1}(\chi)+16\frac{\zeta_2}{\chi}G_{-1,0}(\chi)+42\frac{\zeta_2}{\chi}G_{0,-1}(\chi)-\frac{113}{2}\frac{\zeta_2}{\chi}G_{0,0}(\chi)+9\frac{1}{\chi}G_{-1,0}(\chi)G_{0,0}(\chi)+9\frac{1}{\chi}G_{0,-1}(\chi)G_{0,0}(\chi)\\
&-4\frac{1}{\chi}G_{-1,-1,0,0}(\chi)-15\frac{1}{\chi}G_{-1,0,0,0}(\chi)-13\frac{1}{\chi}G_{0,-1,0,0}(\chi)-27\frac{1}{\chi}G_{0,0,-1,0}(\chi)-27\frac{1}{\chi}G_{0,0,0,-1}(\chi)-\frac{31}{2}\frac{1}{\chi}G_{0,0,0,0}(\chi)\bigg)i\\
&+\bigg(-7\frac{1}{\chi}G_0(\chi)i+4\frac{\pi}{\chi}\bigg)\frac{1}{\epsilon^3}-\frac{62}{3}\frac{\pi\zeta_3}{\chi}-\frac{\pi^3}{\chi}G_{-1}(\chi)+\frac{17}{6}\frac{\pi^3}{\chi}G_0(\chi)+\frac{1}{4}\frac{\pi}{\chi}G_0(\chi)^3+\bigg(\bigg(-30\frac{\zeta_2}{\chi}-\frac{11}{4}\frac{1}{\chi}G_0(\chi)^2+\frac{17}{2}\frac{1}{\chi}G_{0,0}(\chi)\bigg)i\\
&-3\frac{\pi}{\chi}G_0(\chi)\bigg)\frac{1}{\epsilon^2}+\bigg(\bigg(\frac{1}{12}\frac{-15492+10164\zeta_2+3496\zeta_3-1719\zeta_4}{\chi}+\frac{1}{6}\frac{-3819+1143\zeta_2+838\zeta_3}{\chi}G_0(\chi)-\frac{1}{4}\frac{107\zeta_2-451}{\chi}G_0(\chi)^2-\frac{51}{4}\frac{1}{\chi}G_0(\chi)^3\\
&+\frac{47}{48}\frac{1}{\chi}G_0(\chi)^4\bigg)i+\frac{1}{2}\frac{(260\zeta_3+57\pi^2-1266)\pi}{\chi}+\frac{15}{4}\frac{(\pi^2-26)\pi}{\chi}G_0(\chi)+\frac{57}{4}\frac{\pi}{\chi}G_0(\chi)^2-\frac{5}{4}\frac{\pi}{\chi}G_0(\chi)^3\bigg)\epsilon\\
&+\bigg(\bigg(-\frac{1}{6}\frac{-15108+9108\zeta_2+2888\zeta_3-1017\zeta_4}{\chi}-\frac{1}{3}\frac{-3291+867\zeta_2+458\zeta_3}{\chi}G_0(\chi)+\frac{1}{2}\frac{61\zeta_2-363}{\chi}G_0(\chi)^2+\frac{109}{6}\frac{1}{\chi}G_0(\chi)^3-\frac{25}{24}\frac{1}{\chi}G_0(\chi)^4\bigg)i\\
&-3\frac{(52\zeta_3+15\pi^2-390)\pi}{\chi}-\frac{9}{2}\frac{(\pi^2-38)\pi}{\chi}G_0(\chi)-\frac{45}{2}\frac{\pi}{\chi}G_0(\chi)^2+\frac{3}{2}\frac{\pi}{\chi}G_0(\chi)^3\bigg)\epsilon^2+\bigg(\bigg(-38\frac{\zeta_3}{\chi}-12\frac{\zeta_2}{\chi}G_{-1}(\chi)+35\frac{\zeta_2}{\chi}G_0(\chi)\\
&+\frac{11}{12}\frac{1}{\chi}G_0(\chi)^3-4\frac{1}{\chi}G_{-1,0,0}(\chi)-\frac{1}{2}\frac{1}{\chi}G_{0,0,0}(\chi)\bigg)i-\frac{7}{3}\frac{\pi^3}{\chi}-\frac{3}{4}\frac{\pi}{\chi}G_0(\chi)^2+\frac{5}{2}\frac{\pi}{\chi}G_{0,0}(\chi)\bigg)\frac{1}{\epsilon}-2\frac{\pi}{\chi}G_{-1,0,0}(\chi)+\frac{3}{2}\frac{\pi}{\chi}G_{0,0,0}(\chi)\bigg)\mu^{-\epsilon}\\
&+\bigg(-\frac{1}{\epsilon^4}\frac{21}{4}\frac{1}{\chi}i+\bigg(-\frac{827}{16}\frac{\zeta_4}{\chi}-\frac{22}{3}\frac{\zeta_3}{\chi}G_0(\chi)-6\frac{\zeta_2}{\chi}G_{0,-1}(\chi)+\frac{11}{2}\frac{\zeta_2}{\chi}G_{0,0}(\chi)-2\frac{1}{\chi}G_{0,-1,0,0}(\chi)+\frac{3}{2}\frac{1}{\chi}G_{0,0,0,0}(\chi)\bigg)i\\
&+\bigg(-\frac{1}{32}\frac{-43225+44980\zeta_2+11008\zeta_3-17340\zeta_4}{\chi}i-\frac{1}{24}\frac{(3520\zeta_3+1178\pi^2-18657)\pi}{\chi}\bigg)\epsilon+\bigg(\frac{1}{32}\frac{-134045+137124\zeta_2+27904\zeta_3-33780\zeta_4}{\chi}i\\
&+\frac{1}{24}\frac{(7040\zeta_3+3174\pi^2-58365)\pi}{\chi}\bigg)\epsilon^2+\bigg(-\frac{3}{16}\frac{-17913+17940\zeta_2+3328\zeta_3-3636\zeta_4}{\chi}i-\frac{3}{4}\frac{(256\zeta_3+130\pi^2-2595)\pi}{\chi}\bigg)\epsilon^3+\bigg(2\frac{1}{\chi}G_0(\chi)i\\
&-\frac{9}{2}\frac{\pi}{\chi}\bigg)\frac{1}{\epsilon^3}+21\frac{\pi\zeta_3}{\chi}-\frac{7}{12}\frac{\pi^3}{\chi}G_0(\chi)+\bigg(\bigg(\frac{73}{4}\frac{\zeta_2}{\chi}-\frac{1}{2}\frac{1}{\chi}G_{0,0}(\chi)\bigg)i+\frac{3}{2}\frac{\pi}{\chi}G_0(\chi)\bigg)\frac{1}{\epsilon^2}+\bigg(\bigg(20\frac{\zeta_3}{\chi}-5\frac{\zeta_2}{\chi}G_0(\chi)-\frac{1}{2}\frac{1}{\chi}G_{0,0,0}(\chi)\bigg)i+\frac{23}{12}\frac{\pi^3}{\chi}\\
&-\frac{1}{2}\frac{\pi}{\chi}G_{0,0}(\chi)\bigg)\frac{1}{\epsilon}-\frac{1}{2}\frac{\pi}{\chi}G_{0,0,0}(\chi)+\bigg(\epsilon^2\frac{1}{24}\frac{756+540\zeta_2-224\zeta_3+567\zeta_4}{\chi}i-\epsilon^3\frac{1}{4}\frac{124+84\zeta_2-32\zeta_3+63\zeta_4}{\chi}i-\frac{1}{\epsilon^3}\frac{1}{2}\frac{1}{\chi}i+\frac{4}{3}\frac{\zeta_3}{\chi}i-\frac{1}{\epsilon}\frac{3}{2}\frac{\zeta_2}{\chi}i\\
&-\epsilon\frac{63}{8}\frac{\zeta_4}{\chi}i\bigg)\log(\mu)\bigg)\mu^{-2\epsilon}+\bigg(\frac{1}{\epsilon^4}\frac{1}{4}\frac{1}{\chi}i-\frac{1}{\epsilon^2}\frac{7}{4}\frac{\zeta_2}{\chi}i-\frac{201}{16}\frac{\zeta_4}{\chi}i+\bigg(-\frac{7}{6}\frac{\zeta_3}{\chi}i+\frac{1}{12}\frac{\pi^3}{\chi}\bigg)\frac{1}{\epsilon}+\bigg(\frac{1}{24}\frac{-192+336\zeta_2+112\zeta_3+603\zeta_4}{\chi}i\\
&-\frac{1}{3}\frac{(-14\zeta_3+\pi^2+24)\pi}{\chi}\bigg)\epsilon+\frac{1}{\epsilon^3}\frac{1}{2}\frac{\pi}{\chi}-\frac{7}{3}\frac{\pi\zeta_3}{\chi}\bigg)\mu^{-4\epsilon}+O(\mu)
\end{aligned}
\tag{D.9}
$$

I[FP2, 1, 1, 1, 1, 1, 1, 1, 0, 0] =

$$-\frac{1}{\epsilon^4}4\frac{1}{\chi}i+\frac{1}{\epsilon^3}5\frac{1}{\chi}G_0(\chi)i+\frac{1}{\epsilon^2}\left(15\frac{\zeta_2}{\chi}-4\frac{1}{\chi}G_{0,0}(\chi)\right)i+\frac{1}{\epsilon}\left(\frac{65}{3}\frac{\zeta_3}{\chi}+12\frac{\zeta_2}{\chi}G_{-1}(\chi)-33\frac{\zeta_2}{\chi}G_0(\chi)+4\frac{1}{\chi}G_{-1,0,0}(\chi)-4\frac{1}{\chi}G_{0,0,0}(\chi)\right)i$$

$$+\Big(87\frac{\zeta_4}{\chi}+4\frac{\zeta_3}{\chi}G_{-1}(\chi)-\frac{88}{3}\frac{\zeta_3}{\chi}G_0(\chi)+12\frac{\zeta_2}{\chi}G_{-1,-1}(\chi)-20\frac{\zeta_2}{\chi}G_{-1,0}(\chi)-60\frac{\zeta_2}{\chi}G_{0,-1}(\chi)+72\frac{\zeta_2}{\chi}G_{0,0}(\chi)+4\frac{1}{\chi}G_{-1,-1,0,0}(\chi)$$

$$-16\frac{1}{\chi}G_{-1,0,0,0}(\chi)-20\frac{1}{\chi}G_{0,-1,0,0}(\chi)+32\frac{1}{\chi}G_{0,0,0,0}(\chi)\Big)i+\Big(\frac{1}{\epsilon^4}6\frac{1}{\chi}i+\Big(20\frac{\zeta_4}{\chi}-4\frac{\zeta_3}{\chi}G_{-1}(\chi)+\frac{98}{3}\frac{\zeta_3}{\chi}G_0(\chi)+52\frac{\zeta_2}{\chi}G_{-1}(\chi)G_0(\chi)$$

$$-\frac{19}{4}\frac{\zeta_2}{\chi}G_0(\chi)^2+\frac{13}{3}\frac{1}{\chi}G_{-1}(\chi)G_0(\chi)^3-\frac{47}{48}\frac{1}{\chi}G_0(\chi)^4-12\frac{\zeta_2}{\chi}G_{-1,-1}(\chi)-13\frac{1}{\chi}G_0(\chi)^2G_{-1,-1}(\chi)-34\frac{\zeta_2}{\chi}G_{-1,0}(\chi)+2\frac{\zeta_2}{\chi}G_{0,-1}(\chi)$$

$$-5\frac{1}{\chi}G_0(\chi)^2G_{0,-1}(\chi)-\frac{83}{2}\frac{\zeta_2}{\chi}G_{0,0}(\chi)+26\frac{1}{\chi}G_0(\chi)G_{-1,0,-1}(\chi)+26\frac{1}{\chi}G_0(\chi)G_{0,-1,-1}(\chi)-6\frac{1}{\chi}G_0(\chi)G_{0,0,-1}(\chi)+22\frac{1}{\chi}G_{-1,-1,0,0}(\chi)$$

$$-26\frac{1}{\chi}G_{-1,0,0,-1}(\chi)-12\frac{1}{\chi}G_{-1,0,0,0}(\chi)-26\frac{1}{\chi}G_{0,-1,0,-1}(\chi)+2\frac{1}{\chi}G_{0,-1,0,0}(\chi)-26\frac{1}{\chi}G_{0,0,-1,-1}(\chi)+22\frac{1}{\chi}G_{0,0,0,-1}(\chi)$$

$$-\frac{3}{2}\frac{1}{\chi}G_{0,0,0,0}(\chi)\Big)i+\left(-7\frac{1}{\chi}G_0(\chi)i+4\frac{\pi}{\chi}\right)\frac{1}{\epsilon^3}-\frac{62}{3}\frac{\pi\zeta_3}{\chi}-\frac{\pi^3}{\chi}G_{-1}(\chi)+\frac{17}{6}\frac{\pi^3}{\chi}G_0(\chi)-3\frac{\pi}{\chi}G_{-1}(\chi)G_0(\chi)^2+\frac{13}{12}\frac{\pi}{\chi}G_0(\chi)^3$$

$$+\Big(\Big(-36\frac{\zeta_2}{\chi}-\frac{15}{4}\frac{1}{\chi}G_0(\chi)^2+\frac{21}{2}\frac{1}{\chi}G_{0,0}(\chi)\Big)i-3\frac{\pi}{\chi}G_0(\chi)\Big)\frac{1}{\epsilon^2}+\Big(\Big(\frac{1}{8}\frac{-10284+3772\zeta_2+2168\zeta_3-637\zeta_4}{\chi}-\frac{1}{2}\frac{-537+205\zeta_2+42\zeta_3}{\chi}G_0(\chi)$$

$$+\frac{7}{4}\frac{5\zeta_2-21}{\chi}G_0(\chi)^2+\frac{11}{4}\frac{1}{\chi}G_0(\chi)^3-\frac{1}{16}\frac{1}{\chi}G_0(\chi)^4\Big)i+\frac{1}{4}\frac{(260\zeta_3+57\pi^2-1266)\pi}{\chi}-\frac{15}{4}\frac{(\pi^2-26)\pi}{\chi}G_0(\chi)-\frac{57}{4}\frac{\pi}{\chi}G_0(\chi)^2+\frac{5}{4}\frac{\pi}{\chi}G_0(\chi)^3\Big)\epsilon$$

$$+\Big(\Big(-\frac{1}{12}\frac{-30468+11412\zeta_2+6472\zeta_3-9\zeta_4}{\chi}+3\frac{-195+75\zeta_2+26\zeta_3}{\chi}G_0(\chi)-\frac{9}{2}\frac{5\zeta_2-19}{\chi}G_0(\chi)^2-\frac{15}{2}\frac{1}{\chi}G_0(\chi)^3+\frac{3}{8}\frac{1}{\chi}G_0(\chi)^4\Big)i$$

$$-\frac{3}{2}\frac{(52\zeta_3+15\pi^2-390)\pi}{\chi}+\frac{9}{2}\frac{(\pi^2-38)\pi}{\chi}G_0(\chi)+\frac{45}{2}\frac{\pi}{\chi}G_0(\chi)^2-\frac{3}{2}\frac{\pi}{\chi}G_0(\chi)^3\Big)\epsilon^2+6\frac{\pi}{\chi}G_0(\chi)G_{0,-1}(\chi)+\Big(\Big(-28\frac{\zeta_3}{\chi}-12\frac{\zeta_2}{\chi}G_{-1}(\chi)$$

$$+43\frac{\zeta_2}{\chi}G_0(\chi)-5\frac{1}{\chi}G_{-1}(\chi)G_0(\chi)^2+\frac{9}{4}\frac{1}{\chi}G_0(\chi)^3+10\frac{1}{\chi}G_0(\chi)G_{0,-1}(\chi)+6\frac{1}{\chi}G_{-1,0,0}(\chi)-10\frac{1}{\chi}G_{0,0,-1}(\chi)-\frac{13}{2}\frac{1}{\chi}G_{0,0,0}(\chi)\Big)i-\frac{7}{3}\frac{\pi^3}{\chi}$$

$$-\frac{5}{4}\frac{\pi}{\chi}G_0(\chi)^2+\frac{7}{2}\frac{\pi}{\chi}G_{0,0}(\chi)\Big)\frac{1}{\epsilon}+4\frac{\pi}{\chi}G_{-1,0,0}(\chi)-6\frac{\pi}{\chi}G_{0,0,-1}(\chi)-\frac{7}{2}\frac{\pi}{\chi}G_{0,0,0}(\chi)+\Big(\frac{1}{\epsilon^3}4\frac{1}{\chi}i-\frac{1}{\epsilon^2}2\frac{1}{\chi}G_0(\chi)i+\frac{1}{\epsilon}\Big(-6\frac{\zeta_2}{\chi}-\frac{3}{4}\frac{1}{\chi}G_0(\chi)^2$$

$$+\frac{3}{2}\frac{1}{\chi}G_{0,0}(\chi)\Big)i+\Big(-\frac{38}{3}\frac{\zeta_3}{\chi}-6\frac{\zeta_2}{\chi}G_{-1}(\chi)+6\frac{\zeta_2}{\chi}G_0(\chi)+\frac{1}{4}\frac{1}{\chi}G_0(\chi)^3-2\frac{1}{\chi}G_{-1,0,0}(\chi)+\frac{1}{2}\frac{1}{\chi}G_{0,0,0}(\chi)\Big)i-\frac{3}{4}\frac{\pi}{\chi}G_0(\chi)^2$$

$$+\Big(\Big(-\frac{1}{8}\frac{-7980+3900\zeta_2+1976\zeta_3-285\zeta_4}{\chi}+\frac{1}{2}\frac{-633+285\zeta_2+130\zeta_3}{\chi}G_0(\chi)-\frac{15}{4}\frac{5\zeta_2-13}{\chi}G_0(\chi)^2-\frac{19}{4}\frac{1}{\chi}G_0(\chi)^3+\frac{5}{16}\frac{1}{\chi}G_0(\chi)^4\Big)i$$

$$-\frac{1}{4}\frac{(260\zeta_3+57\pi^2-1266)\pi}{\chi}+\frac{15}{4}\frac{(\pi^2-26)\pi}{\chi}G_0(\chi)+\frac{57}{4}\frac{\pi}{\chi}G_0(\chi)^2-\frac{5}{4}\frac{\pi}{\chi}G_0(\chi)^3\Big)\epsilon^2+\Big(\Big(\frac{3}{4}\frac{-2532+1140\zeta_2+520\zeta_3-57\zeta_4}{\chi}$$

$$-3\frac{-195+75\zeta_2+26\zeta_3}{\chi}G_0(\chi)+\frac{9}{2}\frac{5\zeta_2-19}{\chi}G_0(\chi)^2+\frac{15}{2}\frac{1}{\chi}G_0(\chi)^3-\frac{3}{8}\frac{1}{\chi}G_0(\chi)^4\Big)i+\frac{3}{2}\frac{(52\zeta_3+15\pi^2-390)\pi}{\chi}-\frac{9}{2}\frac{(\pi^2-38)\pi}{\chi}G_0(\chi)$$

$$-\frac{45}{2}\frac{\pi}{\chi}G_0(\chi)^2+\frac{3}{2}\frac{\pi}{\chi}G_0(\chi)^3\Big)\epsilon^3+\frac{3}{2}\frac{\pi}{\chi}G_{0,0}(\chi)+\Big(\Big(-18\frac{\zeta_4}{\chi}-2\frac{\zeta_3}{\chi}G_{-1}(\chi)+\frac{22}{3}\frac{\zeta_3}{\chi}G_0(\chi)+\frac{15}{4}\frac{\zeta_2}{\chi}G_0(\chi)^2-\frac{1}{16}\frac{1}{\chi}G_0(\chi)^4-6\frac{\zeta_2}{\chi}G_{-1,-1}(\chi)$$

$$+6\frac{\zeta_2}{\chi}G_{-1,0}(\chi)+6\frac{\zeta_2}{\chi}G_{0,-1}(\chi)-\frac{27}{2}\frac{\zeta_2}{\chi}G_{0,0}(\chi)-2\frac{1}{\chi}G_{-1,-1,0,0}(\chi)+4\frac{1}{\chi}G_{-1,0,0,0}(\chi)+2\frac{1}{\chi}G_{0,-1,0,0}(\chi)-\frac{5}{2}\frac{1}{\chi}G_{0,0,0,0}(\chi)\Big)i$$

$$+\frac{1}{4}\frac{\pi}{\chi}G_0(\chi)^3-\frac{3}{2}\frac{\pi}{\chi}G_{0,0,0}(\chi)\Big)\epsilon\Big)\log(\mu)\Big)\mu^{-\epsilon}+\Big(-\frac{1}{\epsilon^4}\frac{11}{4}\frac{1}{\chi}i+\Big(-\frac{1845}{16}\frac{\zeta_4}{\chi}-6\frac{\zeta_3}{\chi}G_0(\chi)+\frac{1}{4}\frac{\zeta_2}{\chi}G_0(\chi)^2+\frac{1}{48}\frac{1}{\chi}G_0(\chi)^4-6\frac{\zeta_2}{\chi}G_{0,-1}(\chi)$$

$$-\frac{1}{\chi}G_0(\chi)^2G_{0,-1}(\chi)+4\frac{1}{\chi}G_0(\chi)G_{0,0,-1}(\chi)-6\frac{1}{\chi}G_{0,0,0,-1}(\chi)\Big)i+\Big(-\frac{1}{96}\frac{-135819+161436\zeta_2+35840\zeta_3-51156\zeta_4}{\chi}i$$

$$-\frac{1}{24}\frac{(4256\zeta_3+1362\pi^2-20913)\pi}{\chi}\Big)\epsilon+\Big(\frac{1}{32}\frac{-134045+142500\zeta_2+27904\zeta_3-31860\zeta_4}{\chi}i+\frac{1}{8}\frac{(2432\zeta_3+1074\pi^2-19935)\pi}{\chi}\Big)\epsilon^2$$

$$+\Big(-\frac{3}{16}\frac{-17913+17940\zeta_2+3328\zeta_3-3636\zeta_4}{\chi}i-\frac{3}{4}\frac{(256\zeta_3+130\pi^2-2595)\pi}{\chi}\Big)\epsilon^3+\Big(\frac{3}{2}\frac{1}{\chi}G_0(\chi)i-\frac{11}{2}\frac{\pi}{\chi}\Big)\frac{1}{\epsilon^3}+\frac{77}{3}\frac{\pi\zeta_3}{\chi}-\frac{7}{12}\frac{\pi^3}{\chi}G_0(\chi)$$

$$-\frac{1}{12}\frac{\pi}{\chi}G_0(\chi)^3+\Big(\Big(\frac{123}{4}\frac{\zeta_2}{\chi}+\frac{1}{4}\frac{1}{\chi}G_0(\chi)^2\Big)i+\frac{3}{2}\frac{\pi}{\chi}G_0(\chi)\Big)\frac{1}{\epsilon^2}+\Big(\Big(\frac{37}{3}\frac{\zeta_3}{\chi}-\frac{13}{2}\frac{\zeta_2}{\chi}G_0(\chi)-\frac{1}{12}\frac{1}{\chi}G_0(\chi)^3\Big)i+\frac{37}{12}\frac{\pi^3}{\chi}+\frac{1}{4}\frac{\pi}{\chi}G_0(\chi)^2\Big)\frac{1}{\epsilon}$$

$$+\Big(-\frac{1}{\epsilon^3}\frac{3}{2}\frac{1}{\chi}i+4\frac{\zeta_3}{\chi}i+\frac{1}{\epsilon}\frac{15}{2}\frac{\zeta_2}{\chi}i+\frac{1}{3}\frac{\pi^3}{\chi}+\Big(-\frac{1}{8}\frac{-756+900\zeta_2+224\zeta_3+153\zeta_4}{\chi}i-\frac{1}{3}\frac{(48\zeta_3+7\pi^2-186)\pi}{\chi}\Big)\epsilon^2$$

$$+\Big(\frac{3}{4}\frac{-124+140\zeta_2+32\zeta_3+17\zeta_4}{\chi}i+\frac{2}{3}\frac{(16\zeta_3+3\pi^2-90)\pi}{\chi}\Big)\epsilon^3-\frac{1}{\epsilon^2}2\frac{\pi}{\chi}+\Big(\frac{51}{8}\frac{\zeta_4}{\chi}i+\frac{16}{3}\frac{\pi\zeta_3}{\chi}\Big)\epsilon\Big)\log(\mu)\Big)\mu^{-2\epsilon}+\Big(\frac{1}{\epsilon^4}\frac{1}{\chi}i+\Big(6\frac{\zeta_4}{\chi}$$

$$-\frac{14}{3}\frac{\zeta_3}{\chi}G_0(\chi)-5\frac{\zeta_2}{\chi}G_0(\chi)^2+\frac{1}{24}\frac{1}{\chi}G_0(\chi)^4\Big)i+\Big(\frac{1}{\chi}G_0(\chi)i+2\frac{\pi}{\chi}\Big)\frac{1}{\epsilon^3}-\frac{28}{3}\frac{\pi\zeta_3}{\chi}-\frac{2}{3}\frac{\pi^3}{\chi}G_0(\chi)+\frac{1}{3}\frac{\pi}{\chi}G_0(\chi)^3+\Big(\Big(-10\frac{\zeta_2}{\chi}+\frac{1}{2}\frac{1}{\chi}G_0(\chi)^2\Big)i$$

$$+2\frac{\pi}{\chi}G_0(\chi)\Big)\frac{1}{\epsilon^2}+\Big(\Big(-\frac{14}{3}\frac{\zeta_3}{\chi}-10\frac{\zeta_2}{\chi}G_0(\chi)+\frac{1}{6}\frac{1}{\chi}G_0(\chi)^3\Big)i-\frac{2}{3}\frac{\pi^3}{\chi}+\frac{\pi}{\chi}G_0(\chi)^2\Big)\frac{1}{\epsilon}+\Big(\Big(8\frac{-24+40\zeta_2+7\zeta_3-3\zeta_4}{\chi}$$

$$+\frac{8}{3}\frac{-30+45\zeta_2+7\zeta_3}{\chi}G_0(\chi)+4\frac{5\zeta_2-4}{\chi}G_0(\chi)^2-2\frac{1}{\chi}G_0(\chi)^3-\frac{1}{6}\frac{1}{\chi}G_0(\chi)^4\Big)i+\frac{8}{3}\frac{(14\zeta_3+3\pi^2-60)\pi}{\chi}+\frac{8}{3}\frac{(\pi^2-24)\pi}{\chi}G_0(\chi)-12\frac{\pi}{\chi}G_0(\chi)^2$$

$$-\frac{4}{3}\frac{\pi}{\chi}G_0(\chi)^3\Big)\epsilon+\Big(\Big(-\frac{8}{3}\frac{-120+180\zeta_2+28\zeta_3-9\zeta_4}{\chi}-\frac{8}{3}\frac{-48+60\zeta_2+7\zeta_3}{\chi}G_0(\chi)-4\frac{5\zeta_2-6}{\chi}G_0(\chi)^2+\frac{8}{3}\frac{1}{\chi}G_0(\chi)^3+\frac{1}{6}\frac{1}{\chi}G_0(\chi)^4\Big)i$$

$$-\frac{16}{3}\frac{(7\zeta_3+2\pi^2-48)\pi}{\chi}-\frac{8}{3}\frac{(\pi-6)(\pi+6)\pi}{\chi}G_0(\chi)+16\frac{\pi}{\chi}G_0(\chi)^2+\frac{4}{3}\frac{\pi}{\chi}G_0(\chi)^3\Big)\epsilon^2\Big)\mu^{-3\epsilon}+\Big(-\frac{1}{\epsilon^4}\frac{1}{4}\frac{1}{\chi}i+\Big(\frac{305}{16}\frac{\zeta_4}{\chi}+\frac{4}{3}\frac{\zeta_3}{\chi}G_0(\chi)$$

$$+ \frac{5}{2}\frac{\zeta_2}{\chi}G_0(\chi)^2 - \frac{1}{6}\frac{1}{\chi}G_0(\chi)^4\Big)i + \Big(-\frac{1}{2}\frac{1}{\chi}G_0(\chi)i - \frac{1}{2}\frac{\pi}{\chi}\Big)\frac{1}{\epsilon^3} + \frac{4}{3}\frac{\pi\zeta_3}{\chi} - \frac{1}{2}\frac{\pi^3}{\chi}G_0(\chi) - \frac{2}{3}\frac{\pi}{\chi}G_0(\chi)^3 + \Big(\Big(\frac{5}{4}\frac{\zeta_2}{\chi} - \frac{1}{2}\frac{1}{\chi}G_0(\chi)^2\Big)i - \frac{\pi}{\chi}G_0(\chi)\Big)\frac{1}{\epsilon^2}$$
$$+ \Big(\Big(\frac{2}{3}\frac{\zeta_3}{\chi} + \frac{5}{2}\frac{\zeta_2}{\chi}G_0(\chi) - \frac{1}{3}\frac{1}{\chi}G_0(\chi)^3\Big)i - \frac{1}{4}\frac{\pi^3}{\chi} - \frac{\pi}{\chi}G_0(\chi)^2\Big)\frac{1}{\epsilon}\Big)\mu^{-4\epsilon} + O(\mu) \tag{D.10}$$

I[FP3, 1,1, 1, 1, 1, 1, 1, 0, 0] =

$$-\frac{1}{\epsilon^4}4\frac{1}{\chi}i + \frac{1}{\epsilon^3}5\frac{1}{\chi}G_0(\chi)i + \frac{1}{\epsilon^2}\Big(15\frac{1}{\chi}\zeta_2 - 4\frac{1}{\chi}G_{0,0}(\chi)\Big)i + \frac{1}{\epsilon}\Big(\Big(12\frac{1}{\chi}G_{-1}(\chi) - 33\frac{1}{\chi}G_0(\chi)\Big)\zeta_2 + \frac{65}{3}\frac{1}{\chi}\zeta_3 + 4\frac{1}{\chi}G_{-1,0,0}(\chi) - 4\frac{1}{\chi}G_{0,0,0}(\chi)$$
$$\Big)i + \Big(\Big(12\frac{1}{\chi}G_{-1,-1}(\chi) - 20\frac{1}{\chi}G_{-1,0}(\chi) - 60\frac{1}{\chi}G_{0,-1}(\chi) + 72\frac{1}{\chi}G_{0,0}(\chi)\Big)\zeta_2 + \Big(4\frac{1}{\chi}G_{-1}(\chi) - \frac{88}{3}\frac{1}{\chi}G_0(\chi)\Big)\zeta_3 + 87\frac{1}{\chi}\zeta_4$$
$$+ 4\frac{1}{\chi}G_{-1,-1,0,0}(\chi) - 16\frac{1}{\chi}G_{-1,0,0,0}(\chi) - 20\frac{1}{\chi}G_{0,-1,0,0}(\chi) + 32\frac{1}{\chi}G_{0,0,0,0}(\chi)\Big)i + \Big(\frac{1}{\epsilon^4}9\frac{1}{\chi}i + \epsilon^3\Big(-128\frac{(13\chi+21)(\chi+1)^2}{\chi^2}$$
$$+ \Big(32\frac{(\chi+1)(2\chi+3)(4\chi+5)}{\chi^2} - 16\frac{(\chi+1)(2\chi+3)(3\chi+4)}{\chi^2}G_0(\chi) - 192\frac{(\chi+2)^3}{\chi}G_{-1,-1}(\chi) + 160\frac{(\chi+2)^3}{\chi}G_{-1,0}(\chi)$$
$$+ 192\frac{(\chi+2)^3}{\chi}G_{0,-1}(\chi) - 8\frac{20\chi^4+117\chi^3+231\chi^2+155\chi+3}{\chi^2}G_{0,0}(\chi)\Big)\zeta_2 + \Big(\frac{32}{3}\frac{(\chi+1)(46\chi^2+127\chi+84)}{\chi^2} - 64\frac{(\chi+2)^3}{\chi}G_{-1}(\chi)$$
$$+ \frac{16}{3}\frac{20\chi^4+99\chi^3+177\chi^2+125\chi+21}{\chi^2}G_0(\chi)\Big)\zeta_3 - 2\frac{184\chi^4+1017\chi^3+1935\chi^2+1295\chi+63}{\chi^2}\zeta_4 + 64\frac{(\chi+1)(2\chi+3)(5\chi+6)}{\chi^2}G_0(\chi)$$
$$- 32\frac{(\chi+1)(2\chi+3)(4\chi+5)}{\chi^2}G_{0,0}(\chi) + 16\frac{(\chi+1)(2\chi+3)(3\chi+4)}{\chi^2}G_{0,0,0}(\chi) - 64\frac{(\chi+2)^3}{\chi}G_{-1,-1,0,0}(\chi) + 96\frac{(\chi+2)^3}{\chi}G_{-1,0,0,0}(\chi)$$
$$+ 64\frac{(\chi+2)^3}{\chi}G_{0,-1,0,0}(\chi) - 8\frac{12\chi^4+75\chi^3+153\chi^2+101\chi-3}{\chi^2}G_{0,0,0,0}(\chi)\Big)i + \epsilon^4\Big(-256\frac{(\chi+2)(13\chi+21)(\chi+1)^2}{\chi^2}$$
$$+ \Big(64\frac{(\chi+1)(\chi+2)(2\chi+3)(4\chi+5)}{\chi^2} - 32\frac{(\chi+1)(\chi+2)(2\chi+3)(3\chi+4)}{\chi^2}G_0(\chi) - 384\frac{(\chi+2)^4}{\chi}G_{-1,-1}(\chi) + 320\frac{(\chi+2)^4}{\chi}G_{-1,0}(\chi)$$
$$+ 384\frac{(\chi+2)^4}{\chi}G_{0,-1}(\chi) - 16\frac{(\chi+2)(20\chi^4+117\chi^3+231\chi^2+155\chi+3)}{\chi^2}G_{0,0}(\chi)\Big)\zeta_2 + \Big(\frac{64}{3}\frac{(\chi+1)(\chi+2)(46\chi^2+127\chi+84)}{\chi^2}$$
$$- 128\frac{(\chi+2)^4}{\chi}G_{-1}(\chi) + \frac{32}{3}\frac{(\chi+2)(20\chi^4+99\chi^3+177\chi^2+125\chi+21)}{\chi^2}G_0(\chi)\Big)\zeta_3 - 4\frac{(\chi+2)(184\chi^4+1017\chi^3+1935\chi^2+1295\chi+63)}{\chi^2}\zeta_4$$
$$+ 128\frac{(\chi+1)(\chi+2)(2\chi+3)(5\chi+6)}{\chi^2}G_0(\chi) - 64\frac{(\chi+1)(\chi+2)(2\chi+3)(4\chi+5)}{\chi^2}G_{0,0}(\chi) + 32\frac{(\chi+1)(\chi+2)(2\chi+3)(3\chi+4)}{\chi^2}G_{0,0,0}(\chi)$$
$$- 128\frac{(\chi+2)^4}{\chi}G_{-1,-1,0,0}(\chi) + 192\frac{(\chi+2)^4}{\chi}G_{-1,0,0,0}(\chi) + 128\frac{(\chi+2)^4}{\chi}G_{0,-1,0,0}(\chi)$$
$$- 16\frac{(\chi+2)(12\chi^4+75\chi^3+153\chi^2+101\chi-3)}{\chi^2}G_{0,0,0,0}(\chi)\Big)i + \Big(\Big(-12\frac{1}{\chi}G_{-1,-1}(\chi) + 16\frac{1}{\chi}G_{-1,0}(\chi) + 42\frac{1}{\chi}G_{0,-1}(\chi) - 45\frac{1}{\chi}G_{0,0}(\chi)\Big)\zeta_2$$
$$+ \Big(-4\frac{1}{\chi}G_{-1}(\chi) + \frac{98}{3}\frac{1}{\chi}G_0(\chi)\Big)\zeta_3 + 3\frac{1}{\chi}\zeta_4 - 4\frac{1}{\chi}G_{-1,-1,0,0}(\chi) + 12\frac{1}{\chi}G_{-1,0,0,0}(\chi) + 14\frac{1}{\chi}G_{0,-1,0,0}(\chi) - 21\frac{1}{\chi}G_{0,0,0,0}(\chi)\Big)i$$
$$+ \Big(-7\frac{1}{\chi}G_0(\chi)i + 4\frac{\pi}{\chi}\Big)\frac{1}{\epsilon^3} - \frac{62}{3}\frac{\pi}{\chi}\zeta_3 - \frac{\pi^3}{\chi}G_{-1}(\chi) + \frac{17}{6}\frac{\pi^3}{\chi}G_0(\chi) + \Big(\Big(-30\frac{1}{\chi}\zeta_2 + 3\frac{1}{\chi}G_{0,0}(\chi)\Big)i - 3\frac{\pi}{\chi}G_0(\chi)\Big)\frac{1}{\epsilon^2} + \Big(\Big(\Big(-12\frac{1}{\chi}G_{-1}(\chi)$$
$$+ 35\frac{1}{\chi}G_0(\chi)\Big)\zeta_2 - 38\frac{1}{\chi}\zeta_3 - 4\frac{1}{\chi}G_{-1,0,0}(\chi) + 5\frac{1}{\chi}G_{0,0,0}(\chi)\Big)i - \frac{7}{3}\frac{\pi^3}{\chi} + \frac{\pi}{\chi}G_{0,0}(\chi)\Big)\frac{1}{\epsilon} - 2\frac{\pi}{\chi}G_{-1,0,0}(\chi) + 3\frac{\pi}{\chi}G_{0,0,0}(\chi) + \Big(\Big(\frac{2699}{2}\frac{1}{\chi}$$
$$+ \Big(-\frac{1103}{2}\frac{1}{\chi} + \frac{333}{2}\frac{1}{\chi}G_0(\chi) - 48\frac{\chi+2}{\chi}G_{-1,-1}(\chi) + 40\frac{\chi+2}{\chi}G_{-1,0}(\chi) + 48\frac{\chi+2}{\chi}G_{0,-1}(\chi) - \frac{1}{2}\frac{80\chi^2+255\chi+12}{\chi^2}G_{0,0}(\chi)\Big)\zeta_2$$
$$+ \Big(-\frac{1045}{3}\frac{1}{\chi} - 16\frac{\chi+2}{\chi}G_{-1}(\chi) + \frac{1}{3}\frac{80\chi^2+495\chi+84}{\chi^2}G_0(\chi)\Big)\zeta_3 - \frac{1}{8}\frac{736\chi^2+1511\chi+252}{\chi^2}\zeta_4 - \frac{953}{2}\frac{1}{\chi}G_0(\chi) + \frac{323}{2}\frac{1}{\chi}G_{0,0}(\chi)$$
$$- \frac{105}{2}\frac{1}{\chi}G_{0,0,0}(\chi) - 16\frac{\chi+2}{\chi}G_{-1,-1,0,0}(\chi) + 24\frac{\chi+2}{\chi}G_{-1,0,0,0}(\chi) + 16\frac{\chi+2}{\chi}G_{0,-1,0,0}(\chi) - \frac{1}{2}\frac{48\chi^2+61\chi-12}{\chi^2}G_{0,0,0,0}(\chi)\Big)i$$
$$- \frac{3}{4}\frac{(19\pi^2-422)\pi}{\chi} - 65\frac{\pi}{\chi}\zeta_3 + \frac{15}{4}\frac{(\pi^2-26)\pi}{\chi}G_0(\chi) + \frac{57}{2}\frac{\pi}{\chi}G_{0,0}(\chi) - \frac{15}{2}\frac{\pi}{\chi}G_{0,0,0}(\chi)\Big)\epsilon + \Big(\Big(-\frac{832\chi^2+4523\chi+1152}{\chi^2}$$
$$+ \Big(\frac{128\chi^2+1271\chi+192}{\chi^2} - \frac{48\chi^2+377\chi+72}{\chi^2}G_0(\chi) - 96\frac{(\chi+2)^2}{\chi}G_{-1,-1}(\chi) + 80\frac{(\chi+2)^2}{\chi}G_{-1,0}(\chi) + 96\frac{(\chi+2)^2}{\chi}G_{0,-1}(\chi)$$
$$- \frac{80\chi^2+308\chi+247}{\chi}G_{0,0}(\chi)\Big)\zeta_2 + \Big(\frac{2}{3}\frac{368\chi^2+1681\chi+504}{\chi^2} - 32\frac{(\chi+2)^2}{\chi}G_{-1}(\chi) + \frac{2}{3}\frac{80\chi^2+236\chi+7}{\chi}G_0(\chi)\Big)\zeta_3$$
$$- \frac{1}{4}\frac{736\chi^2+2596\chi+2383}{\chi}\zeta_4 + \frac{320\chi^2+1641\chi+480}{\chi^2}G_0(\chi) - \frac{128\chi^2+587\chi+192}{\chi^2}G_{0,0}(\chi) + \frac{48\chi^2+197\chi+72}{\chi^2}G_{0,0,0}(\chi)$$
$$- 32\frac{(\chi+2)^2}{\chi}G_{-1,-1,0,0}(\chi) + 48\frac{(\chi+2)^2}{\chi}G_{-1,0,0,0}(\chi) + 32\frac{(\chi+2)^2}{\chi}G_{0,-1,0,0}(\chi) - \frac{48\chi^2+204\chi+229}{\chi}G_{0,0,0,0}(\chi)\Big)i + \frac{45}{2}\frac{(\pi^2-26)\pi}{\chi}$$
$$+ 78\frac{\pi}{\chi}\zeta_3 - \frac{9}{2}\frac{(\pi^2-38)\pi}{\chi}G_0(\chi) - 45\frac{\pi}{\chi}G_{0,0}(\chi) + 9\frac{\pi}{\chi}G_{0,0,0}(\chi)\Big)\epsilon^2\Big)\mu^{-\epsilon} + \Big(-\frac{1}{\epsilon^4}\frac{21}{4}\frac{1}{\chi}i + \Big(-\frac{20}{3}\frac{1}{\chi}G_0(\chi)\zeta_3 - \frac{379}{16}\frac{1}{\chi}\zeta_4\Big)i + \epsilon^3\Big(\frac{268695}{128}\frac{1}{\chi}$$
$$+ \frac{25875}{32}\frac{1}{\chi}\zeta_2 + \frac{2145}{8}\frac{1}{\chi}\zeta_3 + \frac{13635}{32}\frac{1}{\chi}\zeta_4\Big)i + \epsilon^4\Big(\frac{268695}{32}\frac{1}{\chi} + \frac{25875}{8}\frac{1}{\chi}\zeta_2 + \frac{2145}{2}\frac{1}{\chi}\zeta_3 + \frac{13635}{8}\frac{1}{\chi}\zeta_4\Big)i + \Big(\Big(-\frac{9}{2}\frac{1}{\chi}G_0(\chi)\zeta_2 + 20\frac{1}{\chi}\zeta_3\Big)i$$
$$+ \frac{7}{4}\frac{\pi^3}{\chi}\Big)\frac{1}{\epsilon} + \Big(\frac{5}{2}\frac{1}{\chi}G_0(\chi)i - \frac{7}{2}\frac{\pi}{\chi}\Big)\frac{1}{\epsilon^3} + \frac{37}{3}\frac{\pi}{\chi}\zeta_3 + \Big(\Big(\frac{6633}{256}\frac{1}{\chi} + \frac{17517}{64}\frac{1}{\chi}\zeta_2 + \frac{3255}{16}\frac{1}{\chi}\zeta_3 + \frac{13989}{64}\frac{1}{\chi}\zeta_4\Big)i + \frac{13}{4}\frac{(\pi^2-30)\pi}{\chi} + \frac{52}{3}\frac{\pi}{\chi}\zeta_3\Big)\epsilon^2$$

$$+ \Bigg(\Bigg(\frac{167895}{256}\frac{1}{\chi} - \frac{45}{64}\frac{1}{\chi}\zeta_2 - \frac{2043}{16}\frac{1}{\chi}\zeta_3 - \frac{5985}{64}\frac{1}{\chi}\zeta_4\Bigg)i - \frac{13}{24}\frac{(7\pi^2 - 186)\pi}{\chi} - 26\frac{\pi}{\chi}\zeta_3\Bigg)\epsilon - \frac{1}{3}\frac{\pi^3}{\chi}G_0(\chi) + \Bigg(\frac{65}{4}\frac{1}{\chi}\zeta_2 i + 2\frac{\pi}{\chi}G_0(\chi)\Bigg)\frac{1}{\epsilon^2}$$
$$+ \Bigg(-\frac{1}{\epsilon^3}\frac{5}{2}\frac{1}{\chi}i + \frac{1}{\epsilon}\frac{9}{2}\frac{1}{\chi}\zeta_2 i + \frac{20}{3}\frac{1}{\chi}\zeta_3 i + \epsilon^4\Bigg(\frac{29855}{32}\frac{1}{\chi} + \frac{2875}{8}\frac{1}{\chi}\zeta_2 + \frac{715}{6}\frac{1}{\chi}\zeta_3 + \frac{1515}{8}\frac{1}{\chi}\zeta_4\Bigg)i + \frac{1}{3}\frac{\pi^3}{\chi} - \frac{1}{\epsilon^2}2\frac{\pi}{\chi} + \Bigg(-\frac{75}{8}\frac{1}{\chi}\zeta_4 i + \frac{16}{3}\frac{\pi}{\chi}\zeta_3\Bigg)\epsilon$$
$$+ \Bigg(\Bigg(-\frac{1247}{64}\frac{1}{\chi} + \frac{2357}{16}\frac{1}{\chi}\zeta_2 + \frac{1181}{12}\frac{1}{\chi}\zeta_3 + \frac{1569}{16}\frac{1}{\chi}\zeta_4\Bigg)i + 2\frac{(\pi^2 - 30)\pi}{\chi} + \frac{32}{3}\frac{\pi}{\chi}\zeta_3\Bigg)\epsilon^3 + \Bigg(\Bigg(\frac{20671}{64}\frac{1}{\chi} - \frac{445}{16}\frac{1}{\chi}\zeta_2 - \frac{793}{12}\frac{1}{\chi}\zeta_3 - \frac{687}{16}\frac{1}{\chi}\zeta_4\Bigg)i$$
$$- \frac{1}{3}\frac{(7\pi^2 - 186)\pi}{\chi} - 16\frac{\pi}{\chi}\zeta_3\Bigg)\epsilon^2\Bigg)\log(\mu)\Bigg)\mu^{-2\epsilon} + O(\mu) \tag{D.11}$$

I[FP6, 1,1, 1, 1, 1, 1, 1, 0, 0] =

$$- \frac{1}{\epsilon^4}4\frac{1}{\chi}i + \frac{1}{\epsilon^3}5\frac{1}{\chi}G_0(\chi)i + \frac{1}{\epsilon^2}\Bigg(15\frac{1}{\chi}\zeta_2 - 4\frac{1}{\chi}G_{0,0}(\chi)\Bigg)i + \frac{1}{\epsilon}\Bigg(\Bigg(12\frac{1}{\chi}G_{-1}(\chi) - 33\frac{1}{\chi}G_0(\chi)\Bigg)\zeta_2 + \frac{65}{3}\frac{1}{\chi}\zeta_3 + 4\frac{1}{\chi}G_{-1,0,0}(\chi) - 4\frac{1}{\chi}G_{0,0,0}(\chi)\Bigg)i$$
$$+ \Bigg(\Bigg(12\frac{1}{\chi}G_{-1,-1}(\chi) - 20\frac{1}{\chi}G_{-1,0}(\chi) - 60\frac{1}{\chi}G_{0,-1}(\chi) + 72\frac{1}{\chi}G_{0,0}(\chi)\Bigg)\zeta_2 + \Bigg(4\frac{1}{\chi}G_{-1}(\chi) - \frac{88}{3}\frac{1}{\chi}G_0(\chi)\Bigg)\zeta_3 + 87\frac{1}{\chi}\zeta_4 + 4\frac{1}{\chi}G_{-1,-1,0,0}(\chi)$$
$$- 16\frac{1}{\chi}G_{-1,0,0,0}(\chi) - 20\frac{1}{\chi}G_{0,-1,0,0}(\chi) + 32\frac{1}{\chi}G_{0,0,0,0}(\chi)\Bigg)i + \Bigg(\frac{1}{\epsilon^4}9\frac{1}{\chi}i + \epsilon^3\Bigg(-160\frac{(10\chi^2 + 11\chi - 7)(\chi + 1)^2}{\chi^3}$$
$$+ \Bigg(32\frac{(\chi + 1)(2\chi + 3)(4\chi^2 + 3\chi - 2)}{\chi^3} - 8\frac{(\chi + 1)(2\chi + 3)(6\chi^2 + 5\chi - 3)}{\chi^3}G_0(\chi) - 24\frac{(2\chi^2 + 3\chi - 1)^3}{\chi^4}G_{-1,-1}(\chi)$$
$$+ 20\frac{(2\chi^2 + 3\chi - 1)^3}{\chi^4}G_{-1,0}(\chi) + 24\frac{(2\chi^2 + 3\chi - 1)^3}{\chi^4}G_{0,-1}(\chi) - 4\frac{40\chi^6 + 176\chi^5 + 202\chi^4 - 44\chi^3 - 103\chi^2 + 40\chi - 3}{\chi^4}G_{0,0}(\chi)\Bigg)\zeta_2$$
$$+ \Bigg(\frac{16}{3}\frac{(\chi + 1)(88\chi^3 + 198\chi^2 + 55\chi - 61)}{\chi^3} - 8\frac{(2\chi^2 + 3\chi - 1)^3}{\chi^4}G_{-1}(\chi) + \frac{8}{3}\frac{40\chi^6 + 152\chi^5 + 154\chi^4 - 38\chi^3 - 91\chi^2 + 10\chi + 9}{\chi^4}G_0(\chi)\Bigg)\zeta_3$$
$$- \frac{356\chi^6 + 1488\chi^5 + 1623\chi^4 - 402\chi^3 - 855\chi^2 + 294\chi - 4}{\chi^4}\zeta_4 + 32\frac{(\chi + 1)(2\chi + 3)(10\chi^2 + 7\chi - 5)}{\chi^3}G_0(\chi)$$
$$- 32\frac{(\chi + 1)(2\chi + 3)(4\chi^2 + 3\chi - 2)}{\chi^3}G_{0,0}(\chi) + 8\frac{(\chi + 1)(2\chi + 3)(6\chi^2 + 5\chi - 3)}{\chi^3}G_{0,0,0}(\chi) - 8\frac{(2\chi^2 + 3\chi - 1)^3}{\chi^4}G_{-1,-1,0,0}(\chi)$$
$$+ 12\frac{(2\chi^2 + 3\chi - 1)^3}{\chi^4}G_{-1,0,0,0}(\chi) + 8\frac{(2\chi^2 + 3\chi - 1)^3}{\chi^4}G_{0,-1,0,0}(\chi) - 4\frac{24\chi^6 + 112\chi^5 + 134\chi^4 - 28\chi^3 - 65\chi^2 + 32\chi - 5}{\chi^4}G_{0,0,0,0}(\chi)\Bigg)i$$
$$+ \epsilon^4\Bigg(-160\frac{(2\chi^2 + 3\chi - 1)(10\chi^2 + 11\chi - 7)(\chi + 1)^2}{\chi^4} + \Bigg(32\frac{(\chi + 1)(2\chi + 3)(2\chi^2 + 3\chi - 1)(4\chi^2 + 3\chi - 2)}{\chi^4}$$
$$- 8\frac{(\chi + 1)(2\chi + 3)(2\chi^2 + 3\chi - 1)(6\chi^2 + 5\chi - 3)}{\chi^4}G_0(\chi) - 24\frac{(2\chi^2 + 3\chi - 1)^4}{\chi^5}G_{-1,-1}(\chi) + 20\frac{(2\chi^2 + 3\chi - 1)^4}{\chi^5}G_{-1,0}(\chi)$$
$$+ 24\frac{(2\chi^2 + 3\chi - 1)^4}{\chi^5}G_{0,-1}(\chi) - 4\frac{(2\chi^2 + 3\chi - 1)(40\chi^6 + 176\chi^5 + 202\chi^4 - 44\chi^3 - 103\chi^2 + 40\chi - 3)}{\chi^5}G_{0,0}(\chi)\Bigg)\zeta_2$$
$$+ \Bigg(\frac{16}{3}\frac{(\chi + 1)(2\chi^2 + 3\chi - 1)(88\chi^3 + 198\chi^2 + 55\chi - 61)}{\chi^4} - 8\frac{(2\chi^2 + 3\chi - 1)^4}{\chi^5}G_{-1}(\chi)$$
$$+ \frac{8}{3}\frac{(2\chi^2 + 3\chi - 1)(40\chi^6 + 152\chi^5 + 154\chi^4 - 38\chi^3 - 91\chi^2 + 10\chi + 9)}{\chi^5}G_0(\chi)\Bigg)\zeta_3$$
$$- \frac{(2\chi^2 + 3\chi - 1)(356\chi^6 + 1488\chi^5 + 1623\chi^4 - 402\chi^3 - 855\chi^2 + 294\chi - 4)}{\chi^5}\zeta_4 + 32\frac{(\chi + 1)(2\chi + 3)(2\chi^2 + 3\chi - 1)(10\chi^2 + 7\chi - 5)}{\chi^4}G_0(\chi)$$
$$- 32\frac{(\chi + 1)(2\chi + 3)(2\chi^2 + 3\chi - 1)(4\chi^2 + 3\chi - 2)}{\chi^4}G_{0,0}(\chi) + 8\frac{(\chi + 1)(2\chi + 3)(2\chi^2 + 3\chi - 1)(6\chi^2 + 5\chi - 3)}{\chi^4}G_{0,0,0}(\chi)$$
$$- 8\frac{(2\chi^2 + 3\chi - 1)^4}{\chi^5}G_{-1,-1,0,0}(\chi) + 12\frac{(2\chi^2 + 3\chi - 1)^4}{\chi^5}G_{-1,0,0,0}(\chi) + 8\frac{(2\chi^2 + 3\chi - 1)^4}{\chi^5}G_{0,-1,0,0}(\chi)$$
$$- 4\frac{(2\chi^2 + 3\chi - 1)(24\chi^6 + 112\chi^5 + 134\chi^4 - 28\chi^3 - 65\chi^2 + 32\chi - 5)}{\chi^5}G_{0,0,0,0}(\chi)\Bigg)i + \Bigg(\Bigg(-12\frac{1}{\chi}G_{-1,-1}(\chi) + 16\frac{1}{\chi}G_{-1,0}(\chi)$$
$$+ 42\frac{1}{\chi}G_{0,-1}(\chi) - 45\frac{1}{\chi}G_{0,0}(\chi)\Bigg)\zeta_2 + \Bigg(-4\frac{1}{\chi}G_{-1}(\chi) + \frac{98}{3}\frac{1}{\chi}G_0(\chi)\Bigg)\zeta_3 + 3\frac{1}{\chi}\zeta_4 - 4\frac{1}{\chi}G_{-1,-1,0,0}(\chi) + 12\frac{1}{\chi}G_{-1,0,0,0}(\chi)$$
$$+ 14\frac{1}{\chi}G_{0,-1,0,0}(\chi) - 21\frac{1}{\chi}G_{0,0,0,0}(\chi)\Bigg)i + \Bigg(-7\frac{1}{\chi}G_0(\chi)i + 4\frac{\pi}{\chi}\Bigg)\frac{1}{\epsilon^3} - \frac{62}{3}\frac{\pi}{\chi}\zeta_3 - \frac{\pi^3}{\chi}G_{-1}(\chi) + \frac{17}{6}\frac{\pi^3}{\chi}G_0(\chi) + \Bigg(\Bigg(-30\frac{1}{\chi}\zeta_2 + 3\frac{1}{\chi}G_{0,0}(\chi)\Bigg)i$$
$$- 3\frac{\pi}{\chi}G_0(\chi)\Bigg)\frac{1}{\epsilon^2} + \Bigg(\Bigg(\Bigg(-12\frac{1}{\chi}G_{-1}(\chi) + 35\frac{1}{\chi}G_0(\chi)\Bigg)\zeta_2 - 38\frac{1}{\chi}\zeta_3 - 4\frac{1}{\chi}G_{-1,0,0}(\chi) + 5\frac{1}{\chi}G_{0,0,0}(\chi)\Bigg)i - \frac{7}{3}\frac{\pi^3}{\chi} + \frac{\pi}{\chi}G_{0,0}(\chi)\Bigg)\frac{1}{\epsilon}$$
$$- 2\frac{\pi}{\chi}G_{-1,0,0}(\chi) + 3\frac{\pi}{\chi}G_{0,0,0}(\chi) + \Bigg(\Bigg(2347\frac{1}{\chi} + \Bigg(-1039\frac{1}{\chi} + 309\frac{1}{\chi}G_0(\chi) - 24\frac{2\chi^2 + 3\chi - 1}{\chi^2}G_{-1,-1}(\chi) + 20\frac{2\chi^2 + 3\chi - 1}{\chi^2}G_{-1,0}(\chi)$$
$$+ 24\frac{2\chi^2 + 3\chi - 1}{\chi^2}G_{0,-1}(\chi) - \frac{40\chi^2 + 147\chi - 12}{\chi^2}G_{0,0}(\chi)\Bigg)\zeta_2 + \Bigg(-\frac{1786}{3}\frac{1}{\chi} - 8\frac{2\chi^2 + 3\chi - 1}{\chi^2}G_{-1}(\chi) + \frac{2}{3}\frac{40\chi^2 + 339\chi + 36}{\chi^2}G_0(\chi)\Bigg)\zeta_3$$
$$- \frac{1}{4}\frac{356\chi^2 + 459\chi - 16}{\chi^2}\zeta_4 - 793\frac{1}{\chi}G_0(\chi) + 259\frac{1}{\chi}G_{0,0}(\chi) - 81\frac{1}{\chi}G_{0,0,0}(\chi) - 8\frac{2\chi^2 + 3\chi - 1}{\chi^2}G_{-1,-1,0,0}(\chi) + 12\frac{2\chi^2 + 3\chi - 1}{\chi^2}G_{-1,0,0,0}(\chi)$$
$$+ 8\frac{2\chi^2 + 3\chi - 1}{\chi^2}G_{0,-1,0,0}(\chi) - \frac{24\chi^2 + 9\chi - 20}{\chi^2}G_{0,0,0,0}(\chi)\Bigg)i - \frac{3}{2}\frac{(19\pi^2 - 422)\pi}{\chi} - 130\frac{\pi}{\chi}\zeta_3 + \frac{15}{2}\frac{(\pi^2 - 26)\pi}{\chi}G_0(\chi) + 57\frac{\pi}{\chi}G_{0,0}(\chi)$$
$$- 15\frac{\pi}{\chi}G_{0,0,0}(\chi)\Bigg)\epsilon + \Bigg(\Bigg(-2\frac{400\chi^2 + 3179\chi + 560}{\chi^2} + \Bigg(2\frac{64\chi^2 + 1063\chi + 96}{\chi^2} - 2\frac{24\chi^2 + 301\chi + 36}{\chi^2}G_0(\chi) - 24\frac{(2\chi^2 + 3\chi - 1)^2}{\chi^3}G_{-1,-1}(\chi)$$

$$+ 20\frac{(2\chi^2+3\chi-1)^2}{\chi^3}G_{-1,0}(\chi) + 24\frac{(2\chi^2+3\chi-1)^2}{\chi^3}G_{0,-1}(\chi) - 2\frac{40\chi^4+116\chi^3-5\chi^2-62\chi+6}{\chi^3}G_{0,0}(\chi)\Big)\zeta_2 + \Big(\frac{4}{3}\frac{176\chi^2+1117\chi+244}{\chi^2}$$

$$- 8\frac{(2\chi^2+3\chi-1)^2}{\chi^3}G_{-1}(\chi) + \frac{4}{3}\frac{40\chi^4+92\chi^3-137\chi^2-74\chi-18}{\chi^3}G_0(\chi)\Big)\zeta_3 - \frac{1}{2}\frac{356\chi^4+954\chi^3+373\chi^2-564\chi+8}{\chi^3}\zeta_4$$

$$+ 2\frac{160\chi^2+1113\chi+240}{\chi^2}G_0(\chi) - 2\frac{64\chi^2+379\chi+96}{\chi^2}G_{0,0}(\chi) + 2\frac{24\chi^2+121\chi+36}{\chi^2}G_{0,0,0}(\chi) - 8\frac{(2\chi^2+3\chi-1)^2}{\chi^3}G_{-1,-1,0,0}(\chi)$$

$$+ 12\frac{(2\chi^2+3\chi-1)^2}{\chi^3}G_{-1,0,0,0}(\chi) + 8\frac{(2\chi^2+3\chi-1)^2}{\chi^3}G_{0,-1,0,0}(\chi) - 2\frac{24\chi^4+76\chi^3+49\chi^2-34\chi+10}{\chi^3}G_{0,0,0,0}(\chi)\Big)i + 45\frac{(\pi^2-26)\pi}{\chi}$$

$$+ 156\frac{\pi}{\chi}\zeta_3 - 9\frac{(\pi^2-38)\pi}{\chi}G_0(\chi) - 90\frac{\pi}{\chi}G_{0,0}(\chi) + 18\frac{\pi}{\chi}G_{0,0,0}(\chi)\Big)\epsilon^2\Big)\mu^{-\epsilon} + \Big(-\frac{1}{\epsilon^4}\frac{27}{4}\frac{1}{\chi}i + \Big(\Big(6\frac{1}{\chi}G_{-1,-1}(\chi) - 6\frac{1}{\chi}G_{-1,0}(\chi)$$

$$- 6\frac{1}{\chi}G_{0,-1}(\chi) + 6\frac{1}{\chi}G_{0,0}(\chi)\Big)\zeta_2 + \Big(2\frac{1}{\chi}G_{-1}(\chi) - \frac{26}{3}\frac{1}{\chi}G_0(\chi)\Big)\zeta_3 - \frac{1077}{16}\frac{1}{\chi}\zeta_4 + 2\frac{1}{\chi}G_{-1,-1,0,0}(\chi) - 4\frac{1}{\chi}G_{-1,0,0,0}(\chi) - 2\frac{1}{\chi}G_{0,-1,0,0}(\chi)$$

$$+ 4\frac{1}{\chi}G_{0,0,0,0}(\chi)\Big)i + \Big(\Big(\Big(\Big(6\frac{1}{\chi}G_{-1}(\chi) - \frac{17}{2}\frac{1}{\chi}G_0(\chi)\Big)\zeta_2 + 26\frac{1}{\chi}\zeta_3 + 2\frac{1}{\chi}G_{-1,0,0}(\chi) - 2\frac{1}{\chi}G_{0,0,0}(\chi)\Big)i + \frac{5}{2}\frac{\pi^3}{\chi}\Big)\frac{1}{\epsilon} + \Big(\frac{5}{2}\frac{1}{\chi}G_0(\chi)i - 6\frac{\pi}{\chi}\Big)\frac{1}{\epsilon^3}$$

$$+ 27\frac{\pi}{\chi}\zeta_3 + \Big(\Big(\frac{41209}{32}\frac{1}{\chi} - \frac{10885}{8}\frac{1}{\chi}\zeta_2 - \frac{976}{3}\frac{1}{\chi}\zeta_3 + \frac{4317}{8}\frac{1}{\chi}\zeta_4\Big)i - \frac{1}{24}\frac{(1150\pi^2-17913)\pi}{\chi} - \frac{416}{3}\frac{\pi}{\chi}\zeta_3\Big)\epsilon + \Big(\Big(\frac{53739}{16}\frac{1}{\chi} - \frac{13455}{4}\frac{1}{\chi}\zeta_2$$

$$- 624\frac{1}{\chi}\zeta_3 + \frac{2727}{4}\frac{1}{\chi}\zeta_4\Big)i - \frac{15}{4}\frac{(26\pi^2-519)\pi}{\chi} - 192\frac{\pi}{\chi}\zeta_3\Big)\epsilon^3 + \Big(\Big(-\frac{132061}{32}\frac{1}{\chi} + \frac{33945}{8}\frac{1}{\chi}\zeta_2 + 856\frac{1}{\chi}\zeta_3 - \frac{8433}{8}\frac{1}{\chi}\zeta_4\Big)i + \frac{105}{8}\frac{(10\pi^2-183)\pi}{\chi}$$

$$+ 288\frac{\pi}{\chi}\zeta_3\Big)\epsilon^2 + \frac{\pi^3}{\chi}G_{-1}(\chi) - \frac{\pi^3}{\chi}G_0(\chi) + \Big(\frac{99}{4}\frac{1}{\chi}\zeta_2 i + 2\frac{\pi}{\chi}G_0(\chi)\Big)\frac{1}{\epsilon^2} + 2\frac{\pi}{\chi}G_{-1,0,0}(\chi) - 2\frac{\pi}{\chi}G_{0,0,0}(\chi) + \Big(-\frac{1}{\epsilon^3}\frac{1}{2}\frac{1}{\chi}i - \frac{1}{\epsilon}\frac{3}{2}\frac{1}{\chi}\zeta_2 i + \frac{4}{3}\frac{1}{\chi}\zeta_3 i$$

$$- \epsilon\frac{63}{8}\frac{1}{\chi}\zeta_4 i + \epsilon^3\Big(-31\frac{1}{\chi} - 21\frac{1}{\chi}\zeta_2 + 8\frac{1}{\chi}\zeta_3 - \frac{63}{4}\frac{1}{\chi}\zeta_4\Big)i + \epsilon^2\Big(\frac{63}{2}\frac{1}{\chi} + \frac{45}{2}\frac{1}{\chi}\zeta_2 - \frac{28}{3}\frac{1}{\chi}\zeta_3 + \frac{189}{8}\frac{1}{\chi}\zeta_4\Big)i\Big)\log(\mu)\Big)\mu^{-2\epsilon} + \Big(\frac{1}{\epsilon^4}2\frac{1}{\chi}i - \frac{1}{\epsilon^2}9\frac{1}{\chi}\zeta_2 i$$

$$+ \frac{19}{4}\frac{1}{\chi}\zeta_4 i + \Big(-\frac{22}{3}\frac{1}{\chi}\zeta_3 i - \frac{1}{6}\frac{\pi^3}{\chi}\Big)\frac{1}{\epsilon} + \frac{1}{\epsilon^3}3\frac{\pi}{\chi} - 12\frac{\pi}{\chi}\zeta_3 + \Big(\Big(320\frac{1}{\chi} - 480\frac{1}{\chi}\zeta_2 - \frac{224}{3}\frac{1}{\chi}\zeta_3 + 24\frac{1}{\chi}\zeta_4\Big)i - \frac{32}{3}\frac{(\pi^2-24)\pi}{\chi} - \frac{112}{3}\frac{\pi}{\chi}\zeta_3\Big)\epsilon^2$$

$$+ \Big(\Big(-224\frac{1}{\chi} + 312\frac{1}{\chi}\zeta_2 + \frac{200}{3}\frac{1}{\chi}\zeta_3 - \frac{43}{2}\frac{1}{\chi}\zeta_4\Big)i + 2\frac{(3\pi^2-88)\pi}{\chi} + \frac{128}{3}\frac{\pi}{\chi}\zeta_3\Big)\epsilon\Big)\mu^{-3\epsilon} + \Big(-\frac{1}{\epsilon^4}\frac{1}{4}\frac{1}{\chi}i + \frac{1}{\epsilon^2}\frac{5}{4}\frac{1}{\chi}\zeta_2 i + \frac{305}{16}\frac{1}{\chi}\zeta_4 i + \Big(\frac{2}{3}\frac{1}{\chi}\zeta_3 i$$

$$- \frac{1}{4}\frac{\pi^3}{\chi}\Big)\frac{1}{\epsilon} - \frac{1}{\epsilon^3}\frac{1}{2}\frac{\pi}{\chi} + \frac{4}{3}\frac{\pi}{\chi}\zeta_3\Big)\mu^{-4\epsilon} + O(\mu) \tag{D.12}$$

I[FP7, 1, 1, 1, 1, 1, 1, 1, 0, 0] =

$$- \frac{1}{\epsilon^4}\frac{3}{4}\frac{\chi+1}{\chi^2}i + \frac{1}{\epsilon^3}\Big(-\frac{3}{4}\frac{\chi+1}{\chi^2} + \frac{3}{2}\frac{1}{\chi^2}G_0(\chi)\Big)i + \frac{1}{\epsilon^2}\Big(\frac{3}{2}\frac{\chi+1}{\chi^2} - \frac{3}{4}\frac{\chi+1}{\chi^2}\zeta_2 + \frac{3}{2}\frac{1}{\chi^2}G_0(\chi) - 3\frac{1}{\chi^2}G_{0,0}(\chi)\Big)i + \frac{1}{\epsilon}\Big(-3\frac{\chi+1}{\chi^2} + \Big(-\frac{15}{4}\frac{\chi+1}{\chi^2}$$

$$- 9\frac{\chi+1}{\chi^2}G_{-1}(\chi) + \frac{3}{2}\frac{6\chi+1}{\chi^2}G_0(\chi)\Big)\zeta_2 + \frac{7}{2}\frac{\chi+1}{\chi^2}\zeta_3 - 3\frac{1}{\chi^2}G_0(\chi) - 3\frac{1}{\chi^2}G_{0,0}(\chi) - 3\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi) + 3\frac{\chi+2}{\chi^2}G_{0,0,0}(\chi)\Big)i + \Big(6\frac{\chi+1}{\chi^2}$$

$$+ \Big(\frac{3}{2}\frac{\chi+1}{\chi^2} - 18\frac{\chi+1}{\chi^2}G_{-1}(\chi) + \frac{3}{2}\frac{12\chi+5}{\chi^2}G_0(\chi) - 27\frac{\chi+1}{\chi^2}G_{-1,-1}(\chi) + 3\frac{8\chi+7}{\chi^2}G_{-1,0}(\chi) + 9\frac{3\chi+2}{\chi^2}G_{0,-1}(\chi) - 3\frac{8\chi+1}{\chi^2}G_{0,0}(\chi)\Big)\zeta_2$$

$$+ \Big(-\frac{5}{2}\frac{\chi+1}{\chi^2} - 9\frac{\chi+1}{\chi^2}G_{-1}(\chi) + \frac{9\chi-7}{\chi^2}G_0(\chi)\Big)\zeta_3 - \frac{3}{16}\frac{47\chi-21}{\chi^2}\zeta_4 + 6\frac{1}{\chi^2}G_0(\chi) + 6\frac{1}{\chi^2}G_{0,0}(\chi) - 6\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi) + 6\frac{\chi+1}{\chi^2}G_{0,0,0}(\chi)$$

$$- 9\frac{\chi+1}{\chi^2}G_{-1,-1,0,0}(\chi) + 3\frac{5\chi+4}{\chi^2}G_{-1,0,0,0}(\chi) + 3\frac{3\chi+2}{\chi^2}G_{0,-1,0,0}(\chi) - 3\frac{5\chi+4}{\chi^2}G_{0,0,0,0}(\chi)\Big)i + \epsilon\Big(-\frac{144123}{64}\frac{\chi+1}{\chi^2} + \Big(\frac{711}{16}\frac{\chi+1}{\chi^2}$$

$$- 18\frac{\chi+1}{\chi^2}G_{-1}(\chi) + \frac{9}{8}\frac{3\chi+23}{\chi^2}G_0(\chi) - 72\frac{\chi+1}{\chi^2}G_{-1,-1}(\chi) + 72\frac{\chi+1}{\chi^2}G_{-1,0}(\chi) + 72\frac{\chi+1}{\chi^2}G_{0,-1}(\chi) - \frac{123}{2}\frac{\chi+1}{\chi^2}G_{0,0}(\chi)\Big)\zeta_2$$

$$+ \Big(\frac{897}{2}\frac{\chi+1}{\chi^2} - 24\frac{\chi+1}{\chi^2}G_{-1}(\chi) - 29\frac{3\chi+1}{\chi^2}G_0(\chi)\Big)\zeta_3 + \frac{1257}{16}\frac{\chi+1}{\chi^2}\zeta_4 + \frac{9}{32}\frac{2945\chi+1777}{\chi^2}G_0(\chi) - \frac{3}{8}\frac{1321\chi+833}{\chi^2}G_{0,0}(\chi)$$

$$- 6\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi) + \frac{3}{2}\frac{179\chi+107}{\chi^2}G_{0,0,0}(\chi) - 24\frac{\chi+1}{\chi^2}G_{-1,-1,0,0}(\chi) + 48\frac{\chi+1}{\chi^2}G_{-1,0,0,0}(\chi) + 24\frac{\chi+1}{\chi^2}G_{0,-1,0,0}(\chi)$$

$$- 6\frac{27\chi+19}{\chi^2}G_{0,0,0,0}(\chi)\Big)i + \epsilon^3\Big(\frac{1192665}{64}\frac{\chi+1}{\chi^2} + \Big(-\frac{17001}{16}\frac{\chi+1}{\chi^2} + 90\frac{\chi+1}{\chi^2}G_{-1}(\chi) - \frac{9}{4}\frac{\chi-176}{\chi^2}G_0(\chi) - 18\frac{\chi+1}{\chi^2}G_{-1,-1}(\chi)$$

$$- 18\frac{1}{\chi^2}G_{-1,0}(\chi) + 18\frac{\chi-2}{\chi^2}G_{0,-1}(\chi) - 3\frac{13\chi+16}{\chi^2}G_{0,0}(\chi)\Big)\zeta_2 + \Big(-4014\frac{\chi+1}{\chi^2} - 6\frac{\chi+1}{\chi^2}G_{-1}(\chi) + 14\frac{48\chi+47}{\chi^2}G_0(\chi)\Big)\zeta_3$$

$$- \frac{3}{16}\frac{4957\chi+4549}{\chi^2}\zeta_4 - \frac{45}{16}\frac{1767\chi+2216}{\chi^2}G_0(\chi) + \frac{15}{4}\frac{767\chi+904}{\chi^2}G_{0,0}(\chi) + 30\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi) - 3\frac{503\chi+584}{\chi^2}G_{0,0,0}(\chi)$$

$$- 6\frac{\chi+1}{\chi^2}G_{-1,-1,0,0}(\chi) - 6\frac{\chi+4}{\chi^2}G_{-1,0,0,0}(\chi) + 6\frac{\chi-2}{\chi^2}G_{0,-1,0,0}(\chi) + 6\frac{99\chi+104}{\chi^2}G_{0,0,0,0}(\chi)\Big)i + \epsilon^4\Big(-\frac{494637}{16}\frac{\chi+1}{\chi^2} + \Big(\frac{6309}{4}\frac{\chi+1}{\chi^2}$$

$$- 90\frac{\chi+1}{\chi^2}G_{-1}(\chi) - 9\frac{29\chi+40}{\chi^2}G_0(\chi) + 18\frac{\chi+1}{\chi^2}G_{-1,-1}(\chi) + 18\frac{1}{\chi^2}G_{-1,0}(\chi) - 18\frac{\chi-2}{\chi^2}G_{0,-1}(\chi) + 60\frac{2\chi-1}{\chi^2}G_{0,0}(\chi)\Big)\zeta_2 + \Big(6096\frac{\chi+1}{\chi^2}$$

$$+ 6\frac{\chi+1}{\chi^2}G_{-1}(\chi) - 2\frac{552\chi+331}{\chi^2}G_0(\chi)\Big)\zeta_3 + \frac{3}{4}\frac{1601\chi+1499}{\chi^2}\zeta_4 + \frac{45}{4}\frac{831\chi+796}{\chi^2}G_0(\chi) - 15\frac{347\chi+312}{\chi^2}G_{0,0}(\chi) - 30\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi)$$

$$+ 6\frac{427\chi+388}{\chi^2}G_{0,0,0}(\chi) + 6\frac{\chi+1}{\chi^2}G_{-1,-1,0,0}(\chi) + 6\frac{\chi+4}{\chi^2}G_{-1,0,0,0}(\chi) - 6\frac{\chi-2}{\chi^2}G_{0,-1,0,0}(\chi) - 6\frac{153\chi+112}{\chi^2}G_{0,0,0,0}(\chi)\Big)i$$

$$+ \epsilon^2\Big(\frac{40083}{32}\frac{\chi+1}{\chi^2} + \Big(-\frac{1017}{8}\frac{\chi+1}{\chi^2} - 90\frac{\chi+1}{\chi^2}G_{-1}(\chi) + \frac{63}{8}\frac{17\chi+7}{\chi^2}G_0(\chi) + 18\frac{\chi+1}{\chi^2}G_{-1,-1}(\chi) + 18\frac{1}{\chi^2}G_{-1,0}(\chi) - 18\frac{\chi-2}{\chi^2}G_{0,-1}(\chi)$$

$$- \frac{3}{2}\frac{\chi+49}{\chi^2}G_{0,0}(\chi)\Big)\zeta_2 + \Big(-\frac{363}{2}\frac{\chi+1}{\chi^2} + 6\frac{\chi+1}{\chi^2}G_{-1}(\chi) + \frac{66\chi+19}{\chi^2}G_0(\chi)\Big)\zeta_3 + \frac{51}{16}\frac{\chi-23}{\chi^2}\zeta_4 - \frac{45}{32}\frac{519\chi-35}{\chi^2}G_0(\chi)$$

$$+\frac{3}{8}\frac{907\chi-111}{\chi^2}G_{0,0}(\chi)-30\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi)-\frac{15}{2}\frac{13\chi-17}{\chi^2}G_{0,0,0}(\chi)+6\frac{\chi+1}{\chi^2}G_{-1,-1,0,0}(\chi)+6\frac{\chi+4}{\chi^2}G_{-1,0,0,0}(\chi)$$

$$-6\frac{\chi-2}{\chi^2}G_{0,-1,0,0}(\chi)-114\frac{1}{\chi^2}G_{0,0,0,0}(\chi)\Big)i+\Big(\frac{1}{\epsilon^4}\frac{\chi+1}{\chi^2}i+\Big(-\frac{\chi+1}{\chi^2}+\Big(7\frac{\chi+1}{\chi^2}+12\frac{\chi+1}{\chi^2}G_{-1}(\chi)-\frac{12\chi+1}{\chi^2}G_0(\chi)$$

$$+24\frac{\chi+1}{\chi^2}G_{-1,-1}(\chi)-2\frac{8\chi+7}{\chi^2}G_{-1,0}(\chi)-6\frac{4\chi+1}{\chi^2}G_{0,-1}(\chi)+\frac{16\chi-3}{\chi^2}G_{0,0}(\chi)\Big)\zeta_2+\Big(\frac{58}{3}\frac{\chi+1}{\chi^2}+8\frac{\chi+1}{\chi^2}G_{-1}(\chi)-\frac{4}{3}\frac{6\chi+1}{\chi^2}G_0(\chi)\Big)\zeta_3$$

$$+\frac{1}{4}\frac{193\chi+57}{\chi^2}\zeta_4-\frac{1}{\chi^2}G_0(\chi)-\frac{1}{\chi^2}G_{0,0}(\chi)+4\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi)-\frac{4\chi+1}{\chi^2}G_{0,0,0}(\chi)+8\frac{\chi+1}{\chi^2}G_{-1,-1,0,0}(\chi)-2\frac{4\chi+3}{\chi^2}G_{-1,0,0,0}(\chi)$$

$$-2\frac{4\chi+1}{\chi^2}G_{0,-1,0,0}(\chi)+\frac{8\chi+1}{\chi^2}G_{0,0,0,0}(\chi)\Big)i+\frac{1}{2}\frac{(\chi+1)(\pi^2+2)\pi}{\chi^2}+\Big(\Big(\frac{\chi+1}{\chi^2}-\frac{1}{\chi^2}G_0(\chi)\Big)i+\frac{(\chi+1)\pi}{\chi^2}\Big)\frac{1}{\epsilon^3}+\frac{4}{3}\frac{(\chi+1)\pi}{\chi^2}\zeta_3$$

$$+\frac{(\chi+1)\pi^3}{\chi^2}G_{-1}(\chi)-\frac{1}{6}\frac{(6\chi\pi^2-\pi^2-6)\pi}{\chi^2}G_0(\chi)+\Big(\Big(-\frac{\chi+1}{\chi^2}-3\frac{\chi+1}{\chi^2}\zeta_2-\frac{1}{\chi^2}G_0(\chi)+\frac{1}{\chi^2}G_{0,0}(\chi)\Big)i+\frac{(\chi+1)\pi}{\chi^2}-\frac{\pi}{\chi^2}G_0(\chi)\Big)\frac{1}{\epsilon^2}$$

$$+\frac{\pi}{\chi^2}G_{0,0}(\chi)+\Big(\Big(\frac{\chi+1}{\chi^2}+\Big(\frac{\chi+1}{\chi^2}+6\frac{\chi+1}{\chi^2}G_{-1}(\chi)-3\frac{2\chi-1}{\chi^2}G_0(\chi)\Big)\zeta_2+\frac{4}{3}\frac{\chi+1}{\chi^2}\zeta_3+\frac{1}{\chi^2}G_0(\chi)+\frac{1}{\chi^2}G_{0,0}(\chi)+2\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi)$$

$$-\frac{2\chi+1}{\chi^2}G_{0,0,0}(\chi)\Big)i-\frac{1}{6}\frac{(\chi+1)(\pi^2+6)\pi}{\chi^2}-\frac{\pi}{\chi^2}G_0(\chi)+\frac{\pi}{\chi^2}G_{0,0}(\chi)\Big)\frac{1}{\epsilon}+2\frac{(\chi+1)\pi}{\chi^2}G_{-1,0,0}(\chi)-\frac{(2\chi+1)\pi}{\chi^2}G_{0,0,0}(\chi)$$

$$+\Big(\Big(\frac{\chi+1}{\chi^2}+\Big(-7\frac{\chi+1}{\chi^2}-7\frac{1}{\chi^2}G_0(\chi)+48\frac{\chi+1}{\chi^2}G_{-1,-1}(\chi)-4\frac{8\chi+7}{\chi^2}G_{-1,0}(\chi)-12\frac{4\chi+1}{\chi^2}G_{0,-1}(\chi)+\frac{32\chi+1}{\chi^2}G_{0,0}(\chi)\Big)\zeta_2$$

$$+\Big(\frac{50}{3}\frac{\chi+1}{\chi^2}+16\frac{\chi+1}{\chi^2}G_{-1}(\chi)-\frac{2}{3}\frac{24\chi+29}{\chi^2}G_0(\chi)\Big)\zeta_3+\frac{1}{4}\frac{377\chi+105}{\chi^2}\zeta_4+\frac{1}{\chi^2}G_0(\chi)+\frac{1}{\chi^2}G_{0,0}(\chi)+\frac{1}{\chi^2}G_{0,0,0}(\chi)$$

$$+16\frac{\chi+1}{\chi^2}G_{-1,-1,0,0}(\chi)-4\frac{4\chi+3}{\chi^2}G_{-1,0,0,0}(\chi)-4\frac{4\chi+1}{\chi^2}G_{0,-1,0,0}(\chi)+\frac{16\chi+1}{\chi^2}G_{0,0,0,0}(\chi)\Big)i+\frac{1}{6}\frac{(\chi+1)(5\pi^2-6)\pi}{\chi^2}$$

$$+\frac{58}{3}\frac{(\chi+1)\pi}{\chi^2}\zeta_3+2\frac{(\chi+1)\pi^3}{\chi^2}G_{-1}(\chi)-\frac{1}{2}\frac{(4\chi\pi^2+\pi^2+2)\pi}{\chi^2}G_0(\chi)-\frac{\pi}{\chi^2}G_{0,0}(\chi)+4\frac{(\chi+1)\pi}{\chi^2}G_{-1,0,0}(\chi)-\frac{(4\chi+1)\pi}{\chi^2}G_{0,0,0}(\chi)\Big)\epsilon$$

$$+\Big(\Big(-1484\frac{\chi+1}{\chi^2}+\Big(600\frac{\chi+1}{\chi^2}-140\frac{1}{\chi^2}G_0(\chi)+8\frac{1}{\chi^2}G_{0,0}(\chi)\Big)\zeta_2+\Big(\frac{748}{3}\frac{\chi+1}{\chi^2}-4\frac{1}{\chi^2}G_0(\chi)\Big)\zeta_3-18\frac{\chi+1}{\chi^2}\zeta_4+412\frac{1}{\chi^2}G_0(\chi)$$

$$-100\frac{1}{\chi^2}G_{0,0}(\chi)+20\frac{1}{\chi^2}G_{0,0,0}(\chi)\Big)i+\frac{2}{3}\frac{(\chi+1)(22\pi^2-663)\pi}{\chi^2}+\frac{28}{3}\frac{(\chi+1)\pi}{\chi^2}\zeta_3-\frac{2}{3}\frac{(\pi^2-171)\pi}{\chi^2}G_0(\chi)-26\frac{\pi}{\chi^2}G_{0,0}(\chi)$$

$$+2\frac{\pi}{\chi^2}G_{0,0,0}(\chi)\Big)\epsilon^3+\Big(\Big(1548\frac{\chi+1}{\chi^2}+\Big(-712\frac{\chi+1}{\chi^2}+196\frac{1}{\chi^2}G_0(\chi)-36\frac{1}{\chi^2}G_{0,0}(\chi)\Big)\zeta_2+\Big(-\frac{1148}{3}\frac{\chi+1}{\chi^2}+\frac{212}{3}\frac{1}{\chi^2}G_0(\chi)\Big)\zeta_3$$

$$+27\frac{\chi+1}{\chi^2}\zeta_4-444\frac{1}{\chi^2}G_0(\chi)+116\frac{1}{\chi^2}G_{0,0}(\chi)-28\frac{1}{\chi^2}G_{0,0,0}(\chi)+4\frac{1}{\chi^2}G_{0,0,0,0}(\chi)\Big)i-\frac{2}{3}\frac{(\chi+1)(32\pi^2-711)\pi}{\chi^2}-76\frac{(\chi+1)\pi}{\chi^2}\zeta_3$$

$$+2\frac{(2\pi^2-65)\pi}{\chi^2}G_0(\chi)+34\frac{\pi}{\chi^2}G_{0,0}(\chi)-6\frac{\pi}{\chi^2}G_{0,0,0}(\chi)\Big)\epsilon^2+\Big(\Big(-4584\frac{\chi+1}{\chi^2}+\Big(2052\frac{\chi+1}{\chi^2}-504\frac{1}{\chi^2}G_0(\chi)+108\frac{1}{\chi^2}G_{0,0}(\chi)\Big)\zeta_2$$

$$+\Big(948\frac{\chi+1}{\chi^2}-212\frac{1}{\chi^2}G_0(\chi)\Big)\zeta_3-81\frac{\chi+1}{\chi^2}\zeta_4+1296\frac{1}{\chi^2}G_0(\chi)-336\frac{1}{\chi^2}G_{0,0}(\chi)+72\frac{1}{\chi^2}G_{0,0,0}(\chi)-12\frac{1}{\chi^2}G_{0,0,0,0}(\chi)\Big)i$$

$$+18\frac{(\chi+1)(3\pi^2-77)\pi}{\chi^2}+228\frac{(\chi+1)\pi}{\chi^2}\zeta_3-6\frac{(2\pi^2-63)\pi}{\chi^2}G_0(\chi)-90\frac{\pi}{\chi^2}G_{0,0}(\chi)+18\frac{\pi}{\chi^2}G_{0,0,0}(\chi)\Big)\epsilon^4\Big)\mu^{-\epsilon}+\Big(-\frac{1}{\epsilon^4}\frac{1}{4}\frac{\chi+1}{\chi^2}i$$

$$+\frac{1}{\epsilon}\Big(\Big(\Big(\frac{7}{4}\frac{\chi+1}{\chi^2}-\frac{3}{2}\frac{1}{\chi^2}G_0(\chi)\Big)\zeta_2-\frac{11}{6}\frac{\chi+1}{\chi^2}\zeta_3\Big)i+\Big(\Big(-\frac{7}{2}\frac{\chi+1}{\chi^2}-3\frac{\chi+1}{\chi^2}G_0(2)-\frac{3}{2}\frac{1}{\chi^2}G_0(\chi)\Big)\zeta_2+\Big(-\frac{139}{12}\frac{\chi+1}{\chi^2}+\frac{4}{3}\frac{1}{\chi^2}G_0(\chi)\Big)\zeta_3$$

$$-\frac{145}{16}\frac{\chi+1}{\chi^2}\zeta_4\Big)i+\Big(\epsilon^3\Big(-903\frac{1}{\chi}-375\frac{1}{\chi}\zeta_2-132\frac{1}{\chi}\zeta_3-\frac{909}{4}\frac{1}{\chi}\zeta_4\Big)i+\epsilon^4\Big(-\frac{118461}{32}\frac{1}{\chi}-\frac{11625}{8}\frac{1}{\chi}\zeta_2-\frac{979}{2}\frac{1}{\chi}\zeta_3-\frac{6363}{8}\frac{1}{\chi}\zeta_4\Big)i$$

$$+\Big(-8\frac{Li_4(\frac{1}{2})}{\chi}+16\frac{1}{\chi}G_{0,0}(2)\zeta_2-42\frac{1}{\chi}\zeta_4-8\frac{1}{\chi}G_{0,0,0,0}(2)\Big)i+\Big(\Big(-6\frac{1}{\chi}G_0(2)\zeta_2+\frac{1}{2}\frac{1}{\chi}\zeta_3\Big)i+\frac{1}{3}\frac{\pi^3}{\chi}\Big)\frac{1}{\epsilon}+\frac{19}{2}\frac{\pi}{\chi}\zeta_3+\Big(\Big(-\frac{37725}{64}\frac{1}{\chi}$$

$$-\frac{273}{16}\frac{1}{\chi}\zeta_2+\frac{269}{4}\frac{1}{\chi}\zeta_3+\frac{2175}{16}\frac{1}{\chi}\zeta_4\Big)i+2\frac{(\pi^2-30)\pi}{\chi}+\frac{32}{3}\frac{\pi}{\chi}\zeta_3\Big)\epsilon^2-3\frac{\pi^3}{\chi}G_0(2)+\Big(\Big(\frac{3}{2}\frac{64Li_4(\frac{1}{2})+63}{\chi}+\Big(-\frac{63}{2}\frac{1}{\chi}-48\frac{1}{\chi}G_{0,0}(2)\Big)\zeta_2$$

$$-19\frac{1}{\chi}\zeta_3-\frac{177}{8}\frac{1}{\chi}\zeta_4+96\frac{1}{\chi}G_{0,0,0,0}(2)\Big)i-\frac{8}{3}\frac{(\pi^2-12)\pi}{\chi}-\frac{89}{3}\frac{\pi}{\chi}\zeta_3+6\frac{\pi^3}{\chi}G_0(2)\Big)\epsilon\Big)\mu^{-1}+\Big(\Big(-\frac{1}{4}\frac{\chi+1}{\chi^2}-\frac{1}{2}\frac{1}{\chi^2}G_0(\chi)\Big)i-\frac{(\chi+1)\pi}{\chi^2}\Big)\frac{1}{\epsilon^3}$$

$$+\Big(\Big(-\frac{1}{2}\frac{\chi+1}{\chi^2}+\frac{11}{4}\frac{\chi+1}{\chi^2}\zeta_2-\frac{1}{2}\frac{1}{\chi^2}G_0(\chi)\Big)i-\frac{(\chi+1)\pi}{\chi^2}\Big)\frac{1}{\epsilon^2}-\frac{4}{3}\frac{(\chi+1)\pi}{\chi^2}\zeta_3+\Big(\Big(-\frac{7}{2}\frac{9-18\chi+8Li_4(\frac{1}{2})+8\chi Li_4(\frac{1}{2})}{\chi^2}+\Big(-\frac{1}{2}\frac{94\chi+49}{\chi^2}$$

$$+20\frac{\chi+1}{\chi^2}G_{0,0}(2)\Big)\zeta_2+\Big(-\frac{28}{3}\frac{4\chi+1}{\chi^2}+\frac{4}{3}\frac{1}{\chi^2}G_0(\chi)\Big)\zeta_3-\frac{1}{16}\frac{613\chi+1027}{\chi^2}\zeta_4-28\frac{\chi+1}{\chi^2}G_{0,0,0,0}(2)\Big)i-\frac{1}{6}\frac{(9\chi\pi^2-186\chi+2\pi^2)\pi}{\chi^2}$$

$$-\frac{1}{6}\frac{(47\chi-1)\pi}{\chi^2}\zeta_3-3\frac{(\chi+1)\pi^3}{\chi^2}G_0(2)\Big)\epsilon+\Big(\Big(\frac{1}{64}\frac{-33725-1952\chi+3072Li_4(\frac{1}{2})+3072\chi Li_4(\frac{1}{2})}{\chi^2}+\Big(\frac{1}{16}\frac{1000\chi+391}{\chi^2}$$

$$-24\frac{\chi+1}{\chi^2}G_{0,0}(2)\Big)\zeta_2+\frac{1}{12}\frac{128\chi+647}{\chi^2}\zeta_3+\frac{3}{16}\frac{310\chi+1127}{\chi^2}\zeta_4+48\frac{\chi+1}{\chi^2}G_{0,0,0,0}(2)\Big)i-\frac{1}{3}\frac{(2\chi\pi^2+90\chi-\pi^2+180)\pi}{\chi^2}$$

$$-\frac{1}{6}\frac{(121\chi+89)\pi}{\chi^2}\zeta_3+3\frac{(\chi+1)\pi^3}{\chi^2}G_0(2)\Big)\epsilon^3+\Big(\Big(\frac{94+\chi+48Li_4(\frac{1}{2})+48\chi Li_4(\frac{1}{2})}{\chi^2}+\Big(-5\frac{\chi+7}{\chi^2}-24\frac{\chi+1}{\chi^2}G_{0,0}(2)\Big)\zeta_2+\frac{1}{3}\frac{4\chi-113}{\chi^2}\zeta_3$$

$$+\frac{3}{8}\frac{77\chi-157}{\chi^2}\zeta_4+48\frac{\chi+1}{\chi^2}G_{0,0,0,0}(2)\Big)i-\frac{1}{6}\frac{(11\chi\pi^2-6\chi+18\pi^2-192)\pi}{\chi^2}-\frac{1}{6}\frac{(169\chi+217)\pi}{\chi^2}\zeta_3+3\frac{(\chi+1)\pi^3}{\chi^2}G_0(2)\Big)\epsilon^2$$

$$+\Big(\Big(\frac{-934-31\chi+48Li_4(\frac{1}{2})+48\chi Li_4(\frac{1}{2})}{\chi^2}+\Big(-\frac{23\chi+398}{\chi^2}-24\frac{\chi+1}{\chi^2}G_{0,0}(2)\Big)\zeta_2+12\frac{\chi-10}{\chi^2}\zeta_3+\frac{9}{4}\frac{9\chi-92}{\chi^2}\zeta_4+48\frac{\chi+1}{\chi^2}G_{0,0,0,0}(2)\Big)i$$

$$
\begin{aligned}
&- \frac{5}{3}\frac{(\chi+1)\pi^3}{\chi^2} - \frac{51}{2}\frac{(\chi+1)\pi}{\chi^2}\zeta_3 + 3\frac{(\chi+1)\pi^3}{\chi^2}G_0(2)\Bigg)\epsilon^4 + \Bigg(\frac{1}{\epsilon^3}\frac{1}{2}\frac{\chi+1}{\chi^2}i + \frac{1}{\epsilon^2}\frac{1}{2}\frac{\chi+1}{\chi^2}i + \frac{1}{\epsilon}\frac{3}{2}\frac{\chi+1}{\chi^2}\zeta_2 i + \Bigg(\frac{3}{2}\frac{\chi+1}{\chi^2}\zeta_2 - \frac{4}{3}\frac{\chi+1}{\chi^2}\zeta_3\Bigg)i \\
&+ \epsilon^3\Bigg(-\frac{1}{2}\frac{\chi+1}{\chi^2} - \frac{3}{2}\frac{\chi+1}{\chi^2}\zeta_2 + \frac{4}{3}\frac{\chi+1}{\chi^2}\zeta_3 - \frac{63}{8}\frac{\chi+1}{\chi^2}\zeta_4\Bigg)i + \epsilon\Bigg(-\frac{4}{3}\frac{\chi+1}{\chi^2}\zeta_3 + \frac{63}{8}\frac{\chi+1}{\chi^2}\zeta_4\Bigg)i + \epsilon^2\Bigg(-\frac{63}{2}\frac{\chi+1}{\chi^2} - \frac{45}{2}\frac{\chi+1}{\chi^2}\zeta_2 + \frac{28}{3}\frac{\chi+1}{\chi^2}\zeta_3 \\
&- \frac{63}{4}\frac{\chi+1}{\chi^2}\zeta_4\Bigg)i + \epsilon^4\Bigg(31\frac{\chi+1}{\chi^2} + 21\frac{\chi+1}{\chi^2}\zeta_2 - 8\frac{\chi+1}{\chi^2}\zeta_3 + \frac{63}{4}\frac{\chi+1}{\chi^2}\zeta_4\Bigg)i\Bigg)\log(\mu)\Bigg)\mu^{-2\epsilon} + O(\mu)
\end{aligned} \tag{D.13}
$$

I[FPM2, 1,1, 1, 1, 1, 0, 1, 1, 0] =

$$
\begin{aligned}
&- \frac{1}{\epsilon^4}\frac{3}{4}\frac{\chi+1}{\chi^2}i + \frac{1}{\epsilon^3}\Bigg(-\frac{3}{4}\frac{\chi+1}{\chi^2} + \frac{3}{2}\frac{1}{\chi^2}G_0(\chi)\Bigg)i + \frac{1}{\epsilon^2}\Bigg(\frac{3}{2}\frac{\chi+1}{\chi^2} - \frac{3}{4}\frac{\chi+1}{\chi^2}\zeta_2 + \frac{3}{2}\frac{1}{\chi^2}G_0(\chi) - 3\frac{1}{\chi^2}G_{0,0}(\chi)\Bigg)i + \frac{1}{\epsilon}\Bigg(-3\frac{\chi+1}{\chi^2} \\
&+ \Bigg(-\frac{15}{4}\frac{\chi+1}{\chi^2} - 9\frac{\chi+1}{\chi^2}G_{-1}(\chi) + \frac{3}{2}\frac{6\chi+1}{\chi^2}G_0(\chi)\Bigg)\zeta_2 + \frac{7}{2}\frac{\chi+1}{\chi^2}\zeta_3 - 3\frac{1}{\chi^2}G_0(\chi) - 3\frac{1}{\chi^2}G_{0,0}(\chi) - 3\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi) \\
&+ 3\frac{\chi+2}{\chi^2}G_{0,0,0}(\chi)\Bigg)i + \Bigg(6\frac{\chi+1}{\chi^2} + \Bigg(\frac{3}{2}\frac{\chi+1}{\chi^2} - 18\frac{\chi+1}{\chi^2}G_{-1}(\chi) + \frac{3}{2}\frac{12\chi+5}{\chi^2}G_0(\chi) - 27\frac{\chi+1}{\chi^2}G_{-1,-1}(\chi) + 3\frac{8\chi+7}{\chi^2}G_{-1,0}(\chi) \\
&+ 9\frac{3\chi+2}{\chi^2}G_{0,-1}(\chi) - 3\frac{8\chi+1}{\chi^2}G_{0,0}(\chi)\Bigg)\zeta_2 + \Bigg(-\frac{5}{2}\frac{\chi+1}{\chi^2} - 9\frac{\chi+1}{\chi^2}G_{-1}(\chi) + \frac{9\chi-7}{\chi^2}G_0(\chi)\Bigg)\zeta_3 - \frac{3}{16}\frac{47\chi-21}{\chi^2}\zeta_4 + 6\frac{1}{\chi^2}G_0(\chi) \\
&+ 6\frac{1}{\chi^2}G_{0,0}(\chi) - 6\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi) + 6\frac{\chi+1}{\chi^2}G_{0,0,0}(\chi) - 9\frac{\chi+1}{\chi^2}G_{-1,-1,0,0}(\chi) + 3\frac{5\chi+4}{\chi^2}G_{-1,0,0,0}(\chi) + 3\frac{3\chi+2}{\chi^2}G_{0,-1,0,0}(\chi) \\
&- 3\frac{5\chi+4}{\chi^2}G_{0,0,0,0}(\chi)\Bigg)i + \epsilon\Bigg(-\frac{144123}{64}\frac{\chi+1}{\chi^2} + \Bigg(\frac{711}{16}\frac{\chi+1}{\chi^2} - 18\frac{\chi+1}{\chi^2}G_{-1}(\chi) + \frac{9}{8}\frac{3\chi+23}{\chi^2}G_0(\chi) - 72\frac{\chi+1}{\chi^2}G_{-1,-1}(\chi) \\
&+ 72\frac{\chi+1}{\chi^2}G_{-1,0}(\chi) + 72\frac{\chi+1}{\chi^2}G_{0,-1}(\chi) - \frac{123}{2}\frac{\chi+1}{\chi^2}G_{0,0}(\chi)\Bigg)\zeta_2 + \Bigg(\frac{897}{2}\frac{\chi+1}{\chi^2} - 24\frac{\chi+1}{\chi^2}G_{-1}(\chi) - 29\frac{3\chi+1}{\chi^2}G_0(\chi)\Bigg)\zeta_3 \\
&+ \frac{1257}{16}\frac{\chi+1}{\chi^2}\zeta_4 + \frac{9}{32}\frac{2945\chi+1777}{\chi^2}G_0(\chi) - \frac{3}{8}\frac{1321\chi+833}{\chi^2}G_{0,0}(\chi) - 6\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi) + \frac{3}{2}\frac{179\chi+107}{\chi^2}G_{0,0,0}(\chi) \\
&- 24\frac{\chi+1}{\chi^2}G_{-1,-1,0,0}(\chi) + 48\frac{\chi+1}{\chi^2}G_{-1,0,0,0}(\chi) + 24\frac{\chi+1}{\chi^2}G_{0,-1,0,0}(\chi) - 6\frac{27\chi+19}{\chi^2}G_{0,0,0,0}(\chi)\Bigg)i + \epsilon^3\Bigg(\frac{1192665}{64}\frac{\chi+1}{\chi^2} \\
&+ \Bigg(-\frac{17001}{16}\frac{\chi+1}{\chi^2} + 90\frac{\chi+1}{\chi^2}G_{-1}(\chi) - \frac{9}{4}\frac{\chi-176}{\chi^2}G_0(\chi) - 18\frac{\chi+1}{\chi^2}G_{-1,-1}(\chi) - 18\frac{1}{\chi^2}G_{-1,0}(\chi) + 18\frac{\chi-2}{\chi^2}G_{0,-1}(\chi) \\
&- 3\frac{13\chi+16}{\chi^2}G_{0,0}(\chi)\Bigg)\zeta_2 + \Bigg(-4014\frac{\chi+1}{\chi^2} - 6\frac{\chi+1}{\chi^2}G_{-1}(\chi) + 14\frac{48\chi+47}{\chi^2}G_0(\chi)\Bigg)\zeta_3 - \frac{3}{16}\frac{4957\chi+4549}{\chi^2}\zeta_4 - \frac{45}{16}\frac{1767\chi+2216}{\chi^2}G_0(\chi) \\
&+ \frac{15}{4}\frac{767\chi+904}{\chi^2}G_{0,0}(\chi) + 30\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi) - 3\frac{503\chi+584}{\chi^2}G_{0,0,0}(\chi) - 6\frac{\chi+1}{\chi^2}G_{-1,-1,0,0}(\chi) - 6\frac{\chi+4}{\chi^2}G_{-1,0,0,0}(\chi) \\
&+ 6\frac{\chi-2}{\chi^2}G_{0,-1,0,0}(\chi) + 6\frac{99\chi+104}{\chi^2}G_{0,0,0,0}(\chi)\Bigg)i + \epsilon^4\Bigg(-\frac{494637}{16}\frac{\chi+1}{\chi^2} + \Bigg(\frac{6309}{4}\frac{\chi+1}{\chi^2} - 90\frac{\chi+1}{\chi^2}G_{-1}(\chi) - 9\frac{29\chi+40}{\chi^2}G_0(\chi) \\
&+ 18\frac{\chi+1}{\chi^2}G_{-1,-1}(\chi) + 18\frac{1}{\chi^2}G_{-1,0}(\chi) - 18\frac{\chi-2}{\chi^2}G_{0,-1}(\chi) + 60\frac{2\chi-1}{\chi^2}G_{0,0}(\chi)\Bigg)\zeta_2 + \Bigg(6096\frac{\chi+1}{\chi^2} + 6\frac{\chi+1}{\chi^2}G_{-1}(\chi) \\
&- 2\frac{552\chi+331}{\chi^2}G_0(\chi)\Bigg)\zeta_3 + \frac{3}{4}\frac{1601\chi+1499}{\chi^2}\zeta_4 + \frac{45}{4}\frac{831\chi+796}{\chi^2}G_0(\chi) - 15\frac{347\chi+312}{\chi^2}G_{0,0}(\chi) - 30\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi) \\
&+ 6\frac{427\chi+388}{\chi^2}G_{0,0,0}(\chi) + 6\frac{\chi+1}{\chi^2}G_{-1,-1,0,0}(\chi) + 6\frac{\chi+4}{\chi^2}G_{-1,0,0,0}(\chi) - 6\frac{\chi-2}{\chi^2}G_{0,-1,0,0}(\chi) - 6\frac{153\chi+112}{\chi^2}G_{0,0,0,0}(\chi)\Bigg)i \\
&+ \epsilon^2\Bigg(\frac{40083}{32}\frac{\chi+1}{\chi^2} + \Bigg(-\frac{1017}{8}\frac{\chi+1}{\chi^2} - 90\frac{\chi+1}{\chi^2}G_{-1}(\chi) + \frac{63}{8}\frac{17\chi+7}{\chi^2}G_0(\chi) + 18\frac{\chi+1}{\chi^2}G_{-1,-1}(\chi) + 18\frac{1}{\chi^2}G_{-1,0}(\chi) - 18\frac{\chi-2}{\chi^2}G_{0,-1}(\chi) \\
&- \frac{3}{2}\frac{\chi+49}{\chi^2}G_{0,0}(\chi)\Bigg)\zeta_2 + \Bigg(-\frac{363}{2}\frac{\chi+1}{\chi^2} + 6\frac{\chi+1}{\chi^2}G_{-1}(\chi) + \frac{66\chi+19}{\chi^2}G_0(\chi)\Bigg)\zeta_3 + \frac{51}{16}\frac{\chi-23}{\chi^2}\zeta_4 - \frac{45}{32}\frac{519\chi-35}{\chi^2}G_0(\chi) \\
&+ \frac{3}{8}\frac{907\chi-111}{\chi^2}G_{0,0}(\chi) - 30\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi) - \frac{15}{2}\frac{13\chi-17}{\chi^2}G_{0,0,0}(\chi) + 6\frac{\chi+1}{\chi^2}G_{-1,-1,0,0}(\chi) + 6\frac{\chi+4}{\chi^2}G_{-1,0,0,0}(\chi) \\
&- 6\frac{\chi-2}{\chi^2}G_{0,-1,0,0}(\chi) - 114\frac{1}{\chi^2}G_{0,0,0,0}(\chi)\Bigg)i + \Bigg(\frac{1}{\epsilon^4}2\frac{\chi+1}{\chi^2}i + \frac{1}{\epsilon^3}\Bigg(2\frac{\chi+1}{\chi^2} - 2\frac{1}{\chi^2}G_0(\chi)\Bigg)i + \frac{1}{\epsilon^2}\Bigg(-2\frac{\chi+1}{\chi^2} + \frac{4}{3}\frac{\chi+1}{\chi^2}\zeta_2 - 2\frac{1}{\chi^2}G_0(\chi) \\
&+ 2\frac{1}{\chi^2}G_{0,0}(\chi)\Bigg)i + \frac{1}{\epsilon}\Bigg(-\frac{2}{9}\frac{(\chi+1)(2\mathrm{K}2\pi-9)}{\chi^2} + \Bigg(\frac{16}{3}\frac{\chi+1}{\chi^2} + 6\frac{\chi+1}{\chi^2}G_{-1}(\chi) - \frac{2}{3}\frac{9\chi+2}{\chi^2}G_0(\chi)\Bigg)\zeta_2 - \frac{10}{9}\frac{\chi+1}{\chi^2}\zeta_3 + 2\frac{1}{\chi^2}G_0(\chi) \\
&+ 2\frac{1}{\chi^2}G_{0,0}(\chi) + 2\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi) - 2\frac{\chi+1}{\chi^2}G_{0,0,0}(\chi)\Bigg)i + \epsilon^4\Bigg(-\frac{1}{9}\frac{(\chi+1)(-819+82\mathrm{K}2^2-20\mathrm{K}2\pi-48\mathrm{K}31\pi)}{\chi^2} + \Bigg(\frac{8}{3}\frac{\chi+1}{\chi^2} \\
&+ \frac{8}{3}\frac{1}{\chi^2}G_0(\chi) - \frac{4}{3}\frac{\chi+1}{\chi^2}G_{0,0}(3) + \frac{8}{3}\frac{1}{\chi^2}G_{0,0}(\chi)\Bigg)\zeta_2 + \Bigg(-\frac{352}{9}\frac{\chi+1}{\chi^2} - \frac{64}{9}\frac{1}{\chi^2}G_0(\chi)\Bigg)\zeta_3 - \frac{181}{36}\frac{\chi+1}{\chi^2}\zeta_4 + \frac{20}{9}\frac{(\chi+1)\mathrm{K}2\pi}{\chi^2}G_0(3) \\
&+ \frac{1}{9}\frac{20\mathrm{K}2\pi-423}{\chi^2}G_0(\chi) + 19\frac{1}{\chi^2}G_{0,0}(\chi) - 11\frac{1}{\chi^2}G_{0,0,0}(\chi) + \frac{1}{\chi^2}G_{0,0,0,0}(\chi)\Bigg)i + \epsilon^2\Bigg(-\frac{1}{9}\frac{(\chi+1)(297+82\mathrm{K}2^2-20\mathrm{K}2\pi-48\mathrm{K}31\pi)}{\chi^2} \\
&+ \Bigg(\frac{8}{3}\frac{\chi+1}{\chi^2} + \frac{8}{3}\frac{1}{\chi^2}G_0(\chi) - \frac{4}{3}\frac{\chi+1}{\chi^2}G_{0,0}(3) + \frac{8}{3}\frac{1}{\chi^2}G_{0,0}(\chi)\Bigg)\zeta_2 + \Bigg(-\frac{64}{9}\frac{\chi+1}{\chi^2} - \frac{160}{9}\frac{1}{\chi^2}G_0(\chi)\Bigg)\zeta_3 + \frac{251}{36}\frac{\chi+1}{\chi^2}\zeta_4 + \frac{20}{9}\frac{(\chi+1)\mathrm{K}2\pi}{\chi^2}G_0(3) \\
&+ \frac{1}{9}\frac{20\mathrm{K}2\pi+117}{\chi^2}G_0(\chi) - 9\frac{1}{\chi^2}G_{0,0}(\chi) + \frac{1}{\chi^2}G_{0,0,0}(\chi) - 3\frac{1}{\chi^2}G_{0,0,0,0}(\chi)\Bigg)i + \epsilon^3\Bigg(\frac{1}{9}\frac{(\chi+1)(855+82\mathrm{K}2^2-20\mathrm{K}2\pi-48\mathrm{K}31\pi)}{\chi^2} \\
&+ \Bigg(-\frac{8}{3}\frac{\chi+1}{\chi^2} - \frac{8}{3}\frac{1}{\chi^2}G_0(\chi) + \frac{4}{3}\frac{\chi+1}{\chi^2}G_{0,0}(3) - \frac{8}{3}\frac{1}{\chi^2}G_{0,0}(\chi)\Bigg)\zeta_2 + \Bigg(-\frac{80}{9}\frac{\chi+1}{\chi^2} + \frac{208}{9}\frac{1}{\chi^2}G_0(\chi)\Bigg)\zeta_3 - \frac{467}{36}\frac{\chi+1}{\chi^2}\zeta_4 \\
&- \frac{20}{9}\frac{(\chi+1)\mathrm{K}2\pi}{\chi^2}G_0(3) - \frac{1}{9}\frac{20\mathrm{K}2\pi+387}{\chi^2}G_0(\chi) + 23\frac{1}{\chi^2}G_{0,0}(\chi) - 7\frac{1}{\chi^2}G_{0,0,0}(\chi) + 5\frac{1}{\chi^2}G_{0,0,0,0}(\chi)\Bigg)i
\end{aligned}
$$

$$+ \Bigg(-\frac{2}{45}\,\frac{(\chi+1)(45+65\mathrm{K2}^2+70\mathrm{K2}\pi-24\mathrm{K31}\pi)}{\chi^2} + \Bigg(\frac{8}{3}\,\frac{\chi+1}{\chi^2} + 12\,\frac{\chi+1}{\chi^2}G_{-1}(\chi) - \frac{4}{3}\,\frac{9\chi+4}{\chi^2}G_0(\chi) + 24\,\frac{\chi+1}{\chi^2}G_{-1,-1}(\chi)$$

$$- \frac{2}{3}\,\frac{16\chi+13}{\chi^2}G_{-1,0}(\chi) - \frac{16}{3}\,\frac{\chi+1}{\chi^2}G_{-1,\mathrm{r62}}(\chi) - \frac{16}{3}\,\frac{\chi+1}{\chi^2}G_{-1,\mathrm{r64}}(\chi) - 6\,\frac{4\chi+1}{\chi^2}G_{0,-1}(\chi) - \frac{4}{15}\,\frac{\chi+1}{\chi^2}G_{0,0}(3) + \frac{4}{3}\,\frac{8\chi+1}{\chi^2}G_{0,0}(\chi)$$

$$+ \frac{16}{3}\,\frac{1}{\chi}G_{0,\mathrm{r62}}(\chi) + \frac{16}{3}\,\frac{1}{\chi}G_{0,\mathrm{r64}}(\chi)\Bigg)\zeta_2 + \Bigg(\frac{116}{9}\,\frac{\chi+1}{\chi^2} + \frac{20}{3}\,\frac{\chi+1}{\chi^2}G_{-1}(\chi) - \frac{10}{9}\,\frac{6\chi-1}{\chi^2}G_0(\chi)\Bigg)\zeta_3 + \frac{1}{108}\,\frac{1321\chi-127}{\chi^2}\zeta_4$$

$$- \frac{4}{3}\,\frac{(\chi+1)\mathrm{K2}\pi}{\chi^2}G_{-1}(\chi) + \frac{4}{9}\,\frac{(\chi+1)\mathrm{K2}\pi}{\chi^2}G_0(3) + \frac{2}{9}\,\frac{-9+2\mathrm{K2}\pi+6\chi\mathrm{K2}\pi}{\chi^2}G_0(\chi) - 2\,\frac{1}{\chi^2}G_{0,0}(\chi) + 4\,\frac{\chi+1}{\chi^2}G_{-1,0,0}(\chi) - 2\,\frac{2\chi+1}{\chi^2}G_{0,0,0}(\chi)$$

$$+ 8\,\frac{\chi+1}{\chi^2}G_{-1,-1,0,0}(\chi) - 2\,\frac{3\chi+2}{\chi^2}G_{-1,0,0,0}(\chi) - 2\,\frac{\chi+1}{\chi^2}G_{-1,\mathrm{r62},0,0}(\chi) - 2\,\frac{\chi+1}{\chi^2}G_{-1,\mathrm{r64},0,0}(\chi) - 2\,\frac{4\chi+1}{\chi^2}G_{0,-1,0,0}(\chi)$$

$$+ 2\,\frac{3\chi+1}{\chi^2}G_{0,0,0,0}(\chi) + 2\,\frac{1}{\chi}G_{0,\mathrm{r62},0,0}(\chi) + 2\,\frac{1}{\chi}G_{0,\mathrm{r64},0,0}(\chi)\Bigg)i - \frac{2}{3}\,\frac{(\chi+1)\pi}{\chi^2}G_{-1,\mathrm{r62},0}(\chi) + \frac{2}{3}\,\frac{(\chi+1)\pi}{\chi^2}G_{-1,\mathrm{r64},0}(\chi) + \frac{2}{3}\,\frac{\pi}{\chi}G_{0,\mathrm{r62},0}(\chi)$$

$$- \frac{2}{3}\,\frac{\pi}{\chi}G_{0,\mathrm{r64},0}(\chi) + \Bigg(\Bigg(-\frac{2}{45}\,\frac{(\chi+1)(-45+335\mathrm{K2}^2+50\mathrm{K2}\pi-168\mathrm{K31}\pi)}{\chi^2} + \Bigg(-\frac{8}{3}\,\frac{\chi+1}{\chi^2} - \frac{8}{3}\,\frac{1}{\chi^2}G_0(\chi) + 48\,\frac{\chi+1}{\chi^2}G_{-1,-1}(\chi)$$

$$- \frac{4}{3}\,\frac{16\chi+13}{\chi^2}G_{-1,0}(\chi) - \frac{32}{3}\,\frac{\chi+1}{\chi^2}G_{-1,\mathrm{r62}}(\chi) - \frac{32}{3}\,\frac{\chi+1}{\chi^2}G_{-1,\mathrm{r64}}(\chi) - 12\,\frac{4\chi+1}{\chi^2}G_{0,-1}(\chi) - \frac{28}{15}\,\frac{\chi+1}{\chi^2}G_{0,0}(3) + \frac{16}{3}\,\frac{4\chi+1}{\chi^2}G_{0,0}(\chi)$$

$$+ \frac{32}{3}\,\frac{1}{\chi}G_{0,\mathrm{r62}}(\chi) + \frac{32}{3}\,\frac{1}{\chi}G_{0,\mathrm{r64}}(\chi)\Bigg)\zeta_2 + \Bigg(\frac{136}{9}\,\frac{\chi+1}{\chi^2} + \frac{40}{3}\,\frac{\chi+1}{\chi^2}G_{-1}(\chi) - \frac{4}{9}\,\frac{30\chi+29}{\chi^2}G_0(\chi)\Bigg)\zeta_3 + \frac{1}{108}\,\frac{3071\chi+175}{\chi^2}\zeta_4$$

$$- \frac{8}{3}\,\frac{(\chi+1)\mathrm{K2}\pi}{\chi^2}G_{-1}(\chi) + \frac{28}{9}\,\frac{(\chi+1)\mathrm{K2}\pi}{\chi^2}G_0(3) + \frac{2}{9}\,\frac{9+14\mathrm{K2}\pi+12\chi\mathrm{K2}\pi}{\chi^2}G_0(\chi) + 2\,\frac{1}{\chi^2}G_{0,0}(\chi) + 2\,\frac{1}{\chi^2}G_{0,0,0}(\chi)$$

$$+ 16\,\frac{\chi+1}{\chi^2}G_{-1,-1,0,0}(\chi) - 4\,\frac{3\chi+2}{\chi^2}G_{-1,0,0,0}(\chi) - 4\,\frac{\chi+1}{\chi^2}G_{-1,\mathrm{r62},0,0}(\chi) - 4\,\frac{\chi+1}{\chi^2}G_{-1,\mathrm{r64},0,0}(\chi) - 4\,\frac{4\chi+1}{\chi^2}G_{0,-1,0,0}(\chi)$$

$$+ 2\,\frac{6\chi+1}{\chi^2}G_{0,0,0,0}(\chi) + 4\,\frac{1}{\chi}G_{0,\mathrm{r62},0,0}(\chi) + 4\,\frac{1}{\chi}G_{0,\mathrm{r64},0,0}(\chi)\Bigg)i - \frac{4}{3}\,\frac{(\chi+1)\pi}{\chi^2}G_{-1,\mathrm{r62},0}(\chi) + \frac{4}{3}\,\frac{(\chi+1)\pi}{\chi^2}G_{-1,\mathrm{r64},0}(\chi) + \frac{4}{3}\,\frac{\pi}{\chi}G_{0,\mathrm{r62},0}(\chi)$$

$$- \frac{4}{3}\,\frac{\pi}{\chi}G_{0,\mathrm{r64},0}(\chi)\Bigg)\epsilon + \Bigg(\epsilon^3\Bigg(-62\,\frac{\chi+1}{\chi^2} + \Bigg(16\,\frac{\chi+1}{\chi^2} - \frac{16}{3}\,\frac{1}{\chi^2}G_0(\chi)\Bigg)\zeta_3 + 6\,\frac{\chi+1}{\chi^2}\zeta_4 + 30\,\frac{1}{\chi^2}G_0(\chi) - 14\,\frac{1}{\chi^2}G_{0,0}(\chi) + 6\,\frac{1}{\chi^2}G_{0,0,0}(\chi)$$

$$- 2\,\frac{1}{\chi^2}G_{0,0,0,0}(\chi)\Bigg)i + \epsilon^4\Bigg(62\,\frac{\chi+1}{\chi^2} + \Bigg(-16\,\frac{\chi+1}{\chi^2} + \frac{16}{3}\,\frac{1}{\chi^2}G_0(\chi)\Bigg)\zeta_3 - 6\,\frac{\chi+1}{\chi^2}\zeta_4 - 30\,\frac{1}{\chi^2}G_0(\chi) + 14\,\frac{1}{\chi^2}G_{0,0}(\chi) - 6\,\frac{1}{\chi^2}G_{0,0,0}(\chi)$$

$$+ 2\,\frac{1}{\chi^2}G_{0,0,0,0}(\chi)\Bigg)i\Bigg)\log(\mu)\Bigg)\mu^{-\epsilon} + \Bigg(-\frac{1}{\epsilon^4}\,\frac{5}{4}\,\frac{\chi+1}{\chi^2}i + \frac{1}{\epsilon}\Bigg(\frac{2}{9}\,\frac{(\chi+1)\mathrm{K2}\pi}{\chi^2} + \Bigg(-\frac{35}{12}\,\frac{\chi+1}{\chi^2} + \frac{3}{2}\,\frac{1}{\chi^2}G_0(\chi)\Bigg)\zeta_2 + \frac{13}{18}\,\frac{\chi+1}{\chi^2}\zeta_3\Bigg)i$$

$$+ \Bigg(\frac{2}{45}\,\frac{(\chi+1)(55\mathrm{K2}^2+80\mathrm{K2}\pi-12\mathrm{K31}\pi)}{\chi^2} + \Bigg(-\frac{1}{2}\,\frac{\chi+1}{\chi^2} + \frac{3}{2}\,\frac{1}{\chi^2}G_0(\chi) + \frac{2}{15}\,\frac{\chi+1}{\chi^2}G_{0,0}(3)\Bigg)\zeta_2 + \Bigg(-\frac{119}{18}\,\frac{\chi+1}{\chi^2} - \frac{4}{3}\,\frac{1}{\chi^2}G_0(\chi)\Bigg)\zeta_3$$

$$- \frac{533}{432}\,\frac{\chi+1}{\chi^2}\zeta_4 - \frac{2}{9}\,\frac{(\chi+1)\mathrm{K2}\pi}{\chi^2}G_0(3)\Bigg)i + \frac{1}{\epsilon^3}\Bigg(-\frac{5}{4}\,\frac{\chi+1}{\chi^2} + \frac{1}{2}\,\frac{1}{\chi^2}G_0(\chi)\Bigg)i + \frac{1}{\epsilon^2}\Bigg(\frac{1}{2}\,\frac{\chi+1}{\chi^2} - \frac{23}{12}\,\frac{\chi+1}{\chi^2}\zeta_2 + \frac{1}{2}\,\frac{1}{\chi^2}G_0(\chi)\Bigg)i + \Bigg(\epsilon^3\Bigg(\frac{89565}{16}\,\frac{1}{\chi}$$

$$+ \frac{8625}{4}\,\frac{1}{\chi}\zeta_2 + 715\,\frac{1}{\chi}\zeta_3 + \frac{4545}{4}\,\frac{1}{\chi}\zeta_4\Bigg)i + \epsilon^4\Bigg(\frac{89565}{4}\,\frac{1}{\chi} + 8625\,\frac{1}{\chi}\zeta_2 + 2860\,\frac{1}{\chi}\zeta_3 + 4545\,\frac{1}{\chi}\zeta_4\Bigg)i + \Bigg(\frac{8}{15}\,\frac{5\mathrm{K2}^2-6\mathrm{K31}\pi}{\chi} + \frac{4}{5}\,\frac{1}{\chi}G_{0,0}(3)\zeta_2$$

$$- \frac{331}{27}\,\frac{1}{\chi}\zeta_4 - \frac{4}{3}\,\frac{\mathrm{K2}\pi}{\chi}G_0(3)\Bigg)i$$

$$+ \epsilon^2\Bigg(\frac{1}{2160}\,\frac{1170855+241920\,3^{\frac{1}{2}}\mathrm{K2}+193536\,3^{\frac{1}{2}}\mathrm{K31}+116352\,3^{\frac{1}{2}}\mathrm{K41}+96768\,3^{\frac{1}{2}}\mathrm{K42}+92160\,3^{\frac{1}{2}}\pi+7744\,3^{\frac{1}{2}}\pi^3}{\chi} + \frac{1}{4}\,\frac{1763^{\frac{1}{2}}\mathrm{K2}+1445}{\chi}\zeta_2$$

$$- \frac{1}{9}\,\frac{563^{\frac{1}{2}}\pi-2463}{\chi}\zeta_3 + \frac{2109}{4}\,\frac{1}{\chi}\zeta_4 - \frac{1}{45}\,\frac{(121\pi^2+2160)\pi}{3^{\frac{1}{2}}\chi}G_0(3) + \frac{8}{5}\,\frac{3^{\frac{1}{2}}\pi}{\chi}G_{0,0}(3) - \frac{2}{5}\,\frac{3^{\frac{1}{2}}\pi}{\chi}G_{0,0,0}(3)\Bigg)i + \Bigg(\Bigg(\frac{4}{3}\,\frac{\mathrm{K2}\pi}{\chi} - \frac{2}{3}\,\frac{1}{\chi}\zeta_3\Bigg)i$$

$$+ \frac{2}{9}\,\frac{\pi^3}{\chi}\Bigg)\frac{1}{\epsilon} + \frac{16}{3}\,\frac{\mathrm{K2}}{\chi}\zeta_2 - \frac{4}{9}\,\frac{\pi}{\chi}\zeta_3$$

$$+ \Bigg(\Bigg(-\frac{1}{25920}\,\frac{-29248695+19008003^{\frac{1}{2}}\mathrm{K2}+515520\mathrm{K2}^2+1658880\,3^{\frac{1}{2}}\mathrm{K31}+1213632\,3^{\frac{1}{2}}\mathrm{K41}}{\chi}$$

$$+ \frac{995328\,3^{\frac{1}{2}}\mathrm{K42}+682560\,3^{\frac{1}{2}}\pi+288000\mathrm{K2}\pi-359424\mathrm{K31}\pi+64000\,3^{\frac{1}{2}}\pi^3}{\chi} + \Bigg(-\frac{1}{144}\,\frac{4960\,3^{\frac{1}{2}}\mathrm{K2}-46767}{\chi} - \frac{52}{15}\,\frac{1}{\chi}G_{0,0}(3)\Bigg)\zeta_2$$

$$+ \frac{1}{108}\,\frac{6083^{\frac{1}{2}}\pi-2361}{\chi}\zeta_3 - \frac{78767}{432}\,\frac{1}{\chi}\zeta_4 + \frac{1}{405}\,\frac{(2340\mathrm{K2}+2743^{\frac{1}{2}}\pi^2+41853^{\frac{1}{2}})\pi}{\chi}G_0(3) - \frac{3^{\frac{1}{2}}\pi}{\chi}G_{0,0}(3) + \frac{1}{5}\,\frac{3^{\frac{1}{2}}\pi}{\chi}G_{0,0,0}(3)\Bigg)i - \frac{8}{9}\,\frac{\pi^3}{\chi}$$

$$- \frac{32}{3}\,\frac{\mathrm{K2}}{\chi}\zeta_2 + \frac{8}{9}\,\frac{\pi}{\chi}\zeta_3\Bigg)\epsilon\Bigg)\mu^{-1} + \frac{2}{9}\,\frac{(\chi+1)\pi^3}{\chi^2} + \Bigg(\Bigg(\frac{1}{51840}\,\frac{(\chi+1)(-19037025+1900800\,3^{\frac{1}{2}}\mathrm{K2}+918720\mathrm{K2}^2+1658880\,3^{\frac{1}{2}}\mathrm{K31}+1213632\,3^{\frac{1}{2}}\mathrm{K41}}{\chi^2}$$

$$+ \frac{995328\,3^{\frac{1}{2}}\mathrm{K42}+682560\,3^{\frac{1}{2}}\pi+288000\mathrm{K2}\pi-470016\mathrm{K31}\pi+640003^{\frac{1}{2}}\pi^3)}{\chi^2} + \Bigg(\frac{1}{288}\,\frac{(\chi+1)(49603^{\frac{1}{2}}\mathrm{K2}-29433)}{\chi^2} + \frac{34}{15}\,\frac{\chi+1}{\chi^2}G_{0,0}(3)\Bigg)\zeta_2$$

$$+ \Bigg(-\frac{1}{216}\,\frac{(\chi+1)(6083^{\frac{1}{2}}\pi+5433)}{\chi^2} - \frac{4}{3}\,\frac{1}{\chi^2}G_0(\chi)\Bigg)\zeta_3 + \frac{26923}{864}\,\frac{\chi+1}{\chi^2}\zeta_4 - \frac{1}{810}\,\frac{(\chi+1)(3060\mathrm{K2}+274\,3^{\frac{1}{2}}\pi^2+4185\,3^{\frac{1}{2}})\pi}{\chi^2}G_0(3)$$

$$+ \frac{1}{2}\,\frac{(\chi+1)3^{\frac{1}{2}}\pi}{\chi^2}G_{0,0}(3) - \frac{1}{10}\,\frac{(\chi+1)3^{\frac{1}{2}}\pi}{\chi^2}G_{0,0,0}(3)\Bigg)i + \frac{2}{9}\,\frac{(\chi+1)\pi^3}{\chi^2} + \frac{16}{3}\,\frac{(\chi+1)\mathrm{K2}}{\chi^2}\zeta_2 - \frac{4}{9}\,\frac{(\chi+1)\pi}{\chi^2}\zeta_3\Bigg)\epsilon$$

$$+ \Bigg(\Bigg(\frac{1}{25920}\,\frac{(\chi+1)(414720\,3^{\frac{1}{2}}\mathrm{K42}+129600\,3^{\frac{1}{2}}\pi-126720\mathrm{K2}\pi-27648\mathrm{K31}\pi+17536\,3^{\frac{1}{2}}\pi^3)}{\chi^2}$$

$$+\frac{-19880235+449280\,3^{\frac{1}{2}}\,\mathrm{K2}+308160\mathrm{K2}^{2}+497664\,3^{\frac{1}{2}}\,\mathrm{K31}+515520\,3^{\frac{1}{2}}\,\mathrm{K41}}{\chi^{2}}+\Bigg(\frac{1}{144}\,\frac{(\chi+1)\big(17923^{\frac{1}{2}}\,\mathrm{K2}-43563\big)}{\chi^{2}}+\frac{4}{15}\,\frac{\chi+1}{\chi^{2}}\,G_{0,0}(3)\Bigg)\zeta_{2}$$

$$-\frac{1}{108}\,\frac{(\chi+1)\big(272\,3^{\frac{1}{2}}\,\pi+10599\big)}{\chi^{2}}\zeta_{3}-\frac{22991}{144}\,\frac{\chi+1}{\chi^{2}}\zeta_{4}-\frac{1}{162}\,\frac{(\chi+1)\big(72\mathrm{K2}+37\,3^{\frac{1}{2}}\,\pi^{2}+378\,3^{\frac{1}{2}}\big)\pi}{\chi^{2}}G_{0}(3)+\frac{1}{5}\,\frac{(\chi+1)3^{\frac{1}{2}}\,\pi}{\chi^{2}}G_{0,0}(3)\Bigg)i$$

$$-\frac{2}{3}\,\frac{(\chi+1)\pi^{3}}{\chi^{2}}-\frac{16}{3}\,\frac{(\chi+1)\mathrm{K2}}{\chi^{2}}\zeta_{2}+\frac{4}{9}\,\frac{(\chi+1)\pi}{\chi^{2}}\zeta_{3}\Bigg)\epsilon^{2}$$

$$+\Bigg(\Bigg(\frac{1}{25920}\,\frac{(\chi+1)\big(-94837635+898560\,3^{\frac{1}{2}}\,\mathrm{K2}+408960\mathrm{K2}^{2}+995328\,3^{\frac{1}{2}}\,\mathrm{K31}\big)}{\chi^{2}}$$

$$+\frac{1031040\,3^{\frac{1}{2}}\,\mathrm{K41}+829440\,3^{\frac{1}{2}}\,\mathrm{K42}+259200\,3^{\frac{1}{2}}\,\pi-115200\mathrm{K2}\pi-55296\mathrm{K31}\pi+350723^{\frac{1}{2}}\,\pi^{3}}{\chi^{2}}+\Bigg(\frac{1}{144}\,\frac{(\chi+1)\big(35843^{\frac{1}{2}}\,\mathrm{K2}-196323\big)}{\chi^{2}}$$

$$+\frac{8}{15}\,\frac{\chi+1}{\chi^{2}}G_{0,0}(3)\Bigg)\zeta_{2}-\frac{1}{108}\,\frac{(\chi+1)\big(5443^{\frac{1}{2}}\,\pi+34491\big)}{\chi^{2}}\zeta_{3}-\frac{215827}{432}\,\frac{\chi+1}{\chi^{2}}\zeta_{4}-\frac{1}{81}\,\frac{(\chi+1)\big(72\mathrm{K2}+373^{\frac{1}{2}}\,\pi^{2}+3783^{\frac{1}{2}}\big)\pi}{\chi^{2}}G_{0}(3)$$

$$+\frac{2}{5}\,\frac{(\chi+1)24192033^{\frac{1}{2}}\,\pi}{\chi^{2}}G_{0,0}(3)\Bigg)i-\frac{2}{3}\,\frac{(\chi+1)\pi^{3}}{\chi^{2}}-\frac{16}{3}\,\frac{(\chi+1)\mathrm{K2}}{\chi^{2}}\zeta_{2}+\frac{4}{9}\,\frac{(\chi+1)\pi}{\chi^{2}}\zeta_{3}\Bigg)\epsilon^{3}$$

$$+\Bigg(\Bigg(-\frac{1}{3240}\,\frac{(\chi+1)\big(41472\,3^{\frac{1}{2}}\,\mathrm{K42}+105840\,3^{\frac{1}{2}}\,\pi+14400\mathrm{K2}\pi+6912\mathrm{K31}\pi+72323^{\frac{1}{2}}\,\pi^{3}\big)}{\chi^{2}}+$$

$$\frac{(\chi+1)40074345+250560\,3^{\frac{1}{2}}\,\mathrm{K2}-51120\mathrm{K2}^{2}+165888\,3^{\frac{1}{2}}\,\mathrm{K31}+45648\,3^{\frac{1}{2}}\,\mathrm{K41}\big)}{\chi^{2}}+\Bigg(-\frac{1}{18}\,\frac{(\chi+1)\big(344\,3^{\frac{1}{2}}\,\mathrm{K2}+86583\big)}{\chi^{2}}+\frac{8}{15}\,\frac{\chi+1}{\chi^{2}}G_{0,0}(3)\Bigg)\zeta_{2}$$

$$+\frac{4}{27}\,\frac{(\chi+1)\big(8\,3^{\frac{1}{2}}\,\pi-11019\big)}{\chi^{2}}\zeta_{3}-\frac{276403}{108}\,\frac{\chi+1}{\chi^{2}}\zeta_{4}+\frac{2}{405}\,\frac{(\chi+1)\big(-180\mathrm{K2}+89\,3^{\frac{1}{2}}\,\pi^{2}+2295\,3^{\frac{1}{2}}\big)\pi}{\chi^{2}}G_{0}(3)-\frac{6}{5}\,\frac{(\chi+1)3^{\frac{1}{2}}\,\pi}{\chi^{2}}G_{0,0}(3)$$

$$+\frac{2}{5}\,\frac{(\chi+1)3^{\frac{1}{2}}\,\pi}{\chi^{2}}G_{0,0,0}(3)\Bigg)i-\frac{2}{3}\,\frac{(\chi+1)\pi^{3}}{\chi^{2}}-\frac{16}{3}\,\frac{(\chi+1)\mathrm{K2}}{\chi^{2}}\zeta_{2}+\frac{4}{9}\,\frac{(\chi+1)\pi}{\chi^{2}}\zeta_{3}\Bigg)\epsilon^{4}+\Bigg(-\frac{1}{\epsilon^{3}}\,\frac{1}{2}\,\frac{\chi+1}{\chi^{2}}i-\frac{1}{\epsilon^{2}}\,\frac{1}{2}\,\frac{\chi+1}{\chi^{2}}i-\frac{1}{\epsilon}\,\frac{3}{2}\,\frac{\chi+1}{\chi^{2}}\zeta_{2}i$$

$$+\Bigg(-\frac{3}{2}\,\frac{\chi+1}{\chi^{2}}\zeta_{2}+\frac{4}{3}\,\frac{\chi+1}{\chi^{2}}\zeta_{3}\Bigg)i+\epsilon\Bigg(\frac{4}{3}\,\frac{\chi+1}{\chi^{2}}\zeta_{3}-\frac{63}{8}\,\frac{\chi+1}{\chi^{2}}\zeta_{4}\Bigg)i+\epsilon^{3}\Bigg(\frac{1}{2}\,\frac{\chi+1}{\chi^{2}}+\frac{3}{2}\,\frac{\chi+1}{\chi^{2}}\zeta_{2}-\frac{4}{3}\,\frac{\chi+1}{\chi^{2}}\zeta_{3}+\frac{63}{8}\,\frac{\chi+1}{\chi^{2}}\zeta_{4}\Bigg)i$$

$$+\epsilon^{4}\Bigg(-31\,\frac{\chi+1}{\chi^{2}}-21\,\frac{\chi+1}{\chi^{2}}\zeta_{2}+8\,\frac{\chi+1}{\chi^{2}}\zeta_{3}-\frac{63}{4}\,\frac{\chi+1}{\chi^{2}}\zeta_{4}\Bigg)i+\epsilon^{2}\Bigg(\frac{63}{2}\,\frac{\chi+1}{\chi^{2}}+\frac{45}{2}\,\frac{\chi+1}{\chi^{2}}\zeta_{2}-\frac{28}{3}\,\frac{\chi+1}{\chi^{2}}\zeta_{3}+\frac{63}{4}\,\frac{\chi+1}{\chi^{2}}\zeta_{4}\Bigg)i\Bigg)\log(\mu)\Bigg)\mu^{-2\epsilon}$$

$$+O(\mu) \tag{D.14}$$

I[FPM2, 1, 1, 1, 1, 1, 1, 1, 0, 0] =

$$-\frac{1}{\epsilon^{4}}4\frac{1}{\chi}i+\frac{1}{\epsilon^{3}}5\frac{1}{\chi}G_{0}(\chi)i+\frac{1}{\epsilon^{2}}\Bigg(15\frac{1}{\chi}\zeta_{2}-4\frac{1}{\chi}G_{0,0}(\chi)\Bigg)i+\frac{1}{\epsilon}\Bigg(\Bigg(12\frac{1}{\chi}G_{-1}(\chi)-33\frac{1}{\chi}G_{0}(\chi)\Bigg)\zeta_{2}+\frac{65}{3}\,\frac{1}{\chi}\zeta_{3}+4\frac{1}{\chi}G_{-1,0,0}(\chi)$$

$$-4\frac{1}{\chi}G_{0,0,0}(\chi)\Bigg)i+\Bigg(\Bigg(12\frac{1}{\chi}G_{-1,-1}(\chi)-20\frac{1}{\chi}G_{-1,0}(\chi)-60\frac{1}{\chi}G_{0,-1}(\chi)+72\frac{1}{\chi}G_{0,0}(\chi)\Bigg)\zeta_{2}+\Bigg(4\frac{1}{\chi}G_{-1}(\chi)-\frac{88}{3}\,\frac{1}{\chi}G_{0}(\chi)\Bigg)\zeta_{3}+87\frac{1}{\chi}\zeta_{4}$$

$$+4\frac{1}{\chi}G_{-1,-1,0,0}(\chi)-16\frac{1}{\chi}G_{-1,0,0,0}(\chi)-20\frac{1}{\chi}G_{0,-1,0,0}(\chi)+32\frac{1}{\chi}G_{0,0,0,0}(\chi)\Bigg)i+\Bigg(\frac{1}{\epsilon^{4}}3\frac{1}{\chi}i-\frac{1}{\epsilon^{3}}5\frac{1}{\chi}G_{0}(\chi)i+\frac{1}{\epsilon^{2}}\Bigg(-\frac{40}{3}\,\frac{1}{\chi}\zeta_{2}$$

$$+3\frac{1}{\chi}G_{0,0}(\chi)\Bigg)i+\frac{1}{\epsilon}\Bigg(\frac{16}{9}\,\frac{\mathrm{K2}\pi}{\chi}+\Bigg(-6\frac{1}{\chi}G_{-1}(\chi)+\frac{58}{3}\,\frac{1}{\chi}G_{0}(\chi)\Bigg)\zeta_{2}-\frac{116}{9}\,\frac{1}{\chi}\zeta_{3}+\frac{2}{3}\,\frac{\pi}{3^{\frac{1}{2}}\,\chi}G_{0}(3)G_{0}(\chi)-\frac{1}{3}\,\frac{\pi}{3^{\frac{1}{2}}\,\chi}G_{0}(9)G_{0}(\chi)-2\frac{1}{\chi}G_{-1,0,0}(\chi)$$

$$+3\frac{1}{\chi}G_{0,0,0}(\chi)\Bigg)i+\epsilon^{2}\Bigg(-\frac{1}{45}\,\frac{9180-7980\,3^{\frac{1}{2}}\,\mathrm{K2}+140\mathrm{K2}^{2}-3^{\frac{1}{2}}\big(5040\mathrm{K31}-144\mathrm{K41}-2016\mathrm{K42}-3900\pi\big)+200\mathrm{K2}\pi-96\mathrm{K31}\pi+65\,3^{\frac{1}{2}}\,\pi^{3}}{\chi}$$

$$+\Bigg(-\frac{2}{3}\,\frac{23^{\frac{1}{2}}\,\mathrm{K2}+125}{\chi}-\frac{10}{3}\,\frac{1}{\chi}G_{0}(\chi)-\frac{8}{15}\,\frac{1}{\chi}G_{0,0}(3)+\frac{2}{3}\,\frac{1}{\chi}G_{0,0}(\chi)\Bigg)\zeta_{2}+\Bigg(-\frac{2}{9}\,\frac{543^{\frac{1}{2}}\,\pi+347}{\chi}-\frac{130}{9}\,\frac{1}{\chi}G_{0}(\chi)\Bigg)\zeta_{3}-\frac{187}{6}\,\frac{1}{\chi}\zeta_{4}$$

$$+\frac{1}{45}\,\frac{\big(40\mathrm{K2}+133^{\frac{1}{2}}\,\pi^{2}-11403^{\frac{1}{2}}\big)\pi}{\chi}G_{0}(3)+\frac{1}{45}\,\frac{11700-21003^{\frac{1}{2}}\,\mathrm{K2}-10083^{\frac{1}{2}}\,\mathrm{K31}-11403^{\frac{1}{2}}\,\pi+40\mathrm{K2}\pi+133^{\frac{1}{2}}\,\pi^{3}}{\chi}G_{0}(\chi)$$

$$+20\frac{\pi}{3^{\frac{1}{2}}\,\chi}G_{0}(3)G_{0}(\chi)+2\frac{3^{\frac{1}{2}}\,\pi}{\chi}G_{0,0}(3)-\frac{2}{5}\,\frac{3^{\frac{1}{2}}\,\pi}{\chi}G_{0}(\chi)G_{0,0}(3)+\frac{4}{3}\,\frac{73^{\frac{1}{2}}\,\mathrm{K2}+53^{\frac{1}{2}}\,\pi-57}{\chi}G_{0,0}(\chi)-4\frac{\pi}{3^{\frac{1}{2}}\,\chi}G_{0}(3)G_{0,0}(\chi)$$

$$-\frac{2}{5}\,\frac{3^{\frac{1}{2}}\,\pi}{\chi}G_{0,0,0}(3)-\frac{4}{3}\,\frac{3^{\frac{1}{2}}\,\pi-15}{\chi}G_{0,0,0}(\chi)-4\frac{1}{\chi}G_{0,0,0,0}(\chi)\Bigg)i$$

$$+\epsilon\Bigg(\frac{1}{270}\,\frac{21780-23940\,3^{\frac{1}{2}}\,\mathrm{K2}+4060\mathrm{K2}^{2}-3^{\frac{1}{2}}\big(15120\mathrm{K31}-432\mathrm{K41}-6048\mathrm{K42}-11700\pi\big)+2680\mathrm{K2}\pi-2784\mathrm{K31}\pi+1953^{\frac{1}{2}}\,\pi^{3}}{\chi}$$

$$+\Bigg(\frac{1}{9}\,\frac{63^{\frac{1}{2}}\,\mathrm{K2}-89}{\chi}+\frac{319}{9}\,\frac{1}{\chi}G_{0}(\chi)+\frac{116}{45}\,\frac{1}{\chi}G_{0,0}(3)-\frac{155}{9}\,\frac{1}{\chi}G_{0,0}(\chi)\Bigg)\zeta_{2}+\Bigg(\frac{1}{27}\,\frac{1623^{\frac{1}{2}}\,\pi+241}{\chi}+\frac{1099}{27}\,\frac{1}{\chi}G_{0}(\chi)\Bigg)\zeta_{3}+\frac{71}{36}\,\frac{1}{\chi}\zeta_{4}$$

$$-\frac{1}{270}\,\frac{\big(1160\mathrm{K2}+393^{\frac{1}{2}}\,\pi^{2}-34203^{\frac{1}{2}}\big)\pi}{\chi}G_{0}(3)-\frac{1}{270}\,\frac{40860-63003^{\frac{1}{2}}\,\mathrm{K2}-30243^{\frac{1}{2}}\,\mathrm{K31}-34203^{\frac{1}{2}}\,\pi+1160\mathrm{K2}\pi+393^{\frac{1}{2}}\,\pi^{3}}{\chi}G_{0}(\chi)$$

$$-10\frac{\pi}{3^{\frac{1}{2}}\,\chi}G_{0}(3)G_{0}(\chi)-\frac{3^{\frac{1}{2}}\,\pi}{\chi}G_{0,0}(3)+\frac{1}{5}\,\frac{3^{\frac{1}{2}}\,\pi}{\chi}G_{0}(\chi)G_{0,0}(3)-\frac{2}{3}\,\frac{73^{\frac{1}{2}}\,\mathrm{K2}+53^{\frac{1}{2}}\,\pi-73}{\chi}G_{0,0}(\chi)+2\frac{\pi}{3^{\frac{1}{2}}\,\chi}G_{0}(3)G_{0,0}(\chi)$$

$$+\frac{1}{5}\,\frac{3^{\frac{1}{2}}\,\pi}{\chi}G_{0,0,0}(3)+\frac{2}{3}\,\frac{3^{\frac{1}{2}}\,\pi-23}{\chi}G_{0,0,0}(\chi)+\frac{14}{3}\,\frac{1}{\chi}G_{0,0,0,0}(\chi)\Bigg)i+\Bigg(\frac{32}{45}\,\frac{5\mathrm{K2}^{2}-6\mathrm{K31}\pi}{\chi}+\Bigg(-6\frac{1}{\chi}G_{-1,-1}(\chi)+10\frac{1}{\chi}G_{-1,0}(\chi)$$

$$+ 36\frac{1}{\chi}G_{0,-1}(\chi) + \frac{16}{15}\frac{1}{\chi}G_{0,0}(3) - \frac{76}{3}\frac{1}{\chi}G_{0,0}(\chi) - \frac{16}{3}\frac{1}{\chi}G_{0,\mathrm{r62}}(\chi) - \frac{16}{3}\frac{1}{\chi}G_{0,\mathrm{r64}}(\chi)\Big)\zeta_2 + \Big(-2\frac{1}{\chi}G_{-1}(\chi) + \frac{182}{9}\frac{1}{\chi}G_0(\chi)\Big)\zeta_3 - \frac{835}{54}\frac{1}{\chi}\zeta_4$$
$$- \frac{16}{9}\frac{\pi\mathrm{K2}}{\chi}G_0(3) - \frac{16}{9}\frac{\mathrm{K2}\pi}{\chi}G_0(\chi) - \frac{2}{3}\frac{\pi}{3^{\frac{1}{2}}\chi}G_0(3)G_{0,0}(\chi) + \frac{1}{3}\frac{\pi}{3^{\frac{1}{2}}\chi}G_0(9)G_{0,0}(\chi) - 2\frac{1}{\chi}G_{-1,-1,0,0}(\chi) + 8\frac{1}{\chi}G_{-1,0,0,0}(\chi)$$
$$+ 12\frac{1}{\chi}G_{0,-1,0,0}(\chi) - 13\frac{1}{\chi}G_{0,0,0,0}(\chi) - 2\frac{1}{\chi}G_{0,\mathrm{r62},0,0}(\chi) - 2\frac{1}{\chi}G_{0,\mathrm{r64},0,0}(\chi)\Big)i - \frac{2}{3}\frac{\pi}{\chi}G_{0,\mathrm{r62},0}(\chi) + \frac{2}{3}\frac{\pi}{\chi}G_{0,\mathrm{r64},0}(\chi) + \Big(\frac{1}{\epsilon^3}4\frac{1}{\chi}i$$
$$- \frac{1}{\epsilon}6\frac{1}{\chi}\zeta_2 i - \frac{1}{\epsilon^2}2\frac{1}{\chi}G_0(\chi)i + \Big(\Big(-6\frac{1}{\chi}G_{-1}(\chi) + 6\frac{1}{\chi}G_0(\chi)\Big)\zeta_2 - \frac{38}{3}\frac{1}{\chi}\zeta_3 - 2\frac{1}{\chi}G_{-1,0,0}(\chi) + 2\frac{1}{\chi}G_{0,0,0}(\chi)\Big)i + \epsilon\Big(\Big(-6\frac{1}{\chi}G_{-1,-1}(\chi)$$
$$+ 6\frac{1}{\chi}G_{-1,0}(\chi) + 6\frac{1}{\chi}G_{0,-1}(\chi) - 6\frac{1}{\chi}G_{0,0}(\chi)\Big)\zeta_2 + \Big(-2\frac{1}{\chi}G_{-1}(\chi) + \frac{22}{3}\frac{1}{\chi}G_0(\chi)\Big)\zeta_3 - 18\frac{1}{\chi}\zeta_4 - 2\frac{1}{\chi}G_{-1,-1,0,0}(\chi) + 4\frac{1}{\chi}G_{-1,0,0,0}(\chi)$$
$$+ 2\frac{1}{\chi}G_{0,-1,0,0}(\chi) - 4\frac{1}{\chi}G_{0,0,0,0}(\chi)\Big)i\Big)\log(\mu)\Big)\mu^{-\epsilon} + \Big(\frac{1}{\epsilon^4}\frac{1}{\chi}i + \frac{1}{\epsilon^2}\frac{5}{3}\frac{1}{\chi}\zeta_2 i + \frac{1}{\epsilon}\Big(-\frac{2}{9}\frac{\mathrm{K2}\pi}{\chi} + \frac{3}{2}\frac{1}{\chi}G_0(\chi)\zeta_2 - \frac{23}{9}\frac{1}{\chi}\zeta_3\Big)i + \epsilon^3\Big(-\frac{89565}{32}\frac{1}{\chi}$$
$$- \frac{8625}{8}\frac{1}{\chi}\zeta_2 - \frac{715}{2}\frac{1}{\chi}\zeta_3 - \frac{4545}{8}\frac{1}{\chi}\zeta_4\Big)i + \epsilon^4\Big(-\frac{89565}{8}\frac{1}{\chi} - \frac{8625}{2}\frac{1}{\chi}\zeta_2 - 1430\frac{1}{\chi}\zeta_3 - \frac{4545}{2}\frac{1}{\chi}\zeta_4\Big)i + \Big(-\frac{4}{45}\frac{5\mathrm{K2}^2 - 6\mathrm{K31}\pi}{\chi} - \frac{2}{15}\frac{1}{\chi}G_{0,0}(3)\zeta_2$$
$$- \frac{4}{3}\frac{1}{\chi}G_0(\chi)\zeta_3 + \frac{323}{108}\frac{1}{\chi}\zeta_4 + \frac{2}{9}\frac{\mathrm{K2}\pi}{\chi}G_0(3)\Big)i + \frac{1}{\epsilon^3}\frac{1}{2}\frac{1}{\chi}G_0(\chi)i$$
$$+ \Big(\Big(-\frac{1}{4320}\frac{903015 + 3^{\frac{1}{2}}\left(241920\mathrm{K2} + 193536\mathrm{K31} + 116352\mathrm{K41} + 96768\mathrm{K42} + 92160\pi + 7744\pi^3\right)}{\chi} - \frac{1}{8}\frac{1763^{\frac{1}{2}}\,\mathrm{K2} + 1557}{\chi}\zeta_2$$
$$+ \frac{7}{18}\frac{83^{\frac{1}{2}}\,\pi - 393}{\chi}\zeta_3 - \frac{2017}{8}\frac{1}{\chi}\zeta_4 + \frac{1}{90}\frac{\left(121\pi^2 + 2160\right)\pi}{3^{\frac{1}{2}}\chi}G_0(3) - \frac{4}{5}\frac{3^{\frac{1}{2}}\pi}{\chi}G_{0,0}(3) + \frac{1}{5}\frac{3^{\frac{1}{2}}\pi}{\chi}G_{0,0,0}(3)\Big)i - \frac{2}{3}\frac{\left(\pi^2 - 30\right)\pi}{\chi} - \frac{32}{9}\frac{\pi}{\chi}\zeta_3\Big)\epsilon^2$$
$$+ \Big(\Big(\frac{1}{51840}\frac{-33758775 + 19008003^{\frac{1}{2}}\,\mathrm{K2} + 423360\mathrm{K2}^2 + 16588803^{\frac{1}{2}}\,\mathrm{K31} + 3^{\frac{1}{2}}\left(1213632\mathrm{K41} + 995328\mathrm{K42} + 682560\pi\right)}{\chi}$$
$$+ \frac{1}{51840}\frac{+195840\,\pi\,\mathrm{K2} - 248832\,\pi\,\mathrm{K31} + 64000\,3^{\frac{1}{2}}\,\pi^3}{\chi} + \Big(\frac{1}{288}\frac{4960\,3^{\frac{1}{2}}\,\mathrm{K2} - 49551}{\chi} + \frac{6}{5}\frac{1}{\chi}G_{0,0}(3)\Big)\zeta_2 - \frac{1}{216}\frac{608\,3^{\frac{1}{2}}\,\pi - 8745}{\chi}\zeta_3$$
$$+ \frac{98123}{864}\frac{1}{\chi}\zeta_4 - \frac{1}{810}\frac{\left(1620\mathrm{K2} + 2743^{\frac{1}{2}}\,\pi^2 + 41853^{\frac{1}{2}}\right)\pi}{\chi}G_0(3) + \frac{1}{2}\frac{3^{\frac{1}{2}}\pi}{\chi}G_{0,0}(3) - \frac{1}{10}\frac{3^{\frac{1}{2}}\pi}{\chi}G_{0,0,0}(3)\Big)i - \frac{1}{9}\frac{\left(\pi^2 + 138\right)\pi}{\chi} + \frac{50}{9}\frac{\pi}{\chi}\zeta_3\Big)\epsilon\Big)\mu^{-2\epsilon}$$
$$+ O(\mu) \tag{D.15}$$

I[FNP0, 1, 1, 1, 1, 1, 1, 1, 0, 0] =

$$\Big(\frac{1}{\epsilon^4}\frac{1}{4}\frac{1}{(\chi+1)\chi}i + \Big(\Big(\frac{96\chi+7}{(\chi+1)\chi}G_{-1,-1}(\chi) - \frac{96\chi+7}{(\chi+1)\chi}G_{-1,0}(\chi) - \frac{96\chi+7}{(\chi+1)\chi}G_{0,-1}(\chi) + \frac{96\chi+7}{(\chi+1)\chi}G_{0,0}(\chi)\Big)\zeta_2$$
$$+ \Big(-\frac{40}{3}\frac{2\chi+1}{(\chi+1)\chi}G_{-1}(\chi) + \frac{40}{3}\frac{2\chi+1}{(\chi+1)\chi}G_0(\chi)\Big)\zeta_3 - \frac{1}{16}\frac{960\chi-151}{(\chi+1)\chi}\zeta_4 + 4\frac{1}{(\chi+1)\chi}G_{-1,-1,-1,-1}(\chi) - 4\frac{1}{(\chi+1)\chi}G_{-1,-1,-1,0}(\chi)$$
$$- 4\frac{1}{(\chi+1)\chi}G_{-1,-1,0,-1}(\chi) + 4\frac{1}{(\chi+1)\chi}G_{-1,-1,0,0}(\chi) - 4\frac{1}{(\chi+1)\chi}G_{-1,0,-1,-1}(\chi) + 4\frac{1}{(\chi+1)\chi}G_{-1,0,-1,0}(\chi)$$
$$+ 4\frac{1}{(\chi+1)\chi}G_{-1,0,0,-1}(\chi) - 4\frac{1}{(\chi+1)\chi}G_{-1,0,0,0}(\chi) - 4\frac{1}{(\chi+1)\chi}G_{0,-1,-1,-1}(\chi) + 4\frac{1}{(\chi+1)\chi}G_{0,-1,-1,0}(\chi)$$
$$+ 4\frac{1}{(\chi+1)\chi}G_{0,-1,0,-1}(\chi) - 4\frac{1}{(\chi+1)\chi}G_{0,-1,0,0}(\chi) + 4\frac{1}{(\chi+1)\chi}G_{0,0,-1,-1}(\chi) - 4\frac{1}{(\chi+1)\chi}G_{0,0,-1,0}(\chi) - 4\frac{1}{(\chi+1)\chi}G_{0,0,0,-1}(\chi)$$
$$+ 4\frac{1}{(\chi+1)\chi}G_{0,0,0,0}(\chi)\Big)i + \Big(\Big(\frac{1}{2}\frac{2\chi+1}{(\chi+1)\chi}G_{-1}(\chi) - \frac{1}{2}\frac{2\chi+1}{(\chi+1)\chi}G_0(\chi)\Big)i - \frac{1}{2}\frac{(2\chi-1)\pi}{(\chi+1)\chi}\Big)\frac{1}{\epsilon^3} + \frac{40}{3}\frac{(2\chi-1)\pi}{(\chi+1)\chi}\zeta_3$$
$$- \frac{1}{2}\frac{(12\chi-5)\pi^3}{(\chi+1)\chi}G_{-1}(\chi) + \frac{1}{2}\frac{(12\chi-5)\pi^3}{(\chi+1)\chi}G_0(\chi) + \Big(\Big(\frac{1}{4}\frac{96\chi+7}{(\chi+1)\chi}\zeta_2 + \frac{1}{(\chi+1)\chi}G_{-1,-1}(\chi) - \frac{1}{(\chi+1)\chi}G_{-1,0}(\chi) - \frac{1}{(\chi+1)\chi}G_{0,-1}(\chi)$$
$$+ \frac{1}{(\chi+1)\chi}G_{0,0}(\chi)\Big)i + \frac{(4\chi+1)\pi}{(\chi+1)\chi}G_{-1}(\chi) - \frac{(4\chi+1)\pi}{(\chi+1)\chi}G_0(\chi)\Big)\frac{1}{\epsilon^2} + \Big(\Big(\Big(-\frac{1}{2}\frac{82\chi-7}{(\chi+1)\chi}G_{-1}(\chi) + \frac{1}{2}\frac{82\chi-7}{(\chi+1)\chi}G_0(\chi)\Big)\zeta_2$$
$$- \frac{20}{3}\frac{1}{(\chi+1)\chi}\zeta_3 + 2\frac{2\chi+1}{(\chi+1)\chi}G_{-1,-1,-1}(\chi) - 2\frac{2\chi+1}{(\chi+1)\chi}G_{-1,-1,0}(\chi) - 2\frac{2\chi+1}{(\chi+1)\chi}G_{-1,0,-1}(\chi) + 2\frac{2\chi+1}{(\chi+1)\chi}G_{-1,0,0}(\chi)$$
$$- 2\frac{2\chi+1}{(\chi+1)\chi}G_{0,-1,-1}(\chi) + 2\frac{2\chi+1}{(\chi+1)\chi}G_{0,-1,0}(\chi) + 2\frac{2\chi+1}{(\chi+1)\chi}G_{0,0,-1}(\chi) - 2\frac{2\chi+1}{(\chi+1)\chi}G_{0,0,0}(\chi)\Big)i + \frac{1}{4}\frac{(22\chi+5)\pi^3}{(\chi+1)\chi}$$
$$- 2\frac{(2\chi-1)\pi}{(\chi+1)\chi}G_{-1,-1}(\chi) + 2\frac{(2\chi-1)\pi}{(\chi+1)\chi}G_{-1,0}(\chi) + 2\frac{(2\chi-1)\pi}{(\chi+1)\chi}G_{0,-1}(\chi) - 2\frac{(2\chi-1)\pi}{(\chi+1)\chi}G_{0,0}(\chi)\Big)\frac{1}{\epsilon} + 4\frac{(4\chi+1)\pi}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)$$
$$- 4\frac{(4\chi+1)\pi}{(\chi+1)\chi}G_{-1,-1,0}(\chi) - 4\frac{(4\chi+1)\pi}{(\chi+1)\chi}G_{-1,0,-1}(\chi) + 4\frac{(4\chi+1)\pi}{(\chi+1)\chi}G_{-1,0,0}(\chi) - 4\frac{(4\chi+1)\pi}{(\chi+1)\chi}G_{0,-1,-1}(\chi) + 4\frac{(4\chi+1)\pi}{(\chi+1)\chi}G_{0,-1,0}(\chi)$$
$$+ 4\frac{(4\chi+1)\pi}{(\chi+1)\chi}G_{0,0,-1}(\chi) - 4\frac{(4\chi+1)\pi}{(\chi+1)\chi}G_{0,0,0}(\chi) + \Big(\Big(-\frac{1}{8}\frac{13\chi+1}{(\chi+1)\chi} + \Big(\frac{1}{8}\frac{1145\chi+77}{(\chi+1)\chi} + \frac{1}{4}\frac{949\chi-119}{(\chi+1)\chi}G_{-1}(\chi) - \frac{1}{4}\frac{949\chi-119}{(\chi+1)\chi}G_0(\chi)$$
$$- \frac{1}{2}\frac{1159\chi+91}{(\chi+1)\chi}G_{-1,-1}(\chi) + \frac{1}{2}\frac{1159\chi+91}{(\chi+1)\chi}G_{-1,0}(\chi) + \frac{1}{2}\frac{1159\chi+91}{(\chi+1)\chi}G_{0,-1}(\chi) - \frac{1}{2}\frac{1159\chi+91}{(\chi+1)\chi}G_{0,0}(\chi)\Big)\zeta_2$$
$$+ \Big(\frac{10}{3}\frac{5\chi+17}{(\chi+1)\chi} + \frac{20}{3}\frac{25\chi+13}{(\chi+1)\chi}G_{-1}(\chi) - \frac{20}{3}\frac{25\chi+13}{(\chi+1)\chi}G_0(\chi)\Big)\zeta_3 + \frac{1}{32}\frac{11369\chi-1963}{(\chi+1)\chi}\zeta_4 - \frac{1}{4}\frac{17\chi+5}{(\chi+1)\chi}G_{-1}(\chi) + \frac{1}{4}\frac{17\chi+5}{(\chi+1)\chi}G_0(\chi)$$
$$- \frac{1}{2}\frac{\chi-11}{(\chi+1)\chi}G_{-1,-1}(\chi) + \frac{1}{2}\frac{\chi-11}{(\chi+1)\chi}G_{-1,0}(\chi) + \frac{1}{2}\frac{\chi-11}{(\chi+1)\chi}G_{0,-1}(\chi) - \frac{1}{2}\frac{\chi-11}{(\chi+1)\chi}G_{0,0}(\chi) - \frac{29\chi+17}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)$$

$$+\frac{29\chi+17}{(\chi+1)\chi}G_{-1,-1,0}(\chi)+\frac{29\chi+17}{(\chi+1)\chi}G_{-1,0,-1}(\chi)-\frac{29\chi+17}{(\chi+1)\chi}G_{-1,0,0}(\chi)+\frac{29\chi+17}{(\chi+1)\chi}G_{0,-1,-1}(\chi)-\frac{29\chi+17}{(\chi+1)\chi}G_{0,-1,0}(\chi)$$

$$-\frac{29\chi+17}{(\chi+1)\chi}G_{0,0,-1}(\chi)+\frac{29\chi+17}{(\chi+1)\chi}G_{0,0,0}(\chi)-2\frac{\chi+13}{(\chi+1)\chi}G_{-1,-1,-1,-1}(\chi)+2\frac{\chi+13}{(\chi+1)\chi}G_{-1,-1,-1,0}(\chi)+2\frac{\chi+13}{(\chi+1)\chi}G_{-1,-1,0,-1}(\chi)$$

$$-2\frac{\chi+13}{(\chi+1)\chi}G_{-1,-1,0,0}(\chi)+2\frac{\chi+13}{(\chi+1)\chi}G_{-1,0,-1,-1}(\chi)-2\frac{\chi+13}{(\chi+1)\chi}G_{-1,0,-1,0}(\chi)-2\frac{\chi+13}{(\chi+1)\chi}G_{-1,0,0,-1}(\chi)$$

$$+2\frac{\chi+13}{(\chi+1)\chi}G_{-1,0,0,0}(\chi)+2\frac{\chi+13}{(\chi+1)\chi}G_{0,-1,-1,-1}(\chi)-2\frac{\chi+13}{(\chi+1)\chi}G_{0,-1,-1,0}(\chi)-2\frac{\chi+13}{(\chi+1)\chi}G_{0,-1,0,-1}(\chi)$$

$$+2\frac{\chi+13}{(\chi+1)\chi}G_{0,-1,0,0}(\chi)-2\frac{\chi+13}{(\chi+1)\chi}G_{0,0,-1,-1}(\chi)+2\frac{\chi+13}{(\chi+1)\chi}G_{0,0,-1,0}(\chi)+2\frac{\chi+13}{(\chi+1)\chi}G_{0,0,0,-1}(\chi)-2\frac{\chi+13}{(\chi+1)\chi}G_{0,0,0,0}(\chi)\bigg)i$$

$$-\frac{1}{8}\frac{(289\chi\pi^2-62\chi+85\pi^2+10)\pi}{(\chi+1)\chi}-\frac{20}{3}\frac{(23\chi-13)\pi}{(\chi+1)\chi}\zeta_3+\frac{1}{4}\frac{(139\chi\pi^2+94\chi-65\pi^2+22)\pi}{(\chi+1)\chi}G_{-1}(\chi)$$

$$-\frac{1}{4}\frac{(139\chi\pi^2+94\chi-65\pi^2+22)\pi}{(\chi+1)\chi}G_{0}(\chi)+\frac{(19\chi-17)\pi}{(\chi+1)\chi}G_{-1,-1}(\chi)-\frac{(19\chi-17)\pi}{(\chi+1)\chi}G_{-1,0}(\chi)-\frac{(19\chi-17)\pi}{(\chi+1)\chi}G_{0,-1}(\chi)$$

$$+\frac{(19\chi-17)\pi}{(\chi+1)\chi}G_{0,0}(\chi)-2\frac{(49\chi+13)\pi}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)+2\frac{(49\chi+13)\pi}{(\chi+1)\chi}G_{-1,-1,0}(\chi)+2\frac{(49\chi+13)\pi}{(\chi+1)\chi}G_{-1,0,-1}(\chi)$$

$$-2\frac{(49\chi+13)\pi}{(\chi+1)\chi}G_{-1,0,0}(\chi)+2\frac{(49\chi+13)\pi}{(\chi+1)\chi}G_{0,-1,-1}(\chi)-2\frac{(49\chi+13)\pi}{(\chi+1)\chi}G_{0,-1,0}(\chi)-2\frac{(49\chi+13)\pi}{(\chi+1)\chi}G_{0,0,-1}(\chi)$$

$$+2\frac{(49\chi+13)\pi}{(\chi+1)\chi}G_{0,0,0}(\chi)\bigg)\epsilon+\bigg(\bigg(\frac{3}{64}\frac{6929\chi+1329}{(\chi+1)\chi}+\bigg(-\frac{21}{64}\frac{76267\chi+5067}{(\chi+1)\chi}-\frac{105}{32}\frac{12587\chi-1653}{(\chi+1)\chi}G_{-1}(\chi)+\frac{105}{32}\frac{12587\chi-1653}{(\chi+1)\chi}G_{0}(\chi)$$

$$+\frac{3}{16}\frac{314003\chi+26355}{(\chi+1)\chi}G_{-1,-1}(\chi)-\frac{3}{16}\frac{314003\chi+26355}{(\chi+1)\chi}G_{-1,0}(\chi)-\frac{3}{16}\frac{314003\chi+26355}{(\chi+1)\chi}G_{0,-1}(\chi)+\frac{3}{16}\frac{314003\chi+26355}{(\chi+1)\chi}G_{0,0}(\chi)\bigg)\zeta_2$$

$$+\bigg(-\frac{25}{4}\frac{533\chi+1653}{(\chi+1)\chi}-\frac{5}{2}\frac{6997\chi+3765}{(\chi+1)\chi}G_{-1}(\chi)+\frac{5}{2}\frac{6997\chi+3765}{(\chi+1)\chi}G_{0}(\chi)\bigg)\zeta_3-\frac{3}{256}\frac{3022237\chi-568515}{(\chi+1)\chi}\zeta_4+\frac{21}{32}\frac{1067\chi+267}{(\chi+1)\chi}G_{-1}(\chi)$$

$$-\frac{21}{32}\frac{1067\chi+267}{(\chi+1)\chi}G_{0}(\chi)+\frac{3}{16}\frac{533\chi-5067}{(\chi+1)\chi}G_{-1,-1}(\chi)-\frac{3}{16}\frac{533\chi-5067}{(\chi+1)\chi}G_{-1,0}(\chi)-\frac{3}{16}\frac{533\chi-5067}{(\chi+1)\chi}G_{0,-1}(\chi)$$

$$+\frac{3}{16}\frac{533\chi-5067}{(\chi+1)\chi}G_{0,0}(\chi)+\frac{15}{8}\frac{2773\chi+1653}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)-\frac{15}{8}\frac{2773\chi+1653}{(\chi+1)\chi}G_{-1,-1,0}(\chi)-\frac{15}{8}\frac{2773\chi+1653}{(\chi+1)\chi}G_{-1,0,-1}(\chi)$$

$$+\frac{15}{8}\frac{2773\chi+1653}{(\chi+1)\chi}G_{-1,0,0}(\chi)-\frac{15}{8}\frac{2773\chi+1653}{(\chi+1)\chi}G_{0,-1,-1}(\chi)+\frac{15}{8}\frac{2773\chi+1653}{(\chi+1)\chi}G_{0,-1,0}(\chi)+\frac{15}{8}\frac{2773\chi+1653}{(\chi+1)\chi}G_{0,0,-1}(\chi)$$

$$-\frac{15}{8}\frac{2773\chi+1653}{(\chi+1)\chi}G_{0,0,0}(\chi)+\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{-1,-1,-1,-1}(\chi)-\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{-1,-1,-1,0}(\chi)-\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{-1,-1,0,-1}(\chi)$$

$$+\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{-1,-1,0,0}(\chi)-\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{-1,0,-1,-1}(\chi)+\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{-1,0,-1,0}(\chi)+\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{-1,0,0,-1}(\chi)$$

$$-\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{-1,0,0,0}(\chi)-\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{0,-1,-1,-1}(\chi)+\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{0,-1,-1,0}(\chi)+\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{0,-1,0,-1}(\chi)$$

$$-\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{0,-1,0,0}(\chi)+\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{0,0,-1,-1}(\chi)-\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{0,0,-1,0}(\chi)-\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{0,0,0,-1}(\chi)$$

$$+\frac{3}{4}\frac{533\chi+3765}{(\chi+1)\chi}G_{0,0,0,0}(\chi)\bigg)i+\frac{3}{64}\frac{(136525\chi\pi^2-29862\chi+41325\pi^2+3738)\pi}{(\chi+1)\chi}+\frac{15}{2}\frac{(1977\chi-1255)\pi}{(\chi+1)\chi}\zeta_3$$

$$-\frac{3}{32}\frac{(36119\chi\pi^2+43734\chi-18825\pi^2+10134)\pi}{(\chi+1)\chi}G_{-1}(\chi)+\frac{3}{32}\frac{(36119\chi\pi^2+43734\chi-18825\pi^2+10134)\pi}{(\chi+1)\chi}G_{0}(\chi)$$

$$-\frac{45}{8}\frac{(569\chi-551)\pi}{(\chi+1)\chi}G_{-1,-1}(\chi)+\frac{45}{8}\frac{(569\chi-551)\pi}{(\chi+1)\chi}G_{-1,0}(\chi)+\frac{45}{8}\frac{(569\chi-551)\pi}{(\chi+1)\chi}G_{0,-1}(\chi)-\frac{45}{8}\frac{(569\chi-551)\pi}{(\chi+1)\chi}G_{0,0}(\chi)$$

$$+\frac{9}{4}\frac{(4487\chi+1255)\pi}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)-\frac{9}{4}\frac{(4487\chi+1255)\pi}{(\chi+1)\chi}G_{-1,-1,0}(\chi)-\frac{9}{4}\frac{(4487\chi+1255)\pi}{(\chi+1)\chi}G_{-1,0,-1}(\chi)$$

$$+\frac{9}{4}\frac{(4487\chi+1255)\pi}{(\chi+1)\chi}G_{-1,0,0}(\chi)-\frac{9}{4}\frac{(4487\chi+1255)\pi}{(\chi+1)\chi}G_{0,-1,-1}(\chi)+\frac{9}{4}\frac{(4487\chi+1255)\pi}{(\chi+1)\chi}G_{0,-1,0}(\chi)+\frac{9}{4}\frac{(4487\chi+1255)\pi}{(\chi+1)\chi}G_{0,0,-1}(\chi)$$

$$-\frac{9}{4}\frac{(4487\chi+1255)\pi}{(\chi+1)\chi}G_{0,0,0}(\chi)\bigg)\epsilon^4+\bigg(\bigg(-\frac{3}{32}\frac{715\chi+123}{(\chi+1)\chi}+\bigg(\frac{3}{32}\frac{56447\chi+3759}{(\chi+1)\chi}+\frac{3}{16}\frac{46619\chi-6069}{(\chi+1)\chi}G_{-1}(\chi)-\frac{3}{16}\frac{46619\chi-6069}{(\chi+1)\chi}G_{0}(\chi)$$

$$-\frac{3}{8}\frac{35713\chi+2961}{(\chi+1)\chi}G_{-1,-1}(\chi)+\frac{3}{8}\frac{35713\chi+2961}{(\chi+1)\chi}G_{-1,0}(\chi)+\frac{3}{8}\frac{35713\chi+2961}{(\chi+1)\chi}G_{0,-1}(\chi)-\frac{3}{8}\frac{35713\chi+2961}{(\chi+1)\chi}G_{0,0}(\chi)\bigg)\zeta_2$$

$$+\bigg(\frac{5}{2}\frac{275\chi+867}{(\chi+1)\chi}+5\frac{791\chi+423}{(\chi+1)\chi}G_{-1}(\chi)-5\frac{791\chi+423}{(\chi+1)\chi}G_{0}(\chi)\bigg)\zeta_3+\frac{3}{128}\frac{344975\chi-63873}{(\chi+1)\chi}\zeta_4-\frac{3}{16}\frac{799\chi+207}{(\chi+1)\chi}G_{-1}(\chi)$$

$$+\frac{3}{16}\frac{799\chi+207}{(\chi+1)\chi}G_{0}(\chi)-\frac{3}{8}\frac{55\chi-537}{(\chi+1)\chi}G_{-1,-1}(\chi)+\frac{3}{8}\frac{55\chi-537}{(\chi+1)\chi}G_{-1,0}(\chi)+\frac{3}{8}\frac{55\chi-537}{(\chi+1)\chi}G_{0,-1}(\chi)-\frac{3}{8}\frac{55\chi-537}{(\chi+1)\chi}G_{0,0}(\chi)$$

$$-\frac{3}{4}\frac{1459\chi+867}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)+\frac{3}{4}\frac{1459\chi+867}{(\chi+1)\chi}G_{-1,-1,0}(\chi)+\frac{3}{4}\frac{1459\chi+867}{(\chi+1)\chi}G_{-1,0,-1}(\chi)-\frac{3}{4}\frac{1459\chi+867}{(\chi+1)\chi}G_{-1,0,0}(\chi)$$

$$+\frac{3}{4}\frac{1459\chi+867}{(\chi+1)\chi}G_{0,-1,-1}(\chi)-\frac{3}{4}\frac{1459\chi+867}{(\chi+1)\chi}G_{0,-1,0}(\chi)-\frac{3}{4}\frac{1459\chi+867}{(\chi+1)\chi}G_{0,0,-1}(\chi)+\frac{3}{4}\frac{1459\chi+867}{(\chi+1)\chi}G_{0,0,0}(\chi)$$

$$-\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{-1,-1,-1,-1}(\chi)+\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{-1,-1,-1,0}(\chi)+\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{-1,-1,0,-1}(\chi)-\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{-1,-1,0,0}(\chi)$$

$$+\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{-1,0,-1,-1}(\chi)-\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{-1,0,-1,0}(\chi)-\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{-1,0,0,-1}(\chi)+\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{-1,0,0,0}(\chi)$$

$$
\begin{aligned}
&+\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{0,-1,-1,-1}(\chi)-\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{0,-1,-1,0}(\chi)-\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{0,-1,0,-1}(\chi)+\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{0,-1,0,0}(\chi)\\
&-\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{0,0,-1,-1}(\chi)+\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{0,0,-1,0}(\chi)+\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{0,0,0,-1}(\chi)-\frac{3}{2}\frac{55\chi+423}{(\chi+1)\chi}G_{0,0,0,0}(\chi)\Big)i\\
&-\frac{3}{32}\frac{(14399\chi\pi^2-3138\chi+4335\pi^2+414)\pi}{(\chi+1)\chi}-15\frac{(227\chi-141)\pi}{(\chi+1)\chi}\zeta_3+\frac{3}{16}\frac{(4141\chi\pi^2+4626\chi-2115\pi^2+1074)\pi}{(\chi+1)\chi}G_{-1}(\chi)\\
&-\frac{3}{16}\frac{(4141\chi\pi^2+4626\chi-2115\pi^2+1074)\pi}{(\chi+1)\chi}G_{0}(\chi)+\frac{9}{4}\frac{(303\chi-289)\pi}{(\chi+1)\chi}G_{-1,-1}(\chi)-\frac{9}{4}\frac{(303\chi-289)\pi}{(\chi+1)\chi}G_{-1,0}(\chi)\\
&-\frac{9}{4}\frac{(303\chi-289)\pi}{(\chi+1)\chi}G_{0,-1}(\chi)+\frac{9}{4}\frac{(303\chi-289)\pi}{(\chi+1)\chi}G_{0,0}(\chi)-\frac{9}{2}\frac{(509\chi+141)\pi}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)+\frac{9}{2}\frac{(509\chi+141)\pi}{(\chi+1)\chi}G_{-1,-1,0}(\chi)\\
&+\frac{9}{2}\frac{(509\chi+141)\pi}{(\chi+1)\chi}G_{-1,0,-1}(\chi)-\frac{9}{2}\frac{(509\chi+141)\pi}{(\chi+1)\chi}G_{-1,0,0}(\chi)+\frac{9}{2}\frac{(509\chi+141)\pi}{(\chi+1)\chi}G_{0,-1,-1}(\chi)-\frac{9}{2}\frac{(509\chi+141)\pi}{(\chi+1)\chi}G_{0,-1,0}(\chi)\\
&-\frac{9}{2}\frac{(509\chi+141)\pi}{(\chi+1)\chi}G_{0,0,-1}(\chi)+\frac{9}{2}\frac{(509\chi+141)\pi}{(\chi+1)\chi}G_{0,0,0}(\chi)\Big)\epsilon^3+\Bigg(\Bigg(\Big(\frac{3}{16}\frac{65\chi+9}{(\chi+1)\chi}+\Big(-\frac{21}{16}\frac{763\chi+51}{(\chi+1)\chi}-\frac{21}{8}\frac{631\chi-81}{(\chi+1)\chi}G_{-1}(\chi)\\
&+\frac{21}{8}\frac{631\chi-81}{(\chi+1)\chi}G_{0}(\chi)+\frac{15}{4}\frac{775\chi+63}{(\chi+1)\chi}G_{-1,-1}(\chi)-\frac{15}{4}\frac{775\chi+63}{(\chi+1)\chi}G_{-1,0}(\chi)-\frac{15}{4}\frac{775\chi+63}{(\chi+1)\chi}G_{0,-1}(\chi)+\frac{15}{4}\frac{775\chi+63}{(\chi+1)\chi}G_{0,0}(\chi)\Big)\zeta_2\\
&+\Big(-5\frac{25\chi+81}{(\chi+1)\chi}-50\frac{17\chi+9}{(\chi+1)\chi}G_{-1}(\chi)+50\frac{17\chi+9}{(\chi+1)\chi}G_{0}(\chi)\Big)\zeta_3-\frac{15}{64}\frac{7529\chi-1359}{(\chi+1)\chi}\zeta_4+\frac{21}{8}\frac{11\chi+3}{(\chi+1)\chi}G_{-1}(\chi)-\frac{21}{8}\frac{11\chi+3}{(\chi+1)\chi}G_{0}(\chi)\\
&+\frac{3}{4}\frac{5\chi-51}{(\chi+1)\chi}G_{-1,-1}(\chi)-\frac{3}{4}\frac{5\chi-51}{(\chi+1)\chi}G_{-1,0}(\chi)-\frac{3}{4}\frac{5\chi-51}{(\chi+1)\chi}G_{0,-1}(\chi)+\frac{3}{4}\frac{5\chi-51}{(\chi+1)\chi}G_{0,0}(\chi)+\frac{3}{2}\frac{137\chi+81}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)\\
&-\frac{3}{2}\frac{137\chi+81}{(\chi+1)\chi}G_{-1,-1,0}(\chi)-\frac{3}{2}\frac{137\chi+81}{(\chi+1)\chi}G_{-1,0,-1}(\chi)+\frac{3}{2}\frac{137\chi+81}{(\chi+1)\chi}G_{-1,0,0}(\chi)-\frac{3}{2}\frac{137\chi+81}{(\chi+1)\chi}G_{0,-1,-1}(\chi)\\
&+\frac{3}{2}\frac{137\chi+81}{(\chi+1)\chi}G_{0,-1,0}(\chi)+\frac{3}{2}\frac{137\chi+81}{(\chi+1)\chi}G_{0,0,-1}(\chi)-\frac{3}{2}\frac{137\chi+81}{(\chi+1)\chi}G_{0,0,0}(\chi)+15\frac{\chi+9}{(\chi+1)\chi}G_{-1,-1,-1,-1}(\chi)\\
&-15\frac{\chi+9}{(\chi+1)\chi}G_{-1,-1,-1,0}(\chi)-15\frac{\chi+9}{(\chi+1)\chi}G_{-1,-1,0,-1}(\chi)+15\frac{\chi+9}{(\chi+1)\chi}G_{-1,-1,0,0}(\chi)-15\frac{\chi+9}{(\chi+1)\chi}G_{-1,0,-1,-1}(\chi)\\
&+15\frac{\chi+9}{(\chi+1)\chi}G_{-1,0,-1,0}(\chi)+15\frac{\chi+9}{(\chi+1)\chi}G_{-1,0,0,-1}(\chi)-15\frac{\chi+9}{(\chi+1)\chi}G_{-1,0,0,0}(\chi)-15\frac{\chi+9}{(\chi+1)\chi}G_{0,-1,-1,-1}(\chi)\\
&+15\frac{\chi+9}{(\chi+1)\chi}G_{0,-1,-1,0}(\chi)+15\frac{\chi+9}{(\chi+1)\chi}G_{0,-1,0,-1}(\chi)-15\frac{\chi+9}{(\chi+1)\chi}G_{0,-1,0,0}(\chi)+15\frac{\chi+9}{(\chi+1)\chi}G_{0,0,-1,-1}(\chi)\\
&-15\frac{\chi+9}{(\chi+1)\chi}G_{0,0,-1,0}(\chi)-15\frac{\chi+9}{(\chi+1)\chi}G_{0,0,0,-1}(\chi)+15\frac{\chi+9}{(\chi+1)\chi}G_{0,0,0,0}(\chi)\Bigg)i+\frac{3}{16}\frac{(1357\chi\pi^2-294\chi+405\pi^2+42)\pi}{(\chi+1)\chi}\\
&+150\frac{(5\chi-3)\pi}{(\chi+1)\chi}\zeta_3-\frac{3}{8}\frac{(455\chi\pi^2+438\chi-225\pi^2+102)\pi}{(\chi+1)\chi}G_{-1}(\chi)+\frac{3}{8}\frac{(455\chi\pi^2+438\chi-225\pi^2+102)\pi}{(\chi+1)\chi}G_{0}(\chi)\\
&-\frac{9}{2}\frac{(29\chi-27)\pi}{(\chi+1)\chi}G_{-1,-1}(\chi)+\frac{9}{2}\frac{(29\chi-27)\pi}{(\chi+1)\chi}G_{-1,0}(\chi)+\frac{9}{2}\frac{(29\chi-27)\pi}{(\chi+1)\chi}G_{0,-1}(\chi)-\frac{9}{2}\frac{(29\chi-27)\pi}{(\chi+1)\chi}G_{0,0}(\chi)\\
&+45\frac{(11\chi+3)\pi}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)-45\frac{(11\chi+3)\pi}{(\chi+1)\chi}G_{-1,-1,0}(\chi)-45\frac{(11\chi+3)\pi}{(\chi+1)\chi}G_{-1,0,-1}(\chi)+45\frac{(11\chi+3)\pi}{(\chi+1)\chi}G_{-1,0,0}(\chi)\\
&-45\frac{(11\chi+3)\pi}{(\chi+1)\chi}G_{0,-1,-1}(\chi)+45\frac{(11\chi+3)\pi}{(\chi+1)\chi}G_{0,-1,0}(\chi)+45\frac{(11\chi+3)\pi}{(\chi+1)\chi}G_{0,0,-1}(\chi)-45\frac{(11\chi+3)\pi}{(\chi+1)\chi}G_{0,0,0}(\chi)\Bigg)\epsilon^2\Bigg)\mu^{-1}\mu^{-4\epsilon}\\
&+O(\mu^0)
\end{aligned}
\tag{D.16}
$$

I[FNP1, 1, 1, 0, 1, 1, 0, 1, 1, 0] =

$$
\begin{aligned}
&\frac{1}{\epsilon^4}2\frac{1}{(\chi+1)\chi}i+\Bigg(\Bigg(78\frac{1}{\chi}G_{-1}(\chi)-78\frac{1}{\chi+1}G_{0}(\chi)+12\frac{2\chi+3}{(\chi+1)\chi}G_{-1,-1}(\chi)-12\frac{\chi+4}{(\chi+1)\chi}G_{-1,0}(\chi)-6\frac{4\chi-3}{(\chi+1)\chi}G_{0,-1}(\chi)\\
&+12\frac{\chi-5}{(\chi+1)\chi}G_{0,0}(\chi)\Big)\zeta_2+\Big(24\frac{1}{\chi+1}+\frac{2}{3}\frac{58\chi+53}{(\chi+1)\chi}G_{-1}(\chi)-\frac{4}{3}\frac{29\chi-11}{(\chi+1)\chi}G_{0}(\chi)\Big)\zeta_3-\frac{3}{2}\frac{6\chi+19}{(\chi+1)\chi}\zeta_4-96\frac{1}{\chi}G_{-1}(\chi)\\
&+96\frac{1}{\chi+1}G_{0}(\chi)-48\frac{1}{(\chi+1)\chi}G_{-1,0}(\chi)-48\frac{1}{(\chi+1)\chi}G_{0,-1}(\chi)+24\frac{1}{\chi}G_{-1,-1,-1}(\chi)-24\frac{1}{(\chi+1)\chi}G_{-1,-1,0}(\chi)\\
&-24\frac{1}{(\chi+1)\chi}G_{-1,0,-1}(\chi)+24\frac{1}{\chi}G_{-1,0,0}(\chi)-24\frac{1}{\chi+1}G_{0,-1,-1}(\chi)-24\frac{1}{(\chi+1)\chi}G_{0,-1,0}(\chi)-24\frac{1}{(\chi+1)\chi}G_{0,0,-1}(\chi)\\
&-24\frac{1}{\chi+1}G_{0,0,0}(\chi)-16\frac{\chi+2}{(\chi+1)\chi}G_{-1,-1,-1,-1}(\chi)+8\frac{2\chi+1}{(\chi+1)\chi}G_{-1,-1,-1,0}(\chi)+8\frac{2\chi+1}{(\chi+1)\chi}G_{-1,-1,0,-1}(\chi)\\
&-4\frac{\chi-1}{(\chi+1)\chi}G_{-1,-1,0,0}(\chi)-2\frac{2\chi-7}{(\chi+1)\chi}G_{-1,0,-1,-1}(\chi)+16\frac{1}{\chi}G_{-1,0,-1,0}(\chi)+16\frac{1}{\chi}G_{-1,0,0,-1}(\chi)-16\frac{1}{\chi}G_{-1,0,0,0}(\chi)\\
&+16\frac{1}{\chi+1}G_{0,-1,-1,-1}(\chi)-16\frac{1}{\chi+1}G_{0,-1,-1,0}(\chi)-16\frac{1}{\chi+1}G_{0,-1,0,-1}(\chi)+2\frac{2\chi+9}{(\chi+1)\chi}G_{0,-1,0,0}(\chi)\\
&+4\frac{\chi+2}{(\chi+1)\chi}G_{0,0,-1,-1}(\chi)-8\frac{2\chi+1}{(\chi+1)\chi}G_{0,0,-1,0}(\chi)-8\frac{2\chi+1}{(\chi+1)\chi}G_{0,0,0,-1}(\chi)+16\frac{\chi-1}{(\chi+1)\chi}G_{0,0,0,0}(\chi)\Bigg)i\\
&+\Bigg(\Big(-\frac{1}{2}\frac{2\chi+7}{(\chi+1)\chi}G_{-1}(\chi)+\frac{1}{2}\frac{2\chi-5}{(\chi+1)\chi}G_{0}(\chi)\Big)i+\frac{1}{2}\frac{(2\chi+7)\pi}{(\chi+1)\chi}\Bigg)\frac{1}{\epsilon^3}-\frac{(5\chi\pi^2-96\chi+\pi^2-96)\pi}{(\chi+1)\chi}-\frac{4}{3}\frac{(38\chi+73)\pi}{(\chi+1)\chi}\zeta_3\\
&-\frac{1}{3}\frac{(16\chi+7)\pi^3}{(\chi+1)\chi}G_{-1}(\chi)+\frac{2}{3}\frac{(8\chi\pi^2+13\pi^2+72)\pi}{(\chi+1)\chi}G_{0}(\chi)+\Bigg(\Bigg(-\frac{27}{2}\frac{1}{(\chi+1)\chi}\zeta_2-6\frac{1}{\chi}G_{-1}(\chi)+6\frac{1}{\chi+1}G_{0}(\chi)+2\frac{1}{(\chi+1)\chi}G_{-1,-1}(\chi)
\end{aligned}
$$

$$+4\frac{1}{(\chi+1)\chi}G_{-1,0}(\chi)+4\frac{1}{(\chi+1)\chi}G_{0,-1}(\chi)+2\frac{1}{(\chi+1)\chi}G_{0,0}(\chi)\Big)i+6\frac{\pi}{\chi}-2\frac{\pi}{(\chi+1)\chi}G_{-1}(\chi)-4\frac{\pi}{(\chi+1)\chi}G_{0}(\chi)\Big)\frac{1}{\epsilon^2}$$

$$-24\frac{\pi}{\chi}G_{-1,-1}(\chi)+24\frac{\pi}{(\chi+1)\chi}G_{-1,0}(\chi)+24\frac{\pi}{\chi+1}G_{0,-1}(\chi)+24\frac{\pi}{(\chi+1)\chi}G_{0,0}(\chi)+\Big(\Big(\Big(\Big(\frac{3}{2}\frac{6\chi+13}{(\chi+1)\chi}G_{-1}(\chi)-\frac{3}{2}\frac{6\chi-19}{(\chi+1)\chi}G_{0}(\chi)\Big)\zeta_2$$

$$-\frac{65}{6}\frac{1}{(\chi+1)\chi}\zeta_3+24\frac{1}{\chi}G_{-1}(\chi)-24\frac{1}{\chi+1}G_{0}(\chi)+12\frac{1}{(\chi+1)\chi}G_{-1,0}(\chi)+12\frac{1}{(\chi+1)\chi}G_{0,-1}(\chi)+2\frac{2\chi+3}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)$$

$$-2\frac{2\chi+3}{(\chi+1)\chi}G_{-1,-1,0}(\chi)-2\frac{2\chi+3}{(\chi+1)\chi}G_{-1,0,-1}(\chi)+4\frac{1}{\chi+1}G_{-1,0,0}(\chi)-4\frac{1}{\chi}G_{0,-1,-1}(\chi)+2\frac{2\chi-1}{(\chi+1)\chi}G_{0,-1,0}(\chi)$$

$$+2\frac{2\chi-1}{(\chi+1)\chi}G_{0,0,-1}(\chi)-2\frac{2\chi-1}{(\chi+1)\chi}G_{0,0,0}(\chi)\Big)i-\frac{1}{12}\frac{(34\chi\pi^2+288\chi+55\pi^2+288)\pi}{(\chi+1)\chi}-12\frac{\pi}{(\chi+1)\chi}G_{0}(\chi)-2\frac{(2\chi+3)\pi}{(\chi+1)\chi}G_{-1,-1}(\chi)$$

$$+2\frac{(2\chi+3)\pi}{(\chi+1)\chi}G_{-1,0}(\chi)+4\frac{\pi}{\chi}G_{0,-1}(\chi)-2\frac{(2\chi-1)\pi}{(\chi+1)\chi}G_{0,0}(\chi)\Big)\frac{1}{\epsilon}+16\frac{(\chi+2)\pi}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)-8\frac{(2\chi+1)\pi}{(\chi+1)\chi}G_{-1,-1,0}(\chi)$$

$$+2\frac{(2\chi-7)\pi}{(\chi+1)\chi}G_{-1,0,-1}(\chi)-16\frac{\pi}{\chi}G_{-1,0,0}(\chi)-16\frac{\pi}{\chi+1}G_{0,-1,-1}(\chi)+16\frac{\pi}{\chi+1}G_{0,-1,0}(\chi)-4\frac{(\chi+2)\pi}{(\chi+1)\chi}G_{0,0,-1}(\chi)$$

$$+8\frac{(2\chi+1)\pi}{(\chi+1)\chi}G_{0,0,0}(\chi)+\Big(-\frac{1}{\epsilon^4}\frac{2}{3}\frac{\chi+7}{(\chi+1)\chi}i+\Big(\frac{19\chi+37}{(\chi+1)\chi}+\Big(\frac{5\chi-56}{(\chi+1)\chi}-\frac{1}{3}\frac{108\chi+169}{(\chi+1)\chi}G_{-1}(\chi)+\frac{1}{3}\frac{117\chi-43}{(\chi+1)\chi}G_{0}(\chi)$$

$$-\frac{1}{3}\frac{20\chi+107}{(\chi+1)\chi}G_{-1,-1}(\chi)+\frac{8}{3}\frac{\chi+17}{(\chi+1)\chi}G_{-1,0}(\chi)+\frac{20}{3}\frac{\chi-1}{(\chi+1)\chi}G_{0,-1}(\chi)-\frac{2}{3}\frac{4\chi-77}{(\chi+1)\chi}G_{0,0}(\chi)\Big)\zeta_2+\Big(-\frac{4}{9}\frac{45\chi-4}{(\chi+1)\chi}$$

$$-\frac{2}{9}\frac{83\chi+137}{(\chi+1)\chi}G_{-1}(\chi)+\frac{2}{9}\frac{83\chi-107}{(\chi+1)\chi}G_{0}(\chi)\Big)\zeta_3+\frac{1}{6}\frac{218\chi-1045}{(\chi+1)\chi}\zeta_4+3\frac{13\chi+15}{(\chi+1)\chi}G_{-1}(\chi)-\frac{39\chi-11}{(\chi+1)\chi}G_{0}(\chi)-\frac{5\chi+3}{(\chi+1)\chi}G_{-1,-1}(\chi)$$

$$+2\frac{\chi+10}{(\chi+1)\chi}G_{-1,0}(\chi)+2\frac{\chi+10}{(\chi+1)\chi}G_{0,-1}(\chi)+\frac{\chi+3}{(\chi+1)\chi}G_{0,0}(\chi)-\frac{2}{3}\frac{18\chi+17}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)+\frac{1}{3}\frac{3\chi+23}{(\chi+1)\chi}G_{-1,-1,0}(\chi)$$

$$+\frac{1}{3}\frac{3\chi+23}{(\chi+1)\chi}G_{-1,0,-1}(\chi)-\frac{1}{3}\frac{39\chi+37}{(\chi+1)\chi}G_{-1,0,0}(\chi)+\frac{1}{3}\frac{39\chi-7}{(\chi+1)\chi}G_{0,-1,-1}(\chi)-\frac{1}{3}\frac{3\chi-29}{(\chi+1)\chi}G_{0,-1,0}(\chi)-\frac{1}{3}\frac{3\chi-29}{(\chi+1)\chi}G_{0,0,-1}(\chi)$$

$$+\frac{2}{3}\frac{18\chi+1}{(\chi+1)\chi}G_{0,0,0}(\chi)+\frac{1}{3}\frac{13\chi+46}{(\chi+1)\chi}G_{-1,-1,-1,-1}(\chi)-\frac{1}{3}\frac{13\chi+11}{(\chi+1)\chi}G_{-1,-1,-1,0}(\chi)-\frac{1}{3}\frac{13\chi+11}{(\chi+1)\chi}G_{-1,-1,0,-1}(\chi)$$

$$+\frac{1}{3}\frac{\chi-5}{(\chi+1)\chi}G_{-1,-1,0,0}(\chi)+\frac{1}{3}\frac{11\chi-23}{(\chi+1)\chi}G_{-1,0,-1,-1}(\chi)-\frac{23}{3}\frac{1}{\chi}G_{-1,0,-1,0}(\chi)-\frac{23}{3}\frac{1}{\chi}G_{-1,0,0,-1}(\chi)+\frac{1}{3}\frac{23\chi+25}{(\chi+1)\chi}G_{-1,0,0,0}(\chi)$$

$$-\frac{1}{3}\frac{13\chi-7}{(\chi+1)\chi}G_{0,-1,-1,-1}(\chi)+\frac{1}{3}\frac{13\chi-5}{(\chi+1)\chi}G_{0,-1,-1,0}(\chi)+\frac{1}{3}\frac{13\chi-5}{(\chi+1)\chi}G_{0,-1,0,-1}(\chi)-\frac{1}{3}\frac{\chi+29}{(\chi+1)\chi}G_{0,-1,0,0}(\chi)$$

$$-\frac{11}{3}\frac{1}{\chi}G_{0,0,-1,-1}(\chi)+\frac{1}{3}\frac{23\chi+7}{(\chi+1)\chi}G_{0,0,-1,0}(\chi)+\frac{1}{3}\frac{23\chi+7}{(\chi+1)\chi}G_{0,0,0,-1}(\chi)-\frac{1}{3}\frac{23\chi-31}{(\chi+1)\chi}G_{0,0,0,0}(\chi)\Big)i$$

$$+\frac{1}{2}\frac{(5\chi\pi^2-78\chi+8\pi^2-120)\pi}{(\chi+1)\chi}+\Big(\Big(-\frac{2}{3}\frac{1}{(\chi+1)\chi}+\frac{1}{3}\frac{5\chi+14}{(\chi+1)\chi}G_{-1}(\chi)-\frac{1}{3}\frac{5\chi-11}{(\chi+1)\chi}G_{0}(\chi)\Big)i-\frac{5}{3}\frac{(\chi+5)\pi}{(\chi+1)\chi}\Big)\frac{1}{\epsilon^3}$$

$$+\frac{2}{9}\frac{(101\chi+415)\pi}{(\chi+1)\chi}\zeta_3+\frac{1}{6}\frac{(14\chi\pi^2+12\chi-\pi^2-30)\pi}{(\chi+1)\chi}G_{-1}(\chi)-\frac{1}{3}\frac{(7\chi\pi^2-3\chi+24\pi^2+75)\pi}{(\chi+1)\chi}G_{0}(\chi)+\Big(\Big(\frac{\chi+3}{(\chi+1)\chi}-\frac{2}{3}\frac{\chi-59}{(\chi+1)\chi}\zeta_2$$

$$+\frac{2}{3}\frac{9\chi+10}{(\chi+1)\chi}G_{-1}(\chi)-\frac{2}{3}\frac{9\chi-1}{(\chi+1)\chi}G_{0}(\chi)+\frac{1}{3}\frac{\chi-2}{(\chi+1)\chi}G_{-1,-1}(\chi)-\frac{1}{3}\frac{\chi+11}{(\chi+1)\chi}G_{-1,0}(\chi)-\frac{1}{3}\frac{\chi+11}{(\chi+1)\chi}G_{0,-1}(\chi)$$

$$+\frac{1}{3}\frac{\chi-5}{(\chi+1)\chi}G_{0,0}(\chi)\Big)i-\frac{1}{3}\frac{(18\chi+25)\pi}{(\chi+1)\chi}-\frac{1}{3}\frac{(\chi-10)\pi}{(\chi+1)\chi}G_{-1}(\chi)+\frac{1}{3}\frac{(\chi+19)\pi}{(\chi+1)\chi}G_{0}(\chi)\Big)\frac{1}{\epsilon^2}+\frac{1}{3}\frac{(39\chi+32)\pi}{(\chi+1)\chi}G_{-1,-1}(\chi)$$

$$-\frac{1}{3}\frac{(3\chi+31)\pi}{(\chi+1)\chi}G_{-1,0}(\chi)-\frac{1}{3}\frac{(39\chi+1)\pi}{(\chi+1)\chi}G_{0,-1}(\chi)-\frac{34}{3}\frac{\pi}{(\chi+1)\chi}G_{0,0}(\chi)+\Big(\Big(-\frac{5\chi+11}{(\chi+1)\chi}+\Big(-\frac{1}{3}\frac{9\chi-52}{(\chi+1)\chi}-\frac{1}{3}\frac{16\chi+67}{(\chi+1)\chi}G_{-1}(\chi)$$

$$+\frac{2}{3}\frac{8\chi-65}{(\chi+1)\chi}G_{0}(\chi)\Big)\zeta_2+\frac{2}{9}\frac{8\chi+119}{(\chi+1)\chi}\zeta_3-\frac{15\chi+17}{(\chi+1)\chi}G_{-1}(\chi)+3\frac{5\chi-1}{(\chi+1)\chi}G_{0}(\chi)+\frac{2}{3}\frac{3\chi+2}{(\chi+1)\chi}G_{-1,-1}(\chi)-\frac{1}{3}\frac{3\chi+23}{(\chi+1)\chi}G_{-1,0}(\chi)$$

$$-\frac{1}{3}\frac{3\chi+23}{(\chi+1)\chi}G_{0,-1}(\chi)-\frac{2}{3}\frac{1}{(\chi+1)\chi}G_{0,0}(\chi)-\frac{1}{3}\frac{7\chi+16}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)+\frac{1}{3}\frac{7\chi+11}{(\chi+1)\chi}G_{-1,-1,0}(\chi)+\frac{1}{3}\frac{7\chi+11}{(\chi+1)\chi}G_{-1,0,-1}(\chi)$$

$$-\frac{1}{3}\frac{7\chi+1}{(\chi+1)\chi}G_{-1,0,0}(\chi)+\frac{1}{3}\frac{7\chi+5}{(\chi+1)\chi}G_{0,-1,-1}(\chi)-\frac{1}{3}\frac{7\chi-5}{(\chi+1)\chi}G_{0,-1,0}(\chi)-\frac{1}{3}\frac{7\chi-5}{(\chi+1)\chi}G_{0,0,-1}(\chi)+\frac{7}{3}\frac{\chi-1}{(\chi+1)\chi}G_{0,0,0}(\chi)\Big)i$$

$$+\frac{1}{3}\frac{(5\chi\pi^2+45\chi+21\pi^2+66)\pi}{(\chi+1)\chi}-\frac{1}{3}\frac{(3\chi-4)\pi}{(\chi+1)\chi}G_{-1}(\chi)+\frac{28}{3}\frac{\pi}{(\chi+1)\chi}G_{0}(\chi)+\frac{7}{3}\frac{(\chi+2)\pi}{(\chi+1)\chi}G_{-1,-1}(\chi)-\frac{1}{3}\frac{(7\chi+19)\pi}{(\chi+1)\chi}G_{-1,0}(\chi)$$

$$-\frac{1}{3}\frac{(7\chi+13)\pi}{(\chi+1)\chi}G_{0,-1}(\chi)+\frac{7}{3}\frac{(\chi-1)\pi}{(\chi+1)\chi}G_{0,0}(\chi)\Big)\frac{1}{\epsilon}-\frac{1}{3}\frac{(13\chi+56)\pi}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)+\frac{1}{3}\frac{(13\chi+19)\pi}{(\chi+1)\chi}G_{-1,-1,0}(\chi)$$

$$-\frac{1}{3}\frac{(11\chi-31)\pi}{(\chi+1)\chi}G_{-1,0,-1}(\chi)+\frac{1}{3}\frac{(23\chi+25)\pi}{(\chi+1)\chi}G_{-1,0,0}(\chi)+\frac{1}{3}\frac{(13\chi-5)\pi}{(\chi+1)\chi}G_{0,-1,-1}(\chi)-\frac{13}{3}\frac{(\chi-1)\pi}{(\chi+1)\chi}G_{0,-1,0}(\chi)$$

$$+\frac{1}{3}\frac{(11\chi+19)\pi}{(\chi+1)\chi}G_{0,0,-1}(\chi)-\frac{1}{3}\frac{(23\chi+17)\pi}{(\chi+1)\chi}G_{0,0,0}(\chi)\Big)\mu^{-\epsilon}+\Big(\frac{1}{\epsilon^4}\frac{1}{8}\frac{6\chi+29}{(\chi+1)\chi}i+\Big(-4\frac{2\chi+5}{(\chi+1)\chi}+\Big(-\frac{\chi-44}{(\chi+1)\chi}+\frac{9}{2}\frac{1}{\chi}G_{-1}(\chi)$$

$$-\frac{1}{2}\frac{8\chi-5}{(\chi+1)\chi}G_{0}(\chi)+6\frac{1}{(\chi+1)\chi}G_{0,-1}(\chi)-\frac{11}{2}\frac{1}{(\chi+1)\chi}G_{0,0}(\chi)\Big)\zeta_2+\Big(4\frac{\chi-3}{(\chi+1)\chi}+\frac{2}{3}\frac{2\chi+1}{(\chi+1)\chi}G_{-1}(\chi)-\frac{4}{3}\frac{\chi-4}{(\chi+1)\chi}G_{0}(\chi)\Big)\zeta_3$$

$$-\frac{1}{32}\frac{126\chi-3763}{(\chi+1)\chi}\zeta_4+2\frac{\chi-2}{(\chi+1)\chi}G_{0}(\chi)-\frac{\chi+2}{(\chi+1)\chi}G_{0,0}(\chi)+2\frac{1}{\chi}G_{-1,0,0}(\chi)-\frac{1}{2}\frac{3\chi+2}{(\chi+1)\chi}G_{0,0,0}(\chi)+2\frac{1}{(\chi+1)\chi}G_{0,-1,0,0}(\chi)$$

$$-\frac{3}{2}\frac{1}{(\chi+1)\chi}G_{0,0,0,0}(\chi)\Big)i+\Big(-\frac{1}{\epsilon^2}3\frac{(\mu^2)^{\frac{3}{4}}}{(\chi^2)^{\frac{1}{4}}((\chi+1)^2)^{\frac{1}{4}}}\zeta_2+\frac{1}{\epsilon}24\frac{(\mu^2)^{\frac{3}{4}}}{(\chi^2)^{\frac{1}{4}}((\chi+1)^2)^{\frac{1}{4}}}G_{0}(2)\zeta_2-192\frac{(\mu^2)^{\frac{3}{4}}}{(\chi^2)^{\frac{1}{4}}((\chi+1)^2)^{\frac{1}{4}}}G_{0,0}(2)\zeta_2$$

$$-\frac{105}{2}\frac{(\mu^2)^{\frac{3}{4}}}{(\chi^2)^{\frac{1}{4}}((\chi+1)^2)^{\frac{1}{4}}}\zeta_4)\mu^{-2}+\frac{1}{12}\frac{(6\chi\pi^2+24\chi-25\pi^2+192)\pi}{(\chi+1)\chi}+((\frac{3}{2}\frac{1}{(\chi+1)\chi}-\frac{1}{4}\frac{2\chi+1}{(\chi+1)\chi}G_{-1}(\chi)+\frac{1}{4}\frac{2\chi-5}{(\chi+1)\chi}G_0(\chi))i$$

$$+\frac{1}{4}\frac{(2\chi+23)\pi}{(\chi+1)\chi})\frac{1}{\epsilon^3}-\frac{2}{3}\frac{(2\chi+59)\pi}{(\chi+1)\chi}\zeta_3-\frac{1}{12}\frac{(12\chi-7\pi^2-24)\pi}{(\chi+1)\chi}G_0(\chi)+((-\frac{\chi+4}{(\chi+1)\chi}+\frac{1}{8}\frac{6\chi-209}{(\chi+1)\chi}\zeta_2-\frac{1}{2}\frac{1}{\chi}G_{-1}(\chi)$$

$$+\frac{1}{2}\frac{2\chi-1}{(\chi+1)\chi}G_0(\chi)+\frac{1}{2}\frac{1}{(\chi+1)\chi}G_{0,0}(\chi))i+\frac{1}{2}\frac{(2\chi+9)\pi}{(\chi+1)\chi}-\frac{3}{2}\frac{\pi}{(\chi+1)\chi}G_0(\chi))\frac{1}{\epsilon^2}+\frac{1}{2}\frac{(\chi+2)\pi}{(\chi+1)\chi}G_{0,0}(\chi)+((2\frac{2\chi+5}{(\chi+1)\chi}$$

$$+(\frac{3}{2}\frac{2\chi-13}{(\chi+1)\chi}-\frac{3}{4}\frac{2\chi+1}{(\chi+1)\chi}G_{-1}(\chi)+\frac{1}{4}\frac{6\chi+29}{(\chi+1)\chi}G_0(\chi))\zeta_2+\frac{1}{6}\frac{3\chi-124}{(\chi+1)\chi}\zeta_3-\frac{\chi-2}{(\chi+1)\chi}G_0(\chi)+\frac{1}{2}\frac{\chi+2}{(\chi+1)\chi}G_{0,0}(\chi)$$

$$+\frac{1}{2}\frac{1}{(\chi+1)\chi}G_{0,0,0}(\chi))i+\frac{1}{24}\frac{(6\chi\pi^2-24\chi-67\pi^2-192)\pi}{(\chi+1)\chi}+\frac{1}{2}\frac{(\chi-2)\pi}{(\chi+1)\chi}G_0(\chi)+\frac{1}{2}\frac{\pi}{(\chi+1)\chi}G_{0,0}(\chi))\frac{1}{\epsilon}+\frac{1}{2}\frac{\pi}{(\chi+1)\chi}G_{0,0,0}(\chi)$$

$$+(\frac{1}{\epsilon^3}\frac{1}{4}\frac{2\chi-1}{(\chi+1)\chi}i-\frac{1}{\epsilon^2}\frac{1}{2}\frac{1}{\chi}i+\frac{1}{\epsilon}\frac{3}{4}\frac{2\chi-1}{(\chi+1)\chi}\zeta_2 i+(-\frac{3}{2}\frac{1}{\chi}\zeta_2-\frac{2}{3}\frac{2\chi-1}{(\chi+1)\chi}\zeta_3)i)\log(\mu))\mu^{-2\epsilon}+(-\frac{1}{\epsilon^4}\frac{1}{(\chi+1)\chi}i+(\frac{1}{(\chi+1)\chi}$$

$$+(-\frac{1}{(\chi+1)\chi}+\frac{1}{(\chi+1)\chi}G_{-1}(\chi)+\frac{1}{(\chi+1)\chi}G_{-1,-1}(\chi))\zeta_2+(1+G_{-1}(\chi))\frac{14}{3}\frac{1}{(\chi+1)\chi}\zeta_3+\frac{21}{4}\frac{1}{(\chi+1)\chi}\zeta_4-\frac{1}{(\chi+1)\chi}G_{-1}(\chi)$$

$$+\frac{1}{(\chi+1)\chi}G_{-1,-1}(\chi)-\frac{1}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)-\frac{1}{(\chi+1)\chi}G_{-1,-1,-1,-1}(\chi))i-\frac{1}{6}\frac{(\pi^2+6)\pi}{(\chi+1)\chi}+(-(1+G_{-1}(\chi))\frac{1}{(\chi+1)\chi}i$$

$$-\frac{\pi}{(\chi+1)\chi})\frac{1}{\epsilon^3}+\frac{14}{3}\frac{\pi}{(\chi+1)\chi}\zeta_3-\frac{1}{6}\frac{(\pi^2-6)\pi}{(\chi+1)\chi}G_{-1}(\chi)+((\frac{1}{(\chi+1)\chi}+\frac{1}{(\chi+1)\chi}\zeta_2-\frac{1}{(\chi+1)\chi}G_{-1}(\chi)-\frac{1}{(\chi+1)\chi}G_{-1,-1}(\chi))i$$

$$-(1+G_{-1}(\chi))\frac{\pi}{(\chi+1)\chi})\frac{1}{\epsilon^2}-\frac{\pi}{(\chi+1)\chi}G_{-1,-1}(\chi)+((-\frac{1}{(\chi+1)\chi}+(1+G_{-1}(\chi))\frac{1}{(\chi+1)\chi}\zeta_2+\frac{14}{3}\frac{1}{(\chi+1)\chi}\zeta_3$$

$$+\frac{1}{(\chi+1)\chi}G_{-1}(\chi)-\frac{1}{(\chi+1)\chi}G_{-1,-1}(\chi)-\frac{1}{(\chi+1)\chi}G_{-1,-1,-1}(\chi))i-\frac{1}{6}\frac{(\pi^2-6)\pi}{(\chi+1)\chi}-\frac{\pi}{(\chi+1)\chi}G_{-1}(\chi)-\frac{\pi}{(\chi+1)\chi}G_{-1,-1}(\chi))\frac{1}{\epsilon}$$

$$-\frac{\pi}{(\chi+1)\chi}G_{-1,-1,-1}(\chi))\mu^{-3\epsilon}+(-\frac{1}{\epsilon^4}\frac{1}{24}\frac{2\chi-1}{(\chi+1)\chi}i+((\frac{7}{3}\frac{1}{(\chi+1)\chi}G_{-1}(\chi)-\frac{7}{3}\frac{2\chi+1}{(\chi-1)(\chi+1)\chi}G_0(\chi)-\frac{7}{6}\frac{2\chi-1}{(\chi+1)\chi}G_{-1,-1}(\chi)$$

$$+\frac{7}{6}\frac{2\chi+1}{(\chi+1)\chi}G_{-1,0}(\chi)+\frac{7}{6}\frac{2\chi+1}{(\chi+1)\chi}G_{0,-1}(\chi)-\frac{7}{6}\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{0,0}(\chi))\zeta_2+(-\frac{2}{9}\frac{47\chi^2+95\chi+20}{(\chi+1)(\chi-1)^2\chi}$$

$$+\frac{2}{9}\frac{20\chi^3-23\chi^2-41\chi-10}{(\chi+1)(\chi-1)^2\chi}G_{-1}(\chi)+\frac{1}{9}\frac{68\chi^3-75\chi^2-135\chi-20}{(\chi+1)(\chi-1)^2\chi}G_0(\chi)-6\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_1(\chi))\zeta_3$$

$$+\frac{1}{96}\frac{850\chi^3-253\chi^2-3340\chi+151}{(\chi+1)(\chi-1)^2\chi}\zeta_4+\frac{4}{3}\frac{1}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)-\frac{4}{3}\frac{2\chi+1}{(\chi-1)(\chi+1)\chi}G_{-1,-1,0}(\chi)-\frac{4}{3}\frac{2\chi+1}{(\chi-1)(\chi+1)\chi}G_{-1,0,-1}(\chi)$$

$$+\frac{2}{3}\frac{(\chi+2)(5\chi+1)}{(\chi+1)(\chi-1)^2\chi}G_{-1,0,0}(\chi)-\frac{4}{3}\frac{2\chi+1}{(\chi-1)(\chi+1)\chi}G_{0,-1,-1}(\chi)+\frac{2}{3}\frac{(\chi+2)(5\chi+1)}{(\chi+1)(\chi-1)^2\chi}G_{0,-1,0}(\chi)+\frac{2}{3}\frac{(\chi+2)(5\chi+1)}{(\chi+1)(\chi-1)^2\chi}G_{0,0,-1}(\chi)$$

$$+\frac{1}{3}\frac{19\chi^2+31\chi+4}{(\chi+1)(\chi-1)^2\chi}G_{0,0,0}(\chi)-2\frac{(\chi+2)(5\chi+1)}{(\chi+1)(\chi-1)^2\chi}G_{1,0,0}(\chi)-\frac{2}{3}\frac{2\chi-1}{(\chi+1)\chi}G_{-1,-1,-1,-1}(\chi)+\frac{2}{3}\frac{2\chi+1}{(\chi+1)\chi}G_{-1,-1,-1,0}(\chi)$$

$$+\frac{2}{3}\frac{2\chi+1}{(\chi+1)\chi}G_{-1,-1,0,-1}(\chi)-\frac{2}{3}\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{-1,-1,0,0}(\chi)+\frac{2}{3}\frac{2\chi+1}{(\chi+1)\chi}G_{-1,0,-1,-1}(\chi)-\frac{2}{3}\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{-1,0,-1,0}(\chi)$$

$$-\frac{2}{3}\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{-1,0,0,-1}(\chi)-\frac{1}{3}\frac{8\chi^3-9\chi^2-15\chi-2}{(\chi+1)(\chi-1)^2\chi}G_{-1,0,0,0}(\chi)+2\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{-1,1,0,0}(\chi)+\frac{2}{3}\frac{2\chi+1}{(\chi+1)\chi}G_{0,-1,-1,-1}(\chi)$$

$$-\frac{2}{3}\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{0,-1,-1,0}(\chi)-\frac{2}{3}\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{0,-1,0,-1}(\chi)-\frac{1}{3}\frac{8\chi^3-9\chi^2-15\chi-2}{(\chi+1)(\chi-1)^2\chi}G_{0,-1,0,0}(\chi)$$

$$-\frac{2}{3}\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{0,0,-1,-1}(\chi)-\frac{1}{3}\frac{8\chi^3-9\chi^2-15\chi-2}{(\chi+1)(\chi-1)^2\chi}G_{0,0,-1,0}(\chi)-\frac{1}{3}\frac{8\chi^3-9\chi^2-15\chi-2}{(\chi+1)(\chi-1)^2\chi}G_{0,0,0,-1}(\chi)$$

$$-\frac{1}{6}\frac{20\chi^3-23\chi^2-35\chi-4}{(\chi+1)(\chi-1)^2\chi}G_{0,0,0,0}(\chi)+\frac{8\chi^3-9\chi^2-15\chi-2}{(\chi+1)(\chi-1)^2\chi}G_{0,1,0,0}(\chi)+2\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{1,-1,0,0}(\chi)$$

$$+2\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{1,0,-1,0}(\chi)+2\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{1,0,0,-1}(\chi)+3\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{1,0,0,0}(\chi)-6\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{1,1,0,0}(\chi))i$$

$$+((-\frac{7}{2}\frac{1}{\chi-1}G_{0,0}(\chi)\zeta_2+(6\frac{1}{\chi-1}G_{-1}(\chi)+9\frac{1}{\chi-1}G_0(\chi)-18\frac{1}{\chi-1}G_1(\chi))\zeta_3+\frac{27}{2}\frac{1}{\chi-1}\zeta_4-2\frac{1}{\chi-1}G_{-1,-1,0,0}(\chi)$$

$$-2\frac{1}{\chi-1}G_{-1,0,-1,0}(\chi)-2\frac{1}{\chi-1}G_{-1,0,0,-1}(\chi)-3\frac{1}{\chi-1}G_{-1,0,0,0}(\chi)+6\frac{1}{\chi-1}G_{-1,1,0,0}(\chi)-2\frac{1}{\chi-1}G_{0,-1,-1,0}(\chi)$$

$$-2\frac{1}{\chi-1}G_{0,-1,0,-1}(\chi)-3\frac{1}{\chi-1}G_{0,-1,0,0}(\chi)-2\frac{1}{\chi-1}G_{0,0,-1,-1}(\chi)-3\frac{1}{\chi-1}G_{0,0,-1,0}(\chi)-3\frac{1}{\chi-1}G_{0,0,0,-1}(\chi)$$

$$-\frac{7}{2}\frac{1}{\chi-1}G_{0,0,0,0}(\chi)+9\frac{1}{\chi-1}G_{0,1,0,0}(\chi)+6\frac{1}{\chi-1}G_{1,-1,0,0}(\chi)+6\frac{1}{\chi-1}G_{1,0,-1,0}(\chi)+6\frac{1}{\chi-1}G_{1,0,0,-1}(\chi)+9\frac{1}{\chi-1}G_{1,0,0,0}(\chi)$$

$$-18\frac{1}{\chi-1}G_{1,1,0,0}(\chi))i+9\frac{\pi}{\chi-1}\zeta_3-\frac{\pi^3}{\chi-1}G_{-1}(\chi)-\frac{7}{4}\frac{\pi^3}{\chi-1}G_0(\chi)+(-\frac{1}{2}\frac{1}{\chi-1}G_{0,0}(\chi)i-\frac{1}{2}\frac{\pi}{\chi-1}G_0(\chi))\frac{1}{\epsilon^2}+3\frac{\pi^3}{\chi-1}G_1(\chi)$$

$$+((3\frac{1}{\chi-1}\zeta_3-\frac{1}{\chi-1}G_{-1,0,0}(\chi)-\frac{1}{\chi-1}G_{0,-1,0}(\chi)-\frac{1}{\chi-1}G_{0,0,-1}(\chi)-\frac{3}{2}\frac{1}{\chi-1}G_{0,0,0}(\chi)+3\frac{1}{\chi-1}G_{1,0,0}(\chi))i-\frac{1}{2}\frac{\pi^3}{\chi-1}$$

$$-\frac{\pi}{\chi-1}G_{-1,0}(\chi)-\frac{\pi}{\chi-1}G_{0,-1}(\chi)-\frac{3}{2}\frac{\pi}{\chi-1}G_{0,0}(\chi)+3\frac{\pi}{\chi-1}G_{1,0}(\chi))\frac{1}{\epsilon}-2\frac{\pi}{\chi-1}G_{-1,-1,0}(\chi)-2\frac{\pi}{\chi-1}G_{-1,0,-1}(\chi)$$

$$-3\frac{\pi}{\chi-1}G_{-1,0,0}(\chi)+6\frac{\pi}{\chi-1}G_{-1,1,0}(\chi)-2\frac{\pi}{\chi-1}G_{0,-1,-1}(\chi)-3\frac{\pi}{\chi-1}G_{0,-1,0}(\chi)-3\frac{\pi}{\chi-1}G_{0,0,-1}(\chi)-\frac{7}{2}\frac{\pi}{\chi-1}G_{0,0,0}(\chi)$$

$$
\begin{aligned}
&+ 9\frac{\pi}{\chi-1}G_{0,1,0}(\chi) + 6\frac{\pi}{\chi-1}G_{1,-1,0}(\chi) + 6\frac{\pi}{\chi-1}G_{1,0,-1}(\chi) + 9\frac{\pi}{\chi-1}G_{1,0,0}(\chi) - 18\frac{\pi}{\chi-1}G_{1,1,0}(\chi)\Bigg)\mu^{-1} + \Bigg(\Bigg(\frac{1}{6}\frac{1}{(\chi+1)\chi} \\
&- \frac{1}{12}\frac{2\chi-1}{(\chi+1)\chi}G_{-1}(\chi) + \frac{1}{12}\frac{2\chi+1}{(\chi+1)\chi}G_{0}(\chi)\Bigg)i + \frac{1}{12}\frac{(2\chi+1)\pi}{(\chi+1)\chi}\Bigg)\frac{1}{\epsilon^3} + \frac{1}{6}\frac{(8\chi^2+23\chi+5)\pi^3}{(\chi+1)(\chi-1)^2\chi} + \frac{1}{9}\frac{(68\chi^3-75\chi^2-135\chi-20)\pi}{(\chi+1)(\chi-1)^2\chi}\zeta_3 \\
&- \frac{1}{12}\frac{(6\chi^3-5\chi^2-20\chi-5)\pi^3}{(\chi+1)(\chi-1)^2\chi}G_{-1}(\chi) - \frac{1}{12}\frac{(18\chi^3-20\chi^2-35\chi-5)\pi^3}{(\chi+1)(\chi-1)^2\chi}G_{0}(\chi) + \Bigg(\Bigg(-\frac{7}{24}\frac{2\chi-1}{(\chi+1)\chi}\zeta_2 + \frac{1}{3}\frac{1}{(\chi+1)\chi}G_{-1}(\chi) \\
&- \frac{1}{3}\frac{2\chi+1}{(\chi-1)(\chi+1)\chi}G_{0}(\chi) - \frac{1}{6}\frac{2\chi-1}{(\chi+1)\chi}G_{-1,-1}(\chi) + \frac{1}{6}\frac{2\chi+1}{(\chi+1)\chi}G_{-1,0}(\chi) + \frac{1}{6}\frac{2\chi+1}{(\chi+1)\chi}G_{0,-1}(\chi) - \frac{1}{6}\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{0,0}(\chi)\Bigg)i \\
&- \frac{1}{3}\frac{(2\chi+1)\pi}{(\chi-1)(\chi+1)\chi} + \frac{1}{6}\frac{(2\chi+1)\pi}{(\chi+1)\chi}G_{-1}(\chi) - \frac{1}{6}\frac{(2\chi^2-4\chi-1)\pi}{(\chi-1)^2\chi}G_{0}(\chi)\Bigg)\frac{1}{\epsilon^2} + \frac{(2\chi^2-4\chi-1)\pi^3}{(\chi-1)^2\chi}G_{1}(\chi) \\
&- \frac{4}{3}\frac{(2\chi+1)\pi}{(\chi-1)(\chi+1)\chi}G_{-1,-1}(\chi) + \frac{2}{3}\frac{(\chi+2)(5\chi+1)\pi}{(\chi+1)(\chi-1)^2\chi}G_{-1,0}(\chi) + \frac{2}{3}\frac{(\chi+2)(5\chi+1)\pi}{(\chi+1)(\chi-1)^2\chi}G_{0,-1}(\chi) + \frac{1}{3}\frac{(19\chi^2+31\chi+4)\pi}{(\chi+1)(\chi-1)^2\chi}G_{0,0}(\chi) \\
&- 2\frac{(\chi+2)(5\chi+1)\pi}{(\chi+1)(\chi-1)^2\chi}G_{1,0}(\chi) + \Bigg(\Bigg(\Bigg(\frac{7}{6}\frac{1}{(\chi+1)\chi} - \frac{7}{12}\frac{2\chi-1}{(\chi+1)\chi}G_{-1}(\chi) + \frac{7}{12}\frac{2\chi+1}{(\chi+1)\chi}G_{0}(\chi)\Bigg)\zeta_2 + \frac{1}{9}\frac{20\chi^3-23\chi^2-41\chi-10}{(\chi+1)(\chi-1)^2\chi}\zeta_3 \\
&+ \frac{2}{3}\frac{1}{(\chi+1)\chi}G_{-1,-1}(\chi) - \frac{2}{3}\frac{2\chi+1}{(\chi-1)(\chi+1)\chi}G_{-1,0}(\chi) - \frac{2}{3}\frac{2\chi+1}{(\chi-1)(\chi+1)\chi}G_{0,-1}(\chi) + \frac{1}{3}\frac{(\chi+2)(5\chi+1)}{(\chi+1)(\chi-1)^2\chi}G_{0,0}(\chi) \\
&- \frac{1}{3}\frac{2\chi-1}{(\chi+1)\chi}G_{-1,-1,-1}(\chi) + \frac{1}{3}\frac{2\chi+1}{(\chi+1)\chi}G_{-1,-1,0}(\chi) + \frac{1}{3}\frac{2\chi+1}{(\chi+1)\chi}G_{-1,0,-1}(\chi) - \frac{1}{3}\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{-1,0,0}(\chi) \\
&+ \frac{1}{3}\frac{2\chi+1}{(\chi+1)\chi}G_{0,-1,-1}(\chi) - \frac{1}{3}\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{0,-1,0}(\chi) - \frac{1}{3}\frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{0,0,-1}(\chi) - \frac{1}{6}\frac{8\chi^3-9\chi^2-15\chi-2}{(\chi+1)(\chi-1)^2\chi}G_{0,0,0}(\chi) \\
&+ \frac{2\chi^2-4\chi-1}{(\chi-1)^2\chi}G_{1,0,0}(\chi)\Bigg)i - \frac{1}{24}\frac{(6\chi^3-5\chi^2-20\chi-5)\pi^3}{(\chi+1)(\chi-1)^2\chi} - \frac{2}{3}\frac{(2\chi+1)\pi}{(\chi-1)(\chi+1)\chi}G_{-1}(\chi) + \frac{1}{3}\frac{(\chi+2)(5\chi+1)\pi}{(\chi+1)(\chi-1)^2\chi}G_{0}(\chi) \\
&+ \frac{1}{3}\frac{(2\chi+1)\pi}{(\chi+1)\chi}G_{-1,-1}(\chi) - \frac{1}{3}\frac{(2\chi^2-4\chi-1)\pi}{(\chi-1)^2\chi}G_{-1,0}(\chi) - \frac{1}{3}\frac{(2\chi^2-4\chi-1)\pi}{(\chi-1)^2\chi}G_{0,-1}(\chi) - \frac{1}{6}\frac{(8\chi^3-9\chi^2-15\chi-2)\pi}{(\chi+1)(\chi-1)^2\chi}G_{0,0}(\chi) \\
&+ \frac{(2\chi^2-4\chi-1)\pi}{(\chi-1)^2\chi}G_{1,0}(\chi)\Bigg)\frac{1}{\epsilon} + \frac{2}{3}\frac{(2\chi+1)\pi}{(\chi+1)\chi}G_{-1,-1,-1}(\chi) - \frac{2}{3}\frac{(2\chi^2-4\chi-1)\pi}{(\chi-1)^2\chi}G_{-1,-1,0}(\chi) - \frac{2}{3}\frac{(2\chi^2-4\chi-1)\pi}{(\chi-1)^2\chi}G_{-1,0,-1}(\chi) \\
&- \frac{1}{3}\frac{(8\chi^3-9\chi^2-15\chi-2)\pi}{(\chi+1)(\chi-1)^2\chi}G_{-1,0,0}(\chi) + 2\frac{(2\chi^2-4\chi-1)\pi}{(\chi-1)^2\chi}G_{-1,1,0}(\chi) - \frac{2}{3}\frac{(2\chi^2-4\chi-1)\pi}{(\chi-1)^2\chi}G_{0,-1,-1}(\chi) \\
&- \frac{1}{3}\frac{(8\chi^3-9\chi^2-15\chi-2)\pi}{(\chi+1)(\chi-1)^2\chi}G_{0,-1,0}(\chi) - \frac{1}{3}\frac{(8\chi^3-9\chi^2-15\chi-2)\pi}{(\chi+1)(\chi-1)^2\chi}G_{0,0,-1}(\chi) - \frac{1}{6}\frac{(20\chi^3-23\chi^2-35\chi-4)\pi}{(\chi+1)(\chi-1)^2\chi}G_{0,0,0}(\chi) \\
&+ \frac{(8\chi^3-9\chi^2-15\chi-2)\pi}{(\chi+1)(\chi-1)^2\chi}G_{0,1,0}(\chi) + 2\frac{(2\chi^2-4\chi-1)\pi}{(\chi-1)^2\chi}G_{1,-1,0}(\chi) + 2\frac{(2\chi^2-4\chi-1)\pi}{(\chi-1)^2\chi}G_{1,0,-1}(\chi) + 3\frac{(2\chi^2-4\chi-1)\pi}{(\chi-1)^2\chi}G_{1,0,0}(\chi) \\
&- 6\frac{(2\chi^2-4\chi-1)\pi}{(\chi-1)^2\chi}G_{1,1,0}(\chi)\Bigg)\mu^{-4\epsilon} + O(\mu)
\end{aligned}
\tag{D.17}
$$

I[FNP2, 0, 1, 1, 1, 1, 1, 1, 0, 0] =

$$
\begin{aligned}
&\Bigg(\Bigg(-36\frac{Li_4(\frac{1}{2})}{\chi+2} + \Bigg(-162\frac{1}{\chi+2}G_{-2}(\chi)G_{0}(2) + 81\frac{1}{\chi+2}G_{-1}(\chi)G_{0}(2) + 54\frac{1}{\chi+2}G_{0}(2)G_{0}(\chi) - 162\frac{1}{\chi+2}G_{-2,-2}(\chi) + 9\frac{1}{\chi+2}G_{-2,-1}(\chi) \\
&+ 54\frac{1}{\chi+2}G_{-2,0}(\chi) + 81\frac{1}{\chi+2}G_{-1,-2}(\chi) + 2\frac{1}{\chi+2}G_{-1,-1}(\chi) - 3\frac{1}{\chi+2}G_{-1,0}(\chi) + 54\frac{1}{\chi+2}G_{0,-2}(\chi) - 3\frac{1}{\chi+2}G_{0,-1}(\chi) \\
&- 144\frac{1}{\chi+2}G_{0,0}(2) - 18\frac{1}{\chi+2}G_{0,0}(\chi)\Bigg)\zeta_2 + \Bigg(\frac{63}{2}\frac{1}{\chi+2}G_{-2}(\chi) - \frac{63}{4}\frac{1}{\chi+2}G_{-1}(\chi) - \frac{21}{2}\frac{1}{\chi+2}G_{0}(\chi)\Bigg)\zeta_3 + \frac{171}{8}\frac{1}{\chi+2}\zeta_4 \\
&+ 18\frac{1}{\chi+2}G_{-2,-2,-1,-1}(\chi) - 9\frac{1}{\chi+2}G_{-2,-1,-1,-1}(\chi) - 6\frac{1}{\chi+2}G_{-2,-1,-1,0}(\chi) - 6\frac{1}{\chi+2}G_{-2,-1,0,-1}(\chi) - 6\frac{1}{\chi+2}G_{-2,0,-1,-1}(\chi) \\
&- 9\frac{1}{\chi+2}G_{-1,-2,-1,-1}(\chi) + \frac{7}{2}\frac{1}{\chi+2}G_{-1,-1,-1,-1}(\chi) + 3\frac{1}{\chi+2}G_{-1,-1,-1,0}(\chi) + 3\frac{1}{\chi+2}G_{-1,-1,0,-1}(\chi) + 2\frac{1}{\chi+2}G_{-1,-1,0,0}(\chi) \\
&+ 3\frac{1}{\chi+2}G_{-1,0,-1,-1}(\chi) + 2\frac{1}{\chi+2}G_{-1,0,-1,0}(\chi) + 2\frac{1}{\chi+2}G_{-1,0,0,-1}(\chi) - 6\frac{1}{\chi+2}G_{0,-2,-1,-1}(\chi) + 3\frac{1}{\chi+2}G_{0,-1,-1,-1}(\chi) \\
&+ 2\frac{1}{\chi+2}G_{0,-1,-1,0}(\chi) + 2\frac{1}{\chi+2}G_{0,-1,0,-1}(\chi) + 2\frac{1}{\chi+2}G_{0,0,-1,-1}(\chi) - 36\frac{1}{\chi+2}G_{0,0,0,0}(2)\Bigg)i - \frac{21}{2}\frac{\pi}{\chi+2}\zeta_3 + 9\frac{\pi^3}{\chi+2}G_{-2}(\chi) \\
&- 4\frac{\pi^3}{\chi+2}G_{-1}(\chi) + \Bigg(\Bigg(-\frac{9}{2}\frac{1}{\chi+2}\zeta_2 + \frac{1}{2}\frac{1}{\chi+2}G_{-1,-1}(\chi)\Bigg)i - \frac{\pi}{\chi+2}G_{-1}(\chi)\Bigg)\frac{1}{\epsilon^2} + 9\frac{\pi^3}{\chi+2}G_{0}(2) - 3\frac{\pi^3}{\chi+2}G_{0}(\chi) + \Bigg(\Bigg(\Bigg(27\frac{1}{\chi+2}G_{-2}(\chi) \\
&- \frac{3}{2}\frac{1}{\chi+2}G_{-1}(\chi) + 27\frac{1}{\chi+2}G_{0}(2) - 9\frac{1}{\chi+2}G_{0}(\chi)\Bigg)\zeta_2 - \frac{21}{4}\frac{1}{\chi+2}\zeta_3 - 3\frac{1}{\chi+2}G_{-2,-1,-1}(\chi) + \frac{3}{2}\frac{1}{\chi+2}G_{-1,-1,-1}(\chi) \\
&+ \frac{1}{\chi+2}G_{-1,-1,0}(\chi) + \frac{1}{\chi+2}G_{-1,0,-1}(\chi) + \frac{1}{\chi+2}G_{0,-1,-1}(\chi)\Bigg)i - \frac{3}{2}\frac{\pi^3}{\chi+2} + 6\frac{\pi}{\chi+2}G_{-2,-1}(\chi) - 2\frac{\pi}{\chi+2}G_{-1,-1}(\chi) \\
&- 2\frac{\pi}{\chi+2}G_{-1,0}(\chi) - 2\frac{\pi}{\chi+2}G_{0,-1}(\chi)\Bigg)\frac{1}{\epsilon} - 36\frac{\pi}{\chi+2}G_{-2,-2,-1}(\chi) + 12\frac{\pi}{\chi+2}G_{-2,-1,-1}(\chi) + 12\frac{\pi}{\chi+2}G_{-2,-1,0}(\chi) \\
&+ 12\frac{\pi}{\chi+2}G_{-2,0,-1}(\chi) + 18\frac{\pi}{\chi+2}G_{-1,-2,-1}(\chi) - 4\frac{\pi}{\chi+2}G_{-1,-1,-1}(\chi) - 4\frac{\pi}{\chi+2}G_{-1,-1,0}(\chi) - 4\frac{\pi}{\chi+2}G_{-1,0,-1}(\chi) \\
&- 4\frac{\pi}{\chi+2}G_{-1,0,0}(\chi) + 12\frac{\pi}{\chi+2}G_{0,-2,-1}(\chi) - 4\frac{\pi}{\chi+2}G_{0,-1,-1}(\chi) - 4\frac{\pi}{\chi+2}G_{0,-1,0}(\chi) - 4\frac{\pi}{\chi+2}G_{0,0,-1}(\chi)\Bigg)\mu^{-4\epsilon}\mu^{-1}
\end{aligned}
$$

$$+ \Bigg(\Bigg(-39\frac{1}{\chi}G_{-1}(\chi) + 39\frac{1}{\chi+1}G_0(\chi)\Bigg)\zeta_2 - 12\frac{1}{\chi+1}\zeta_3 + 48\frac{1}{\chi}G_{-1}(\chi) - 48\frac{1}{\chi+1}G_0(\chi) + 24\frac{1}{(\chi+1)\chi}G_{-1,0}(\chi) + 24\frac{1}{(\chi+1)\chi}G_{0,-1}(\chi)$$

$$- 12\frac{1}{\chi}G_{-1,-1,-1}(\chi) + 12\frac{1}{(\chi+1)\chi}G_{-1,-1,0}(\chi) + 12\frac{1}{(\chi+1)\chi}G_{-1,0,-1}(\chi) - 12\frac{1}{\chi}G_{-1,0,0}(\chi) + 12\frac{1}{\chi+1}G_{0,-1,-1}(\chi)$$

$$+ 12\frac{1}{(\chi+1)\chi}G_{0,-1,0}(\chi) + 12\frac{1}{(\chi+1)\chi}G_{0,0,-1}(\chi) + 12\frac{1}{\chi+1}G_{0,0,0}(\chi)\Bigg)i + \frac{1}{2}\frac{(5\chi\pi^2 - 96\chi + \pi^2 - 96)\pi}{(\chi+1)\chi} + \Bigg(\Bigg(3\frac{1}{\chi}G_{-1}(\chi)$$

$$- 3\frac{1}{\chi+1}G_0(\chi)\Bigg)i - 3\frac{\pi}{\chi}\Bigg)\frac{1}{\epsilon^2} - 24\frac{\pi}{(\chi+1)\chi}G_0(\chi) + \Bigg(\Bigg(-12\frac{1}{\chi}G_{-1}(\chi) + 12\frac{1}{\chi+1}G_0(\chi) - 6\frac{1}{(\chi+1)\chi}G_{-1,0}(\chi) - 6\frac{1}{(\chi+1)\chi}G_{0,-1}(\chi)\Bigg)i$$

$$+ 12\frac{\pi}{\chi} + 6\frac{\pi}{(\chi+1)\chi}G_0(\chi)\Bigg)\frac{1}{\epsilon} + 12\frac{\pi}{\chi}G_{-1,-1}(\chi) - 12\frac{\pi}{(\chi+1)\chi}G_{-1,0}(\chi) - 12\frac{\pi}{\chi+1}G_{0,-1}(\chi) - 12\frac{\pi}{(\chi+1)\chi}G_{0,0}(\chi) + \Bigg(-\frac{1}{\epsilon^3}\frac{1}{3}\frac{5\chi+2}{(\chi+1)\chi}i$$

$$+ \Bigg(9\frac{5\chi+2}{(\chi+1)\chi} + \Bigg(\frac{14\chi+17}{(\chi+1)\chi} + \frac{1}{3}\frac{79\chi+82}{(\chi+1)\chi}G_{-1}(\chi) - \frac{1}{3}\frac{70\chi-17}{(\chi+1)\chi}G_0(\chi)\Bigg)\zeta_2 + \frac{4}{9}\frac{37\chi+22}{(\chi+1)\chi}\zeta_3 - 3\frac{11\chi+14}{(\chi+1)\chi}G_{-1}(\chi) + 3\frac{11\chi+2}{(\chi+1)\chi}G_0(\chi)$$

$$+ \frac{5\chi+2}{(\chi+1)\chi}G_{-1,-1}(\chi) - 2\frac{\chi+7}{(\chi+1)\chi}G_{-1,0}(\chi) - 2\frac{\chi+7}{(\chi+1)\chi}G_{0,-1}(\chi) - \frac{\chi-2}{(\chi+1)\chi}G_{0,0}(\chi) + \frac{1}{3}\frac{31\chi+28}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)$$

$$- \frac{2}{3}\frac{2\chi+11}{(\chi+1)\chi}G_{-1,-1,0}(\chi) - \frac{2}{3}\frac{2\chi+11}{(\chi+1)\chi}G_{-1,0,-1}(\chi) + \frac{2}{3}\frac{11\chi+14}{(\chi+1)\chi}G_{-1,0,0}(\chi) - \frac{2}{3}\frac{17\chi-1}{(\chi+1)\chi}G_{0,-1,-1}(\chi) + \frac{2}{3}\frac{2\chi-7}{(\chi+1)\chi}G_{0,-1,0}(\chi)$$

$$+ \frac{2}{3}\frac{2\chi-7}{(\chi+1)\chi}G_{0,0,-1}(\chi) - \frac{1}{3}\frac{19\chi-2}{(\chi+1)\chi}G_{0,0,0}(\chi)\Bigg)i - \frac{1}{6}\frac{(18\chi\pi^2 - 108\chi + 11\pi^2 - 162)\pi}{(\chi+1)\chi} + \Bigg(\Bigg(\frac{5\chi+2}{(\chi+1)\chi} - \frac{1}{3}\frac{11\chi+14}{(\chi+1)\chi}G_{-1}(\chi)$$

$$+ \frac{1}{3}\frac{11\chi+2}{(\chi+1)\chi}G_0(\chi)\Bigg)i + \frac{(2\chi+3)\pi}{(\chi+1)\chi}\Bigg)\frac{1}{\epsilon^2} + 3\frac{(\chi+2)\pi}{(\chi+1)\chi}G_{-1}(\chi) - 3\frac{(2\chi-3)\pi}{(\chi+1)\chi}G_0(\chi) + \Bigg(\Bigg(-3\frac{5\chi+2}{(\chi+1)\chi} - \frac{1}{3}\frac{14\chi+17}{(\chi+1)\chi}\zeta_2$$

$$+ \frac{11\chi+14}{(\chi+1)\chi}G_{-1}(\chi) - \frac{11\chi+2}{(\chi+1)\chi}G_0(\chi) - \frac{1}{3}\frac{5\chi+2}{(\chi+1)\chi}G_{-1,-1}(\chi) + \frac{2}{3}\frac{\chi+7}{(\chi+1)\chi}G_{-1,0}(\chi) + \frac{2}{3}\frac{\chi+7}{(\chi+1)\chi}G_{0,-1}(\chi) + \frac{1}{3}\frac{\chi-2}{(\chi+1)\chi}G_{0,0}(\chi)\Bigg)i$$

$$- 3\frac{(2\chi+3)\pi}{(\chi+1)\chi} - \frac{(\chi+2)\pi}{(\chi+1)\chi}G_{-1}(\chi) + \frac{(2\chi-3)\pi}{(\chi+1)\chi}G_0(\chi)\Bigg)\frac{1}{\epsilon} - \frac{(11\chi+10)\pi}{(\chi+1)\chi}G_{-1,-1}(\chi) + 2\frac{(\chi+5)\pi}{(\chi+1)\chi}G_{-1,0}(\chi) + 2\frac{(6\chi+1)\pi}{(\chi+1)\chi}G_{0,-1}(\chi)$$

$$- \frac{(2\chi-3)\pi}{(\chi+1)\chi}G_{0,0}(\chi)\Bigg)\mu^{-\epsilon} + \Bigg(\frac{1}{\epsilon^3}\frac{1}{2}\frac{5\chi+3}{(\chi+1)\chi}i + \Bigg(-4\frac{5\chi+3}{(\chi+1)\chi} + \Bigg(5\frac{7\chi+6}{(\chi+1)\chi} + 2\frac{\chi-1}{(\chi+1)\chi}G_{-1}(\chi)\Bigg)\zeta_2 - \frac{8}{3}\frac{7\chi+6}{(\chi+1)\chi}\zeta_3$$

$$+ 2\frac{2\chi+3}{(\chi+1)\chi}G_{-1}(\chi) - \frac{2\chi+1}{(\chi+1)\chi}G_{-1,-1}(\chi) - \frac{1}{2}\frac{2\chi+1}{(\chi+1)\chi}G_{-1,-1,-1}(\chi) + 2\frac{1}{\chi+1}G_{0,-1,-1}(\chi)\Bigg)i - \frac{1}{6}\frac{(13\chi\pi^2 - 72\chi + 11\pi^2 - 48)\pi}{(\chi+1)\chi}$$

$$+ \Bigg(\Bigg(-\frac{5\chi+3}{(\chi+1)\chi} + \frac{1}{2}\frac{2\chi+3}{(\chi+1)\chi}G_{-1}(\chi)\Bigg)i + \frac{(3\chi+2)\pi}{(\chi+1)\chi}\Bigg)\frac{1}{\epsilon^2} - 2\frac{\pi}{(\chi+1)\chi}G_{-1}(\chi) + \Bigg(\Bigg(2\frac{5\chi+3}{(\chi+1)\chi} - \frac{5}{2}\frac{7\chi+6}{(\chi+1)\chi}\zeta_2 - \frac{2\chi+3}{(\chi+1)\chi}G_{-1}(\chi)$$

$$+ \frac{1}{2}\frac{2\chi+1}{(\chi+1)\chi}G_{-1,-1}(\chi)\Bigg)i - 2\frac{(3\chi+2)\pi}{(\chi+1)\chi} + \frac{\pi}{(\chi+1)\chi}G_{-1}(\chi)\Bigg)\frac{1}{\epsilon} + \frac{(2\chi+1)\pi}{(\chi+1)\chi}G_{-1,-1}(\chi) - 2\frac{\pi}{\chi+1}G_{0,-1}(\chi)\Bigg)\mu^{-2\epsilon} + \Bigg(-\frac{1}{\epsilon^3}\frac{1}{\chi}i + \Bigg(\frac{1}{\chi}$$

$$+ \Bigg(-10\frac{1}{\chi} + 10\frac{1}{\chi}G_0(\chi)\Bigg)\zeta_2 + \frac{14}{3}\frac{1}{\chi}\zeta_3 - \frac{1}{\chi}G_0(\chi) + \frac{1}{\chi}G_{0,0}(\chi) - \frac{1}{\chi}G_{0,0,0}(\chi)\Bigg)i + \frac{2}{3}\frac{(\pi^2-3)\pi}{\chi} + \Bigg(\Bigg(\frac{1}{\chi} - \frac{1}{\chi}G_0(\chi)\Bigg)i - 2\frac{\pi}{\chi}\Bigg)\frac{1}{\epsilon^2}$$

$$+ 2\frac{\pi}{\chi}G_0(\chi) + \Bigg(\Bigg(-\frac{1}{\chi} + 10\frac{1}{\chi}\zeta_2 + \frac{1}{\chi}G_0(\chi) - \frac{1}{\chi}G_{0,0}(\chi)\Bigg)i + 2\frac{\pi}{\chi} - 2\frac{\pi}{\chi}G_0(\chi)\Bigg)\frac{1}{\epsilon} - 2\frac{\pi}{\chi}G_{0,0}(\chi)\Bigg)\mu^{-3\epsilon} + \Bigg(\frac{1}{\epsilon^3}\frac{1}{6}\frac{1}{\chi}i + \Bigg(72\frac{Li_4(\frac{1}{2})}{(\chi+2)^2}$$

$$+ \Bigg(9\frac{4\chi^3 - 13\chi^2 - 4\chi + 4}{(\chi+1)\chi(\chi+2)^2}G_{-2}(\chi) - \frac{1}{3}\frac{19\chi^3 - 13\chi^2 - 58\chi - 20}{(\chi+1)\chi(\chi+2)^2}G_{-1}(\chi) + 9\frac{4\chi^3 - 13\chi^2 - 4\chi + 4}{(\chi+1)\chi(\chi+2)^2}G_0(2) + 324\frac{1}{(\chi+2)^2}G_{-2}(\chi)G_0(2)$$

$$- 162\frac{1}{(\chi+2)^2}G_{-1}(\chi)G_0(2) - \frac{1}{3}\frac{89\chi^3 - 68\chi^2 - 188\chi - 40}{(\chi+1)\chi(\chi+2)^2}G_0(\chi) - 108\frac{1}{(\chi+2)^2}G_0(2)G_0(\chi) + 324\frac{1}{(\chi+2)^2}G_{-2,-2}(\chi)$$

$$- 18\frac{1}{(\chi+2)^2}G_{-2,-1}(\chi) - 108\frac{1}{(\chi+2)^2}G_{-2,0}(\chi) - 162\frac{1}{(\chi+2)^2}G_{-1,-2}(\chi) - 4\frac{1}{(\chi+2)^2}G_{-1,-1}(\chi) + 6\frac{1}{(\chi+2)^2}G_{-1,0}(\chi)$$

$$- 108\frac{1}{(\chi+2)^2}G_{0,-2}(\chi) + 6\frac{1}{(\chi+2)^2}G_{0,-1}(\chi) + 288\frac{1}{(\chi+2)^2}G_{0,0}(2) + 36\frac{1}{(\chi+2)^2}G_{0,0}(\chi)\Bigg)\zeta_2 + \Bigg(-\frac{1}{36}\frac{268\chi^3 - 739\chi^2 - 124\chi + 316}{(\chi+1)\chi(\chi+2)^2}$$

$$- 63\frac{1}{(\chi+2)^2}G_{-2}(\chi) + \frac{63}{2}\frac{1}{(\chi+2)^2}G_{-1}(\chi) + 21\frac{1}{(\chi+2)^2}G_0(\chi)\Bigg)\zeta_3 - \frac{171}{4}\frac{1}{(\chi+2)^2}\zeta_4 - \frac{4\chi^3 - 13\chi^2 - 4\chi + 4}{(\chi+1)\chi(\chi+2)^2}G_{-2,-1,-1}(\chi)$$

$$+ \frac{1}{6}\frac{16\chi^3 - 37\chi^2 - 28\chi + 4}{(\chi+1)\chi(\chi+2)^2}G_{-1,-1,-1}(\chi) + \frac{1}{3}\frac{4\chi^3 - 13\chi^2 - 4\chi + 4}{(\chi+1)\chi(\chi+2)^2}G_{-1,-1,0}(\chi) + \frac{1}{3}\frac{4\chi^3 - 13\chi^2 - 4\chi + 4}{(\chi+1)\chi(\chi+2)^2}G_{-1,0,-1}(\chi)$$

$$- \frac{2}{3}\frac{(\chi-2)(2\chi+1)}{(\chi+1)(\chi+2)\chi}G_{-1,0,0}(\chi) + \frac{1}{3}\frac{4\chi^3 - 13\chi^2 - 4\chi + 4}{(\chi+1)\chi(\chi+2)^2}G_{0,-1,-1}(\chi) - \frac{2}{3}\frac{(\chi-2)(2\chi+1)}{(\chi+1)(\chi+2)\chi}G_{0,-1,0}(\chi)$$

$$- \frac{2}{3}\frac{(\chi-2)(2\chi+1)}{(\chi+1)(\chi+2)\chi}G_{0,0,-1}(\chi) + \frac{4}{3}\frac{1}{\chi}G_{0,0,0}(\chi) - 36\frac{1}{(\chi+2)^2}G_{-2,-2,-1,-1}(\chi) + 18\frac{1}{(\chi+2)^2}G_{-2,-1,-1,-1}(\chi)$$

$$+ 12\frac{1}{(\chi+2)^2}G_{-2,-1,-1,0}(\chi) + 12\frac{1}{(\chi+2)^2}G_{-2,-1,0,-1}(\chi) + 12\frac{1}{(\chi+2)^2}G_{-2,0,-1,-1}(\chi) + 18\frac{1}{(\chi+2)^2}G_{-1,-2,-1,-1}(\chi)$$

$$- 7\frac{1}{(\chi+2)^2}G_{-1,-1,-1,-1}(\chi) - 6\frac{1}{(\chi+2)^2}G_{-1,-1,-1,0}(\chi) - 6\frac{1}{(\chi+2)^2}G_{-1,-1,0,-1}(\chi) - 4\frac{1}{(\chi+2)^2}G_{-1,-1,0,0}(\chi)$$

$$- 6\frac{1}{(\chi+2)^2}G_{-1,0,-1,-1}(\chi) - 4\frac{1}{(\chi+2)^2}G_{-1,0,-1,0}(\chi) - 4\frac{1}{(\chi+2)^2}G_{-1,0,0,-1}(\chi) + 12\frac{1}{(\chi+2)^2}G_{0,-2,-1,-1}(\chi)$$

$$- 6\frac{1}{(\chi+2)^2}G_{0,-1,-1,-1}(\chi) - 4\frac{1}{(\chi+2)^2}G_{0,-1,-1,0}(\chi) - 4\frac{1}{(\chi+2)^2}G_{0,-1,0,-1}(\chi) - 4\frac{1}{(\chi+2)^2}G_{0,0,-1,-1}(\chi)$$

$$+ 72\frac{1}{(\chi+2)^2}G_{0,0,0,0}(2)\Bigg)i - \frac{1}{6}\frac{(13\chi^3 - 43\chi^2 - 28\chi + 4)\pi^3}{(\chi+1)\chi(\chi+2)^2} + 21\frac{\pi}{(\chi+2)^2}\zeta_3 - 18\frac{\pi^3}{(\chi+2)^2}G_{-2}(\chi) + 8\frac{\pi^3}{(\chi+2)^2}G_{-1}(\chi)$$

$$+ \Bigg(\bigg(9 \frac{1}{(\chi+2)^2}\zeta_2 - \frac{1}{6}\frac{(\chi-2)(2\chi+1)}{(\chi+1)(\chi+2)\chi}G_{-1}(\chi) + \frac{1}{3}\frac{1}{\chi}G_0(\chi) - \frac{1}{(\chi+2)^2}G_{-1,-1}(\chi)\bigg) i + \frac{\chi\pi}{(\chi+1)(\chi+2)} + 2\frac{\pi}{(\chi+2)^2}G_{-1}(\chi)\Bigg)\frac{1}{\epsilon^2}$$

$$- 18\frac{\pi^3}{(\chi+2)^2}G_0(2) + 6\frac{\pi^3}{(\chi+2)^2}G_0(\chi) + 2\frac{(4\chi^3-13\chi^2-4\chi+4)\pi}{(\chi+1)\chi(\chi+2)^2}G_{-2,-1}(\chi) - 4\frac{(\chi^2-2\chi-2)\pi}{(\chi+1)(\chi+2)^2}G_{-1,-1}(\chi)$$

$$- 4\frac{(\chi^2-2\chi-2)\pi}{(\chi+1)(\chi+2)^2}G_{-1,0}(\chi) - 4\frac{(\chi^2-2\chi-2)\pi}{(\chi+1)(\chi+2)^2}G_{0,-1}(\chi) + \Bigg(\bigg(\Big(-\frac{1}{6}\frac{89\chi^3-68\chi^2-188\chi-40}{(\chi+1)\chi(\chi+2)^2} - 54\frac{1}{(\chi+2)^2}G_{-2}(\chi)$$

$$+ 3\frac{1}{(\chi+2)^2}G_{-1}(\chi) - 54\frac{1}{(\chi+2)^2}G_0(2) + 18\frac{1}{(\chi+2)^2}G_0(\chi)\Big)\zeta_2 + \frac{21}{2}\frac{1}{(\chi+2)^2}\zeta_3 + \frac{1}{6}\frac{4\chi^3-13\chi^2-4\chi+4}{(\chi+1)\chi(\chi+2)^2}G_{-1,-1}(\chi)$$

$$- \frac{1}{3}\frac{(\chi-2)(2\chi+1)}{(\chi+1)(\chi+2)\chi}G_{-1,0}(\chi) - \frac{1}{3}\frac{(\chi-2)(2\chi+1)}{(\chi+1)(\chi+2)\chi}G_{0,-1}(\chi) + \frac{2}{3}\frac{1}{\chi}G_{0,0}(\chi) + 6\frac{1}{(\chi+2)^2}G_{-2,-1,-1}(\chi) - 3\frac{1}{(\chi+2)^2}G_{-1,-1,-1}(\chi)$$

$$- 2\frac{1}{(\chi+2)^2}G_{-1,-1,0}(\chi) - 2\frac{1}{(\chi+2)^2}G_{-1,0,-1}(\chi) - 2\frac{1}{(\chi+2)^2}G_{0,-1,-1}(\chi)\bigg) i + 3\frac{\pi^3}{(\chi+2)^2} - 2\frac{(\chi^2-2\chi-2)\pi}{(\chi+1)(\chi+2)^2}G_{-1}(\chi)$$

$$+ 2\frac{\chi\pi}{(\chi+1)(\chi+2)}G_0(\chi) - 12\frac{\pi}{(\chi+2)^2}G_{-2,-1}(\chi) + 4\frac{\pi}{(\chi+2)^2}G_{-1,-1}(\chi) + 4\frac{\pi}{(\chi+2)^2}G_{-1,0}(\chi) + 4\frac{\pi}{(\chi+2)^2}G_{0,-1}(\chi)\Bigg)\frac{1}{\epsilon}$$

$$+ 4\frac{\chi\pi}{(\chi+1)(\chi+2)}G_{0,0}(\chi) + 72\frac{\pi}{(\chi+2)^2}G_{-2,-2,-1}(\chi) - 24\frac{\pi}{(\chi+2)^2}G_{-2,-1,-1}(\chi) - 24\frac{\pi}{(\chi+2)^2}G_{-2,-1,0}(\chi)$$

$$- 24\frac{\pi}{(\chi+2)^2}G_{-2,0,-1}(\chi) - 36\frac{\pi}{(\chi+2)^2}G_{-1,-2,-1}(\chi) + 8\frac{\pi}{(\chi+2)^2}G_{-1,-1,-1}(\chi) + 8\frac{\pi}{(\chi+2)^2}G_{-1,-1,0}(\chi) + 8\frac{\pi}{(\chi+2)^2}G_{-1,0,-1}(\chi)$$

$$+ 8\frac{\pi}{(\chi+2)^2}G_{-1,0,0}(\chi) - 24\frac{\pi}{(\chi+2)^2}G_{0,-2,-1}(\chi) + 8\frac{\pi}{(\chi+2)^2}G_{0,-1,-1}(\chi) + 8\frac{\pi}{(\chi+2)^2}G_{0,-1,0}(\chi) + 8\frac{\pi}{(\chi+2)^2}G_{0,0,-1}(\chi)\Bigg)\mu^{-4\epsilon}$$

$$+ O(\mu) \tag{D.18}$$

I[FNP6, 0, 1, 1, 1, 1, 1, 1, 0, 0] =

$$\Bigg(\bigg(\Big(2\frac{1}{2\chi+1}G_{-1,-1}(\chi) + 81\frac{1}{2\chi+1}G_{-1,-\frac{1}{2}}(\chi) - 83\frac{1}{2\chi+1}G_{-1,0}(\chi) + 9\frac{1}{2\chi+1}G_{-\frac{1}{2},-1}(\chi) - 162\frac{1}{2\chi+1}G_{-\frac{1}{2},-\frac{1}{2}}(\chi)$$

$$+ 153\frac{1}{2\chi+1}G_{-\frac{1}{2},0}(\chi) - 11\frac{1}{2\chi+1}G_{0,-1}(\chi) + 81\frac{1}{2\chi+1}G_{0,-\frac{1}{2}}(\chi) - 70\frac{1}{2\chi+1}G_{0,0}(\chi)\Big)\zeta_2 + \Big(-9\frac{1}{2\chi+1}G_{-1}(\chi) + 18\frac{1}{2\chi+1}G_{-\frac{1}{2}}(\chi)$$

$$- 9\frac{1}{2\chi+1}G_0(\chi)\Big)\zeta_3 + \frac{747}{8}\frac{1}{2\chi+1}\zeta_4 + \frac{7}{2}\frac{1}{2\chi+1}G_{-1,-1,-1,-1}(\chi) - \frac{7}{2}\frac{1}{2\chi+1}G_{-1,-1,-1,0}(\chi) - \frac{7}{2}\frac{1}{2\chi+1}G_{-1,-1,0,-1}(\chi)$$

$$+ \frac{7}{2}\frac{1}{2\chi+1}G_{-1,-1,0,0}(\chi) - 9\frac{1}{2\chi+1}G_{-1,-\frac{1}{2},-1,-1}(\chi) + 9\frac{1}{2\chi+1}G_{-1,-\frac{1}{2},-1,0}(\chi) + 9\frac{1}{2\chi+1}G_{-1,-\frac{1}{2},0,-1}(\chi)$$

$$- 9\frac{1}{2\chi+1}G_{-1,-\frac{1}{2},0,0}(\chi) + \frac{11}{2}\frac{1}{2\chi+1}G_{-1,0,-1,-1}(\chi) - \frac{11}{2}\frac{1}{2\chi+1}G_{-1,0,-1,0}(\chi) - \frac{11}{2}\frac{1}{2\chi+1}G_{-1,0,0,-1}(\chi)$$

$$+ \frac{11}{2}\frac{1}{2\chi+1}G_{-1,0,0,0}(\chi) - 9\frac{1}{2\chi+1}G_{-\frac{1}{2},-1,-1,-1}(\chi) + 9\frac{1}{2\chi+1}G_{-\frac{1}{2},-1,-1,0}(\chi) + 9\frac{1}{2\chi+1}G_{-\frac{1}{2},-1,0,-1}(\chi)$$

$$- 9\frac{1}{2\chi+1}G_{-\frac{1}{2},-1,0,0}(\chi) + 18\frac{1}{2\chi+1}G_{-\frac{1}{2},-\frac{1}{2},-1,-1}(\chi) - 18\frac{1}{2\chi+1}G_{-\frac{1}{2},-\frac{1}{2},-1,0}(\chi) - 18\frac{1}{2\chi+1}G_{-\frac{1}{2},-\frac{1}{2},0,-1}(\chi)$$

$$+ 18\frac{1}{2\chi+1}G_{-\frac{1}{2},-\frac{1}{2},0,0}(\chi) - 9\frac{1}{2\chi+1}G_{-\frac{1}{2},0,-1,-1}(\chi) + 9\frac{1}{2\chi+1}G_{-\frac{1}{2},0,-1,0}(\chi) + 9\frac{1}{2\chi+1}G_{-\frac{1}{2},0,0,-1}(\chi) - 9\frac{1}{2\chi+1}G_{-\frac{1}{2},0,0,0}(\chi)$$

$$+ \frac{11}{2}\frac{1}{2\chi+1}G_{0,-1,-1,-1}(\chi) - \frac{11}{2}\frac{1}{2\chi+1}G_{0,-1,-1,0}(\chi) - \frac{11}{2}\frac{1}{2\chi+1}G_{0,-1,0,-1}(\chi) + \frac{11}{2}\frac{1}{2\chi+1}G_{0,-1,0,0}(\chi)$$

$$- 9\frac{1}{2\chi+1}G_{0,-\frac{1}{2},-1,-1}(\chi) + 9\frac{1}{2\chi+1}G_{0,-\frac{1}{2},-1,0}(\chi) + 9\frac{1}{2\chi+1}G_{0,-\frac{1}{2},0,-1}(\chi) - 9\frac{1}{2\chi+1}G_{0,-\frac{1}{2},0,0}(\chi) + \frac{7}{2}\frac{1}{2\chi+1}G_{0,0,-1,-1}(\chi)$$

$$- \frac{7}{2}\frac{1}{2\chi+1}G_{0,0,-1,0}(\chi) - \frac{7}{2}\frac{1}{2\chi+1}G_{0,0,0,-1}(\chi) + \frac{7}{2}\frac{1}{2\chi+1}G_{0,0,0,0}(\chi)\bigg) i - 24\frac{\pi}{2\chi+1}\zeta_3 - \frac{11}{2}\frac{\pi^3}{2\chi+1}G_{-1}(\chi) + 12\frac{\pi^3}{2\chi+1}G_{-\frac{1}{2}}(\chi)$$

$$- \frac{13}{2}\frac{\pi^3}{2\chi+1}G_0(\chi) + \bigg(\Big(-\frac{9}{2}\frac{1}{2\chi+1}\zeta_2 + \frac{1}{2}\frac{1}{2\chi+1}G_{-1,-1}(\chi) - \frac{1}{2}\frac{1}{2\chi+1}G_{-1,0}(\chi) - \frac{1}{2}\frac{1}{2\chi+1}G_{0,-1}(\chi) + \frac{1}{2}\frac{1}{2\chi+1}G_{0,0}(\chi)\Big) i$$

$$- \frac{\pi}{2\chi+1}G_{-1}(\chi) + \frac{\pi}{2\chi+1}G_0(\chi)\bigg)\frac{1}{\epsilon^2} + \bigg(\Big(\big(-\frac{3}{2}\frac{1}{2\chi+1}G_{-1}(\chi) + 27\frac{1}{2\chi+1}G_{-\frac{1}{2}}(\chi) - \frac{51}{2}\frac{1}{2\chi+1}G_0(\chi)\big)\zeta_2 - 3\frac{1}{2\chi+1}\zeta_3$$

$$+ \frac{3}{2}\frac{1}{2\chi+1}G_{-1,-1,-1}(\chi) - \frac{3}{2}\frac{1}{2\chi+1}G_{-1,-1,0}(\chi) - \frac{3}{2}\frac{1}{2\chi+1}G_{-1,0,-1}(\chi) + \frac{3}{2}\frac{1}{2\chi+1}G_{-1,0,0}(\chi) - 3\frac{1}{2\chi+1}G_{-\frac{1}{2},-1,-1}(\chi)$$

$$+ 3\frac{1}{2\chi+1}G_{-\frac{1}{2},-1,0}(\chi) + 3\frac{1}{2\chi+1}G_{-\frac{1}{2},0,-1}(\chi) - 3\frac{1}{2\chi+1}G_{-\frac{1}{2},0,0}(\chi) + \frac{3}{2}\frac{1}{2\chi+1}G_{0,-1,-1}(\chi) - \frac{3}{2}\frac{1}{2\chi+1}G_{0,-1,0}(\chi)$$

$$- \frac{3}{2}\frac{1}{2\chi+1}G_{0,0,-1}(\chi) + \frac{3}{2}\frac{1}{2\chi+1}G_{0,0,0}(\chi)\Big) i - 2\frac{\pi^3}{2\chi+1} - 2\frac{\pi}{2\chi+1}G_{-1,-1}(\chi) + 2\frac{\pi}{2\chi+1}G_{-1,0}(\chi) + 6\frac{\pi}{2\chi+1}G_{-\frac{1}{2},-1}(\chi)$$

$$- 6\frac{\pi}{2\chi+1}G_{-\frac{1}{2},0}(\chi) - 4\frac{\pi}{2\chi+1}G_{0,-1}(\chi) + 4\frac{\pi}{2\chi+1}G_{0,0}(\chi)\bigg)\frac{1}{\epsilon} - 4\frac{\pi}{2\chi+1}G_{-1,-1,-1}(\chi) + 4\frac{\pi}{2\chi+1}G_{-1,-1,0}(\chi)$$

$$+ 18\frac{\pi}{2\chi+1}G_{-1,-\frac{1}{2},-1}(\chi) - 18\frac{\pi}{2\chi+1}G_{-1,-\frac{1}{2},0}(\chi) - 14\frac{\pi}{2\chi+1}G_{-1,0,-1}(\chi) + 14\frac{\pi}{2\chi+1}G_{-1,0,0}(\chi) + 12\frac{\pi}{2\chi+1}G_{-\frac{1}{2},-1,-1}(\chi)$$

$$- 12\frac{\pi}{2\chi+1}G_{-\frac{1}{2},-1,0}(\chi) - 36\frac{\pi}{2\chi+1}G_{-\frac{1}{2},-\frac{1}{2},-1}(\chi) + 36\frac{\pi}{2\chi+1}G_{-\frac{1}{2},-\frac{1}{2},0}(\chi) + 24\frac{\pi}{2\chi+1}G_{-\frac{1}{2},0,-1}(\chi) - 24\frac{\pi}{2\chi+1}G_{-\frac{1}{2},0,0}(\chi)$$

$$- 8\frac{\pi}{2\chi+1}G_{0,-1,-1}(\chi) + 8\frac{\pi}{2\chi+1}G_{0,-1,0}(\chi) + 18\frac{\pi}{2\chi+1}G_{0,-\frac{1}{2},-1}(\chi) - 18\frac{\pi}{2\chi+1}G_{0,-\frac{1}{2},0}(\chi) - 10\frac{\pi}{2\chi+1}G_{0,0,-1}(\chi)$$

$$+ 10\frac{\pi}{2\chi+1}G_{0,0,0}(\chi)\Bigg)\mu^{-4\epsilon}\mu^{-1} + \Bigg(\bigg(-39\frac{1}{\chi}G_{-1}(\chi) + 39\frac{1}{\chi+1}G_0(\chi)\bigg)\zeta_2 - 12\frac{1}{\chi+1}\zeta_3 + 48\frac{1}{\chi}G_{-1}(\chi) - 48\frac{1}{\chi+1}G_0(\chi)$$

$$
\begin{aligned}
&+ 24\frac{1}{(\chi+1)\chi}G_{-1,0}(\chi) + 24\frac{1}{(\chi+1)\chi}G_{0,-1}(\chi) - 12\frac{1}{\chi}G_{-1,-1,-1}(\chi) + 12\frac{1}{(\chi+1)\chi}G_{-1,-1,0}(\chi) + 12\frac{1}{(\chi+1)\chi}G_{-1,0,-1}(\chi)\\
&- 12\frac{1}{\chi}G_{-1,0,0}(\chi) + 12\frac{1}{\chi+1}G_{0,-1,-1}(\chi) + 12\frac{1}{(\chi+1)\chi}G_{0,-1,0}(\chi) + 12\frac{1}{(\chi+1)\chi}G_{0,0,-1}(\chi) + 12\frac{1}{\chi+1}G_{0,0,0}(\chi)\bigg)i\\
&+ \frac{1}{2}\frac{(5\chi\pi^2-96\chi+\pi^2-96)\pi}{(\chi+1)\chi} + \bigg(\bigg(3\frac{1}{\chi}G_{-1}(\chi) - 3\frac{1}{\chi+1}G_0(\chi)\bigg)i - 3\frac{\pi}{\chi}\bigg)\frac{1}{\epsilon^2} - 24\frac{\pi}{(\chi+1)\chi}G_0(\chi) + \bigg(\bigg(-12\frac{1}{\chi}G_{-1}(\chi) + 12\frac{1}{\chi+1}G_0(\chi)\\
&- 6\frac{1}{(\chi+1)\chi}G_{-1,0}(\chi) - 6\frac{1}{(\chi+1)\chi}G_{0,-1}(\chi)\bigg)i + 12\frac{\pi}{\chi} + 6\frac{\pi}{(\chi+1)\chi}G_0(\chi)\bigg)\frac{1}{\epsilon} + 12\frac{\pi}{\chi}G_{-1,-1}(\chi) - 12\frac{\pi}{(\chi+1)\chi}G_{-1,0}(\chi)\\
&- 12\frac{\pi}{\chi+1}G_{0,-1}(\chi) - 12\frac{\pi}{(\chi+1)\chi}G_{0,0}(\chi) + \bigg(-\frac{1}{\epsilon^3}\frac{1}{3}\frac{2\chi+5}{(\chi+1)\chi}i + \bigg(9\frac{2\chi+5}{(\chi+1)\chi} + \bigg(\frac{17\chi+14}{(\chi+1)\chi} + \frac{1}{3}\frac{82\chi+79}{(\chi+1)\chi}G_{-1}(\chi)\\
&- \frac{1}{3}\frac{82\chi-5}{(\chi+1)\chi}G_0(\chi)\bigg)\zeta_2 + \frac{2}{9}\frac{80\chi+29}{(\chi+1)\chi}\zeta_3 - 3\frac{14\chi+11}{(\chi+1)\chi}G_{-1}(\chi) + 3\frac{14\chi+5}{(\chi+1)\chi}G_0(\chi) + \frac{2\chi+5}{(\chi+1)\chi}G_{-1,-1}(\chi) - 2\frac{\chi+7}{(\chi+1)\chi}G_{-1,0}(\chi)\\
&- 2\frac{\chi+7}{(\chi+1)\chi}G_{0,-1}(\chi) + \frac{2\chi+5}{(\chi+1)\chi}G_{0,0}(\chi) + \frac{1}{3}\frac{28\chi+31}{(\chi+1)\chi}G_{-1,-1,-1}(\chi) - \frac{2}{3}\frac{2\chi+11}{(\chi+1)\chi}G_{-1,-1,0}(\chi) - \frac{2}{3}\frac{2\chi+11}{(\chi+1)\chi}G_{-1,0,-1}(\chi)\\
&+ \frac{2}{3}\frac{11\chi+14}{(\chi+1)\chi}G_{-1,0,0}(\chi) - \frac{4}{3}\frac{7\chi-2}{(\chi+1)\chi}G_{0,-1,-1}(\chi) + \frac{4}{3}\frac{\chi-5}{(\chi+1)\chi}G_{0,-1,0}(\chi) + \frac{4}{3}\frac{\chi-5}{(\chi+1)\chi}G_{0,0,-1}(\chi) - \frac{1}{3}\frac{22\chi-5}{(\chi+1)\chi}G_{0,0,0}(\chi)\bigg)i\\
&- \frac{1}{6}\frac{(19\chi\pi^2-162\chi+8\pi^2-108)\pi}{(\chi+1)\chi} + \bigg(\bigg(\frac{2\chi+5}{(\chi+1)\chi} - \frac{1}{3}\frac{14\chi+11}{(\chi+1)\chi}G_{-1}(\chi) + \frac{1}{3}\frac{14\chi+5}{(\chi+1)\chi}G_0(\chi)\bigg)i + \frac{(3\chi+2)\pi}{(\chi+1)\chi}\bigg)\frac{1}{\epsilon^2} + 3\frac{(2\chi+1)\pi}{(\chi+1)\chi}G_{-1}(\chi)\\
&- 3\frac{(2\chi-3)\pi}{(\chi+1)\chi}G_0(\chi) + \bigg(\bigg(-3\frac{2\chi+5}{(\chi+1)\chi} - \frac{1}{3}\frac{17\chi+14}{(\chi+1)\chi}\zeta_2 + \frac{14\chi+11}{(\chi+1)\chi}G_{-1}(\chi) - \frac{14\chi+5}{(\chi+1)\chi}G_0(\chi) - \frac{1}{3}\frac{2\chi+5}{(\chi+1)\chi}G_{-1,-1}(\chi)\\
&+ \frac{2}{3}\frac{\chi+7}{(\chi+1)\chi}G_{-1,0}(\chi) + \frac{2}{3}\frac{\chi+7}{(\chi+1)\chi}G_{0,-1}(\chi) - \frac{1}{3}\frac{2\chi+5}{(\chi+1)\chi}G_{0,0}(\chi)\bigg)i - 3\frac{(3\chi+2)\pi}{(\chi+1)\chi} - \frac{(2\chi+1)\pi}{(\chi+1)\chi}G_{-1}(\chi) + \frac{(2\chi-3)\pi}{(\chi+1)\chi}G_0(\chi)\bigg)\frac{1}{\epsilon}\\
&- \frac{(10\chi+11)\pi}{(\chi+1)\chi}G_{-1,-1}(\chi) + 2\frac{(\chi+5)\pi}{(\chi+1)\chi}G_{-1,0}(\chi) + 10\frac{\pi}{\chi+1}G_{0,-1}(\chi) - \frac{(2\chi-5)\pi}{(\chi+1)\chi}G_{0,0}(\chi)\bigg)\mu^{-\epsilon} + \bigg(\frac{1}{\epsilon^3}\frac{1}{2}\frac{3\chi+5}{(\chi+1)\chi}i + \bigg(-4\frac{3\chi+5}{(\chi+1)\chi}\\
&+ \bigg(5\frac{6\chi+7}{(\chi+1)\chi} - 2\frac{\chi-1}{(\chi+1)\chi}G_{-1}(\chi) + 2\frac{\chi-1}{(\chi+1)\chi}G_0(\chi)\bigg)\zeta_2 - \frac{2}{3}\frac{24\chi+25}{(\chi+1)\chi}\zeta_3 + 2\frac{3\chi+2}{(\chi+1)\chi}G_{-1}(\chi) - 2\frac{3\chi+2}{(\chi+1)\chi}G_0(\chi)\\
&- \frac{\chi+2}{(\chi+1)\chi}G_{-1,-1}(\chi) + \frac{\chi+2}{(\chi+1)\chi}G_{-1,0}(\chi) + \frac{\chi+2}{(\chi+1)\chi}G_{0,-1}(\chi) - \frac{\chi+2}{(\chi+1)\chi}G_{0,0}(\chi) - \frac{1}{2}\frac{\chi+2}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)\\
&+ \frac{1}{2}\frac{\chi+2}{(\chi+1)\chi}G_{-1,-1,0}(\chi) + \frac{1}{2}\frac{\chi+2}{(\chi+1)\chi}G_{-1,0,-1}(\chi) - \frac{1}{2}\frac{\chi+2}{(\chi+1)\chi}G_{-1,0,0}(\chi) + \frac{1}{2}\frac{\chi-2}{(\chi+1)\chi}G_{0,-1,-1}(\chi) - \frac{1}{2}\frac{\chi-2}{(\chi+1)\chi}G_{0,-1,0}(\chi)\\
&- \frac{1}{2}\frac{\chi-2}{(\chi+1)\chi}G_{0,0,-1}(\chi) + \frac{1}{2}\frac{\chi-2}{(\chi+1)\chi}G_{0,0,0}(\chi)\bigg)i - \frac{1}{6}\frac{(11\chi\pi^2-48\chi+15\pi^2-72)\pi}{(\chi+1)\chi} + \bigg(\bigg(-\frac{3\chi+5}{(\chi+1)\chi} + \frac{1}{2}\frac{3\chi+2}{(\chi+1)\chi}G_{-1}(\chi)\\
&- \frac{1}{2}\frac{3\chi+2}{(\chi+1)\chi}G_0(\chi)\bigg)i + \frac{(2\chi+3)\pi}{(\chi+1)\chi}\bigg)\frac{1}{\epsilon^2} - 2\frac{\pi}{\chi+1}G_{-1}(\chi) + 2\frac{\pi}{\chi+1}G_0(\chi) + \bigg(\bigg(2\frac{3\chi+5}{(\chi+1)\chi} - \frac{5}{2}\frac{6\chi+7}{(\chi+1)\chi}\zeta_2 - \frac{3\chi+2}{(\chi+1)\chi}G_{-1}(\chi)\\
&+ \frac{3\chi+2}{(\chi+1)\chi}G_0(\chi) + \frac{1}{2}\frac{\chi+2}{(\chi+1)\chi}G_{-1,-1}(\chi) - \frac{1}{2}\frac{\chi+2}{(\chi+1)\chi}G_{-1,0}(\chi) - \frac{1}{2}\frac{\chi+2}{(\chi+1)\chi}G_{0,-1}(\chi) + \frac{1}{2}\frac{\chi+2}{(\chi+1)\chi}G_{0,0}(\chi)\bigg)i - 2\frac{(2\chi+3)\pi}{(\chi+1)\chi}\\
&+ \frac{\pi}{\chi+1}G_{-1}(\chi) - \frac{\pi}{\chi+1}G_0(\chi)\bigg)\frac{1}{\epsilon} + \frac{(\chi+2)\pi}{(\chi+1)\chi}G_{-1,-1}(\chi) - \frac{(\chi+2)\pi}{(\chi+1)\chi}G_{-1,0}(\chi) - \frac{\pi}{\chi+1}G_{0,-1}(\chi) + \frac{\pi}{\chi+1}G_{0,0}(\chi)\bigg)\mu^{-2\epsilon} + \bigg(-\frac{1}{\epsilon^3}\frac{1}{\chi}i\\
&+ \bigg(\frac{1}{\chi} - 10\frac{1}{\chi}\zeta_2 + \frac{14}{3}\frac{1}{\chi}\zeta_3\bigg)i + \frac{2}{3}\frac{(\pi^2-3)\pi}{\chi} + \bigg(\frac{1}{\chi}i - 2\frac{\pi}{\chi}\bigg)\frac{1}{\epsilon^2} + \bigg(\bigg(-\frac{1}{\chi} + 10\frac{1}{\chi}\zeta_2\bigg)i + 2\frac{\pi}{\chi}\bigg)\frac{1}{\epsilon}\bigg)\mu^{-3\epsilon} + \bigg(\frac{1}{\epsilon^3}\frac{1}{6}\frac{1}{\chi}i\\
&+ \bigg(\bigg(\frac{1}{3}\frac{20\chi^3+58\chi^2+13\chi-19}{(\chi+1)\chi(2\chi+1)^2}G_{-1}(\chi) + 9\frac{4\chi^3-4\chi^2-13\chi+4}{(\chi+1)\chi(2\chi+1)^2}G_{-\frac{1}{2}}(\chi) - \frac{1}{3}\frac{128\chi^3-50\chi^2-338\chi+89}{(\chi+1)\chi(2\chi+1)^2}G_0(\chi)\\
&- 4\frac{1}{(2\chi+1)^2}G_{-1,-1}(\chi) - 162\frac{1}{(2\chi+1)^2}G_{-1,-\frac{1}{2}}(\chi) + 166\frac{1}{(2\chi+1)^2}G_{-1,0}(\chi) - 18\frac{1}{(2\chi+1)^2}G_{-\frac{1}{2},-1}(\chi) + 324\frac{1}{(2\chi+1)^2}G_{-\frac{1}{2},-\frac{1}{2}}(\chi)\\
&- 306\frac{1}{(2\chi+1)^2}G_{-\frac{1}{2},0}(\chi) + 22\frac{1}{(2\chi+1)^2}G_{0,-1}(\chi) - 162\frac{1}{(2\chi+1)^2}G_{0,-\frac{1}{2}}(\chi) + 140\frac{1}{(2\chi+1)^2}G_{0,0}(\chi)\bigg)\zeta_2\\
&+ \bigg(-\frac{1}{9}\frac{52\chi^3-4\chi^2-97\chi+40}{(\chi+1)\chi(2\chi+1)^2} + 18\frac{1}{(2\chi+1)^2}G_{-1}(\chi) - 36\frac{1}{(2\chi+1)^2}G_{-\frac{1}{2}}(\chi) + 18\frac{1}{(2\chi+1)^2}G_0(\chi)\bigg)\zeta_3 - \frac{747}{4}\frac{1}{(2\chi+1)^2}\zeta_4\\
&+ \frac{1}{6}\frac{4\chi^3-28\chi^2-37\chi+16}{(\chi+1)\chi(2\chi+1)^2}G_{-1,-1,-1}(\chi) - \frac{1}{6}\frac{4\chi^3-28\chi^2-37\chi+16}{(\chi+1)\chi(2\chi+1)^2}G_{-1,-1,0}(\chi) - \frac{1}{6}\frac{4\chi^3-28\chi^2-37\chi+16}{(\chi+1)\chi(2\chi+1)^2}G_{-1,0,-1}(\chi)\\
&+ \frac{1}{6}\frac{4\chi^3-28\chi^2-37\chi+16}{(\chi+1)\chi(2\chi+1)^2}G_{-1,0,0}(\chi) - \frac{4\chi^3-4\chi^2-13\chi+4}{(\chi+1)\chi(2\chi+1)^2}G_{-\frac{1}{2},-1,-1}(\chi) + \frac{4\chi^3-4\chi^2-13\chi+4}{(\chi+1)\chi(2\chi+1)^2}G_{-\frac{1}{2},-1,0}(\chi)\\
&+ \frac{4\chi^3-4\chi^2-13\chi+4}{(\chi+1)\chi(2\chi+1)^2}G_{-\frac{1}{2},0,-1}(\chi) - \frac{4\chi^3-4\chi^2-13\chi+4}{(\chi+1)\chi(2\chi+1)^2}G_{-\frac{1}{2},0,0}(\chi) + \frac{1}{6}\frac{20\chi^3+4\chi^2-41\chi+8}{(\chi+1)\chi(2\chi+1)^2}G_{0,-1,-1}(\chi)\\
&- \frac{1}{6}\frac{20\chi^3+4\chi^2-41\chi+8}{(\chi+1)\chi(2\chi+1)^2}G_{0,-1,0}(\chi) - \frac{1}{6}\frac{20\chi^3+4\chi^2-41\chi+8}{(\chi+1)\chi(2\chi+1)^2}G_{0,0,-1}(\chi) + \frac{1}{6}\frac{20\chi^3+4\chi^2-41\chi+8}{(\chi+1)\chi(2\chi+1)^2}G_{0,0,0}(\chi)\\
&- 7\frac{1}{(2\chi+1)^2}G_{-1,-1,-1,-1}(\chi) + 7\frac{1}{(2\chi+1)^2}G_{-1,-1,-1,0}(\chi) + 7\frac{1}{(2\chi+1)^2}G_{-1,-1,0,-1}(\chi) - 7\frac{1}{(2\chi+1)^2}G_{-1,-1,0,0}(\chi)\\
&+ 18\frac{1}{(2\chi+1)^2}G_{-1,-\frac{1}{2},-1,-1}(\chi) - 18\frac{1}{(2\chi+1)^2}G_{-1,-\frac{1}{2},-1,0}(\chi) - 18\frac{1}{(2\chi+1)^2}G_{-1,-\frac{1}{2},0,-1}(\chi) + 18\frac{1}{(2\chi+1)^2}G_{-1,-\frac{1}{2},0,0}(\chi)\\
&- 11\frac{1}{(2\chi+1)^2}G_{-1,0,-1,-1}(\chi) + 11\frac{1}{(2\chi+1)^2}G_{-1,0,-1,0}(\chi) + 11\frac{1}{(2\chi+1)^2}G_{-1,0,0,-1}(\chi) - 11\frac{1}{(2\chi+1)^2}G_{-1,0,0,0}(\chi)
\end{aligned}
$$

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\begin{aligned}
&+ 18 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},-1,-1,-1}(\chi) - 18 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},-1,-1,0}(\chi) - 18 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},-1,0,-1}(\chi) + 18 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},-1,0,0}(\chi) \\
&- 36 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},-\frac{1}{2},-1,-1}(\chi) + 36 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},-\frac{1}{2},-1,0}(\chi) + 36 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},-\frac{1}{2},0,-1}(\chi) - 36 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},-\frac{1}{2},0,0}(\chi) \\
&+ 18 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},0,-1,-1}(\chi) - 18 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},0,-1,0}(\chi) - 18 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},0,0,-1}(\chi) + 18 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},0,0,0}(\chi) \\
&- 11 \frac{1}{(2\chi+1)^2} G_{0,-1,-1,-1}(\chi) + 11 \frac{1}{(2\chi+1)^2} G_{0,-1,-1,0}(\chi) + 11 \frac{1}{(2\chi+1)^2} G_{0,-1,0,-1}(\chi) - 11 \frac{1}{(2\chi+1)^2} G_{0,-1,0,0}(\chi) \\
&+ 18 \frac{1}{(2\chi+1)^2} G_{0,-\frac{1}{2},-1,-1}(\chi) - 18 \frac{1}{(2\chi+1)^2} G_{0,-\frac{1}{2},-1,0}(\chi) - 18 \frac{1}{(2\chi+1)^2} G_{0,-\frac{1}{2},0,-1}(\chi) + 18 \frac{1}{(2\chi+1)^2} G_{0,-\frac{1}{2},0,0}(\chi) \\
&- 7 \frac{1}{(2\chi+1)^2} G_{0,0,-1,-1}(\chi) + 7 \frac{1}{(2\chi+1)^2} G_{0,0,-1,0}(\chi) + 7 \frac{1}{(2\chi+1)^2} G_{0,0,0,-1}(\chi) - 7 \frac{1}{(2\chi+1)^2} G_{0,0,0,0}(\chi) \Big) i \\
&- \frac{1}{6} \frac{(8\chi^3 - 32\chi^2 - 56\chi + 17)\pi^3}{(\chi+1)\chi(2\chi+1)^2} + 48 \frac{\pi}{(2\chi+1)^2} \zeta_3 + 11 \frac{\pi^3}{(2\chi+1)^2} G_{-1}(\chi) - 24 \frac{\pi^3}{(2\chi+1)^2} G_{-\frac{1}{2}}(\chi) + 13 \frac{\pi^3}{(2\chi+1)^2} G_0(\chi) \\
&+ \Big(\Big(9 \frac{1}{(2\chi+1)^2} \zeta_2 + \frac{1}{6} \frac{(\chi+2)(2\chi-1)}{(\chi+1)(2\chi+1)\chi} G_{-1}(\chi) - \frac{1}{6} \frac{(\chi+2)(2\chi-1)}{(\chi+1)(2\chi+1)\chi} G_0(\chi) - \frac{1}{(2\chi+1)^2} G_{-1,-1}(\chi) + \frac{1}{(2\chi+1)^2} G_{-1,0}(\chi) \\
&+ \frac{1}{(2\chi+1)^2} G_{0,-1}(\chi) - \frac{1}{(2\chi+1)^2} G_{0,0}(\chi)\Big) i + \frac{\pi}{(\chi+1)(2\chi+1)\chi} + 2 \frac{\pi}{(2\chi+1)^2} G_{-1}(\chi) - 2 \frac{\pi}{(2\chi+1)^2} G_0(\chi)\Big) \frac{1}{\epsilon^2} \\
&+ 4 \frac{(2\chi^2 + 2\chi - 1)\pi}{(\chi+1)\chi(2\chi+1)^2} G_{-1,-1}(\chi) - 4 \frac{(2\chi^2 + 2\chi - 1)\pi}{(\chi+1)\chi(2\chi+1)^2} G_{-1,0}(\chi) + 2 \frac{(4\chi^3 - 4\chi^2 - 13\chi + 4)\pi}{(\chi+1)\chi(2\chi+1)^2} G_{-\frac{1}{2},-1}(\chi) \\
&- 2 \frac{(4\chi^3 - 4\chi^2 - 13\chi + 4)\pi}{(\chi+1)\chi(2\chi+1)^2} G_{-\frac{1}{2},0}(\chi) - 2 \frac{(4\chi^3 - 9\chi + 2)\pi}{(\chi+1)\chi(2\chi+1)^2} G_{0,-1}(\chi) + 2 \frac{(4\chi^3 - 9\chi + 2)\pi}{(\chi+1)\chi(2\chi+1)^2} G_{0,0}(\chi) \\
&+ \Big(\Big(\Big(\Big(\frac{1}{6} \frac{40\chi^3 + 188\chi^2 + 68\chi - 89}{(\chi+1)\chi(2\chi+1)^2} + 3 \frac{1}{(2\chi+1)^2} G_{-1}(\chi) - 54 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2}}(\chi) + 51 \frac{1}{(2\chi+1)^2} G_0(\chi)\Big) \zeta_2 + 6 \frac{1}{(2\chi+1)^2} \zeta_3 \\
&+ \frac{1}{6} \frac{4\chi^3 - 4\chi^2 - 13\chi + 4}{(\chi+1)\chi(2\chi+1)^2} G_{-1,-1}(\chi) - \frac{1}{6} \frac{4\chi^3 - 4\chi^2 - 13\chi + 4}{(\chi+1)\chi(2\chi+1)^2} G_{-1,0}(\chi) - \frac{1}{6} \frac{4\chi^3 - 4\chi^2 - 13\chi + 4}{(\chi+1)\chi(2\chi+1)^2} G_{0,-1}(\chi) \\
&+ \frac{1}{6} \frac{4\chi^3 - 4\chi^2 - 13\chi + 4}{(\chi+1)\chi(2\chi+1)^2} G_{0,0}(\chi) - 3 \frac{1}{(2\chi+1)^2} G_{-1,-1,-1}(\chi) + 3 \frac{1}{(2\chi+1)^2} G_{-1,-1,0}(\chi) + 3 \frac{1}{(2\chi+1)^2} G_{-1,0,-1}(\chi) \\
&- 3 \frac{1}{(2\chi+1)^2} G_{-1,0,0}(\chi) + 6 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},-1,-1}(\chi) - 6 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},-1,0}(\chi) - 6 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},0,-1}(\chi) \\
&+ 6 \frac{1}{(2\chi+1)^2} G_{-\frac{1}{2},0,0}(\chi) - 3 \frac{1}{(2\chi+1)^2} G_{0,-1,-1}(\chi) + 3 \frac{1}{(2\chi+1)^2} G_{0,-1,0}(\chi) + 3 \frac{1}{(2\chi+1)^2} G_{0,0,-1}(\chi) - 3 \frac{1}{(2\chi+1)^2} G_{0,0,0}(\chi)\Big) i \\
&+ 4 \frac{\pi^3}{(2\chi+1)^2} + 2 \frac{(2\chi^2 + 2\chi - 1)\pi}{(\chi+1)\chi(2\chi+1)^2} G_{-1}(\chi) - 2 \frac{(2\chi^2 + 2\chi - 1)\pi}{(\chi+1)\chi(2\chi+1)^2} G_0(\chi) + 4 \frac{\pi}{(2\chi+1)^2} G_{-1,-1}(\chi) - 4 \frac{\pi}{(2\chi+1)^2} G_{-1,0}(\chi) \\
&- 12 \frac{\pi}{(2\chi+1)^2} G_{-\frac{1}{2},-1}(\chi) + 12 \frac{\pi}{(2\chi+1)^2} G_{-\frac{1}{2},0}(\chi) + 8 \frac{\pi}{(2\chi+1)^2} G_{0,-1}(\chi) - 8 \frac{\pi}{(2\chi+1)^2} G_{0,0}(\chi)\Big) \frac{1}{\epsilon} + 8 \frac{\pi}{(2\chi+1)^2} G_{-1,-1,-1}(\chi) \\
&- 8 \frac{\pi}{(2\chi+1)^2} G_{-1,-1,0}(\chi) - 36 \frac{\pi}{(2\chi+1)^2} G_{-1,-\frac{1}{2},-1}(\chi) + 36 \frac{\pi}{(2\chi+1)^2} G_{-1,-\frac{1}{2},0}(\chi) + 28 \frac{\pi}{(2\chi+1)^2} G_{-1,0,-1}(\chi) \\
&- 28 \frac{\pi}{(2\chi+1)^2} G_{-1,0,0}(\chi) - 24 \frac{\pi}{(2\chi+1)^2} G_{-\frac{1}{2},-1,-1}(\chi) + 24 \frac{\pi}{(2\chi+1)^2} G_{-\frac{1}{2},-1,0}(\chi) + 72 \frac{\pi}{(2\chi+1)^2} G_{-\frac{1}{2},-\frac{1}{2},-1}(\chi) \\
&- 72 \frac{\pi}{(2\chi+1)^2} G_{-\frac{1}{2},-\frac{1}{2},0}(\chi) - 48 \frac{\pi}{(2\chi+1)^2} G_{-\frac{1}{2},0,-1}(\chi) + 48 \frac{\pi}{(2\chi+1)^2} G_{-\frac{1}{2},0,0}(\chi) + 16 \frac{\pi}{(2\chi+1)^2} G_{0,-1,-1}(\chi) \\
&- 16 \frac{\pi}{(2\chi+1)^2} G_{0,-1,0}(\chi) - 36 \frac{\pi}{(2\chi+1)^2} G_{0,-\frac{1}{2},-1}(\chi) + 36 \frac{\pi}{(2\chi+1)^2} G_{0,-\frac{1}{2},0}(\chi) + 20 \frac{\pi}{(2\chi+1)^2} G_{0,0,-1}(\chi) \\
&- 20 \frac{\pi}{(2\chi+1)^2} G_{0,0,0}(\chi)\Big) \mu^{-4\epsilon} + O(\mu)
\end{aligned}
\tag{D.19}
$$

I[FNP9, 1, 1, 0, 0, 1, 0, 1, 1, 1] =

$$
\begin{aligned}
&\Big(\frac{1}{\epsilon^4} \frac{1}{4} \frac{1}{\chi} i + \Big(\Big(7 \frac{1}{\chi} G_{-1,-1}(\chi) - 7 \frac{1}{\chi} G_{-1,0}(\chi) - 7 \frac{1}{\chi} G_{0,-1}(\chi) + 7 \frac{1}{\chi} G_{0,0}(\chi)\Big) \zeta_2 + \Big(-\frac{40}{3} \frac{1}{\chi} G_{-1}(\chi) + \frac{40}{3} \frac{1}{\chi} G_0(\chi)\Big) \zeta_3 + \frac{151}{16} \frac{1}{\chi} \zeta_4 \\
&+ 4 \frac{1}{\chi} G_{-1,-1,-1,-1}(\chi) - 4 \frac{1}{\chi} G_{-1,-1,-1,0}(\chi) - 4 \frac{1}{\chi} G_{-1,-1,0,-1}(\chi) + 4 \frac{1}{\chi} G_{-1,-1,0,0}(\chi) - 4 \frac{1}{\chi} G_{-1,0,-1,-1}(\chi) + 4 \frac{1}{\chi} G_{-1,0,-1,0}(\chi) \\
&+ 4 \frac{1}{\chi} G_{-1,0,0,-1}(\chi) - 4 \frac{1}{\chi} G_{-1,0,0,0}(\chi) - 4 \frac{1}{\chi} G_{0,-1,-1,-1}(\chi) + 4 \frac{1}{\chi} G_{0,-1,-1,0}(\chi) + 4 \frac{1}{\chi} G_{0,-1,0,-1}(\chi) - 4 \frac{1}{\chi} G_{0,-1,0,0}(\chi) \\
&+ 4 \frac{1}{\chi} G_{0,0,-1,-1}(\chi) - 4 \frac{1}{\chi} G_{0,0,-1,0}(\chi) - 4 \frac{1}{\chi} G_{0,0,0,-1}(\chi) + 4 \frac{1}{\chi} G_{0,0,0,0}(\chi)\Big) i + \Big(\Big(\frac{1}{2} \frac{1}{\chi} G_{-1}(\chi) - \frac{1}{2} \frac{1}{\chi} G_0(\chi)\Big) i - \frac{1}{2} \frac{\pi}{\chi}\Big) \frac{1}{\epsilon^3} + \frac{40}{3} \frac{\pi}{\chi} \zeta_3 \\
&- \frac{5}{2} \frac{\pi^3}{\chi} G_{-1}(\chi) + \frac{5}{2} \frac{\pi^3}{\chi} G_0(\chi) + \Big(\Big(\frac{7}{4} \frac{1}{\chi} \zeta_2 + \frac{1}{\chi} G_{-1,-1}(\chi) - \frac{1}{\chi} G_{-1,0}(\chi) - \frac{1}{\chi} G_{0,-1}(\chi) + \frac{1}{\chi} G_{0,0}(\chi)\Big) i - \frac{\pi}{\chi} G_{-1}(\chi) + \frac{\pi}{\chi} G_0(\chi)\Big) \frac{1}{\epsilon^2} \\
&+ \Big(\Big(\Big(\Big(\frac{7}{2} \frac{1}{\chi} G_{-1}(\chi) - \frac{7}{2} \frac{1}{\chi} G_0(\chi)\Big) \zeta_2 - \frac{20}{3} \frac{1}{\chi} \zeta_3 + 2 \frac{1}{\chi} G_{-1,-1,-1}(\chi) - 2 \frac{1}{\chi} G_{-1,-1,0}(\chi) - 2 \frac{1}{\chi} G_{-1,0,-1}(\chi) + 2 \frac{1}{\chi} G_{-1,0,0}(\chi) \\
&- 2 \frac{1}{\chi} G_{0,-1,-1}(\chi) + 2 \frac{1}{\chi} G_{0,-1,0}(\chi) + 2 \frac{1}{\chi} G_{0,0,-1}(\chi) - 2 \frac{1}{\chi} G_{0,0,0}(\chi)\Big) i - \frac{5}{4} \frac{\pi^3}{\chi} - 2 \frac{\pi}{\chi} G_{-1,-1}(\chi) + 2 \frac{\pi}{\chi} G_{-1,0}(\chi) + 2 \frac{\pi}{\chi} G_{0,-1}(\chi)
\end{aligned}
$$

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\begin{aligned}
&-2\frac{\pi}{\chi}G_{0,0}(\chi)\Big)\frac{1}{\epsilon}-4\frac{\pi}{\chi}G_{-1,-1,-1}(\chi)+4\frac{\pi}{\chi}G_{-1,-1,0}(\chi)+4\frac{\pi}{\chi}G_{-1,0,-1}(\chi)-4\frac{\pi}{\chi}G_{-1,0,0}(\chi)+4\frac{\pi}{\chi}G_{0,-1,-1}(\chi)-4\frac{\pi}{\chi}G_{0,-1,0}(\chi)\\
&-4\frac{\pi}{\chi}G_{0,0,-1}(\chi)+4\frac{\pi}{\chi}G_{0,0,0}(\chi)\Big)\mu^{-4\epsilon}\mu^{-1}+\Big(\Big(-39\frac{1}{\chi}G_{-1}(\chi)+39\frac{1}{\chi+1}G_{0}(\chi)\Big)\zeta_2-12\frac{1}{\chi+1}\zeta_3+48\frac{1}{\chi}G_{-1}(\chi)-48\frac{1}{\chi+1}G_{0}(\chi)\\
&+24\frac{1}{(\chi+1)\chi}G_{-1,0}(\chi)+24\frac{1}{(\chi+1)\chi}G_{0,-1}(\chi)-12\frac{1}{\chi}G_{-1,-1,-1}(\chi)+12\frac{1}{(\chi+1)\chi}G_{-1,-1,0}(\chi)+12\frac{1}{(\chi+1)\chi}G_{-1,0,-1}(\chi)\\
&-12\frac{1}{\chi}G_{-1,0,0}(\chi)+12\frac{1}{\chi+1}G_{0,-1,-1}(\chi)+12\frac{1}{(\chi+1)\chi}G_{0,-1,0}(\chi)+12\frac{1}{(\chi+1)\chi}G_{0,0,-1}(\chi)+12\frac{1}{\chi+1}G_{0,0,0}(\chi)\Big)i\\
&+\frac{1}{2}\frac{(5\chi\pi^2-96\chi+\pi^2-96)\pi}{(\chi+1)\chi}+\Big(\Big(3\frac{1}{\chi}G_{-1}(\chi)-3\frac{1}{\chi+1}G_{0}(\chi)\Big)i-3\frac{\pi}{\chi}\Big)\frac{1}{\epsilon^2}-24\frac{\pi}{(\chi+1)\chi}G_{0}(\chi)+\Big(\Big(-12\frac{1}{\chi}G_{-1}(\chi)+12\frac{1}{\chi+1}G_{0}(\chi)\\
&-6\frac{1}{(\chi+1)\chi}G_{-1,0}(\chi)-6\frac{1}{(\chi+1)\chi}G_{0,-1}(\chi)\Big)i+12\frac{\pi}{\chi}+6\frac{\pi}{(\chi+1)\chi}G_{0}(\chi)\Big)\frac{1}{\epsilon}+12\frac{\pi}{\chi}G_{-1,-1}(\chi)-12\frac{\pi}{(\chi+1)\chi}G_{-1,0}(\chi)\\
&-12\frac{\pi}{\chi+1}G_{0,-1}(\chi)-12\frac{\pi}{(\chi+1)\chi}G_{0,0}(\chi)+\Big(\frac{1}{\epsilon^3}\frac{1}{\chi+1}i+\Big(-27\frac{1}{\chi+1}+\Big(-3\frac{2\chi+3}{(\chi+1)\chi}+\frac{12\chi+11}{(\chi+1)\chi}G_{-1}(\chi)-6\frac{2\chi+1}{(\chi+1)\chi}G_{0}(\chi)\Big)\zeta_2\\
&+\frac{16}{3}\frac{1}{\chi+1}\zeta_3-9\frac{2\chi+1}{(\chi+1)\chi}G_{-1}(\chi)+18\frac{1}{\chi+1}G_{0}(\chi)+3\frac{1}{(\chi+1)\chi}G_{-1,-1}(\chi)-6\frac{1}{(\chi+1)\chi}G_{-1,0}(\chi)-6\frac{1}{(\chi+1)\chi}G_{0,-1}(\chi)\\
&+\frac{2\chi+3}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)-2\frac{1}{(\chi+1)\chi}G_{-1,-1,0}(\chi)-2\frac{1}{(\chi+1)\chi}G_{-1,0,-1}(\chi)+4\frac{1}{\chi}G_{-1,0,0}(\chi)-2\frac{\chi-1}{(\chi+1)\chi}G_{0,-1,-1}(\chi)\\
&-4\frac{1}{(\chi+1)\chi}G_{0,-1,0}(\chi)-4\frac{1}{(\chi+1)\chi}G_{0,0,-1}(\chi)-4\frac{1}{\chi+1}G_{0,0,0}(\chi)\Big)i-\frac{1}{6}\frac{(2\chi\pi^2-108\chi-3\pi^2-54)\pi}{(\chi+1)\chi}+\Big(\Big(-3\frac{1}{\chi+1}\\
&-\frac{2\chi+1}{(\chi+1)\chi}G_{-1}(\chi)+2\frac{1}{\chi+1}G_{0}(\chi)\Big)i+\frac{(2\chi+1)\pi}{(\chi+1)\chi}\Big)\frac{1}{\epsilon^2}-3\frac{\pi}{(\chi+1)\chi}G_{-1}(\chi)+6\frac{\pi}{(\chi+1)\chi}G_{0}(\chi)+\Big(\Big(9\frac{1}{\chi+1}+\frac{2\chi+3}{(\chi+1)\chi}\zeta_2\\
&+3\frac{2\chi+1}{(\chi+1)\chi}G_{-1}(\chi)-6\frac{1}{\chi+1}G_{0}(\chi)-\frac{1}{(\chi+1)\chi}G_{-1,-1}(\chi)+2\frac{1}{(\chi+1)\chi}G_{-1,0}(\chi)+2\frac{1}{(\chi+1)\chi}G_{0,-1}(\chi)\Big)i-3\frac{(2\chi+1)\pi}{(\chi+1)\chi}\\
&+\frac{\pi}{(\chi+1)\chi}G_{-1}(\chi)-2\frac{\pi}{(\chi+1)\chi}G_{0}(\chi)\Big)\frac{1}{\epsilon}-\frac{(2\chi+3)\pi}{(\chi+1)\chi}G_{-1,-1}(\chi)+2\frac{\pi}{(\chi+1)\chi}G_{-1,0}(\chi)+2\frac{(\chi-1)\pi}{(\chi+1)\chi}G_{0,-1}(\chi)\\
&+4\frac{\pi}{(\chi+1)\chi}G_{0,0}(\chi)\Big)\mu^{-\epsilon}+\Big(-\frac{1}{\epsilon^3}\frac{1}{2}\frac{1}{\chi+1}i+\frac{1}{\epsilon^2}\frac{1}{\chi+1}i+\frac{1}{\epsilon}\Big(-2\frac{1}{\chi+1}-\frac{3}{2}\frac{1}{\chi+1}\zeta_2\Big)i+\Big(4\frac{1}{\chi+1}+3\frac{1}{\chi+1}\zeta_2+\frac{4}{3}\frac{1}{\chi+1}\zeta_3\Big)i\Big)\mu^{-2\epsilon}\\
&+\Big(\frac{1}{\epsilon^3}\frac{1}{2}\frac{1}{(\chi+1)\chi}i+\Big(\Big(7\frac{1}{(\chi+1)\chi}G_{-1}(\chi)-7\frac{1}{(\chi+1)\chi}G_{0}(\chi)\Big)\zeta_2-\frac{40}{3}\frac{1}{(\chi+1)\chi}\zeta_3+4\frac{1}{(\chi+1)\chi}G_{-1,-1,-1}(\chi)\\
&-4\frac{1}{(\chi+1)\chi}G_{-1,-1,0}(\chi)-4\frac{1}{(\chi+1)\chi}G_{-1,0,-1}(\chi)+4\frac{1}{(\chi+1)\chi}G_{-1,0,0}(\chi)-4\frac{1}{(\chi+1)\chi}G_{0,-1,-1}(\chi)\\
&+4\frac{1}{(\chi+1)\chi}G_{0,-1,0}(\chi)+4\frac{1}{(\chi+1)\chi}G_{0,0,-1}(\chi)-4\frac{1}{(\chi+1)\chi}G_{0,0,0}(\chi)\Big)i+\Big(\Big(\frac{1}{(\chi+1)\chi}G_{-1}(\chi)-\frac{1}{(\chi+1)\chi}G_{0}(\chi)\Big)i-\frac{\pi}{(\chi+1)\chi}\Big)\frac{1}{\epsilon^2}\\
&-\frac{5}{2}\frac{\pi^3}{(\chi+1)\chi}+\Big(\Big(\frac{7}{2}\frac{1}{(\chi+1)\chi}\zeta_2+2\frac{1}{(\chi+1)\chi}G_{-1,-1}(\chi)-2\frac{1}{(\chi+1)\chi}G_{-1,0}(\chi)-2\frac{1}{(\chi+1)\chi}G_{0,-1}(\chi)+2\frac{1}{(\chi+1)\chi}G_{0,0}(\chi)\Big)i\\
&-2\frac{\pi}{(\chi+1)\chi}G_{-1}(\chi)+2\frac{\pi}{(\chi+1)\chi}G_{0}(\chi)\Big)\frac{1}{\epsilon}-4\frac{\pi}{(\chi+1)\chi}G_{-1,-1}(\chi)+4\frac{\pi}{(\chi+1)\chi}G_{-1,0}(\chi)+4\frac{\pi}{(\chi+1)\chi}G_{0,-1}(\chi)\\
&-4\frac{\pi}{(\chi+1)\chi}G_{0,0}(\chi)\Big)\mu^{-4\epsilon}+O(\mu)
\end{aligned}
\tag{D.20}
$$

I[FNPM2, 1, 1, 0, 1, 1, 0, 0, 1, 1] =

$$
\begin{aligned}
&-\frac{1}{\epsilon^4}\frac{3}{4}\frac{1}{\chi+1}i+\Big(\Big(33\frac{1}{\chi+1}G_{-1,-1}(\chi)-15\frac{1}{\chi+1}G_{0,-1}(\chi)\Big)\zeta_2-13\frac{1}{\chi+1}G_{-1}(\chi)\zeta_3-\frac{957}{16}\frac{1}{\chi+1}\zeta_4-12\frac{1}{\chi+1}G_{-1,-1,-1,-1}(\chi)\\
&+6\frac{1}{\chi+1}G_{-1,0,-1,-1}(\chi)+12\frac{1}{\chi+1}G_{0,-1,-1,-1}(\chi)-9\frac{1}{\chi+1}G_{0,0,-1,-1}(\chi)\Big)i+\Big(\frac{3}{2}\frac{1}{\chi+1}G_{-1}(\chi)i-\frac{3}{2}\frac{1}{\chi+1}\pi\Big)\frac{1}{\epsilon^3}\\
&+\Big(\Big(\frac{33}{4}\frac{1}{\chi+1}\zeta_2-3\frac{1}{\chi+1}G_{-1,-1}(\chi)\Big)i+3\frac{1}{\chi+1}G_{-1}(\chi)\pi\Big)\frac{1}{\epsilon^2}+\Big(12\frac{1}{\chi+1}G_{-1,-1,-1}(\chi)-6\frac{1}{\chi+1}G_{-1,0,-1}(\chi)\\
&-12\frac{1}{\chi+1}G_{0,-1,-1}(\chi)+9\frac{1}{\chi+1}G_{0,0,-1}(\chi)\Big)\pi+\Big(\Big(-\frac{33}{2}\frac{1}{\chi+1}G_{-1}(\chi)\zeta_2+\frac{13}{2}\frac{1}{\chi+1}\zeta_3+6\frac{1}{\chi+1}G_{-1,-1,-1}(\chi)\\
&-3\frac{1}{\chi+1}G_{0,-1,-1}(\chi)\Big)i+\Big(-6\frac{1}{\chi+1}G_{-1,-1}(\chi)+3\frac{1}{\chi+1}G_{0,-1}(\chi)\Big)\pi+\frac{5}{4}\frac{1}{\chi+1}\pi^3\Big)\frac{1}{\epsilon}-\frac{5}{2}\frac{1}{\chi+1}G_{-1}(\chi)\pi^3+16\frac{1}{\chi+1}\pi\zeta_3\\
&+\Big(\Big(-\frac{143355}{64}\frac{1}{\chi+1}+\Big(\frac{15081}{16}\frac{1}{\chi+1}-\frac{3909}{8}\frac{1}{\chi+1}G_{-1}(\chi)+\frac{519}{2}\frac{1}{\chi+1}G_{-1,-1}(\chi)-42\frac{1}{\chi+1}G_{0,-1}(\chi)\Big)\zeta_2\\
&+\Big(\frac{935}{2}\frac{1}{\chi+1}-46\frac{1}{\chi+1}G_{-1}(\chi)\Big)\zeta_3-\frac{519}{8}\frac{1}{\chi+1}\zeta_4+\frac{16377}{32}\frac{1}{\chi+1}G_{-1}(\chi)-\frac{2403}{8}\frac{1}{\chi+1}G_{-1,-1}(\chi)+\frac{345}{2}\frac{1}{\chi+1}G_{-1,-1,-1}(\chi)\\
&-12\frac{1}{\chi+1}G_{0,-1,-1}(\chi)-102\frac{1}{\chi+1}G_{-1,-1,-1,-1}(\chi)+12\frac{1}{\chi+1}G_{-1,0,-1,-1}(\chi)+24\frac{1}{\chi+1}G_{0,-1,-1,-1}(\chi)\\
&-6\frac{1}{\chi+1}G_{0,0,-1,-1}(\chi)\Big)i+\Big(-\frac{16377}{32}\frac{1}{\chi+1}+\frac{2403}{8}\frac{1}{\chi+1}G_{-1}(\chi)-\frac{345}{2}\frac{1}{\chi+1}G_{-1,-1}(\chi)+12\frac{1}{\chi+1}G_{0,-1}(\chi)\\
&+102\frac{1}{\chi+1}G_{-1,-1,-1}(\chi)-12\frac{1}{\chi+1}G_{-1,0,-1}(\chi)-24\frac{1}{\chi+1}G_{0,-1,-1}(\chi)+6\frac{1}{\chi+1}G_{0,0,-1}(\chi)\Big)\pi+\Big(\frac{415}{16}\frac{1}{\chi+1}\\
&-\frac{45}{4}\frac{1}{\chi+1}G_{-1}(\chi)\Big)\pi^3+40\frac{1}{\chi+1}\pi\zeta_3\Big)\epsilon+\Big(\Big(\frac{450999}{64}\frac{1}{\chi+1}+\Big(-\frac{54009}{16}\frac{1}{\chi+1}+\frac{12285}{8}\frac{1}{\chi+1}G_{-1}(\chi)-\frac{1053}{2}\frac{1}{\chi+1}G_{-1,-1}(\chi)\Big)\zeta_2
\end{aligned}
$$

$$
\begin{aligned}
&+\bigg(-1455\frac{1}{\chi+1}+216\frac{1}{\chi+1}G_{-1}(\chi)\bigg)\zeta_3+\frac{9225}{16}\frac{1}{\chi+1}\zeta_4-\frac{57645}{32}\frac{1}{\chi+1}G_{-1}(\chi)+\frac{7911}{8}\frac{1}{\chi+1}G_{-1,-1}(\chi)-\frac{945}{2}\frac{1}{\chi+1}G_{-1,-1,-1}(\chi)\\
&+162\frac{1}{\chi+1}G_{-1,-1,-1,-1}(\chi)\bigg)i+\bigg(\frac{57645}{32}\frac{1}{\chi+1}-\frac{7911}{8}\frac{1}{\chi+1}G_{-1}(\chi)+\frac{945}{2}\frac{1}{\chi+1}G_{-1,-1}(\chi)-162\frac{1}{\chi+1}G_{-1,-1,-1}(\chi)\bigg)\pi\\
&+\bigg(-\frac{1575}{16}\frac{1}{\chi+1}+\frac{135}{4}\frac{1}{\chi+1}G_{-1}(\chi)\bigg)\pi^3-216\frac{1}{\chi+1}\pi\zeta_3\bigg)\epsilon^2+\bigg(\frac{1}{\epsilon^4}\frac{1}{\chi+1}i+\bigg(\frac{49}{9}\frac{1}{\chi+1}K_2^2-\bigg(G_{-1}(\chi)+G_0(3)\bigg)\frac{14}{9}\frac{1}{\chi+1}K_2\,\pi\\
&-\frac{56}{15}\frac{1}{\chi+1}K_{31}\pi+\bigg(-\frac{47}{3}\frac{1}{\chi+1}G_{-1,-1}(\chi)+4\frac{1}{\chi+1}G_{0,-1}(\chi)+\frac{14}{15}\frac{1}{\chi+1}G_{0,0}(3)\bigg)\zeta_2+\frac{112}{9}\frac{1}{\chi+1}G_{-1}(\chi)\zeta_3+\frac{287}{8}\frac{1}{\chi+1}\zeta_4\\
&+5\frac{1}{\chi+1}G_{-1,-1,-1,-1}(\chi)-2\frac{1}{\chi+1}G_{-1,0,-1,-1}(\chi)-4\frac{1}{\chi+1}G_{0,-1,-1,-1}(\chi)+4\frac{1}{\chi+1}G_{0,0,-1,-1}(\chi)\bigg)i+\bigg(-2\frac{1}{\chi+1}G_{-1}(\chi)i\\
&+2\frac{1}{\chi+1}\pi\bigg)\frac{1}{\epsilon^3}+\bigg(\bigg(-\frac{29}{3}\frac{1}{\chi+1}\zeta_2+3\frac{1}{\chi+1}G_{-1,-1}(\chi)\bigg)i-3\frac{1}{\chi+1}G_{-1}(\chi)\pi\bigg)\frac{1}{\epsilon^2}+\bigg(-5\frac{1}{\chi+1}G_{-1,-1,-1}(\chi)+2\frac{1}{\chi+1}G_{-1,0,-1}(\chi)\\
&+4\frac{1}{\chi+1}G_{0,-1,-1}(\chi)-4\frac{1}{\chi+1}G_{0,0,-1}(\chi)\bigg)\pi+\bigg(\bigg(\frac{14}{9}\frac{1}{\chi+1}K_2\pi+\frac{38}{3}\frac{1}{\chi+1}G_{-1}(\chi)\zeta_2-\frac{88}{9}\frac{1}{\chi+1}\zeta_3-4\frac{1}{\chi+1}G_{-1,-1,-1}(\chi)\\
&+2\frac{1}{\chi+1}G_{0,-1,-1}(\chi)\bigg)i+\bigg(4\frac{1}{\chi+1}G_{-1,-1}(\chi)-2\frac{1}{\chi+1}G_{0,-1}(\chi)\bigg)\pi-\frac{10}{9}\frac{1}{\chi+1}\pi^3\bigg)\frac{1}{\epsilon}+\frac{23}{18}\frac{1}{\chi+1}G_{-1}(\chi)\pi^3+\frac{28}{3}\frac{1}{\chi+1}K_2\zeta_2\\
&-\frac{130}{9}\frac{1}{\chi+1}\pi\zeta_3\bigg)\mu^{-\epsilon}+\bigg(\frac{1}{\epsilon^3}\frac{1}{\chi+1}i+\bigg(3\frac{1}{\chi+1}G_{-1}(\chi)\zeta_2-\frac{8}{3}\frac{1}{\chi+1}\zeta_3-\frac{1}{\chi+1}G_{-1,-1,-1}(\chi)\bigg)i+\bigg(-\frac{1}{\chi+1}G_{-1}(\chi)i+\frac{1}{\chi+1}\pi\bigg)\frac{1}{\epsilon^2}\\
&+\bigg(\bigg(-3\frac{1}{\chi+1}\zeta_2+\frac{1}{\chi+1}G_{-1,-1}(\chi)\bigg)i-\frac{1}{\chi+1}G_{-1}(\chi)\pi\bigg)\frac{1}{\epsilon}+\frac{1}{\chi+1}G_{-1,-1}(\chi)\pi-\frac{1}{6}\frac{1}{\chi+1}\pi^3+\bigg(\bigg(-3\frac{1}{\chi+1}G_{-1,-1}(\chi)\zeta_2\\
&+\frac{8}{3}\frac{1}{\chi+1}G_{-1}(\chi)\zeta_3+\frac{3}{4}\frac{1}{\chi+1}\zeta_4+\frac{1}{\chi+1}G_{-1,-1,-1,-1}(\chi)\bigg)i-\frac{1}{\chi+1}G_{-1,-1,-1}(\chi)\pi+\frac{1}{6}\frac{1}{\chi+1}G_{-1}(\chi)\pi^3-\frac{8}{3}\frac{1}{\chi+1}\pi\zeta_3\bigg)\epsilon\\
&+\bigg(\bigg(-32\frac{1}{\chi+1}+\bigg(24\frac{1}{\chi+1}-12\frac{1}{\chi+1}G_{-1}(\chi)+6\frac{1}{\chi+1}G_{-1,-1}(\chi)\bigg)\zeta_2+\bigg(\frac{32}{3}\frac{1}{\chi+1}-\frac{16}{3}\frac{1}{\chi+1}G_{-1}(\chi)\bigg)\zeta_3-\frac{3}{2}\frac{1}{\chi+1}\zeta_4\\
&+16\frac{1}{\chi+1}G_{-1}(\chi)-8\frac{1}{\chi+1}G_{-1,-1}(\chi)+4\frac{1}{\chi+1}G_{-1,-1,-1}(\chi)-2\frac{1}{\chi+1}G_{-1,-1,-1,-1}(\chi)\bigg)i+\bigg(-16\frac{1}{\chi+1}+8\frac{1}{\chi+1}G_{-1}(\chi)\\
&-4\frac{1}{\chi+1}G_{-1,-1}(\chi)+2\frac{1}{\chi+1}G_{-1,-1,-1}(\chi)\bigg)\pi+\bigg(\frac{2}{3}\frac{1}{\chi+1}-\frac{1}{3}\frac{1}{\chi+1}G_{-1}(\chi)\bigg)\pi^3+\frac{16}{3}\frac{1}{\chi+1}\pi\zeta_3\bigg)\epsilon^2\bigg)\log(\mu)\mu^{-\epsilon}+\bigg(-\frac{1}{\epsilon^4}\frac{1}{4}\frac{1}{\chi+1}i\\
&+\frac{1}{\epsilon^2}\frac{17}{12}\frac{1}{\chi+1}\zeta_2 i+\epsilon\bigg(-16\frac{1}{\chi+1}+\frac{28}{9}\frac{1}{\chi+1}K_2^2+\bigg(\frac{16}{9}\frac{1}{\chi+1}-\frac{8}{9}\frac{1}{\chi+1}G_0(3)\bigg)K_2\pi-\frac{32}{15}\frac{1}{\chi+1}K_{31}\pi+\bigg(-\frac{20}{3}\frac{1}{\chi+1}\\
&+\frac{8}{15}\frac{1}{\chi+1}G_{0,0}(3)\bigg)\zeta_2+\frac{16}{9}\frac{1}{\chi+1}\zeta_3-\frac{1}{36}\frac{1}{\chi+1}\zeta_4\bigg)i+\bigg(-\frac{28}{9}\frac{1}{\chi+1}K_2^2+\frac{8}{9}\frac{1}{\chi+1}G_0(3)K_2\pi+\frac{32}{15}\frac{1}{\chi+1}K_{31}\pi-\frac{8}{15}\frac{1}{\chi+1}G_{0,0}(3)\zeta_2\\
&-\frac{4}{3}\frac{1}{\chi+1}G_{-1}(\chi)\zeta_3+\frac{2605}{144}\frac{1}{\chi+1}\zeta_4\bigg)i+\bigg(\frac{1}{2}\frac{1}{\chi+1}G_{-1}(\chi)i-\frac{1}{2}\frac{1}{\chi+1}\pi\bigg)\frac{1}{\epsilon^3}+\bigg(\bigg(-\frac{8}{9}\frac{1}{\chi+1}K_2\pi+\frac{3}{2}\frac{1}{\chi+1}G_{-1}(\chi)\zeta_2+\frac{53}{18}\frac{1}{\chi+1}\zeta_3\bigg)i\\
&-\frac{1}{4}\frac{1}{\chi+1}\pi^3\bigg)\frac{1}{\epsilon}+\frac{4}{3}\frac{1}{\chi+1}\pi\zeta_3\bigg)\mu^{-2\epsilon}+\bigg(-\frac{1}{\epsilon^3}\frac{1}{2}\frac{1}{\chi+1}i-\frac{1}{\epsilon}\frac{3}{2}\frac{1}{\chi+1}\zeta_2 i+\frac{4}{3}\frac{1}{\chi+1}\zeta_3 i-\epsilon\frac{63}{8}\frac{1}{\chi+1}\zeta_4 i+\epsilon^2\bigg(\frac{63}{2}\frac{1}{\chi+1}+\frac{45}{2}\frac{1}{\chi+1}\zeta_2\\
&-\frac{28}{3}\frac{1}{\chi+1}\zeta_3+\frac{189}{8}\frac{1}{\chi+1}\zeta_4\bigg)i\bigg)\log(\mu)\mu^{-2\epsilon}+O(\mu)
\end{aligned}
\tag{D.21}
$$

Appendix E

The $H + j$: results

In this appendix, we present results related to the $H + j$ productions. In particular, we show few examples of subtraction terms for form factors that were described in Section 2.4. We list all helicity coefficients which are needed to calculate one- and two-loop helicity amplitudes.

E.1. Subtraction terms

$$
\begin{aligned}
F^{\mathrm{IR}}_{1,g} = \; & \Bigg(\Bigg(-\frac{37}{4}\frac{z^2+z+1}{sz} + \Bigg(3\frac{z^2+z+1}{sz} + \frac{3}{16}\frac{3z-2}{sz}G_{-1}(z) - \frac{3}{16}\frac{3z+5}{sz}G_0(z) - \frac{3}{16}\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1,-1}(z) + \frac{3}{8}\frac{z+1}{s}G_{-1,0}(z) \\
& + \frac{3}{8}\frac{z+1}{s}G_{0,-1}(z) - \frac{3}{16}\frac{2z^3+4z^2-z-1}{(z+1)sz}G_{0,0}(z)\Bigg)\gamma_E + \Bigg(\frac{3}{16}\frac{6z^3+6z^2-9z+11}{(z+1)sz} + \frac{3}{16}\frac{6z^4+16z^3-5z^2-15z-6}{(z+1)sz^2}G_{-1}(z) \\
& - \frac{3}{16}\frac{6z^3+16z^2+5z+1}{(z+1)sz}G_0(z)\Bigg)\zeta_2 + \frac{9}{8}\frac{1}{(z+1)s}\gamma_E\zeta_2 + \frac{3}{16}\frac{8z^3+17z^2+10z+3}{(z+1)sz}\zeta_3 - \frac{1}{8}\frac{15z-13}{sz}G_{-1}(z) + \frac{1}{8}\frac{15z+28}{sz}G_0(z) \\
& + \frac{1}{8}\frac{10z^3+13z^2-15z-10}{sz^2}G_{-1,-1}(z) - \frac{5}{4}\frac{z+1}{s}G_{-1,0}(z) - \frac{5}{4}\frac{z+1}{s}G_{0,-1}(z) + \frac{1}{8}\frac{10z^3+17z^2-11z-8}{(z+1)sz}G_{0,0}(z) \\
& - \frac{3}{16}\frac{4z^3+3z^2-6z-4}{sz^2}G_{-1,-1,-1}(z) + \frac{3}{8}\frac{z+1}{s}G_{-1,-1,0}(z) + \frac{3}{8}\frac{z+1}{s}G_{-1,0,-1}(z) + \frac{3}{16}\frac{z-1}{(z+1)s}G_{-1,0,0}(z) \\
& - \frac{3}{16}\frac{(z+1)(z+2)}{sz^2}G_{0,-1,-1}(z) + \frac{3}{8}\frac{z+1}{s}G_{0,-1,0}(z) + \frac{3}{8}\frac{z+1}{s}G_{0,0,-1}(z) - \frac{3}{16}\frac{4z^3+9z^2-1}{(z+1)sz}G_{0,0,0}(z) - \frac{9}{8}\frac{1}{(z+1)s}\zeta_2\log(4\pi) \\
& + \Bigg(-3\frac{z^2+z+1}{sz} - \frac{3}{16}\frac{3z-2}{sz}G_{-1}(z) + \frac{3}{16}\frac{3z+5}{sz}G_0(z) + \frac{3}{16}\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1,-1}(z) - \frac{3}{8}\frac{z+1}{s}G_{-1,0}(z) - \frac{3}{8}\frac{z+1}{s}G_{0,-1}(z) \\
& + \frac{3}{16}\frac{2z^3+4z^2-z-1}{(z+1)sz}G_{0,0}(z)\Bigg)\log(4\pi)\Bigg)i\frac{1}{\pi^2} + \Bigg(\Bigg(-3\frac{z^2+z+1}{sz} - \frac{9}{8}\frac{1}{(z+1)s}\zeta_2 - \frac{3}{16}\frac{3z-2}{sz}G_{-1}(z) + \frac{3}{16}\frac{3z+5}{sz}G_0(z) \\
& + \frac{3}{16}\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1,-1}(z) - \frac{3}{8}\frac{z+1}{s}G_{-1,0}(z) - \frac{3}{8}\frac{z+1}{s}G_{0,-1}(z) + \frac{3}{16}\frac{2z^3+4z^2-z-1}{(z+1)sz}G_{0,0}(z)\Bigg)i\frac{1}{\pi^2} \\
& + \Bigg(\frac{3}{16}\frac{3z-2}{sz} - \frac{3}{16}\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1}(z) + \frac{3}{8}\frac{z+1}{s}G_0(z)\Bigg)\frac{1}{\pi}\Bigg)\frac{1}{\pi} + \Bigg(\frac{1}{8}\frac{15z-13}{sz} + \Bigg(-\frac{3}{16}\frac{3z-2}{sz} \\
& + \frac{3}{16}\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1}(z) - \frac{3}{8}\frac{z+1}{s}G_0(z)\Bigg)\gamma_E - \frac{1}{8}\frac{10z^3+13z^2-15z-10}{sz^2}G_{-1}(z) + \frac{5}{4}\frac{z+1}{s}G_0(z) \\
& + \frac{3}{16}\frac{4z^3+3z^2-6z-4}{sz^2}G_{-1,-1}(z) - \frac{3}{8}\frac{z+1}{s}G_{-1,0}(z) + \frac{3}{16}\frac{(z+1)(z+2)}{sz^2}G_{0,-1}(z) - \frac{3}{8}\frac{z+1}{s}G_{0,0}(z) + \Bigg(\frac{3}{16}\frac{3z-2}{sz} \\
& - \frac{3}{16}\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1}(z) + \frac{3}{8}\frac{z+1}{s}G_0(z)\Bigg)\log(4\pi)\Bigg)\frac{1}{\pi} - \frac{1}{16}\frac{z+2}{s}\pi + \Bigg(\Bigg(\frac{1}{4}\frac{9z^2+9z+10}{sz} + \Bigg(\frac{3}{16}\frac{6z^2+6z+5}{sz} + \frac{3}{16}\frac{1}{s}G_{-1}(z) \\
& - \frac{3}{16}\frac{z+1}{sz}G_0(z)\Bigg)\gamma_E + \frac{3}{16}\frac{2z^3+4z^2+9z+1}{(z+1)sz}\zeta_2 + \frac{1}{8}\frac{4z-3}{sz}G_{-1}(z) - \frac{1}{8}\frac{4z+7}{sz}G_0(z) - \frac{3}{16}\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1,-1}(z) \\
& + \frac{3}{8}\frac{z+1}{s}G_{-1,0}(z) + \frac{3}{8}\frac{z+1}{s}G_{0,-1}(z) - \frac{3}{16}\frac{2z^3+4z^2-z-1}{(z+1)sz}G_{0,0}(z) + \Bigg(-\frac{3}{16}\frac{6z^2+6z+5}{sz} - \frac{3}{16}\frac{1}{s}G_{-1}(z) \\
& + \frac{3}{16}\frac{z+1}{sz}G_0(z)\Bigg)\log(4\pi)\Bigg)i\frac{1}{\pi^2} + \Bigg(\Bigg(-\frac{3}{16}\frac{6z^2+6z+5}{sz} - \frac{3}{16}\frac{1}{s}G_{-1}(z) + \frac{3}{16}\frac{z+1}{sz}G_0(z)\Bigg)i\frac{1}{\pi^2} + \frac{3}{16}\frac{1}{s}\frac{1}{\pi}\Bigg)\frac{1}{\pi} + \Bigg(-\frac{1}{8}\frac{4z-3}{sz} \\
& - \frac{3}{16}\frac{1}{s}\gamma_E + \frac{3}{16}\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1}(z) - \frac{3}{8}\frac{z+1}{s}G_0(z) + \frac{3}{16}\frac{1}{s}\log(4\pi)\Bigg)\frac{1}{\pi}\Bigg)\log(-\frac{m_t^2}{s}) + \Bigg(-\frac{1}{\pi}\frac{3}{32}\frac{2z^2+2z+1}{sz}i\frac{1}{\pi^2} \\
& + \Bigg(\frac{1}{8}\frac{13z^2+13z+11}{sz} + \frac{3}{32}\frac{2z^2+2z+1}{sz}\gamma_E + \frac{9}{32}\frac{1}{s}G_{-1}(z) - \frac{9}{32}\frac{z+1}{sz}G_0(z) - \frac{3}{32}\frac{2z^2+2z+1}{sz}\log(4\pi)\Bigg)i\frac{1}{\pi^2} - \frac{9}{32}\frac{1}{s}\frac{1}{\pi}\Bigg)\log^2(-\frac{m_t^2}{s})
\end{aligned}
$$

$$+\frac{5}{32}\frac{2z^2+2z+1}{sz}i\frac{1}{\pi^2}\log^3(-\frac{m_t^2}{s})+\Bigg(\bigg(6\frac{z^2+z+1}{sz}+\frac{9}{4}\frac{1}{(z+1)s}\zeta_2+\frac{3}{8}\frac{3z-2}{sz}G_{-1}(z)-\frac{3}{8}\frac{3z+5}{sz}G_0(z)$$

$$-\frac{3}{8}\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1,-1}(z)+\frac{3}{4}\frac{z+1}{s}G_{-1,0}(z)+\frac{3}{4}\frac{z+1}{s}G_{0,-1}(z)-\frac{3}{8}\frac{2z^3+4z^2-z-1}{(z+1)sz}G_{0,0}(z)\bigg)i\frac{1}{\pi^2}+\bigg(-\frac{3}{8}\frac{3z-2}{sz}$$

$$+\frac{3}{8}\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1}(z)-\frac{3}{4}\frac{z+1}{s}G_0(z)\bigg)\frac{1}{\pi}\Bigg)\log(-\frac{s}{\mu_R^2})+\Bigg(\bigg(\Big(\frac{3}{8}\frac{6z^2+6z+5}{sz}+\frac{3}{8}\frac{1}{s}G_{-1}(z)-\frac{3}{8}\frac{z+1}{sz}G_0(z)\Big)i\frac{1}{\pi^2}$$

$$-\frac{3}{8}\frac{1}{s}\frac{1}{\pi}\bigg)\log(-\frac{m_t^2}{s})\log(-\frac{s}{\mu_R^2})+\frac{3}{16}\frac{2z^2+2z+1}{sz}i\frac{1}{\pi^2}\log^2(-\frac{m_t^2}{s})\log(-\frac{s}{\mu_R^2})\Bigg)\frac{1}{N_c}+\Bigg(\bigg(-\frac{85}{4}\frac{z^2+z+1}{sz}+\Big(\frac{21}{2}\frac{z^2+z+1}{sz}$$

$$-\frac{1}{16}\frac{12z^2-19z+46}{sz}G_{-1}(z)-\frac{1}{16}\frac{12z^2+43z+77}{sz}G_0(z)-\frac{1}{8}\frac{14z^3+19z^2-23z-14}{sz^2}G_{-1,-1}(z)+\frac{1}{8}\frac{14z^2+14z+3}{sz}G_{-1,0}(z)$$

$$+\frac{1}{8}\frac{14z^2+14z+3}{sz}G_{0,-1}(z)-\frac{1}{8}\frac{14z^3+23z^2-19z-14}{(z+1)sz}G_{0,0}(z)+\frac{3}{16}\frac{6z^3+5z^2-9z-6}{sz^2}G_{-1,-1,-1}(z)$$

$$-\frac{1}{16}\frac{8z^3+8z^2+3z+2}{sz^2}G_{-1,-1,0}(z)-\frac{1}{16}\frac{8z^3+8z^2+3z+2}{sz^2}G_{-1,0,-1}(z)-\frac{1}{16}\frac{2z^3+7z^2+2z+1}{(z+1)sz}G_{-1,0,0}(z)$$

$$-\frac{1}{16}\frac{2z^3-z^2-6z-4}{sz^2}G_{0,-1,-1}(z)-\frac{1}{16}\frac{8z^3+16z^2+11z+1}{(z+1)sz}G_{0,-1,0}(z)-\frac{1}{16}\frac{8z^3+16z^2+11z+1}{(z+1)sz}G_{0,0,-1}(z)$$

$$+\frac{3}{16}\frac{6z^3+13z^2-z-2}{(z+1)sz}G_{0,0,0}(z)\Big)\gamma_E+\Big(-\frac{9}{8}\frac{z^2+z+1}{sz}-\frac{3}{16}\frac{z-1}{sz}G_{-1}(z)+\frac{3}{16}\frac{z+2}{sz}G_0(z)+\frac{3}{32}\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1,-1}(z)$$

$$-\frac{3}{16}\frac{z+1}{s}G_{-1,0}(z)-\frac{3}{16}\frac{z+1}{s}G_{0,-1}(z)+\frac{3}{32}\frac{2z^3+4z^2-z-1}{(z+1)sz}G_{0,0}(z)\Big)\gamma_E^2+\Big(\frac{1}{16}\frac{98z^3+36z^2+55z+261}{(z+1)sz}$$

$$+\frac{1}{16}\frac{126z^4+344z^3-72z^2-359z-130}{(z+1)sz^2}G_{-1}(z)-\frac{1}{16}\frac{134z^3+306z^2+107z+42}{(z+1)sz}G_0(z)-\frac{1}{8}\frac{54z^4+100z^3-29z^2-135z-54}{(z+1)sz^2}G_{-1,-1}(z)$$

$$+\frac{1}{16}\frac{54z^4+99z^3+73z^2+16z+6}{(z+1)sz^2}G_{-1,0}(z)+\frac{1}{16}\frac{24z^4+24z^3-17z^2-59z-24}{(z+1)sz^2}G_{0,-1}(z)+\frac{1}{16}\frac{30z^3+77z^2+31z+8}{(z+1)sz}G_{0,0}(z)\Big)\zeta_2$$

$$+\Big(-\frac{3}{8}\frac{2z^3-z^2-10z+7}{(z+1)sz}-\frac{3}{16}\frac{10z^4+24z^3-5z^2-25z-10}{(z+1)sz^2}G_{-1}(z)+\frac{3}{16}\frac{10z^3+24z^2+7z+1}{(z+1)sz}G_0(z)\Big)\gamma_E\zeta_2-\frac{9}{16}\frac{1}{(z+1)s}\gamma_E^2\zeta_2$$

$$+\Big(\frac{1}{4}\frac{25z^3+52z^2+32z+12}{(z+1)sz}-\frac{1}{16}\frac{(7z+3)(2z^2+2z+1)}{(z+1)sz}G_{-1}(z)-\frac{1}{16}\frac{14z^3+38z^2+25z+9}{(z+1)sz}G_0(z)\Big)\zeta_3-\frac{3}{16}\frac{8z^3+17z^2+10z+3}{(z+1)sz}\gamma_E\zeta_3$$

$$+\frac{3}{64}\frac{104z^3+275z^2+272z+5}{(z+1)sz}\zeta_4+\frac{1}{8}\frac{25z^2-14z+66}{sz}G_{-1}(z)+\frac{1}{8}\frac{25z^2+64z+105}{sz}G_0(z)+\frac{1}{12}\frac{27z^3+67z^2-82z-36}{sz^2}G_{-1,-1}(z)$$

$$-\frac{1}{16}\frac{48z^2+48z+29}{sz}G_{-1,0}(z)-\frac{1}{16}\frac{48z^2+48z+29}{sz}G_{0,-1}(z)+\frac{1}{12}\frac{27z^3+14z^2-135z-86}{(z+1)sz}G_{0,0}(z)$$

$$-\frac{1}{32}\frac{158z^3+178z^2-237z-158}{sz^2}G_{-1,-1,-1}(z)+\frac{1}{96}\frac{214z^3+214z^2+93z+46}{sz^2}G_{-1,-1,0}(z)+\frac{1}{96}\frac{214z^3+214z^2+93z+46}{sz^2}G_{-1,0,-1}(z)$$

$$+\frac{1}{96}\frac{46z^3+140z^2+19z+47}{(z+1)sz}G_{-1,0,0}(z)+\frac{1}{96}\frac{46z^3-2z^2-123z-122}{sz^2}G_{0,-1,-1}(z)+\frac{1}{96}\frac{214z^3+428z^2+307z+47}{(z+1)sz}G_{0,-1,0}(z)$$

$$+\frac{1}{96}\frac{214z^3+428z^2+307z+47}{(z+1)sz}G_{0,0,-1}(z)-\frac{1}{32}\frac{158z^3+296z^2-119z-99}{(z+1)sz}G_{0,0,0}(z)+\frac{1}{16}\frac{46z^3+36z^2-69z-46}{sz^2}G_{-1,-1,-1,-1}(z)$$

$$-\frac{1}{16}\frac{14z^3+15z^2+6z+4}{sz^2}G_{-1,-1,-1,0}(z)-\frac{1}{16}\frac{14z^3+15z^2+6z+4}{sz^2}G_{-1,-1,0,-1}(z)+\frac{1}{16}\frac{z^3-7z^2-6z-2}{(z+1)sz^2}G_{-1,-1,0,0}(z)$$

$$-\frac{1}{16}\frac{6z^3+3z^2-6z-4}{sz^2}G_{-1,0,-1,-1}(z)-\frac{1}{16}\frac{8z^3+9z^2+4z+2}{sz^2}G_{-1,0,-1,0}(z)-\frac{1}{16}\frac{8z^3+9z^2+4z+2}{sz^2}G_{-1,0,0,-1}(z)$$

$$+\frac{1}{16}\frac{4z^3+9z-1}{(z+1)sz}G_{-1,0,0,0}(z)+\frac{1}{16}\frac{4z^3+12z^2+21z+14}{sz^2}G_{0,-1,-1,-1}(z)-\frac{1}{16}\frac{8z^3+15z^2+10z+1}{(z+1)sz}G_{0,-1,-1,0}(z)$$

$$-\frac{1}{16}\frac{8z^3+15z^2+10z+1}{(z+1)sz}G_{0,-1,0,-1}(z)-\frac{1}{16}\frac{6z^3+15z^2+6z+1}{(z+1)sz}G_{0,-1,0,0}(z)-\frac{1}{16}\frac{z^3+10z^2+11z+4}{(z+1)sz^2}G_{0,0,-1,-1}(z)$$

$$-\frac{1}{16}\frac{14z^3+27z^2+18z+1}{(z+1)sz}G_{0,0,-1,0}(z)-\frac{1}{16}\frac{14z^3+27z^2+18z+1}{(z+1)sz}G_{0,0,0,-1}(z)+\frac{1}{16}\frac{46z^3+102z^2-3z-13}{(z+1)sz}G_{0,0,0,0}(z)$$

$$+\Big(\frac{9}{4}\frac{z^2+z+1}{sz}+\frac{3}{8}\frac{z-1}{sz}G_{-1}(z)-\frac{3}{8}\frac{z+2}{sz}G_0(z)-\frac{3}{16}\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1,-1}(z)+\frac{3}{8}\frac{z+1}{s}G_{-1,0}(z)+\frac{3}{8}\frac{z+1}{s}G_{0,-1}(z)$$

$$-\frac{3}{16}\frac{2z^3+4z^2-z-1}{(z+1)sz}G_{0,0}(z)\Big)\gamma_E\log(4\pi)+\Big(\frac{3}{8}\frac{2z^3-z^2-10z+7}{(z+1)sz}+\frac{3}{16}\frac{10z^4+24z^3-5z^2-25z-10}{(z+1)sz^2}G_{-1}(z)$$

$$-\frac{3}{16}\frac{10z^3+24z^2+7z+1}{(z+1)sz}G_0(z)\Big)\zeta_2\log(4\pi)+\frac{9}{8}\frac{1}{(z+1)s}\gamma_E\zeta_2\log(4\pi)+\frac{3}{16}\frac{8z^3+17z^2+10z+3}{(z+1)sz}\zeta_3\log(4\pi)+\Big(-\frac{21}{2}\frac{z^2+z+1}{sz}$$

$$+\frac{1}{16}\frac{12z^2-19z+46}{sz}G_{-1}(z)+\frac{1}{16}\frac{12z^2+43z+77}{sz}G_0(z)+\frac{1}{8}\frac{14z^3+19z^2-23z-14}{sz^2}G_{-1,-1}(z)-\frac{1}{8}\frac{14z^2+14z+3}{sz}G_{-1,0}(z)$$

$$-\frac{1}{8}\frac{14z^2+14z+3}{sz}G_{0,-1}(z)+\frac{1}{8}\frac{14z^3+23z^2-19z-14}{(z+1)sz}G_{0,0}(z)-\frac{3}{16}\frac{6z^3+5z^2-9z-6}{sz^2}G_{-1,-1,-1}(z)$$

$$+\frac{1}{16}\frac{8z^3+8z^2+3z+2}{sz^2}G_{-1,-1,0}(z)+\frac{1}{16}\frac{8z^3+8z^2+3z+2}{sz^2}G_{-1,0,-1}(z)+\frac{1}{16}\frac{2z^3+7z^2+2z+1}{(z+1)sz}G_{-1,0,0}(z)$$

$$+\frac{1}{16}\frac{2z^3-z^2-6z-4}{sz^2}G_{0,-1,-1}(z)+\frac{1}{16}\frac{8z^3+16z^2+11z+1}{(z+1)sz}G_{0,-1,0}(z)+\frac{1}{16}\frac{8z^3+16z^2+11z+1}{(z+1)sz}G_{0,0,-1}(z)$$

$$- \frac{3}{16} \frac{6z^3 + 13z^2 - z - 2}{(z+1)sz} G_{0,0,0}(z) \Big) \log(4\pi) - \frac{9}{16} \frac{1}{(z+1)s} \zeta_2 \log^2(4\pi) + \Big(- \frac{9}{8} \frac{z^2 + z + 1}{sz} - \frac{3}{16} \frac{z - 1}{sz} G_{-1}(z) + \frac{3}{16} \frac{z + 2}{sz} G_0(z)$$

$$+ \frac{3}{32} \frac{2z^3 + 2z^2 - 3z - 2}{sz^2} G_{-1,-1}(z) - \frac{3}{16} \frac{z + 1}{s} G_{-1,0}(z) - \frac{3}{16} \frac{z + 1}{s} G_{0,-1}(z) + \frac{3}{32} \frac{2z^3 + 4z^2 - z - 1}{(z+1)sz} G_{0,0}(z) \Big) \log^2(4\pi) \Big) i \frac{1}{\pi^2}$$

$$+ \Big(\Big(\Big(- \frac{9}{4} \frac{z^2 + z + 1}{sz} - \frac{9}{8} \frac{1}{(z+1)s} \zeta_2 - \frac{3}{8} \frac{z - 1}{sz} G_{-1}(z) + \frac{3}{8} \frac{z + 2}{sz} G_0(z) + \frac{3}{16} \frac{2z^3 + 2z^2 - 3z - 2}{sz^2} G_{-1,-1}(z) - \frac{3}{8} \frac{z + 1}{s} G_{-1,0}(z)$$

$$- \frac{3}{8} \frac{z + 1}{s} G_{0,-1}(z) + \frac{3}{16} \frac{2z^3 + 4z^2 - z - 1}{(z+1)sz} G_{0,0}(z) \Big) i \frac{1}{\pi^2} + \Big(\frac{3}{8} \frac{z - 1}{sz} - \frac{3}{16} \frac{2z^3 + 2z^2 - 3z - 2}{sz^2} G_{-1}(z) + \frac{3}{8} \frac{z + 1}{s} G_0(z) \Big) \frac{1}{\pi} \Big) \frac{1}{\epsilon^2}$$

$$+ \Big(- \frac{1}{8} \frac{25z^2 - 14z + 66}{sz} + \Big(\frac{1}{16} \frac{12z^2 - 19z + 46}{sz} + \frac{1}{8} \frac{14z^3 + 19z^2 - 23z - 14}{sz^2} G_{-1}(z) - \frac{1}{8} \frac{14z^2 + 14z + 3}{sz} G_0(z)$$

$$- \frac{3}{16} \frac{6z^3 + 5z^2 - 9z - 6}{sz^2} G_{-1,-1}(z) + \frac{1}{16} \frac{8z^3 + 8z^2 + 3z + 2}{sz^2} G_{-1,0}(z) + \frac{1}{16} \frac{2z^3 - z^2 - 6z - 4}{sz^2} G_{0,-1}(z)$$

$$+ \frac{1}{16} \frac{8z^3 + 16z^2 + 11z + 1}{(z+1)sz} G_{0,0}(z) \Big) \gamma_E + \Big(\frac{3}{16} \frac{z - 1}{sz} - \frac{3}{32} \frac{2z^3 + 2z^2 - 3z - 2}{sz^2} G_{-1}(z) + \frac{3}{16} \frac{z + 1}{s} G_0(z) \Big) \gamma_E^2 + \frac{1}{16} \frac{8z^3 + 11z^2 + 4z + 3}{(z+1)sz} \zeta_3$$

$$- \frac{1}{12} \frac{27z^3 + 67z^2 - 82z - 36}{sz^2} G_{-1}(z) + \frac{1}{16} \frac{48z^2 + 48z + 29}{sz} G_0(z) + \frac{1}{32} \frac{158z^3 + 178z^2 - 237z - 158}{sz^2} G_{-1,-1}(z)$$

$$- \frac{1}{96} \frac{214z^3 + 214z^2 + 93z + 46}{sz^2} G_{-1,0}(z) - \frac{1}{96} \frac{46z^3 - 2z^2 - 123z - 122}{sz^2} G_{0,-1}(z) - \frac{1}{96} \frac{214z^3 + 428z^2 + 307z + 47}{(z+1)sz} G_{0,0}(z)$$

$$- \frac{1}{16} \frac{46z^3 + 36z^2 - 69z - 46}{sz^2} G_{-1,-1,-1}(z) + \frac{1}{16} \frac{14z^3 + 15z^2 + 6z + 4}{sz^2} G_{-1,-1,0}(z) + \frac{1}{16} \frac{6z^3 + 3z^2 - 6z - 4}{sz^2} G_{-1,0,-1}(z)$$

$$+ \frac{1}{16} \frac{8z^3 + 9z^2 + 4z + 2}{sz^2} G_{-1,0,0}(z) - \frac{1}{16} \frac{4z^3 + 12z^2 + 21z + 14}{sz^2} G_{0,-1,-1}(z) + \frac{1}{16} \frac{8z^3 + 15z^2 + 10z + 1}{(z+1)sz} G_{0,-1,0}(z)$$

$$+ \frac{1}{16} \frac{z^3 + 10z^2 + 11z + 4}{(z+1)sz^2} G_{0,0,-1}(z) + \frac{1}{16} \frac{14z^3 + 27z^2 + 18z + 1}{(z+1)sz} G_{0,0,0}(z) + \Big(- \frac{3}{8} \frac{z - 1}{sz} + \frac{3}{16} \frac{2z^3 + 2z^2 - 3z - 2}{sz^2} G_{-1}(z)$$

$$- \frac{3}{8} \frac{z + 1}{s} G_0(z) \Big) \gamma_E \log(4\pi) + \Big(- \frac{1}{16} \frac{12z^2 - 19z + 46}{sz} - \frac{1}{8} \frac{14z^3 + 19z^2 - 23z - 14}{sz^2} G_{-1}(z) + \frac{1}{8} \frac{14z^2 + 14z + 3}{sz} G_0(z)$$

$$+ \frac{3}{16} \frac{6z^3 + 5z^2 - 9z - 6}{sz^2} G_{-1,-1}(z) - \frac{1}{16} \frac{8z^3 + 8z^2 + 3z + 2}{sz^2} G_{-1,0}(z) - \frac{1}{16} \frac{2z^3 - z^2 - 6z - 4}{sz^2} G_{0,-1}(z)$$

$$- \frac{1}{16} \frac{8z^3 + 16z^2 + 11z + 1}{(z+1)sz} G_{0,0}(z) \Big) \log(4\pi) + \Big(\frac{3}{16} \frac{z - 1}{sz} - \frac{3}{32} \frac{2z^3 + 2z^2 - 3z - 2}{sz^2} G_{-1}(z) + \frac{3}{16} \frac{z + 1}{s} G_0(z) \Big) \log^2(4\pi) \Big) \frac{1}{\pi}$$

$$+ \Big(\Big(- \frac{21}{2} \frac{z^2 + z + 1}{sz} + \Big(\frac{9}{4} \frac{z^2 + z + 1}{sz} + \frac{3}{8} \frac{z - 1}{sz} G_{-1}(z) - \frac{3}{8} \frac{z + 2}{sz} G_0(z) - \frac{3}{16} \frac{2z^3 + 2z^2 - 3z - 2}{sz^2} G_{-1,-1}(z) + \frac{3}{8} \frac{z + 1}{s} G_{-1,0}(z)$$

$$+ \frac{3}{8} \frac{z + 1}{s} G_{0,-1}(z) - \frac{3}{16} \frac{2z^3 + 4z^2 - z - 1}{(z+1)sz} G_{0,0}(z) \Big) \gamma_E + \Big(\frac{3}{8} \frac{2z^3 - z^2 - 10z + 7}{(z+1)sz} + \frac{3}{16} \frac{10z^4 + 24z^3 - 5z^2 - 25z - 10}{(z+1)sz^2} G_{-1}(z)$$

$$- \frac{3}{16} \frac{10z^3 + 24z^2 + 7z + 1}{(z+1)sz} G_0(z) \Big) \zeta_2 + \frac{9}{8} \frac{1}{(z+1)s} \gamma_E \zeta_2 + \frac{3}{16} \frac{8z^3 + 17z^2 + 10z + 3}{(z+1)sz} \zeta_3 + \frac{1}{16} \frac{12z^2 - 19z + 46}{sz} G_{-1}(z)$$

$$+ \frac{1}{16} \frac{12z^2 + 43z + 77}{sz} G_0(z) + \frac{1}{8} \frac{14z^3 + 19z^2 - 23z - 14}{sz^2} G_{-1,-1}(z) - \frac{1}{8} \frac{14z^2 + 14z + 3}{sz} G_{-1,0}(z) - \frac{1}{8} \frac{14z^2 + 14z + 3}{sz} G_{0,-1}(z)$$

$$+ \frac{1}{8} \frac{14z^3 + 23z^2 - 19z - 14}{(z+1)sz} G_{0,0}(z) - \frac{3}{16} \frac{6z^3 + 5z^2 - 9z - 6}{sz^2} G_{-1,-1,-1}(z) + \frac{1}{16} \frac{8z^3 + 8z^2 + 3z + 2}{sz^2} G_{-1,-1,0}(z)$$

$$+ \frac{1}{16} \frac{8z^3 + 8z^2 + 3z + 2}{sz^2} G_{-1,0,-1}(z) + \frac{1}{16} \frac{2z^3 + 7z^2 + 2z + 1}{(z+1)sz} G_{-1,0,0}(z) + \frac{1}{16} \frac{2z^3 - z^2 - 6z - 4}{sz^2} G_{0,-1,-1}(z)$$

$$+ \frac{1}{16} \frac{8z^3 + 16z^2 + 11z + 1}{(z+1)sz} G_{0,-1,0}(z) + \frac{1}{16} \frac{8z^3 + 16z^2 + 11z + 1}{(z+1)sz} G_{0,0,-1}(z) - \frac{3}{16} \frac{6z^3 + 13z^2 - z - 2}{(z+1)sz} G_{0,0,0}(z) - \frac{9}{8} \frac{1}{(z+1)s} \zeta_2 \log(4\pi)$$

$$+ \Big(- \frac{9}{4} \frac{z^2 + z + 1}{sz} - \frac{3}{8} \frac{z - 1}{sz} G_{-1}(z) + \frac{3}{8} \frac{z + 2}{sz} G_0(z) + \frac{3}{16} \frac{2z^3 + 2z^2 - 3z - 2}{sz^2} G_{-1,-1}(z) - \frac{3}{8} \frac{z + 1}{s} G_{-1,0}(z) - \frac{3}{8} \frac{z + 1}{s} G_{0,-1}(z)$$

$$+ \frac{3}{16} \frac{2z^3 + 4z^2 - z - 1}{(z+1)sz} G_{0,0}(z) \Big) \log(4\pi) \Big) i \frac{1}{\pi^2} + \Big(- \frac{1}{16} \frac{12z^2 - 19z + 46}{sz} + \Big(- \frac{3}{8} \frac{z - 1}{sz} + \frac{3}{16} \frac{2z^3 + 2z^2 - 3z - 2}{sz^2} G_{-1}(z) - \frac{3}{8} \frac{z + 1}{s} G_0(z) \Big) \gamma_E$$

$$- \frac{1}{8} \frac{14z^3 + 19z^2 - 23z - 14}{sz^2} G_{-1}(z) + \frac{1}{8} \frac{14z^2 + 14z + 3}{sz} G_0(z) + \frac{3}{16} \frac{6z^3 + 5z^2 - 9z - 6}{sz^2} G_{-1,-1}(z) - \frac{1}{16} \frac{8z^3 + 8z^2 + 3z + 2}{sz^2} G_{-1,0}(z)$$

$$- \frac{1}{16} \frac{2z^3 - z^2 - 6z - 4}{sz^2} G_{0,-1}(z) - \frac{1}{16} \frac{8z^3 + 16z^2 + 11z + 1}{(z+1)sz} G_{0,0}(z) + \Big(\frac{3}{8} \frac{z - 1}{sz} - \frac{3}{16} \frac{2z^3 + 2z^2 - 3z - 2}{sz^2} G_{-1}(z)$$

$$+ \frac{3}{8} \frac{z + 1}{s} G_0(z) \Big) \log(4\pi) \Big) \frac{1}{\pi} - \frac{1}{16} \frac{z^2 + 3z + 3}{(z+1)s} \pi \Big) \frac{1}{\pi} + \Big(- \frac{1}{96} \frac{24z^3 + 88z^2 + 59z - 28}{(z+1)sz} + \frac{1}{16} \frac{z^2 + 3z + 3}{(z+1)s} \gamma_E$$

$$+ \frac{1}{48} \frac{16z^3 + 19z^2 - 18z - 12}{sz^2} G_{-1}(z) - \frac{1}{96} \frac{16z^3 + 34z^2 + 13z + 1}{(z+1)sz} G_0(z) - \frac{1}{16} \frac{z^2 + 3z + 3}{(z+1)s} \log(4\pi) \Big) \pi + \Big(\Big(- \frac{1}{4} \frac{9z^2 + 9z + 10}{sz}$$

$$+ \Big(\frac{1}{16} \frac{26z^2 + 26z + 29}{sz} - \frac{1}{4} \frac{z^2 - z + 1}{sz} G_{-1}(z) - \frac{1}{4} \frac{z^2 + 3z + 3}{sz} G_0(z) - \frac{1}{8} \frac{1}{s} G_{-1,-1}(z) + \frac{1}{16} \frac{1}{sz} G_{-1,0}(z) + \frac{1}{16} \frac{1}{sz} G_{0,-1}(z)$$

$$+ \frac{1}{8} \frac{z + 1}{sz} G_{0,0}(z) \Big) \gamma_E + \Big(- \frac{3}{8} \frac{z^2 + z + 1}{sz} - \frac{3}{32} \frac{1}{s} G_{-1}(z) + \frac{3}{32} \frac{z + 1}{sz} G_0(z) \Big) \gamma_E^2 + \Big(\frac{1}{16} \frac{28z^3 + 45z^2 + 19z + 20}{(z+1)sz} - \frac{1}{16} \frac{2z^2 - 7z + 1}{sz} G_{-1}(z)$$

$$- \frac{1}{16} \frac{2z^2 + 5z + 4}{sz} G_0(z) \Big) \zeta_2 - \frac{3}{16} \frac{2z^2 + 1}{sz} \gamma_E \zeta_2 - \frac{3}{16} \frac{2z^2 + 2z + 1}{sz} \zeta_3 + \frac{1}{24} \frac{11z^2 - z + 20}{sz} G_{-1}(z) + \frac{1}{24} \frac{11z^2 + 23z + 32}{sz} G_0(z)$$

$$+ \frac{1}{48} \frac{6z^3 + 17z^2 - 39z - 18}{sz^2} G_{-1,-1}(z) - \frac{1}{96} \frac{36z^2 + 36z + 11}{sz} G_{-1,0}(z) - \frac{1}{96} \frac{36z^2 + 36z + 11}{sz} G_{0,-1}(z) + \frac{1}{48} \frac{6z^3 + z^2 - 55z - 32}{(z+1)sz} G_{0,0}(z)$$

$$- \frac{3}{16}\frac{1}{s}G_{-1,-1,-1}(z) + \frac{1}{16}\frac{z+1}{sz}G_{-1,-1,0}(z) + \frac{1}{16}\frac{z+1}{sz}G_{-1,0,-1}(z) - \frac{1}{16}\frac{1}{s}G_{-1,0,0}(z) + \frac{1}{16}\frac{z+1}{sz}G_{0,-1,-1}(z) - \frac{1}{16}\frac{1}{s}G_{0,-1,0}(z)$$

$$- \frac{1}{16}\frac{1}{s}G_{0,0,-1}(z) + \frac{3}{16}\frac{z+1}{sz}G_{0,0,0}(z) + \Big(\frac{3}{4}\frac{z^2+z+1}{sz} + \frac{3}{16}\frac{1}{s}G_{-1}(z) - \frac{3}{16}\frac{z+1}{sz}G_0(z)\Big)\gamma_E\log(4\pi) + \frac{3}{16}\frac{2z^2+1}{sz}\zeta_2\log(4\pi)$$

$$+ \Big(-\frac{1}{16}\frac{26z^2+26z+29}{sz} + \frac14\frac{z^2-z+1}{sz}G_{-1}(z) + \frac14\frac{z^2+3z+3}{sz}G_0(z) + \frac18\frac1s G_{-1,-1}(z) - \frac1{16}\frac1{sz}G_{-1,0}(z) - \frac1{16}\frac1{sz}G_{0,-1}(z)$$

$$- \frac18\frac{z+1}{sz}G_{0,0}(z)\Big)\log(4\pi) + \Big(-\frac38\frac{z^2+z+1}{sz} - \frac3{32}\frac1s G_{-1}(z) + \frac3{32}\frac{z+1}{sz}G_0(z)\Big)\log^2(4\pi)\Big) i\frac{1}{\pi^2} + \Big(\Big(-\frac34\frac{z^2+z+1}{sz} - \frac3{16}\frac1s G_{-1}(z)$$

$$+ \frac3{16}\frac{z+1}{sz}G_0(z)\Big)i\frac1{\pi^2} + \frac3{16}\frac1s\frac1\pi\Big)\frac1{\epsilon^2} + \Big(\Big(-\frac1{16}\frac{26z^2+26z+29}{sz} + \Big(\frac34\frac{z^2+z+1}{sz} + \frac3{16}\frac1s G_{-1}(z) - \frac3{16}\frac{z+1}{sz}G_0(z)\Big)\gamma_E + \frac3{16}\frac{2z^2+1}{sz}\zeta_2$$

$$+ \frac14\frac{z^2-z+1}{sz}G_{-1}(z) + \frac14\frac{z^2+3z+3}{sz}G_0(z) + \frac18\frac1s G_{-1,-1}(z) - \frac1{16}\frac1{sz}G_{-1,0}(z) - \frac1{16}\frac1{sz}G_{0,-1}(z) - \frac18\frac{z+1}{sz}G_{0,0}(z) + \Big(-\frac34\frac{z^2+z+1}{sz}$$

$$- \frac3{16}\frac1s G_{-1}(z) + \frac3{16}\frac{z+1}{sz}G_0(z)\Big)\log(4\pi)\Big)i\frac1{\pi^2} + \Big(-\frac14\frac{z^2-z+1}{sz}$$

$$- \frac3{16}\frac1s\gamma_E - \frac18\frac1s G_{-1}(z) + \frac1{16}\frac1{sz}G_0(z) + \frac3{16}\frac1s\log(4\pi)\Big)\frac1\pi\Big)\frac1\pi + \Big(-\frac1{24}\frac{11z^2-z+20}{sz} + \Big(\frac14\frac{z^2-z+1}{sz} + \frac18\frac1s G_{-1}(z) - \frac1{16}\frac1{sz}G_0(z)\Big)\gamma_E$$

$$+ \frac3{32}\frac1s\gamma_E^2 - \frac1{48}\frac{6z^3+17z^2-39z-18}{sz^2}G_{-1}(z) + \frac1{96}\frac{36z^2+36z+11}{sz}G_0(z) + \frac3{16}\frac1s G_{-1,-1}(z) - \frac1{16}\frac{z+1}{sz}G_{-1,0}(z) - \frac1{16}\frac{z+1}{sz}G_{0,-1}(z)$$

$$+ \frac1{16}\frac1s G_{0,0}(z) - \frac3{16}\frac1s\gamma_E\log(4\pi) + \Big(-\frac14\frac{z^2-z+1}{sz} - \frac18\frac1s G_{-1}(z) + \frac1{16}\frac1{sz}G_0(z)\Big)\log(4\pi) + \frac3{32}\frac1s\log^2(4\pi)\Big)\frac1\pi$$

$$+ \frac1{96}\frac{2z^2-z+1}{sz}\pi\Big)\log(-\frac{m_t^2}{s}) + \Big(-\frac1{\epsilon^2}\frac3{32}\frac{2z^2+2z+1}{sz}i\frac1{\pi^2} + \Big(-\frac14\frac{z+1}{s} + \Big(\frac18\frac{z^2+z-1}{sz} - \frac1{32}\frac{2z^2+5z+1}{sz}G_{-1}(z)$$

$$- \frac1{32}\frac{2z^2-z-2}{sz}G_0(z)\Big)\gamma_E - \frac3{64}\frac{2z^2+2z+1}{sz}\gamma_E^2 + \frac3{16}\frac1s\zeta_2 - \frac1{192}\frac{2z^2-10z+13}{sz}G_{-1}(z) - \frac1{192}\frac{2z^2+14z+25}{sz}G_0(z)$$

$$- \frac1{32}\frac{2z^2+4z+1}{sz}G_{-1,-1}(z) + \frac1{32}\frac1{sz}G_{-1,0}(z) + \frac1{32}\frac1{sz}G_{0,-1}(z) - \frac1{32}\frac{2z^2-1}{sz}G_{0,0}(z) + \frac3{32}\frac{2z^2+2z+1}{sz}\gamma_E\log(4\pi) + \Big(-\frac18\frac{z^2+z-1}{sz}$$

$$+ \frac1{32}\frac{2z^2+5z+1}{sz}G_{-1}(z) + \frac1{32}\frac{2z^2-z-2}{sz}G_0(z)\Big)\log(4\pi) - \frac3{64}\frac{2z^2+2z+1}{sz}\log^2(4\pi)\Big)i\frac1{\pi^2} + \Big(\Big(-\frac18\frac{z^2+z-1}{sz} + \frac3{32}\frac{2z^2+2z+1}{sz}\gamma_E$$

$$+ \frac1{32}\frac{2z^2+5z+1}{sz}G_{-1}(z) + \frac1{32}\frac{2z^2-z-2}{sz}G_0(z) - \frac3{32}\frac{2z^2+2z+1}{sz}\log(4\pi)\Big)i\frac1{\pi^2} - \frac1{32}\frac{2z^2+5z+1}{sz}\frac1\pi\Big)\frac1\pi + \Big(\frac1{192}\frac{2z^2-10z+13}{sz}$$

$$+ \frac1{32}\frac{2z^2+5z+1}{sz}\gamma_E + \frac1{32}\frac{2z^2+4z+1}{sz}G_{-1}(z) - \frac1{32}\frac1{sz}G_0(z) - \frac1{32}\frac{2z^2+5z+1}{sz}\log(4\pi)\Big)\frac1\pi\Big)\log^2(-\frac{m_t^2}{s}) + \Big(\frac1\pi\frac1{16}\frac{2z^2+2z+1}{sz}i\frac1{\pi^2}$$

$$+ \Big(\frac1{96}\frac{2z^2+2z-5}{sz} - \frac1{16}\frac{2z^2+2z+1}{sz}\gamma_E - \frac1{96}\frac{4z^2+7z+2}{sz}G_{-1}(z) - \frac1{96}\frac{4z^2+z-1}{sz}G_0(z) + \frac1{16}\frac{2z^2+2z+1}{sz}\log(4\pi)\Big)i\frac1{\pi^2}$$

$$+ \frac1{96}\frac{4z^2+7z+2}{sz}\frac1\pi\Big)\log^3(-\frac{m_t^2}{s}) - \frac3{128}\frac{2z^2+2z+1}{sz}i\frac1{\pi^2}\log^4(-\frac{m_t^2}{s}) + \Big(\Big(\Big(\frac{135}8\frac{z^2+z+1}{sz} + \Big(-\frac92\frac{z^2+z+1}{sz} - \frac34\frac{z-1}{sz}G_{-1}(z)$$

$$+ \frac34\frac{z+2}{sz}G_0(z) + \frac38\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1,-1}(z) - \frac34\frac{z+1}{s}G_{-1,0}(z) - \frac34\frac{z+1}{s}G_{0,-1}(z) + \frac38\frac{2z^3+4z^2-z-1}{(z+1)sz}G_{0,0}(z)\Big)\gamma_E$$

$$+ \Big(-\frac3{16}\frac{8z^3-4z^2-29z+28}{(z+1)sz} - \frac38\frac{10z^4+24z^3-5z^2-25z-10}{(z+1)sz^2}G_{-1}(z) + \frac38\frac{10z^3+24z^2+7z+1}{(z+1)sz}G_0(z)\Big)\zeta_2 - \frac94\frac1{(z+1)s}\gamma_E\zeta_2$$

$$- \frac38\frac{8z^3+17z^2+10z+3}{(z+1)sz}\zeta_3 - \frac3{16}\frac{8z^2-9z+27}{sz}G_{-1}(z) - \frac3{16}\frac{8z^2+25z+44}{sz}G_0(z) - \frac1{32}\frac{90z^3+130z^2-151z-90}{sz^2}G_{-1,-1}(z)$$

$$+ \frac3{16}\frac{15z^2+15z+4}{sz}G_{-1,0}(z) + \frac3{16}\frac{15z^2+15z+4}{sz}G_{0,-1}(z) - \frac1{32}\frac{90z^3+140z^2-141z-101}{(z+1)sz}G_{0,0}(z)$$

$$+ \frac38\frac{6z^3+5z^2-9z-6}{sz^2}G_{-1,-1,-1}(z) - \frac18\frac{8z^3+8z^2+3z+2}{sz^2}G_{-1,-1,0}(z) - \frac18\frac{8z^3+8z^2+3z+2}{sz^2}G_{-1,0,-1}(z)$$

$$- \frac18\frac{2z^3+7z^2+2z+1}{(z+1)sz}G_{-1,0,0}(z) - \frac18\frac{2z^3-z^2-6z-4}{sz^2}G_{0,-1,-1}(z) - \frac18\frac{8z^3+16z^2+11z+1}{(z+1)sz}G_{0,-1,0}(z)$$

$$- \frac18\frac{8z^3+16z^2+11z+1}{(z+1)sz}G_{0,0,-1}(z) + \frac38\frac{6z^3+13z^2-z-2}{(z+1)sz}G_{0,0,0}(z) + \frac94\frac1{(z+1)s}\zeta_2\log(4\pi) + \Big(\frac92\frac{z^2+z+1}{sz} + \frac34\frac{z-1}{sz}G_{-1}(z)$$

$$- \frac34\frac{z+2}{sz}G_0(z) - \frac38\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1,-1}(z) + \frac34\frac{z+1}{s}G_{-1,0}(z) + \frac34\frac{z+1}{s}G_{0,-1}(z) - \frac38\frac{2z^3+4z^2-z-1}{(z+1)sz}G_{0,0}(z)\Big)\log(4\pi)\Big)i\frac1{\pi^2}$$

$$+ \Big(\Big(\Big(\frac92\frac{z^2+z+1}{sz} + \frac94\frac1{(z+1)s}\zeta_2 + \frac34\frac{z-1}{sz}G_{-1}(z) - \frac34\frac{z+2}{sz}G_0(z) - \frac38\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1,-1}(z) + \frac34\frac{z+1}{s}G_{-1,0}(z)$$

$$+ \frac34\frac{z+1}{s}G_{0,-1}(z) - \frac38\frac{2z^3+4z^2-z-1}{(z+1)sz}G_{0,0}(z)\Big)i\frac1{\pi^2} + \Big(-\frac34\frac{z-1}{sz} + \frac38\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1}(z) - \frac34\frac{z+1}{s}G_0(z)\Big)\frac1\pi\Big)\frac1\pi$$

$$+ \Big(\frac3{16}\frac{8z^2-9z+27}{sz} + \Big(\frac34\frac{z-1}{sz} - \frac38\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1}(z) + \frac34\frac{z+1}{s}G_0(z)\Big)\gamma_E + \frac1{32}\frac{90z^3+130z^2-151z-90}{sz^2}G_{-1}(z)$$

$$- \frac3{16}\frac{15z^2+15z+4}{sz}G_0(z) - \frac38\frac{6z^3+5z^2-9z-6}{sz^2}G_{-1,-1}(z) + \frac18\frac{8z^3+8z^2+3z+2}{sz^2}G_{-1,0}(z) + \frac18\frac{2z^3-z^2-6z-4}{sz^2}G_{0,-1}(z)$$

$$+ \frac18\frac{8z^3+16z^2+11z+1}{(z+1)sz}G_{0,0}(z) + \Big(-\frac34\frac{z-1}{sz} + \frac38\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1}(z) - \frac34\frac{z+1}{s}G_0(z)\Big)\log(4\pi)\Big)\frac1\pi + \frac18\frac{z^2+3z+3}{(z+1)s}\pi\Big)\log(-\frac{s}{\mu_R^2})$$

$$+ \Big(\Big(\frac38\frac{5z^2+5z+6}{sz} + \Big(-\frac32\frac{z^2+z+1}{sz} - \frac38\frac1s G_{-1}(z) + \frac38\frac{z+1}{sz}G_0(z)\Big)\gamma_E - \frac38\frac{2z^2+1}{sz}\zeta_2 - \frac1{32}\frac{16z^2-5z+16}{sz}G_{-1}(z)$$

$$- \frac{1}{32}\frac{16z^2+37z+37}{sz}G_0(z) - \frac14\frac1s G_{-1,-1}(z) + \frac18\frac{1}{sz}G_{-1,0}(z) + \frac18\frac{1}{sz}G_{0,-1}(z) + \frac14\frac{z+1}{sz}G_{0,0}(z) + \Big(\frac32\frac{z^2+z+1}{sz} + \frac38\frac1s G_{-1}(z)$$

$$- \frac38\frac{z+1}{sz}G_0(z)\Big)\log(4\pi)\Big) i\frac{1}{\pi^2} + \Big(\Big(\frac32\frac{z^2+z+1}{sz}+\frac38\frac1sG_{-1}(z) - \frac38\frac{z+1}{sz}G_0(z)\Big)i\frac1{\pi^2} - \frac38\frac1s\frac1\pi\Big)\frac1\pi + \Big(\frac1{32}\frac{16z^2-5z+16}{sz}+\frac38\frac1s\gamma_E$$

$$+ \frac14\frac1sG_{-1}(z) - \frac18\frac1{sz}G_0(z) - \frac38\frac1s\log(4\pi)\Big)\frac1\pi\Big)\log(-\frac{m_t^2}{s})\log(-\frac{s}{\mu_R^2}) + \Big(\frac1\pi\frac3{16}\frac{2z^2+2z+1}{sz}i\frac1{\pi^2} + \Big(-\frac3{64}\frac{2z^2+2z+9}{sz} - \frac3{16}\frac{2z^2+2z+1}{sz}\gamma_E$$

$$- \frac1{16}\frac{2z^2+5z+1}{sz}G_{-1}(z) - \frac1{16}\frac{2z^2-z-2}{sz}G_0(z) + \frac3{16}\frac{2z^2+2z+1}{sz}\log(4\pi)\Big)i\frac1{\pi^2} + \frac1{16}\frac{2z^2+5z+1}{sz}\frac1\pi\Big)\log^2(-\frac{m_t^2}{s})\log(-\frac{s}{\mu_R^2})$$

$$- \frac18\frac{2z^2+2z+1}{sz}i\frac1{\pi^2}\log^3(-\frac{m_t^2}{s})\log(-\frac{s}{\mu_R^2}) + \Big(\Big(-\frac92\frac{z^2+z+1}{sz} - \frac94\frac{1}{(z+1)s}\zeta_2 - \frac34\frac{z-1}{sz}G_{-1}(z) + \frac34\frac{z+2}{sz}G_0(z)$$

$$+ \frac38\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1,-1}(z) - \frac34\frac{z+1}{s}G_{-1,0}(z) - \frac34\frac{z+1}{s}G_{0,-1}(z) + \frac38\frac{2z^3+4z^2-z-1}{(z+1)sz}G_{0,0}(z)\Big)i\frac1{\pi^2} + \Big(\frac34\frac{z-1}{sz}$$

$$- \frac38\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1}(z) + \frac34\frac{z+1}{s}G_0(z)\Big)\frac1\pi\Big)\log^2(-\frac{s}{\mu_R^2}) + \Big(\Big(-\frac32\frac{z^2+z+1}{sz} - \frac38\frac1sG_{-1}(z) + \frac38\frac{z+1}{sz}G_0(z)\Big)i\frac1{\pi^2}$$

$$+ \frac38\frac1s\frac1\pi\Big)\log(-\frac{m_t^2}{s})\log^2(-\frac{s}{\mu_R^2}) - \frac3{16}\frac{2z^2+2z+1}{sz}i\frac1{\pi^2}\log^2(-\frac{m_t^2}{s})\log^2(-\frac{s}{\mu_R^2})\Big)N_c + \Big(\Big(\frac72\frac{z^2+z+1}{sz} + \Big(-\frac32\frac{z^2+z+1}{sz} - \frac14\frac{z-1}{sz}G_{-1}(z)$$

$$+ \frac14\frac{z+2}{sz}G_0(z) + \frac18\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1,-1}(z) - \frac14\frac{z+1}{s}G_{-1,0}(z) - \frac14\frac{z+1}{s}G_{0,-1}(z) + \frac18\frac{2z^3+4z^2-z-1}{(z+1)sz}G_{0,0}(z)\Big)\gamma_E$$

$$+ \Big(-\frac12\frac{z^3-z+3}{(z+1)sz} - \frac18\frac{8z^4+20z^3-5z^2-20z-8}{(z+1)sz^2}G_{-1}(z) + \frac18\frac{8z^3+20z^2+6z+1}{(z+1)sz}G_0(z)\Big)\zeta_2$$

$$- \frac34\frac1{(z+1)s}\gamma_E\zeta_2 - \frac18\frac{8z^3+17z^2+10z+3}{(z+1)sz}\zeta_3 - \frac14\frac{z^2-2z+4}{sz}G_{-1}(z) - \frac14\frac{z^2+4z+7}{sz}G_0(z) - \frac16\frac{(z-1)(3z^2+8z+3)}{sz^2}G_{-1,-1}(z)$$

$$+ \frac18\frac{(2z+1)^2}{sz}G_{-1,0}(z) + \frac18\frac{(2z+1)^2}{sz}G_{0,-1}(z) - \frac16\frac{(z+2)(3z^2-2z-2)}{(z+1)sz}G_{0,0}(z) + \frac1{16}\frac{10z^3+8z^2-15z-10}{sz^2}G_{-1,-1,-1}(z)$$

$$- \frac1{48}\frac{14z^3+14z^2+3z+2}{sz^2}G_{-1,-1,0}(z) - \frac1{48}\frac{14z^3+14z^2+3z+2}{sz^2}G_{-1,0,-1}(z) - \frac1{48}\frac{2z^3+10z^2-z+1}{(z+1)sz}G_{-1,0,0}(z)$$

$$- \frac1{48}\frac{2z^3-4z^2-15z-10}{sz^2}G_{0,-1,-1}(z) - \frac1{48}\frac{14z^3+28z^2+17z+1}{(z+1)sz}G_{0,-1,0}(z) - \frac1{48}\frac{14z^3+28z^2+17z+1}{(z+1)sz}G_{0,0,-1}(z)$$

$$+ \frac1{16}\frac{10z^3+22z^2-z-3}{(z+1)sz}G_{0,0,0}(z) + \frac34\frac1{(z+1)s}\zeta_2\log(4\pi) + \Big(\frac32\frac{z^2+z+1}{sz} + \frac14\frac{z-1}{sz}G_{-1}(z) - \frac14\frac{z+2}{sz}G_0(z)$$

$$- \frac18\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1,-1}(z) + \frac14\frac{z+1}{s}G_{-1,0}(z) + \frac14\frac{z+1}{s}G_{0,-1}(z) - \frac18\frac{2z^3+4z^2-z-1}{(z+1)sz}G_{0,0}(z)\Big)\log(4\pi)\Big)i\frac1{\pi^2}$$

$$+ \Big(\Big(\frac32\frac{z^2+z+1}{sz} + \frac34\frac1{(z+1)s}\zeta_2 + \frac14\frac{z-1}{sz}G_{-1}(z) - \frac14\frac{z+2}{sz}G_0(z) - \frac18\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1,-1}(z) + \frac14\frac{z+1}{s}G_{-1,0}(z)$$

$$+ \frac14\frac{z+1}{s}G_{0,-1}(z) - \frac18\frac{2z^3+4z^2-z-1}{(z+1)sz}G_{0,0}(z)\Big)i\frac1{\pi^2} + \Big(-\frac14\frac{z-1}{sz} + \frac18\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1}(z) - \frac14\frac{z+1}{s}G_0(z)\Big)\frac1\pi\Big)\frac1\pi$$

$$+ \Big(\frac14\frac{z^2-2z+4}{sz} + \Big(\frac14\frac{z-1}{sz} - \frac18\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1}(z) + \frac14\frac{z+1}{s}G_0(z)\Big)\gamma_E + \frac16\frac{(z-1)(3z^2+8z+3)}{sz^2}G_{-1}(z) - \frac18\frac{(2z+1)^2}{sz}G_0(z)$$

$$- \frac1{16}\frac{10z^3+8z^2-15z-10}{sz^2}G_{-1,-1}(z) + \frac1{48}\frac{14z^3+14z^2+3z+2}{sz^2}G_{-1,0}(z) + \frac1{48}\frac{2z^3-4z^2-15z-10}{sz^2}G_{0,-1}(z)$$

$$+ \frac1{48}\frac{14z^3+28z^2+17z+1}{(z+1)sz}G_{0,0}(z) + \Big(-\frac14\frac{z-1}{sz} + \frac18\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1}(z) - \frac14\frac{z+1}{s}G_0(z)\Big)\log(4\pi)\Big)\frac1\pi + \frac1{48}\frac{2z^2+6z+5}{(z+1)s}\pi$$

$$+ \Big(\Big(\Big(\Big(-\frac12\frac{z^2+z+1}{sz} - \frac18\frac1sG_{-1}(z) + \frac18\frac{z+1}{sz}G_0(z)\Big)\gamma_E - \frac18\frac{2z^2+z+1}{sz}\zeta_2 - \frac1{12}\frac{z^2+z+1}{sz}G_{-1}(z) - \frac1{12}\frac{z^2+z+1}{sz}G_0(z)$$

$$- \frac1{24}\frac1sG_{-1,-1}(z) + \frac1{48}\frac1{sz}G_{-1,0}(z) + \frac1{48}\frac1{sz}G_{0,-1}(z) + \frac1{24}\frac{z+1}{sz}G_{0,0}(z) + \Big(\frac12\frac{z^2+z+1}{sz} + \frac18\frac1sG_{-1}(z) - \frac18\frac{z+1}{sz}G_0(z)\Big)\log(4\pi)\Big)i\frac1{\pi^2}$$

$$+ \Big(\Big(\frac12\frac{z^2+z+1}{sz} + \frac18\frac1sG_{-1}(z) - \frac18\frac{z+1}{sz}G_0(z)\Big)i\frac1{\pi^2} - \frac18\frac1s\frac1\pi\Big)\frac1\pi + \Big(\frac1{12}\frac{z^2+z+1}{sz} + \frac18\frac1s\gamma_E + \frac1{24}\frac1sG_{-1}(z) - \frac1{48}\frac1{sz}G_0(z)$$

$$- \frac18\frac1s\log(4\pi)\Big)\frac1\pi\Big)\log(-\frac{m_t^2}{s}) + \Big(\frac1\pi\frac1{16}\frac{2z^2+2z+1}{sz}i\frac1{\pi^2} + \Big(-\frac14\frac{z^2+z+1}{sz} - \frac1{16}\frac{2z^2+2z+1}{sz}\gamma_E - \frac1{96}\frac{2z^2+8z+1}{sz}G_{-1}(z)$$

$$- \frac1{96}\frac{2z^2-4z-5}{sz}G_0(z) + \frac1{16}\frac{2z^2+2z+1}{sz}\log(4\pi)\Big)i\frac1{\pi^2} + \frac1{96}\frac{2z^2+8z+1}{sz}\frac1\pi\Big)\log^2(-\frac{m_t^2}{s}) - \frac1{24}\frac{2z^2+2z+1}{sz}i\frac1{\pi^2}\log^3(-\frac{m_t^2}{s})$$

$$+ \Big(\Big(-\frac94\frac{z^2+z+1}{sz} - \frac98\frac1{(z+1)s}\zeta_2 - \frac38\frac{z-1}{sz}G_{-1}(z) + \frac38\frac{z+2}{sz}G_0(z) + \frac3{16}\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1,-1}(z) - \frac38\frac{z+1}{s}G_{-1,0}(z)$$

$$- \frac38\frac{z+1}{s}G_{0,-1}(z) + \frac3{16}\frac{2z^3+4z^2-z-1}{(z+1)sz}G_{0,0}(z)\Big)i\frac1{\pi^2} + \Big(\frac38\frac{z-1}{sz} - \frac3{16}\frac{2z^3+2z^2-3z-2}{sz^2}G_{-1}(z) + \frac38\frac{z+1}{s}G_0(z)\Big)\frac1\pi\Big)\log(-\frac{s}{\mu_R^2})$$

$$+ \Big(\Big(-\frac34\frac{z^2+z+1}{sz} - \frac3{16}\frac1sG_{-1}(z) + \frac3{16}\frac{z+1}{sz}G_0(z)\Big)i\frac1{\pi^2} + \frac3{16}\frac1s\frac1\pi\Big)\log(-\frac{m_t^2}{s})\log(-\frac{s}{\mu_R^2}) - \frac3{32}\frac{2z^2+2z+1}{sz}i\frac1{\pi^2}\log^2(-\frac{m_t^2}{s})\log(-\frac{s}{\mu_R^2})\Big)N_f$$

$$+ \Big(\Big(-\frac18\frac{30z^4+60z^3+72z^2+42z+35}{(z+1)s^2z^2} + \Big(\frac38\frac{6z^4+12z^3+12z^2+6z+5}{(z+1)s^2z^2} + \frac38\frac{3z^2+3z-2}{(z+1)s^2z}G_{-1}(z) - \frac38\frac{3z^2+3z-2}{s^2z^2}G_0(z)$$

$$+ \frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z) - \frac{3}{2}\frac{1}{s^2z}G_{-1,0}(z) - \frac{3}{2}\frac{1}{s^2z}G_{0,-1}(z) + \frac{3}{2}\frac{z+2}{s^2z^2}G_{0,0}(z)\Big)\gamma_E + \Big(\frac{3}{4}\frac{31z^2+12z-1}{(z+1)s^2z^2} + \frac{3}{4}\frac{4z^2+5z+17}{(z+1)s^2z}G_{-1}(z)$$

$$- \frac{3}{2}\frac{2z^2+3z+2}{s^2z^2}G_0(z)\Big)\zeta_2 - \frac{9}{4}\frac{5z+1}{(z+1)s^2z}\gamma_E\zeta_2 + \frac{3}{4}\frac{12z^4+25z^3+38z^2+25z+12}{(z+1)s^2z^2}\zeta_3 - \frac{1}{8}\frac{9z^2+9z-14}{(z+1)s^2z}G_{-1}(z) + \frac{1}{8}\frac{9z^2+9z-14}{s^2z^2}G_0(z)$$

$$- \frac{1}{4}\frac{11z-8}{(z+1)s^2z}G_{-1,-1}(z) + 2\frac{1}{s^2z}G_{-1,0}(z) + 2\frac{1}{s^2z}G_{0,-1}(z) - \frac{1}{4}\frac{11z+19}{s^2z^2}G_{0,0}(z) + \frac{3}{4}\frac{z^2+3z-2}{(z+1)s^2z}G_{-1,-1,-1}(z) - \frac{3}{2}\frac{1}{s^2z}G_{-1,-1,0}(z)$$

$$- \frac{3}{2}\frac{1}{s^2z}G_{-1,0,-1}(z) + \frac{3}{4}\frac{z+3}{s^2z}G_{-1,0,0}(z) - \frac{3}{4}\frac{z-2}{s^2z}G_{0,-1,-1}(z) - \frac{3}{2}\frac{1}{s^2z}G_{0,-1,0}(z) - \frac{3}{2}\frac{1}{s^2z}G_{0,0,-1}(z) - \frac{3}{4}\frac{z^2-z-4}{s^2z^2}G_{0,0,0}(z)$$

$$+ \frac{9}{4}\frac{5z+1}{(z+1)s^2z}\zeta_2\log(4\pi) + \Big(-\frac{3}{8}\frac{6z^4+12z^3+12z^2+6z+5}{(z+1)s^2z^2} - \frac{3}{8}\frac{3z^2+3z-2}{(z+1)s^2z}G_{-1}(z) + \frac{3}{8}\frac{3z^2+3z-2}{s^2z^2}G_0(z) - \frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z)$$

$$+ \frac{3}{2}\frac{1}{s^2z}G_{-1,0}(z) + \frac{3}{2}\frac{1}{s^2z}G_{0,-1}(z) - \frac{3}{2}\frac{z+2}{s^2z^2}G_{0,0}(z)\Big)\log(4\pi)\Big)i\frac{1}{\pi^2} + \Big(\Big(-\frac{3}{8}\frac{6z^4+12z^3+12z^2+6z+5}{(z+1)s^2z^2} + \frac{9}{4}\frac{5z+1}{(z+1)s^2z}\zeta_2$$

$$- \frac{3}{8}\frac{3z^2+3z-2}{(z+1)s^2z}G_{-1}(z) + \frac{3}{8}\frac{3z^2+3z-2}{s^2z^2}G_0(z) - \frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z) + \frac{3}{2}\frac{1}{s^2z}G_{-1,0}(z) + \frac{3}{2}\frac{1}{s^2z}G_{0,-1}(z) - \frac{3}{2}\frac{z+2}{s^2z^2}G_{0,0}(z)\Big)i\frac{1}{\pi^2}$$

$$+ \Big(\frac{3}{8}\frac{3z^2+3z-2}{(z+1)s^2z} + \frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1}(z) - \frac{3}{2}\frac{1}{s^2z}G_0(z)\Big)\frac{1}{\pi}\Big)\frac{1}{\epsilon} + \Big(\frac{1}{8}\frac{9z^2+9z-14}{(z+1)s^2z} + \Big(-\frac{3}{8}\frac{3z^2+3z-2}{(z+1)s^2z} - \frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1}(z)$$

$$+ \frac{3}{2}\frac{1}{s^2z}G_0(z)\Big)\gamma_E + \frac{1}{4}\frac{11z-8}{(z+1)s^2z}G_{-1}(z) - 2\frac{1}{s^2z}G_0(z) - \frac{3}{4}\frac{z^2+3z-2}{(z+1)s^2z}G_{-1,-1}(z) + \frac{3}{2}\frac{1}{s^2z}G_{-1,0}(z) + \frac{3}{4}\frac{z-2}{s^2z}G_{0,-1}(z) + \frac{3}{2}\frac{1}{s^2z}G_{0,0}(z)$$

$$+ \Big(\frac{3}{8}\frac{3z^2+3z-2}{(z+1)s^2z} + \frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1}(z) - \frac{3}{2}\frac{1}{s^2z}G_0(z)\Big)\log(4\pi)\Big)\frac{1}{\pi} - \frac{1}{4}\frac{(z-1)^2}{(z+1)s^2z}\pi + \Big(\Big(\frac{1}{2}\frac{9z^4+18z^3+17z^2+8z+11}{(z+1)s^2z^2}$$

$$+ \Big(\frac{3}{4}\frac{2z^2+2z-1}{(z+1)s^2z^2} + \frac{3}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z) - \frac{3}{4}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)\gamma_E + \frac{3}{4}\frac{(2z+1)(2z^3+3z^2-3z+4)}{(z+1)s^2z^2}\zeta_2 - \frac{1}{4}\frac{z^2-10z+14}{(z+1)s^2z}G_{-1}(z)$$

$$+ \frac{1}{4}\frac{z^2+12z+25}{s^2z^2}G_0(z) + \frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z) - \frac{3}{2}\frac{1}{s^2z}G_{-1,0}(z) - \frac{3}{2}\frac{1}{s^2z}G_{0,-1}(z) + \frac{3}{2}\frac{z+2}{s^2z^2}G_{0,0}(z) + \Big(-\frac{3}{4}\frac{2z^2+2z-1}{(z+1)s^2z^2}$$

$$- \frac{3}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z) + \frac{3}{4}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)\log(4\pi)\Big)i\frac{1}{\pi^2} + \Big(\Big(-\frac{3}{4}\frac{2z^2+2z-1}{(z+1)s^2z^2} - \frac{3}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z) + \frac{3}{4}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)i\frac{1}{\pi^2}$$

$$+ \frac{3}{4}\frac{z^2-z+2}{(z+1)s^2z}\frac{1}{\pi}\Big)\frac{1}{\epsilon} + \Big(\frac{1}{4}\frac{z^2-10z+14}{(z+1)s^2z} - \frac{3}{4}\frac{z^2-z+2}{(z+1)s^2z}\gamma_E - \frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1}(z) + \frac{3}{2}\frac{1}{s^2z}G_0(z)$$

$$+ \frac{3}{4}\frac{z^2-z+2}{(z+1)s^2z}\log(4\pi)\Big)\frac{1}{\pi}\Big)\log(-\frac{m_t^2}{s}) + \Big(-\frac{1}{\epsilon}\frac{3}{2}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}i\frac{1}{\pi^2} + \Big(-\frac{1}{8}\frac{22z^4+44z^3+48z^2+26z+31}{(z+1)s^2z^2} + \frac{3}{2}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}\gamma_E$$

$$+ \frac{9}{8}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z) - \frac{9}{8}\frac{z^2+3z+4}{s^2z^2}G_0(z) - \frac{3}{2}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}\log(4\pi)\Big)i\frac{1}{\pi^2} - \frac{9}{8}\frac{z^2-z+2}{(z+1)s^2z}\frac{1}{\pi}\Big)\log^2(-\frac{m_t^2}{s})$$

$$+ \frac{5}{2}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}i\frac{1}{\pi^2}\log^3(-\frac{m_t^2}{s}) + \Big(\Big(\Big(\frac{3}{4}\frac{6z^4+12z^3+12z^2+6z+5}{(z+1)s^2z^2} - \frac{9}{2}\frac{5z+1}{(z+1)s^2z}\zeta_2 + \frac{3}{4}\frac{3z^2+3z-2}{(z+1)s^2z}G_{-1}(z)$$

$$- \frac{3}{4}\frac{3z^2+3z-2}{s^2z^2}G_0(z) + 3\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z) - 3\frac{1}{s^2z}G_{-1,0}(z) - 3\frac{1}{s^2z}G_{0,-1}(z) + 3\frac{z+2}{s^2z^2}G_{0,0}(z)\Big)i\frac{1}{\pi^2} + \Big(-\frac{3}{4}\frac{3z^2+3z-2}{(z+1)s^2z}$$

$$- 3\frac{z-1}{(z+1)s^2z}G_{-1}(z) + 3\frac{1}{s^2z}G_0(z)\Big)\frac{1}{\pi}\Big)\log(-\frac{s}{\mu_R^2}) + \Big(\Big(\frac{3}{2}\frac{2z^2+2z-1}{(z+1)s^2z^2} + \frac{3}{2}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z) - \frac{3}{2}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)i\frac{1}{\pi^2}$$

$$- \frac{3}{2}\frac{z^2-z+2}{(z+1)s^2z}\frac{1}{\pi}\Big)\log(-\frac{m_t^2}{s})\log(-\frac{s}{\mu_R^2}) + 3\frac{(z^2+z+1)^2}{(z+1)s^2z^2}i\frac{1}{\pi^2}\log^2(-\frac{m_t^2}{s})\log(-\frac{s}{\mu_R^2})\Big)m_t^2\frac{1}{N_c} + \Big(\Big(\Big(-\frac{1}{8}\frac{50z^4+100z^3+156z^2+106z+67}{(z+1)s^2z^2}$$

$$+ \Big(\frac{1}{8}\frac{38z^4+76z^3+88z^2+50z+47}{(z+1)s^2z^2} - \frac{1}{8}\frac{4z^4+9z^3-5z^2+26z+4}{(z+1)s^2z^2}G_{-1}(z) - \frac{1}{8}\frac{4z^4+7z^3-8z^2-47z-32}{(z+1)s^2z^2}G_0(z)$$

$$- \frac{1}{8}\frac{2z^2-9z+6}{(z+1)s^2z}G_{-1,-1}(z) - \frac{1}{8}\frac{11z^2+11z+3}{(z+1)s^2z^2}G_{-1,0}(z) - \frac{1}{8}\frac{11z^2+11z+3}{(z+1)s^2z^2}G_{0,-1}(z) + \frac{1}{8}\frac{2z^2+13z+17}{s^2z^2}G_{0,0}(z)$$

$$- \frac{3}{8}\frac{z^2+5z-4}{(z+1)s^2z}G_{-1,-1,-1}(z) + \frac{1}{2}\frac{2z+3}{(z+1)s^2z}G_{-1,-1,0}(z) + \frac{1}{2}\frac{2z+3}{(z+1)s^2z}G_{-1,0,-1}(z) - \frac{1}{8}\frac{(z+1)(3z+4)}{s^2z^2}G_{-1,0,0}(z)$$

$$+ \frac{1}{8}\frac{3z-1}{(z+1)s^2}G_{0,-1,-1}(z) + \frac{1}{2}\frac{2z-1}{s^2z^2}G_{0,-1,0}(z) + \frac{1}{2}\frac{2z-1}{s^2z^2}G_{0,0,-1}(z) + \frac{3}{8}\frac{z^2-3z-8}{s^2z^2}G_{0,0,0}(z)\Big)\gamma_E + \Big(-\frac{3}{4}\frac{z^4+2z^3+2z^2+z+1}{(z+1)s^2z^2}$$

$$- \frac{3}{16}\frac{z^2+2z-2}{(z+1)s^2z}G_{-1}(z) + \frac{3}{16}\frac{z^2-3}{s^2z^2}G_0(z) - \frac{3}{8}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z) + \frac{3}{8}\frac{1}{s^2z}G_{-1,0}(z) + \frac{3}{8}\frac{1}{s^2z}G_{0,-1}(z) - \frac{3}{8}\frac{z+2}{s^2z^2}G_{0,0}(z)\Big)\gamma_E^2$$

$$+ \Big(-\frac{1}{8}\frac{16z^4+37z^3+144z^2+24}{(z+1)s^2z^2} + \frac{1}{16}\frac{4z^4+114z^3+25z^2+195z+6}{(z+1)s^2z^2}G_{-1}(z) + \frac{1}{16}\frac{4z^4-86z^3-229z^2-117z-22}{(z+1)s^2z^2}G_0(z)$$

$$+ \frac{1}{4}\frac{8z^2+61z-35}{(z+1)s^2z}G_{-1,-1}(z) - \frac{1}{8}\frac{9z^3+85z^2+88z-4}{(z+1)s^2z^2}G_{-1,0}(z) - \frac{1}{2}\frac{6z^3+10z^2+7z-1}{(z+1)s^2z^2}G_{0,-1}(z)$$

$$+ \frac{1}{8}\frac{17z^3+53z^2+56z+32}{(z+1)s^2z^2}G_{0,0}(z)\Big)\zeta_2 + \Big(\frac{3}{8}\frac{2z^4+6z^3-26z^2-13z+3}{(z+1)s^2z^2} - \frac{3}{2}\frac{z^2-z+5}{(z+1)s^2z}G_{-1}(z)$$

$$+ \frac{3}{8}\frac{4z^3+11z^2+7z+4}{(z+1)s^2z^2}G_0(z)\Big)\gamma_E\zeta_2 + \frac{9}{16}\frac{5z+1}{(z+1)s^2z}\gamma_E^2\zeta_2 + \Big(\frac{1}{8}\frac{84z^4+179z^3+286z^2+191z+78}{(z+1)s^2z^2} - \frac{1}{4}\frac{6z^4+8z^3+13z^2-z+6}{(z+1)s^2z^2}G_{-1}(z)$$

$$- \frac{1}{4}\frac{6z^4+17z^3+34z^2+35z+18}{(z+1)s^2z^2}G_0(z)\Big)\zeta_3 - \frac{3}{8}\frac{12z^4+25z^3+38z^2+25z+12}{(z+1)s^2z^2}\gamma_E\zeta_3 + \frac{3}{32}\frac{20z^4+107z^3-341z^2+32z+20}{(z+1)s^2z^2}\zeta_4$$

$$+\frac{1}{24}\frac{34z^4+137z^3+89z^2+88z+40}{(z+1)s^2z^2}G_{-1}(z)+\frac{1}{24}\frac{34z^4-z^3-118z^2-185z-62}{(z+1)s^2z^2}G_{0}(z)-\frac{1}{24}\frac{12z^4+19z^3-22z^2+94z+12}{(z+1)s^2z^2}G_{-1,-1}(z)$$

$$-\frac{1}{48}\frac{67z^2+67z-51}{(z+1)s^2z^2}G_{-1,0}(z)-\frac{1}{48}\frac{67z^2+67z-51}{(z+1)s^2z^2}G_{0,-1}(z)-\frac{1}{24}\frac{12z^4+29z^3-7z^2-147z-111}{(z+1)s^2z^2}G_{0,0}(z)$$

$$+\frac{1}{8}\frac{8z^2+32z-15}{(z+1)s^2z}G_{-1,-1,-1}(z)+\frac{1}{24}\frac{3z^3-35z^2-72z-9}{(z+1)s^2z^2}G_{-1,-1,0}(z)+\frac{1}{24}\frac{3z^3-35z^2-72z-9}{(z+1)s^2z^2}G_{-1,0,-1}(z)$$

$$+\frac{1}{24}\frac{30z^3+82z^2+86z+25}{(z+1)s^2z^2}G_{-1,0,0}(z)-\frac{1}{24}\frac{30z^3+8z^2+12z+9}{(z+1)s^2z^2}G_{0,-1,-1}(z)-\frac{1}{24}\frac{3z^3+44z^2+7z-25}{(z+1)s^2z^2}G_{0,-1,0}(z)$$

$$-\frac{1}{24}\frac{3z^3+44z^2+7z-25}{(z+1)s^2z^2}G_{0,0,-1}(z)-\frac{1}{8}\frac{8z^2-16z-39}{s^2z^2}G_{0,0,0}(z)-\frac{1}{4}\frac{5z^2+18z-13}{(z+1)s^2z}G_{-1,-1,-1,-1}(z)$$

$$-\frac{1}{8}\frac{z^2-15z-20}{(z+1)s^2z}G_{-1,-1,-1,0}(z)-\frac{1}{8}\frac{z^2-15z-20}{(z+1)s^2z}G_{-1,-1,0,-1}(z)+\frac{1}{8}\frac{z^3+4z^2+3z-4}{(z+1)s^2z^2}G_{-1,-1,0,0}(z)$$

$$+\frac{1}{8}\frac{3z^2+3z+4}{(z+1)s^2z}G_{-1,0,-1,-1}(z)-\frac{1}{8}\frac{z^3-4z^2-5z+4}{(z+1)s^2z^2}G_{-1,0,-1,0}(z)-\frac{1}{8}\frac{z^3-4z^2-5z+4}{(z+1)s^2z^2}G_{-1,0,0,-1}(z)-\frac{1}{2}\frac{2z^2+7z+1}{s^2z^2}G_{-1,0,0,0}(z)$$

$$+\frac{1}{2}\frac{2z^2-3z-4}{(z+1)s^2z}G_{0,-1,-1,-1}(z)+\frac{1}{8}\frac{z^3+7z^2+6z-4}{(z+1)s^2z^2}G_{0,-1,-1,0}(z)+\frac{1}{8}\frac{z^3+7z^2+6z-4}{(z+1)s^2z^2}G_{0,-1,0,-1}(z)-\frac{1}{8}\frac{3z^2+3z+4}{s^2z^2}G_{0,-1,0,0}(z)$$

$$-\frac{1}{8}\frac{z^3-z^2-2z+4}{(z+1)s^2z^2}G_{0,0,-1,-1}(z)+\frac{1}{8}\frac{z^2+17z-4}{s^2z^2}G_{0,0,-1,0}(z)+\frac{1}{8}\frac{z^2+17z-4}{s^2z^2}G_{0,0,0,-1}(z)+\frac{1}{4}\frac{5z^2-8z-26}{s^2z^2}G_{0,0,0,0}(z)$$

$$+\bigg(\frac{3}{2}\frac{z^4+2z^3+2z^2+z+1}{(z+1)s^2z^2}+\frac{3}{8}\frac{z^2+2z-2}{(z+1)s^2z}G_{-1}(z)-\frac{3}{8}\frac{z^2-3}{s^2z^2}G_{0}(z)+\frac{3}{4}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z)-\frac{3}{4}\frac{1}{s^2z}G_{-1,0}(z)-\frac{3}{4}\frac{1}{s^2z}G_{0,-1}(z)$$

$$+\frac{3}{4}\frac{z+2}{s^2z^2}G_{0,0}(z)\bigg)\gamma_E\log(4\pi)+\bigg(-\frac{3}{8}\frac{2z^4+6z^3-26z^2-13z+3}{(z+1)s^2z^2}+\frac{3}{2}\frac{z^2-z+5}{(z+1)s^2z}G_{-1}(z)-\frac{3}{8}\frac{4z^3+11z^2+7z+4}{(z+1)s^2z^2}G_{0}(z)\bigg)\zeta_2\log(4\pi)$$

$$-\frac{9}{8}\frac{5z+1}{(z+1)s^2z}\gamma_E\zeta_2\log(4\pi)+\frac{3}{8}\frac{12z^4+25z^3+38z^2+25z+12}{(z+1)s^2z^2}\zeta_3\log(4\pi)+\bigg(-\frac{1}{8}\frac{38z^4+76z^3+88z^2+50z+47}{(z+1)s^2z^2}$$

$$+\frac{1}{8}\frac{4z^4+9z^3-5z^2+26z+4}{(z+1)s^2z^2}G_{-1}(z)+\frac{1}{8}\frac{4z^4+7z^3-8z^2-47z-32}{(z+1)s^2z^2}G_{0}(z)+\frac{1}{8}\frac{2z^2-9z+6}{(z+1)s^2z}G_{-1,-1}(z)+\frac{1}{8}\frac{11z^2+11z+3}{(z+1)s^2z^2}G_{-1,0}(z)$$

$$+\frac{1}{8}\frac{11z^2+11z+3}{(z+1)s^2z^2}G_{0,-1}(z)-\frac{1}{8}\frac{2z^2+13z+17}{s^2z^2}G_{0,0}(z)+\frac{3}{8}\frac{z^2+5z-4}{(z+1)s^2z}G_{-1,-1,-1}(z)-\frac{1}{2}\frac{2z+3}{(z+1)s^2z}G_{-1,-1,0}(z)$$

$$-\frac{1}{2}\frac{2z+3}{(z+1)s^2z}G_{-1,0,-1}(z)+\frac{1}{8}\frac{(z+1)(3z+4)}{s^2z^2}G_{-1,0,0}(z)-\frac{1}{8}\frac{3z-1}{(z+1)s^2}G_{0,-1,-1}(z)-\frac{1}{2}\frac{2z-1}{s^2z^2}G_{0,-1,0}(z)-\frac{1}{2}\frac{2z-1}{s^2z^2}G_{0,0,-1}(z)$$

$$-\frac{3}{8}\frac{z^2-3z-8}{s^2z^2}G_{0,0,0}(z)\bigg)\log(4\pi)+\frac{9}{16}\frac{5z+1}{(z+1)s^2z}\zeta_2\log^2(4\pi)+\bigg(-\frac{3}{4}\frac{z^4+2z^3+2z^2+z+1}{(z+1)s^2z^2}-\frac{3}{16}\frac{z^2+2z-2}{(z+1)s^2z}G_{-1}(z)$$

$$+\frac{3}{16}\frac{z^2-3}{s^2z^2}G_{0}(z)-\frac{3}{8}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z)+\frac{3}{8}\frac{1}{s^2z}G_{-1,0}(z)+\frac{3}{8}\frac{1}{s^2z}G_{0,-1}(z)-\frac{3}{8}\frac{z+2}{s^2z^2}G_{0,0}(z)\bigg)\log^2(4\pi)\bigg)i\frac{1}{\pi^2}$$

$$+\bigg(\bigg(-\frac{3}{2}\frac{z^4+2z^3+2z^2+z+1}{(z+1)s^2z^2}+\frac{9}{8}\frac{5z+1}{(z+1)s^2z}\zeta_2-\frac{3}{8}\frac{z^2+2z-2}{(z+1)s^2z}G_{-1}(z)+\frac{3}{8}\frac{z^2-3}{s^2z^2}G_{0}(z)-\frac{3}{4}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z)+\frac{3}{4}\frac{1}{s^2z}G_{-1,0}(z)$$

$$+\frac{3}{4}\frac{1}{s^2z}G_{0,-1}(z)-\frac{3}{4}\frac{z+2}{s^2z^2}G_{0,0}(z)\bigg)i\frac{1}{\pi^2}+\bigg(\frac{3}{8}\frac{z^2+2z-2}{(z+1)s^2z}+\frac{3}{4}\frac{z-1}{(z+1)s^2z}G_{-1}(z)-\frac{3}{4}\frac{1}{s^2z}G_{0}(z)\bigg)\frac{1}{\pi}\bigg)\frac{1}{\epsilon^2}$$

$$+\bigg(-\frac{1}{24}\frac{34z^4+137z^3+89z^2+88z+40}{(z+1)s^2z^2}+\bigg(\frac{1}{8}\frac{4z^4+9z^3-5z^2+26z+4}{(z+1)s^2z^2}+\frac{1}{8}\frac{2z^2-9z+6}{(z+1)s^2z}G_{-1}(z)+\frac{1}{8}\frac{11z^2+11z+3}{(z+1)s^2z^2}G_{0}(z)$$

$$+\frac{3}{8}\frac{z^2+5z-4}{(z+1)s^2z}G_{-1,-1}(z)-\frac{1}{2}\frac{2z+3}{(z+1)s^2z}G_{-1,0}(z)-\frac{1}{8}\frac{3z-1}{(z+1)s^2}G_{0,-1}(z)-\frac{1}{2}\frac{2z-1}{s^2z^2}G_{0,0}(z)\bigg)\gamma_E+\bigg(\frac{3}{16}\frac{z^2+2z-2}{(z+1)s^2z}$$

$$+\frac{3}{8}\frac{z-1}{(z+1)s^2z}G_{-1}(z)-\frac{3}{8}\frac{1}{s^2z}G_{0}(z)\bigg)\gamma_E^2+\frac{1}{8}\frac{12z^4+19z^3+44z^2+13z+12}{(z+1)s^2z^2}\zeta_3+\frac{1}{24}\frac{12z^4+19z^3-22z^2+94z+12}{(z+1)s^2z^2}G_{-1}(z)$$

$$+\frac{1}{48}\frac{67z^2+67z-51}{(z+1)s^2z^2}G_{0}(z)-\frac{1}{8}\frac{8z^2+32z-15}{(z+1)s^2z}G_{-1,-1}(z)-\frac{1}{24}\frac{3z^3-35z^2-72z-9}{(z+1)s^2z^2}G_{-1,0}(z)+\frac{1}{24}\frac{30z^3+8z^2+12z+9}{(z+1)s^2z^2}G_{0,-1}(z)$$

$$+\frac{1}{24}\frac{3z^3+44z^2+7z-25}{(z+1)s^2z^2}G_{0,0}(z)+\frac{1}{4}\frac{5z^2+18z-13}{(z+1)s^2z}G_{-1,-1,-1}(z)+\frac{1}{8}\frac{z^2-15z-20}{(z+1)s^2z}G_{-1,-1,0}(z)-\frac{1}{8}\frac{3z^2+3z+4}{(z+1)s^2z}G_{-1,0,-1}(z)$$

$$+\frac{1}{8}\frac{z^3-4z^2-5z+4}{(z+1)s^2z^2}G_{-1,0,0}(z)-\frac{1}{2}\frac{2z^2-3z-4}{(z+1)s^2z}G_{0,-1,-1}(z)-\frac{1}{8}\frac{z^3+7z^2+6z-4}{(z+1)s^2z^2}G_{0,-1,0}(z)+\frac{1}{8}\frac{z^3-z^2-2z+4}{(z+1)s^2z^2}G_{0,0,-1}(z)$$

$$-\frac{1}{8}\frac{z^2+17z-4}{s^2z^2}G_{0,0,0}(z)+\bigg(-\frac{3}{8}\frac{z^2+2z-2}{(z+1)s^2z}-\frac{3}{4}\frac{z-1}{(z+1)s^2z}G_{-1}(z)+\frac{3}{4}\frac{1}{s^2z}G_{0}(z)\bigg)\gamma_E\log(4\pi)+\bigg(-\frac{1}{8}\frac{4z^4+9z^3-5z^2+26z+4}{(z+1)s^2z^2}$$

$$-\frac{1}{8}\frac{2z^2-9z+6}{(z+1)s^2z}G_{-1}(z)-\frac{1}{8}\frac{11z^2+11z+3}{(z+1)s^2z^2}G_{0}(z)-\frac{3}{8}\frac{z^2+5z-4}{(z+1)s^2z}G_{-1,-1}(z)+\frac{1}{2}\frac{2z+3}{(z+1)s^2z}G_{-1,0}(z)+\frac{1}{8}\frac{3z-1}{(z+1)s^2}G_{0,-1}(z)$$

$$+\frac{1}{2}\frac{2z-1}{s^2z^2}G_{0,0}(z)\bigg)\log(4\pi)+\bigg(\frac{3}{16}\frac{z^2+2z-2}{(z+1)s^2z}+\frac{3}{8}\frac{z-1}{(z+1)s^2z}G_{-1}(z)-\frac{3}{8}\frac{1}{s^2z}G_{0}(z)\bigg)\log^2(4\pi)\bigg)\frac{1}{\pi}$$

$$+\bigg(\bigg(-\frac{1}{8}\frac{38z^4+76z^3+88z^2+50z+47}{(z+1)s^2z^2}+\bigg(\frac{3}{2}\frac{z^4+2z^3+2z^2+z+1}{(z+1)s^2z^2}+\frac{3}{8}\frac{z^2+2z-2}{(z+1)s^2z}G_{-1}(z)-\frac{3}{8}\frac{z^2-3}{s^2z^2}G_{0}(z)+\frac{3}{4}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z)$$

$$-\frac{3}{4}\frac{1}{s^2z}G_{-1,0}(z)-\frac{3}{4}\frac{1}{s^2z}G_{0,-1}(z)+\frac{3}{4}\frac{z+2}{s^2z^2}G_{0,0}(z)\bigg)\gamma_E+\bigg(-\frac{3}{8}\frac{2z^4+6z^3-26z^2-13z+3}{(z+1)s^2z^2}+\frac{3}{2}\frac{z^2-z+5}{(z+1)s^2z}G_{-1}(z)$$

$$-\frac{3}{8}\frac{4z^3+11z^2+7z+4}{(z+1)s^2z^2}G_{0}(z)\bigg)\zeta_2-\frac{9}{8}\frac{5z+1}{(z+1)s^2z}\gamma_E\zeta_2+\frac{3}{8}\frac{12z^4+25z^3+38z^2+25z+12}{(z+1)s^2z^2}\zeta_3+\frac{1}{8}\frac{4z^4+9z^3-5z^2+26z+4}{(z+1)s^2z^2}G_{-1}(z)$$

$$+\frac{1}{8}\frac{4z^4+7z^3-8z^2-47z-32}{(z+1)s^2z^2}G_0(z)+\frac{1}{8}\frac{2z^2-9z+6}{(z+1)s^2z}G_{-1,-1}(z)+\frac{1}{8}\frac{11z^2+11z+3}{(z+1)s^2z^2}G_{-1,0}(z)+\frac{1}{8}\frac{11z^2+11z+3}{(z+1)s^2z^2}G_{0,-1}(z)$$
$$-\frac{1}{8}\frac{2z^2+13z+17}{s^2z^2}G_{0,0}(z)+\frac{3}{8}\frac{z^2+5z-4}{(z+1)s^2z}G_{-1,-1,-1}(z)-\frac{1}{2}\frac{2z+3}{(z+1)s^2z}G_{-1,-1,0}(z)-\frac{1}{2}\frac{2z+3}{(z+1)s^2z}G_{-1,0,-1}(z)$$
$$+\frac{1}{8}\frac{(z+1)(3z+4)}{s^2z^2}G_{-1,0,0}(z)-\frac{1}{8}\frac{3z-1}{(z+1)s^2}G_{0,-1,-1}(z)-\frac{1}{2}\frac{2z-1}{s^2z^2}G_{0,-1,0}(z)-\frac{1}{2}\frac{2z-1}{s^2z^2}G_{0,0,-1}(z)-\frac{3}{8}\frac{z^2-3z-8}{s^2z^2}G_{0,0,0}(z)$$
$$+\frac{9}{8}\frac{5z+1}{(z+1)s^2z}\zeta_2\log(4\pi)+\Big(-\frac{3}{2}\frac{z^4+2z^3+2z^2+z+1}{(z+1)s^2z^2}-\frac{3}{8}\frac{z^2+2z-2}{(z+1)s^2z}G_{-1}(z)+\frac{3}{8}\frac{z^2-3}{s^2z^2}G_0(z)-\frac{3}{4}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z)$$
$$+\frac{3}{4}\frac{1}{s^2z}G_{-1,0}(z)+\frac{3}{4}\frac{1}{s^2z}G_{0,-1}(z)-\frac{3}{4}\frac{z+2}{s^2z^2}G_{0,0}(z)\Big)\log(4\pi)\Big)i\frac{1}{\pi^2}+\Big(-\frac{1}{8}\frac{4z^4+9z^3-5z^2+26z+4}{(z+1)s^2z^2}$$
$$+\Big(-\frac{3}{8}\frac{z^2+2z-2}{(z+1)s^2z}-\frac{3}{4}\frac{z-1}{(z+1)s^2z}G_{-1}(z)+\frac{3}{4}\frac{1}{s^2z}G_0(z)\Big)\gamma_E-\frac{1}{8}\frac{2z^2-9z+6}{(z+1)s^2z}G_{-1}(z)-\frac{1}{8}\frac{11z^2+11z+3}{(z+1)s^2z^2}G_0(z)$$
$$-\frac{3}{8}\frac{z^2+5z-4}{(z+1)s^2z}G_{-1,-1}(z)+\frac{1}{2}\frac{2z+3}{(z+1)s^2z}G_{-1,0}(z)+\frac{1}{8}\frac{3z-1}{(z+1)s^2}G_{0,-1}(z)+\frac{1}{2}\frac{2z-1}{s^2z^2}G_{0,0}(z)+\Big(\frac{3}{8}\frac{z^2+2z-2}{(z+1)s^2z}+\frac{3}{4}\frac{z-1}{(z+1)s^2z}G_{-1}(z)$$
$$-\frac{3}{4}\frac{1}{s^2z}G_0(z)\Big)\log(4\pi)\Big)\frac{1}{\pi}-\frac{1}{16}\frac{2z^2-9z+1}{(z+1)s^2z}\pi\Big)\frac{1}{\epsilon}+\Big(-\frac{1}{96}\frac{4z^4+58z^3-81z^2-11z+6}{(z+1)s^2z^2}+\frac{1}{16}\frac{2z^2-9z+1}{(z+1)s^2z}\gamma_E$$
$$+\frac{1}{24}\frac{3z^2-19z+14}{(z+1)s^2z}G_{-1}(z)-\frac{1}{24}\frac{z^3+z^2-2z+2}{(z+1)s^2z^2}G_0(z)-\frac{1}{16}\frac{2z^2-9z+1}{(z+1)s^2z}\log(4\pi)\Big)\pi+\Big(\Big(\frac{3}{4}\frac{10z^4+20z^3+32z^2+22z+11}{(z+1)s^2z^2}$$
$$+\Big(-\frac{5z^4+10z^3+13z^2+8z+6}{(z+1)s^2z^2}+\frac{1}{8}\frac{2z^4+12z^3-4z^2+18z+3}{(z+1)s^2z^2}G_{-1}(z)+\frac{1}{8}\frac{2z^4-4z^3-28z^2-54z-29}{(z+1)s^2z^2}G_0(z)-\frac{1}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1,-1}(z)$$
$$+\frac{1}{8}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_{-1,0}(z)+\frac{1}{8}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_{0,-1}(z)+\frac{1}{4}\frac{z^2+3z+4}{s^2z^2}G_{0,0}(z)\Big)\gamma_E+\Big(\frac{3}{16}\frac{2z^4+4z^3+4z^2+2z+3}{(z+1)s^2z^2}$$
$$-\frac{3}{16}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)+\frac{3}{16}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)\gamma_E^2+\Big(\frac{1}{8}\frac{26z^4+35z^3+191z^2+42z+23}{(z+1)s^2z^2}-\frac{1}{8}\frac{4z^4-z^3+21z^2-10z+4}{(z+1)s^2z^2}G_{-1}(z)$$
$$-\frac{1}{8}\frac{4z^4+11z^3+24z^2+29z+16}{(z+1)s^2z^2}G_0(z)\Big)\zeta_2-\frac{3}{4}\frac{2z^4+3z^3+7z^2+2z+2}{(z+1)s^2z^2}\gamma_E\zeta_2-\frac{3}{2}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}\zeta_3$$
$$-\frac{1}{48}\frac{58z^4+188z^3+176z^2+82z+75}{(z+1)s^2z^2}G_{-1}(z)-\frac{1}{48}\frac{58z^4+44z^3-40z^2-62z+39}{(z+1)s^2z^2}G_0(z)+\frac{1}{24}\frac{6z^4+29z^3-41z^2+76z+9}{(z+1)s^2z^2}G_{-1,-1}(z)$$
$$-\frac{1}{48}\frac{13z^2+13z+68}{(z+1)s^2z^2}G_{-1,0}(z)-\frac{1}{48}\frac{13z^2+13z+68}{(z+1)s^2z^2}G_{0,-1}(z)+\frac{1}{24}\frac{6z^4-5z^3-92z^2-221z-131}{(z+1)s^2z^2}G_{0,0}(z)-\frac{3}{8}\frac{z^2-z+2}{(z+1)s^2z}G_{-1,-1,-1}(z)$$
$$+\frac{1}{8}\frac{z^2+3z+4}{s^2z^2}G_{-1,-1,0}(z)+\frac{1}{8}\frac{z^2+3z+4}{s^2z^2}G_{-1,0,-1}(z)-\frac{1}{8}\frac{z^2-z+2}{(z+1)s^2z}G_{-1,0,0}(z)+\frac{1}{8}\frac{z^2+3z+4}{s^2z^2}G_{0,-1,-1}(z)$$
$$-\frac{1}{8}\frac{z^2-z+2}{(z+1)s^2z}G_{0,-1,0}(z)-\frac{1}{8}\frac{z^2-z+2}{(z+1)s^2z}G_{0,0,-1}(z)+\frac{3}{8}\frac{z^2+3z+4}{s^2z^2}G_{0,0,0}(z)+\Big(-\frac{3}{8}\frac{2z^4+4z^3+4z^2+2z+3}{(z+1)s^2z^2}+\frac{3}{8}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)$$
$$-\frac{3}{8}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)\gamma_E\log(4\pi)+\frac{3}{4}\frac{2z^4+3z^3+7z^2+2z+2}{(z+1)s^2z^2}\zeta_2\log(4\pi)+\Big(\frac{5z^4+10z^3+13z^2+8z+6}{(z+1)s^2z^2}$$
$$-\frac{1}{8}\frac{2z^4+12z^3-4z^2+18z+3}{(z+1)s^2z^2}G_{-1}(z)-\frac{1}{8}\frac{2z^4-4z^3-28z^2-54z-29}{(z+1)s^2z^2}G_0(z)+\frac{1}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1,-1}(z)-\frac{1}{8}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_{-1,0}(z)$$
$$-\frac{1}{8}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_{0,-1}(z)-\frac{1}{4}\frac{z^2+3z+4}{s^2z^2}G_{0,0}(z)\Big)\log(4\pi)+\Big(\frac{3}{16}\frac{2z^4+4z^3+4z^2+2z+3}{(z+1)s^2z^2}-\frac{3}{16}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)$$
$$+\frac{3}{16}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)\log^2(4\pi)\Big)i\frac{1}{\pi^2}+\Big(\Big(\frac{3}{8}\frac{2z^4+4z^3+4z^2+2z+3}{(z+1)s^2z^2}-\frac{3}{8}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)+\frac{3}{8}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)i\frac{1}{\pi^2}$$
$$+\frac{3}{8}\frac{z^2-z+2}{(z+1)s^2z}\frac{1}{\pi}\Big)\frac{1}{\epsilon^2}+\Big(\Big(\frac{5z^4+10z^3+13z^2+8z+6}{(z+1)s^2z^2}+\Big(-\frac{3}{8}\frac{2z^4+4z^3+4z^2+2z+3}{(z+1)s^2z^2}+\frac{3}{8}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)$$
$$-\frac{3}{8}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)\gamma_E+\frac{3}{4}\frac{2z^4+3z^3+7z^2+2z+2}{(z+1)s^2z^2}\zeta_2-\frac{1}{8}\frac{2z^4+12z^3-4z^2+18z+3}{(z+1)s^2z^2}G_{-1}(z)-\frac{1}{8}\frac{2z^4-4z^3-28z^2-54z-29}{(z+1)s^2z^2}G_0(z)$$
$$+\frac{1}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1,-1}(z)-\frac{1}{8}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_{-1,0}(z)-\frac{1}{8}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_{0,-1}(z)-\frac{1}{4}\frac{z^2+3z+4}{s^2z^2}G_{0,0}(z)+\Big(\frac{3}{8}\frac{2z^4+4z^3+4z^2+2z+3}{(z+1)s^2z^2}$$
$$-\frac{3}{8}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)+\frac{3}{8}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)\log(4\pi)\Big)i\frac{1}{\pi^2}+\Big(\frac{1}{8}\frac{2z^4+12z^3-4z^2+18z+3}{(z+1)s^2z^2}-\frac{3}{8}\frac{z^2-z+2}{(z+1)s^2z}\gamma_E-\frac{1}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)$$
$$+\frac{1}{8}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_0(z)+\frac{3}{8}\frac{z^2-z+2}{(z+1)s^2z}\log(4\pi)\Big)\frac{1}{\pi}\Big)\frac{1}{\epsilon}+\Big(\frac{1}{48}\frac{58z^4+188z^3+176z^2+82z+75}{(z+1)s^2z^2}+\Big(-\frac{1}{8}\frac{2z^4+12z^3-4z^2+18z+3}{(z+1)s^2z^2}$$
$$+\frac{1}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)-\frac{1}{8}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_0(z)\Big)\gamma_E+\frac{3}{16}\frac{z^2-z+2}{(z+1)s^2z}\gamma_E^2-\frac{1}{24}\frac{6z^4+29z^3-41z^2+76z+9}{(z+1)s^2z^2}G_{-1}(z)$$
$$+\frac{1}{48}\frac{13z^2+13z+68}{(z+1)s^2z^2}G_0(z)+\frac{3}{8}\frac{z^2-z+2}{(z+1)s^2z}G_{-1,-1}(z)-\frac{1}{8}\frac{z^2+3z+4}{s^2z^2}G_{-1,0}(z)-\frac{1}{8}\frac{z^2+3z+4}{s^2z^2}G_{0,-1}(z)+\frac{1}{8}\frac{z^2-z+2}{(z+1)s^2z}G_{0,0}(z)$$
$$-\frac{3}{8}\frac{z^2-z+2}{(z+1)s^2z}\gamma_E\log(4\pi)+\Big(\frac{1}{8}\frac{2z^4+12z^3-4z^2+18z+3}{(z+1)s^2z^2}-\frac{1}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)+\frac{1}{8}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_0(z)\Big)\log(4\pi)$$
$$+\frac{3}{16}\frac{z^2-z+2}{(z+1)s^2z}\log^2(4\pi)\Big)\frac{1}{\pi}+\frac{1}{48}\frac{4z^4+5z^3+15z^2+2z+4}{(z+1)s^2z^2}\pi\Big)\log(-\frac{-m_t^2}{s})+\Big(-\frac{1}{\epsilon^2}\frac{3}{4}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}i\frac{1}{\pi^2}$$
$$+\Big(-\frac{1}{4}\frac{13z^4+26z^3+41z^2+28z+12}{(z+1)s^2z^2}+\Big(\frac{1}{16}\frac{38z^4+76z^3+108z^2+70z+41}{(z+1)s^2z^2}-\frac{1}{16}\frac{4z^4+11z^3+9z^2+14z+4}{(z+1)s^2z^2}G_{-1}(z)$$

$$- \frac{1}{16}\frac{4z^4+5z^3-13z-8}{(z+1)s^2z^2}G_0(z)\Big)\gamma_E - \frac{3}{8}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}\gamma_E^2 + \frac{3}{8}\frac{z^2-z+2}{(z+1)s^2z}\zeta_2 + \frac{1}{48}\frac{40z^4+68z^3+126z^2+50z+43}{(z+1)s^2z^2}G_{-1}(z)$$

$$+ \frac{1}{48}\frac{40z^4+92z^3+162z^2+158z+91}{(z+1)s^2z^2}G_0(z) - \frac{1}{8}\frac{2z^4+5z^3+5z^2+6z+2}{(z+1)s^2z^2}G_{-1,-1}(z) + \frac{1}{16}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_{-1,0}(z)$$

$$+ \frac{1}{16}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_{0,-1}(z) - \frac{1}{8}\frac{2z^4+3z^3+2z^2-3z-2}{(z+1)s^2z^2}G_{0,0}(z) + \frac{3}{4}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}\gamma_E\log(4\pi) + \Big(-\frac{1}{16}\frac{38z^4+76z^3+108z^2+70z+41}{(z+1)s^2z^2}$$

$$+ \frac{1}{16}\frac{4z^4+11z^3+9z^2+14z+4}{(z+1)s^2z^2}G_{-1}(z) + \frac{1}{16}\frac{4z^4+5z^3-13z-8}{(z+1)s^2z^2}G_0(z)\Big)\log(4\pi) - \frac{3}{8}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}\log^2(4\pi)\Big)i\frac{1}{\pi^2}$$

$$+ \Big(\Big(\Big(-\frac{1}{16}\frac{38z^4+76z^3+108z^2+70z+41}{(z+1)s^2z^2} + \frac{3}{4}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}\gamma_E + \frac{1}{16}\frac{4z^4+11z^3+9z^2+14z+4}{(z+1)s^2z^2}G_{-1}(z) + \frac{1}{16}\frac{4z^4+5z^3-13z-8}{(z+1)s^2z^2}G_0(z)$$

$$- \frac{3}{4}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}\log(4\pi)\Big)i\frac{1}{\pi^2} - \frac{1}{16}\frac{4z^4+11z^3+9z^2+14z+4}{(z+1)s^2z^2}\frac{1}{\pi}\Big)\frac{1}{\epsilon} + \Big(-\frac{1}{48}\frac{40z^4+68z^3+126z^2+50z+43}{(z+1)s^2z^2}$$

$$+ \frac{1}{16}\frac{4z^4+11z^3+9z^2+14z+4}{(z+1)s^2z^2}\gamma_E + \frac{1}{8}\frac{2z^4+5z^3+5z^2+6z+2}{(z+1)s^2z^2}G_{-1}(z) - \frac{1}{16}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_0(z)$$

$$- \frac{1}{16}\frac{4z^4+11z^3+9z^2+14z+4}{(z+1)s^2z^2}\log(4\pi)\Big)\frac{1}{\pi}\Big)\log^2(-\frac{m_t^2}{s}) + \Big(\frac{1}{\epsilon}\frac{1}{2}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}i\frac{1}{\pi^2} + \Big(-\frac{1}{48}\frac{2z^4+4z^3+12z^2+10z-1}{(z+1)s^2z^2}$$

$$- \frac{1}{2}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}\gamma_E - \frac{1}{48}\frac{8z^4+19z^3+21z^2+22z+8}{(z+1)s^2z^2}G_{-1}(z) - \frac{1}{48}\frac{8z^4+13z^3+12z^2-5z-4}{(z+1)s^2z^2}G_0(z) + \frac{1}{2}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}\log(4\pi)\Big)i\frac{1}{\pi^2}$$

$$+ \frac{1}{48}\frac{8z^4+19z^3+21z^2+22z+8}{(z+1)s^2z^2}\frac{1}{\pi}\Big)\log^3(-\frac{m_t^2}{s}) - \frac{3}{16}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}i\frac{1}{\pi^2}\log^4(-\frac{m_t^2}{s}) + \Big(\Big(\Big(\frac{3}{4}\frac{9z^4+18z^3+22z^2+13z+12}{(z+1)s^2z^2}$$

$$+ \Big(-3\frac{z^4+2z^3+2z^2+z+1}{(z+1)s^2z^2} - \frac{3}{4}\frac{z^2+2z-2}{(z+1)s^2z}G_{-1}(z) + \frac{3}{4}\frac{z^2-3}{s^2z^2}G_0(z) - \frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z) + \frac{3}{2}\frac{1}{s^2z}G_{-1,0}(z) + \frac{3}{2}\frac{1}{s^2z}G_{0,-1}(z)$$

$$- \frac{3}{2}\frac{z+2}{s^2z^2}G_{0,0}(z)\Big)\gamma_E + \Big(\frac{3}{16}\frac{8z^4+24z^3-49z^2-41z+12}{(z+1)s^2z^2} - 3\frac{z^2-z+5}{(z+1)s^2z}G_{-1}(z) + \frac{3}{4}\frac{4z^3+11z^2+7z+4}{(z+1)s^2z^2}G_0(z)\Big)\zeta_2 + \frac{9}{4}\frac{5z+1}{(z+1)s^2z}\gamma_E\zeta_2$$

$$- \frac{3}{4}\frac{12z^4+25z^3+38z^2+25z+12}{(z+1)s^2z^2}\zeta_3 - \frac{1}{16}\frac{16z^4+47z^3+2z^2+82z+16}{(z+1)s^2z^2}G_{-1}(z) - \frac{1}{16}\frac{16z^4+17z^3-43z^2-155z-95}{(z+1)s^2z^2}G_0(z)$$

$$- \frac{1}{8}\frac{4z^2-7z+1}{(z+1)s^2z}G_{-1,-1}(z) - \frac{1}{8}\frac{11z^2+11z+6}{(z+1)s^2z^2}G_{-1,0}(z) - \frac{1}{8}\frac{11z^2+11z+6}{(z+1)s^2z^2}G_{0,-1}(z) + \frac{1}{8}\frac{4z^2+15z+12}{s^2z^2}G_{0,0}(z)$$

$$- \frac{3}{4}\frac{z^2+5z-4}{(z+1)s^2z}G_{-1,-1,-1}(z) + \frac{2z+3}{(z+1)s^2z}G_{-1,-1,0}(z) + \frac{2z+3}{(z+1)s^2z}G_{-1,0,-1}(z) - \frac{1}{4}\frac{(z+1)(3z+4)}{s^2z^2}G_{-1,0,0}(z)$$

$$+ \frac{1}{4}\frac{3z-1}{(z+1)s^2}G_{0,-1,-1}(z) + \frac{2z-1}{s^2z^2}G_{0,-1,0}(z) + \frac{2z-1}{s^2z^2}G_{0,0,-1}(z) + \frac{3}{4}\frac{z^2-3z-8}{s^2z^2}G_{0,0,0}(z) - \frac{9}{4}\frac{5z+1}{(z+1)s^2z}\zeta_2\log(4\pi)$$

$$+ \Big(3\frac{z^4+2z^3+2z^2+z+1}{(z+1)s^2z^2} + \frac{3}{4}\frac{z^2+2z-2}{(z+1)s^2z}G_{-1}(z) - \frac{3}{4}\frac{z^2-3}{s^2z^2}G_0(z) + \frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z) - \frac{3}{2}\frac{1}{s^2z}G_{-1,0}(z) - \frac{3}{2}\frac{1}{s^2z}G_{0,-1}(z)$$

$$+ \frac{3}{2}\frac{z+2}{s^2z^2}G_{0,0}(z)\Big)\log(4\pi)\Big)i\frac{1}{\pi^2} + \Big(\Big(3\frac{z^4+2z^3+2z^2+z+1}{(z+1)s^2z^2} - \frac{9}{4}\frac{5z+1}{(z+1)s^2z}\zeta_2 + \frac{3}{4}\frac{z^2+2z-2}{(z+1)s^2z}G_{-1}(z) - \frac{3}{4}\frac{z^2-3}{s^2z^2}G_0(z)$$

$$+ \frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z) - \frac{3}{2}\frac{1}{s^2z}G_{-1,0}(z) - \frac{3}{2}\frac{1}{s^2z}G_{0,-1}(z) + \frac{3}{2}\frac{z+2}{s^2z^2}G_{0,0}(z)\Big)i\frac{1}{\pi^2} + \Big(-\frac{3}{4}\frac{z^2+2z-2}{(z+1)s^2z} - \frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1}(z)$$

$$+ \frac{3}{2}\frac{1}{s^2z}G_0(z)\Big)\frac{1}{\pi}\Big)\frac{1}{\epsilon} + \Big(\frac{1}{16}\frac{16z^4+47z^3+2z^2+82z+16}{(z+1)s^2z^2} + \Big(\frac{3}{4}\frac{z^2+2z-2}{(z+1)s^2z} + \frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1}(z) - \frac{3}{2}\frac{1}{s^2z}G_0(z)\Big)\gamma_E$$

$$+ \frac{1}{8}\frac{4z^2-7z+1}{(z+1)s^2z}G_{-1}(z) + \frac{1}{8}\frac{11z^2+11z+6}{(z+1)s^2z^2}G_0(z) + \frac{3}{4}\frac{z^2+5z-4}{(z+1)s^2z}G_{-1,-1}(z) - \frac{2z+3}{(z+1)s^2z}G_{-1,0}(z) - \frac{1}{4}\frac{3z-1}{(z+1)s^2}G_{0,-1}(z)$$

$$- \frac{2z-1}{s^2z^2}G_{0,0}(z) + \Big(-\frac{3}{4}\frac{z^2+2z-2}{(z+1)s^2z} - \frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1}(z) + \frac{3}{2}\frac{1}{s^2z}G_0(z)\Big)\log(4\pi)\Big)\frac{1}{\pi} + \frac{1}{8}\frac{2z^2-9z+1}{(z+1)s^2z}\pi\Big)\log(-\frac{s}{\mu_R^2})$$

$$+ \Big(\Big(\Big(-\frac{3}{16}\frac{46z^4+92z^3+124z^2+78z+53}{(z+1)s^2z^2} + \Big(\frac{3}{4}\frac{2z^4+4z^3+4z^2+2z+3}{(z+1)s^2z^2} - \frac{3}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z) + \frac{3}{4}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)\gamma_E$$

$$- \frac{3}{2}\frac{2z^4+3z^3+7z^2+2z+2}{(z+1)s^2z^2}\zeta_2 + \frac{1}{16}\frac{8z^4+37z^3-5z^2+50z+12}{(z+1)s^2z^2}G_{-1}(z) + \frac{1}{16}\frac{8z^4-5z^3-68z^2-139z-72}{(z+1)s^2z^2}G_0(z)$$

$$- \frac{1}{2}\frac{z^2-z+2}{(z+1)s^2z}G_{-1,-1}(z) + \frac{1}{4}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_{-1,0}(z) + \frac{1}{4}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_{0,-1}(z) + \frac{1}{2}\frac{z^2+3z+4}{s^2z^2}G_{0,0}(z) + \Big(-\frac{3}{4}\frac{2z^4+4z^3+4z^2+2z+3}{(z+1)s^2z^2}$$

$$+ \frac{3}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z) - \frac{3}{4}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)\log(4\pi)\Big)i\frac{1}{\pi^2} + \Big(\Big(-\frac{3}{4}\frac{2z^4+4z^3+4z^2+2z+3}{(z+1)s^2z^2} + \frac{3}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)$$

$$- \frac{3}{4}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)i\frac{1}{\pi^2} - \frac{3}{4}\frac{z^2-z+2}{(z+1)s^2z}\frac{1}{\pi}\Big)\frac{1}{\epsilon} + \Big(-\frac{1}{16}\frac{8z^4+37z^3-5z^2+50z+12}{(z+1)s^2z^2} + \frac{3}{4}\frac{z^2-z+2}{(z+1)s^2z}\gamma_E + \frac{1}{2}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)$$

$$- \frac{1}{4}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_0(z) - \frac{3}{4}\frac{z^2-z+2}{(z+1)s^2z}\log(4\pi)\Big)\frac{1}{\pi}\Big)\log(-\frac{m_t^2}{s})\log(-\frac{s}{\mu_R^2}) + \Big(\frac{1}{\epsilon}\frac{3}{2}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}i\frac{1}{\pi^2} + \Big(\frac{3}{8}\frac{9z^4+18z^3+25z^2+16z+10}{(z+1)s^2z^2}$$

$$- \frac{3}{2}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}\gamma_E - \frac{1}{8}\frac{4z^4+11z^3+9z^2+14z+4}{(z+1)s^2z^2}G_{-1}(z) - \frac{1}{8}\frac{4z^4+5z^3-13z-8}{(z+1)s^2z^2}G_0(z) + \frac{3}{2}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}\log(4\pi)\Big)i\frac{1}{\pi^2}$$

$$+ \frac{1}{8}\frac{4z^4+11z^3+9z^2+14z+4}{(z+1)s^2z^2}\frac{1}{\pi}\Big)\log^2(-\frac{m_t^2}{s})\log(-\frac{s}{\mu_R^2}) - \frac{(z^2+z+1)^2}{(z+1)s^2z^2}i\frac{1}{\pi^2}\log^3(-\frac{m_t^2}{s})\log(-\frac{s}{\mu_R^2}) + \Big(\Big(-3\frac{z^4+2z^3+2z^2+z+1}{(z+1)s^2z^2}$$

$$+\frac{9}{4}\frac{5z+1}{(z+1)s^2z}\zeta_2-\frac{3}{4}\frac{z^2+2z-2}{(z+1)s^2z}G_{-1}(z)+\frac{3}{4}\frac{z^2-3}{s^2z^2}G_0(z)-\frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z)+\frac{3}{2}\frac{1}{s^2z}G_{-1,0}(z)+\frac{3}{2}\frac{1}{s^2z}G_{0,-1}(z)$$

$$-\frac{3}{2}\frac{z+2}{s^2z^2}G_{0,0}(z)\Big)i\frac{1}{\pi^2}+\Big(\frac{3}{4}\frac{z^2+2z-2}{(z+1)s^2z}+\frac{3}{2}\frac{z-1}{(z+1)s^2z}G_{-1}(z)-\frac{3}{2}\frac{1}{s^2z}G_0(z)\Big)\frac{1}{\pi}\Big)\log^2(-\frac{s}{\mu_R^2})+\Big(\Big(\frac{3}{4}\frac{2z^4+4z^3+4z^2+2z+3}{(z+1)s^2z^2}$$

$$-\frac{3}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)+\frac{3}{4}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)i\frac{1}{\pi^2}+\frac{3}{4}\frac{z^2-z+2}{(z+1)s^2z}\frac{1}{\pi}\Big)\log(-\frac{m_t^2}{s})\log^2(-\frac{s}{\mu_R^2})$$

$$-\frac{3}{2}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}i\frac{1}{\pi^2}\log^2(-\frac{m_t^2}{s})\log^2(-\frac{s}{\mu_R^2})\Big)m_t^2N_c+\Big(\Big(\frac{1}{2}\frac{2z^4+4z^3+6z^2+4z+3}{(z+1)s^2z^2}+\Big(-\frac{z^4+2z^3+2z^2+z+1}{(z+1)s^2z^2}-\frac{1}{4}\frac{z^2+2z-2}{(z+1)s^2z}G_{-1}(z)$$

$$+\frac{1}{4}\frac{z^2-3}{s^2z^2}G_0(z)-\frac{1}{2}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z)+\frac{1}{2}\frac{1}{s^2z}G_{-1,0}(z)+\frac{1}{2}\frac{1}{s^2z}G_{0,-1}(z)-\frac{1}{2}\frac{z+2}{s^2z^2}G_{0,0}(z)\Big)\gamma_E+\Big(\frac{1}{4}\frac{(2z-1)(z^3+3z^2-3)}{(z+1)s^2z^2}$$

$$-\frac{1}{8}\frac{8z^2+z+37}{(z+1)s^2z}G_{-1}(z)+\frac{1}{8}\frac{8z^3+21z^2+17z+8}{(z+1)s^2z^2}G_0(z)\Big)\zeta_2+\frac{3}{4}\frac{5z+1}{(z+1)s^2z}\gamma_E\zeta_2-\frac{1}{4}\frac{12z^4+25z^3+38z^2+25z+12}{(z+1)s^2z^2}\zeta_3$$

$$-\frac{1}{12}\frac{2z^4+7z^3+4z^2+8z+2}{(z+1)s^2z^2}G_{-1}(z)-\frac{1}{12}\frac{2z^4+z^3-5z^2-13z-7}{(z+1)s^2z^2}G_0(z)-\frac{1}{12}\frac{z-2}{s^2z}G_{-1,-1}(z)-\frac{1}{24}\frac{z^2+z+3}{(z+1)s^2z^2}G_{-1,0}(z)$$

$$-\frac{1}{24}\frac{z^2+z+3}{(z+1)s^2z^2}G_{0,-1}(z)+\frac{1}{12}\frac{z+3}{s^2z}G_{0,0}(z)-\frac{1}{4}\frac{z^2+4z-3}{(z+1)s^2z}G_{-1,-1,-1}(z)+\frac{1}{12}\frac{7z+9}{(z+1)s^2z}G_{-1,-1,0}(z)+\frac{1}{12}\frac{7z+9}{(z+1)s^2z}G_{-1,0,-1}(z)$$

$$-\frac{1}{12}\frac{3z^2+8z+2}{s^2z^2}G_{-1,0,0}(z)+\frac{1}{12}\frac{3z^2-2z-3}{(z+1)s^2z}G_{0,-1,-1}(z)+\frac{1}{12}\frac{7z-2}{s^2z^2}G_{0,-1,0}(z)+\frac{1}{12}\frac{7z-2}{s^2z^2}G_{0,0,-1}(z)+\frac{1}{4}\frac{z^2-2z-6}{s^2z^2}G_{0,0,0}(z)$$

$$-\frac{3}{4}\frac{5z+1}{(z+1)s^2z}\zeta_2\log(4\pi)+\Big(\frac{z^4+2z^3+2z^2+z+1}{(z+1)s^2z^2}+\frac{1}{4}\frac{z^2+2z-2}{(z+1)s^2z}G_{-1}(z)-\frac{1}{4}\frac{z^2-3}{s^2z^2}G_0(z)+\frac{1}{2}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z)$$

$$-\frac{1}{2}\frac{1}{s^2z}G_{-1,0}(z)-\frac{1}{2}\frac{1}{s^2z}G_{0,-1}(z)+\frac{1}{2}\frac{z+2}{s^2z^2}G_{0,0}(z)\Big)\log(4\pi)\Big)i\frac{1}{\pi^2}+\Big(\Big(\Big(\frac{z^4+2z^3+2z^2+z+1}{(z+1)s^2z^2}-\frac{3}{4}\frac{5z+1}{(z+1)s^2z}\zeta_2$$

$$+\frac{1}{4}\frac{z^2+2z-2}{(z+1)s^2z}G_{-1}(z)-\frac{1}{4}\frac{z^2-3}{s^2z^2}G_0(z)+\frac{1}{2}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z)-\frac{1}{2}\frac{1}{s^2z}G_{-1,0}(z)-\frac{1}{2}\frac{1}{s^2z}G_{0,-1}(z)+\frac{1}{2}\frac{z+2}{s^2z^2}G_{0,0}(z)\Big)i\frac{1}{\pi^2}$$

$$+\Big(-\frac{1}{4}\frac{z^2+2z-2}{(z+1)s^2z}-\frac{1}{2}\frac{z-1}{(z+1)s^2z}G_{-1}(z)+\frac{1}{2}\frac{1}{s^2z}G_0(z)\Big)\frac{1}{\pi}\Big)\frac{1}{\epsilon}+\Big(\frac{1}{12}\frac{2z^4+7z^3+4z^2+8z+2}{(z+1)s^2z^2}+\Big(\frac{1}{4}\frac{z^2+2z-2}{(z+1)s^2z}$$

$$+\frac{1}{2}\frac{z-1}{(z+1)s^2z}G_{-1}(z)-\frac{1}{2}\frac{1}{s^2z}G_0(z)\Big)\gamma_E+\frac{1}{12}\frac{z-2}{s^2z}G_{-1}(z)+\frac{1}{24}\frac{z^2+z+3}{(z+1)s^2z^2}G_0(z)+\frac{1}{4}\frac{z^2+4z-3}{(z+1)s^2z}G_{-1,-1}(z)-\frac{1}{12}\frac{7z+9}{(z+1)s^2z}G_{-1,0}(z)$$

$$-\frac{1}{12}\frac{3z^2-2z-3}{(z+1)s^2z}G_{0,-1}(z)-\frac{1}{12}\frac{7z-2}{s^2z^2}G_{0,0}(z)+\Big(-\frac{1}{4}\frac{z^2+2z-2}{(z+1)s^2z}-\frac{1}{2}\frac{z-1}{(z+1)s^2z}G_{-1}(z)+\frac{1}{2}\frac{1}{s^2z}G_0(z)\Big)\log(4\pi)\Big)\frac{1}{\pi}$$

$$+\frac{1}{48}\frac{(z-3)(4z-1)}{(z+1)s^2z}\pi+\Big(\Big(\Big(-\frac{1}{4}\frac{6z^4+12z^3+16z^2+10z+7}{(z+1)s^2z^2}+\Big(\frac{1}{4}\frac{2z^4+4z^3+4z^2+2z+3}{(z+1)s^2z^2}-\frac{1}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)$$

$$+\frac{1}{4}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)\gamma_E-\frac{1}{4}\frac{4z^4+7z^3+13z^2+6z+4}{(z+1)s^2z^2}\zeta_2+\frac{1}{24}\frac{2z^4+10z^3-2z^2+14z+3}{(z+1)s^2z^2}G_{-1}(z)$$

$$+\frac{1}{24}\frac{2z^4-2z^3-20z^2-40z-21}{(z+1)s^2z^2}G_0(z)-\frac{1}{12}\frac{z^2-z+2}{(z+1)s^2z}G_{-1,-1}(z)+\frac{1}{24}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_{-1,0}(z)+\frac{1}{24}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_{0,-1}(z)$$

$$+\frac{1}{12}\frac{z^2+3z+4}{s^2z^2}G_{0,0}(z)+\Big(-\frac{1}{4}\frac{2z^4+4z^3+4z^2+2z+3}{(z+1)s^2z^2}+\frac{1}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)-\frac{1}{4}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)\log(4\pi)\Big)i\frac{1}{\pi^2}$$

$$+\Big(\Big(-\frac{1}{4}\frac{2z^4+4z^3+4z^2+2z+3}{(z+1)s^2z^2}+\frac{1}{4}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)-\frac{1}{4}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)i\frac{1}{\pi^2}-\frac{1}{4}\frac{z^2-z+2}{(z+1)s^2z}\frac{1}{\pi}\Big)\frac{1}{\epsilon}$$

$$+\Big(-\frac{1}{24}\frac{2z^4+10z^3-2z^2+14z+3}{(z+1)s^2z^2}+\frac{1}{4}\frac{z^2-z+2}{(z+1)s^2z}\gamma_E+\frac{1}{12}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)-\frac{1}{24}\frac{5z^2+5z+4}{(z+1)s^2z^2}G_0(z)$$

$$-\frac{1}{4}\frac{z^2-z+2}{(z+1)s^2z}\log(4\pi)\Big)\frac{1}{\pi}\Big)\log(-\frac{m_t^2}{s})+\Big(\frac{1}{\epsilon}\frac{1}{2}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}i\frac{1}{\pi^2}+\Big(\frac{1}{8}\frac{6z^4+12z^3+16z^2+10z+7}{(z+1)s^2z^2}-\frac{1}{2}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}\gamma_E$$

$$-\frac{1}{24}\frac{2z^4+7z^3+3z^2+10z+2}{(z+1)s^2z^2}G_{-1}(z)-\frac{1}{24}\frac{2z^4+z^3-6z^2-17z-10}{(z+1)s^2z^2}G_0(z)+\frac{1}{2}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}\log(4\pi)\Big)i\frac{1}{\pi^2}$$

$$+\frac{1}{24}\frac{2z^4+7z^3+3z^2+10z+2}{(z+1)s^2z^2}\frac{1}{\pi}\Big)\log^2(-\frac{m_t^2}{s})-\frac{1}{3}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}i\frac{1}{\pi^2}\log^3(-\frac{m_t^2}{s})+\Big(\Big(\Big(-\frac{3}{2}\frac{z^4+2z^3+2z^2+z+1}{(z+1)s^2z^2}+\frac{9}{8}\frac{5z+1}{(z+1)s^2z}\zeta_2$$

$$-\frac{3}{8}\frac{z^2+2z-2}{(z+1)s^2z}G_{-1}(z)+\frac{3}{8}\frac{z^2-3}{s^2z^2}G_0(z)-\frac{3}{4}\frac{z-1}{(z+1)s^2z}G_{-1,-1}(z)+\frac{3}{4}\frac{1}{s^2z}G_{-1,0}(z)+\frac{3}{4}\frac{1}{s^2z}G_{0,-1}(z)-\frac{3}{4}\frac{z+2}{s^2z^2}G_{0,0}(z)\Big)i\frac{1}{\pi^2}$$

$$+\Big(\frac{3}{8}\frac{z^2+2z-2}{(z+1)s^2z}+\frac{3}{4}\frac{z-1}{(z+1)s^2z}G_{-1}(z)-\frac{3}{4}\frac{1}{s^2z}G_0(z)\Big)\frac{1}{\pi}\Big)\log(-\frac{s}{\mu_R^2})+\Big(\Big(\frac{3}{8}\frac{2z^4+4z^3+4z^2+2z+3}{(z+1)s^2z^2}-\frac{3}{8}\frac{z^2-z+2}{(z+1)s^2z}G_{-1}(z)$$

$$+\frac{3}{8}\frac{z^2+3z+4}{s^2z^2}G_0(z)\Big)i\frac{1}{\pi^2}+\frac{3}{8}\frac{z^2-z+2}{(z+1)s^2z}\frac{1}{\pi}\Big)\log(-\frac{m_t^2}{s})\log(-\frac{s}{\mu_R^2})$$

$$-\frac{3}{4}\frac{(z^2+z+1)^2}{(z+1)s^2z^2}i\frac{1}{\pi^2}\log^2(-\frac{m_t^2}{s})\log(-\frac{s}{\mu_R^2})\Big)m_t^2N_f+O((m_H^2)^0)+O((m_t^2)^2) \tag{E.1}$$

$$F_{1,q}^{\mathrm{IR}}=F_{2,q}^{\mathrm{IR}}=$$

$$\Big(-\frac{1}{\epsilon^2}\frac{3}{4}\frac{1}{s^2}\frac{1}{\pi^2}+\frac{1}{\epsilon}\Big(\frac{1}{8}\frac{1+6\gamma_E-6\log(4\pi)}{s^2}+\frac{1}{4}\frac{1}{s^2}\zeta_2+\frac{3}{8}\frac{1}{s^2}\zeta_3\Big)\frac{1}{\pi^2}+\Big(-\frac{1}{8}\frac{-23+\gamma_E+3\gamma_E^2-\log(4\pi)-6\gamma_E\log(4\pi)+3\log^2(4\pi)}{s^2}$$

$$- \frac{1}{4}\frac{\gamma_E - \log(4\pi)}{s^2}\zeta_2 - \frac{1}{16}\frac{1+6\gamma_E - 6\log(4\pi)}{s^2}\zeta_3 + \frac{5}{32}\frac{1}{s^2}\zeta_4\Big)\frac{1}{\pi^2} + \Big(-\frac{1}{\epsilon^2}\frac{1}{2}\frac{1}{s^3}\frac{1}{\pi^2} + \frac{1}{\epsilon}\Big(\frac{1}{2}\frac{2+\gamma_E-\log(4\pi)}{s^3} - \frac{1}{4}\frac{1}{s^3}\zeta_2 + \frac{3}{2}\frac{1}{s^3}\zeta_3\Big)\frac{1}{\pi^2}$$

$$+\Big(-\frac{1}{4}\frac{-6+4\gamma_E+\gamma_E^2-4\log(4\pi)-2\gamma_E\log(4\pi)+\log^2(4\pi)}{s^3} + \frac{1}{8}\frac{-5+2\gamma_E-2\log(4\pi)}{s^3}\zeta_2 - \frac{1}{4}\frac{23+6\gamma_E-6\log(4\pi)}{s^3}\zeta_3 + \frac{5}{8}\frac{1}{s^3}\zeta_4\Big)\frac{1}{\pi^2}$$

$$+\Big(\frac{1}{\epsilon^2}\frac{1}{4}\frac{1}{s^3}\frac{1}{\pi^2} + \frac{1}{\epsilon}\Big(-\frac{1}{8}\frac{-9+2\gamma_E-2\log(4\pi)}{s^3} + \frac{1}{2}\frac{1}{s^3}\zeta_2\Big)\frac{1}{\pi^2} + \Big(\frac{1}{8}\frac{-17-9\gamma_E+\gamma_E^2+9\log(4\pi)-2\gamma_E\log(4\pi)+\log^2(4\pi)}{s^3}$$

$$-\frac{1}{4}\frac{7+2\gamma_E-2\log(4\pi)}{s^3}\zeta_2 - \frac{1}{2}\frac{1}{s^3}\zeta_3\Big)\frac{1}{\pi^2}\Big)\log(-\frac{m_t^2}{s}) + \Big(-\frac{1}{\epsilon^2}\frac{1}{4}\frac{1}{s^3}\frac{1}{\pi^2} + \frac{1}{\epsilon}\frac{1}{4}\frac{3+\gamma_E-\log(4\pi)}{s^3}\frac{1}{\pi^2}$$

$$+\Big(-\frac{1}{16}\frac{-25+12\gamma_E+2\gamma_E^2-12\log(4\pi)-4\gamma_E\log(4\pi)+2\log^2(4\pi)}{s^3} - \frac{1}{4}\frac{1}{s^3}\zeta_2\Big)\frac{1}{\pi^2}\Big)\log^2(-\frac{m_t^2}{s}) + \Big(-\frac{1}{24}\frac{49+4\gamma_E-4\log(4\pi)}{s^3}\frac{1}{\pi^2}$$

$$+\frac{1}{\epsilon}\frac{1}{6}\frac{1}{s^3}\frac{1}{\pi^2}\Big)\log^3(-\frac{m_t^2}{s}) - \frac{1}{16}\frac{1}{s^3}\frac{1}{\pi^2}\log^4(-\frac{m_t^2}{s}) + \Big(-\frac{1}{\epsilon}\frac{1}{s^3}\frac{1}{\pi^2} + \Big(\frac{2+\gamma_E-\log(4\pi)}{s^3} - \frac{1}{2}\frac{1}{s^3}\zeta_2 + 3\frac{1}{s^3}\zeta_3\Big)\frac{1}{\pi^2}\Big)\log(-\frac{\mu_R^2}{s}) + \Big(\frac{1}{\epsilon}\frac{1}{2}\frac{1}{s^3}\frac{1}{\pi^2}$$

$$+\Big(-\frac{1}{4}\frac{-9+2\gamma_E-2\log(4\pi)}{s^3} + \frac{1}{s^3}\zeta_2\Big)\frac{1}{\pi^2}\Big)\log(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s}) + \Big(-\frac{1}{\epsilon}\frac{1}{2}\frac{1}{s^3}\frac{1}{\pi^2} + \frac{1}{2}\frac{3+\gamma_E-\log(4\pi)}{s^3}\frac{1}{\pi^2}\Big)\log^2(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s})$$

$$+\frac{1}{3}\frac{1}{s^3}\frac{1}{\pi^2}\log^3(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s}) - \frac{1}{s^3}\frac{1}{\pi^2}\log^2(-\frac{\mu_R^2}{s}) + \frac{1}{2}\frac{1}{s^3}\frac{1}{\pi^2}\log(-\frac{m_t^2}{s})\log^2(-\frac{\mu_R^2}{s}) - \frac{1}{2}\frac{1}{s^3}\frac{1}{\pi^2}\log^2(-\frac{m_t^2}{s})\log^2(-\frac{\mu_R^2}{s})\Big)m_t^2$$

$$+\Big(-\frac{1}{\epsilon^2}\frac{1}{4}\frac{1}{s^2}\frac{1}{\pi^2} + \frac{1}{\epsilon}\Big(\frac{1}{4}\frac{3+\gamma_E-\log(4\pi)}{s^2} + \frac{1}{8}\frac{1}{s^2}\zeta_2\Big)\frac{1}{\pi^2} + \Big(-\frac{1}{8}\frac{18+6\gamma_E+\gamma_E^2-6\log(4\pi)-2\gamma_E\log(4\pi)+\log^2(4\pi)}{s^2}$$

$$-\frac{1}{16}\frac{3+2\gamma_E-2\log(4\pi)}{s^2}\zeta_2 - \frac{1}{8}\frac{1}{s^2}\zeta_3\Big)\frac{1}{\pi^2}\Big)\log(-\frac{m_t^2}{s}) + \Big(\frac{1}{\epsilon}\frac{1}{32}\frac{7+2\gamma_E-2\log(4\pi)}{s^2}\frac{1}{\pi^2} - \frac{1}{\epsilon^2}\frac{1}{16}\frac{1}{s^2}\frac{1}{\pi^2}$$

$$+\Big(-\frac{1}{32}\frac{46+7\gamma_E+\gamma_E^2-7\log(4\pi)-2\gamma_E\log(4\pi)+\log^2(4\pi)}{s^2} - \frac{1}{16}\frac{1}{s^2}\zeta_2\Big)\frac{1}{\pi^2}\Big)\log^2(-\frac{m_t^2}{s}) + \Big(\frac{1}{\epsilon}\frac{1}{24}\frac{1}{s^2}\frac{1}{\pi^2}$$

$$-\frac{1}{24}\frac{7+\gamma_E-\log(4\pi)}{s^2}\frac{1}{\pi^2}\Big)\log^3(-\frac{m_t^2}{s}) - \frac{1}{64}\frac{1}{s^2}\frac{1}{\pi^2}\log^4(-\frac{m_t^2}{s}) + \Big(-\frac{1}{\epsilon}\frac{3}{2}\frac{1}{s^2}\frac{1}{\pi^2} + \Big(\frac{1}{4}\frac{1+6\gamma_E-6\log(4\pi)}{s^2} + \frac{1}{2}\frac{1}{s^2}\zeta_2 + \frac{3}{4}\frac{1}{s^2}\zeta_3\Big)\frac{1}{\pi^2}\Big)\log(-\frac{\mu_R^2}{s})$$

$$+\Big(-\frac{1}{\epsilon}\frac{1}{2}\frac{1}{s^2}\frac{1}{\pi^2} + \Big(\frac{1}{2}\frac{3+\gamma_E-\log(4\pi)}{s^2} + \frac{1}{4}\frac{1}{s^2}\zeta_2\Big)\frac{1}{\pi^2}\Big)\log(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s}) + \Big(\frac{1}{16}\frac{7+2\gamma_E-2\log(4\pi)}{s^2}\frac{1}{\pi^2}$$

$$-\frac{1}{\epsilon}\frac{1}{8}\frac{1}{s^2}\frac{1}{\pi^2}\Big)\log^2(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s}) + \frac{1}{12}\frac{1}{s^2}\frac{1}{\pi^2}\log^3(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s}) - \frac{3}{2}\frac{1}{s^2}\frac{1}{\pi^2}\log^2(-\frac{\mu_R^2}{s}) - \frac{1}{2}\frac{1}{s^2}\frac{1}{\pi^2}\log(-\frac{m_t^2}{s})\log^2(-\frac{\mu_R^2}{s})$$

$$-\frac{1}{8}\frac{1}{s^2}\frac{1}{\pi^2}\log^2(-\frac{m_t^2}{s})\log^2(-\frac{\mu_R^2}{s})\Big)\frac{1}{N_c} + \Big(\frac{1}{\epsilon^2}\frac{3}{2}\frac{1}{s^2}\frac{1}{\pi^2} + \Big(\frac{1}{24}\frac{-133+27\gamma_E+18\gamma_E^2-27\log(4\pi)-36\gamma_E\log(4\pi)+18\log^2(4\pi)}{s^2}$$

$$+\Big(\frac{1}{12}\frac{-25+6\gamma_E-6\log(4\pi)}{s^2} + \frac{1}{4}\frac{1}{s^2}G_{-1}(z) + \frac{1}{4}\frac{1}{s^2}G_0(z)\Big)\zeta_2 + \Big(\frac{3}{16}\frac{5+4\gamma_E-4\log(4\pi)}{s^2} + \frac{3}{8}\frac{1}{s^2}G_{-1}(z) + \frac{3}{8}\frac{1}{s^2}G_0(z)\Big)\zeta_3 - \frac{5}{16}\frac{1}{s^2}\zeta_4$$

$$+\frac{3}{4}\frac{-4+\gamma_E-\log(4\pi)}{s^2}G_{-1}(z) + \frac{3}{8}\frac{1}{s^2}G_{-1}(z)^2 + \frac{3}{4}\frac{-4+\gamma_E-\log(4\pi)}{s^2}G_0(z) + \frac{3}{8}\frac{1}{s^2}G_0(z)^2\Big)\frac{1}{\pi^2} + \Big(\Big(-\frac{3}{8}\frac{3+4\gamma_E-4\log(4\pi)}{s^2}$$

$$-\frac{1}{2}\frac{1}{s^2}\zeta_2 - \frac{3}{4}\frac{1}{s^2}\zeta_3 - \frac{3}{4}\frac{1}{s^2}G_{-1}(z) - \frac{3}{4}\frac{1}{s^2}G_0(z)\Big)\frac{1}{\pi^2} - \frac{3}{4}\frac{1}{s^2}i\frac{1}{\pi}\Big)\frac{1}{\epsilon} + \Big(\frac{3}{4}\frac{-4+\gamma_E-\log(4\pi)}{s^2} + \frac{3}{8}\frac{1}{s^2}\zeta_3 + \frac{3}{4}\frac{1}{s^2}G_{-1}(z)\Big)i\frac{1}{\pi} + \frac{1}{24}\frac{1}{s^2}i\pi$$

$$+\Big(\frac{1}{\epsilon^2}\frac{1}{s^3}\frac{1}{\pi^2} + \Big(\frac{1}{6}\frac{-11+14\gamma_E+3\gamma_E^2-14\log(4\pi)-6\gamma_E\log(4\pi)+3\log^2(4\pi)}{s^3} + \Big(-\frac{1}{24}\frac{37+12\gamma_E-12\log(4\pi)}{s^3} - \frac{1}{4}\frac{1}{s^3}G_{-1}(z)$$

$$-\frac{1}{4}\frac{1}{s^3}G_0(z)\Big)\zeta_2 + \Big(\frac{1}{4}\frac{41+12\gamma_E-12\log(4\pi)}{s^3} + \frac{3}{2}\frac{1}{s^3}G_{-1}(z) + \frac{3}{2}\frac{1}{s^3}G_0(z)\Big)\zeta_3 - \frac{5}{4}\frac{1}{s^3}\zeta_4 + \frac{1}{6}\frac{-8+3\gamma_E-3\log(4\pi)}{s^3}G_{-1}(z)$$

$$+\frac{1}{4}\frac{1}{s^3}G_{-1}(z)^2 + \frac{1}{6}\frac{-8+3\gamma_E-3\log(4\pi)}{s^3}G_0(z) + \frac{1}{4}\frac{1}{s^3}G_0(z)^2\Big)\frac{1}{\pi^2} + \Big(\Big(-\frac{1}{3}\frac{7+3\gamma_E-3\log(4\pi)}{s^3} + \frac{1}{2}\frac{1}{s^3}\zeta_2 - 3\frac{1}{s^3}\zeta_3 - \frac{1}{2}\frac{1}{s^3}G_{-1}(z)$$

$$-\frac{1}{2}\frac{1}{s^3}G_0(z)\Big)\frac{1}{\pi^2} - \frac{1}{2}\frac{1}{s^3}i\frac{1}{\pi}\Big)\frac{1}{\epsilon} + \Big(\frac{1}{6}\frac{-8+3\gamma_E-3\log(4\pi)}{s^3} + \frac{3}{2}\frac{1}{s^3}\zeta_3 + \frac{1}{2}\frac{1}{s^3}G_{-1}(z)\Big)i\frac{1}{\pi} - \frac{1}{24}\frac{1}{s^3}i\pi + \Big(-\frac{1}{\epsilon^2}\frac{1}{2}\frac{1}{s^3}\frac{1}{\pi^2}$$

$$+\Big(-\frac{1}{24}\frac{-87-23\gamma_E+6\gamma_E^2+23\log(4\pi)-12\gamma_E\log(4\pi)+6\log^2(4\pi)}{s^3} + \Big(\frac{1}{6}\frac{23+6\gamma_E-6\log(4\pi)}{s^3} + \frac{1}{2}\frac{1}{s^3}G_{-1}(z) + \frac{1}{2}\frac{1}{s^3}G_0(z)\Big)\zeta_2 + \frac{1}{s^3}\zeta_3$$

$$-\frac{1}{12}\frac{-14+3\gamma_E-3\log(4\pi)}{s^3}G_{-1}(z) - \frac{1}{8}\frac{1}{s^3}G_{-1}(z)^2 - \frac{1}{12}\frac{-14+3\gamma_E-3\log(4\pi)}{s^3}G_0(z) - \frac{1}{8}\frac{1}{s^3}G_0(z)^2\Big)\frac{1}{\pi^2}$$

$$+\Big(\Big(\frac{1}{24}\frac{-23+12\gamma_E-12\log(4\pi)}{s^3} - \frac{1}{s^3}\zeta_2 + \frac{1}{4}\frac{1}{s^3}G_{-1}(z) + \frac{1}{4}\frac{1}{s^3}G_0(z)\Big)\frac{1}{\pi^2} + \frac{1}{4}\frac{1}{s^3}i\frac{1}{\pi}\Big)\frac{1}{\epsilon} + \Big(-\frac{1}{12}\frac{-14+3\gamma_E-3\log(4\pi)}{s^3}$$

$$-\frac{1}{4}\frac{1}{s^3}G_{-1}(z)\Big)i\frac{1}{\pi} + \frac{1}{12}\frac{1}{s^3}i\pi\Big)\log(-\frac{m_t^2}{s}) + \Big(\frac{1}{\epsilon^2}\frac{1}{2}\frac{1}{s^3}\frac{1}{\pi^2} + \Big(\frac{1}{48}\frac{-111+62\gamma_E+12\gamma_E^2-62\log(4\pi)-24\gamma_E\log(4\pi)+12\log^2(4\pi)}{s^3}$$

$$-\frac{1}{4}\frac{1}{s^3}\zeta_2 + \frac{1}{24}\frac{-19+6\gamma_E-6\log(4\pi)}{s^3}G_{-1}(z) + \frac{1}{8}\frac{1}{s^3}G_{-1}(z)^2 + \frac{1}{24}\frac{-19+6\gamma_E-6\log(4\pi)}{s^3}G_0(z) + \frac{1}{8}\frac{1}{s^3}G_0(z)^2\Big)\frac{1}{\pi^2}$$

$$+\Big(\Big(-\frac{1}{24}\frac{31+12\gamma_E-12\log(4\pi)}{s^3} - \frac{1}{4}\frac{1}{s^3}G_{-1}(z) - \frac{1}{4}\frac{1}{s^3}G_0(z)\Big)\frac{1}{\pi^2} - \frac{1}{4}\frac{1}{s^3}i\frac{1}{\pi}\Big)\frac{1}{\epsilon} + \Big(\frac{1}{24}\frac{-19+6\gamma_E-6\log(4\pi)}{s^3} + \frac{1}{4}\frac{1}{s^3}G_{-1}(z)$$

$$\Big)i\frac{1}{\pi}\Big)\log^2(-\frac{m_t^2}{s}) + \Big(-\frac{1}{\epsilon}\frac{1}{3}\frac{1}{s^3}\frac{1}{\pi^2} + \Big(\frac{1}{9}\frac{22+3\gamma_E-3\log(4\pi)}{s^3} + \frac{1}{6}\frac{1}{s^3}G_{-1}(z) + \frac{1}{6}\frac{1}{s^3}G_0(z)\Big)\frac{1}{\pi^2} + \frac{1}{6}\frac{1}{s^3}i\frac{1}{\pi}\Big)\log^3(-\frac{m_t^2}{s}) + \frac{1}{8}\frac{1}{s^3}\frac{1}{\pi^2}\log^4(-\frac{m_t^2}{s})$$

$$+\Big(\frac{1}{\epsilon}2\frac{1}{s^3}\frac{1}{\pi^2} + \Big(-\frac{1}{12}\frac{23+24\gamma_E-24\log(4\pi)}{s^3} + \frac{1}{s^3}\zeta_2 - 6\frac{1}{s^3}\zeta_3 - \frac{1}{s^3}G_{-1}(z) - \frac{1}{s^3}G_0(z)\Big)\frac{1}{\pi^2} - \frac{1}{s^3}i\frac{1}{\pi}\Big)\log(-\frac{\mu_R^2}{s})$$

$$+\Big(-\frac{1}{\epsilon}\frac{1}{s^3}\frac{1}{\pi^2} + \Big(\frac{1}{24}\frac{-79+24\gamma_E-24\log(4\pi)}{s^3} - 2\frac{1}{s^3}\zeta_2 + \frac{1}{2}\frac{1}{s^3}G_{-1}(z) + \frac{1}{2}\frac{1}{s^3}G_0(z)\Big)\frac{1}{\pi^2} + \frac{1}{2}\frac{1}{s^3}i\frac{1}{\pi}\Big)\log(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s}) + \Big(\frac{1}{\epsilon}\frac{1}{s^3}\frac{1}{\pi^2}$$

$$
\begin{aligned}
&+\Bigg(-\frac{1}{24}\frac{29+24\gamma_E-24\log(4\pi)}{s^3}-\frac{1}{2}\frac{1}{s^3}G_{-1}(z)-\frac{1}{2}\frac{1}{s^3}G_0(z)\Bigg)\frac{1}{\pi^2}-\frac{1}{2}\frac{1}{s^3}i\frac{1}{\pi}\Bigg)\log^2(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s})-\frac{2}{3}\frac{1}{s^3}\frac{1}{\pi^2}\log^3(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s})\\
&+2\frac{1}{s^3}\frac{1}{\pi^2}\log^2(-\frac{\mu_R^2}{s})-\frac{1}{s^3}\frac{1}{\pi^2}\log(-\frac{m_t^2}{s})\log^2(-\frac{\mu_R^2}{s})+\frac{1}{s^3}\frac{1}{\pi^2}\log^2(-\frac{m_t^2}{s})\log^2(-\frac{\mu_R^2}{s})\Bigg)m_t^2+\Bigg(\frac{1}{\epsilon^2}\frac{1}{2}\frac{1}{s^2}\frac{1}{\pi^2}\\
&+\Bigg(\frac{1}{12}\frac{27+20\gamma_E+3\gamma_E^2-20\log(4\pi)-6\gamma_E\log(4\pi)+3\log^2(4\pi)}{s^2}+\Bigg(\frac{1}{48}\frac{-5+12\gamma_E-12\log(4\pi)}{s^2}+\frac{1}{8}\frac{1}{s^2}G_{-1}(z)+\frac{1}{8}\frac{1}{s^2}G_0(z)\Bigg)\zeta_2+\frac{1}{4}\frac{1}{s^2}\zeta_3\\
&+\frac{1}{12}\frac{-5+3\gamma_E-3\log(4\pi)}{s^2}G_{-1}(z)+\frac{1}{8}\frac{1}{s^2}G_{-1}(z)^2+\frac{1}{12}\frac{-5+3\gamma_E-3\log(4\pi)}{s^2}G_0(z)+\frac{1}{8}\frac{1}{s^2}G_0(z)^2\Bigg)\frac{1}{\pi^2}+\Bigg(\Bigg(-\frac{1}{6}\frac{10+3\gamma_E-3\log(4\pi)}{s^2}\\
&-\frac{1}{4}\frac{1}{s^2}\zeta_2-\frac{1}{4}\frac{1}{s^2}G_{-1}(z)-\frac{1}{4}\frac{1}{s^2}G_0(z)\Bigg)\frac{1}{\pi^2}-\frac{1}{4}\frac{1}{s^2}i\frac{1}{\pi}\Bigg)\frac{1}{\epsilon}+\Bigg(\frac{1}{12}\frac{-5+3\gamma_E-3\log(4\pi)}{s^2}+\frac{1}{4}\frac{1}{s^2}G_{-1}(z)\Bigg)i\frac{1}{\pi}+\frac{1}{48}\frac{1}{s^2}i\pi\Bigg)\log(-\frac{m_t^2}{s})\\
&+\Bigg(\frac{1}{\epsilon^2}\frac{1}{8}\frac{1}{s^2}\frac{1}{\pi^2}+\Bigg(\frac{1}{96}\frac{182+55\gamma_E+6\gamma_E^2-55\log(4\pi)-12\gamma_E\log(4\pi)+6\log^2(4\pi)}{s^2}-\frac{1}{16}\frac{1}{s^2}\zeta_2+\frac{1}{48}\frac{1+3\gamma_E-3\log(4\pi)}{s^2}G_{-1}(z)\\
&+\frac{1}{32}\frac{1}{s^2}G_{-1}(z)^2+\frac{1}{48}\frac{1+3\gamma_E-3\log(4\pi)}{s^2}G_0(z)+\frac{1}{32}\frac{1}{s^2}G_0(z)^2\Bigg)\frac{1}{\pi^2}+\Bigg(\Bigg(-\frac{1}{96}\frac{55+12\gamma_E-12\log(4\pi)}{s^2}-\frac{1}{16}\frac{1}{s^2}G_{-1}(z)-\frac{1}{16}\frac{1}{s^2}G_0(z)\Bigg)\frac{1}{\pi^2}\\
&-\frac{1}{16}\frac{1}{s^2}i\frac{1}{\pi}\Bigg)\frac{1}{\epsilon}+\Bigg(\frac{1}{48}\frac{1+3\gamma_E-3\log(4\pi)}{s^2}+\frac{1}{16}\frac{1}{s^2}G_{-1}(z)\Bigg)i\frac{1}{\pi}\Bigg)\log^2(-\frac{m_t^2}{s})+\Bigg(-\frac{1}{\epsilon}\frac{1}{12}\frac{1}{s^2}\frac{1}{\pi^2}+\Bigg(\frac{1}{72}\frac{35+6\gamma_E-6\log(4\pi)}{s^2}\\
&+\frac{1}{24}\frac{1}{s^2}G_{-1}(z)+\frac{1}{24}\frac{1}{s^2}G_0(z)\Bigg)\frac{1}{\pi^2}+\frac{1}{24}\frac{1}{s^2}i\frac{1}{\pi}\Bigg)\log^3(-\frac{m_t^2}{s})+\frac{1}{32}\frac{1}{s^2}\frac{1}{\pi^2}\log^4(-\frac{m_t^2}{s})+\Bigg(\frac{1}{\epsilon}3\frac{1}{s^2}\frac{1}{\pi^2}+\Bigg(-\frac{3}{8}\frac{-5+8\gamma_E-8\log(4\pi)}{s^2}-\frac{1}{s^2}\zeta_2\\
&-\frac{3}{2}\frac{1}{s^2}\zeta_3-\frac{3}{2}\frac{1}{s^2}G_{-1}(z)-\frac{3}{2}\frac{1}{s^2}G_0(z)\Bigg)\frac{1}{\pi^2}-\frac{3}{2}\frac{1}{s^2}i\frac{1}{\pi}\Bigg)\log(-\frac{\mu_R^2}{s})+\Bigg(\frac{1}{\epsilon}\frac{1}{s^2}\frac{1}{\pi^2}+\Bigg(-\frac{1}{24}\frac{47+24\gamma_E-24\log(4\pi)}{s^2}-\frac{1}{2}\frac{1}{s^2}\zeta_2-\frac{1}{2}\frac{1}{s^2}G_{-1}(z)\\
&-\frac{1}{2}\frac{1}{s^2}G_0(z)\Bigg)\frac{1}{\pi^2}-\frac{1}{2}\frac{1}{s^2}i\frac{1}{\pi}\Bigg)\log(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s})+\Bigg(\frac{1}{\epsilon}\frac{1}{4}\frac{1}{s^2}\frac{1}{\pi^2}+\Bigg(-\frac{1}{96}\frac{77+24\gamma_E-24\log(4\pi)}{s^2}-\frac{1}{8}\frac{1}{s^2}G_{-1}(z)-\frac{1}{8}\frac{1}{s^2}G_0(z)\Bigg)\frac{1}{\pi^2}\\
&-\frac{1}{8}\frac{1}{s^2}i\frac{1}{\pi}\Bigg)\log^2(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s})-\frac{1}{6}\frac{1}{s^2}\frac{1}{\pi^2}\log^3(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s})+3\frac{1}{s^2}\frac{1}{\pi^2}\log^2(-\frac{\mu_R^2}{s})+\frac{1}{s^2}\frac{1}{\pi^2}\log(-\frac{m_t^2}{s})\log^2(-\frac{\mu_R^2}{s})\\
&+\frac{1}{4}\frac{1}{s^2}\frac{1}{\pi^2}\log^2(-\frac{m_t^2}{s})\log^2(-\frac{\mu_R^2}{s})\Bigg)N_c+\Bigg(\frac{1}{\epsilon}\frac{1}{2}\frac{1}{s^2}\frac{1}{\pi^2}+\Bigg(-\frac{1}{6}\frac{-7+3\gamma_E-3\log(4\pi)}{s^2}-\frac{1}{6}\frac{1}{s^2}\zeta_2-\frac{1}{4}\frac{1}{s^2}\zeta_3+\frac{1}{8}\frac{1}{s^2}G_{-1}(z)+\frac{1}{8}\frac{1}{s^2}G_0(z)\Bigg)\frac{1}{\pi^2}\\
&+\frac{1}{8}\frac{1}{s^2}i\frac{1}{\pi}+\Bigg(\frac{1}{\epsilon}\frac{1}{3}\frac{1}{s^3}\frac{1}{\pi^2}+\Bigg(-\frac{1}{3}\frac{-1+\gamma_E-\log(4\pi)}{s^3}+\frac{1}{6}\frac{1}{s^3}\zeta_2-\frac{1}{s^3}\zeta_3+\frac{1}{12}\frac{1}{s^3}G_{-1}(z)+\frac{1}{12}\frac{1}{s^3}G_0(z)\Bigg)\frac{1}{\pi^2}+\frac{1}{12}\frac{1}{s^3}i\frac{1}{\pi}+\Bigg(-\frac{1}{\epsilon}\frac{1}{6}\frac{1}{s^3}\frac{1}{\pi^2}\\
&+\Bigg(\frac{1}{6}\frac{-3+\gamma_E-\log(4\pi)}{s^3}-\frac{1}{3}\frac{1}{s^3}\zeta_2-\frac{1}{24}\frac{1}{s^3}G_{-1}(z)-\frac{1}{24}\frac{1}{s^3}G_0(z)\Bigg)\frac{1}{\pi^2}-\frac{1}{24}\frac{1}{s^3}i\frac{1}{\pi}\Bigg)\log(-\frac{m_t^2}{s})+\Bigg(\frac{1}{\epsilon}\frac{1}{6}\frac{1}{s^3}\frac{1}{\pi^2}\\
&+\Bigg(-\frac{1}{12}\frac{-3+2\gamma_E-2\log(4\pi)}{s^3}+\frac{1}{24}\frac{1}{s^3}G_{-1}(z)+\frac{1}{24}\frac{1}{s^3}G_0(z)\Bigg)\frac{1}{\pi^2}+\frac{1}{24}\frac{1}{s^3}i\frac{1}{\pi}\Bigg)\log^2(-\frac{m_t^2}{s})-\frac{1}{9}\frac{1}{s^3}\frac{1}{\pi^2}\log^3(-\frac{m_t^2}{s})+\frac{1}{6}\frac{1}{s^3}\frac{1}{\pi^2}\log(-\frac{\mu_R^2}{s})\\
&-\frac{1}{12}\frac{1}{s^3}\frac{1}{\pi^2}\log(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s})+\frac{1}{12}\frac{1}{s^3}\frac{1}{\pi^2}\log^2(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s})\Bigg)m_t^2+\Bigg(\frac{1}{\epsilon}\frac{1}{6}\frac{1}{s^2}\frac{1}{\pi^2}+\Bigg(-\frac{1}{6}\frac{\gamma_E-\log(4\pi)}{s^2}-\frac{1}{12}\frac{1}{s^2}\zeta_2+\frac{1}{24}\frac{1}{s^2}G_{-1}(z)\\
&+\frac{1}{24}\frac{1}{s^2}G_0(z)\Bigg)\frac{1}{\pi^2}+\frac{1}{24}\frac{1}{s^2}i\frac{1}{\pi}\Bigg)\log(-\frac{m_t^2}{s})+\Bigg(\frac{1}{\epsilon}\frac{1}{24}\frac{1}{s^2}\frac{1}{\pi^2}+\Bigg(-\frac{1}{24}\frac{2+\gamma_E-\log(4\pi)}{s^2}+\frac{1}{96}\frac{1}{s^2}G_{-1}(z)+\frac{1}{96}\frac{1}{s^2}G_0(z)\Bigg)\frac{1}{\pi^2}\\
&+\frac{1}{96}\frac{1}{s^2}i\frac{1}{\pi}\Bigg)\log^2(-\frac{m_t^2}{s})-\frac{1}{36}\frac{1}{s^2}\frac{1}{\pi^2}\log^3(-\frac{m_t^2}{s})+\frac{1}{4}\frac{1}{s^2}\frac{1}{\pi^2}\log(-\frac{\mu_R^2}{s})+\frac{1}{12}\frac{1}{s^2}\frac{1}{\pi^2}\log(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s})+\frac{1}{48}\frac{1}{s^2}\frac{1}{\pi^2}\log^2(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s})\Bigg)N_f\\
&+\frac{1}{\epsilon}\frac{1}{2}\frac{1}{s^2}\frac{1}{\pi^2}+\Bigg(-\frac{1}{6}\frac{-7+3\gamma_E-3\log(4\pi)}{s^2}-\frac{1}{6}\frac{1}{s^2}\zeta_2-\frac{1}{4}\frac{1}{s^2}\zeta_3\Bigg)\frac{1}{\pi^2}+\Bigg(\frac{1}{\epsilon}\frac{1}{3}\frac{1}{s^3}\frac{1}{\pi^2}+\Bigg(-\frac{1}{3}\frac{-1+\gamma_E-\log(4\pi)}{s^3}+\frac{1}{6}\frac{1}{s^3}\zeta_2-\frac{1}{s^3}\zeta_3\Bigg)\frac{1}{\pi^2}\\
&+\Bigg(-\frac{1}{\epsilon}\frac{1}{6}\frac{1}{s^3}\frac{1}{\pi^2}+\Bigg(\frac{1}{6}\frac{-5+\gamma_E-\log(4\pi)}{s^3}-\frac{1}{3}\frac{1}{s^3}\zeta_2\Bigg)\frac{1}{\pi^2}\Bigg)\log(-\frac{m_t^2}{s})+\Bigg(-\frac{1}{12}\frac{-5+2\gamma_E-2\log(4\pi)}{s^3}\frac{1}{\pi^2}+\frac{1}{\epsilon}\frac{1}{6}\frac{1}{s^3}\frac{1}{\pi^2}\Bigg)\log^2(-\frac{m_t^2}{s})\\
&-\frac{5}{18}\frac{1}{s^3}\frac{1}{\pi^2}\log^3(-\frac{m_t^2}{s})+\frac{2}{3}\frac{1}{s^3}\frac{1}{\pi^2}\log(-\frac{\mu_R^2}{s})-\frac{1}{3}\frac{1}{s^3}\frac{1}{\pi^2}\log(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s})+\frac{1}{3}\frac{1}{s^3}\frac{1}{\pi^2}\log^2(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s})\Bigg)m_t^2+\Bigg(\frac{1}{\epsilon}\frac{1}{6}\frac{1}{s^2}\frac{1}{\pi^2}\\
&+\Bigg(-\frac{1}{6}\frac{3+\gamma_E-\log(4\pi)}{s^2}-\frac{1}{12}\frac{1}{s^2}\zeta_2\Bigg)\frac{1}{\pi^2}\Bigg)\log(-\frac{m_t^2}{s})+\Bigg(\frac{1}{\epsilon}\frac{1}{24}\frac{1}{s^2}\frac{1}{\pi^2}-\frac{1}{24}\frac{6+\gamma_E-\log(4\pi)}{s^2}\frac{1}{\pi^2}\Bigg)\log^2(-\frac{m_t^2}{s})-\frac{5}{72}\frac{1}{s^2}\frac{1}{\pi^2}\log^3(-\frac{m_t^2}{s})\\
&+\frac{1}{s^2}\frac{1}{\pi^2}\log(-\frac{\mu_R^2}{s})+\frac{1}{3}\frac{1}{s^2}\frac{1}{\pi^2}\log(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s})+\frac{1}{12}\frac{1}{s^2}\frac{1}{\pi^2}\log^2(-\frac{m_t^2}{s})\log(-\frac{\mu_R^2}{s})+O(m_h^2)+O((m_t^2)^2)
\end{aligned}
\tag{E.2}
$$

E.2. One-loop helicity coefficients

$$
\begin{aligned}
(\Omega^g_{++-})^{1l} =&\Bigg(-12\frac{u_3^2-u_3+1}{u_3-1}+6(u_3-1)\zeta_2+4\frac{2u_3-1}{(u_3-1)u_3}G_1(u_3)-G_{0,0}(u_3)+1G_{0,1}(u_3)+1G_{1,0}(u_3)-\frac{u_3^3+5u_3^2-8u_3+4}{(u_3-1)u_3^2}G_{1,1}(u_3)\Bigg)i\\
&+\Bigg(-4\frac{(u_3-1)(u_3+1)}{u_3}-G_0(u_3)+\frac{(u_3-2)^2}{u_3^2}G_1(u_3)\Bigg)\pi+\Bigg(\Bigg(8\frac{u_3^4-2u_3^3+u_3^2+2u_3-1}{s(u_3-1)^2}-12\frac{2u_3^2-1}{s}\zeta_2+2\frac{u_3^2-3u_3+3}{(u_3-1)s}G_0(u_3)\\
&-2\frac{u_3^3-4u_3^2+1}{s(u_3-1)^2}G_1(u_3)+4\frac{1}{s}G_{0,0}(u_3)-2\frac{2u_3-1}{(u_3-1)s}G_{0,1}(u_3)-2\frac{2u_3-1}{(u_3-1)s}G_{1,0}(u_3)+4\frac{u_3^2+2u_3-1}{s(u_3-1)^2}G_{1,1}(u_3)\Bigg)i\\
&+\Bigg(-2\frac{2u_3^3-u_3^2-3u_3+1}{(u_3-1)s}+2\frac{u_3+1}{s}G_0(u_3)
\end{aligned}
$$

$$
\begin{aligned}
&- 2\frac{u_3^2 + 2u_3 - 2}{(u_3 - 1)s} G_1(u_3)\Big)\pi + \Big(\Big(-2\frac{(u_3^2 + u_3 - 1)(2u_3^2 - 5u_3 + 5)}{s(u_3 - 1)^2} + 2\frac{u_3^2}{(u_3 - 1)s} G_0(u_3) - 2\frac{u_3^3 + u_3^2 + 3u_3 - 1}{s(u_3 - 1)^2} G_1(u_3)\Big) i \\
&+ 2\frac{4u_3^3 - 4u_3^2 + 1}{(u_3 - 1)s}\pi\Big)\log(\frac{m_t^2}{t}) + 2\frac{2u_3^4 - 4u_3^3 + 3u_3^2 + 2u_3 - 1}{s(u_3 - 1)^2} i\log^2(\frac{m_t^2}{t})\Big) m_t^2 + \Big(\Big(-4\frac{u_3^2 - u_3 + 1}{u_3 - 1} - G_0(u_3) + \frac{u_3 + 1}{u_3 - 1} G_1(u_3)\Big) i \\
&+ (-2u_3 + 1)\pi\Big)\log(\frac{m_t^2}{t}) - \frac{1}{2}\frac{2u_3^2 - 3u_3 + 3}{u_3 - 1} i\log^2(\frac{m_t^2}{t}) + O((m_t^2)^2) + O(m_H^2)
\end{aligned}
\tag{E.3}
$$

$$
\begin{aligned}
(\Omega^g_{+-+})^{1l} =& \Big(12\frac{u_3^2 - u_3 + 1}{u_3 - 1} + 12(u_3 - 1)u_3^2\zeta_2 - 4(u_3 - 1)(u_3 + 1)G_0(u_3) + 4\frac{(u_3 - 2)u_3^2}{u_3 - 1} G_1(u_3) + (4u_3^3 - 4u_3^2 + u_3 - 2)G_{0,0}(u_3) \\
&- u_3(2u_3 - 1)^2 G_{0,1}(u_3) - u_3(2u_3 - 1)^2 G_{1,0}(u_3) + \frac{(4u_3^3 - 8u_3^2 + 5u_3 + 1)u_3}{u_3 - 1} G_{1,1}(u_3)\Big) i - \Big(G_0(u_3) + G_1(u_3)\Big) u_3\pi \\
&+ \Big(\Big(8\frac{u_3^4 - 2u_3^3 - u_3^2 + 2u_3 - 1}{s(u_3 - 1)^2} + 12\frac{u_3^2}{s}\zeta_2 + 2\frac{u_3^3 - 3u_3^2 - u_3 + 2}{(u_3 - 1)s} G_0(u_3) + 2\frac{(u_3^3 - 4u_3 + 1)u_3}{s(u_3 - 1)^2} G_1(u_3) + 4\frac{u_3^2 - 2}{s} G_{0,0}(u_3) \\
&- 2\frac{(2u_3^2 - 2u_3 - 1)u_3}{(u_3 - 1)s} G_{0,1}(u_3) - 2\frac{(2u_3^2 - 2u_3 - 1)u_3}{(u_3 - 1)s} G_{1,0}(u_3) + 4\frac{(u_3^2 - 2u_3 - 1)u_3^2}{s(u_3 - 1)^2} G_{1,1}(u_3)\Big) i + \Big(-2\frac{(3u_3^2 - 3u_3 + 1)u_3}{(u_3 - 1)s} \\
&- 2\frac{(u_3 + 1)u_3}{s} G_0(u_3) - 2\frac{(u_3 - 2)u_3^2}{(u_3 - 1)s} G_1(u_3)\Big)\pi + \Big(\Big(-2\frac{(u_3^2 - u_3 - 1)(5u_3^2 - 5u_3 + 2)}{s(u_3 - 1)^2} - 2\frac{u_3^3 - 4u_3 + 4}{(u_3 - 1)s} G_0(u_3) \\
&- 2\frac{(u_3^3 - 3u_3^2 - u_3 - 1)u_3}{s(u_3 - 1)^2} G_1(u_3)\Big) i - 2\frac{u_3}{(u_3 - 1)s}\pi\Big)\log(\frac{m_t^2}{t}) + 2\frac{u_3^4 - 2u_3^3 - 3u_3^2 + 4u_3 - 2}{s(u_3 - 1)^2} i\log^2(\frac{m_t^2}{t})\Big) m_t^2 \\
&+ \Big(\Big(4\frac{u_3^2 - u_3 + 1}{u_3 - 1} + (-u_3 + 2)G_0(u_3) - \frac{(u_3 + 1)u_3}{u_3 - 1} G_1(u_3)\Big) i + u_3\pi\Big)\log(\frac{m_t^2}{t}) \\
&+ \frac{1}{2}\frac{3u_3^2 - 3u_3 + 2}{u_3 - 1} i\log^2(\frac{m_t^2}{t}) + O((m_t^2)^2) + O(m_H^2)
\end{aligned}
\tag{E.4}
$$

$$
\begin{aligned}
(\Omega^q_{-++})^{1l} =& \Big(\frac{8}{s} + \frac{4}{s} G_0(u_3) + \frac{8}{s} G_{0,0}(u_3) + \Big(-\frac{4}{s} - \frac{8}{s} G_0(u_3)\Big)\log(\frac{m_t^2}{t}) + \frac{4}{s}\log^2(\frac{m_t^2}{t})\Big) m_t^2 + 12 - 4G_0(u_3) + 2G_{0,0}(u_3) \\
&+ \Big(4 - 2G_0(u_3)\Big)\log(\frac{m_t^2}{t}) + \log^2(\frac{m_t^2}{t}) + O(m_h^2) + O((m_t^2)^2)
\end{aligned}
\tag{E.5}
$$

E.3. Two-loop helicity coefficients

$$
\begin{aligned}
(\Omega^g_{++-})^{2l} =& \Big(\Big(10\frac{u_3^2 - u_3 + 1}{u_3 - 1} + \Big(-\frac{1}{2}\frac{58u_3^3 - 69u_3^2 - 9u_3 + 24}{(u_3 - 1)u_3} + \frac{1}{2}\frac{(2u_3 - 5)(3u_3 - 5)}{u_3 - 1} G_0(u_3) + \frac{1}{2}\frac{6u_3^4 - 20u_3^3 + 9u_3^2 + 33u_3 - 24}{(u_3 - 1)u_3^2} G_1(u_3) \\
&- \frac{7}{2} G_{0,0}(u_3) - 3G_{0,1}(u_3) - \frac{1}{2}\frac{u_3 + 4}{u_3} G_{1,0}(u_3) + \frac{1}{2}\frac{2u_3^3 - 38u_3^2 + 55u_3 - 25}{(u_3 - 1)u_3^2} G_{1,1}(u_3)\Big)\zeta_2 + \Big(-\frac{17u_3^2 - 24u_3 + 24}{u_3 - 1} - 3G_0(u_3) \\
&+ \frac{1}{2}\frac{3u_3^2 + 23u_3 - 10}{(u_3 - 1)u_3} G_1(u_3)\Big)\zeta_3 + \frac{1}{4}\frac{49u_3^2 - 73u_3 - 2}{u_3 - 1}\zeta_4 + 8G_0(u_3) - 4\frac{2u_3^2 + 2u_3 - 1}{(u_3 - 1)u_3} G_1(u_3) - \frac{1}{4}\frac{16u_3^2 - 13u_3 + 13}{u_3 - 1} G_{0,0}(u_3) \\
&+ \frac{1}{4}\frac{16u_3^2 - 9u_3 - 2}{u_3} G_{0,1}(u_3) + \frac{1}{4}\frac{16u_3^2 - 9u_3 - 2}{u_3} G_{1,0}(u_3) - \frac{1}{4}\frac{16u_3^3 - 37u_3^2 - 15u_3 + 16}{(u_3 - 1)u_3} G_{1,1}(u_3) \\
&+ \frac{1}{2}\frac{4u_3^2 - 17u_3 + 17}{u_3 - 1} G_{0,0,0}(u_3) + \frac{1}{4}(-4u_3 + 13)G_{0,0,1}(u_3) + \frac{1}{4}(-4u_3 + 13)G_{0,1,0}(u_3) - \frac{3u_3^2 - 4u_3 + 4}{u_3^2} G_{0,1,1}(u_3) \\
&+ \frac{1}{4}\frac{5u_3^2 - u_3 - 8}{(u_3 - 1)u_3} G_{1,0,0}(u_3) - \frac{(u_3 - 1)^2}{u_3} G_{1,0,1}(u_3) - \frac{(u_3 - 1)^2}{u_3} G_{1,1,0}(u_3) + \frac{1}{4}\frac{8u_3^4 - 17u_3^3 - 31u_3^2 + 66u_3 - 32}{(u_3 - 1)u_3^2} G_{1,1,1}(u_3) \\
&- \frac{1}{2} G_{0,0,0,0}(u_3) - 1G_{0,0,0,1}(u_3) - 1G_{0,0,1,0}(u_3) + \frac{3}{2} G_{0,0,1,1}(u_3) - 1G_{0,1,0,0}(u_3) + 2G_{0,1,0,1}(u_3) + 2G_{0,1,1,0}(u_3) \\
&- \frac{3}{2} G_{0,1,1,1}(u_3) - 1G_{1,0,0,0}(u_3) + 2G_{1,0,0,1}(u_3) + 2G_{1,0,1,0}(u_3) - \frac{1}{2}\frac{u_3 + 2}{u_3} G_{1,0,1,1}(u_3) - 2\frac{u_3 - 1}{u_3^2} G_{1,1,0,0}(u_3) \\
&- \frac{1}{2}\frac{u_3^2 - 4u_3 + 2}{u_3^2} G_{1,1,0,1}(u_3) - \frac{1}{2}\frac{u_3^2 - 4u_3 + 2}{u_3^2} G_{1,1,1,0}(u_3) - \frac{1}{2}\frac{4u_3^3 - 2u_3^2 + 3u_3 - 3}{(u_3 - 1)u_3^2} G_{1,1,1,1}(u_3)\Big) i + \Big(4\frac{3u_3^2 - 1}{u_3} \\
&- 4(2u_3 - 1)\zeta_3 - \frac{1}{4}\frac{5u_3^2 + 13u_3 - 2}{(u_3 - 1)u_3} G_0(u_3) + \frac{5}{4} G_1(u_3) - \frac{1}{4}\frac{9u_3 - 13}{u_3 - 1} G_{0,0}(u_3) - \frac{1}{2}\frac{3u_3^2 - 8u_3 + 8}{u_3^2} G_{0,1}(u_3) \\
&+ \frac{1}{2}\frac{3u_3^2 - 3u_3 - 2}{(u_3 - 1)u_3} G_{1,0}(u_3) + \frac{1}{4}\frac{(u_3 - 2)(9u_3 - 8)}{u_3^2} G_{1,1}(u_3) + 1G_{0,0,0}(u_3) + \frac{1}{2} G_{0,1,0}(u_3) + \frac{1}{2} G_{1,0,0}(u_3) + \frac{1}{2}\frac{u_3 - 2}{u_3} G_{1,0,1}(u_3) \\
&- \frac{1}{2}\frac{3u_3^2 - 2}{u_3^2} G_{1,1,0}(u_3) - \frac{(u_3 - 2)^2}{u_3^2} G_{1,1,1}(u_3)\Big)\pi + \Big(-\frac{1}{12}\frac{7u_3^3 - 7u_3^2 + u_3 - 3}{(u_3 - 1)u_3} + \frac{1}{12} G_0(u_3) + \frac{1}{4}\frac{(u_3 - 1)(u_3 + 1)}{u_3^2} G_1(u_3)\Big)\pi^3
\end{aligned}
$$

$$+\Bigg(\Bigg(\Bigg(\frac{1}{2}\frac{72u_3^4-165u_3^3+152u_3^2+26u_3-13}{s(u_3-1)^2}+\Bigg(-\frac{1}{4}\frac{152u_3^4-205u_3^3-73u_3^2+178u_3-44}{s(u_3-1)^2}+\frac{1}{4}\frac{u_3^2+320u_3-356}{(u_3-1)s}G_0(u_3)$$

$$-\frac{1}{4}\frac{23u_3^4-41u_3^3+83u_3^2-189u_3+108}{s(u_3-1)^2u_3}G_1(u_3)+\frac{1}{2}\frac{47u_3^2-42u_3+33}{(u_3-1)s}G_{0,0}(u_3)+\frac{9}{2}\frac{2u_3^3+4u_3^2+3u_3-2}{(u_3-1)su_3}G_{0,1}(u_3)$$

$$-\frac{5}{2}\frac{6u_3^3-8u_3^2-21u_3+14}{(u_3-1)su_3}G_{1,0}(u_3)-\frac{1}{2}\frac{35u_3^4-39u_3^3-94u_3^2+142u_3-60}{s(u_3-1)^2u_3}G_{1,1}(u_3)\Bigg)\zeta_2+\Bigg(\frac{1}{2}\frac{32u_3^4-97u_3^3+5u_3^2+184u_3-92}{s(u_3-1)^2}$$

$$-2\frac{3u_3^2-7u_3+7}{(u_3-1)s}G_0(u_3)+\frac{1}{2}\frac{13u_3^4-29u_3^3-184u_3^2+232u_3-96}{s(u_3-1)^2u_3}G_1(u_3)\Bigg)\zeta_3-\frac{1}{4}\frac{116u_3^4-528u_3^3+807u_3^2-618u_3+159}{s(u_3-1)^2}\zeta_4$$

$$+\frac{1}{4}\frac{53u_3^2-96u_3+96}{(u_3-1)s}G_0(u_3)-\frac{1}{4}\frac{43u_3^4-151u_3^3+66u_3^2-10u_3-4}{s(u_3-1)^2u_3}G_1(u_3)-\frac{21}{2}\frac{1}{s}G_{0,0}(u_3)+\frac{1}{2}\frac{3u_3^2+8u_3+2}{(u_3-1)su_3}G_{0,1}(u_3)$$

$$+\frac{1}{2}\frac{3u_3^2+8u_3+2}{(u_3-1)su_3}G_{1,0}(u_3)-\frac{3}{2}\frac{(3u_3-11)(3u_3-2)}{s(u_3-1)^2}G_{1,1}(u_3)-\frac{7}{2}\frac{u_3^2-12u_3+12}{(u_3-1)s}G_{0,0,0}(u_3)-\frac{1}{2}\frac{29u_3-38}{(u_3-1)s}G_{0,0,1}(u_3)$$

$$-\frac{1}{2}\frac{29u_3-38}{(u_3-1)s}G_{0,1,0}(u_3)+\frac{1}{2}\frac{9u_3^2+61u_3-67}{(u_3-1)s}G_{0,1,1}(u_3)+\frac{1}{2}\frac{5u_3^3-37u_3^2+32u_3+4}{(u_3-1)su_3}G_{1,0,0}(u_3)-\frac{1}{2}\frac{31u_3^2-32u_3+2}{(u_3-1)su_3}G_{1,0,1}(u_3)$$

$$-\frac{1}{2}\frac{31u_3^2-32u_3+2}{(u_3-1)su_3}G_{1,1,0}(u_3)-\frac{1}{2}\frac{7u_3^4+15u_3^3-129u_3^2+89u_3-22}{s(u_3-1)^2u_3}G_{1,1,1}(u_3)+2\frac{7u_3^2-10u_3+10}{(u_3-1)s}G_{0,0,0,0}(u_3)$$

$$+\frac{4u_3^2-14u_3+3}{(u_3-1)su_3}G_{0,0,0,1}(u_3)+\frac{4u_3^2-14u_3+3}{(u_3-1)su_3}G_{0,0,1,0}(u_3)-\frac{1}{2}\frac{8u_3^2+12u_3-9}{(u_3-1)s}G_{0,0,1,1}(u_3)$$

$$-\frac{1}{2}\frac{15u_3^3-32u_3^2+10u_3+6}{(u_3-1)su_3}G_{0,1,0,0}(u_3)-\frac{10u_3^2+2u_3-3}{(u_3-1)su_3}G_{0,1,0,1}(u_3)-\frac{10u_3^2+2u_3-3}{(u_3-1)su_3}G_{0,1,1,0}(u_3)$$

$$+\frac{1}{2}\frac{13u_3^3+16u_3^2+15u_3-6}{(u_3-1)su_3}G_{0,1,1,1}(u_3)-\frac{1}{2}\frac{29u_3^3-52u_3^2-14u_3+22}{(u_3-1)su_3}G_{1,0,0,0}(u_3)-2\frac{5u_3^2+2u_3-2}{(u_3-1)su_3}G_{1,0,0,1}(u_3)$$

$$-2\frac{5u_3^2+2u_3-2}{(u_3-1)su_3}G_{1,0,1,0}(u_3)+\frac{1}{2}\frac{7u_3^3+33u_3-18}{(u_3-1)su_3}G_{1,0,1,1}(u_3)+\frac{1}{2}\frac{16u_3^3-12u_3^2-19u_3+8}{(u_3-1)su_3}G_{1,1,0,0}(u_3)$$

$$-\frac{10u_3-11}{(u_3-1)su_3}G_{1,1,0,1}(u_3)-\frac{10u_3-11}{(u_3-1)su_3}G_{1,1,1,0}(u_3)-2\frac{3u_3^4-10u_3^3-4u_3^2+18u_3-9}{s(u_3-1)^2u_3}G_{1,1,1,1}(u_3)\Bigg)i$$

$$+\Bigg(-\frac{1}{4}\frac{56u_3^4-65u_3^3-12u_3^2-26u_3+4}{(u_3-1)su_3}+\frac{1}{2}\frac{64u_3^3-101u_3^2+48u_3-8}{(u_3-1)s}\zeta_3-\frac{1}{2}\frac{13u_3^2-12u_3+2}{su_3}G_0(u_3)+\frac{5u_3^2-38u_3+38}{(u_3-1)s}G_1(u_3)$$

$$-\frac{1}{2}\frac{9u_3-38}{s}G_{0,0}(u_3)+\frac{1}{2}\frac{9u_3^2+56u_3-56}{(u_3-1)s}G_{0,1}(u_3)+\frac{1}{2}\frac{5u_3^2-39u_3-2}{su_3}G_{1,0}(u_3)-\frac{1}{2}\frac{3u_3^3+61u_3^2-82u_3+24}{(u_3-1)su_3}G_{1,1}(u_3)$$

$$+\frac{7u_3^2-8u_3-3}{su_3}G_{0,0,0}(u_3)-4\frac{u_3^2}{(u_3-1)s}G_{0,0,1}(u_3)-\frac{1}{2}\frac{(3u_3-2)(5u_3+3)}{su_3}G_{0,1,0}(u_3)+\frac{7}{2}\frac{u_3^2}{(u_3-1)s}G_{0,1,1}(u_3)$$

$$-\frac{1}{2}\frac{17u_3^2-u_3-8}{su_3}G_{1,0,0}(u_3)+\frac{1}{2}\frac{7u_3^3-8u_3^2+24u_3-16}{(u_3-1)su_3}G_{1,0,1}(u_3)+4\frac{2u_3^2+2u_3-3}{su_3}G_{1,1,0}(u_3)-2\frac{u_3^2-2u_3+2}{(u_3-1)s}G_{1,1,1}(u_3)\Bigg)\pi$$

$$+\Bigg(\frac{1}{24}\frac{64u_3^3-120u_3^2+186u_3-121}{(u_3-1)s}+\frac{1}{6}\frac{5u_3-7}{s}G_0(u_3)-\frac{1}{12}\frac{5u_3^3+4u_3^2-52u_3+32}{(u_3-1)su_3}G_1(u_3)\Bigg)\pi^3$$

$$+\Bigg(\Bigg(-\frac{1}{4}\frac{56u_3^4-111u_3^3+55u_3^2+112u_3-56}{s(u_3-1)^2}+\Bigg(\frac{1}{2}\frac{112u_3^4-270u_3^3+146u_3^2+53u_3-49}{s(u_3-1)^2}-\frac{1}{2}\frac{17u_3^2+18u_3+3}{(u_3-1)s}G_0(u_3)$$

$$+\frac{1}{2}\frac{35u_3^4-79u_3^3+38u_3^2-8u_3-2}{s(u_3-1)^2u_3}G_1(u_3)\Bigg)\zeta_2+\frac{32u_3^4-75u_3^3+69u_3^2+12u_3-6}{s(u_3-1)^2}\zeta_3-\frac{1}{2}\frac{13u_3^2-43u_3+43}{(u_3-1)s}G_0(u_3)$$

$$+\frac{5u_3^4-31u_3^3+15u_3^2-2u_3+1}{s(u_3-1)^2u_3}G_1(u_3)-\frac{u_3^2+4u_3-4}{(u_3-1)s}G_{0,0}(u_3)+\frac{3}{2}\frac{8u_3-9}{(u_3-1)s}G_{0,1}(u_3)+\frac{3}{2}\frac{8u_3-9}{(u_3-1)s}G_{1,0}(u_3)$$

$$+\frac{1}{2}\frac{4u_3^3-12u_3^2-49u_3+17}{s(u_3-1)^2}G_{1,1}(u_3)-\frac{7u_3^2-u_3+1}{(u_3-1)s}G_{0,0,0}(u_3)+\frac{1}{2}\frac{4u_3^2+19u_3-6}{(u_3-1)su_3}G_{0,0,1}(u_3)+\frac{1}{2}\frac{4u_3^2+19u_3-6}{(u_3-1)su_3}G_{0,1,0}(u_3)$$

$$-\frac{1}{2}\frac{6u_3^2-4u_3+11}{(u_3-1)s}G_{0,1,1}(u_3)+\frac{1}{2}\frac{12u_3^3-14u_3^2+u_3+6}{(u_3-1)su_3}G_{1,0,0}(u_3)+\frac{1}{2}\frac{(3u_3+2)(4u_3-3)}{(u_3-1)su_3}G_{1,0,1}(u_3)$$

$$+\frac{1}{2}\frac{(3u_3+2)(4u_3-3)}{(u_3-1)su_3}G_{1,1,0}(u_3)+\frac{4u_3^4-14u_3^3-6u_3^2+19u_3-7}{s(u_3-1)^2u_3}G_{1,1,1}(u_3)\Bigg)i+\Bigg(\frac{12u_3^4-3u_3^3-15u_3^2+2u_3-1}{(u_3-1)su_3}$$

$$+\frac{3}{2}\frac{u_3-9}{s}G_0(u_3)+18\frac{1}{s}G_1(u_3)-\frac{1}{2}\frac{17u_3^2-7u_3-6}{su_3}G_{0,0}(u_3)-3\frac{u_3^2}{(u_3-1)s}G_{0,1}(u_3)+\frac{6u_3^2+u_3-3}{su_3}G_{1,0}(u_3)$$

$$+\frac{1}{2}\frac{11u_3^3+4u_3^2-12u_3+8}{(u_3-1)su_3}G_{1,1}(u_3)\Bigg)\pi+\frac{1}{6}\frac{4u_3^3-22u_3^2+24u_3-9}{(u_3-1)s}\pi^3\Bigg)\log(\frac{m_t^2}{t})+\Bigg(\Bigg(\frac{3}{4}\frac{8u_3^4-9u_3^3-6u_3^2+30u_3-15}{s(u_3-1)^2}$$

$$-\frac{1}{4}\frac{8u_3^4+48u_3^3-158u_3^2+121u_3-35}{s(u_3-1)^2}\zeta_2+\frac{1}{4}\frac{5u_3^2-46u_3+46}{(u_3-1)s}G_0(u_3)-\frac{u_3^3-6u_3^2-7u_3+2}{s(u_3-1)^2}G_1(u_3)-\frac{3}{4}\frac{u_3^2-6u_3+6}{(u_3-1)s}G_{0,0}(u_3)$$

$$-\frac{1}{2}\frac{(2u_3-1)(2u_3+3)}{(u_3-1)su_3}G_{0,1}(u_3)-\frac{1}{2}\frac{(2u_3-1)(2u_3+3)}{(u_3-1)su_3}G_{1,0}(u_3)+\frac{1}{4}\frac{3u_3^3+9u_3^2+9u_3-13}{s(u_3-1)^2}G_{1,1}(u_3)\Bigg)i$$

$$+\Big(-\frac{10u_3^3-14u_3^2+u_3+2}{(u_3-1)s}+\frac{1}{2}\frac{(u_3+1)(5u_3-3)}{su_3}G_0(u_3)-4\frac{u_3^2}{(u_3-1)s}G_1(u_3)\Big)\pi\Big)\log^2(\frac{m_t^2}{t})$$

$$+\Big(\Big(-\frac{1}{12}\frac{40u_3^4-95u_3^3+33u_3^2+124u_3-62}{s(u_3-1)^2}+\frac{1}{12}\frac{13u_3^2-16u_3+16}{(u_3-1)s}G_0(u_3)-\frac{1}{12}\frac{19u_3^4-15u_3^3+9u_3^2+u_3-6}{s(u_3-1)^2u_3}G_1(u_3)\Big)i$$

$$+\frac{1}{12}\frac{8u_3^4+18u_3^3-24u_3^2+23u_3-6}{(u_3-1)su_3}\pi\Big)\log^3(\frac{m_t^2}{t})+\frac{1}{12}\frac{2u_3^4+4u_3^3-7u_3^2+6u_3-3}{s(u_3-1)^2}i\log^4(\frac{m_t^2}{t})\Big)m_t^2+\Big(\Big(\Big(12\frac{u_3^2-u_3+1}{u_3-1}$$

$$+\Big(-\frac{1}{2}\frac{19u_3^3-37u_3^2+28u_3-6}{(u_3-1)u_3}+\frac{7}{2}G_0(u_3)+\frac{3u_3^3+2u_3^2-5u_3+3}{(u_3-1)u_3^2}G_1(u_3)\Big)\zeta_2-\frac{8u_3^2-11u_3+11}{u_3-1}\zeta_3+\frac{7}{4}G_0(u_3)$$

$$-\frac{1}{4}\frac{7u_3^2+27u_3-14}{(u_3-1)u_3}G_1(u_3)-\frac{u_3^2-2u_3+2}{u_3-1}G_{0,0}(u_3)+\frac{1}{4}(4u_3-7)G_{0,1}(u_3)+\frac{1}{4}(4u_3-7)G_{1,0}(u_3)$$

$$-\frac{1}{2}\frac{2u_3^4-7u_3^3-4u_3^2+14u_3-8}{(u_3-1)u_3^2}G_{1,1}(u_3)+\frac{5}{2}G_{0,0,0}(u_3)-\frac{1}{2}G_{0,0,1}(u_3)-\frac{1}{2}G_{0,1,0}(u_3)-\frac{1}{2}G_{0,1,1}(u_3)-\frac{1}{2}G_{1,0,0}(u_3)$$

$$-1G_{1,0,1}(u_3)-1G_{1,1,0}(u_3)+\frac{1}{2}\frac{3u_3^3+3u_3^2-7u_3+3}{(u_3-1)u_3^2}G_{1,1,1}(u_3)\Big)i+\Big(\frac{1}{4}\frac{20u_3^2+u_3-14}{u_3}+\frac{1}{4}\frac{3u_3-7}{u_3-1}G_0(u_3)$$

$$-\frac{1}{2}\frac{u_3^2-8u_3+8}{u_3^2}G_1(u_3)+\frac{1}{2}G_{0,0}(u_3)-\frac{1}{2}G_{1,0}(u_3)-\frac{1}{2}\frac{(u_3-1)^2}{u_3^2}G_{1,1}(u_3)\Big)\pi+\frac{1}{12}(-4u_3+3)\pi^3\Big)\log(\frac{m_t^2}{t})$$

$$+\Big(\Big(\frac{1}{8}\frac{20u_3^2-19u_3+19}{u_3-1}-\frac{1}{4}\frac{5u_3+1}{u_3-1}\zeta_2+\frac{3}{4}G_0(u_3)-\frac{1}{8}\frac{5u_3^2-3u_3+4}{(u_3-1)u_3}G_1(u_3)-\frac{3}{4}G_{0,0}(u_3)+\frac{1}{2}G_{0,1}(u_3)+\frac{1}{2}G_{1,0}(u_3)$$

$$-\frac{1}{2}\frac{u_3^3+u_3^2-2u_3+1}{(u_3-1)u_3^2}G_{1,1}(u_3)\Big)i+\Big(\frac{1}{8}\frac{6u_3^2-5u_3+4}{u_3}-\frac{1}{2}G_0(u_3)+\frac{1}{4}\frac{u_3^2-2u_3+2}{u_3^2}G_1(u_3)\Big)\pi\Big)\log^2(\frac{m_t^2}{t})+\Big(\Big(\frac{1}{4}\frac{u_3^2-2u_3+2}{u_3-1}$$

$$-\frac{1}{12}G_0(u_3)+\frac{1}{12}\frac{u_3+1}{u_3-1}G_1(u_3)\Big)i+\frac{1}{12}(-2u_3+1)\pi\Big)\log^3(\frac{m_t^2}{t})-\frac{1}{48}\frac{2u_3^2-3u_3+3}{u_3-1}i\log^4(\frac{m_t^2}{t})\Big)\frac{1}{N_c}$$

$$+\Big(\Big(2\frac{15u_3^4-15u_3^3+16u_3^2-2u_3+1}{(u_3-1)u_3^2}+\Big(-\frac{1}{2}\frac{102u_3^3-117u_3^2+217u_3-116}{(u_3-1)u_3}+\frac{1}{2}\frac{61u_3^2-7}{u_3-1}G_0(u_3)$$

$$+\frac{1}{2}\frac{61u_3^4-121u_3^3+223u_3^2-235u_3+92}{(u_3-1)u_3^2}G_1(u_3)+\frac{u_3^2+u_3-1}{u_3-1}G_{0,0}(u_3)+\frac{1}{2}\frac{4u_3^4-5u_3^3+19u_3^2-48u_3+24}{(u_3-1)u_3^2}G_{0,1}(u_3)$$

$$+\frac{2u_3^2-4u_3+7}{u_3}G_{1,0}(u_3)+\frac{1}{2}\frac{2u_3^4-12u_3^3-19u_3^2+50u_3-33}{(u_3-1)u_3^2}G_{1,1}(u_3)\Big)\zeta_2+\Big(\frac{14u_3^2-27u_3+27}{u_3-1}-1G_0(u_3)$$

$$+\frac{7u_3^2-31u_3+22}{(u_3-1)u_3}G_1(u_3)\Big)\zeta_3+\frac{1}{4}\frac{14u_3^2+u_3-76}{u_3-1}\zeta_4+\frac{1}{3}\frac{33u_3^4+57u_3^3-59u_3^2+4u_3-2}{(u_3-1)u_3^2}G_0(u_3)+\frac{11u_3^3-41u_3^2+55u_3-22}{(u_3-1)u_3}G_1(u_3)$$

$$-\frac{1}{12}\frac{252u_3^4-99u_3^3+95u_3^2+8u_3-4}{(u_3-1)u_3^2}G_{0,0}(u_3)+\frac{1}{12}\frac{180u_3^3-243u_3^2+89u_3+2}{(u_3-1)u_3}G_{0,1}(u_3)+\frac{1}{12}\frac{180u_3^3-243u_3^2+89u_3+2}{(u_3-1)u_3}G_{1,0}(u_3)$$

$$-\frac{1}{12}\frac{252u_3^4-531u_3^3+947u_3^2-664u_3+216}{(u_3-1)u_3^2}G_{1,1}(u_3)+\frac{1}{4}\frac{68u_3^2-3u_3+3}{u_3-1}G_{0,0,0}(u_3)-\frac{1}{6}\frac{48u_3^2-47u_3-13}{u_3-1}G_{0,0,1}(u_3)$$

$$-\frac{1}{6}\frac{48u_3^2-47u_3-13}{u_3-1}G_{0,1,0}(u_3)-\frac{1}{12}\frac{12u_3^4-109u_3^3+223u_3^2-200u_3+100}{(u_3-1)u_3^2}G_{0,1,1}(u_3)-\frac{1}{6}\frac{6u_3^3+115u_3^2-247u_3+156}{(u_3-1)u_3}G_{1,0,0}(u_3)$$

$$-\frac{1}{12}\frac{96u_3^4-133u_3^3+175u_3^2-68u_3-44}{(u_3-1)u_3^2}G_{1,0,1}(u_3)-\frac{1}{12}\frac{96u_3^4-133u_3^3+175u_3^2-68u_3-44}{(u_3-1)u_3^2}G_{1,1,0}(u_3)$$

$$+\frac{1}{2}\frac{34u_3^4-54u_3^3+139u_3^2-139u_3+58}{(u_3-1)u_3^2}G_{1,1,1}(u_3)+\frac{1}{2}\frac{2u_3^2-15u_3+15}{u_3-1}G_{0,0,0,0}(u_3)+\frac{1}{2}\frac{5u_3-7}{u_3-1}G_{0,0,0,1}(u_3)$$

$$+\frac{1}{2}\frac{5u_3-7}{u_3-1}G_{0,0,1,0}(u_3)-\frac{3}{2}G_{0,0,1,1}(u_3)+\frac{1}{2}\frac{5u_3-7}{u_3-1}G_{0,1,0,0}(u_3)-2\frac{u_3^2+2u_3-2}{u_3^2}G_{0,1,0,1}(u_3)-2\frac{u_3^2+2u_3-2}{u_3^2}G_{0,1,1,0}(u_3)$$

$$-\frac{u_3^4-5u_3^3-3u_3^2+8u_3-4}{(u_3-1)u_3^2}G_{0,1,1,1}(u_3)+\frac{1}{2}(-2u_3+7)G_{1,0,0,0}(u_3)-\frac{3u_3-2}{u_3-1}G_{1,0,0,1}(u_3)-\frac{3u_3-2}{u_3-1}G_{1,0,1,0}(u_3)$$

$$+\frac{1}{2}\frac{u_3^2+15u_3-10}{(u_3-1)u_3}G_{1,0,1,1}(u_3)-\frac{27u_3^2-52u_3+26}{(u_3-1)u_3^2}G_{1,1,0,0}(u_3)+\frac{1}{2}\frac{6u_3^3+23u_3^2-53u_3+30}{(u_3-1)u_3^2}G_{1,1,0,1}(u_3)$$

$$+\frac{1}{2}\frac{6u_3^3+23u_3^2-53u_3+30}{(u_3-1)u_3^2}G_{1,1,1,0}(u_3)+\frac{1}{2}\frac{2u_3^4-9u_3^3-56u_3^2+84u_3-45}{(u_3-1)u_3^2}G_{1,1,1,1}(u_3)\Big)i+\Big(-\frac{3u_3^3-3u_3^2-11u_3+22}{(u_3-1)u_3}$$

$$+\frac{3u_3^2-2u_3-2}{u_3-1}\zeta_3-\frac{1}{12}\frac{28u_3^3+59u_3^2+95u_3-2}{(u_3-1)u_3}G_0(u_3)-\frac{1}{12}\frac{28u_3^4-115u_3^3+331u_3^2-432u_3+216}{(u_3-1)u_3^2}G_1(u_3)$$

$$-\frac{1}{6}\frac{12u_3^2-73u_3+13}{u_3-1}G_{0,0}(u_3)+\frac{25}{12}\frac{5u_3^2-4u_3+4}{u_3^2}G_{0,1}(u_3)-\frac{1}{12}\frac{175u_3^3-291u_3^2+244u_3-44}{(u_3-1)u_3^2}G_{1,0}(u_3)$$

$$-\frac{1}{6}\frac{12u_3^4+24u_3^3-116u_3^2+149u_3-82}{(u_3-1)u_3^2}G_{1,1}(u_3)+\frac{1}{2}(2u_3-7)G_{0,0,0}(u_3)+\frac{3}{2}G_{0,0,1}(u_3)+\frac{(u_3-2)^2}{u_3^2}G_{0,1,0}(u_3)$$

$$-\frac{1}{2}\frac{2u_3^2-3u_3-1}{u_3-1}G_{0,1,1}(u_3)+(-u_3+1)G_{1,0,0}(u_3)-\frac{3u_3-5}{u_3}G_{1,0,1}(u_3)+\frac{1}{2}\frac{5u_3^2-29u_3+22}{u_3^2}G_{1,1,0}(u_3)$$

$$
\begin{aligned}
&+\frac{1}{2}\frac{2u_3^4-2u_3^3-21u_3^2+27u_3-12}{(u_3-1)u_3^2}G_{1,1,1}(u_3)\bigg)\pi+\bigg(\frac{1}{12}\frac{6u_3^3+25u_3^2-56u_3+39}{(u_3-1)u_3}+\frac{1}{6}(3u_3-2)G_0(u_3)\\
&+\frac{1}{4}\frac{(u_3-1)(2u_3^2-3u_3+13)}{u_3^2}G_1(u_3)\bigg)\pi^3+\bigg(\bigg(-\frac{1}{12}\frac{1692u_3^6-3999u_3^5+2853u_3^4+2276u_3^3-1098u_3^2-48u_3+16}{s(u_3-1)^2u_3^2}\\
&+\bigg(\frac{1}{4}\frac{816u_3^4-1669u_3^3+685u_3^2+1080u_3-520}{s(u_3-1)^2}+\frac{1}{4}\frac{136u_3^4-367u_3^3-51u_3^2+806u_3-404}{s(u_3-1)^2}G_0(u_3)\\
&+\frac{1}{4}\frac{136u_3^5-97u_3^4-235u_3^3+743u_3^2-387u_3+80}{s(u_3-1)^2u_3}G_1(u_3)-\frac{1}{2}\frac{8u_3^4-40u_3^3+67u_3^2-30u_3+3}{s(u_3-1)^2}G_{0,0}(u_3)\\
&-\frac{1}{2}\frac{16u_3^4-76u_3^3+93u_3^2-75u_3+18}{s(u_3-1)^2}G_{0,1}(u_3)-\frac{1}{2}\frac{16u_3^4+12u_3^3-101u_3^2+168u_3-64}{(u_3-1)su_3}G_{1,0}(u_3)\\
&-\frac{1}{2}\frac{8u_3^5+24u_3^4-157u_3^3+209u_3^2-204u_3+76}{s(u_3-1)^2u_3}G_{1,1}(u_3)\bigg)\zeta_2+\bigg(-\frac{1}{2}\frac{144u_3^4-287u_3^3+u_3^2+572u_3-286}{s(u_3-1)^2}-\frac{8u_3^2+9u_3-9}{(u_3-1)s}G_0(u_3)\\
&+2\frac{2u_3^4-8u_3^3+72u_3^2-81u_3+34}{s(u_3-1)^2u_3}G_1(u_3)\bigg)\zeta_3+\frac{1}{8}\frac{418u_3^4-1143u_3^3+1419u_3^2-12u_3-234}{s(u_3-1)^2}\zeta_4\\
&-\frac{1}{12}\frac{88u_3^6-59u_3^5-677u_3^4+1464u_3^3-712u_3^2-24u_3+8}{s(u_3-1)^2u_3^2}G_0(u_3)-\frac{1}{12}\frac{88u_3^5-233u_3^4+703u_3^3+572u_3^2-142u_3-12}{s(u_3-1)^2u_3}G_1(u_3)\\
&+\frac{1}{6}\frac{24u_3^6-76u_3^5+199u_3^4-238u_3^3+99u_3^2+24u_3-8}{s(u_3-1)^2u_3^2}G_{0,0}(u_3)-\frac{1}{6}\frac{3u_3^4-42u_3^3+213u_3^2-134u_3+6}{s(u_3-1)^2u_3}G_{0,1}(u_3)\\
&-\frac{1}{6}\frac{3u_3^4-42u_3^3+213u_3^2-134u_3+6}{s(u_3-1)^2u_3}G_{1,0}(u_3)+\frac{1}{6}\frac{24u_3^4-14u_3^3-103u_3^2-210u_3+115}{s(u_3-1)^2}G_{1,1}(u_3)\\
&+\frac{1}{2}\frac{20u_3^4-47u_3^3-13u_3^2+120u_3-60}{s(u_3-1)^2}G_{0,0,0}(u_3)-\frac{1}{3}\frac{(u_3-3)(3u_3^2-23u_3+8)}{s(u_3-1)^2}G_{0,0,1}(u_3)-\frac{1}{3}\frac{(u_3-3)(3u_3^2-23u_3+8)}{s(u_3-1)^2}G_{0,1,0}(u_3)\\
&-\frac{1}{6}\frac{60u_3^4-102u_3^3+212u_3^2-205u_3+103}{s(u_3-1)^2}G_{0,1,1}(u_3)-\frac{1}{6}\frac{60u_3^5-159u_3^4+59u_3^3+112u_3^2-108u_3+24}{s(u_3-1)^2u_3}G_{1,0,0}(u_3)\\
&+\frac{1}{3}\frac{65u_3^3-169u_3^2+64u_3+6}{s(u_3-1)^2u_3}G_{1,0,1}(u_3)+\frac{1}{3}\frac{65u_3^3-169u_3^2+64u_3+6}{s(u_3-1)^2u_3}G_{1,1,0}(u_3)\\
&+\frac{1}{2}\frac{20u_3^5-36u_3^4+9u_3^3-75u_3^2+62u_3-30}{s(u_3-1)^2u_3}G_{1,1,1}(u_3)-\frac{1}{2}\frac{8u_3^4-25u_3^3-31u_3^2+112u_3-56}{s(u_3-1)^2}G_{0,0,0,0}(u_3)\\
&-\frac{1}{2}\frac{15u_3^3-24u_3^2-23u_3+24}{s(u_3-1)^2}G_{0,0,0,1}(u_3)-\frac{1}{2}\frac{15u_3^3-24u_3^2-23u_3+24}{s(u_3-1)^2}G_{0,0,1,0}(u_3)+\frac{1}{2}\frac{(3u_3-2)(5u_3+2)}{(u_3-1)s}G_{0,0,1,1}(u_3)\\
&+\frac{1}{2}\frac{5u_3^3-36u_3^2+63u_3-24}{s(u_3-1)^2}G_{0,1,0,0}(u_3)-\frac{1}{2}\frac{17u_3^2-30u_3+18}{(u_3-1)s}G_{0,1,0,1}(u_3)-\frac{1}{2}\frac{17u_3^2-30u_3+18}{(u_3-1)s}G_{0,1,1,0}(u_3)\\
&+\frac{1}{2}\frac{8u_3^4-12u_3^3-39u_3^2+18u_3+1}{s(u_3-1)^2}G_{0,1,1,1}(u_3)+\frac{1}{2}\frac{8u_3^3-u_3^2-43u_3+24}{(u_3-1)s}G_{1,0,0,0}(u_3)+\frac{1}{2}\frac{15u_3^3-15u_3^2+21u_3-13}{s(u_3-1)^2}G_{1,0,0,1}(u_3)\\
&+\frac{1}{2}\frac{15u_3^3-15u_3^2+21u_3-13}{s(u_3-1)^2}G_{1,0,1,0}(u_3)-\frac{1}{2}\frac{7u_3^4+8u_3^3+76u_3^2-91u_3+24}{s(u_3-1)^2u_3}G_{1,0,1,1}(u_3)-\frac{1}{2}\frac{21u_3^3-55u_3^2+36u_3-10}{s(u_3-1)^2}G_{1,1,0,0}(u_3)\\
&+\frac{1}{2}\frac{17u_3^4-64u_3^3+59u_3^2-68u_3+32}{s(u_3-1)^2u_3}G_{1,1,0,1}(u_3)+\frac{1}{2}\frac{17u_3^4-64u_3^3+59u_3^2-68u_3+32}{s(u_3-1)^2u_3}G_{1,1,1,0}(u_3)\\
&-\frac{1}{2}\frac{8u_3^5-4u_3^4-74u_3^3-47u_3^2-53u_3+44}{s(u_3-1)^2u_3}G_{1,1,1,1}(u_3)\bigg)i+\bigg(\frac{1}{12}\frac{976u_3^5-2261u_3^4+1447u_3^3+116u_3^2-202u_3+12}{s(u_3-1)^2u_3}\\
&-\frac{1}{2}\frac{84u_3^4-249u_3^3+303u_3^2-162u_3+16}{s(u_3-1)^2}\zeta_3+\frac{1}{6}\frac{46u_3^4-6u_3^3-153u_3^2+110u_3+6}{(u_3-1)su_3}G_0(u_3)\\
&+\frac{1}{6}\frac{46u_3^4-83u_3^3+178u_3^2-190u_3+95}{s(u_3-1)^2}G_1(u_3)+\frac{1}{3}\frac{(2u_3-3)(12u_3^2-7u_3-8)}{(u_3-1)s}G_{0,0}(u_3)-\frac{10u_3^4-17u_3^3+41u_3^2-48u_3+24}{s(u_3-1)^2}G_{0,1}(u_3)\\
&-\frac{1}{2}\frac{20u_3^3-15u_3^2-41u_3+4}{su_3}G_{1,0}(u_3)+\frac{1}{6}\frac{48u_3^5-95u_3^4+323u_3^3-407u_3^2+277u_3-78}{s(u_3-1)^2u_3}G_{1,1}(u_3)\\
&-\frac{1}{2}\frac{8u_3^3+2u_3^2-49u_3+24}{(u_3-1)s}G_{0,0,0}(u_3)+\frac{1}{2}\frac{7u_3^2-11u_3+11}{(u_3-1)s}G_{0,0,1}(u_3)-\frac{1}{2}\frac{5u_3^2+17u_3+3}{(u_3-1)s}G_{0,1,0}(u_3)\\
&+\frac{1}{2}\frac{8u_3^4-28u_3^3+31u_3^2-26u_3+7}{s(u_3-1)^2}G_{0,1,1}(u_3)+\frac{1}{2}\frac{8u_3^3+12u_3^2-41u_3+28}{(u_3-1)s}G_{1,0,0}(u_3)+\frac{1}{2}\frac{3u_3^3+u_3^2-43u_3+24}{(u_3-1)su_3}G_{1,0,1}(u_3)\\
&-\frac{1}{2}\frac{9u_3^3-3u_3^2-37u_3+32}{(u_3-1)su_3}G_{1,1,0}(u_3)-\frac{1}{2}\frac{8u_3^4-22u_3^3+29u_3^2-70u_3+31}{s(u_3-1)^2}G_{1,1,1}(u_3)\bigg)\pi\\
&+\bigg(-\frac{1}{24}\frac{244u_3^4-530u_3^3+406u_3^2-103u_3+23}{s(u_3-1)^2}-\frac{1}{4}\frac{8u_3^3+3u_3^2-23u_3+4}{(u_3-1)s}G_0(u_3)-\frac{1}{12}\frac{24u_3^4-75u_3^3+27u_3^2+36u_3-44}{(u_3-1)su_3}G_1(u_3)\bigg)\pi^3\\
&+\bigg(\bigg(\frac{1}{12}\frac{888u_3^6-2025u_3^5+1209u_3^4+1624u_3^3-792u_3^2-24u_3+8}{s(u_3-1)^2u_3^2}+\bigg(-\frac{1}{2}\frac{286u_3^4-667u_3^3+478u_3^2+128u_3-105}{s(u_3-1)^2}
\end{aligned}
$$

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\begin{aligned}
&-\frac{1}{2}\,\frac{48u_3^3+13u_3^2-81u_3+21}{(u_3-1)s}\,G_0(u_3)-\frac{1}{2}\,\frac{48u_3^5-151u_3^4+158u_3^3+58u_3^2-93u_3+24}{s(u_3-1)^2u_3}\,G_1(u_3)\Big)\zeta_2\\
&-\frac{38u_3^4-100u_3^3+107u_3^2-14u_3+7}{s(u_3-1)^2}\,\zeta_3+\frac{1}{3}\,\frac{11u_3^6-3u_3^5-128u_3^4+258u_3^3-119u_3^2-12u_3+4}{s(u_3-1)^2u_3^2}\,G_0(u_3)\\
&+\frac{1}{6}\,\frac{22u_3^5-33u_3^4+107u_3^3+233u_3^2-121u_3+6}{s(u_3-1)^2u_3}\,G_1(u_3)-\frac{1}{6}\,\frac{12u_3^4+u_3^3-19u_3^2+36u_3-18}{s(u_3-1)^2}\,G_{0,0}(u_3)\\
&+\frac{1}{3}\,\frac{3u_3^3-16u_3^2+69u_3-28}{s(u_3-1)^2}\,G_{0,1}(u_3)+\frac{1}{3}\,\frac{3u_3^3-16u_3^2+69u_3-28}{s(u_3-1)^2}\,G_{1,0}(u_3)-\frac{1}{6}\,\frac{12u_3^4-13u_3^3-100u_3^2-24u_3-29}{s(u_3-1)^2}\,G_{1,1}(u_3)\\
&-2\,\frac{u_3^2+2u_3-2}{(u_3-1)s}\,G_{0,0,0}(u_3)+\frac{1}{2}\,\frac{7u_3^3-23u_3^2-u_3+9}{s(u_3-1)^2}\,G_{0,0,1}(u_3)+\frac{1}{2}\,\frac{7u_3^3-23u_3^2-u_3+9}{s(u_3-1)^2}\,G_{0,1,0}(u_3)\\
&+\frac{1}{2}\,\frac{u_3^3+23u_3^2+4u_3-12}{s(u_3-1)^2}\,G_{0,1,1}(u_3)-\frac{1}{2}\,\frac{2u_3^3-4u_3^2+19u_3-9}{s(u_3-1)^2}\,G_{1,0,0}(u_3)-\frac{1}{2}\,\frac{5u_3^3-21u_3^2-11u_3+11}{s(u_3-1)^2}\,G_{1,0,1}(u_3)\\
&-\frac{1}{2}\,\frac{5u_3^3-21u_3^2-11u_3+11}{s(u_3-1)^2}\,G_{1,1,0}(u_3)+\frac{1}{2}\,\frac{u_3^4-39u_3^3-36u_3^2-16u_3+12}{s(u_3-1)^2u_3}\,G_{1,1,1}(u_3)\Big)i\\
&+\Big(-\frac{1}{6}\,\frac{214u_3^5-426u_3^4+140u_3^3+191u_3^2-91u_3-6}{s(u_3-1)^2u_3}-\frac{1}{3}\,\frac{28u_3^3-38u_3^2-15u_3+28}{(u_3-1)s}\,G_0(u_3)\\
&-\frac{1}{3}\,\frac{28u_3^4-37u_3^3+14u_3^2+46u_3-23}{s(u_3-1)^2}\,G_1(u_3)+\frac{1}{2}\,\frac{8u_3^3+2u_3^2-26u_3+9}{(u_3-1)s}\,G_{0,0}(u_3)+\frac{1}{2}\,\frac{5u_3^2-3u_3+3}{(u_3-1)s}\,G_{0,1}(u_3)\\
&-\frac{1}{2}\,\frac{10u_3^2-7u_3-2}{(u_3-1)s}\,G_{1,0}(u_3)+\frac{1}{2}\,\frac{8u_3^5-21u_3^4+5u_3^3+7u_3^2-27u_3+12}{s(u_3-1)^2u_3}\,G_{1,1}(u_3)\Big)\pi-\frac{1}{12}\,\frac{10u_3^3-59u_3^2+85u_3-12}{(u_3-1)s}\,\pi^3\Big)\log(\frac{m_t^2}{t})\\
&+\Big(\Big(-\frac{1}{12}\,\frac{192u_3^6-366u_3^5+57u_3^4+610u_3^3-285u_3^2-24u_3+8}{s(u_3-1)^2u_3^2}-\frac{1}{4}\,\frac{70u_3^4-249u_3^3+270u_3^2-174u_3+39}{s(u_3-1)^2}\,\zeta_2\\
&-\frac{1}{6}\,\frac{22u_3^4-47u_3^3+94u_3-47}{s(u_3-1)^2}\,G_0(u_3)-\frac{1}{12}\,\frac{(u_3+1)(44u_3^3-132u_3^2+195u_3-40)}{s(u_3-1)^2}\,G_1(u_3)+\frac{1}{4}\,\frac{8u_3^4-9u_3^3-u_3^2+20u_3-10}{s(u_3-1)^2}\,G_{0,0}(u_3)\\
&-\frac{1}{4}\,\frac{(u_3+3)(3u_3-1)u_3}{s(u_3-1)^2}\,G_{0,1}(u_3)-\frac{1}{4}\,\frac{(u_3+3)(3u_3-1)u_3}{s(u_3-1)^2}\,G_{1,0}(u_3)+\frac{1}{4}\,\frac{(8u_3^3-17u_3^2+23u_3+32)u_3}{s(u_3-1)^2}\,G_{1,1}(u_3)\Big)i\\
&+\Big(\frac{1}{12}\,\frac{(2u_3-1)(67u_3^3-138u_3^2+75u_3+40)}{s(u_3-1)^2}+\frac{1}{4}\,\frac{(2u_3+1)(4u_3-3)u_3}{(u_3-1)s}\,G_0(u_3)+\frac{1}{4}\,\frac{8u_3^4-22u_3^3+19u_3^2+6u_3-3}{s(u_3-1)^2}\,G_1(u_3)\Big)\pi\Big)\log^2(\frac{m_t^2}{t})\\
&+\Big(\Big(\frac{1}{4}\,\frac{10u_3^4-29u_3^3+17u_3^2+24u_3-12}{s(u_3-1)^2}+\frac{1}{6}\,\frac{u_3^2+5u_3-5}{(u_3-1)s}\,G_0(u_3)-\frac{1}{12}\,\frac{2u_3^3-4u_3^2+29u_3+3}{s(u_3-1)^2}\,G_1(u_3)\Big)i\\
&+\frac{1}{12}\,\frac{30u_3^3-63u_3^2+38u_3-3}{(u_3-1)s}\,\pi\Big)\log^3(\frac{m_t^2}{t})+\frac{1}{48}\,\frac{30u_3^4-93u_3^3+89u_3^2+8u_3-4}{s(u_3-1)^2}\,i\log^4(\frac{m_t^2}{t})\Big)m_t^2\\
&+\Big(\Big(\frac{2}{3}\,\frac{12u_3^4-12u_3^3+13u_3^2-2u_3+1}{(u_3-1)u_3^2}+\Big(-\frac{1}{2}\,\frac{31u_3^3-22u_3^2+17u_3-6}{(u_3-1)u_3}+3(2u_3-1)G_0(u_3)\\
&+\frac{6u_3^4-13u_3^3+18u_3^2-8u_3+3}{(u_3-1)u_3^2}\,G_1(u_3)\Big)\zeta_2+\frac{2u_3^2+5u_3-5}{u_3-1}\,\zeta_3+\frac{1}{12}\,\frac{44u_3^4+85u_3^3-89u_3^2+8u_3-4}{(u_3-1)u_3^2}\,G_0(u_3)\\
&+\frac{1}{12}\,\frac{44u_3^3-173u_3^2+263u_3-102}{(u_3-1)u_3}\,G_1(u_3)-\frac{1}{6}\,\frac{66u_3^2-47u_3+47}{u_3-1}\,G_{0,0}(u_3)+\frac{1}{12}\,\frac{108u_3^2-111u_3+5}{u_3-1}\,G_{0,1}(u_3)\\
&+\frac{1}{12}\,\frac{108u_3^2-111u_3+5}{u_3-1}\,G_{1,0}(u_3)-\frac{1}{6}\,\frac{66u_3^4-88u_3^3+131u_3^2-102u_3+48}{(u_3-1)u_3^2}\,G_{1,1}(u_3)+\frac{1}{2}\,G_{0,0,0}(u_3)+\frac{1}{2}\,\frac{u_3+1}{u_3-1}\,G_{0,0,1}(u_3)\\
&+\frac{1}{2}\,\frac{u_3+1}{u_3-1}\,G_{0,1,0}(u_3)-\frac{1}{2}\,\frac{3u_3+1}{u_3-1}\,G_{0,1,1}(u_3)+\frac{1}{2}\,\frac{u_3+1}{u_3-1}\,G_{1,0,0}(u_3)-\frac{1}{2}\,\frac{u_3+3}{u_3-1}\,G_{1,0,1}(u_3)-\frac{1}{2}\,\frac{u_3+3}{u_3-1}\,G_{1,1,0}(u_3)\\
&+\frac{1}{2}\,\frac{u_3^3+12u_3^2-4u_3+3}{(u_3-1)u_3^2}\,G_{1,1,1}(u_3)\Big)i+\Big(-\frac{1}{12}\,\frac{32u_3^3-47u_3^2-43u_3+102}{(u_3-1)u_3}-\frac{1}{12}\,\frac{2u_3^2+101u_3+5}{u_3-1}\,G_0(u_3)\\
&-\frac{1}{6}\,\frac{u_3^4-34u_3^3+82u_3^2-96u_3+48}{(u_3-1)u_3^2}\,G_1(u_3)+\frac{1}{2}(-2u_3+1)G_{0,0}(u_3)-\frac{1}{2}\,G_{0,1}(u_3)+\frac{1}{2}\,G_{1,0}(u_3)-\frac{1}{2}\,\frac{2u_3^4-5u_3^3-1}{(u_3-1)u_3^2}\,G_{1,1}(u_3)\Big)\pi\\
&+\frac{1}{6}(-3u_3+1)\pi^3\Big)\log(\frac{m_t^2}{t})+\Big(\Big(\frac{1}{24}\,\frac{12u_3^4+3u_3^3+u_3^2-8u_3+4}{(u_3-1)u_3^2}-\frac{3}{2}\,\frac{u_3^2+1}{u_3-1}\,\zeta_2+\frac{1}{24}\,\frac{22u_3^2-51u_3+51}{u_3-1}\,G_0(u_3)\\
&+\frac{1}{12}\,\frac{11u_3^3-15u_3^2+30u_3-6}{(u_3-1)u_3}\,G_1(u_3)-\frac{1}{4}\,\frac{2u_3^2-u_3+1}{u_3-1}\,G_{0,0}(u_3)+\frac{1}{4}\,\frac{u_3+1}{u_3-1}\,G_{0,1}(u_3)+\frac{1}{4}\,\frac{u_3+1}{u_3-1}\,G_{1,0}(u_3)\\
&-\frac{1}{4}\,\frac{2u_3^4-5u_3^3+9u_3^2-4u_3+2}{(u_3-1)u_3^2}\,G_{1,1}(u_3)\Big)i+\Big(-\frac{1}{12}\,\frac{20u_3^3-27u_3^2+12u_3+6}{(u_3-1)u_3}+\frac{1}{4}(-2u_3+1)G_0(u_3)\\
&-\frac{1}{4}\,\frac{(u_3^2-2u_3+2)(2u_3^2-u_3+1)}{(u_3-1)u_3^2}\,G_1(u_3)\Big)\pi\Big)\log^2(\frac{m_t^2}{t})+\Big(\Big(-\frac{1}{4}\,\frac{u_3^2-2u_3+2}{u_3-1}+\frac{1}{12}\,G_0(u_3)-\frac{1}{12}\,G_1(u_3)\Big)i+\frac{1}{12}\,\pi\Big)\log^3(\frac{m_t^2}{t})\\
&+\frac{1}{48}\,i\log^4(\frac{m_t^2}{t})\Big)N_c+\Big(\Big(-2\,\frac{u_3-1}{u_3^2}+\Big(-4\,\frac{(u_3-1)(u_3+1)}{u_3}+(u_3-2)G_0(u_3)+\frac{u_3^3-4u_3+4}{u_3^2}\,G_1(u_3)\Big)\zeta_2
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{2}{3}\frac{3u_3^4-3u_3^3+2u_3^2+2u_3-1}{(u_3-1)u_3^2}G_0(u_3)-2\frac{u_3^2-u_3+1}{u_3-1}G_1(u_3)-\frac{1}{3}\frac{u_3-1}{u_3^2}G_{0,0}(u_3)+\frac{2}{3}\frac{2u_3-1}{(u_3-1)u_3}G_{0,1}(u_3)\\
&+\frac{2}{3}\frac{2u_3-1}{(u_3-1)u_3}G_{1,0}(u_3)+\frac{4}{3}\frac{2u_3-1}{(u_3-1)u_3}G_{1,1}(u_3)-\frac{1}{2}G_{0,0,0}(u_3)+\frac{1}{6}G_{0,0,1}(u_3)+\frac{1}{6}G_{0,1,0}(u_3)+\frac{1}{6}\frac{u_3^3-7u_3^2+8u_3-4}{(u_3-1)u_3^2}G_{0,1,1}(u_3)\\
&+\frac{1}{6}G_{1,0,0}(u_3)+\frac{1}{6}\frac{u_3^3-7u_3^2+8u_3-4}{(u_3-1)u_3^2}G_{1,0,1}(u_3)+\frac{1}{6}\frac{u_3^3-7u_3^2+8u_3-4}{(u_3-1)u_3^2}G_{1,1,0}(u_3)-\frac{1}{2}\frac{u_3^3+5u_3^2-8u_3+4}{(u_3-1)u_3^2}G_{1,1,1}(u_3)\Big)i\\
&+\Big(2\frac{u_3^2-u_3+1}{u_3-1}-\frac{2}{3}\frac{(u_3-1)(u_3+1)}{u_3}G_0(u_3)-\frac{2}{3}\frac{u_3^2-u_3+1}{u_3-1}G_1(u_3)-\frac{1}{6}G_{0,0}(u_3)-\frac{1}{6}\frac{u_3^2+4u_3-4}{u_3^2}G_{0,1}(u_3)\\
&-\frac{1}{6}\frac{u_3^2+4u_3-4}{u_3^2}G_{1,0}(u_3)+\frac{1}{6}\frac{(3u_3-2)(u_3^2-u_3+2)}{(u_3-1)u_3^2}G_{1,1}(u_3)\Big)\pi+\frac{1}{6}(-u_3+1)\pi^3+\Big(\Big(\Big(-\frac{4}{3}\frac{u_3-1}{su_3^2}+\Big(-2\frac{2u_3^3-u_3^2-3u_3+1}{(u_3-1)s}\\
&-2\frac{2u_3^2-u_3-2}{s}G_0(u_3)-2\frac{2u_3^3-u_3^2+u_3-1}{(u_3-1)s}G_1(u_3)\Big)\zeta_2+\frac{2}{3}\frac{2u_3^6-4u_3^5+2u_3^4+3u_3^3+u_3^2-3u_3+1}{s(u_3-1)^2u_3^2}G_0(u_3)\\
&+\frac{4}{3}\frac{u_3^4-2u_3^3+u_3^2+2u_3-1}{s(u_3-1)^2}G_1(u_3)+\frac{2}{3}\frac{u_3^4-3u_3^3+u_3^2+4u_3-2}{(u_3-1)su_3^2}G_{0,0}(u_3)+\frac{2}{3}\frac{3u_3-2}{s(u_3-1)^2}G_{0,1}(u_3)+\frac{2}{3}\frac{3u_3-2}{s(u_3-1)^2}G_{1,0}(u_3)\\
&-\frac{2}{3}\frac{u_3^3-4u_3^2+1}{s(u_3-1)^2}G_{1,1}(u_3)+2\frac{1}{s}G_{0,0,0}(u_3)-\frac{2}{3}\frac{u_3}{(u_3-1)s}G_{0,0,1}(u_3)-\frac{2}{3}\frac{u_3}{(u_3-1)s}G_{0,1,0}(u_3)-\frac{2}{3}\frac{u_3^2-5u_3+2}{s(u_3-1)^2}G_{0,1,1}(u_3)\\
&-\frac{2}{3}\frac{u_3}{(u_3-1)s}G_{1,0,0}(u_3)-\frac{2}{3}\frac{u_3^2-5u_3+2}{s(u_3-1)^2}G_{1,0,1}(u_3)-\frac{2}{3}\frac{u_3^2-5u_3+2}{s(u_3-1)^2}G_{1,1,0}(u_3)+2\frac{u_3^2+2u_3-1}{s(u_3-1)^2}G_{1,1,1}(u_3)\Big)i\\
&+\Big(-\frac{4}{3}\frac{u_3^4-2u_3^3+u_3^2+2u_3-1}{s(u_3-1)^2}-\frac{2}{3}\frac{(u_3-1)(u_3+2)}{s}G_0(u_3)-\frac{2}{3}\frac{u_3^4-2u_3^3+u_3^2+2u_3-1}{s(u_3-1)^2}G_1(u_3)+\frac{2}{3}\frac{u_3}{s}G_{0,0}(u_3)\\
&-\frac{2}{3}\frac{u_3^3+2u_3^2-2u_3+1}{s(u_3-1)^2}G_{1,1}(u_3)\Big)\pi+\frac{1}{3}\frac{2u_3^2-1}{s}\pi^3+\Big(\Big(\Big(\frac{2}{3}\frac{u_3-1}{su_3^2}+2\frac{4u_3^3-4u_3^2+1}{(u_3-1)s}\zeta_2\\
&-\frac{1}{3}\frac{2u_3^6-3u_3^5-2u_3^4+6u_3^3+7u_3^2-12u_3+4}{s(u_3-1)^2u_3^2}G_0(u_3)-\frac{1}{3}\frac{(u_3^2+u_3-1)(2u_3^2-5u_3+5)}{s(u_3-1)^2}G_1(u_3)+\frac{2}{3}\frac{u_3^2}{(u_3-1)s}G_{0,0}(u_3)\\
&-\frac{1}{3}\frac{2u_3^2+3u_3-1}{s(u_3-1)^2}G_{0,1}(u_3)-\frac{1}{3}\frac{2u_3^2+3u_3-1}{s(u_3-1)^2}G_{1,0}(u_3)-\frac{2}{3}\frac{u_3^3+u_3^2+3u_3-1}{s(u_3-1)^2}G_{1,1}(u_3)\Big)i+\Big(\frac{1}{3}\frac{(u_3^2+u_3-1)(2u_3^2-5u_3+5)}{s(u_3-1)^2}\\
&+\frac{1}{3}\frac{4u_3^2-u_3-1}{s}G_0(u_3)+\frac{1}{3}\frac{(u_3^2-2u_3+2)(4u_3^2+u_3-1)}{s(u_3-1)^2}G_1(u_3)\Big)\pi\Big)\log(\frac{m_t^2}{t})+\Big(\Big(-\frac{2}{3}\frac{u_3-1}{su_3^2}\\
&+\frac{1}{3}\frac{2u_3^4-4u_3^3+3u_3^2+2u_3-1}{s(u_3-1)^2}G_0(u_3)+\frac{1}{3}\frac{2u_3^4-4u_3^3+3u_3^2+2u_3-1}{s(u_3-1)^2}G_1(u_3)\Big)i-\frac{1}{3}\frac{2u_3^4-4u_3^3+3u_3^2+2u_3-1}{s(u_3-1)^2}\pi\Big)\log^2(\frac{m_t^2}{t})\Big)m_t^2\\
&+\Big(\Big(\Big(-\frac{2}{3}\frac{u_3-1}{u_3^2}+(-2u_3+1)\zeta_2-\frac{1}{3}\frac{2u_3^4-2u_3^3+u_3^2+2u_3-1}{(u_3-1)u_3^2}G_0(u_3)-\frac{2}{3}\frac{u_3^2-u_3+1}{u_3-1}G_1(u_3)-\frac{1}{3}G_{0,0}(u_3)+\frac{1}{3}\frac{1}{u_3-1}G_{0,1}(u_3)\\
&+\frac{1}{3}\frac{1}{u_3-1}G_{1,0}(u_3)+\frac{1}{3}\frac{u_3+1}{u_3-1}G_{1,1}(u_3)\Big)i+\Big(\frac{2}{3}\frac{u_3^2-u_3+1}{u_3-1}+\frac{1}{3}(-u_3+1)G_0(u_3)-\frac{1}{3}\frac{u_3^2-u_3+1}{u_3-1}G_1(u_3)\Big)\pi\Big)\log(\frac{m_t^2}{t})\\
&+\Big(\Big(-\frac{1}{6}\frac{u_3-1}{u_3^2}-\frac{1}{12}\frac{2u_3^2-3u_3+3}{u_3-1}G_0(u_3)-\frac{1}{12}\frac{2u_3^2-3u_3+3}{u_3-1}G_1(u_3)\Big)i+\frac{1}{12}\frac{2u_3^2-3u_3+3}{u_3-1}\pi\Big)\log^2(\frac{m_t^2}{t})\Big)N_f+\Big(-2\frac{u_3-1}{u_3^2}\\
&+\frac{2}{3}\frac{u_3-1}{u_3^2}G_0(u_3)-\frac{1}{3}\frac{u_3-1}{u_3^2}G_{0,0}(u_3)\Big)i+\Big(\Big(-\frac{148}{3}\frac{u_3-1}{su_3^2}+\frac{118}{3}\frac{u_3-1}{su_3^2}G_0(u_3)-\frac{112}{3}\frac{u_3-1}{su_3^2}G_{0,0}(u_3)+24\frac{u_3-1}{su_3^2}G_{0,0,0}(u_3)\\
&-12\frac{u_3-1}{su_3^2}G_{0,0,0,0}(u_3)\Big)i+\Big(-\frac{118}{3}\frac{u_3-1}{su_3^2}+\frac{112}{3}\frac{u_3-1}{su_3^2}G_0(u_3)-24\frac{u_3-1}{su_3^2}G_{0,0}(u_3)+12\frac{u_3-1}{su_3^2}G_{0,0,0}(u_3)\Big)i\log(\frac{m_t^2}{t})\\
&+\Big(-\frac{56}{3}\frac{u_3-1}{su_3^2}+12\frac{u_3-1}{su_3^2}G_0(u_3)-6\frac{u_3-1}{su_3^2}G_{0,0}(u_3)\Big)i\log^2(\frac{m_t^2}{t})+\Big(-4\frac{u_3-1}{su_3^2}+2\frac{u_3-1}{su_3^2}G_0(u_3)\Big)i\log^3(\frac{m_t^2}{t})\\
&-\frac{1}{2}\frac{u_3-1}{su_3^2}i\log^4(\frac{m_t^2}{t})\Big)m_t^2+\Big(-\frac{2}{3}\frac{u_3-1}{u_3^2}+\frac{1}{3}\frac{u_3-1}{u_3^2}G_0(u_3)\Big)i\log(\frac{m_t^2}{t})-\frac{1}{6}\frac{u_3-1}{u_3^2}i\log^2(\frac{m_t^2}{t})+O((m_t^2)^2)+O(m_H^2)
\end{aligned}
\tag{E.6}
$$

$$
\begin{aligned}
(\Omega^g_{+-+})^{2l}=&\Big(\Big(-10\frac{u_3^2-u_3+1}{u_3-1}+\Big(\frac{1}{2}\frac{21u_3^2-21u_3+4}{u_3-1}+\frac{12u_3^4-21u_3^3+3u_3^2+5u_3-2}{u_3-1}G_0(u_3)+\frac{12u_3^4-27u_3^3+12u_3^2+4u_3-3}{u_3-1}G_1(u_3)\\
&+(-4u_3^3+2u_3^2-3u_3-3)G_{0,0}(u_3)+\frac{1}{2}(11u_3^2-16u_3+7)u_3G_{0,1}(u_3)+\frac{1}{2}(11u_3^2-6u_3+2)u_3G_{1,0}(u_3)\\
&-\frac{(4u_3^3-10u_3^2+11u_3-8)u_3}{u_3-1}G_{1,1}(u_3)\Big)\zeta_2+\Big(-\frac{4u_3^4-9u_3^3-14u_3^2+18u_3-16}{u_3-1}+\frac{1}{2}(-2u_3^3-4u_3^2-3u_3+16)G_0(u_3)\\
&+\frac{1}{2}\frac{(2u_3^3+2u_3^2-17u_3-3)u_3}{u_3-1}G_1(u_3)\Big)\zeta_3-\frac{1}{8}\frac{183u_3^4-281u_3^3+83u_3^2+15u_3-52}{u_3-1}\zeta_4+4(u_3^2-3)G_0(u_3)\\
&-4\frac{(u_3^2-2u_3-2)u_3}{u_3-1}G_1(u_3)-\frac{1}{4}\frac{16u_3^3-33u_3^2-19u_3+20}{u_3-1}G_{0,0}(u_3)-\frac{5}{4}u_3G_{0,1}(u_3)-\frac{5}{4}u_3G_{1,0}(u_3)\\
&+\frac{1}{4}\frac{16u_3^3-15u_3^2-37u_3+16}{u_3-1}G_{1,1}(u_3)+\frac{1}{4}\frac{32u_3^4-62u_3^3+25u_3^2-9u_3+6}{u_3-1}G_{0,0,0}(u_3)-\frac{1}{4}(2u_3-1)(8u_3+1)u_3G_{0,0,1}(u_3)
\end{aligned}
$$

$$-\frac{1}{4}\left(2u_3-1\right)\left(8u_3+1\right)u_3G_{0,1,0}(u_3)+\frac{1}{4}\left(6u_3^2+11u_3-4\right)G_{0,1,1}(u_3)-\frac{1}{4}\frac{\left(6u_3^2-23u_3+13\right)u_3}{u_3-1}G_{1,0,0}(u_3)$$
$$-\frac{1}{4}\left(2u_3-1\right)\left(8u_3-9\right)u_3G_{1,0,1}(u_3)-\frac{1}{4}\left(2u_3-1\right)\left(8u_3-9\right)u_3G_{1,1,0}(u_3)+\frac{1}{4}\frac{32u_3^4-66u_3^3+31u_3^2+17u_3-8}{u_3-1}G_{1,1,1}(u_3)$$
$$+\frac{1}{2}\left(-3u_3^3+6u_3^2-5u_3-2\right)G_{0,0,0,0}(u_3)+u_3\left(2u_3-1\right)^2G_{0,0,0,1}(u_3)+u_3\left(2u_3-1\right)^2G_{0,0,1,0}(u_3)$$
$$-\frac{1}{2}\left(7u_3^2-4u_3+3\right)u_3G_{0,0,1,1}(u_3)+\left(3u_3^2-5u_3+3\right)u_3G_{0,1,0,0}(u_3)-\frac{1}{2}\left(9u_3^2-10u_3+3\right)u_3G_{0,1,0,1}(u_3)$$
$$-\frac{1}{2}\left(9u_3^2-10u_3+3\right)u_3G_{0,1,1,0}(u_3)+\left(3u_3^2-2u_3+2\right)u_3G_{0,1,1,1}(u_3)+\left(3u_3^2-4u_3+3\right)u_3G_{1,0,0,0}(u_3)$$
$$-\frac{1}{2}\left(9u_3^2-8u_3+2\right)u_3G_{1,0,0,1}(u_3)-\frac{1}{2}\left(9u_3^2-8u_3+2\right)u_3G_{1,0,1,0}(u_3)+\left(3u_3^2-u_3+1\right)u_3G_{1,0,1,1}(u_3)$$
$$-\frac{1}{2}\left(7u_3^2-10u_3+6\right)u_3G_{1,1,0,0}(u_3)+u_3\left(2u_3-1\right)^2G_{1,1,0,1}(u_3)+u_3\left(2u_3-1\right)^2G_{1,1,1,0}(u_3)$$
$$-\frac{1}{2}\frac{\left(3u_3^3-3u_3^2+2u_3-4\right)u_3}{u_3-1}G_{1,1,1,1}(u_3)\Bigg)i+\Bigg(-8u_3-\left(2u_3^2-2u_3-5\right)u_3\zeta_3-\frac{1}{4}\frac{\left(2u_3^2-13u_3-5\right)u_3}{u_3-1}G_0(u_3)$$
$$+\frac{1}{4}\left(2u_3^2+9u_3-16\right)G_1(u_3)-\frac{u_3^3}{u_3-1}G_{0,0}(u_3)+\frac{1}{2}\left(-2u_3^2+7u_3-2\right)G_{0,1}(u_3)+\frac{1}{2}\frac{\left(2u_3^2+3u_3-3\right)u_3}{u_3-1}G_{1,0}(u_3)$$
$$+\left(u_3-1\right)^2G_{1,1}(u_3)+\frac{1}{2}\left(2u_3^2-1\right)u_3G_{0,0,0}(u_3)+\frac{1}{2}\left(2u_3^2-4u_3-1\right)u_3G_{0,0,1}(u_3)-\frac{1}{2}\left(2u_3^2-3\right)u_3G_{0,1,0}(u_3)$$
$$-\frac{1}{2}\left(2u_3^2-4u_3-1\right)u_3G_{0,1,1}(u_3)-\frac{1}{2}\left(2u_3^2-3\right)u_3G_{1,0,0}(u_3)-\frac{1}{2}\left(2u_3^2-4u_3-1\right)u_3G_{1,0,1}(u_3)+\frac{1}{2}\left(2u_3^2-3\right)u_3G_{1,1,0}(u_3)$$
$$+\frac{1}{2}\left(2u_3^2-4u_3+1\right)u_3G_{1,1,1}(u_3)\Bigg)\pi+\Bigg(-\frac{1}{6}\frac{\left(u_3^2-u_3+1\right)u_3}{u_3-1}+\frac{1}{6}\left(u_3-1\right)\left(u_3+1\right)u_3G_0(u_3)-\frac{1}{6}\left(u_3^2-2\right)u_3G_1(u_3)\Bigg)\pi^3$$
$$+\Bigg(\Bigg(\Bigg(\frac{1}{2}\frac{13u_3^4-26u_3^3-152u_3^2+165u_3-72}{s\left(u_3-1\right)^2}+\Bigg(\frac{1}{4}\frac{226u_3^4-452u_3^3+233u_3^2-7u_3+8}{s\left(u_3-1\right)^2}$$
$$+\frac{1}{4}\frac{48u_3^4+81u_3^3-116u_3^2-70u_3+16}{\left(u_3-1\right)s}G_0(u_3)-\frac{1}{4}\frac{\left(48u_3^4-273u_3^3+415u_3^2-133u_3-41\right)u_3}{s\left(u_3-1\right)^2}G_1(u_3)$$
$$-\frac{1}{2}\frac{12u_3^4+5u_3^3+14u_3^2-u_3-16}{\left(u_3-1\right)s}G_{0,0}(u_3)-\frac{1}{2}\frac{\left(20u_3^3-22u_3^2+44u_3+27\right)u_3}{\left(u_3-1\right)s}G_{0,1}(u_3)$$
$$+\frac{1}{2}\frac{\left(20u_3^3-38u_3^2+60u_3-69\right)u_3}{\left(u_3-1\right)s}G_{1,0}(u_3)+\frac{1}{2}\frac{\left(12u_3^4-53u_3^3+101u_3^2-90u_3+14\right)u_3}{s\left(u_3-1\right)^2}G_{1,1}(u_3)\Bigg)\zeta_2$$
$$+\Bigg(\frac{1}{2}\frac{2u_3^5-9u_3^4+7u_3^3-88u_3^2+88u_3-32}{s\left(u_3-1\right)^2}-2\frac{14u_3^4-24u_3^3+16u_3^2-23u_3+16}{\left(u_3-1\right)s}G_0(u_3)$$
$$+\frac{1}{2}\frac{\left(56u_3^4-139u_3^3+131u_3^2+36u_3-20\right)u_3}{s\left(u_3-1\right)^2}G_1(u_3)\Bigg)\zeta_3-\frac{1}{4}\frac{260u_3^5-383u_3^4+172u_3^3+41u_3^2-90u_3+64}{s\left(u_3-1\right)^2}\zeta_4$$
$$+\frac{1}{4}\frac{4u_3^4-26u_3^3-12u_3^2-65u_3+56}{\left(u_3-1\right)s}G_0(u_3)-\frac{1}{4}\frac{\left(4u_3^4+10u_3^3-66u_3^2+151u_3-43\right)u_3}{s\left(u_3-1\right)^2}G_1(u_3)+\frac{3}{2}\frac{\left(2u_3+1\right)\left(11u_3-8\right)}{s}G_{0,0}(u_3)$$
$$-\frac{\left(38u_3^2-38u_3+5\right)u_3}{\left(u_3-1\right)s}G_{0,1}(u_3)-\frac{\left(38u_3^2-38u_3+5\right)u_3}{\left(u_3-1\right)s}G_{1,0}(u_3)+\frac{3}{2}\frac{\left(2u_3-3\right)\left(11u_3-3\right)u_3^2}{s\left(u_3-1\right)^2}G_{1,1}(u_3)$$
$$+\frac{1}{2}\frac{22u_3^4+u_3^3-6u_3^2-64u_3+40}{\left(u_3-1\right)s}G_{0,0,0}(u_3)-\frac{1}{2}\frac{\left(24u_3^3+10u_3^2-31u_3-6\right)u_3}{\left(u_3-1\right)s}G_{0,0,1}(u_3)$$
$$-\frac{1}{2}\frac{\left(24u_3^3+10u_3^2-31u_3-6\right)u_3}{\left(u_3-1\right)s}G_{0,1,0}(u_3)+\frac{1}{2}\frac{\left(22u_3^3+17u_3^2-31u_3-6\right)u_3}{\left(u_3-1\right)s}G_{0,1,1}(u_3)$$
$$-\frac{1}{2}\frac{\left(22u_3^3-83u_3^2+69u_3-2\right)u_3}{\left(u_3-1\right)s}G_{1,0,0}(u_3)+\frac{1}{2}\frac{\left(24u_3^3-82u_3^2+61u_3+3\right)u_3}{\left(u_3-1\right)s}G_{1,0,1}(u_3)$$
$$+\frac{1}{2}\frac{\left(24u_3^3-82u_3^2+61u_3+3\right)u_3}{\left(u_3-1\right)s}G_{1,1,0}(u_3)-\frac{1}{2}\frac{\left(22u_3^4-89u_3^3+129u_3^2-15u_3-7\right)u_3}{s\left(u_3-1\right)^2}G_{1,1,1}(u_3)$$
$$+2\frac{9u_3^4-18u_3^3+4u_3^2+2}{\left(u_3-1\right)s}G_{0,0,0,0}(u_3)+2\frac{\left(2u_3^2-2u_3+1\right)u_3}{\left(u_3-1\right)s}G_{0,0,0,1}(u_3)+2\frac{\left(2u_3^2-2u_3+1\right)u_3}{\left(u_3-1\right)s}G_{0,0,1,0}(u_3)$$
$$-\frac{1}{2}\frac{\left(32u_3^3-36u_3^2-12u_3-7\right)u_3}{\left(u_3-1\right)s}G_{0,0,1,1}(u_3)-\frac{1}{2}\frac{\left(40u_3^3-75u_3^2+22u_3+16\right)u_3}{\left(u_3-1\right)s}G_{0,1,0,0}(u_3)$$
$$+\frac{\left(4u_3^3-10u_3^2+6u_3+7\right)u_3}{\left(u_3-1\right)s}G_{0,1,0,1}(u_3)+\frac{\left(4u_3^3-10u_3^2+6u_3+7\right)u_3}{\left(u_3-1\right)s}G_{0,1,1,0}(u_3)+\frac{1}{2}\frac{\left(28u_3^3-27u_3^2-24u_3-9\right)u_3}{\left(u_3-1\right)s}G_{0,1,1,1}(u_3)$$
$$-\frac{1}{2}\frac{\left(28u_3^3-57u_3^2+6u_3+32\right)u_3}{\left(u_3-1\right)s}G_{1,0,0,0}(u_3)-\frac{\left(4u_3^3-2u_3^2-2u_3-7\right)u_3}{\left(u_3-1\right)s}G_{1,0,0,1}(u_3)-\frac{\left(4u_3^3-2u_3^2-2u_3-7\right)u_3}{\left(u_3-1\right)s}G_{1,0,1,0}(u_3)$$
$$+\frac{1}{2}\frac{\left(40u_3^3-45u_3^2-8u_3-3\right)u_3}{\left(u_3-1\right)s}G_{1,0,1,1}(u_3)+\frac{1}{2}\frac{\left(32u_3^3-60u_3^2+12u_3+23\right)u_3}{\left(u_3-1\right)s}G_{1,1,0,0}(u_3)+2\frac{\left(2u_3^2-2u_3+1\right)u_3}{\left(u_3-1\right)s}G_{1,1,0,1}(u_3)$$
$$+2\frac{\left(2u_3^2-2u_3+1\right)u_3}{\left(u_3-1\right)s}G_{1,1,1,0}(u_3)-2\frac{\left(9u_3^4-18u_3^3+4u_3^2+10u_3-3\right)u_3}{s\left(u_3-1\right)^2}G_{1,1,1,1}(u_3)\Bigg)i+\Bigg(-\frac{1}{4}\frac{\left(96u_3^2-96u_3+53\right)u_3}{\left(u_3-1\right)s}$$
$$+\frac{1}{2}\frac{\left(2u_3^3+35u_3^2-36u_3-4\right)u_3}{\left(u_3-1\right)s}\zeta_3+\frac{1}{2}\frac{\left(2u_3^2-12u_3+13\right)u_3}{s}G_0(u_3)-\frac{1}{2}\frac{\left(2u_3^2+8u_3+3\right)u_3^2}{\left(u_3-1\right)s}G_1(u_3)$$

$$- \frac{1}{2} \frac{(2u_3^2 + 28u_3 + 1)u_3}{s} G_{0,0}(u_3) - \frac{1}{2} \frac{(2u_3^2 - 43u_3 + 36)u_3^2}{(u_3 - 1)s} G_{0,1}(u_3) + \frac{1}{2} \frac{(2u_3^2 + 39u_3 - 5)u_3}{s} G_{1,0}(u_3)$$

$$+ \frac{1}{2} \frac{(2u_3^2 - 32u_3 + 31)u_3^2}{(u_3 - 1)s} G_{1,1}(u_3) + \frac{(u_3 - 1)(11u_3 - 1)u_3}{s} G_{0,0,0}(u_3) - 4 \frac{(3u_3^2 - 4u_3 - 1)u_3^2}{(u_3 - 1)s} G_{0,0,1}(u_3)$$

$$- \frac{1}{2} \frac{(24u_3^2 - 19u_3 - 13)u_3}{s} G_{0,1,0}(u_3) + \frac{1}{2} \frac{(22u_3^2 - 25u_3 - 8)u_3^2}{(u_3 - 1)s} G_{0,1,1}(u_3) - \frac{1}{2} \frac{(22u_3^2 - 19u_3 - 11)u_3}{s} G_{1,0,0}(u_3)$$

$$+ \frac{1}{2} \frac{(24u_3^2 - 29u_3 - 8)u_3^2}{(u_3 - 1)s} G_{1,0,1}(u_3) + 4 \frac{(3u_3^2 - 2u_3 - 2)u_3}{s} G_{1,1,0}(u_3) - \frac{(11u_3 - 10)u_3^3}{(u_3 - 1)s} G_{1,1,1}(u_3) \bigg) \pi$$

$$+ \bigg(- \frac{1}{24} \frac{(4u_3^3 - 20u_3^2 + 6u_3 - 1)u_3}{(u_3 - 1)s} + \frac{1}{6} \frac{(9u_3^2 - 13u_3 + 2)u_3}{s} G_0(u_3) - \frac{1}{12} \frac{(18u_3^3 - 21u_3^2 + 4u_3 + 16)u_3}{(u_3 - 1)s} G_1(u_3) \bigg) \pi^3$$

$$+ \bigg(\bigg(- \frac{1}{4} \frac{56u_3^4 - 112u_3^3 - 55u_3^2 + 111u_3 - 56}{s(u_3 - 1)^2} + \bigg(- \frac{1}{2} \frac{26u_3^4 - 52u_3^3 - 7u_3^2 + 33u_3 - 8}{s(u_3 - 1)^2} - \frac{1}{2} \frac{26u_3^4 - 25u_3^3 - 32u_3^2 - 5u_3 + 16}{(u_3 - 1)s} G_0(u_3)$$

$$+ \frac{1}{2} \frac{(26u_3^4 - 79u_3^3 + 49u_3^2 + 40u_3 - 20)u_3}{s(u_3 - 1)^2} G_1(u_3) \bigg) \zeta_2 + \frac{3u_3^5 - 2u_3^4 - 11u_3^3 - 68u_3^2 + 78u_3 - 32}{s(u_3 - 1)^2} \zeta_3$$

$$+ \frac{u_3^4 - 2u_3^3 + 15u_3^2 + 3u_3 - 12}{(u_3 - 1)s} G_0(u_3) - \frac{(u_3^4 - 2u_3^3 + 15u_3^2 - 31u_3 + 5)u_3}{s(u_3 - 1)^2} G_1(u_3) - \frac{1}{2} \frac{17u_3^3 - 2u_3^2 - 59u_3 + 40}{(u_3 - 1)s} G_{0,0}(u_3)$$

$$+ 18 \frac{u_3^2}{s} G_{0,1}(u_3) + 18 \frac{u_3^2}{s} G_{1,0}(u_3) - \frac{1}{2} \frac{(17u_3^3 - 49u_3^2 - 12u_3 + 4)u_3}{s(u_3 - 1)^2} G_{1,1}(u_3) - \frac{7u_3^4 - 9u_3^3 - 9u_3^2 + 3u_3 + 4}{(u_3 - 1)s} G_{0,0,0}(u_3)$$

$$+ \frac{1}{2} \frac{(8u_3^3 - 12u_3^2 + 4u_3 - 11)u_3}{(u_3 - 1)s} G_{0,0,1}(u_3) + \frac{1}{2} \frac{(8u_3^3 - 12u_3^2 + 4u_3 - 11)u_3}{(u_3 - 1)s} G_{0,1,0}(u_3) - \frac{1}{2} \frac{(14u_3^3 - 22u_3^2 - 4u_3 + 5)u_3}{(u_3 - 1)s} G_{0,1,1}(u_3)$$

$$+ \frac{1}{2} \frac{(14u_3^3 - 20u_3^2 - 6u_3 + 7)u_3}{(u_3 - 1)s} G_{1,0,0}(u_3) - \frac{1}{2} \frac{(8u_3^3 - 12u_3^2 + 4u_3 + 11)u_3}{(u_3 - 1)s} G_{1,0,1}(u_3) - \frac{1}{2} \frac{(8u_3^3 - 12u_3^2 + 4u_3 + 11)u_3}{(u_3 - 1)s} G_{1,1,0}(u_3)$$

$$+ \frac{(7u_3^4 - 19u_3^3 + 6u_3^2 + 14u_3 - 4)u_3}{s(u_3 - 1)^2} G_{1,1,1}(u_3) \bigg) i + \bigg(\frac{1}{2} \frac{(43u_3^2 - 43u_3 + 13)u_3}{(u_3 - 1)s} + \frac{3}{2} \frac{(9u_3 - 1)u_3}{s} G_0(u_3) + \frac{3}{2} \frac{(9u_3 - 8)u_3^2}{(u_3 - 1)s} G_1(u_3)$$

$$- \frac{1}{2} \frac{(2u_3 - 5)(3u_3 + 1)u_3}{s} G_{0,0}(u_3) - \frac{(3u_3^2 - 5u_3 - 4)u_3^2}{(u_3 - 1)s} G_{0,1}(u_3) + \frac{(3u_3^2 - u_3 - 6)u_3}{s} G_{1,0}(u_3)$$

$$+ \frac{1}{2} \frac{(2u_3 + 3)(3u_3 - 4)u_3^2}{(u_3 - 1)s} G_{1,1}(u_3) \bigg) \pi - \frac{1}{6} \frac{(3u_3^2 - 20u_3 + 11)u_3^2}{(u_3 - 1)s} \pi^3 \bigg) \log(\frac{m_t^2}{t}) + \bigg(\bigg(\frac{3}{4} \frac{15u_3^4 - 30u_3^3 + 6u_3^2 + 9u_3 - 8}{s(u_3 - 1)^2}$$

$$+ \frac{1}{4} \frac{58u_3^4 - 116u_3^3 + 31u_3^2 + 27u_3 - 16}{s(u_3 - 1)^2} \zeta_2 + \frac{2u_3^3 + u_3^2 - 14u_3 + 10}{(u_3 - 1)s} G_0(u_3) + \frac{(2u_3^3 - 7u_3^2 - 6u_3 + 1)u_3}{s(u_3 - 1)^2} G_1(u_3)$$

$$+ \frac{1}{4} \frac{13u_3^3 - 30u_3^2 + 12u_3 + 8}{(u_3 - 1)s} G_{0,0}(u_3) + 4 \frac{u_3}{(u_3 - 1)s} G_{0,1}(u_3) + 4 \frac{u_3}{(u_3 - 1)s} G_{1,0}(u_3) + \frac{1}{4} \frac{(13u_3^3 - 9u_3^2 - 9u_3 - 3)u_3}{s(u_3 - 1)^2} G_{1,1}(u_3) \bigg) i$$

$$+ \bigg(- \frac{1}{4} \frac{(46u_3^2 - 46u_3 + 5)u_3}{(u_3 - 1)s} + \frac{1}{2} \frac{(u_3 + 1)(3u_3 - 5)u_3}{s} G_0(u_3) - \frac{1}{2} \frac{(u_3 - 2)(3u_3 + 2)u_3^2}{(u_3 - 1)s} G_1(u_3) \bigg) \pi \bigg) \log^2(\frac{m_t^2}{t})$$

$$+ \bigg(\bigg(- \frac{1}{12} \frac{62u_3^4 - 124u_3^3 - 33u_3^2 + 95u_3 - 40}{s(u_3 - 1)^2} + \frac{1}{12} \frac{6u_3^4 - 23u_3^3 + 24u_3^2 - 18u_3 - 8}{(u_3 - 1)s} G_0(u_3)$$

$$- \frac{1}{12} \frac{(6u_3^4 - u_3^3 - 9u_3^2 + 15u_3 - 19)u_3}{s(u_3 - 1)^2} G_1(u_3) \bigg) i - \frac{1}{12} \frac{(16u_3^2 - 16u_3 + 13)u_3}{(u_3 - 1)s} \pi \bigg) \log^3(\frac{m_t^2}{t})$$

$$+ \frac{1}{12} \frac{3u_3^4 - 6u_3^3 + 7u_3^2 - 4u_3 - 2}{s(u_3 - 1)^2} i \log^4(\frac{m_t^2}{t}) \bigg) m_t^2 + \bigg(\bigg(-12 \frac{u_3^2 - u_3 + 1}{u_3 - 1} + \bigg(- \frac{(3u_3^2 - 3u_3 - 2)(2u_3 - 1)^2}{u_3 - 1}$$

$$+ \frac{1}{2} (- 3u_3^3 + 2u_3^2 + u_3 + 6) G_0(u_3) - \frac{1}{2} \frac{(3u_3^3 - 7u_3^2 + 4u_3 + 6)u_3}{u_3 - 1} G_1(u_3) \bigg) \zeta_2 + \frac{1}{2} \frac{23u_3^2 - 23u_3 + 16}{u_3 - 1} \zeta_3 + \frac{1}{4} (14u_3^2 - u_3 - 20) G_0(u_3)$$

$$- \frac{1}{4} \frac{(14u_3^2 - 27u_3 - 7)u_3}{u_3 - 1} G_1(u_3) - \frac{1}{2} \frac{8u_3^4 - 18u_3^3 + 10u_3^2 - 5u_3 + 3}{u_3 - 1} G_{0,0}(u_3) + \frac{1}{2} (8u_3^2 - 8u_3 + 1) u_3 G_{0,1}(u_3)$$

$$+ \frac{1}{2} (8u_3^2 - 8u_3 + 1) u_3 G_{1,0}(u_3) - \frac{1}{2} \frac{8u_3^4 - 14u_3^3 + 4u_3^2 + 7u_3 - 2}{u_3 - 1} G_{1,1}(u_3) + \frac{1}{2} (- 3u_3^3 + 2u_3^2 + 2u_3 + 2) G_{0,0,0}(u_3)$$

$$+ \frac{1}{2} u_3^3 G_{0,0,1}(u_3) + \frac{1}{2} u_3^3 G_{0,1,0}(u_3) + \frac{1}{2} (u_3 - 2) u_3^2 G_{0,1,1}(u_3) + \frac{1}{2} (u_3 - 1)(u_3 + 1) u_3 G_{1,0,0}(u_3) + \frac{1}{2} (u_3 - 1)^2 u_3 G_{1,0,1}(u_3)$$

$$+ \frac{1}{2} (u_3 - 1)^2 u_3 G_{1,1,0}(u_3) - \frac{1}{2} \frac{(3u_3^3 - 7u_3^2 + 3u_3 + 3)u_3}{u_3 - 1} G_{1,1,1}(u_3) \bigg) i + \bigg(- \frac{7}{4} u_3 + \frac{1}{4} \frac{(7u_3 - 3)u_3}{u_3 - 1} G_0(u_3)$$

$$+ \frac{1}{4} (7u_3 - 4) G_1(u_3) + u_3 G_{0,0}(u_3) - \frac{1}{2} u_3 G_{0,1}(u_3) - \frac{1}{2} u_3 G_{1,0}(u_3) + u_3 G_{1,1}(u_3) \bigg) \pi + \frac{1}{3} u_3 \pi^3 \bigg) \log(\frac{m_t^2}{t})$$

$$+ \bigg(\bigg(- \frac{1}{8} \frac{19u_3^2 - 19u_3 + 20}{u_3 - 1} + \frac{1}{2} \frac{3u_3^4 - 6u_3^3 + 8u_3^2 - 5u_3 + 3}{u_3 - 1} \zeta_2 + \frac{1}{8} (- 4u_3^2 + 5u_3 - 6) G_0(u_3) + \frac{1}{8} \frac{(4u_3^2 - 3u_3 + 5)u_3}{u_3 - 1} G_1(u_3)$$

$$+ \frac{1}{2} (u_3^3 - u_3^2 - 1) G_{0,0}(u_3) - \frac{1}{4} (2u_3^2 - 2u_3 + 1) u_3 G_{0,1}(u_3) - \frac{1}{4} (2u_3^2 - 2u_3 + 1) u_3 G_{1,0}(u_3) + \frac{1}{2} \frac{(u_3^3 - 2u_3^2 + u_3 + 1)u_3}{u_3 - 1} G_{1,1}(u_3) \bigg) i$$

$$+ \bigg(- \frac{3}{4} u_3 - \frac{1}{2} u_3 G_0(u_3) - \frac{1}{2} u_3 G_1(u_3) \bigg) \pi \bigg) \log^2(\frac{m_t^2}{t}) + \bigg(\bigg(- \frac{1}{4} \frac{2u_3^2 - 2u_3 + 1}{u_3 - 1} + \frac{1}{12} (- u_3 + 2) G_0(u_3) - \frac{1}{12} \frac{(u_3 + 1)u_3}{u_3 - 1} G_1(u_3) \bigg) i$$

$$+\frac{1}{12}u_3\pi\Big)\log^3\Big(\frac{m_t^2}{t}\Big)+\frac{1}{48}\frac{3u_3^2-3u_3+2}{u_3-1}i\log^4\Big(\frac{m_t^2}{t}\Big)\Big)\frac{1}{N_c}+\Big(\Big(-2\frac{u_3^4-2u_3^3+16u_3^2-15u_3+15}{u_3-1}$$

$$+\Big(\frac12\frac{110u_3^4-220u_3^3+295u_3^2-185u_3+86}{u_3-1}-\frac12\frac{82u_3^4-206u_3^3+199u_3^2-41u_3+20}{u_3-1}G_0(u_3)$$

$$-\frac12\frac{82u_3^4-122u_3^3+73u_3^2-67u_3+54}{u_3-1}G_1(u_3)-\frac{12u_3^4-34u_3^3+19u_3^2+10u_3-6}{u_3-1}G_{0,0}(u_3)+\frac12\frac{(27u_3^3-32u_3^2+3u_3+8)u_3}{u_3-1}G_{0,1}(u_3)$$

$$+\frac12(27u_3^3-49u_3^2+20u_3-6)G_{1,0}(u_3)-\frac{12u_3^4-14u_3^3-11u_3^2+6u_3+1}{u_3-1}G_{1,1}(u_3)\Big)\zeta_2+\Big(\frac{12u_3^4-11u_3^3-23u_3^2+29u_3-21}{u_3-1}$$

$$+\frac12(-22u_3^3+49u_3^2-20u_3-4)G_0(u_3)+\frac12\frac{(30u_3^3-87u_3^2+70u_3-9)u_3}{u_3-1}G_1(u_3)\Big)\zeta_3-\frac18\frac{1431u_3^4-3236u_3^3+1870u_3^2-131u_3-56}{u_3-1}\zeta_4$$

$$-\frac{22u_3^3-11u_3^2-3u_3+3}{u_3-1}G_0(u_3)+\frac{22u_3^3-55u_3^2+41u_3-11}{u_3-1}G_1(u_3)+\frac{1}{12}\frac{216u_3^4-200u_3^3+251u_3^2-235u_3+220}{u_3-1}G_{0,0}(u_3)$$

$$-\frac{1}{12}\frac{216u_3^4-432u_3^3+331u_3^2-115u_3+28}{u_3-1}G_{0,1}(u_3)-\frac{1}{12}\frac{216u_3^4-432u_3^3+331u_3^2-115u_3+28}{u_3-1}G_{1,0}(u_3)$$

$$+\frac{1}{12}\frac{216u_3^4-664u_3^3+947u_3^2-531u_3+252}{u_3-1}G_{1,1}(u_3)-\frac12\frac{58u_3^4-93u_3^3+70u_3^2-39u_3+38}{u_3-1}G_{0,0,0}(u_3)$$

$$+\frac16\frac{82u_3^4-179u_3^3+161u_3^2-89u_3+13}{u_3-1}G_{0,0,1}(u_3)+\frac16\frac{82u_3^4-179u_3^3+161u_3^2-89u_3+13}{u_3-1}G_{0,1,0}(u_3)$$

$$+\frac16\frac{10u_3^4-83u_3^3+92u_3^2-36u_3+30}{u_3-1}G_{0,1,1}(u_3)+\frac16\frac{10u_3^4+43u_3^3-97u_3^2+61u_3+13}{u_3-1}G_{1,0,0}(u_3)$$

$$+\frac16\frac{82u_3^4-149u_3^3+116u_3^2-24u_3-12}{u_3-1}G_{1,0,1}(u_3)+\frac16\frac{82u_3^4-149u_3^3+116u_3^2-24u_3-12}{u_3-1}G_{1,1,0}(u_3)$$

$$-\frac12\frac{58u_3^4-139u_3^3+139u_3^2-54u_3+34}{u_3-1}G_{1,1,1}(u_3)+\frac12\frac{45u_3^4-96u_3^3+74u_3^2-49u_3+24}{u_3-1}G_{0,0,0,0}(u_3)$$

$$-\frac12\frac{12u_3^4-21u_3^3+12u_3^2-11u_3+6}{u_3-1}G_{0,0,0,1}(u_3)-\frac12\frac{12u_3^4-21u_3^3+12u_3^2-11u_3+6}{u_3-1}G_{0,0,1,0}(u_3)$$

$$+\frac12(33u_3^3-21u_3^2-2u_3-2)G_{0,0,1,1}(u_3)-\frac12\frac{42u_3^4-98u_3^3+71u_3^2-23u_3+6}{u_3-1}G_{0,1,0,0}(u_3)+\frac12(11u_3^3-16u_3^2+4u_3-2)G_{0,1,0,1}(u_3)$$

$$+\frac12(11u_3^3-16u_3^2+4u_3-2)G_{0,1,1,0}(u_3)-\frac{(17u_3^3-32u_3^2+19u_3-1)u_3}{u_3-1}G_{0,1,1,1}(u_3)+(-17u_3^3+19u_3^2-6u_3+3)G_{1,0,0,0}(u_3)$$

$$+\frac12\frac{(11u_3^3-17u_3^2+5u_3+3)u_3}{u_3-1}G_{1,0,0,1}(u_3)+\frac12\frac{(11u_3^3-17u_3^2+5u_3+3)u_3}{u_3-1}G_{1,0,1,0}(u_3)$$

$$-\frac12\frac{42u_3^4-70u_3^3+29u_3^2+7u_3-2}{u_3-1}G_{1,0,1,1}(u_3)+\frac12\frac{(33u_3^3-78u_3^2+55u_3-8)u_3}{u_3-1}G_{1,1,0,0}(u_3)$$

$$-\frac12\frac{12u_3^4-27u_3^3+21u_3^2+2u_3-2}{u_3-1}G_{1,1,0,1}(u_3)-\frac12\frac{12u_3^4-27u_3^3+21u_3^2+2u_3-2}{u_3-1}G_{1,1,1,0}(u_3)$$

$$+\frac12\frac{45u_3^4-84u_3^3+56u_3^2+9u_3-2}{u_3-1}G_{1,1,1,1}(u_3)\Big)i+\Big(-\frac13\frac{2u_3^4-4u_3^3+59u_3^2-57u_3-33}{u_3-1}-\frac{(30u_3^3-60u_3^2+32u_3-3)u_3}{u_3-1}\zeta_3$$

$$-\frac{1}{12}\frac{2u_3^3-95u_3^2-59u_3-28}{u_3-1}G_0(u_3)+\frac{1}{12}\frac{2u_3^3+89u_3^2-243u_3+180}{u_3-1}G_1(u_3)+\frac{1}{12}\frac{44u_3^4-244u_3^3+293u_3^2-163u_3-26}{u_3-1}G_{0,0}(u_3)$$

$$+\frac{1}{12}(-44u_3^3-112u_3^2+65u_3-84)G_{0,1}(u_3)-\frac{1}{12}\frac{(44u_3^3-244u_3^2+291u_3-175)u_3}{u_3-1}G_{1,0}(u_3)$$

$$+\frac{1}{12}\frac{44u_3^4+68u_3^3-175u_3^2+133u_3-96}{u_3-1}G_{1,1}(u_3)+\frac12(30u_3^3-37u_3^2+7u_3-6)G_{0,0,0}(u_3)+\frac12(22u_3^2-15u_3-2)u_3G_{0,0,1}(u_3)$$

$$-\frac12(30u_3^2-37u_3+8)u_3G_{0,1,0}(u_3)-\frac12\frac{(22u_3^3-37u_3^2+16u_3+1)u_3}{u_3-1}G_{0,1,1}(u_3)+\frac12(-22u_3^3+29u_3^2-8u_3+2)G_{1,0,0}(u_3)$$

$$-\frac12(30u_3^2-23u_3+1)u_3G_{1,0,1}(u_3)+\frac12(22u_3^2-29u_3+5)u_3G_{1,1,0}(u_3)+\frac12\frac{(30u_3^3-53u_3^2+23u_3+6)u_3}{u_3-1}G_{1,1,1}(u_3)\Big)\pi$$

$$+\Big(\frac16\frac{(11u_3^3-35u_3^2+30u_3-13)u_3}{u_3-1}+\frac{1}{12}(38u_3^2-45u_3+10)u_3G_0(u_3)-\frac{1}{12}(14u_3^2-21u_3+3)u_3G_1(u_3)\Big)\pi^3$$

$$+\Big(\Big(\Big(\frac{1}{12}\frac{16u_3^6-48u_3^5-1098u_3^4+2276u_3^3+2853u_3^2-3999u_3+1692}{s(u_3-1)^2}+\Big(-\frac14\frac{16u_3^6-48u_3^5-488u_3^4+1056u_3^3+201u_3^2-737u_3+392}{s(u_3-1)^2}$$

$$-\frac14\frac{52u_3^5-55u_3^4-107u_3^3+484u_3^2-494u_3+240}{s(u_3-1)^2}G_0(u_3)+\frac14\frac{52u_3^5-205u_3^4+193u_3^3-353u_3^2+193u_3-120}{s(u_3-1)^2}G_1(u_3)$$

$$+\frac12\frac{56u_3^5-73u_3^4-36u_3^3+30u_3^2+75u_3-44}{s(u_3-1)^2}G_{0,0}(u_3)+\frac12\frac{(68u_3^4-162u_3^3+123u_3^2-43u_3-10)u_3}{s(u_3-1)^2}G_{0,1}(u_3)$$

$$-\frac12\frac{68u_3^4-110u_3^3+45u_3^2+11u_3-24}{(u_3-1)s}G_{1,0}(u_3)-\frac12\frac{56u_3^5-207u_3^4+232u_3^3-44u_3^2+15u_3-8}{s(u_3-1)^2}G_{1,1}(u_3)\Big)\zeta_2$$

$$+\Big(\frac12\frac{4u_3^5-209u_3^4+391u_3^3+115u_3^2-321u_3+164}{s(u_3-1)^2}+\frac{40u_3^4-66u_3^3+63u_3^2-65u_3+38}{(u_3-1)s}G_0(u_3)$$

$$-\frac{1}{2}\frac{\left(80u_3^4-165u_3^3+153u_3^2+24u_3-16\right)u_3}{s\left(u_3-1\right)^2}G_1(u_3)\Big)\zeta_3+\frac{1}{8}\frac{1156u_3^5-1735u_3^4-103u_3^3+625u_3^2-207u_3-184}{s\left(u_3-1\right)^2}\zeta_4$$

$$+\frac{1}{12}\frac{12u_3^5-202u_3^4+116u_3^3+1447u_3^2-2261u_3+976}{s\left(u_3-1\right)^2}G_0(u_3)-\frac{1}{12}\frac{12u_3^5+142u_3^4-572u_3^3-703u_3^2+233u_3-88}{s\left(u_3-1\right)^2}G_1(u_3)$$

$$-\frac{1}{6}\frac{115u_3^4-250u_3^3-43u_3^2+390u_3-188}{s\left(u_3-1\right)^2}G_{0,0}(u_3)+\frac{1}{6}\frac{95u_3^4-190u_3^3+178u_3^2-83u_3+46}{s\left(u_3-1\right)^2}G_{0,1}(u_3)$$

$$+\frac{1}{6}\frac{95u_3^4-190u_3^3+178u_3^2-83u_3+46}{s\left(u_3-1\right)^2}G_{1,0}(u_3)-\frac{1}{6}\frac{115u_3^4-210u_3^3-103u_3^2-14u_3+24}{s\left(u_3-1\right)^2}G_{1,1}(u_3)$$

$$-\frac{1}{2}\frac{30u_3^5-88u_3^4+127u_3^3-144u_3^2+145u_3-50}{s\left(u_3-1\right)^2}G_{0,0,0}(u_3)+\frac{1}{6}\frac{78u_3^5-113u_3^4+79u_3^3-16u_3^2-48u_3+68}{s\left(u_3-1\right)^2}G_{0,0,1}(u_3)$$

$$+\frac{1}{6}\frac{78u_3^5-113u_3^4+79u_3^3-16u_3^2-48u_3+68}{s\left(u_3-1\right)^2}G_{0,1,0}(u_3)-\frac{1}{6}\frac{90u_3^5-46u_3^4-136u_3^3+19u_3^2-7u_3+12}{s\left(u_3-1\right)^2}G_{0,1,1}(u_3)$$

$$+\frac{1}{6}\frac{90u_3^5-404u_3^4+580u_3^3-235u_3^2-111u_3+68}{s\left(u_3-1\right)^2}G_{1,0,0}(u_3)-\frac{1}{6}\frac{78u_3^5-277u_3^4+407u_3^3-323u_3^2+95u_3-48}{s\left(u_3-1\right)^2}G_{1,0,1}(u_3)$$

$$-\frac{1}{6}\frac{78u_3^5-277u_3^4+407u_3^3-323u_3^2+95u_3-48}{s\left(u_3-1\right)^2}G_{1,1,0}(u_3)+\frac{1}{2}\frac{30u_3^5-62u_3^4+75u_3^3-9u_3^2+36u_3-20}{s\left(u_3-1\right)^2}G_{1,1,1}(u_3)$$

$$-\frac{1}{2}\frac{44u_3^5-167u_3^4+181u_3^3+93u_3^2-285u_3+126}{s\left(u_3-1\right)^2}G_{0,0,0,0}(u_3)-\frac{1}{2}\frac{31u_3^4-54u_3^3+5u_3^2+50u_3-24}{s\left(u_3-1\right)^2}G_{0,0,0,1}(u_3)$$

$$-\frac{1}{2}\frac{31u_3^4-54u_3^3+5u_3^2+50u_3-24}{s\left(u_3-1\right)^2}G_{0,0,1,0}(u_3)+\frac{1}{2}\frac{44u_3^4-49u_3^3-6u_3^2-5u_3+8}{\left(u_3-1\right)s}G_{0,0,1,1}(u_3)$$

$$+\frac{1}{2}\frac{56u_3^5-152u_3^4+113u_3^3+40u_3^2-89u_3+24}{s\left(u_3-1\right)^2}G_{0,1,0,0}(u_3)-\frac{1}{2}\frac{12u_3^4-39u_3^3+12u_3^2+19u_3-8}{\left(u_3-1\right)s}G_{0,1,0,1}(u_3)$$

$$-\frac{1}{2}\frac{12u_3^4-39u_3^3+12u_3^2+19u_3-8}{\left(u_3-1\right)s}G_{0,1,1,0}(u_3)-\frac{1}{2}\frac{\left(32u_3^4-38u_3^3-29u_3^2+4u_3+7\right)u_3}{s\left(u_3-1\right)^2}G_{0,1,1,1}(u_3)$$

$$+\frac{1}{2}\frac{32u_3^4-90u_3^3+49u_3^2+40u_3-24}{\left(u_3-1\right)s}G_{1,0,0,0}(u_3)+\frac{1}{2}\frac{\left(12u_3^4-9u_3^3-33u_3^2+26u_3-4\right)u_3}{s\left(u_3-1\right)^2}G_{1,0,0,1}(u_3)$$

$$+\frac{1}{2}\frac{\left(12u_3^4-9u_3^3-33u_3^2+26u_3-4\right)u_3}{s\left(u_3-1\right)^2}G_{1,0,1,0}(u_3)-\frac{1}{2}\frac{56u_3^5-128u_3^4+65u_3^3-27u_3^2+2u_3+8}{s\left(u_3-1\right)^2}G_{1,0,1,1}(u_3)$$

$$-\frac{1}{2}\frac{\left(44u_3^4-127u_3^3+111u_3^2-12u_3-8\right)u_3}{s\left(u_3-1\right)^2}G_{1,1,0,0}(u_3)-\frac{1}{2}\frac{31u_3^4-70u_3^3+29u_3^2-22u_3+8}{s\left(u_3-1\right)^2}G_{1,1,0,1}(u_3)$$

$$-\frac{1}{2}\frac{31u_3^4-70u_3^3+29u_3^2-22u_3+8}{s\left(u_3-1\right)^2}G_{1,1,1,0}(u_3)+\frac{1}{2}\frac{44u_3^5-53u_3^4-47u_3^3-74u_3^2-4u_3+8}{s\left(u_3-1\right)^2}G_{1,1,1,1}(u_3)\Big)i$$

$$+\Big(-\frac{1}{12}\frac{8u_3^6-24u_3^5-712u_3^4+1464u_3^3-677u_3^2-59u_3+88}{s\left(u_3-1\right)^2}+\frac{\left(18u_3^3-45u_3^2+65u_3-42\right)u_3}{s\left(u_3-1\right)^2}\zeta_3$$

$$+\frac{1}{6}\frac{6u_3^4+110u_3^3-153u_3^2-6u_3+46}{\left(u_3-1\right)s}G_0(u_3)-\frac{1}{6}\frac{\left(6u_3^4-134u_3^3+213u_3^2-42u_3+3\right)u_3}{s\left(u_3-1\right)^2}G_1(u_3)-\frac{1}{3}\frac{6u_3^3-82u_3^2-23u_3+34}{s}G_{0,0}(u_3)$$

$$-\frac{1}{2}\frac{\left(4u_3^3+29u_3^2-85u_3+32\right)u_3^2}{s\left(u_3-1\right)^2}G_{0,1}(u_3)+\frac{1}{2}\frac{4u_3^3-41u_3^2-15u_3+20}{s}G_{1,0}(u_3)+\frac{1}{3}\frac{\left(6u_3^3+64u_3^2-169u_3+65\right)u_3^2}{s\left(u_3-1\right)^2}G_{1,1}(u_3)$$

$$-\frac{1}{2}\frac{32u_3^4-60u_3^3+47u_3^2+22u_3-24}{\left(u_3-1\right)s}G_{0,0,0}(u_3)+\frac{1}{2}\frac{\left(32u_3^3-59u_3^2+19u_3-1\right)u_3}{\left(u_3-1\right)s}G_{0,0,1}(u_3)$$

$$+\frac{1}{2}\frac{\left(32u_3^3-36u_3^2+7u_3+24\right)u_3}{\left(u_3-1\right)s}G_{0,1,0}(u_3)-\frac{1}{2}\frac{\left(32u_3^4-61u_3^3+22u_3^2+4u_3-5\right)u_3}{s\left(u_3-1\right)^2}G_{0,1,1}(u_3)$$

$$+\frac{1}{2}\frac{32u_3^4-67u_3^3+31u_3^2+7u_3-8}{\left(u_3-1\right)s}G_{1,0,0}(u_3)-\frac{1}{2}\frac{\left(32u_3^3-60u_3^2+31u_3-27\right)u_3}{\left(u_3-1\right)s}G_{1,0,1}(u_3)$$

$$-\frac{1}{2}\frac{\left(32u_3^3-37u_3^2-3u_3+9\right)u_3}{\left(u_3-1\right)s}G_{1,1,0}(u_3)+\frac{1}{2}\frac{\left(32u_3^4-68u_3^3+59u_3^2-64u_3+17\right)u_3}{s\left(u_3-1\right)^2}G_{1,1,1}(u_3)\Big)\pi$$

$$+\Big(-\frac{1}{24}\frac{\left(8u_3^4-52u_3^3+50u_3^2-65u_3+19\right)u_3}{s\left(u_3-1\right)^2}-\frac{1}{12}\frac{\left(32u_3^3-41u_3^2+3u_3+46\right)u_3}{\left(u_3-1\right)s}G_0(u_3)+\frac{1}{12}\frac{\left(32u_3^3-32u_3^2-17u_3-4\right)u_3}{\left(u_3-1\right)s}G_1(u_3)\Big)\pi^3$$

$$+\Big(\Big(-\frac{1}{12}\frac{8u_3^6-24u_3^5-792u_3^4+1624u_3^3+1209u_3^2-2025u_3+888}{s\left(u_3-1\right)^2}+\Big(-\frac{1}{2}\frac{116u_3^4-232u_3^3-151u_3^2+267u_3-120}{s\left(u_3-1\right)^2}$$

$$+\frac{1}{2}\frac{12u_3^4-18u_3^3+5u_3^2+43u_3-44}{\left(u_3-1\right)s}G_0(u_3)-\frac{1}{2}\frac{\left(12u_3^4-30u_3^3+23u_3^2-47u_3-2\right)u_3}{s\left(u_3-1\right)^2}G_1(u_3)\Big)\zeta_2$$

$$+\frac{1}{2}\frac{15u_3^4-21u_3^3+212u_3^2-206u_3+76}{s\left(u_3-1\right)^2}\zeta_3+\frac{1}{6}\frac{6u_3^5+91u_3^4-191u_3^3-140u_3^2+426u_3-214}{s\left(u_3-1\right)^2}G_0(u_3)$$

$$-\frac{1}{6}\frac{6u_3^5-121u_3^4+233u_3^3+107u_3^2-33u_3+22}{s\left(u_3-1\right)^2}G_1(u_3)-\frac{1}{6}\frac{29u_3^4-140u_3^3+346u_3^2-401u_3+154}{s\left(u_3-1\right)^2}G_{0,0}(u_3)$$

$$+\frac{1}{3}\frac{23u_3^4-46u_3^3-14u_3^2+37u_3-28}{s\big(u_3-1\big)^2}G_{0,1}(u_3)+\frac{1}{3}\frac{23u_3^4-46u_3^3-14u_3^2+37u_3-28}{s\big(u_3-1\big)^2}G_{1,0}(u_3)$$

$$-\frac{1}{6}\frac{29u_3^4+24u_3^3+100u_3^2+13u_3-12}{s\big(u_3-1\big)^2}G_{1,1}(u_3)+\frac{1}{2}\frac{12u_3^4-32u_3^3-12u_3^2+111u_3-78}{(u_3-1)s}G_{0,0,0}(u_3)$$

$$-\frac{1}{2}\frac{12u_3^5-33u_3^4+19u_3^3+16u_3^2-38u_3+16}{s\big(u_3-1\big)^2}G_{0,0,1}(u_3)-\frac{1}{2}\frac{12u_3^5-33u_3^4+19u_3^3+16u_3^2-38u_3+16}{s\big(u_3-1\big)^2}G_{0,1,0}(u_3)$$

$$+\frac{1}{2}\frac{12u_3^5-28u_3^4+7u_3^2-15u_3+8}{s\big(u_3-1\big)^2}G_{0,1,1}(u_3)-\frac{1}{2}\frac{12u_3^5-32u_3^4+8u_3^3+41u_3^2-53u_3+16}{s\big(u_3-1\big)^2}G_{1,0,0}(u_3)$$

$$+\frac{1}{2}\frac{12u_3^5-27u_3^4+7u_3^3+5u_3^2-21u_3+8}{s\big(u_3-1\big)^2}G_{1,0,1}(u_3)+\frac{1}{2}\frac{12u_3^5-27u_3^4+7u_3^3+5u_3^2-21u_3+8}{s\big(u_3-1\big)^2}G_{1,1,0}(u_3)$$

$$-\frac{1}{2}\frac{\big(12u_3^4-16u_3^3-36u_3^2-39u_3+1\big)u_3}{s\big(u_3-1\big)^2}G_{1,1,1}(u_3)\Big)i+\Big(\frac{1}{3}\frac{4u_3^6-12u_3^5-119u_3^4+258u_3^3-128u_3^2-3u_3+11}{s\big(u_3-1\big)^2}$$

$$-\frac{1}{3}\frac{28u_3^3-15u_3^2-38u_3+28}{(u_3-1)s}G_0(u_3)-\frac{1}{3}\frac{\big(28u_3^3-69u_3^2+16u_3-3\big)u_3}{s\big(u_3-1\big)^2}G_1(u_3)-\frac{1}{2}\frac{11u_3^3-22u_3^2-10u_3+16}{(u_3-1)s}G_{0,0}(u_3)$$

$$+\frac{1}{2}\frac{\big(2u_3^2-11u_3-1\big)u_3}{(u_3-1)s}G_{0,1}(u_3)+\frac{1}{2}\frac{\big(2u_3^2+7u_3-10\big)u_3}{(u_3-1)s}G_{1,0}(u_3)-\frac{1}{2}\frac{\big(11u_3^3-11u_3^2-21u_3+5\big)u_3}{s\big(u_3-1\big)^2}G_{1,1}(u_3)\Big)\pi$$

$$-\frac{1}{12}\frac{\big(4u_3^2+5u_3-30\big)u_3}{(u_3-1)s}\pi^3\Big)\log(\frac{m_t^2}{t})+\Big(\Big(\frac{1}{12}\frac{8u_3^6-24u_3^5-285u_3^4+610u_3^3+57u_3^2-366u_3+192}{s\big(u_3-1\big)^2}$$

$$+\frac{1}{4}\frac{9u_3^4-18u_3^3-90u_3^2+99u_3-44}{s\big(u_3-1\big)^2}\zeta_2-\frac{1}{12}\frac{\big(u_3-2\big)\big(40u_3^3+75u_3^2-138u_3+67\big)}{s\big(u_3-1\big)^2}G_0(u_3)$$

$$-\frac{1}{12}\frac{\big(u_3+1\big)\big(40u_3^3-195u_3^2+132u_3-44\big)}{s\big(u_3-1\big)^2}G_1(u_3)+\frac{1}{4}\frac{32u_3^3-119u_3^2+125u_3-46}{s\big(u_3-1\big)^2}G_{0,0}(u_3)$$

$$-\frac{1}{4}\frac{3u_3^4-6u_3^3-19u_3^2+22u_3-8}{s\big(u_3-1\big)^2}G_{0,1}(u_3)-\frac{1}{4}\frac{3u_3^4-6u_3^3-19u_3^2+22u_3-8}{s\big(u_3-1\big)^2}G_{1,0}(u_3)-\frac{1}{4}\frac{32u_3^3+23u_3^2-17u_3+8}{s\big(u_3-1\big)^2}G_{1,1}(u_3)\Big)i$$

$$+\Big(\frac{1}{6}\frac{47u_3^4-94u_3^3+47u_3-22}{s\big(u_3-1\big)^2}-\frac{1}{4}\frac{\big(u_3+2\big)\big(3u_3-4\big)}{(u_3-1)s}G_0(u_3)+\frac{1}{4}\frac{\big(u_3-3\big)\big(3u_3+1\big)u_3}{s\big(u_3-1\big)^2}G_1(u_3)\Big)\pi\Big)\log^2(\frac{m_t^2}{t})$$

$$+\Big(\Big(\frac{1}{4}\frac{12u_3^4-24u_3^3-17u_3^2+29u_3-10}{s\big(u_3-1\big)^2}+\frac{1}{12}\frac{3u_3^3-38u_3^2+63u_3-30}{(u_3-1)s}G_0(u_3)+\frac{1}{12}\frac{\big(3u_3^3+29u_3^2-4u_3+2\big)u_3}{s\big(u_3-1\big)^2}G_1(u_3)\Big)i$$

$$+\frac{1}{6}\frac{\big(5u_3^2-5u_3-1\big)u_3}{(u_3-1)s}\pi\Big)\log^3(\frac{m_t^2}{t})+\frac{1}{48}\frac{4u_3^4-8u_3^3-89u_3^2+93u_3-30}{s\big(u_3-1\big)^2}i\log^4(\frac{m_t^2}{t})\Big)m_t^2+\Big(\Big(-\frac{2}{3}\frac{u_3^4-2u_3^3+13u_3^2-12u_3+12}{u_3-1}$$

$$+\Big(\frac{1}{2}\frac{48u_3^4-96u_3^3+83u_3^2-35u_3+20}{u_3-1}+\frac{1}{2}\big(-3u_3^3+5u_3^2-10u_3+12\big)G_0(u_3)-\frac{1}{2}\frac{\big(3u_3^3-4u_3^2+9u_3+4\big)u_3}{u_3-1}G_1(u_3)\Big)\zeta_2$$

$$+2\frac{2u_3^2-2u_3-1}{u_3-1}\zeta_3-\frac{1}{12}\frac{102u_3^3-43u_3^2-47u_3+32}{u_3-1}G_0(u_3)+\frac{1}{12}\frac{102u_3^3-263u_3^2+173u_3-44}{u_3-1}G_1(u_3)$$

$$+\frac{1}{6}\frac{48u_3^4-90u_3^3+113u_3^2-60u_3+55}{u_3-1}G_{0,0}(u_3)-\frac{1}{6}\frac{48u_3^4-96u_3^3+82u_3^2-34u_3+1}{u_3-1}G_{0,1}(u_3)$$

$$-\frac{1}{6}\frac{48u_3^4-96u_3^3+82u_3^2-34u_3+1}{u_3-1}G_{1,0}(u_3)+\frac{1}{6}\frac{48u_3^4-102u_3^3+131u_3^2-88u_3+66}{u_3-1}G_{1,1}(u_3)$$

$$+\frac{1}{2}\big(-3u_3^3+5u_3^2-13u_3+12\big)G_{0,0,0}(u_3)+\frac{1}{2}\frac{u_3^4-4u_3^3+6u_3^2-9u_3+4}{u_3-1}G_{0,0,1}(u_3)+\frac{1}{2}\frac{u_3^4-4u_3^3+6u_3^2-9u_3+4}{u_3-1}G_{0,1,0}(u_3)$$

$$+\frac{1}{2}\frac{u_3^4+2u_3^2+3u_3-2}{u_3-1}G_{0,1,1}(u_3)+\frac{1}{2}\frac{u_3^4-4u_3^3+8u_3^2-11u_3+4}{u_3-1}G_{1,0,0}(u_3)+\frac{1}{2}\frac{u_3^4+5u_3-2}{u_3-1}G_{1,0,1}(u_3)$$

$$+\frac{1}{2}\frac{u_3^4+5u_3-2}{u_3-1}G_{1,1,0}(u_3)-\frac{1}{2}\frac{\big(3u_3^3-4u_3^2+12u_3+1\big)u_3}{u_3-1}G_{1,1,1}(u_3)\Big)i+\Big(-\frac{1}{12}\frac{4u_3^4-8u_3^3+89u_3^2-85u_3-44}{u_3-1}$$

$$+\frac{1}{12}\frac{5u_3^2+101u_3+2}{u_3-1}G_0(u_3)+\frac{1}{12}\frac{5u_3^2-111u_3+108}{u_3-1}G_1(u_3)+\frac{1}{2}\big(-3u_3+4\big)G_{0,0}(u_3)+\frac{1}{2}u_3G_{0,1}(u_3)+\frac{1}{2}u_3G_{1,0}(u_3)$$

$$-\frac{1}{2}\frac{\big(3u_3+1\big)u_3}{u_3-1}G_{1,1}(u_3)\Big)\pi-\frac{1}{6}u_3\pi^3\Big)\log(\frac{m_t^2}{t})+\Big(\Big(-\frac{1}{24}\frac{4u_3^4-8u_3^3+u_3^2+3u_3+12}{u_3-1}+\frac{3}{2}\frac{u_3^4-2u_3^3+5u_3^2-4u_3+2}{u_3-1}\zeta_2$$

$$-\frac{1}{12}\frac{6u_3^3+12u_3^2-27u_3+20}{u_3-1}G_0(u_3)+\frac{1}{12}\frac{6u_3^3-30u_3^2+15u_3-11}{u_3-1}G_1(u_3)+\frac{1}{4}\frac{2u_3^4-4u_3^3+9u_3^2-9u_3+4}{u_3-1}G_{0,0}(u_3)$$

$$-\frac{1}{4}\frac{\big(u_3^2-u_3+2\big)\big(2u_3^2-2u_3+1\big)}{u_3-1}G_{0,1}(u_3)-\frac{1}{4}\frac{\big(u_3^2-u_3+2\big)\big(2u_3^2-2u_3+1\big)}{u_3-1}G_{1,0}(u_3)+\frac{1}{4}\frac{2u_3^4-4u_3^3+9u_3^2-5u_3+2}{u_3-1}G_{1,1}(u_3)\Big)i$$

$$+\Big(\frac{1}{24}\frac{51u_3^2-51u_3+22}{u_3-1}+\frac{1}{4}\big(u_3-2\big)G_0(u_3)+\frac{1}{4}\frac{\big(u_3+1\big)u_3}{u_3-1}G_1(u_3)\Big)\pi\Big)\log^2(\frac{m_t^2}{t})+\Big(\Big(\frac{1}{4}\frac{2u_3^2-2u_3+1}{u_3-1}-\frac{1}{12}u_3G_0(u_3)$$

$$-\frac{1}{12}u_3G_1(u_3)\Big)i-\frac{1}{12}u_3\pi\Big)\log^3(\frac{m_t^2}{t})+\frac{1}{48}u_3i\log^4(\frac{m_t^2}{t})\Big)N_c+\Big(\Big(2\big(u_3-1\big)u_3^2+\Big(-\big(u_3-1\big)u_3^2+\big(2u_3^2-2u_3-1\big)u_3G_0(u_3)$$

$$+\big(2u_3^2-2u_3-1\big)u_3G_1(u_3)\Big)\zeta_2+2\frac{u_3^2-u_3+1}{u_3-1}G_0(u_3)+2\frac{u_3^2-u_3+1}{u_3-1}G_1(u_3)-\frac{4}{3}\big(u_3-1\big)\big(u_3+1\big)G_{0,0}(u_3)$$

$$
\begin{aligned}
&- \frac{2}{3} \frac{u_3^2 - u_3 + 1}{u_3 - 1} G_{0,1}(u_3) - \frac{2}{3} \frac{u_3^2 - u_3 + 1}{u_3 - 1} G_{1,0}(u_3) + \frac{4}{3} \frac{(u_3 - 2)u_3^2}{u_3 - 1} G_{1,1}(u_3) + \frac{1}{2} (4u_3^3 - 4u_3^2 + u_3 - 2) G_{0,0,0}(u_3) \\
&- \frac{1}{6} (2u_3 + 1)(2u_3^2 - 3u_3 + 2) G_{0,0,1}(u_3) - \frac{1}{6} (2u_3 + 1)(2u_3^2 - 3u_3 + 2) G_{0,1,0}(u_3) - \frac{1}{6} \frac{(2u_3 - 3)(2u_3^2 - u_3 + 1)u_3}{u_3 - 1} G_{0,1,1}(u_3) \\
&- \frac{1}{6} (2u_3 + 1)(2u_3^2 - 3u_3 + 2) G_{1,0,0}(u_3) - \frac{1}{6} \frac{(2u_3 - 3)(2u_3^2 - u_3 + 1)u_3}{u_3 - 1} G_{1,0,1}(u_3) - \frac{1}{6} \frac{(2u_3 - 3)(2u_3^2 - u_3 + 1)u_3}{u_3 - 1} G_{1,1,0}(u_3) \\
&+ \frac{1}{2} \frac{(4u_3^3 - 8u_3^2 + 5u_3 + 1)u_3}{u_3 - 1} G_{1,1,1}(u_3) \Big) i + \Big(\frac{2}{3} \frac{u_3^4 - 2u_3^3 - 2u_3^2 + 3u_3 - 3}{u_3 - 1} + \frac{2}{3} (u_3 - 1)(u_3 + 1) G_0(u_3) - \frac{2}{3} \frac{(u_3 - 2)u_3^2}{u_3 - 1} G_1(u_3) \\
&+ \frac{1}{6} (- 4u_3^3 + 4u_3^2 - 3u_3 + 2) G_{0,0}(u_3) + \frac{1}{6} (4u_3^2 - 4u_3 - 1) u_3 G_{0,1}(u_3) + \frac{1}{6} (4u_3^2 - 4u_3 - 1) u_3 G_{1,0}(u_3) \\
&- \frac{1}{6} \frac{(4u_3^3 - 8u_3^2 + 7u_3 - 1)u_3}{u_3 - 1} G_{1,1}(u_3) \Big) \pi - \frac{1}{3} (u_3 - 1) u_3^2 \pi^3 + \Big(\Big(- \frac{4}{3} \frac{(u_3 - 1)u_3^3}{s} + \Big(2 \frac{(2u_3^4 - 4u_3^3 - u_3^2 + 3u_3 - 1)u_3}{(u_3 - 1)s} - 2 \frac{u_3}{s} G_0(u_3) \\
&+ 2 \frac{u_3^2}{(u_3 - 1)s} G_1(u_3) \Big) \zeta_2 + \frac{4}{3} \frac{u_3^4 - 2u_3^3 - u_3^2 + 2u_3 - 1}{s(u_3 - 1)^2} G_0(u_3) + \frac{4}{3} \frac{u_3^4 - 2u_3^3 - u_3^2 + 2u_3 - 1}{s(u_3 - 1)^2} G_1(u_3) + \frac{2}{3} \frac{u_3^3 - 3u_3^2 - u_3 + 2}{(u_3 - 1)s} G_{0,0}(u_3) \\
&+ \frac{2}{3} \frac{u_3^4 - 2u_3^3 - u_3^2 + 2u_3 - 1}{s(u_3 - 1)^2} G_{0,1}(u_3) + \frac{2}{3} \frac{u_3^4 - 2u_3^3 - u_3^2 + 2u_3 - 1}{s(u_3 - 1)^2} G_{1,0}(u_3) + \frac{2}{3} \frac{(u_3^3 - 4u_3 + 1)u_3}{s(u_3 - 1)^2} G_{1,1}(u_3) + 2 \frac{u_3^2 - 2}{s} G_{0,0,0}(u_3) \\
&- \frac{2}{3} \frac{u_3^3 - u_3^2 + u_3 - 2}{(u_3 - 1)s} G_{0,0,1}(u_3) - \frac{2}{3} \frac{u_3^3 - u_3^2 + u_3 - 2}{(u_3 - 1)s} G_{0,1,0}(u_3) - \frac{2}{3} \frac{(u_3^3 - 2u_3^2 + 2u_3 + 1)u_3}{s(u_3 - 1)^2} G_{0,1,1}(u_3) \\
&- \frac{2}{3} \frac{u_3^3 - u_3^2 + u_3 - 2}{(u_3 - 1)s} G_{1,0,0}(u_3) - \frac{2}{3} \frac{(u_3^3 - 2u_3^2 + 2u_3 + 1)u_3}{s(u_3 - 1)^2} G_{1,0,1}(u_3) - \frac{2}{3} \frac{(u_3^3 - 2u_3^2 + 2u_3 + 1)u_3}{s(u_3 - 1)^2} G_{1,1,0}(u_3) \\
&+ 2 \frac{(u_3^2 - 2u_3 - 1)u_3^2}{s(u_3 - 1)^2} G_{1,1,1}(u_3) \Big) i + \Big(\frac{2}{3} \frac{u_3^6 - 3u_3^5 + u_3^4 + 3u_3^3 + 2u_3^2 - 4u_3 + 2}{s(u_3 - 1)^2} - \frac{2}{3} \frac{(u_3 - 1)(2u_3 + 1)}{s} G_0(u_3) \\
&- \frac{2}{3} \frac{(2u_3 - 3)u_3^3}{s(u_3 - 1)^2} G_1(u_3) - \frac{2}{3} \frac{2u_3^2 + u_3 - 2}{s} G_{0,0}(u_3) - \frac{2}{3} \frac{(2u_3^2 - 5u_3 + 1)u_3^2}{s(u_3 - 1)^2} G_{1,1}(u_3) \Big) \pi - \frac{1}{3} \frac{u_3^2}{s} \pi^3 + \Big(\Big(\frac{2}{3} \frac{(u_3 - 1)u_3^3}{s} \\
&- 2 \frac{u_3}{(u_3 - 1)s} \zeta_2 - \frac{1}{3} \frac{(u_3^2 - u_3 - 1)(5u_3^2 - 5u_3 + 2)}{s(u_3 - 1)^2} G_0(u_3) - \frac{1}{3} \frac{(u_3^2 - u_3 - 1)(5u_3^2 - 5u_3 + 2)}{s(u_3 - 1)^2} G_1(u_3) - \frac{2}{3} \frac{u_3^3 - 4u_3 + 4}{(u_3 - 1)s} G_{0,0}(u_3) \\
&- \frac{1}{3} \frac{(u_3^2 - u_3 - 4)(2u_3^2 - 2u_3 + 1)}{s(u_3 - 1)^2} G_{0,1}(u_3) - \frac{1}{3} \frac{(u_3^2 - u_3 - 4)(2u_3^2 - 2u_3 + 1)}{s(u_3 - 1)^2} G_{1,0}(u_3) - \frac{2}{3} \frac{(u_3^3 - 3u_3^2 - u_3 - 1)u_3}{s(u_3 - 1)^2} G_{1,1}(u_3) \Big) i \\
&+ \Big(- \frac{1}{3} \frac{4u_3^6 - 12u_3^5 + 7u_3^4 + 6u_3^3 - 2u_3^2 - 3u_3 + 2}{s(u_3 - 1)^2} + \frac{1}{3} \frac{u_3^2 + u_3 - 4}{s} G_0(u_3) + \frac{1}{3} \frac{(u_3^2 - 3u_3 - 2)u_3^2}{s(u_3 - 1)^2} G_1(u_3) \Big) \pi \Big) \log(\frac{m_t^2}{t}) \\
&+ \Big(\Big(- \frac{2}{3} \frac{(u_3 - 1)u_3^3}{s} + \frac{1}{3} \frac{u_3^4 - 2u_3^3 - 3u_3^2 + 4u_3 - 2}{s(u_3 - 1)^2} G_0(u_3) + \frac{1}{3} \frac{u_3^4 - 2u_3^3 - 3u_3^2 + 4u_3 - 2}{s(u_3 - 1)^2} G_1(u_3) \Big) i \\
&- \frac{1}{3} \frac{u_3^4 - 2u_3^3 - 3u_3^2 + 4u_3 - 2}{s(u_3 - 1)^2} \pi \Big) \log^2(\frac{m_t^2}{t}) \Big) m_t^2 + \Big(\Big(\frac{2}{3} (u_3 - 1) u_3^2 + u_3 \zeta_2 + \frac{2}{3} \frac{u_3^2 - u_3 + 1}{u_3 - 1} G_0(u_3) + \frac{2}{3} \frac{u_3^2 - u_3 + 1}{u_3 - 1} G_1(u_3) \\
&+ \frac{1}{3} (- u_3 + 2) G_{0,0}(u_3) - \frac{1}{3} \frac{u_3^2 - u_3 + 1}{u_3 - 1} G_{0,1}(u_3) - \frac{1}{3} \frac{u_3^2 - u_3 + 1}{u_3 - 1} G_{1,0}(u_3) - \frac{1}{3} \frac{(u_3 + 1)u_3}{u_3 - 1} G_{1,1}(u_3) \Big) i \\
&+ \Big(\frac{1}{3} \frac{u_3^4 - 2u_3^3 - u_3^2 + 2u_3 - 2}{u_3 - 1} + \frac{1}{3} (u_3 - 1) G_0(u_3) + \frac{1}{3} \frac{u_3^2}{u_3 - 1} G_1(u_3) \Big) \pi \Big) \log(\frac{m_t^2}{t}) + \Big(\Big(\frac{1}{6} (u_3 - 1) u_3^2 + \frac{1}{12} \frac{3u_3^2 - 3u_3 + 2}{u_3 - 1} G_0(u_3) \\
&+ \frac{1}{12} \frac{3u_3^2 - 3u_3 + 2}{u_3 - 1} G_1(u_3) \Big) i - \frac{1}{12} \frac{3u_3^2 - 3u_3 + 2}{u_3 - 1} \pi \Big) \log^2(\frac{m_t^2}{t}) \Big) N_f + \Big(2(u_3 - 1) u_3^2 - (u_3 - 1) u_3^2 \zeta_2 \Big) i + \frac{2}{3} (u_3 - 1) u_3^2 \pi \\
&+ \Big(\Big(- \frac{148}{3} \frac{(u_3 - 1)u_3^3}{s} + 112 \frac{(u_3 - 1)u_3^3}{s} \zeta_2 - 45 \frac{(u_3 - 1)u_3^3}{s} \zeta_4 \Big) i - \frac{118}{3} \frac{(u_3 - 1)u_3^3}{s} \pi + 4 \frac{(u_3 - 1)u_3^3}{s} \pi^3 + \Big(\Big(- \frac{118}{3} \frac{(u_3 - 1)u_3^3}{s} \\
&+ 72 \frac{(u_3 - 1)u_3^3}{s} \zeta_2 \Big) i - \frac{112}{3} \frac{(u_3 - 1)u_3^3}{s} \pi + 2 \frac{(u_3 - 1)u_3^3}{s} \pi^3 \Big) \log(\frac{m_t^2}{t}) + \Big(\Big(- \frac{56}{3} \frac{(u_3 - 1)u_3^3}{s} + 18 \frac{(u_3 - 1)u_3^3}{s} \zeta_2 \Big) i \\
&- 12 \frac{(u_3 - 1)u_3^3}{s} \pi \Big) \log^2(\frac{m_t^2}{t}) + \Big(-4 \frac{(u_3 - 1)u_3^3}{s} i - 2 \frac{(u_3 - 1)u_3^3}{s} \pi \Big) \log^3(\frac{m_t^2}{t}) - \frac{1}{2} \frac{(u_3 - 1)u_3^3}{s} i \log^4(\frac{m_t^2}{t}) \Big) m_t^2 + \Big(\frac{2}{3} (u_3 - 1) u_3^2 i \\
&+ \frac{1}{3} (u_3 - 1) u_3^2 \pi \Big) \log(\frac{m_t^2}{t}) + \frac{1}{6} (u_3 - 1) u_3^2 i \log^2(\frac{m_t^2}{t}) + O((m_t^2)^2) + O(m_H^2)
\end{aligned}
\tag{E.7}
$$

$$
\begin{aligned}
(\Omega^q_{-++})^{2l} = &\Big(\Big(4 \frac{(3u_3 - 5)u_3}{u_3 - 1} + \frac{13u_3 - 14}{u_3 - 1} G_1(u_3) + \frac{u_3^2 - 6}{u_3 - 1} G_{0,1}(u_3) - \frac{u_3^2 + 4u_3 - 6}{u_3 - 1} G_{1,1}(u_3) - 2 \frac{u_3}{u_3 - 1} G_{0,0,1}(u_3) + 2 \frac{u_3}{u_3 - 1} G_{0,1,1}(u_3) \\
&+ 2 \frac{u_3}{u_3 - 1} G_{1,0,1}(u_3) \Big) i\pi + \Big(- \frac{2}{3} \frac{u_3^2}{u_3 - 1} + \frac{1}{3} \frac{u_3}{u_3 - 1} G_1(u_3) \Big) i\pi^3 + \Big(\Big(2 \frac{(11u_3 - 12)u_3^3}{s(u_3 - 1)^2} + 4 \frac{u_3^2}{(u_3 - 1)s} G_1(u_3) \\
&- 2 \frac{(u_3^2 - 2u_3 - 2)u_3^2}{s(u_3 - 1)^2} G_{0,1}(u_3) + 2 \frac{(u_3^2 - 2u_3 + 2)u_3^2}{s(u_3 - 1)^2} G_{1,1}(u_3) - 4 \frac{u_3^2}{s(u_3 - 1)^2} G_{0,0,1}(u_3) + 4 \frac{u_3^2}{s(u_3 - 1)^2} G_{0,1,1}(u_3) \\
&+ 4 \frac{u_3^2}{s(u_3 - 1)^2} G_{1,0,1}(u_3) \Big) i\pi + \Big(- \frac{(u_3 - 2)u_3^3}{s(u_3 - 1)^2} + \frac{2}{3} \frac{u_3^2}{s(u_3 - 1)^2} G_1(u_3) \Big) i\pi^3 + 2 \frac{21u_3^4 - 24u_3^3 + 26u_3^2 - 52u_3 + 26}{s(u_3 - 1)^2} \\
&+ \Big(-2 \frac{20u_3^4 - 28u_3^3 + u_3^2 - 2u_3 + 1}{s(u_3 - 1)^2} + 4 \frac{1}{s} G_0(u_3) + 4 \frac{u_3^2}{s(u_3 - 1)^2} G_1(u_3) + 8 \frac{1}{s} G_{0,0}(u_3) - 12 \frac{u_3^2}{s(u_3 - 1)^2} G_{0,1}(u_3)
\end{aligned}
$$

$$+ 4\frac{u_3^2}{s(u_3-1)^2}G_{1,1}(u_3)\Big)\zeta_2 + \Big(-4\frac{u_3^4-2u_3^3-4u_3^2+8u_3-4}{s(u_3-1)^2} - 32\frac{1}{s}G_0(u_3)\Big)\zeta_3 + \frac{23u_3^4-46u_3^3+16u_3^2-32u_3+16}{s(u_3-1)^2}\zeta_4$$

$$+ 22\frac{1}{s}G_0(u_3) + 2\frac{u_3^3}{s(u_3-1)^2}G_1(u_3) + 28\frac{1}{s}G_{0,0}(u_3) - 2\frac{(u_3-2)u_3^2}{s(u_3-1)^2}G_{1,1}(u_3) + 20\frac{1}{s}G_{0,0,0}(u_3) + 2\frac{(u_3^2-2u_3-2)u_3^2}{s(u_3-1)^2}G_{0,1,1}(u_3)$$

$$- 2\frac{(u_3^2-2u_3+3)u_3^2}{s(u_3-1)^2}G_{1,1,1}(u_3) + 4\frac{1}{s}G_{0,0,0,0}(u_3) + 4\frac{u_3^2}{s(u_3-1)^2}G_{0,0,1,1}(u_3) - 8\frac{u_3^2}{s(u_3-1)^2}G_{0,1,1,1}(u_3)$$

$$- 4\frac{u_3^2}{s(u_3-1)^2}G_{1,0,1,1}(u_3) + \Big(\Big(-8\frac{u_3^3}{(u_3-1)s} - 2\frac{u_3^2}{s(u_3-1)^2}G_1(u_3) - 4\frac{u_3^2}{s(u_3-1)^2}G_{0,1}(u_3)\Big)i\pi + \frac{2}{3}\frac{(u_3-2)u_3^3}{s(u_3-1)^2}i\pi^3$$

$$- 22\frac{u_3^3+u_3-1}{(u_3-1)s} + \Big(4\frac{u_3^4-2u_3^3-u_3^2+2u_3-1}{s(u_3-1)^2} - 8\frac{1}{s}G_0(u_3) - 4\frac{u_3^2}{s(u_3-1)^2}G_1(u_3)\Big)\zeta_2 + 32\frac{1}{s}\zeta_3 - 28\frac{1}{s}G_0(u_3) - 2\frac{u_3^3}{s(u_3-1)^2}G_1(u_3)$$

$$- 20\frac{1}{s}G_{0,0}(u_3) + 2\frac{u_3^2}{s(u_3-1)^2}G_{1,1}(u_3) - 4\frac{1}{s}G_{0,0,0}(u_3) + 4\frac{u_3^2}{s(u_3-1)^2}G_{0,1,1}(u_3)\Big)\log(\frac{m_t^2}{t}) + \Big(-\frac{(u_3-2)u_3^3}{s(u_3-1)^2}i\pi$$

$$+ \frac{4u_3^4-3u_3^3+14u_3^2-28u_3+14}{s(u_3-1)^2} - 2\frac{u_3^4-2u_3^3-2u_3^2+4u_3-2}{s(u_3-1)^2}\zeta_2 + 10\frac{1}{s}G_0(u_3) + 2\frac{1}{s}G_{0,0}(u_3)\Big)\log^2(\frac{m_t^2}{t})$$

$$+ \Big(\frac{1}{3}\frac{u_3^4-2u_3^3-10u_3^2+20u_3-10}{s(u_3-1)^2} - \frac{2}{3}\frac{1}{s}G_0(u_3)\Big)\log^3(\frac{m_t^2}{t}) + \frac{1}{6}\frac{1}{s}\log^4(\frac{m_t^2}{t})\Big)m_t^2 - 2\frac{15u_3^2-7u_3+7}{u_3-1} + \Big(\frac{25u_3^2-16u_3-2}{u_3-1}$$

$$- 2G_0(u_3) + 6\frac{2u_3-3}{u_3-1}G_1(u_3) + 3G_{0,0}(u_3) - 6\frac{u_3}{u_3-1}G_{0,1}(u_3) + 2\frac{u_3}{u_3-1}G_{1,1}(u_3)\Big)\zeta_2 + \Big(16-8G_0(u_3)\Big)\zeta_3 - \frac{1}{2}\frac{23u_3^2-13u_3+13}{u_3-1}\zeta_4$$

$$+ 4G_0(u_3) + 20\frac{u_3}{u_3-1}G_1(u_3) - G_{0,0}(u_3) - \frac{19u_3-14}{u_3-1}G_{1,1}(u_3) + \frac{3}{2}G_{0,0,0}(u_3) - \frac{u_3^2-6}{u_3-1}G_{0,1,1}(u_3) + \frac{u_3^2+9u_3-12}{u_3-1}G_{1,1,1}(u_3)$$

$$+ G_{0,0,0,0}(u_3) + 2\frac{u_3}{u_3-1}G_{0,0,1,1}(u_3) - 4\frac{u_3}{u_3-1}G_{0,1,1,1}(u_3) - 2\frac{u_3}{u_3-1}G_{1,0,1,1}(u_3) + \Big(\Big(2\frac{(2u_3-3)u_3}{u_3-1} + \frac{5u_3-6}{u_3-1}G_1(u_3)$$

$$- 2\frac{u_3}{u_3-1}G_{0,1}(u_3)\Big)i\pi - \frac{1}{3}\frac{u_3^2}{u_3-1}i\pi^3 - 4\frac{3u_3^2+u_3-1}{u_3-1} + \Big(\frac{3u_3^2+2u_3-2}{u_3-1} - 3G_0(u_3) - 2\frac{u_3}{u_3-1}G_1(u_3)\Big)\zeta_2 + 8\zeta_3 + G_0(u_3)$$

$$+ 6\frac{u_3}{u_3-1}G_1(u_3) - \frac{3}{2}G_{0,0}(u_3) - \frac{5u_3-6}{u_3-1}G_{1,1}(u_3) - G_{0,0,0}(u_3) + 2\frac{u_3}{u_3-1}G_{0,1,1}(u_3)\Big)\log(\frac{m_t^2}{t}) + \Big(-\frac{1}{2}\frac{4u_3^2+u_3-1}{u_3-1}$$

$$+ \frac{1}{2}\frac{2u_3^2+3u_3-3}{u_3-1}\zeta_2 + \frac{3}{4}G_0(u_3) + \frac{1}{2}G_{0,0}(u_3)\Big)\log^2(\frac{m_t^2}{t}) + \Big(-\frac{1}{4} - \frac{1}{6}G_0(u_3)\Big)\log^3(\frac{m_t^2}{t}) + \frac{1}{24}\log^4(\frac{m_t^2}{t})\Big)\frac{1}{N_c}$$

$$+ \Big(\Big(-2\frac{6u_3^2-5u_3-5}{u_3-1} - 4\frac{(u_3-2)u_3}{(u_3-1)^2}\zeta_3 - \frac{2}{3}\frac{7u_3^2-8u_3+4}{(u_3-1)^2}G_0(u_3) - 3\frac{u_3}{u_3-1}G_1(u_3) + \frac{1}{6}\frac{6u_3^3-4u_3^2+5u_3+14}{(u_3-1)^2}G_{0,0}(u_3)$$

$$- \frac{1}{2}\frac{(u_3-2)(2u_3+1)}{(u_3-1)^2}G_{0,1}(u_3) - \frac{(u_3^2-3u_3+4)u_3}{(u_3-1)^2}G_{1,0}(u_3) + \frac{u_3}{u_3-1}G_{1,1}(u_3) - 3G_{0,0,0}(u_3) - \frac{(u_3-2)u_3}{(u_3-1)^2}G_{0,0,1}(u_3)$$

$$+ \frac{(u_3-2)u_3}{(u_3-1)^2}G_{0,1,0}(u_3) + \frac{(u_3-2)u_3}{(u_3-1)^2}G_{0,1,1}(u_3) + G_{1,0,0}(u_3)\Big)i\pi + \Big(\frac{1}{12}\frac{(10u_3^2-21u_3+18)u_3}{(u_3-1)^2} + \frac{1}{12}\frac{(u_3-2)u_3}{(u_3-1)^2}G_0(u_3)\Big)i\pi^3$$

$$+ \Big(\Big(-\frac{1}{6}\frac{132u_3^4-177u_3^3+52u_3^2-41u_3+40}{s(u_3-1)^2} + 2\frac{(6u_3-11)u_3}{s(u_3-1)^2}\zeta_3 + \frac{1}{6}\frac{18u_3^3-47u_3^2+97u_3-44}{s(u_3-1)^2}G_0(u_3) - \frac{(5u_3-4)u_3^2}{s(u_3-1)^2}G_1(u_3)$$

$$- \frac{1}{6}\frac{12u_3^4-33u_3^3+82u_3^2-149u_3+64}{s(u_3-1)^2}G_{0,0}(u_3) + 3\frac{u_3^3}{s(u_3-1)^2}G_{0,1}(u_3) + \frac{1}{2}\frac{4u_3^3-9u_3^2+11u_3-20}{(u_3-1)s}G_{1,0}(u_3) - \frac{(2u_3-1)u_3^2}{s(u_3-1)^2}G_{1,1}(u_3)$$

$$- \frac{10u_3^2-25u_3+12}{s(u_3-1)^2}G_{0,0,0}(u_3) - 4\frac{u_3^2}{s(u_3-1)^2}G_{0,0,1}(u_3) + \frac{(4u_3-3)u_3}{s(u_3-1)^2}G_{0,1,0}(u_3) + 2\frac{(u_3+1)u_3}{s(u_3-1)^2}G_{0,1,1}(u_3) + 2\frac{u_3-2}{(u_3-1)s}G_{1,0,0}(u_3)$$

$$+ 6\frac{u_3}{(u_3-1)s}G_{1,0,1}(u_3) - 4\frac{u_3}{(u_3-1)s}G_{1,1,1}(u_3)\Big)i\pi + \Big(\frac{1}{6}\frac{(4u_3^3-3u_3^2+2)u_3}{s(u_3-1)^2} - \frac{(u_3-2)u_3}{s(u_3-1)^2}G_0(u_3) - \frac{1}{3}\frac{u_3}{(u_3-1)s}G_1(u_3)\Big)i\pi^3$$

$$- \frac{1}{18}\frac{756u_3^4-1026u_3^3+2083u_3^2-4139u_3+2218}{s(u_3-1)^2} + \Big(2\frac{20u_3^4-29u_3^3+43u_3^2-91u_3+49}{s(u_3-1)^2} + \frac{1}{2}\frac{6u_3^3+41u_3^2-212u_3+120}{s(u_3-1)^2}G_0(u_3)$$

$$+ \frac{1}{2}\frac{2u_3^3+46u_3^2-117u_3+60}{s(u_3-1)^2}G_1(u_3) - \frac{u_3^2+37u_3-22}{s(u_3-1)^2}G_{0,0}(u_3) - \frac{(11u_3+1)u_3}{s(u_3-1)^2}G_{0,1}(u_3) + 12\frac{u_3^2-3u_3+1}{s(u_3-1)^2}G_{1,0}(u_3)$$

$$- \frac{u_3^2-u_3+4}{s(u_3-1)^2}G_{1,1}(u_3)\Big)\zeta_2 + \Big(\frac{2u_3^4-5u_3^3-79u_3^2+168u_3-82}{s(u_3-1)^2} + 2\frac{18u_3^2-33u_3+19}{s(u_3-1)^2}G_0(u_3) + 2\frac{(u_3+1)u_3}{s(u_3-1)^2}G_1(u_3)\Big)\zeta_3$$

$$- \frac{23u_3^4-46u_3^3+32u_3^2+21u_3-23}{s(u_3-1)^2}\zeta_4 + \frac{1}{9}\frac{63u_3^3-601u_3^2+1229u_3-664}{s(u_3-1)^2}G_0(u_3) - \frac{1}{6}\frac{60u_3^3+61u_3^2-149u_3+40}{s(u_3-1)^2}G_1(u_3)$$

$$- \frac{1}{18}\frac{36u_3^3+169u_3^2-401u_3+268}{s(u_3-1)^2}G_{0,0}(u_3) - \frac{1}{6}\frac{47u_3^2-97u_3+44}{s(u_3-1)^2}G_{0,1}(u_3) - \frac{1}{6}\frac{47u_3^2-97u_3+44}{s(u_3-1)^2}G_{1,0}(u_3) - 2\frac{u_3^2+2u_3-2}{s(u_3-1)^2}G_{1,1}(u_3)$$

$$+ \frac{1}{2}\frac{4u_3^4-11u_3^3-69u_3^2+108u_3-58}{s(u_3-1)^2}G_{0,0,0}(u_3) - \frac{1}{6}\frac{6u_3^3+22u_3^2-125u_3+64}{s(u_3-1)^2}G_{0,0,1}(u_3) - \frac{1}{6}\frac{6u_3^3+22u_3^2-125u_3+64}{s(u_3-1)^2}G_{0,1,0}(u_3)$$

$$- \frac{1}{2}\frac{4u_3^3+15u_3^2+4u_3-4}{s(u_3-1)^2}G_{0,1,1}(u_3) - \frac{1}{6}\frac{12u_3^3-21u_3^2+37u_3-64}{(u_3-1)s}G_{1,0,0}(u_3) + \frac{1}{2}\frac{2u_3^3-23u_3^2+35u_3-16}{s(u_3-1)^2}G_{1,0,1}(u_3)$$

$$+\frac{1}{2}\frac{2u_3^3-23u_3^2+35u_3-16}{s(u_3-1)^2}G_{1,1,0}(u_3)+\frac{1}{2}\frac{4u_3^3+24u_3^2-43u_3+20}{s(u_3-1)^2}G_{1,1,1}(u_3)+\frac{44u_3^2-121u_3+63}{s(u_3-1)^2}G_{0,0,0,0}(u_3)$$

$$-\frac{(u_3-4)(5u_3-3)}{s(u_3-1)^2}G_{0,0,0,1}(u_3)-\frac{(u_3-4)(5u_3-3)}{s(u_3-1)^2}G_{0,0,1,0}(u_3)+\frac{u_3-4}{(u_3-1)s}G_{0,0,1,1}(u_3)-\frac{16u_3^2-29u_3+12}{s(u_3-1)^2}G_{0,1,0,0}(u_3)$$

$$+4\frac{1}{s}G_{0,1,0,1}(u_3)+4\frac{1}{s}G_{0,1,1,0}(u_3)-\frac{(7u_3+1)u_3}{s(u_3-1)^2}G_{0,1,1,1}(u_3)-\frac{11u_3^2-19u_3+12}{s(u_3-1)^2}G_{1,0,0,0}(u_3)-\frac{(u_3-5)u_3}{s(u_3-1)^2}G_{1,0,0,1}(u_3)$$

$$-\frac{(u_3-5)u_3}{s(u_3-1)^2}G_{1,0,1,0}(u_3)-2\frac{u_3^2+3u_3-2}{s(u_3-1)^2}G_{1,0,1,1}(u_3)-2\frac{u_3}{(u_3-1)s}G_{1,1,0,0}(u_3)+\frac{5u_3-4}{(u_3-1)s}G_{1,1,0,1}(u_3)$$

$$+\frac{5u_3-4}{(u_3-1)s}G_{1,1,1,0}(u_3)+\frac{3u_3+4}{(u_3-1)s}G_{1,1,1,1}(u_3)+\Bigg(\bigg(\bigg(\frac{1}{6}\frac{48u_3^4-48u_3^3+11u_3^2-25u_3+20}{s(u_3-1)^2}+\frac{1}{6}\frac{3u_3^3+25u_3^2-101u_3+52}{s(u_3-1)^2}G_0(u_3)$$

$$+\frac{u_3^3}{s(u_3-1)^2}G_1(u_3)+2\frac{2u_3^2-7u_3+4}{s(u_3-1)^2}G_{0,0}(u_3)-2\frac{u_3^2}{s(u_3-1)^2}G_{0,1}(u_3)+2\frac{u_3}{(u_3-1)s}G_{1,1}(u_3)\bigg)i\pi-\frac{1}{3}\frac{(2u_3^3-4u_3^2-u_3+4)u_3}{s(u_3-1)^2}i\pi^3$$

$$+\frac{1}{18}\frac{396u_3^3-81u_3^2+1064u_3-1172}{(u_3-1)s}+\bigg(-\frac{1}{2}\frac{8u_3^4+79u_3^2-216u_3+120}{s(u_3-1)^2}-\frac{(3u_3-2)(5u_3-11)}{s(u_3-1)^2}G_0(u_3)-2\frac{(u_3-3)u_3}{s(u_3-1)^2}G_1(u_3)\bigg)\zeta_2$$

$$-2\frac{17u_3^2-33u_3+19}{s(u_3-1)^2}\zeta_3-\frac{1}{18}\frac{18u_3^3-253u_3^2+587u_3-334}{s(u_3-1)^2}G_0(u_3)+\frac{1}{6}\frac{30u_3^3+35u_3^2-73u_3+20}{s(u_3-1)^2}G_1(u_3)$$

$$+\frac{1}{6}\frac{6u_3^3+179u_3^2-334u_3+170}{s(u_3-1)^2}G_{0,0}(u_3)+\frac{1}{6}\frac{67u_3^2-113u_3+52}{s(u_3-1)^2}G_{0,1}(u_3)+\frac{1}{6}\frac{67u_3^2-113u_3+52}{s(u_3-1)^2}G_{1,0}(u_3)$$

$$-\frac{1}{2}\frac{2u_3^3+3u_3^2-8u_3+4}{s(u_3-1)^2}G_{1,1}(u_3)-\frac{29u_3^2-73u_3+39}{s(u_3-1)^2}G_{0,0,0}(u_3)+2\frac{4u_3^2-9u_3+4}{s(u_3-1)^2}G_{0,0,1}(u_3)+2\frac{4u_3^2-9u_3+4}{s(u_3-1)^2}G_{0,1,0}(u_3)$$

$$+\frac{u_3^2+7u_3-4}{s(u_3-1)^2}G_{0,1,1}(u_3)+2\frac{5u_3-4}{(u_3-1)s}G_{1,0,0}(u_3)-\frac{3u_3-4}{(u_3-1)s}G_{1,0,1}(u_3)-\frac{3u_3-4}{(u_3-1)s}G_{1,1,0}(u_3)-4\frac{u_3}{(u_3-1)s}G_{1,1,1}(u_3)\bigg)\log(\frac{m_t^2}{t})$$

$$+\bigg(\bigg(\frac{1}{12}\frac{12u_3^4-45u_3^3+2u_3^2+65u_3-40}{s(u_3-1)^2}+\frac{1}{2}\frac{5u_3-4}{s(u_3-1)^2}G_0(u_3)\bigg)i\pi-\frac{1}{36}\frac{144u_3^4-72u_3^3+193u_3^2-485u_3+256}{s(u_3-1)^2}$$

$$+\frac{1}{2}\frac{4u_3^4-8u_3^3+27u_3^2-49u_3+22}{s(u_3-1)^2}\zeta_2-\frac{1}{12}\frac{9u_3^3+139u_3^2-284u_3+142}{s(u_3-1)^2}G_0(u_3)-\frac{1}{12}\frac{58u_3^2-89u_3+40}{s(u_3-1)^2}G_1(u_3)$$

$$+\frac{1}{2}\frac{17u_3^2-41u_3+23}{s(u_3-1)^2}G_{0,0}(u_3)-\frac{1}{2}\frac{7u_3-4}{(u_3-1)s}G_{0,1}(u_3)-\frac{1}{2}\frac{7u_3-4}{(u_3-1)s}G_{1,0}(u_3)+\frac{1}{2}\frac{5u_3-4}{(u_3-1)s}G_{1,1}(u_3)\bigg)\log^2(\frac{m_t^2}{t})$$

$$+\bigg(-\frac{1}{3}\frac{u_3}{(u_3-1)s}i\pi-\frac{1}{12}\frac{4u_3^4-18u_3^3-21u_3^2+58u_3-30}{s(u_3-1)^2}-\frac{5}{6}\frac{2u_3-3}{(u_3-1)s}G_0(u_3)+\frac{1}{3}\frac{u_3}{(u_3-1)s}G_1(u_3)\bigg)\log^3(\frac{m_t^2}{t})$$

$$+\frac{5}{24}\frac{2u_3-3}{(u_3-1)s}\log^4(\frac{m_t^2}{t})\Bigg)m_t^2+\frac{10}{3}\frac{9u_3^2-u_3+1}{u_3-1}+\bigg(-\frac{37u_3^2-63u_3+43}{u_3-1}+\frac{1}{2}\frac{24u_3^3-56u_3^2+63u_3-20}{(u_3-1)^2}G_0(u_3)$$

$$+\frac{(3u_3-1)(4u_3-3)}{u_3-1}G_1(u_3)+2\frac{3u_3^2-u_3+3}{(u_3-1)^2}G_{0,0}(u_3)-2\frac{(u_3-3)u_3}{(u_3-1)^2}G_{0,1}(u_3)-3\frac{u_3+1}{u_3-1}G_{1,0}(u_3)-\frac{3u_3-1}{u_3-1}G_{1,1}(u_3)\bigg)\zeta_2$$

$$+\bigg(-\frac{1}{2}\frac{6u_3^3+29u_3^2-84u_3+42}{(u_3-1)^2}+\frac{1}{2}\frac{5u_3^2-22u_3+4}{(u_3-1)^2}G_0(u_3)+2\frac{u_3}{u_3-1}G_1(u_3)\bigg)\zeta_3+\frac{1}{2}\frac{23u_3^3-8u_3^2+11u_3+14}{(u_3-1)^2}\zeta_4-\frac{125}{9}G_0(u_3)$$

$$-10\frac{3u_3-1}{u_3-1}G_1(u_3)-\frac{1}{9}\frac{36u_3^3-271u_3^2+434u_3-217}{(u_3-1)^2}G_{0,0}(u_3)+\frac{4}{3}\frac{3u_3^2-2u_3+2}{u_3-1}G_{0,1}(u_3)+\frac{4}{3}\frac{3u_3^2-2u_3+2}{u_3-1}G_{1,0}(u_3)$$

$$-\frac{4u_3^2-15u_3+6}{u_3-1}G_{1,1}(u_3)+\frac{1}{2}\frac{14u_3^3-63u_3^2+83u_3-40}{(u_3-1)^2}G_{0,0,0}(u_3)-\frac{1}{6}\frac{24u_3^3-65u_3^2+64u_3-14}{(u_3-1)^2}G_{0,0,1}(u_3)$$

$$-\frac{1}{6}\frac{24u_3^3-65u_3^2+64u_3-14}{(u_3-1)^2}G_{0,1,0}(u_3)-3G_{0,1,1}(u_3)+\frac{1}{6}\frac{6u_3^3-u_3^2-7u_3+14}{(u_3-1)^2}G_{1,0,0}(u_3)-2\frac{2u_3^2-1}{u_3-1}G_{1,0,1}(u_3)$$

$$-2\frac{2u_3^2-1}{u_3-1}G_{1,1,0}(u_3)+\frac{8u_3^2-7u_3+1}{u_3-1}G_{1,1,1}(u_3)+\frac{11u_3^2-18u_3+12}{(u_3-1)^2}G_{0,0,0,0}(u_3)-\frac{3}{2}\frac{3u_3^2-2u_3+2}{(u_3-1)^2}G_{0,0,0,1}(u_3)$$

$$-\frac{3}{2}\frac{3u_3^2-2u_3+2}{(u_3-1)^2}G_{0,0,1,0}(u_3)+\frac{5u_3^2-2u_3+1}{(u_3-1)^2}G_{0,0,1,1}(u_3)-\frac{1}{2}\frac{7u_3^2-14u_3+6}{(u_3-1)^2}G_{0,1,0,0}(u_3)+\frac{2u_3^2-2u_3+1}{(u_3-1)^2}G_{0,1,0,1}(u_3)$$

$$+\frac{2u_3^2-2u_3+1}{(u_3-1)^2}G_{0,1,1,0}(u_3)-4\frac{u_3}{u_3-1}G_{0,1,1,1}(u_3)-\frac{5u_3-3}{u_3-1}G_{1,0,0,0}(u_3)+2\frac{u_3}{u_3-1}G_{1,0,0,1}(u_3)+2\frac{u_3}{u_3-1}G_{1,0,1,0}(u_3)$$

$$-\frac{u_3+1}{u_3-1}G_{1,0,1,1}(u_3)+G_{1,1,0,1}(u_3)+G_{1,1,1,0}(u_3)-G_{1,1,1,1}(u_3)+\bigg(\bigg(\bigg(-\frac{2}{3}\frac{6u_3^2-4u_3-5}{u_3-1}+\frac{1}{3}\frac{2u_3^2-4u_3-1}{(u_3-1)^2}G_0(u_3)$$

$$-\frac{1}{2}\frac{3u_3-2}{u_3-1}G_1(u_3)+\frac{1}{2}\frac{3u_3^2-6u_3+4}{(u_3-1)^2}G_{0,0}(u_3)-\frac{1}{2}\frac{(u_3-2)u_3}{(u_3-1)^2}G_{0,1}(u_3)\bigg)i\pi+\frac{1}{12}\frac{(4u_3^2-9u_3+6)u_3}{(u_3-1)^2}i\pi^3+\frac{4}{9}\frac{27u_3^2+2u_3-2}{u_3-1}$$

$$+\bigg(-\frac{1}{2}\frac{30u_3^2-37u_3+20}{u_3-1}-\frac{1}{2}\frac{u_3^2-14u_3+12}{(u_3-1)^2}G_0(u_3)+2\frac{u_3}{u_3-1}G_1(u_3)\bigg)\zeta_2-\frac{1}{2}\frac{9u_3^2-14u_3+4}{(u_3-1)^2}\zeta_3-\frac{70}{9}G_0(u_3)-\frac{2}{3}\frac{14u_3-5}{u_3-1}G_1(u_3)$$

$$-\frac{1}{6}\frac{24u_3^3-95u_3^2+124u_3-59}{(u_3-1)^2}G_{0,0}(u_3)+\frac{1}{6}\frac{24u_3^2-17u_3+2}{u_3-1}G_{0,1}(u_3)+\frac{1}{6}\frac{24u_3^2-17u_3+2}{u_3-1}G_{1,0}(u_3)-\frac{4u_3^2-6u_3+3}{u_3-1}G_{1,1}(u_3)$$

$$
\begin{aligned}
&- \frac{1}{2}\,\frac{7u_3^2 - 18u_3 + 12}{\left(u_3 - 1\right)^2} G_{0,0,0}(u_3) + \frac{1}{2}\,\frac{u_3^2 - 6u_3 + 4}{\left(u_3 - 1\right)^2} G_{0,0,1}(u_3) + \frac{1}{2}\,\frac{u_3^2 - 6u_3 + 4}{\left(u_3 - 1\right)^2} G_{0,1,0}(u_3) + \frac{u_3 + 1}{u_3 - 1} G_{0,1,1}(u_3) + 2G_{1,0,0}(u_3) \\
&- G_{1,0,1}(u_3) - G_{1,1,0}(u_3)\bigg) \log(\frac{m_t^2}{t}) + \bigg(\bigg(- \frac{1}{12}\,\frac{7u_3 - 10}{u_3 - 1} - \frac{1}{4}\,\frac{u_3^2 - 2u_3 + 2}{\left(u_3 - 1\right)^2} G_0(u_3)\bigg) i\pi + \frac{1}{18}\,\frac{36u_3^2 + 31u_3 - 31}{u_3 - 1} - \frac{u_3^2 - 3u_3 + 3}{u_3 - 1} \zeta_2 \\
&- \frac{1}{12}\,\frac{25u_3 - 22}{u_3 - 1} G_0(u_3) - \frac{5}{6} G_1(u_3) + \frac{1}{4}\,\frac{3u_3^2 - 6u_3 + 4}{\left(u_3 - 1\right)^2} G_{0,0}(u_3) - \frac{1}{2} G_{0,1}(u_3) - \frac{1}{2} G_{1,0}(u_3) + \frac{1}{2} G_{1,1}(u_3)\bigg) \log^2(\frac{m_t^2}{t}) \\
&+ \frac{1}{4} \log^3(\frac{m_t^2}{t})\bigg) N_c + \bigg(\bigg(1 - \frac{1}{3} G_0(u_3) + \frac{1}{6} G_{0,0}(u_3)\bigg) i\pi + \bigg(\bigg(\frac{2}{3}\,\frac{1}{s} + \frac{1}{3}\,\frac{1}{s} G_0(u_3) + \frac{2}{3}\,\frac{1}{s} G_{0,0}(u_3)\bigg) i\pi - \frac{40}{9}\,\frac{1}{s} + \frac{4}{9}\,\frac{1}{s} G_0(u_3) + \frac{2}{3}\,\frac{1}{s} G_1(u_3) \\
&- \frac{16}{9}\,\frac{1}{s} G_{0,0}(u_3) + \frac{1}{3}\,\frac{1}{s} G_{0,1}(u_3) + \frac{1}{3}\,\frac{1}{s} G_{1,0}(u_3) + 8\frac{1}{s} G_{0,0,0}(u_3) + \frac{2}{3}\,\frac{1}{s} G_{0,0,1}(u_3) + \frac{2}{3}\,\frac{1}{s} G_{0,1,0}(u_3) + \frac{2}{3}\,\frac{1}{s} G_{1,0,0}(u_3) + \bigg(\bigg(- \frac{1}{3}\,\frac{1}{s} \\
&- \frac{2}{3}\,\frac{1}{s} G_0(u_3)\bigg) i\pi + \frac{20}{9}\,\frac{1}{s} + \frac{28}{9}\,\frac{1}{s} G_0(u_3) - \frac{1}{3}\,\frac{1}{s} G_1(u_3) - \frac{16}{3}\,\frac{1}{s} G_{0,0}(u_3) - \frac{2}{3}\,\frac{1}{s} G_{0,1}(u_3) - \frac{2}{3}\,\frac{1}{s} G_{1,0}(u_3)\bigg) \log(\frac{m_t^2}{t}) + \bigg(\frac{1}{3}\,\frac{1}{s} i\pi \\
&- \frac{20}{9}\,\frac{1}{s} + \frac{4}{3}\,\frac{1}{s} G_0(u_3) + \frac{1}{3}\,\frac{1}{s} G_1(u_3)\bigg) \log^2(\frac{m_t^2}{t})\bigg) m_t^2 - \frac{20}{3} + \frac{56}{9} G_0(u_3) + G_1(u_3) - \frac{34}{9} G_{0,0}(u_3) - \frac{1}{3} G_{0,1}(u_3) - \frac{1}{3} G_{1,0}(u_3) \\
&+ 2G_{0,0,0}(u_3) + \frac{1}{6} G_{0,0,1}(u_3) + \frac{1}{6} G_{0,1,0}(u_3) + \frac{1}{6} G_{1,0,0}(u_3) + \bigg(\bigg(\frac{1}{3} - \frac{1}{6} G_0(u_3)\bigg) i\pi - \frac{20}{9} + \frac{22}{9} G_0(u_3) + \frac{1}{3} G_1(u_3) - \frac{4}{3} G_{0,0}(u_3) \\
&- \frac{1}{6} G_{0,1}(u_3) - \frac{1}{6} G_{1,0}(u_3)\bigg) \log(\frac{m_t^2}{t}) + \bigg(\frac{1}{12} i\pi - \frac{5}{9} + \frac{1}{3} G_0(u_3) + \frac{1}{12} G_1(u_3)\bigg) \log^2(\frac{m_t^2}{t})\bigg) N_f + \bigg(- \frac{256}{9}\,\frac{1}{s} + \frac{76}{9}\,\frac{1}{s} G_0(u_3) \\
&- \frac{52}{9}\,\frac{1}{s} G_{0,0}(u_3) + 8\frac{1}{s} G_{0,0,0}(u_3) + \bigg(- \frac{76}{9}\,\frac{1}{s} + \frac{52}{9}\,\frac{1}{s} G_0(u_3) - 8\frac{1}{s} G_{0,0}(u_3)\bigg) \log(\frac{m_t^2}{t}) + \bigg(- \frac{26}{9}\,\frac{1}{s} + 4\frac{1}{s} G_0(u_3)\bigg) \log^2(\frac{m_t^2}{t}) \\
&- \frac{4}{3}\,\frac{1}{s} \log^3(\frac{m_t^2}{t})\bigg) m_t^2 - \frac{20}{3} + \frac{56}{9} G_0(u_3) - \frac{34}{9} G_{0,0}(u_3) + 2G_{0,0,0}(u_3) + \bigg(- \frac{56}{9} + \frac{34}{9} G_0(u_3) - 2G_{0,0}(u_3)\bigg) \log(\frac{m_t^2}{t}) \\
&+ \bigg(- \frac{17}{9} + G_0(u_3)\bigg) \log^2(\frac{m_t^2}{t}) - \frac{1}{3} \log^3(\frac{m_t^2}{t}) + O((m_t^2)^2) + O(m_H^2)
\end{aligned}
\tag{E.8}
$$