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Using composite indicators in econometric decision models with application to occupational health

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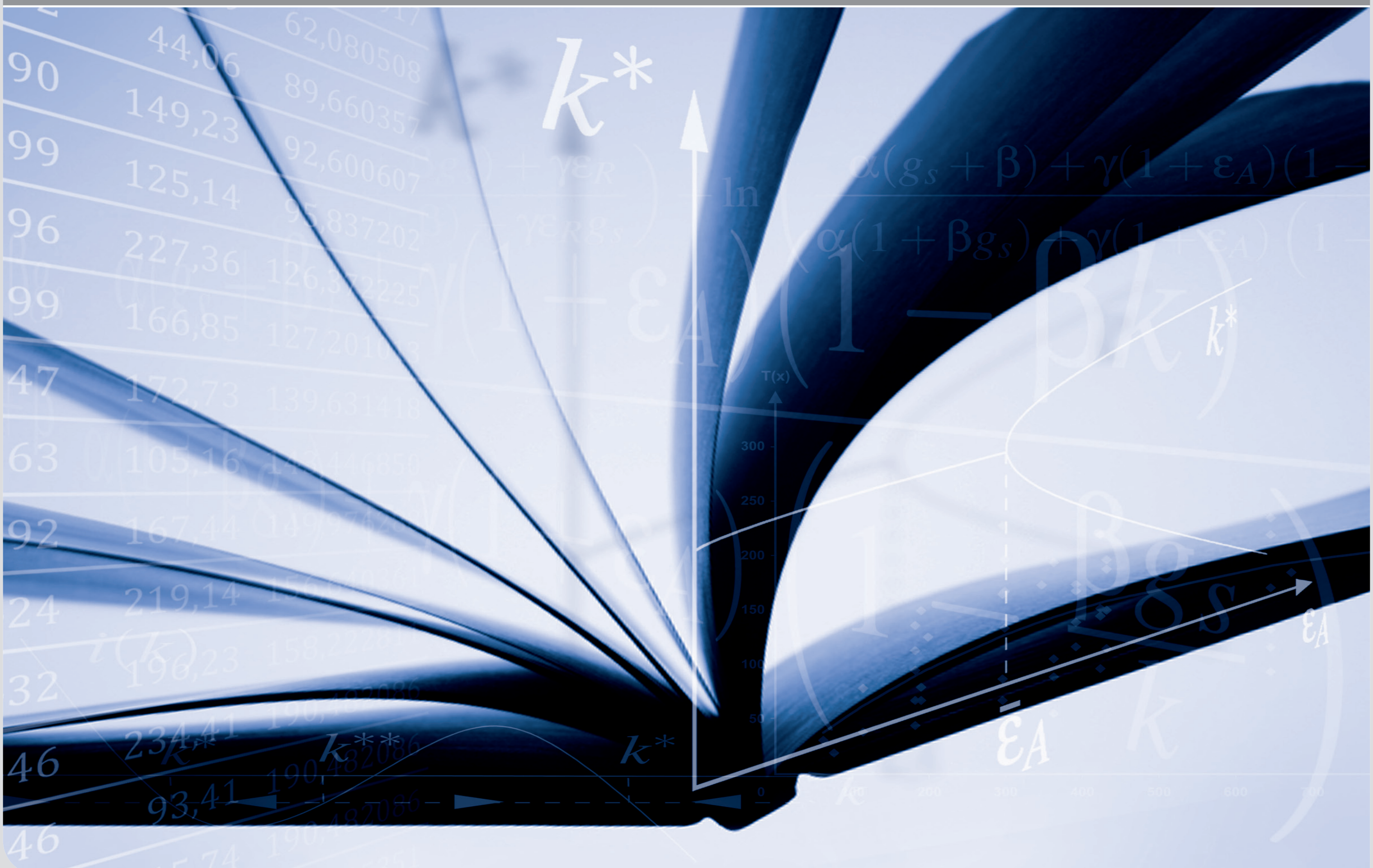

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Using Composite Indicators in Econometric Decision Models

With Application to Occupational Health[1]

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[1]Paper for the InGRID-2 Expert Workshop *Occupational Safety and Health Policy Indicators* organized by the Central Institute for Labour Protection – National Research Institute (CIOP–PIB), Warsaw, 16 October 2020

Abstract

The paper explains the use of composite indicators as objective functions in econometric decision models. We consider an example of minimizing the harm from two kinds of pollution at the workplace, which can be reduced by two types of anti-pollution measures restricted to a given budget. At first, we define the pollution harm as a sum of two factors and find the optimal budget distribution to minimize it. Next, the harm is regarded as a probabilistic risk, and the cumulative effects from the interaction of two factors is taken into account. The corresponding indicator of the occupational hazard becomes a quadratic function, resulting in a different budget distribution. We also discuss the methodology of using composite indicators and refer to advanced methods of their construction.

Keywords: occupational health; occupational hazard; composite indicators; objective functions; multiple criteria decision making; decision support aid; budgeting

JEL classification: D81 — Criteria for decision-making under risk and uncertainty; J28 — Safety; Q52 — Pollution control adaptation and costs; Q53 — Air pollution

Contents

A doctrine... can only contain so much science proper as there is in it of applied mathematics.

Immanuel Kant (1724–1804)[2]

1 Introduction

At the 2018 InGRID-2 expert meeting in Rome, we have discussed composite indicators for policy monitoring using the example of job quality [Tangian 2018]. At the 2019 meeting in Noisy-le-Grand, the issue was their commensuration, in particular, to evaluate the 'deal fairness' in collective bargaining [Tangian 2019]. At the current event, the subject is their use as objective functions in econometric decision models to minimize health risks at the workplace.

To be specific, I consider a conditional application. In March 2013, I visited the Gibson electric guitar factory in Memphis, Tennessee (closed in 2018). It occupied a single ten-meter high room divided by three-meter partitions into specialized workshops: electronic, metalworking, woodworking, painting, assembling and quality control ones. During the guided tour, it was explained that the types of pickups and electronic circles that determined the sound and even the kind of body — solid, semi-acoustic, etc. — had no significant impact on the guitar price, which primarily depended on finishing. The production of one guitar took a month or longer due to several stages of coating, polishing and drying, which were all harmful to workers' health. Indeed, the air was polluted with industrial dust — wooden or metal — and chemical fumes and vapors from painting, drying, electrolytic processing, quenching and other works.

Let us suppose that a certain budget is intended for the pollution reduction. It must be optimally distributed between dust collectors and chemical neutralizers to minimize the pollution harm. In this paper, we solve this problem in terms of econometric decision models, which include three elements:

- space of alternatives represented by n-dimensional vectors,
- constraints that restrict the space of alternatives to a feasible domain, which are given by equalities and/or inequalities in the n variables, and
- objective function on the space of alternatives — a composite indicator in the n variables, which operationalizes the decision maker's preference.

The optimal decision is found from the objective function's maximization or minimization, depending on the problem, subject to the constraints.

Section 2, 'Objective functions in econometric decision models', contains a few historical and methodological notes.

In Section 3, 'Econometric decision model to minimize occupational hazards', we describe the construction and solution of the econometric decision model for our example.

In Section 4, 'Harm to health as probabilistic risk', the optimization model is considered under the assumption that the occupational hazards are no longer indices but probabilistic risks.

In Section 5, 'Composite indicators as preference functions', some reservations about our approach are made.

In Section 6, 'Conclusion', the statements of the paper are recapitulated and placed into context.

2 Objective functions in econometric decision models

The idea of economic optimization stems from the mid 19th century [Gossen 1854, Menger 1871, Jevons 1871, Edgeworth 1881, Walras 1874]. Since then, a great effort has been made to develop

[2]Kant I (1786) Metaphysische Anfangsgründe der Naturwissenschaft. Bey Johann Friederich Harknoch, Riga. (Metaphysical Foundations of Natural Sciences. Engl trsansl Ernest Belfort Bax). Preface.

methods for solving optimization problems, which is the subject of mathematical programming. Next, relations between various economic indices have been implemented in econometric equation systems, which can be regarded as optimization constraints.

The third model element, the objective function, has been studied much less. A certain role played the Cold War's Western promotion of the self-organizing market economy, as opposed to the socialist centrally planned economy with its explicitly formulated objectives. This ideological bias has been stimulated by Nobel Prizes in economics, which have been almost exclusively awarded to the market equilibrium theorists and econometricians.

Contrary to the post World War II mainstream, two founding fathers of econometrics and the winners of the first Nobel Prize in economics (1969), Ragnar Frisch (1895–1973) and Jan Tinbergen (1903–1994), both theorists and practitioners, have been deeply involved in the problem of constructing objective functions [Bjerkholt and Strøm 2002, Merkies 2002]. Moreover, Frisch's Nobel Prize lecture was dedicated just to this particular topic [Frisch 1970].

Unlike econometric equations, which are derived from economic statistics, objective functions can hardly be obtained empirically. Instead, they primarily depend on 'external' normative goal settings. For example, all economic policies are aimed at reducing unemployment but some are unconditional, and some are targeted at a certain figure not to be lowered down below this limit. The corresponding objective functions are clearly different. Policy priorities, for instance, of unemployment and inflation reduction, can differ as well, affecting their weighting in the objective function.

'Subjective' objective functions (sorry for the pun) require revealing the decision maker's preference. The techniques of preference elicitation by specially designed interviews are reviewed in [Tangian and Gruber 2002], where also practical examples are considered. These techniques, which are equally relevant to constructing composite indicators, are however beyond the scope of this paper devoted to *using* the latter in econometric decision models.

3 Econometric decision model to minimize occupational hazards

In this section we trace the construction of the related econometric decision model step-by-step.

Space of alternatives Let the space of alternatives S be the set of two-dimensional vectors that represent the dust pollution and chemical pollution levels below the initial 20 g/m^3 of dust and 10 g/m^3 of chemicals from fumes and vapors:

$$S = \{(d, f) : 0 \leq d \leq 20,\ 0 \leq f \leq 10\} ,$$

where

d — dusts, in g/m^3 of particles, and

f — fumes and vapors, in g/m^3 of chemicals.

Constraints Let the costs for pollution reduction be estimated as follows:

$C_d(d) \quad = (20 - d)^2$ K USD is needed to reduce the dusts to d-value,

$C_f(f) \quad = 2^{10-f}$ K USD is needed to reduce the fumes and vapors to f-value, and

$C(d, f) = C_d(d) + C_f(f) \leq 150$ K USD, that is, the total costs are restricted to 150K USD.

Figures 1–3 show these one-variable and two-variable cost functions, and Figure 4 shows the level curves of the surface from Figure 3. The feasible domain for the budget of 150K USD is located above the level curve $C = 150$.

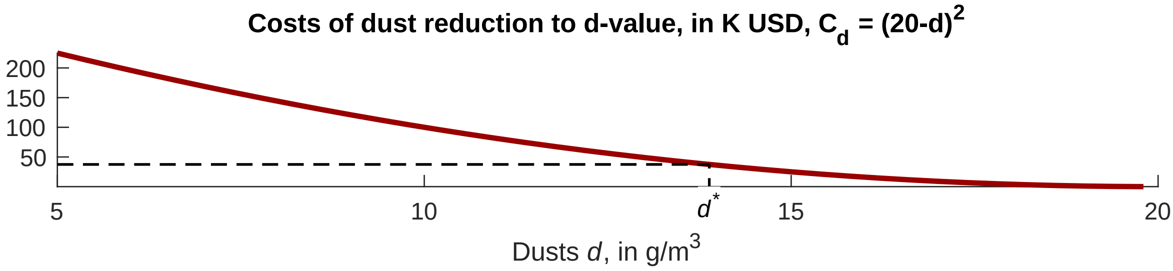

Figure 1: Costs of dust reduction to d-value, in K USD, $C_d = (20 - d)^2$

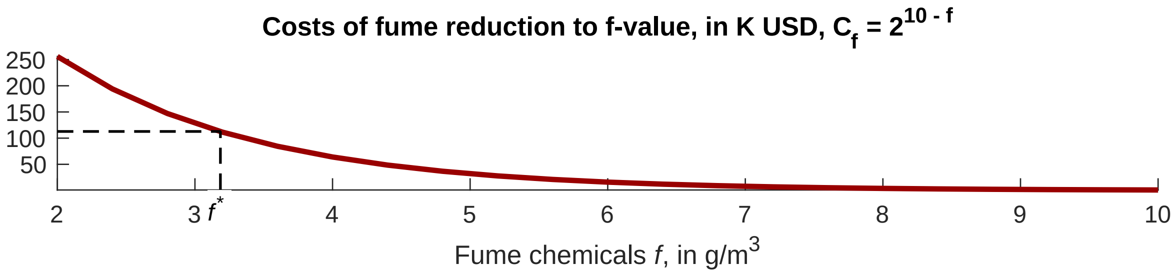

Figure 2: Costs of fume reduction to f-value, in K USD, $C_f = 2^{10-f}$

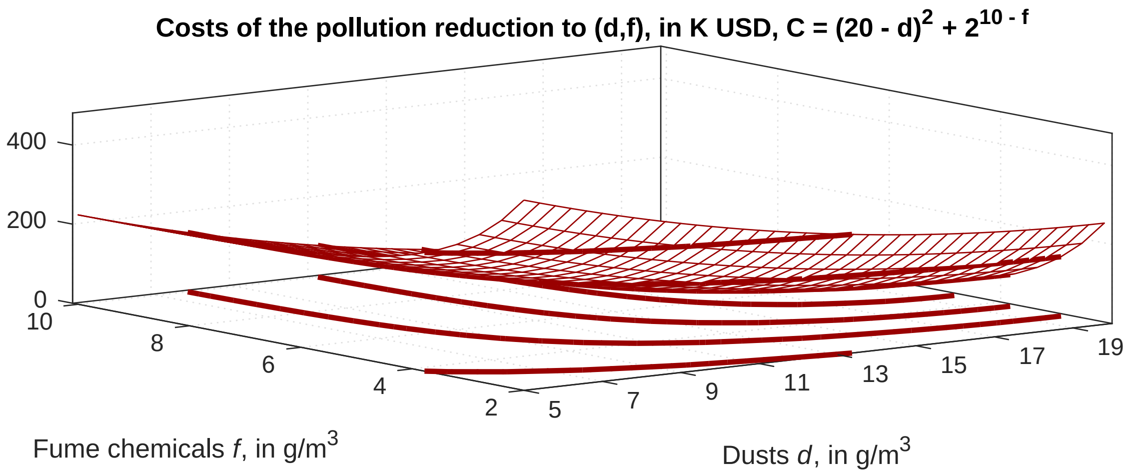

Figure 3: Costs of the pollution reduction to (d, f), in K USD, $C = (20 - d)^2 + 2^{10-f}$

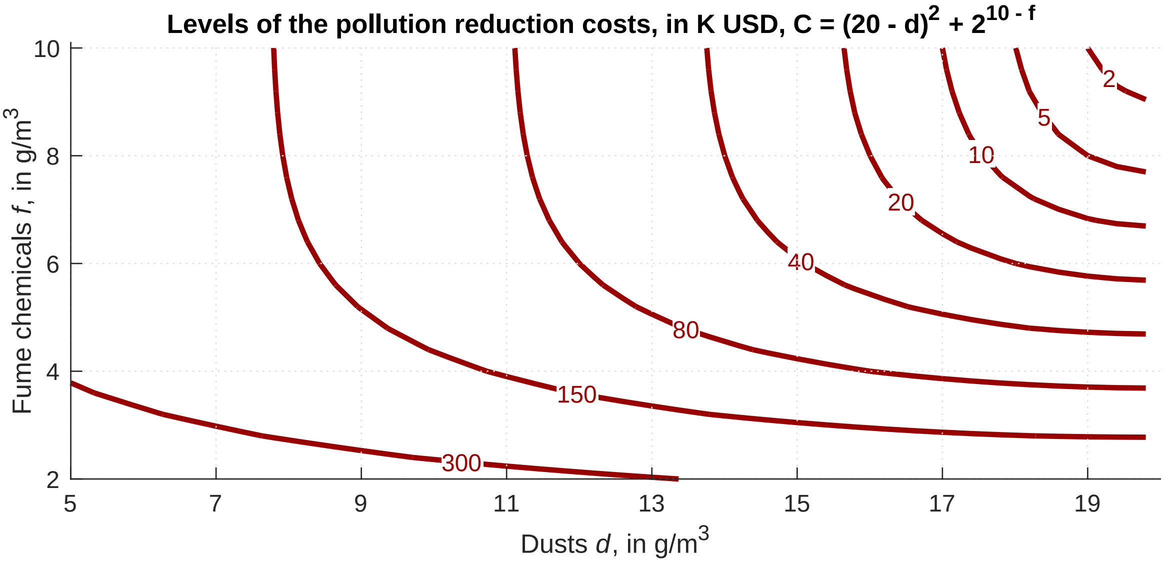

Figure 4: Levels of the pollution reduction costs, in K USD, $C = (20 - d)^2 + 2^{10-f}$

Composite indicator of the pollution harm as scalar-valued objective function Let the dust and fume harm to health be estimated in conditional units, and the total pollution harm be defined as their sum as follows:

$$
\begin{aligned}
H_d(d) &= d && \text{(dust harm)},\\
H_f(f) &= f^2 && \text{(fume harm)},\\
H(d,v) &= H_d(d) + H_f(f) && \text{(total pollution harm)}.
\end{aligned}
$$

Figures 5–7 show these one-variable and two-variable harm functions. Figure 8 shows the blue level curves of the surface from Figure 7 and the red levels of the cost function from Figure 4.

Econometric decision model Under the given budget, the least attainable pollution harm is found from the following optimization problem

$$
\begin{array}{rcll}
H(d,v) = d + f^2 & \to & \min & \text{(minimizing harm to health)}\\
\text{subject to constraints} & & & \\
C(d,f) = (20-d)^2 + 2^{10-f} & \leq & 150 & \text{(budget constraint)}\\
0 \leq d & \leq & 20 & \text{(belonging to space } S)\\
0 \leq f & \leq & 10 & \text{(belonging to space } S)
\end{array}
$$

Least pollution harm under the given budget The solution to the above problem

$$(d^*, f^*) = (13.884, 3.185)$$

is shown in Figure 8 by dashed lines. It is the tangent point of level curves

$$
\begin{array}{llll}
L_C = \{(d,f): \ C(d,f) & = & C(d^*,f^*) = 150\} & \text{(red cost level curve)}\\
L_H = \{(d,f): \ H(d,f) & = & H(d^*,f^*) = \ 24.0283\} & \text{(blue harm level curve)}
\end{array}
$$

For an arbitrary budget c, the optimal solution (d, f) is found in the same way. It is the tangent point of the budget level curve $C(d, f) = c$ and a certain pollution harm level curve. These solutions for the variable budget are shown in Figure 8 by the orange curve.

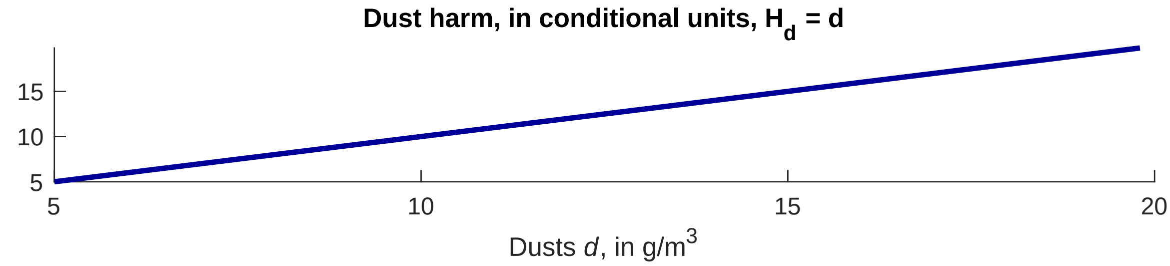

Figure 5: Dust harm, in conditional units, $H_d = d$

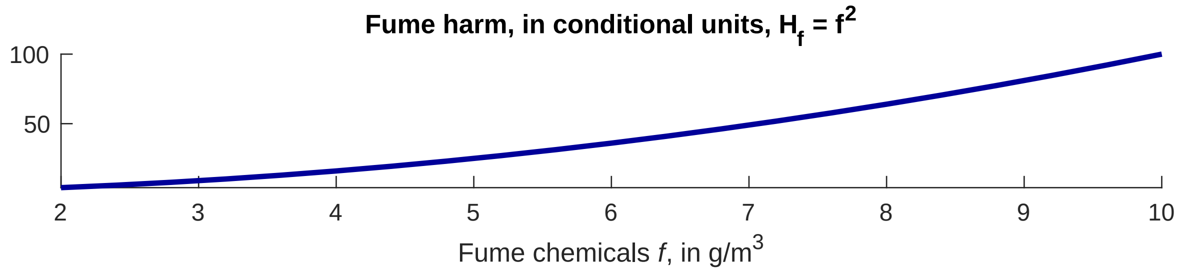

Figure 6: Fume harm, in conditional units, $H_f = f^2$

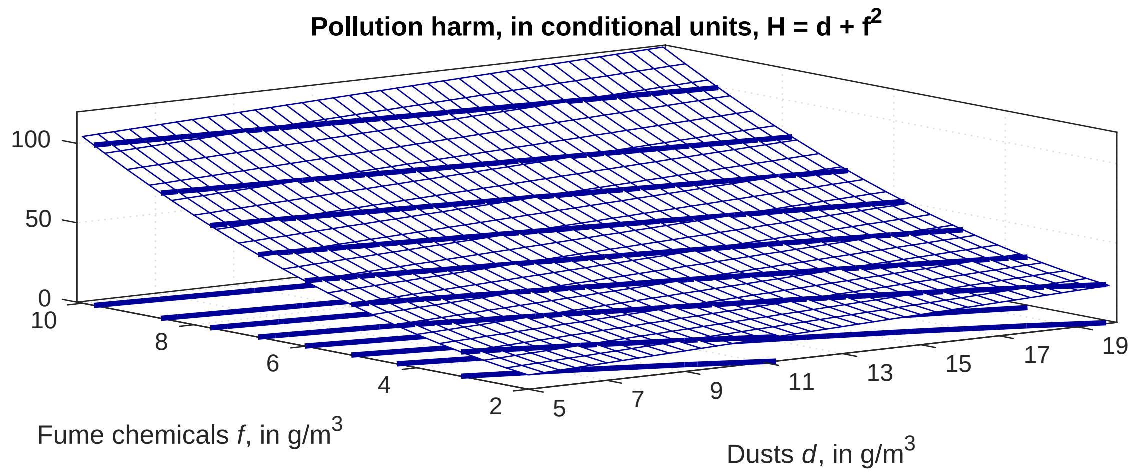

Figure 7: Pollution harm, in conditional units, $H = d + f^2$

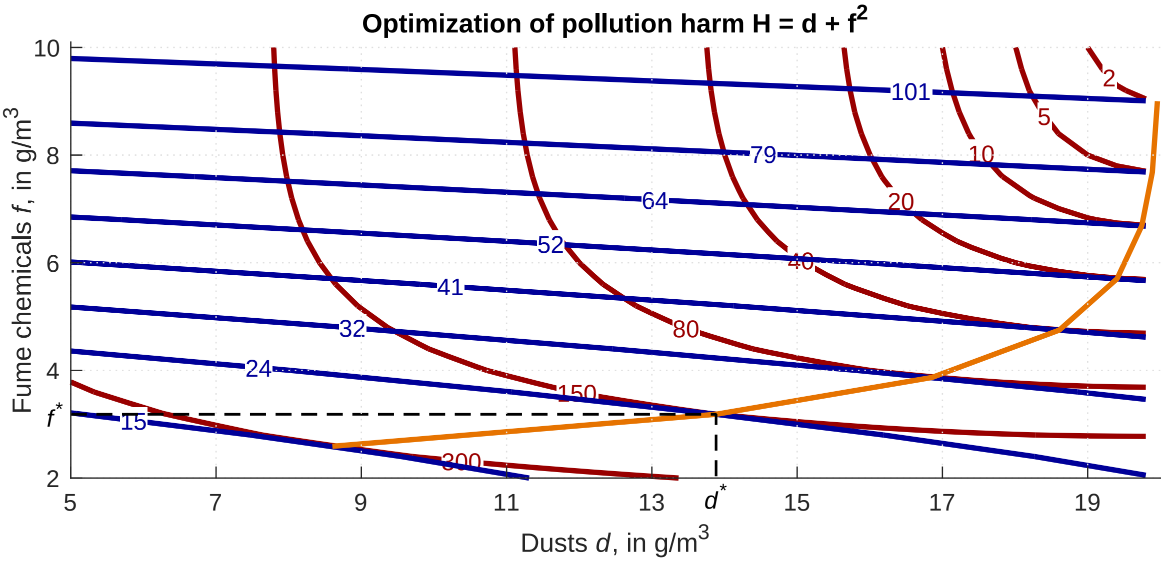

Figure 8: Optimization of pollution harm $H = d + f^2$

Optimal budget distribution To find the optimal budget distribution between two types of anti-pollution measures, we substitute d^* and f^* into functions C_d and C_f, respectively:

$$\begin{array}{rcl} \text{Costs of dust collectors } C_d(d^*) = & (20 - 13.884)^2 & = \ 37,405 \text{ USD} \\ \text{Costs of fume neutralizers } C_f(f^*) = & 2^{10-3.185} & = 112,595 \text{ USD} . \end{array}$$

Finding these costs is illustrated in Figures 1 and 2 by dashed lines.

4 Harm to health as probabilistic risk

Health risks Now the dust and fume harm to health are regarded as independent probabilistic risks r_d and r_f, respectively. Then the joint risk R is as follows:

$$\underbrace{R}_{\substack{\text{Probability of}\\ \text{harm to health}\\ \text{from dust or fumes}}} = 1 - \underbrace{\underbrace{\left(1 - \underbrace{r_d}_{\substack{\text{Probability of}\\ \text{harm to health}\\ \text{from dust}}}\right)}_{\substack{\text{Probability of no harm}\\ \text{to health from dust}}} \underbrace{\left(1 - \underbrace{r_f}_{\substack{\text{Probability of}\\ \text{harm to health}\\ \text{from fumes}}}\right)}_{\substack{\text{Probability of no harm}\\ \text{to health from fumes}}}}_{\text{Probability of no harm to health from dust or fumes}} .$$

For consistency with indices H_d and H_f, we 'transform' them into probabilities (which take values from the segment $[0; 1]$) by dividing H_d and H_f by their maxima. Hence, the joint health risk is as follows (the subscript r at H indicates health *risks*):

$$\begin{aligned} H_r(d, v) &= 1 - \left(1 - \frac{H_d(d)}{\max H_d}\right)\left(1 - \frac{H_f(f)}{\max H_f}\right) \\ &= \frac{H_d(d)}{\max H_d} + \frac{H_f(f)}{\max H_f} - \frac{H_d(d)\, H_f(f)}{\max H_d \max H_f} \\ &= \frac{d}{20} + \frac{f^2}{100} - \frac{d\, f^2}{2000} . \end{aligned}$$

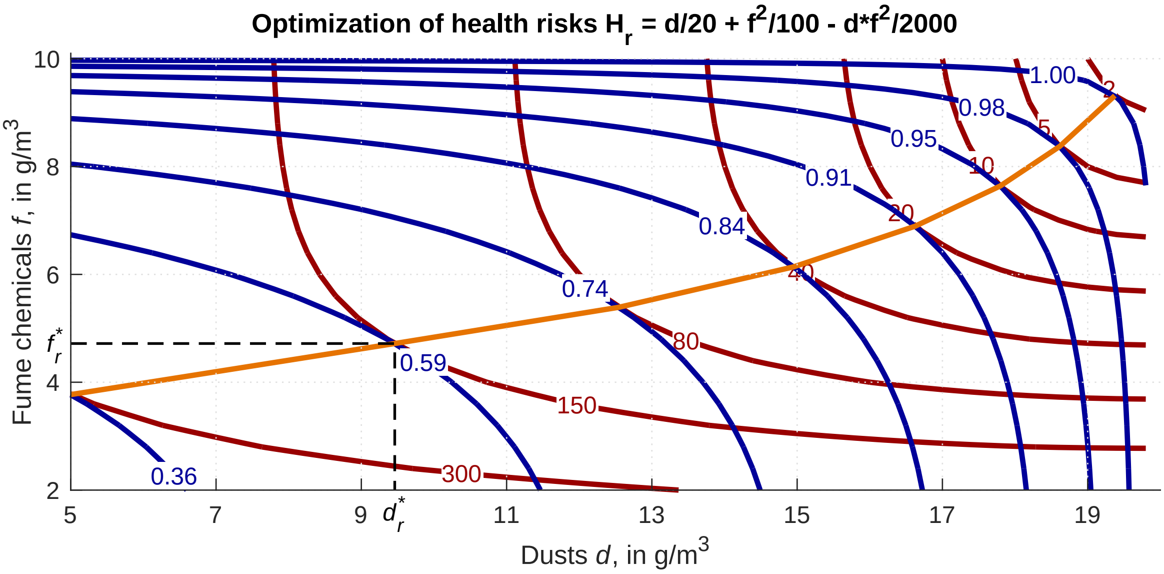

Figure 9: Optimization of health risks $H_r = d/20 + f^2/100 - d * f^2/2000$

Least health risk For the modified objective function $H_r(d, f)$, the econometric decision model and its solution look as follows:

$$H_r(d, v) = \frac{d}{20} + \frac{f^2}{100} - \frac{d\,f^2}{2000} \quad \to \quad \min \qquad \text{(minimizing health risks)}$$

subject to constraints

$$\begin{aligned} C(d, f) = (20 - d)^2 + 2^{10-f} &\leq 150 && \text{(budget constraint)} \\ 0 \leq d &\leq 20 && \text{(belonging to space } S) \\ 0 \leq f &\leq 10 && \text{(belonging to space } S) \end{aligned}$$

$$(d_r^*, f_r^*) \quad = \quad (9.460, 4.718) \quad \text{instead of} \quad (13.884, 3.185)\ .$$

This solution as well as the curve of optimal solutions for a variable budget are shown in Figure 9, which is analogous to Figure 8. The red level curves of the cost function $C(d, f)$ remain the same, but the blue level curves of the health risk function $H_r(d, f)$ are very different. The level values of function $H_r(d, f)$, being probabilities, are now from the segment $[0; 1]$.

Optimal budget distribution to minimize health risks As previously, the optimal budget distribution between two types of anti-pollution measures is found by substituting d_r^* and f_r^* into functions C_d and C_f, respectively:

$$\begin{aligned} \text{Costs of dust collectors } C_d(d_r^*) = &\quad (20 - 9.460)^2 &&= 111,092 \text{ USD (instead of 37,405)} \\ \text{Costs of fume neutralizers } C_f(f_r^*) = &\quad 2^{10-4.718} &&= \ 38,908 \text{ USD (instead of 112,595)}\ . \end{aligned}$$

This optimal budget distribution drastically differs from the former one, illustrating the composite indicator/objective function's decisive role.

5 Composite indicators as preference functions

In our consideration, the optimal solutions are found as tangent points of two families of level curves. The (blue) health harm level curves, consisting of equivalent harm kinds combinations, are called *indifference curves* of the decision-maker's (medical expert's) preference, and only they are needed to find the optimum, not their level values.

A set of indifference curves determine an *ordinal preference*, whose *degree* is not defined, and only its direction is important. An ordinal preference can be *represented* by a numerical function, which takes greater values at higher levels. The emerging numerical degree of preference is irrelevant, because numerical representations are not unique and depend on scaling. For example, the same temperature preference can be represented in Celsius or Fahrenheit scales.

An ordinal preference suffices for optimization, because minimization or maximization depends on the direction of preference decrease/increase but not its steepness. If a preference is used for benchmarking, ranking or other ordinal comparisons, its scaling plays no role either.

The situation is different if a preference function is intended for evaluation, that is, when its numerical differences are interpreted as quality proportions. For example, a composite indicator of occupational health that summarizes the firm's losses from professional diseases has a clear numerical meaning. Then the statement 'this year is twice better than the previous one' means that the losses are twice lower. On the other hand, a composite indicator of a country's health (if not simplistically the life expectancy) can only suggest country ranks but not the health *degrees*. Indeed, Denmark's population can be regarded more healthy than, say, that of Congo, but would you trust the statement that its health is '2.3 times better'?

The reservations concerning numerical preference functions/composite indicators do not call into question their usefulness. In many cases, one can skip scaling problems and construct ordinal preferences from indifference levels, using their numerical representations exclusively for computing and rankings rather than evaluations. The composite indicators constructed this way are accurate, surmount methodological bottle-necks and are acceptable for most applications.

6 Conclusion

Composite indicators as objective functions We have illustrated how to use composite indicators for optimization, in particular as objective functions in econometric decision models. As we have seen, the low-level indices must be selected and weighted carefully, because optimal decisions depend strongly on the composite indicator's accuracy.

Alternative use of the model In this paper, we optimally distribute a fixed budget to minimize occupational hazards. Conversely, one can start from a target value of pollution harm and find the minimal budget to attain it. For example, the pollution harm reduced to 24 conditional units for 150 K USD can still be considered too high and the target value of 15 'harm units' be adopted. As seen in Figure 8, this task requires at least 300 K USD. If such an expenditure is unaffordable, a compromise can be found along the orange curve.

Occupational hazards as probabilities If harms to health are regarded as probabilistic risks, the interactions of several risk factors should be taken into account. Then the composite indicator of occupational hazards contains cross products of low-level indices (Section 4). In this case, a weighted sum of low-level indices is insufficient, and at least a quadratic function is required. Relevant theoretical issues are studied in the lottery-based expected utility theory [von Neumann and Morgenstern 1944, Keeney and Raiffa 2014].

Constructing composite indicators from interviews Practically constructing linear and quadratic composite indicators/objective functions from interviews, as well as models to design such interviews, are collected in the already cited [Tangian and Gruber 2002]. We may consider them on some other occasion.

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