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A multi-scale fatigue-damage model for fiber-reinforced polymers

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Experimental studies of Chebbi et al. [1] on fatigue loading of fiber-reinforced polymers have shown that there is a phase of stable stiffness decrease prior to growing fatigue cracks. Modeling this stiffness degradation is an essential step in understanding fatigue effects of these materials.

The constitutive behavior of short-fiber reinforced polymers depends on numerous factors, such as fiber-volume content, the aspect ratio of the fibers, the fiber-orientation tensor and the loading direction. Accounting for these influence factors on a purely experimental basis is very time and resource demanding.

As a remedy, we follow a multi-scale approach for simulating the fatigue-damage evolution in short-fiber reinforced polymers. Using a simple damage model for the polymer matrix, the model inherently accounts for the influence of the fiber micro-structure through homogenization. We show that the stiffness degradation predicted by this model is of anisotropic nature and depends strongly on loading direction and fiber-orientation tensor.

Due to its specific structure, the model permits a straightforward model-order-reduction strategy and can be efficiently employed for component-scale simulations, see Köbler et al. [3].

1 Fatigue-damage model for the polymer matrix

We present a material model for the matrix of a fiber-reinforced polymer material. The fibers are considered to be purely linear elastic. The matrix-material model is formulated in the framework of generalized standard materials proposed by Halphen-Nguyen [2]. The free energy potential ψ and the dissipation potential ϕ are defined as

$$\psi(\varepsilon, D) = \frac{1}{2}[\eta + (1-D)^2]\varepsilon : \mathbf{C} : \varepsilon + w\,\ell^2\,\|\nabla D\|^2 \quad \text{and} \quad \phi(D') = \frac{1}{2\alpha}D'^2,$$

where ε denotes the strain tensor and the relative residual stiffness $\eta > 0$ is introduced for numerical reasons, D is the scalar damage variable ranging from 0 (undamaged) to 1 (fully damaged), $\mathbf{C}$ is the initial stiffness tensor of the material, w is a material parameter, ℓ is a length-scale parameter and α governs the speed of the damage evolution.

The model is a priori formulated in cycle space. We denote by $(.)' \equiv \frac{\mathrm{d}}{\mathrm{d}N}$ differentiation w.r.t. the cycle variable N .

The model is applied to different fiber orientations realized as representative volume elements that were generated by the SAM algorithm, see Schneider [5]. A typical representative volume element for a uni-directional (left) and a planar-isotropic fiber orientation, respectively, is shown in figure 1.

The model is discretized in time and space, and solved by a fast Fourier-transform based approach, see Köbler et al. [4], [3].

a) Uni-directional fiber structure **b)** Planar-isotropic fiber structure

Fig. 1: Two unit cells with different fiber orientations

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2 Stiffness degradation for different stress amplitudes and fiber orientations

After carrying out several verification steps, a representative volume with an edge length of twice a fiber length was found to be sufficient for studying the primary stiffness degradation (loss in the effective moduli of several 10 %), see Köbler et al. [4]. A fiber is resolved by 128 voxels per length or, at a chosen aspect ratio of 20, by 6.4 voxels across the fiber diameter.

Subsequently, a study on the fiber-orientation influence on the stiffness degradation was conducted. The results for the above shown uni-directional and planar-isotropic structures are shown in figure 2 and 3.

Figure 2 shows the dependence of the effective Young's modulus decrease in x-, y- and z-direction subjected to loading in x-direction. The considered second-order fiber orientations interpolate between a uni-directional ($\mu = 0$) and a planar-isotropic ($\mu = 1$) second-order fiber-orientation tensor via

$$A(\mu) = \mu e_1 \otimes e_1 + (1-\mu)\frac{1}{2}(e_2 \otimes e_2 + e_3 \otimes e_3), \quad \mu \in [0,1]. \tag{1}$$

The evolution curves for different values of μ lie in between the extreme structures, i.e., for $\mu = 0$ and $\mu = 1$, and depend continuously on the parameter μ.

Figure 3 depicts the evolution of the Young's modulus bodies of the planar-isotropic and the uni-directional structure for three different cutting planes. The decrease in loading direction, i.e. the x-direction, is the most significant for both scenarios. The degradation processes is thus found to be of anisotropic nature.

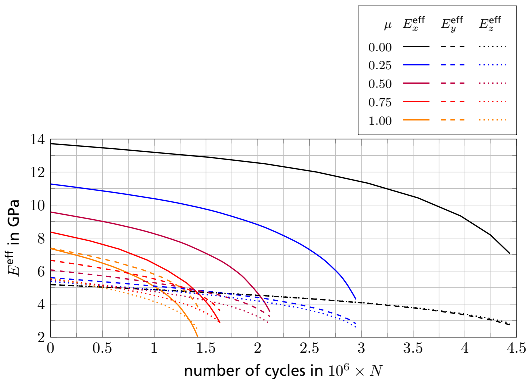
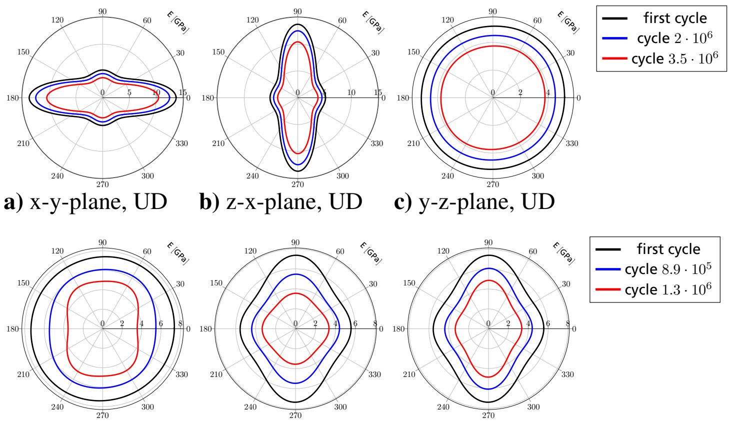


Fig. 2: Decrease of the absolute Young's modulus under loading in x-direction for fiber orientations parametrized between uni-directional and planar-isotropic



a) x-y-plane, UD **b)** z-x-plane, UD **c)** y-z-plane, UD

d) x-y-plane, PISO **e)** z-x-plane, PISO **f)** y-z-plane, PISO

Fig. 3: Evolution of the Young's modulus body of the uni-directional (UD, top) and the planar-isotropic (PISO, bottom) structure for different cutting planes

How to incorporate the presented material model into a multi-scale simulation framework is discussed in Köbler et al. [3]. This data-driven method enables efficient computations on the component scale, predicting the stiffness degradation for engineering applications on component scale. This is part of ongoing research.

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