Karlsruhe Institute of Technology

# Optimal Investment in Energy Efficiency as a Problem of Growth-Rate Maximisation

Anthony Britto, Joris Dehler-Holland, and Wolf Fichtner

No. 56 | March 2021

WORKING PAPER SERIES IN PRODUCTION AND ENERGY

# Optimal Investment in Energy Efficiency as a Problem of Growth-Rate Maximisation

Anthony Britto, Joris Dehler-Holland, and Wolf Fichtner

Chair of Energy Economics, Institute for Industrial Production (IIP)
Karlsruhe Institute of Technology (KIT)
Hertzstraße 16 - Building 06.33
76187 Karlsruhe, Germany

**Abstract.** Despite the vast literature on the energy-efficiency gap, there is a general dearth of investment models which incorporate the consumer's temporal freedom in the investment decision. Focusing on the building sector, we formulate optimal investment in energy efficiency as a problem of wealth growth-rate maximisation under uncertainty, subject to the diminishing marginal utility of retrofitting. The resulting model provides an unambiguous answer to the question of how much, and at what point in time, consumers with given wealth dynamics and parameters should invest in energy-efficiency measures for their particular dwelling. We treat in detail two foundational wealth dynamics: consumers who solely earn a fixed income, and those whose wealth grows multiplicatively. The differences in decision-making between these cases is seen to be substantial, with the latter group exhibiting further significant heterogeneity. All of this has profound implications for the social planner: on the one hand, we show how he must work harder to influence wealthier consumers; on the other, the model provides a methodology for crafting highly targeted policy interventions.

# Optimal Investment in Energy Efficiency as a Problem of Growth-Rate Maximisation

Anthony Britto[*], Joris Dehler-Holland, and Wolf Fichtner

Chair of Energy Economics, Karlsruhe Institute of Technology
Hertzstr. 16 – Building 06.33
76187 Karlsruhe, Germany

March 9, 2021

**Abstract**

Despite the vast literature on the energy-efficiency gap, there is a general dearth of investment models which incorporate the consumer's temporal freedom in the investment decision. Focusing on the building sector, we formulate optimal investment in energy efficiency as a problem of wealth growth-rate maximisation under uncertainty, subject to the diminishing marginal utility of retrofitting. The resulting model provides an unambiguous answer to the question of how much, and at what point in time, consumers with given wealth dynamics and parameters should invest in energy-efficiency measures for their particular dwelling. We treat in detail two foundational wealth dynamics: consumers who solely earn a fixed income, and those whose wealth grows multiplicatively. The differences in decision-making between these cases is seen to be substantial, with the latter group exhibiting further significant heterogeneity. All of this has profound implications for the social planner: on the one hand, we show how he must work harder to influence wealthier consumers; on the other, the model provides a methodology for crafting highly targeted policy interventions.

*Keywords:* the energy-efficiency gap; energetic building retrofits; diminishing marginal utility; investment under uncertainty; growth-rate maximisation; social planning; retrofit incentives.

*JEL classification:* D01; D04; D11; D15; G11; G51; O33.

[*]Correspondence address: anthony.britto@kit.edu.

# 1. Introduction and Background

The energy-efficiency gap is commonly understood as the suggestion that the way individuals make decisions about investing in energy efficiency leads to a slower-than-optimal diffusion of conservation technology. In this article we contribute to the theoretical underpinnings of this hypothesis by proposing a decision-making procedure for optimal investment in energy efficiency. We focus on a prototypical and salient example of the same, namely the energetic retrofit of a dwelling, and provide a definitive answer to a surprisingly non-trivial question: "When and how much should a risk-neutral consumer invest in energy efficiency?"

Although it is undeniable that only a minuscule share of the total building stock undergoes meaningful energy retrofits each year,[1] this fact on its own cannot constitute evidence for an energy-efficiency gap; an additional definition of optimal consumer behaviour is required. The broad consensus in the literature is that a discounted cash flow analysis is the appropriate method of investment appraisal. The debate surrounding the gap therefore tends to focus on discount rates, transaction costs, performance deviation of the technology, behavioural phenomena such as the rebound effect, and other such factors (Allcott & Greenstone, 2012; Gerarden et al., 2017). That is, each of the variables, or missing variables, in a net-present-value–type model is investigated, and discussion around the existence or size of the gap framed in this context.

A critical drawback to using the above class of models for baselining the energy-efficiency gap is the following: a discounted cash flow appraisal simply cannot answer the basic question of whether it is advantageous for the consumer to wait a year or two or five before investing. This matters greatly for the energy-efficiency gap, since conservation technology, like any other technology, is subject to a diffusion process, and the consumer's ability to put off an investment to a later date is a fundamental degree of freedom in the description of this dynamic unfolding. Another relevant, and as it turns out related, weakness of the discounted cash flow model is that the burden of quantifying the investment risk typically falls on a single variable, the discount rate, which consequently often becomes the most debated and prescient aspect of the gap (Gillingham & Palmer, 2014). This substantially diminishes the usefulness of the model as a baseline for optimal technological diffusion, since discount rates are known to

[1] In the EU, for instance, only around 0.2% of the building stock undergoes deep-energy retrofits per annum (Esser et al., 2019).

exhibit considerable heterogeneity (Newell & Siikamäki, 2015).[2] It is our contention that these shortcomings make a discounted cash flow analysis unsuitable for use as a yardstick for optimal consumer investment in energy efficiency.

Given the overwhelming dominance in the literature of this type of model, alternative proposals are hard to come by. A prominent attempt is that of Hassett and Metcalf (1992), who simultaneously address both deficiencies outlined in the previous paragraph with an innovative retrofit investment model based on real option theory. The main source of risk in such an investment in their estimation, is the stochastic nature of the price of the energy carrier and the cost of capital. Using mathematical machinery developed by McDonald and Siegel (1986), who quantify the real option of waiting to invest, Hassett and Metcalf prescribe a fuel-price trigger to inform optimal investment. In a subsequent publication, they demonstrate that their model leads to a slower diffusion of conservation technology than a simple net-present-value–type decision framework, and thereby make a case against the existence of a pervasive energy-efficiency gap (Hassett & Metcalf, 1993). We discuss their work in some detail in an appendix to this article.

Despite their differences, the above expected-utility investment model does hold one central assumption in common with a discounted-cash-flow analysis, namely, that the consumer seeks to minimise the net present value of their heating expenses. But this is not the only possible framing of the investment problem. A notable alternative is found in the finance literature, where the long-run wealth of the investor is maximised via a *logarithmic* utility function.[3] We shall see in this article how this method, when combined with the assumption of diminishing marginal utility in retrofitting, provides an unambiguous solution to our motivating question of how much a risk-neutral consumer should optimally invest in energy efficiency, and when. An additional benefit of this method is a natural integration of the consumer's wealth parameters into the investment problem.

We were nevertheless led us to ask if there was not a larger framework within which we could situate this calculation. We discovered just such a schema in ergodicity economics (Peters, 2019), which has enjoyed much success in resolving long-standing microeconomic puzzles in an integrated, fundamental way (Adamou et al., 2020; Peters & Gell-Mann, 2016; Peters, 2011a, 2011b). The approach seeks to provide new foundations for utility theory, among which is

[2] Conversely, appealing to this heterogeneity to explain the energy-efficiency gap simply begs the question (Jaffe & Stavins, 1994a).

[3] This approach was in fact central to the development of utility theory, and enjoys a long and fascinating history (MacLean et al., 2011; Peters, 2011b).

the axiom that the consumer's specific utility function is related to the growth rate of their wealth, and can be derived from the specified wealth dynamic. Accordingly, a logarithmic utility function corresponds to wealth that is growing exponentially; in contrast, for a consumer with an additive wealth dynamic, i.e. one who only earns a fixed income, cost minimisation is in fact equivalent to growth-rate maximisation.

In this article therefore, we show how optimal investment in energy efficiency can be formulated as a problem of wealth-growth-rate maximisation. The resulting model is rich in economic and policy insight, demonstrating not only how consumers differ in their decision making as a result of their wealth dynamic, but further that the social planner's task of incentivising the take-up of energy efficiency will differ markedly according to the consumer's wealth and wealth dynamic. Answering the question of optimal retrofit depth, i.e. investment size, required us to mathematically formalise the concept of diminishing marginal utility of energy retrofitting (Galvin, 2010) and quantify its implications for investment strategy. To the best of our knowledge, this article is the first to do so.

We proceed as follows. Section 2 contains a brief treatment of diminishing marginal utility in retrofitting. In the section following, we discuss how uncertainty enters the investment decision, and introduce the relevant concepts from ergodicity economics. We bring these preliminaries together in section 4, where we frame the growth-rate-maximisation problem. Section 5 is a numerical proof of concept. We conclude in section 6.

*Related literature.*

It is naturally with the work of Hassett and Metcalf (1992) and related literature, i.e. prescriptive investment-decision models concerned with technological adoption, that we most closely identify our contribution (e.g. Sunding & Zilberman, 2001; Grenadier & Weiss, 1997). On the other hand, descriptive models of technology diffusion abound; Geroski (2000) provides an overview of the literature. The most relevant example of the same is by Jaffe and Stavins (1994b), authors of an extremely influential summary (Jaffe & Stavins, 1994a) of the economic theory underlying the energy-efficiency gap. More recent surveys of the discourse around the gap can be found in the widely-cited articles by Gerarden et al. (2017) and Gillingham and Palmer (2014).

Other retrofit-investment decision tools of varying scope and potential for scalability exist in the literature. A typical application of the real option framework (McDonald & Siegel, 1986) to a single building is the simulation model of Kumbaroğlu and Madlener (2012). Articles by Gabrielli and Ruggeri (2019) and Hong et al. (2014) are exemplary of the large class of decision support models with specific foci (here, large building portfolios and multi-family housing complexes respectively). Friege and Chappin (2014) and Ma et al. (2012) review this vast literature.

Finally, a detailed treatment of the history and theory of long-run wealth maximisation via the logarithmic utility function can be found in the book edited by MacLean et al. (2011). The article by Pirvu and Žitković (2009) is exemplary of the questions in this field that continue to stimulate research in finance.

## 2. The Diminishing Marginal Utility of Retrofit Investments

It is a straightforward consequence of building physics and economic reality that investments in energy efficiency diminish in marginal utility. Consider thermal insulation, the quintessential retrofit measure: building physics tells us that the energy saved increases linearly with the thickness of the insulation; ceteris paribus, this means that each $x\%$ increase in insulation thickness reduces energy loss through that medium by $x\%$; but as the $x\%$ increase in insulation thickness is absolute, whereas the $x\%$ decrease is relative to the energy consumption before the new insulation was added on, it is clear that we have diminishing marginal energy savings for increasing insulation thickness. Colloquially, "the first centimetre of insulation is the most important". See Galvin (2012) for a further discussion of the non-linearity in retrofit costs.

To capture this concept, we assume that the energy saved post-retrofit $s$ (kWh/m$^2$/yr) as a function of retrofit cost $k$ (€/m$^2$) has the form of a classic diminishing-marginal-utility function:

$$s(k) := s_1 k - s_2 k^2 , \tag{1}$$

for constants $s_1, s_2 > 0$. In the case of retrofitting, we usually have $s1 \gg s2 > 0$; see table 1.

We make a few mathematical observations. The function $s(k)$ is maximised for $k_m := s_1/(2s_2)$, taking on the value $s_m := s_1^2/(4s_2)$, and begins to decrease after this point. This decrease is not physically sensible, so we must restrict the domain of our function to $[0, k_m]$.[4] This is of course in keeping with the intuition of diminishing marginal utility; in fact, it is straightforward to demonstrate that on this domain the conditions $s_1, s_2 > 0$ are necessary and sufficient to

[4]When fitting real data, this effectively reduces the number of degrees of freedom from two to one.

guarantee both that $s(k)$ is positive, and that it satisfies the assumption of diminishing marginal utility on the investment, i.e. $\partial_k s > 0$ and $\partial_k^2 s < 0$.

The variable $s$ is in a one-to-one relationship with thermal standards for dwellings, usually advertised by their expected final-energy consumption, e.g. 45 kWh/m$^2$/yr for a Passivhaus for heating and hot-water usage (Georges et al., 2012). Indeed, $s$ is exactly equal to the difference between the initial final-energy consumption and the thermal standard to which the consumer chooses to renovate, with higher standards, and hence higher energy savings, incurring increasing marginal costs.

It is obvious that it is not possible to retrofit with arbitrary precision. That is, there are usually only a handful of retrofit options available to the consumer: she may have the choice of adding 8 cm of insulation to her roof, or increasing that to 12 cm and installing basement insulation too, and so on (see table 2). We assume here that the domain and range of $s(k)$ are subsets of $\mathbb{R}$ so that we can carry out standard algebraic manipulations. It will become clear that this assumption does not affect the conclusions of our analysis in any meaningful way.

## 3. Energy Efficiency Investments and Uncertainty

In order to introduce a stochastic element into the analysis, we take as a starting point the central assumption in Hassett and Metcalf (1992), with which we are in agreement, that the chief source of uncertainty in energy efficiency investments is the price of the energy carrier.[5]

Suppose then that the price $p_t$ of the fuel that the consumer uses for heating follows a geometric Brownian motion with trend $\mu$ and variance $\sigma > 0$,

$$\mathrm{d}p_t / p_t = \mu \, \mathrm{d}t + \sigma \, \mathrm{d}z_t \,, \tag{2}$$

where $z_t$ is a standard Wiener process with mean zero and unit variance. To simplify the discussion we will assume that $\mu > 0$ throughout. Now in order to work with this stochastic process, it is necessary to somehow remove the randomness to discover what happens "on average". But this turns out to be more challenging than one might suspect, and the consequent divergent definitions of averaging lie at the heart of ergodicity economics. What follows is the briefest of summaries of these ideas, restricted to geometric Brownian motion and the two

[5]Hassett and Metcalf assume further that the cost of capital also follows a geometric Brownian motion. We avoid this assumption, which does not affect our argument, in order to simplify the presentation. See also the discussion in section 6.

wealth dynamics that we will consider in this article.

Consider that there are at least two ways of computing what happens to $p_t$ as time passes. On the one hand, its expectation value at time $t$ is given by

$$\mathbb{E}\left[p_t\right] = p_0 \exp \mu t\ , \tag{3}$$

whence we would conclude that the price of the fuel grows exponentially at rate $\mu$. But it is possible to compute another growth rate, namely that of the logarithm of $p_t$,

$$\mathbb{E}\left[\log p_t\right] = \log p_0 + \left(\mu - \sigma^2/2\right) t\ . \tag{4}$$

From this equation, one would again conclude that the fuel price grows exponentially, but now with rate $\mu - \sigma^2/2$. The existence of these two growth rates for the same stochastic process is a fundamental result of Itô calculus, and is universally employed to estimate returns in finance (Hull, 2017). Its explanation is usually couched in terms of the non-commutativity of the log and expected-value operators, or as the difference between the arithmetic and geometric means of the multiplicative process $p_t$. For ergodicity economics however, this phenomenon is an example of *ergodicity breaking* (Peters & Klein, 2013), and forms the cornerstone of the argument for a new axiom of economic decision-making (Peters & Gell-Mann, 2016).

The rate $\mu$ computed in equation 3 is the familiar average growth rate over the ensemble of all possible realisations of the process $p_t$. On the other hand, the rate $\mu - \sigma^2/2$ computed in equation 5 answers the question, "what happens to any *single realisation* of this stochastic process as time passes?". One way to see this is as follows. If we rearrange equation 5 to isolate for the growth rate, we can in fact write

$$\mu - \sigma^2/2 = \mathbb{E}\left[\frac{1}{t} \log \frac{p_t}{p_0}\right]\ ; \tag{5}$$

that is, the growth rate can be thought of as being computed from the observable $(\log p_t/p_0)/t$. This particular observable turns out to be *ergodic*, i.e. its expectation value equals its time average (Peters & Gell-Mann, 2016).[6] This means that by definition, the expectation value of

[6]Formally, an observable $f : \omega \longrightarrow \mathbb{R}$ on the probability space $(\Omega, \omega, \mathbb{P})$ is called ergodic if it satisfies Birkhoff's equation:

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T f(\omega(t))\,\mathrm{d}t = \int_\Omega f(\omega)\mathbb{P}(\omega)\,\mathrm{d}\omega\ . \tag{6}$$

On the left is the time average of $f$, and on the right, its expectation value (Peters, 2019).

$(\log p_t/p_0)/t$, which is constant in time, is observed with probability one in any single, sufficiently long, random sample of the process $p_t$.

We come to the first premise of ergodicity economics for optimal decision-making under uncertainty: for an individual contemplating a stochastic investment opportunity, the average over an infinite ensemble is often irrelevant; what matters in the long run is the time-average growth rate of a single realisation of the random process. This is computed via an ergodic growth rate function.

The second premise of the decision axiom of ergodicity economics is that a consumer acts so as to maximise the growth rate of her wealth. A consequence of this axiom is a natural definition of the consumer's utility function. To build intuition, we consider the two foundational cases: the multiplicative and additive wealth dynamics. In the first is assumed that the decision-maker's wealth is continuously reinvested at return $\delta$ so that the increase in her wealth after a passage of time $\Delta t$ is given by

$$w(t + \Delta t) = w(t) \exp \delta \Delta t \; . \tag{7}$$

In the additive dynamic in contrast, the consumer simply earns a fixed income per unit time $r$, and does not reinvest her wealth, so that wealth grows as

$$w(t + \Delta t) = w(t) + r\Delta t \; . \tag{8}$$

In each case, there is a natural growth rate of wealth, and extracting it calls for different mathematical operations. In the multiplicative case we have

$$\delta = \frac{1}{\Delta t} \log \frac{w(t + \Delta t)}{w(t)} \; , \tag{9}$$

while for the additive we compute

$$r = \frac{w(t + \Delta t) - w(t)}{\Delta t} \; . \tag{10}$$

We see that the growth rate in the multiplicative dynamic is in fact computed from a logarithmic function of the wealth; ergodicity economics argues that this provides a first-principles explanation for the logarithmic utility function (Peters, 2011b, cf. MacLean et al., 2011). In contrast, the additive dynamic corresponds to a linear utility function. We restrict ourselves to these cases in this article, but more complicated wealth dynamics are of course possible, and each dynamic corresponds to a utility function; appendix B contains an illustrative example.

Given these ideas, we can state the decision axiom of ergodicity economics: *the consumer acts so as to maximise the time-averaged growth rate of their wealth under the specified wealth dynamic.*[7] The connection to our topic of interest, investments in energy efficiency, consists in recognising that energy expenses are a "drag" on the wealth growth rate, a drag which may be eased by investing in conservation technology.

A final point of interest, given the prominence of the discount rate in the energy-efficiency gap debate, is that the above decision axiom has been framed free of any discount function: it is a remarkable result of ergodicity economics that the proper discount function is in fact automatically specified once the wealth dynamic and decision time-frame are fixed (Adamou et al., 2020; see also appendix B).

## 4. Investment in Energy Efficiency as a Problem of Wealth Growth-Rate Maximisation

Given these preliminaries, it is straightforward to formulate optimal investment in energy efficiency as a problem of maximising the growth-rate of wealth.

We assume first of all that the consumer retrofits "anyway" in cycles of $L$ years (e.g. 25 years), which may be thought of as the lifetime of the building insulation or heating system (Galvin, 2014). The consumer wishes to know if there exists some optimal investment time $t^\star < L$ such that her wealth growth-rate would be maximised if she invested at $t^\star$ rather than letting the lifetime of her equipment run its course. Hence in this setup, "no investment" corresponds to $t^\star = L$.

In addition, given the assumption of diminishing marginal utility in retrofitting, the consumer is also looking for the optimal investment $k^\star \in [0, k_m]$, where the $s(k)$ parameters $s_1$ and $s_2$ are specific to her dwelling.

We further assume that all energy-carrier prices follow geometric Brownian motions. As per the discussion in the previous section, we define the action of the time-average operator

[7] Whether decision-making in the real world proceeds according to this axiom is the subject of active research, with promising initial results (Meder et al., 2020; Peters, 2019).

$\mathbb{T}[\cdot]$ on a geometric Brownian motion $(p_t, \mu, \sigma)$ as

$$\mathbb{T}\left[p_t\right] := p_0 \exp \nu t\,, \tag{11}$$

where $\nu := \mu - \sigma^2/2$ is then the *effective trend* of the fuel price (see equation 5).

Finally, we allow that the consumer might wish to upgrade, at cost $h$, her boiler with efficiency $\eta$ to one with efficiency $\zeta > \eta$, including a possible switch of the energy carrier from one with current price and effective trend $(p_0, \nu)$ to another with parameters $(q_0, \alpha)$.

*The multiplicative wealth dynamic.*

In the case of the multiplicative wealth dynamic, since all wealth is reinvested at return $\delta$, we compute the net future value of the energy expenses and the retrofit investment using this rate (cf. Adamou et al., 2020). If the consumer makes her decision at time $t$, and the lifetime of the new equipment is again $L$, the relevant future reference point becomes $L + t$. The consumer's expenses stream is then divided into three logical parts. First we have the net future value of the time-averaged energy costs per square meter for the period up to the retrofit:

$$\mathbb{T}\left[\int_0^t p_s \frac{u}{\eta} \exp \delta(L + t - s)\, \mathrm{d}s\right] = p_0 \frac{u}{\eta} \frac{e^{\delta L}\left(e^{\delta t} - e^{\nu t}\right)}{\delta - \nu}\,, \tag{12}$$

where we introduce the consumer's per-square-meter final-energy need $u$. Then, the per-square-meter cost of the retrofit and heater at the temporal reference point $L + t$ is simply given by

$$(k + h)e^{\delta L}\,. \tag{13}$$

Finally, post retrofit, the net future value of the time-averaged energy costs per square meter is

$$\mathbb{T}\left[\int_t^{L+t} q_s \frac{u - s(ke^{\delta L})}{\zeta} \exp \delta(L + t - s)\, \mathrm{d}s\right] = q_0 \frac{u - s(ke^{\delta L})}{\zeta} \frac{e^{\alpha t}\left(e^{\delta L} - e^{\alpha L}\right)}{\delta - \alpha}\,. \tag{14}$$

Assuming now that the consumer starts off with wealth $w_0$, the growth rate of wealth in the presence of energy costs over the period $L + t$ can be computed as (cf. equation 9)

$$\begin{aligned} g_m(k, t) = \frac{1}{L + t} \log \Bigg[ & \exp \delta(L + t) \;- \\ & \frac{a}{w_0}\left(p_0 \frac{u}{\eta} \frac{e^{\delta L}\left(e^{\delta t} - e^{\nu t}\right)}{\delta - \nu} + (k + h)e^{\delta L} + q_0 \frac{u - s(ke^{\delta L})}{\zeta} \frac{e^{\alpha t}\left(e^{\delta L} - e^{\alpha L}\right)}{\delta - \alpha}\right)\Bigg]\,, \end{aligned} \tag{15}$$

where $a$ is the heated area of the dwelling in square meters.

It is this growth rate that the consumer seeks to maximise.[8] We see that the consumer's wealth parameters $w_0$ and $\delta$ are naturally integrated into the optimisation problem.

*The additive wealth dynamic.*

In the case of the additive wealth dynamic, the linear utility function means that there is no discounting of future costs (Adamou et al., 2020). The sum of the time-averaged energy costs per squared-meter up to the investment time $t$ is therefore given by

$$\mathbb{T}\left[\int_0^t p_s \frac{u}{\eta} \mathrm{d}s\right] = p_0 \frac{u}{\eta} \frac{e^{\nu t} - 1}{\nu} , \tag{16}$$

and the sum of the time-averaged energy costs per squared-meter post-retrofit is computed as

$$\mathbb{T}\left[\int_t^{L+t} q_s \frac{u - s(k)}{\zeta} \mathrm{d}s\right] = q_0 \frac{u - s(k)}{\zeta} \frac{e^{\alpha(L+t)} - e^{\alpha t}}{\alpha} . \tag{17}$$

Hence from equation 10, we can write the growth rate of the consumer's wealth over the relevant time period as

$$g_a(k,t) = r - \frac{a}{L+t} \left( p_0 \frac{u}{\eta} \frac{e^{\nu t} - 1}{\nu} + k + h + q_0 \frac{u - s(k)}{\zeta} \frac{e^{\alpha(L+t)} - e^{\alpha t}}{\alpha} \right) . \tag{18}$$

Maximising this function is the same as minimising the subtrahend, i.e. minimising the annualised total expenses on energy for heating.[9] We see that in contrast to the case of the multiplicative wealth dynamic, reference to the consumer's wealth has disappeared, and the growth rate $r$ is also irrelevant for optimisation. We address this conceptual challenge in section 6.

*Features of the solution.*

We consider the multiplicative and additive cases together in discussing the solution to the optimisation problem . Firstly, the existence of an optimal tuple $(k^\star, t^\star)$ maximising the growth rates $g_m$ and $g_a$ is guaranteed since they are continuous functions on a bounded domain. How-

[8]Note that in order for the problem to be well-defined, we must have the argument of the logarithm be greater than unity, which simply translates to the reasonable requirement that the total costs (energy plus retrofit) up to time $L + t$ not exceed the total wealth accumulated by that point.

[9]This is the the derivation of what was claimed in the introduction, namely, that the expected-utility and net-present-value models assume a linear utility function.

ever, given their analytical complexity, closed-form solutions to the growth-rate–maximisation problem could not be located. This is ultimately no great loss for the practicability of our framework, since the optimisation problem is numerically tractable. Nonetheless, in this section we present the features of the solution that could be determined analytically, in order that the behaviour of the growth-rate functions may be understood to the greatest extent possible. In particular, we examine the conditions under which we expect to find non-trivial solutions, i.e. conditions which ensure that $t^\star < L$.[10]

We begin with the optimal investment size $k^\star$. Due to the assumption of diminishing marginal utility, there exists a unique optimal investment $k^\star < k_m$ for each moment in time $t$, both in the multiplicative and additive cases:

$$k^\star_m(t) := \frac{s_1}{2s_2}\left(1 - \frac{(\delta-\alpha)\zeta\, e^{\delta L - \alpha t}}{q_0 s_1 (e^{\delta L} - e^{\alpha L})}\right), \tag{19}$$

$$k^\star_a(t) := \frac{s_1}{2s_2}\left(1 - \frac{\alpha\zeta\, e^{-\alpha t}}{q_0 s_1 \left(e^{\alpha L} - 1\right)}\right). \tag{20}$$

The expressions lend themselves to intuitive interpretation: as the fuel price or its trend increases, or as the cost of renovation or the efficiency of the new heater decreases, $k^\star$ gets larger, approaching its upper bound $k_m = s_1/(2s_2)$ asymptotically. The expressions correctly contain no reference to the previous energy carrier. Further, we see that $k^\star > 0$, i.e an optimal investment in insulation exists, if and only if the expression in parentheses is positive. This can be rewritten, for instance, as a lower bound on the fuel price of the energy-carrier post retrofit: we must have

$$q_0 \overset{!}{>} q_{min,m}(t) := \frac{(\delta-\alpha)\zeta\, e^{\delta L - \alpha t}}{s_1 (e^{\delta L} - e^{\alpha L})} \tag{21}$$

in the multiplicative case, and

$$q_0 \overset{!}{>} q_{min,a}(t) := \frac{\alpha\zeta\, e^{-\alpha t}}{s_1 \left(e^{\alpha L} - 1\right)} \tag{22}$$

in the additive.

For ease of terminology, we refer to the above as the "$q_{min}$-criterion". If the set of constants in the functions $g_a$ and $g_m$ are positive, which is the usual case and which we will assume throughout in what follows, the strictest possible lower bound on $q_0$ corresponds to taking limit

[10] Recall that the consumer's total investment is always $a \cdot (k^\star + h)$; this means that if $k^\star = 0$, we can only conclude that the consumer should not invest in *insulation*. She may still invest in energy efficiency by upgrading her heater at cost $a \cdot h$ if $t^\star < L$.

$t \longrightarrow 0$, and the laxest to taking the limit $t \longrightarrow L$, in equations 21 and 22. That is, if $q_0 > q_{min}(0)$, the new fuel is expensive enough to make an investment in insulation worthwhile no matter the value of $t^\star$; on the other hand, if $q_0 < q_{min}(L)$, the new fuel is cheap enough that insulation is not economically viable, no matter the value of $t^\star$.

Given the above solutions $k^\star(t)$ to the insulation-depth problem, we have two cases: either the $q_{min}$-criterion is satisfied, or it is not. We accordingly have the following pair of results for the additive dynamic, the proofs of which are given in appendix A.

**Proposition 1.** *If the set of constants in the wealth growth-rate $g_a$ are such that the $q_{min,a}$-criterion is not satisfied, then $k^\star = 0$ and the consumer should not invest in insulation. The optimal investment time $t^\star$ to upgrade her heating system at cost $a \cdot h$ is given by the implicit equation*

$$p_0 = \frac{\eta\nu\left(\alpha\zeta h - q_0 u\left(e^{\alpha L} - 1\right)e^{\alpha t^\star}(\alpha(L + t^\star) - 1)\right)}{\alpha\zeta u\left(e^{\nu t^\star}(\nu(L + t^\star) - 1) + 1\right)}\,. \tag{23}$$

*In particular, if*

$$p_0 > \frac{\eta\left(\alpha\zeta h - q_0 u\left(e^{\alpha L} - 1\right)(\alpha L - 1)\right)}{\alpha\zeta L u}\,, \tag{24}$$

*the consumer should invest immediately; conversely, if*

$$p_0 < \frac{\eta\nu\left(\alpha\zeta h - q_0 u e^{\alpha L}\left(e^{\alpha L} - 1\right)(2\alpha L - 1)\right)}{\alpha\zeta u\left(e^{L\nu}(2L\nu - 1) + 1\right)}\,, \tag{25}$$

*she should not invest at all.*

**Proposition 2.** *If the set of constants in the wealth growth-rate $g_a$ are such that the $q_{min,a}$-criterion is satisfied, the optimal investment time $t^\star$ for a consumer to upgrade her heating system and invest in insulation at cost $a \cdot (k_a^\star(t^\star) + h)$ is given by the implicit equation*

$$p_0 = \frac{\eta\nu\Big(q_0\left(e^{\alpha L} - 1\right)e^{\alpha t^\star}\left(-2\alpha\zeta\left(2hs_2 + s_1\right) - q_0\left(e^{\alpha L} - 1\right)\left(s_1^2 - 4s_2 u\right)e^{\alpha t^\star}(\alpha(L + t^\star) - 1)\right) + \alpha^2\zeta^2(\alpha(L + t^\star) + 1)\Big)}{4\alpha\zeta q_0 s_2 u(e^{\alpha t^\star} - e^{\alpha(L+t^\star)})(e^{\nu t^\star}(\nu(L + t^\star) - 1) + 1)}\,, \tag{26}$$

*In particular, if*

$$p_0 > \frac{\eta\Big(\alpha^2\zeta^2(\alpha L + 1) - 2\alpha\zeta q_0\left(2hs_2 + s_1\right)\left(e^{\alpha L} - 1\right) - q_0^2\left(e^{\alpha L} - 1\right)^2(\alpha L - 1)\left(s_1^2 - 4s_2 u\right)\Big)}{4\alpha\zeta L q_0 s_2 u\left(1 - e^{\alpha L}\right)}\,, \tag{27}$$

*the consumer should invest the amount $a \cdot (k_a^\star(0) + h)$ immediately; conversely, if*

$$p_0 < \frac{\eta\nu\Big(\alpha^2\zeta^2(2\alpha L + 1) - 2\alpha\zeta q_0\,(2hs_2 + s_1)\,e^{\alpha L}\left(e^{\alpha L} - 1\right) - q_0^2 e^{2\alpha L}\left(e^{\alpha L} - 1\right)^2(2\alpha L - 1)\left(s_1^2 - 4s_2 u\right)\Big)}{4\alpha\zeta q_0 s_2 u\left(e^{\alpha L} - e^{2\alpha L}\right)\left(e^{L\nu}(2L\nu - 1) + 1\right)}\,, \tag{28}$$

*she should not invest at all.*

The unwieldy expressions in these propositions perhaps mask the intuitive nature of the result: if the current energy carrier is expensive enough (equations 25 and 28) it is beneficial to undertake an energy-efficiency investment according to either equation 23 or 26. That is, propositions 1 and 2 are the resolution to the motivating question of this article in the case of the additive dynamic.

Corresponding results exist in the multiplicative case, but these could only be numerically studied; the equivalent algebraic expressions are unfortunately analytically intractable (see appendix A).

*The role of the social planner.*

Consider now that the function $g_a$ contains 12 relevant degrees of freedom. It is helpful for our purposes to divide these 12 into two sets: a first set of five, $\{a, L, u, \eta, \zeta\}$, describing the immutable physical aspects of the consumer's dwelling and equipment, and a second set of seven, $\{p_0, \nu, q_0, \alpha, s_1, s_2, h\}$, describing parameters over which the consumer has no control, but which the social planner may control to some degree.[11] The above propositions imply that if two constraints, namely the $q_{min}$-constraint plus one of equations 25 or 28, are satisfied, a non-trivial solution to the optimisation problem exists, and consumer sees incentive to retrofit. In the case of multiplicative dynamics, a similar statement holds, except that there a total of three constraints are required to completely specify the solution to the optimisation problem (see appendix A). We hence have the following result.

**Corollary 1.** *For the social planner to be able to maximise the growth-rate function $g_m$ (resp. $g_a$) at a tuple $(k^\star, t^\star)$ of his choosing, it is necessary that the set of parameters $\{p_0,\nu,q_0,\alpha,s_1,s_2,h\}$ allows him at least three (resp. two) of degrees of freedom.*

This result qualifies the intuition that if the fuel and retrofit prices and trends can be changed arbitrarily, any desired goal can be achieved: in point of fact, the social planner needs to pull

[11] Although $s_1$ and $s_2$ depend on physical characteristics of the dwelling, we include them here since they could be changed by introducing subsidies on energetic retrofits.

exactly two levers in the additive case (increasing for example the energy-carrier price trends $\nu$ and $\alpha$ via a carbon tax), and three in the multiplicative (increasing $\nu$ and $\alpha$ via a tax, and reducing the cost of retrofitting $s_1$ via a subsidy, say). That the social planner has to pull a whole extra lever to influence the decision-making of consumers with multiplicatively-growing wealth is a non-trivial result, which simply falls out of the mathematics of growth-rate maximisation. It confirms the idea that a wealthier consumer is harder to influence than a one less well-off: the wealth dynamic of the wealthier consumer corresponds to a utility function which is simply more robust against changes in the environment.

The effectiveness of these policy instruments are is another matter entirely. Mathematically, this involves comparing derivatives: for instance, the relative sizes of $\partial g/\partial \nu$ and $\partial g/\partial s_1$ could be used to give a rough indication of the effectiveness of a carbon tax relative to a subsidy scheme in generating consumer incentive. We expound on these points in the discussion in section 6.

## 5. Case Study: German Single Family Dwelling

By way of example, consider the parameters in table 1 for a typical single-family home built in Germany between 1979–1983, which currently relies on an older oil boiler for heat. The retrofit measures used to estimate the parameters $s_1$ and $s_2$ are listed in table 2. We list the parameters for the various options for an upgrade of the heating system, along with arbitrarily selected effective price trends $\alpha$ for the corresponding energy carriers.

*The additive wealth dynamic.*

To understand the mechanics of the optimisation problem, let us begin in the simpler case of additive dynamics, and further imagine that the consumer only has a single choice for her new energy carrier, say gas. We leave the price parameters $\{p_0, \nu, q_0, \alpha\}$ unspecified for the moment.

Given the dwelling and heating-system parameters in tables 1 and 3, and additionally setting $\alpha = 0.02$, we first compute $q_{min,a}(t)$; this curve is depicted in figure 1. If a $(q_0, t)$ tuple lies in the shaded region depicted in the figure, $k_a^\star(t)$ will be positive.

Next, consider figure 2, which depicts the surface $r - g_a(k, t)$, i.e. the annualised energy expenses, for different values of $p_0$ and $q_0$, where we set $(\nu, \alpha) = (0.01, 0.02)$; also depicted is the point on the surface where energy expenses are minimised. For the consumer, this highlighted point is the precise answer to the question, “how much should I invest in energy efficiency and

TABLE 1. Physical retrofit parameters for a typical single-family home built in Germany between 1979–1983, obtained from the EPISCOPE database of representative buildings in the European Union (Loga et al., 2016). In the expression for $\eta$, the denominator 1.67 is the *energy-expenditure coefficient* and includes the efficiency of the heating system and any transmission losses. Note also the intuitive interpretation of the fit constant $s_1$: it is the cost of saving a kilowatt-hour of final energy consumption per year.

| Physical Parameters of Dwelling | | | |
|---|---|---|---|
| *Description* | *Variable* | *Value* | *Unit* |
| Retrofit cycle | $L$ | 25. | yrs |
| Heated area | $a$ | 142. | $m^2$ |
| Final-energy need | $u$ | 108.9 | $kWh/m^2$ |
| Heater efficiency | $\eta$ | 1/1.67 | – |
| Energy carrier | – | Oil | – |
| Max. retrofit investment | $k_m$ | 257.4 | €/$m^2$ |
| Max. final-energy saved | $s_m$ | 55. | $kWh/m^2/yr$ |
| $s(k)$ fit constant | $s_1$ | 0.38 | (kWh/yr)/€ |
| $s(k)$ fit constant | $s_2$ | $7.2 \times 10^{-4}$ | (kWh $m^2$/yr)/€$^2$ |

TABLE 2. Example combinations of retrofit measures, in increasing order of price, with corresponding reductions in final-energy need for the dwelling described in table 1. The data were obtained from a freely-available retrofit calculator developed by Bosch Thermotechnik GmbH and the Fraunhofer Institute for Building Physics IBP. (Available at https://www.effizienzhaus-online.de/sanierungsrechner/. Last accessed 10.12.2020.)

| Retrofit measures | | | | *Price* $k$ (€/$m^2$) | *Final energy saved* $s(k)$ ($kWh/m^2/yr$) |
|---|---|---|---|---|---|
| Insulation (cm) | | | | | |
| *Roof* | *Basement* | *External wall* | *Windows* | | |
| – | – | – | – | 0. | 0. |
| 8 | – | – | – | 14.4 | 13.2 |
| 8 | 4 | – | – | 33.3 | 19.1 |
| 8 | 4 | 8 | – | 150.9 | 31.1 |
| 8 | 4 | 8 | Double glazing | 206.9 | 43.1 |
| 28 | 4 | 8 | Double glazing | 231. | 49.1 |
| 28 | 12 | 16 | Double glazing | 257.4 | 55. |

TABLE 3. Typical options for a heating system upgrade. Analogous to table 1, the denominator in the "efficiency" column is the energy-expenditure coefficient. Note that in the case of electricity, the efficiency is the *coefficient of performance* of the heat pump, which is greater than unity.

| Heating System Upgrade Options | | |
|---|---|---|
| *Energy carrier* | Parameters | |
| | *Investment* $a \cdot h$ (€) | *Efficiency* $\zeta$ (−) |
| Oil | 9,000 | 1/1.16 |
| Gas | 15,000 | 1/1.08 |
| Elec. | 25,000 | 3 |

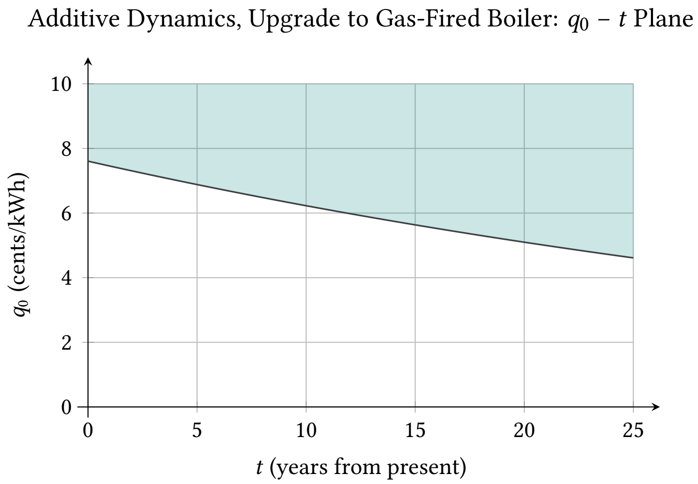


FIGURE 1. The curve is equation 22, with the parameters taken from tables 1 and 3, and additionally setting $\alpha = 0.02$. The shaded region depicts the $(q_0, t)$ tuples that lead to $k_a^\star > 0$.

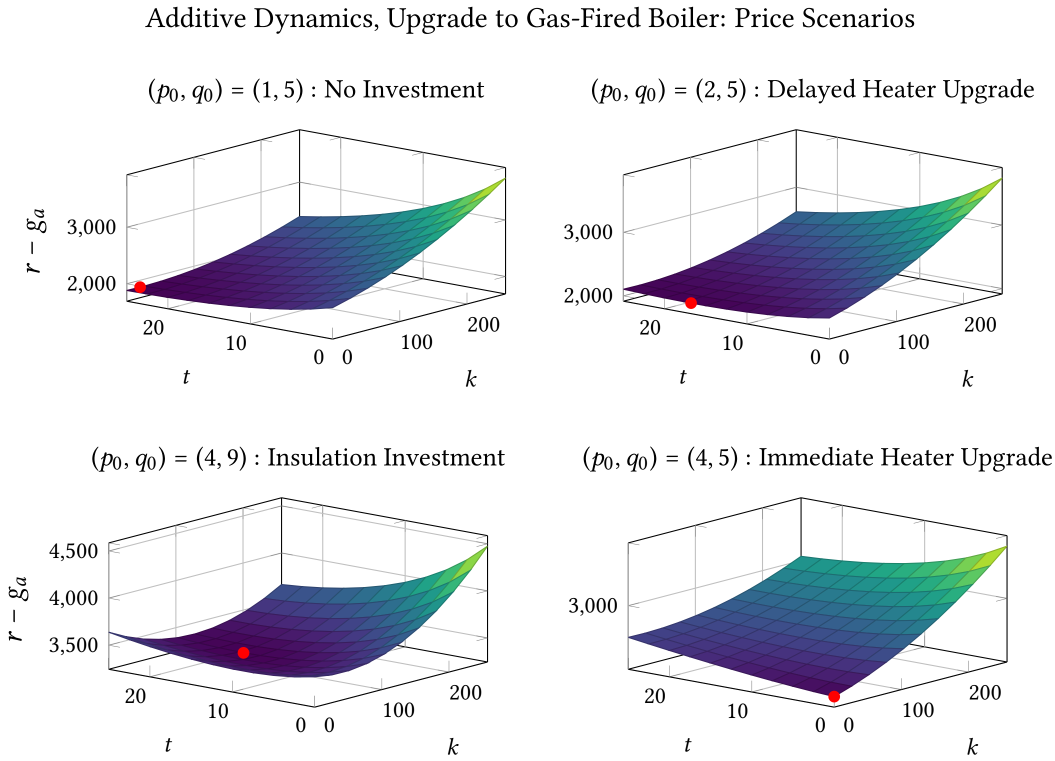


FIGURE 2. Depicted here are the annualised expenses for heating energy from equation 18, $r - g_a(k, t)$, in four different cases, with $(\nu, \alpha) = (0.01, 0.02)$. These expenses are minimised at the point indicated in red. The title of each figure gives the $(p_0, q_0)$ tuple in cents/kWh; the remaining parameters come from tables 1 and 3. The axis labels $k$, $t$ and $r - g_a$ have units €/m$^2$, "years from present time", and €/yr respectively. Clockwise from the top-left, the first three figures have $q_0 < q_{min,a}(L)$, and illustrate the three cases in proposition 1. Finally, the figure in the bottom-right is an illustration of the general case, equation 28 in proposition 2.

when?".

Simultaneously, figure 2 is an illustration of corollary 1 in action. The top-left scenario is one in which neither the $q_{min,a}$-criterion nor equation 25 are satisfied. Proceeding clockwise from this case we see how, since all other variables are held fixed, increasing $p_0$ alone is enough to ensure that equation 25 is satisfied (top-right scenario), and if increased still further (bottom-right scenario), that equation 24 is fulfilled. But to now get from this scenario to one in which the consumer has incentive to additionally invest in energy-efficiency measures requires that social planner introduce another degree of freedom (here, $q_0$) in order that the $q_{min,a}$-bound be satisfied. In sum, two degrees of freedom are required to produce a $(k^\star, t^\star)$ of the social planner's choosing.

In the general case, our consumer has several choices for a new energy carrier. She should proceed by computing $g_a^\star$ for each energy carrier $(q_0, \alpha)$, including the case $(q_0, \alpha) = (p_0, \nu)$ for the insulation-only option, and rank-order them; her optimal investment strategy is the one that results in the lowest annualised energy costs. An example calculation is presented in table

TABLE 4. Numerically-obtained solutions to the optimisation problem specified by tables 1 and 3 in the case of additive wealth dynamics, for the fuel parameters listed below. The status-quo parameters ($p_0$, $v$) are the same as those in the "insulation-only" row. The optimal investment strategy (lowest annualised energy costs) is highlighted.

| Additive Dynamics, Example Investment Decision | | | | | |
|---|---|---|---|---|---|
| Energy carrier scenario | | | Optimisation result | | |
| *Energy carrier* | Price parameters | | | | |
| | *Price* | *Price trend* | *Min. annul. energy expenses* | *Opt. invest.* | *Opt. invest. time* |
| | $q_0$ (cents/kWh) | $\alpha$ (–) | $r - g_a^\star$ (€/yr) | $k^\star$ (€/m$^2$) | $t^\star$ (yrs. from present) |
| Insul. only | 4 | 0.01 | 1785.1 | 0. | 0. |
| Oil | 4 | 0.01 | 1599.9 | 0. | 0. |
| Gas | 9 | 0.02 | 3356.4 | 102.6 | 17.0 |
| Elec. | 23 | 0.015 | 2951.1 | 47.3 | 22.6 |

4. We see that the best strategy for the given set of constants is an immediate investment in a new oil boiler, with no further investment in insulation.

### *The multiplicative wealth dynamic.*

We turn now to the multiplicative dynamic, where the growth rate function $g_m(k, t)$ contains express reference to the wealth parameters $w_0$ and $\delta$. It is an interesting and essential feature of this function that it is ill-defined if the argument of the logarithm in the expression (equation 15) is negative; this imposes an immediate restriction on $w_0$ and $\delta$: they must both be large enough, relative to each other, to ensure that the argument of the logarithm remains positive.

We hence restrict ourselves to the upper wealth quantiles in Germany for the calculation presented in table 5, where we show how the same physical parameters result in starkly different optimal investment decisions depending on $w_0$ and $\delta$.[12] If one looks along a given row (i.e. for fixed $w_0$), one sees that as $\delta$ increases, $k^\star$ decreases while $t^\star$ increases; i.e. the consumer should invest less and later, the faster her wealth grows. On the other hand, if one looks down a given column (i.e. for fixed $\delta$), as $w_0$ increases, both $k^\star$ and $t^\star$ increase, though the increase in $k^\star$ is far less significant (cf. equation 19). That is, the wealthier consumer should invest slightly more, but at a significantly later point in time.

[12] As corollary 1 specifies, three degrees of freedom were required to generate table 5.

TABLE 5. Numerically-obtained solutions to the optimisation problem specified by table 1 for an upgrade to a heat pump (i.e. oil to electricity) in the case of multiplicative wealth dynamics. Various wealth scenarios are considered. The fuel price parameters, listed in the title of the table in the units we have been using throughout, were chosen (cf. corollary 1) so that non-trivial solutions resulted.

| Multiplicative Dynamics, Optimal Investment in a Heat Pump $(p_0, \nu, q_0, \alpha, \zeta) = (8, 0.04, 40, 0.015, 3)$ | | | |
|---|---|---|---|
| | *Optimal investment* $g_m^\star$ $(k^\star, t^\star)$ | | |
| *Wealth*, $w_0$ (€) | Growth rate, $\delta$ | | |
| | *2.5%* | *3%* | *5%* |
| 215,400 | 0.56% (78.2, 7.4) | 1.25% (74.3, 10.1) | 3.87% (55.6, 22.4) |
| 428,400 | 1.68% (82.1, 12.0) | 2.26% (77.5, 14.8) | 4.51% (56.3, 25) |
| 861,600 | 2.13% (84.2, 14.5) | 2.66% (79.2, 17.4) | 4.77% (56.3, 25) |

## 6. Policy Implications and Outlook

In this article, we have presented a novel investment-decision model that answers the question "when and how much should a risk-neutral consumer invest in energy efficiency?" Our framework can be readily employed to model the diffusion of energy-efficiency technology in the absence of market barriers, thereby generating a baseline against which an energy-efficiency gap may be measured.

We address here a perceived shortcoming of the preceding calculation, namely, that the model appears to be less prescriptive in the additive case than in the multiplicative because the wealth parameters in the former case drop out. This turns out to not be a shortcoming of the model, but rather of the additive dynamic itself, which is an extremely simplified description of wealth growth. In appendix B, we derive the utility function and growth rate for a more realistic wealth dynamic: a consumer one who earns a fixed income while regularly saving a portion of wealth. Instead of the linear utility function of the additive dynamic we obtain a mixed log-linear utility function, and in contrast to a strict "no discounting", we derive a power-law discount function (see figure 3). The optimisation problem as described in section 4 considered for this wealth dynamic will now be a function of the consumer's wealth parameters instead of independent of them, as in the additive case. We see the application of our framework to different wealth dynamics as an exciting avenue of research.

Indeed, the first main policy takeaway is that heterogeneity in consumer decision-making, down to the very form of the discount function itself, can be understood as stemming from underlying wealth dynamics. This knowledge empowers the social planner to model the effects of policy instruments along multiple dimensions in novel ways. For instance, in contrast to the dominant discounted-cash-flow approach, where the burden of explaining consumer heterogeneity falls almost exclusively on a single variable, the hard-to-measure discount rate (Newell & Siikamäki, 2015; Allcott & Greenstone, 2012; Jaffe & Stavins, 1994b), modelling policy measures in our framework relies instead on household wealth data and dynamics, which are usually readily available (e.g. Deutsche Bundesbank, 2019).

A second immediate implication for policy planning comes from corollary 1, where we demonstrated that the social planner must deploy additional policy measures to generate investment incentive for consumers, typically wealthier, whose wealth grows multiplicatively as compared to those whose wealth grows additively; table 5 further showed how consumers within the multiplicative case itself can differ significantly in their decision-making due to their specific wealth parameters. These ideas can be extended and generalised to other, more realistic, wealth dynamics, such as the one described in appendix B, so that the workings of policy instruments can be better differentiated both between and within wealth quantiles.

Another aspect of corollary 1 that bears highlighting is the lower bound on the number of policy levers that the social planner must exercise to generate a desired level of consumer incentive. Considering again the set of seven variables $\{p_0, v, q_0, \alpha, s_1, s_2, h\}$ ostensibly within the planner's sphere of influence, it is clear in reality, he should expect to have far fewer than seven degrees of freedom. For instance, if we consider the two classic policy levers of a carbon tax and retrofit subsidies, the planner will be left with just three degrees of freedom, since the four fuel-price parameters $\{p_0, v, q_0, \alpha\}$ will typically move in tandem in response to a carbon tax,[13] and the retrofit parameters $s_1$ and $s_2$ are jointly influenced by subsidies. Once again, extensions of this article to more complex wealth dynamics will reveal both if the number of policy levers is sufficient, and the extent to which they need to move independently from each other, in order to generate retrofit incentive for different wealth quantiles. Targeted and creative policy instruments can be thereby conceived and implemented. As an example, the German government recently began to offer steep discounts on the upgrade of heating systems (BMWi,

[13]Except of course if the consumer is upgrading to an electric heating system, in which case $q_0$ and $\alpha$ are unaffected by a carbon tax.

2021): in our framework, this can be interpreted as the social planner taking advantage of the fact that the cost of the heating system $h$ can be subsidised independently of retrofit-measure costs $s_1$ and $s_2$.

We turn now to the other key assumption in our model, namely, diminishing marginal utility in retrofitting. Our formalisation of this economic and physical reality, and subsequent discovery of an optimal retrofit depth for given dwelling and wealth parameters, provides a quantitative method of corroborating the observed fact that consumers everywhere do invest in shallow and medium retrofit measures, while resisting deep retrofits (Esser et al., 2019). Table 5 indicated strikingly that even the very wealthy may not stand to gain by retrofitting to the maximum thermal standard possible.

Our decision framework may also be used for other analytical exercises, such as locating the Pareto-optimum level of greenhouse gas emissions due to heating (Garcia, 2017), with the consumer's utility function specified by the wealth dynamic (see appendix B). The socially-optimal level of subsidies for retrofitting can be similarly determined (Allcott & Greenstone, 2012). Additionally, we indicated in section 4 how the derivatives of the growth rate function can be used to compare the effectiveness of different policy instruments.

Finally, at the macroeconomic level, our investment criterion can form the basis of a representative-agent diffusion model. For instance, since our framework can be used to model the diffusion of energy efficiency technology in the absence of market barriers, increasing the trend in fossil-fuel prices to mimic a carbon tax would give an upper-bound on the effects of such a tax on the uptake of conservation technology. Similar policy experiments could be carried out, scenarios charted, and even optimisation problems framed.

## Acknowledgements

This work was supported by the Research Training Group 2153 of the German Research Foundation (DFG): "Energy Status Data – Informatics Methods for its Collection, Analysis and Exploitation". A.B. thanks Ole Peters for helpful comments on the draft, and Alex Adamou, Andrew Fleck, Elias Naber, Max Kleinebrahm, and the participants of the *Ergodicity Economics 2021* conference for stimulating discussions.

## Declaration of Interests

The authors declare no competing interests.

## CRediT Author Statement

*Anthony Britto*: Conceptualisation, Methodology, Software, Formal Analysis, Writing – Original Draft; *Joris Dehler-Holland*: Supervision, Validation, Writing – Review and Editing; *Wolf Fichtner*: Supervision, Writing – Review and Editing

## Appendix A. Proofs of Mathematical Results

We briefly sketch the proofs of propositions 1 and 2, and corollary 1.

We consider the additive and multiplicative dynamics together, differentiating by a subscript where necessary. Consider first the case where the $q_{min}$-criterion is not satisfied. A straightforward application of the Karush-Kuhn-Tucker conditions (Kemp, 1978) reveals that in order for $g$ to attain a maximum for $0 < t < L$, the following conditions must be satisfied:

$$\partial_k g|_{k=0} = \lambda \,, \tag{29}$$

$$\partial_t g|_{k=0} = 0 \,. \tag{30}$$

Here $\lambda \neq 0$ is a constant. Eliminating $\lambda$ between the two equations gives us equation 23 (to wit, one condition) for the additive growth function $g_a$. Hence, the $q_{min}$-criterion together with equation 23 determine the existence of a non-trivial solution to the optimisation problem. On the other hand, eliminating $\lambda$ in the above system of equations for $g_m$ leads again to two conditions instead of one,[14] leading to a total of three degrees of freedom. The expressions here, and in the paragraph below, are too unwieldy to display in print, but are contained in a Mathematica notebook which is available upon request.

On the other hand, if the $q_{min}$-criterion is not satisfied, locating a growth-rate-maximising

[14]This is due to the appearance of a fraction with a non-trivial denominator, which cannot be allowed to vanish.

tuple $(k^\star, t^\star)$ requires first of all solving the simultaneous equations

$$\partial_k g = 0 \,, \tag{31}$$

$$\partial_t g = 0 \,, \tag{32}$$

to find a critical point $(k^\star, t^\star)$. We further require that at the point $(k^\star, t^\star)$, one of the two second partial derivatives be negative, and that the determinant of the Hessian matrix be positive; i.e.

$$\left.\frac{\partial^2 g}{\partial k^2}\right|_{(k^\star,t^\star)} \overset{!}{<} 0 \,, \tag{33}$$

$$\left[\frac{\partial^2 g}{\partial k^2}\frac{\partial^2 g}{\partial t^2} - \left(\frac{\partial^2 g}{\partial k \partial t}\right)^2\right]_{(k^\star,t^\star)} \overset{!}{>} 0 \,. \tag{34}$$

It turns out that equation 33 is trivially satisfied in both the additive and multiplicative cases, while equation 34 is only trivially satisfied in the additive case. Hence, we end up at the same result as in the previous paragraph: two degrees of freedom in the additive case, and three in the multiplicative.

## Appendix B. Derivation of the Utility Function, Growth Rate and Discount Function for a General Wealth Dynamic

We sketch, with recourse to an example, how one computes the utility function, growth rate and discount function for a general wealth dynamic. The interested reader is referred to Peters and Gell-Mann (2016) for further intuition and details.

We consider a non-trivial wealth dynamic, a hybrid of the additive and multiplicative cases: a consumer with net annual income $r$ who continually invests a share $\sigma < 1$ of her wealth (e.g. 20%) to grow at some annual rate $\delta$ (e.g. 2%). That is, we have the recurrence relations

$$w(0) = w_0 \,, \tag{35}$$

$$w(n+1) = \sigma w(n) e^{\delta} + (1-\sigma) w(n) + r \,, \tag{36}$$

which can be solved to yield

$$w(t) = \frac{\left(\left(e^{\delta}-1\right)\sigma + 1\right)^t \left(r + \left(e^{\delta}-1\right)\sigma w_0\right) - r}{\left(e^{\delta}-1\right)\sigma} \,. \tag{37}$$

For $\delta \ll 1$, if we define $\epsilon := \delta\sigma$, this simplifies to

$$w(t) \approx \frac{(1+\epsilon)^t (r + \epsilon w_0) - r}{\epsilon} \,. \tag{38}$$

*Computing the growth rate and utility function.*

In essence, the growth rate $g$ is a quantity that we extract out of the wealth dynamic $w(t)$ which conveys *time-independent* information about the growth of our wealth. Intuitively, it must therefore have units "something/time". In the additive case the "something" is "dollars", and in the multiplicative it is "annual percentage growth". For a general dynamic it may be something more complicated; it is usually difficult to say simply by inspection. Indeed, although it is clear that the consumer's wealth in equation 37 grows over time, it is not obvious how the growth rate should be defined.

But since we know that the growth rate must have units "something/time", the following procedure suffices: we simply linearise the relation $w_t = w(t)$ to bring it to the form $u(w_t) = gt$ for $g$ a constant; then $u$ is the consumer's utility function, and $g$ is the growth rate of her wealth per unit time $t$.[15] This is easily seen to work in the additive and multiplicative cases:

$$w_t = w_0 + rt \quad \longrightarrow \quad u(w_t) = w_t - w_0 = rt \,, \tag{39}$$

$$w_t = w_0 e^{\delta t} \quad \longrightarrow \quad u(w_t) = \log \frac{w_t}{w_0} = \delta t \,. \tag{40}$$

For our dynamic, equation 37, we arrive at

$$\log \frac{r + \left(e^{\delta} - 1\right) \sigma w_t}{r + \left(e^{\delta} - 1\right) \sigma w_0} = \log\left(\left(e^{\delta} - 1\right) \sigma + 1\right) t \,, \tag{41}$$

whence we can read off the utility function and growth rate. In the small-$\delta$ limit, the expression is approximated by

$$\log \frac{r + \epsilon w_t}{r + \epsilon w_0} = \epsilon t \,, \tag{42}$$

which means that the growth rate is in fact approximated by $\epsilon$. On the left-hand-side of the equation, we see that the consumer's utility function is a hybrid of the additive and dynamic cases. Crucially, in contrast to the pure additive dynamic, optimal investment in energy

[15] In practice, one accomplishes this by solving $w_t = w(t)$ for $t$ and then reading off the constant; in other words, we invert $w(t)$. The invertibility of $w(t)$ is guaranteed by the inverse function theorem as long as $\partial_t w(t)$ is continuous and non-vanishing. Whether a closed-form expression for the inverse exists is of course another matter.

efficiency will not be simply a problem of cost minimisation (cf. equation 18), but will involve the consumer's wealth parameters.

*Computing the discount function.*

The calculation of the discount function for a given wealth dynamic within the framework of ergodicity economics is detailed in Adamou et al. (2020). We briefly sketch the argument. Since the fundamental premise in ergodicity economics is that consumers should maximise the growth-rate of their wealth, it is necessary to specify the *time frame* of the economic decision in order to compute a discount function. This can be either fixed or adaptive, where the former corresponds to a decision that does not affect future choices, whereas in the latter, future choices do in fact depend on the present decision. An energy-efficiency investment corresponds to a fixed-time-frame decision, so we restrict ourselves to this case here.

Mathematically, the discount function is computed via a so-called *riskless inter-temporal payment problem* (RIPP): the consumer is simply asked to compare two cash payments $p_1 < p_2$ to be received at times $t_1 < t_2$ respectively; she should choose the one which make her wealth grow faster over the specified time frame. The *horizon* of the decision is $t_1$, and the *delay* between the two payments is $t_2 - t_1$. The discount function is that function which when applied to the later payment $p_2$ makes the consumer indifferent between the payments $p_1$ and $p_2$.

We will stick to equation 38, the small-$\delta$ limit for $w(t)$, in what follows. For the fixed–time-frame decision, we must compare the growth rate of the consumer's wealth up to time of the later payment $t_2$ for each of the two payments. Wealth in each case grows to[16]

$$w_1(t_2) = w(t_2) + p_1(1+\epsilon)^{t_2-t_1} \,, \tag{43}$$

$$w_2(t_2) = w(t_2) + p_2 \,, \tag{44}$$

whence the corresponding growth rates are

$$g_1(t_2) = \frac{1}{t_2}\log\frac{r+\epsilon\big(w(t_2)+p_1(1+\epsilon)^{t_2-t_1}\big)}{r+\epsilon w_0} = \frac{1}{t_2}\log\left((1+\epsilon)^{t_2} + \frac{\epsilon p_1(1+\epsilon)^{t_2-t_1}}{r+\epsilon w_0}\right) \,, \tag{45}$$

$$g_2(t_2) = \frac{1}{t_2}\log\frac{r+\epsilon\big(w(t_2)+p_2\big)}{r+\epsilon w_0} = \frac{1}{t_2}\log\left((1+\epsilon)^{t_2} + \frac{\epsilon p_2}{r+\epsilon w_0}\right) . \tag{46}$$

The expressions are identical except for the term $(1+\epsilon)^{-(t_2-t_1)}$, which we immediately identify

[16]The derivation of the expression for the growth of $p_1$ follows easily from equation 38 by setting $r$ to zero.

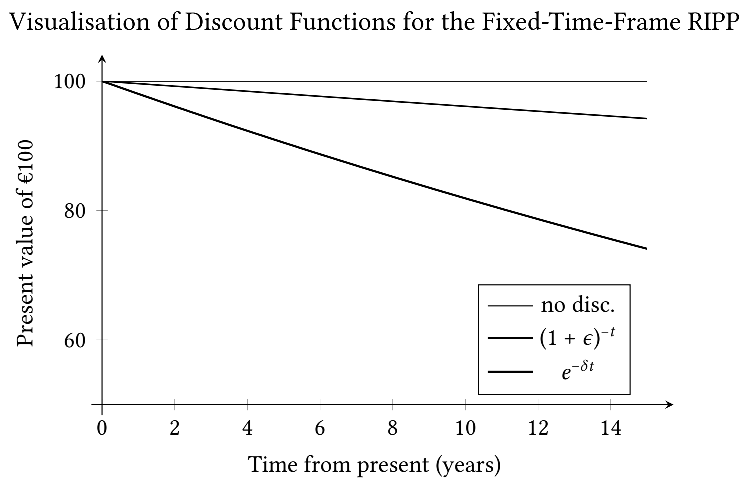


FIGURE 3. Parameter values: $\delta = 0.02$, $\sigma = 0.2$. We see that the in contrast to the pure additive dynamic where no discounting function exists, the power-law function we derived here leads to a more realistic gentle discounting of money.

as the discount function; see figure 3 for a comparison with the additive and exponential cases.

The discount function $(1+\epsilon)^{-t}$, in combination with the utility function in equation 42, can now be straightforwardly used to formulate the growth-rate maximisation problem as in section 4.

## Appendix C. Revisiting the Expected-Utility Investment Model

Hassett and Metcalf situate their investment model within expected utility theory, which is the standard framework for dealing with choice under uncertainty in economics. They formulate the decision to invest in energy efficiency as a problem of minimising the expected value of the net present value of energy and retrofit expenses. The mathematics is as follows.

Suppose that the consumer has some delivered-energy need[17] $x$, which she can reduce by a factor $\epsilon \in [0,1]$ at cost $k$. We assume again that the price $p_t$ of the energy carrier that the consumer uses for heating follows a geometric Brownian motion with trend $\mu$ and variance $\sigma$ (equation 2). The optimal time $t$ to invest in energy efficiency should then be chosen such that the expected present value of the energy expenses plus the investment itself is minimised:

$$\min_{0<t<\infty} \mathbb{E}\left[\int_0^t p_s x e^{-\gamma s}\,\mathrm{d}s + k e^{-\gamma t} + \int_t^\infty p_s(1-\epsilon)x e^{-\gamma s}\,\mathrm{d}s\right], \tag{47}$$

where $\gamma$ is the consumer's discount rate. Hence, in this setup, "do not invest" corresponds to

[17]We distinguish between delivered energy need $x$ and final-energy need $u$, which we used in the main text. The two are related by $x = u/\eta$.

the optimal $t$ being infinity.[18]

The proposed solution, based on a model of irreversible investment by McDonald and Siegel (1986), is that the optimal time to invest is when the price of the energy carrier crosses the threshold

$$p^\star := \frac{\beta}{\beta - 1} \frac{(\gamma - \mu)k}{\epsilon x} \,, \tag{48}$$

where

$$\beta := \frac{\sigma^2 - 2\mu + \sqrt{8\gamma\sigma^2 + (\sigma^2 - 2\mu)^2}}{2\sigma^2} > 1 \,. \tag{49}$$

We briefly outline the idea. Firstly, the expression $(\gamma - \mu)k/(\epsilon x)$ in equation 48, the so-called *Marshallian investment trigger*, is obtained by setting the net-present value of the monetary savings due to the reduced energy consumption minus the investment to null, and solving for the minimum-viable price $p$:

$$0 = \frac{p\epsilon x}{\gamma - \mu} - k \,. \tag{50}$$

The mathematics of real option theory requires that this price trigger be made larger by the factor $\beta/(\beta - 1)$ to account for the value of waiting due to the uncertainty in the path of the fuel price. Notice that $\beta \longrightarrow \infty$ as $\sigma \longrightarrow 0$, collapsing this investment rule to the Marshallian criterion in the limit of zero uncertainty.

The result is plausible and the mathematics elegant, but equation 48 is unfortunately not a solution to the optimisation problem posed in equation 47. The direct argument is as follows. Consider that one can directly simplify equation 47 to obtain

$$\begin{aligned} \min_{0 \le t \le \infty} \mathbb{E} \left[ \int_0^t p_s x e^{-\gamma s} \, \mathrm{d}s + k e^{-\gamma t} + \int_t^\infty p_s (1 - \epsilon) x e^{-\gamma s} \, \mathrm{d}s \right] = \\ \min_{0 \le t \le \infty} \left( p_0 x \frac{1 - e^{-(\gamma - \mu)t}}{\gamma - \mu} + k e^{-\gamma t} + p_0 (1 - \epsilon) x \frac{e^{-(\gamma - \mu)t}}{\gamma - \mu} \right) \,, \end{aligned} \tag{51}$$

where for convergence of the latter integral the economically natural condition $\gamma > \mu$ is required. We further employed Fubini's theorem and equation 3. The right-hand side of equation 51 now contains no reference to $\sigma$. Hence, the proposed solution, equation 48, which does contain $\sigma$, has no direct mathematical connection to the cost-minimisation problem.

A second, more intuitive, argument against equation 48 as a solution can also be formulated from equation 51 above. A direct comparison of the Marshallian criterion, equation 50, and the

[18] Note that the energy carrier is not switched here, though allowing for such a consideration is straightforward.

real-option–derived equation 48 reveals that the former, by definition, simply leaves out the energy expenses incurred up to the moment of investment. Hence, although the Marshallian criterion is appropriate for the irreversible investment opportunity which McDonald and Siegel (1986) consider, it is not directly applicable to the case at hand, where the consumer has a continuous stream of expenses which could be staunched by making an investment in energy efficiency. Indeed, here the task becomes to check if if the three terms in the consumer's cost stream, equation 51, might conspire to produce a non-trivial optimal investment time. In point of fact, an elementary calculation produces the following result.

**Lemma 1.** *If all the constants in the cost-minimisation problem equation 51 are assumed to be positive, with $\mu < \gamma$ and $0 < \epsilon < 1$, and if*

$$x < \frac{\gamma k}{p_0}, \tag{52}$$

*a positive optimal investment time exists and is given by*

$$t^\star = \frac{1}{\mu} \log \frac{\gamma k}{p_0 \epsilon x} . \tag{53}$$

In other words, if her current energy need is not too large, the above optimisation problem produces an optimal investment time for the consumer. Nevertheless, notice that $\sigma$, the only explicit quantifier of uncertainty in the model, is conspicuously absent from the above result.

We have addressed this puzzle in our discussion of ergodicity breaking in section 3. The solution is to switch from the using the expected-value operator to the time-average operator (equation 11); our optimisation problem is then written as

$$\min_{0 \le t \le \infty} \left( p_0 x \frac{1 - e^{-(\gamma - \nu)t}}{\gamma - \nu} + k e^{-\gamma t} + p_0 (1 - \epsilon) x \frac{e^{-(\gamma - \nu)t}}{\gamma - \nu} \right) , \tag{54}$$

where we once again bring in the effective trend $\nu = \mu - \sigma^2/2$. We further impose the requirement that the discount rate $\gamma > \nu$. This leads to the main result of this section, which is a straight-forward modification of lemma 1 above, and the correct resolution of the cost-minimisation problem posed by Hassett and Metcalf.

**Proposition 3.** *Suppose that the price of the energy carrier $p_t$ follows a geometric Brownian motion $(\mu, \sigma)$ such that $0 < \mu - \sigma^2/2$. If all the other constants in equation 54, the optimisation problem, are positive, with $\nu < \gamma$ and $0 < \epsilon < 1$, the optimal time for the consumer to invest so* *that the net present value of her energy and retrofit expenses is minimised is*

$$t^{\star} = \frac{1}{\nu} \log \frac{\gamma k}{p_0 \epsilon x}, \tag{55}$$

*provided that*

$$x < \frac{\gamma k}{p_0}. \tag{56}$$

## References


Adamou, A. T. I., Berman, Y., Mavroyiannis, D. P., & Peters, O. B. (2020). *Microfoundations of discounting.* Retrieved from https://arxiv.org/abs/1910.02137

Allcott, H., & Greenstone, M. (2012). Is there an energy efficiency gap? *Journal of Economic Perspectives*, *26*(1), 3–28. https://doi.org/10.1257/jep.26.1.3

BMWi. (2021). *FAQs: Market incentive programme (MAP).* Berlin: Federal Ministry for Economic Affairs and Energy. Retrieved from https://www.bmwi.de/Redaktion/EN/FAQ/Market-Incentive-Programme-MAP/faq-map-02.html

Deutsche Bundesbank. (2019). Household wealth and finances in Germany: Results of the 2017 survey. In *Monthly report: April 2019.* Frankfurt: Author. Retrieved from https://www.bundesbank.de/en/publications/search/household-wealth-and-finances-in-germany-results-of-the-2017-survey-796280

Esser, A., Dunne, A., Meeusen, T., Quaschning, S., & Wegge, D. (2019). *Comprehensive study of building energy renovation activities and the uptake of nearly zero-energy buildings in the EU.* Brussels: Directorate-General for Energy, European Commision. Retrieved from https://op.europa.eu/s/oNma

Friege, J., & Chappin, E. (2014). Modelling decisions on energy-efficient renovations: A review. *Renewable and Sustainable Energy Reviews*, *39*, 196–208. https://doi.org/10.1016/j.rser.2014.07.091

Gabrielli, L., & Ruggeri, A. G. (2019). Developing a model for energy retrofit in large building portfolios: Energy assessment, optimization and uncertainty. *Energy and Buildings*, *202*, 109–356. https://doi.org/10.1016/j.enbuild.2019.109356

Galvin, R. (2010). Thermal upgrades of existing homes in Germany: The building code, subsidies, and economic efficiency. *Energy and Buildings*, *42*(6), 834–844. https://doi.org/10.1016/j.enbuild.2009.12.004

Galvin, R. (2012). German federal policy on thermal renovation of existing homes: A policy evaluation. *Sustainable Cities and Society*, *4*, 58–66. https://doi.org/10.1016/j.scs.2012.05.003

Galvin, R. (2014). Why German homeowners are reluctant to retrofit. *Building Research & Information*, *42*(4), 398–408. https://doi.org/10.1080/09613218.2014.882738

Garcia, F. (2017). Externalities and public goods. In *Advanced microeconomic theory: an intuitive approach with examples.* Cambridge, Massachusetts: The MIT Press.

Georges, L., Massart, C., Van Moeseke, G., & De Herde, A. (2012). Environmental and economic performance of heating systems for energy-efficient dwellings: Case of passive and low-energy single-family houses. *Energy Policy*, *40*, 452–464. https://doi.org/10.1016/j.enpol.2011.10.037

Gerarden, T., Newell, R., & Stavins, R. (2017). Assessing the energy-efficiency gap. *Journal of Economic Literature*, *55*(4), 1486–1525. https://doi.org/10.1257/jel.20161360

Geroski, P. (2000). Models of technology diffusion. *Research Policy*, *29*(4-5), 603–625. https://doi.org/10.1016/s0048-7333(99)00092-x

Gillingham, K., & Palmer, K. (2014). Bridging the energy efficiency gap: Policy insights from economic theory and empirical evidence. *Review of Environmental Economics and Policy*, *8*(1), 18–38. https://doi.org/10.1093/reep/ret021

Grenadier, S., & Weiss, A. (1997). Investment in technological innovations: An option pricing approach. *Journal of Financial Economics*, *44*(3), 397–416. https://doi.org/10.1016/s0304-405x(97)00009-3

Hassett, K. A., & Metcalf, G. E. (1992). *Energy tax credits and residential conservation investment* (Working Paper No. 4020). National Bureau of Economic Research. https://doi.org/10.3386/w4020

Hassett, K. A., & Metcalf, G. E. (1993). Energy conservation investment. *Energy Policy*, *21*(6), 710–716. https://doi.org/10.1016/0301-4215(93)90294-p

Hong, T., Koo, C., Kim, H., & Park, H. (2014). Decision support model for establishing the optimal energy retrofit strategy for existing multi-family housing complexes. *Energy Policy*, *66*, 157–169. https://doi.org/10.1016/j.enpol.2013.10.057

Hull, J. C. (2017). *Options, futures, and other derivatives.* New York: Pearson.

Jaffe, A., & Stavins, R. (1994a). The energy-efficiency gap: What does it mean? *Energy Policy*, *22*(10), 804–810. https://doi.org/10.1016/0301-4215(94)90138-4

Jaffe, A., & Stavins, R. (1994b). The energy paradox and the diffusion of conservation technology. *Resource and Energy Economics*, *16*(2), 91–122. https://doi.org/10.1016/0928-7655(94)90001-9

Kemp, M. (1978). *Introduction to mathematical economics.* New York: Springer-Verlag.

Kumbaroğlu, G., & Madlener, R. (2012). Evaluation of economically optimal retrofit investment options for energy savings in buildings. *Energy and Buildings*, *49*, 327–334. https://doi.org/10.1016/j.enbuild.2012.02.022

Loga, T., Stein, B., & Diefenbach, N. (2016). TABULA building typologies in 20 European countries—making energy-related features of residential building stocks comparable. *Energy and Buildings*, *132*, 4–12. https://doi.org/10.1016/j.enbuild.2016.06.094

Ma, Z., Cooper, P., Daly, D., & Ledo, L. (2012). Existing building retrofits: Methodology and state-of-the-art. *Energy and Buildings*, *55*, 889–902. https://doi.org/10.1016/j.enbuild.2012.08.018

MacLean, L., Thorp, E., & Ziemba, W. (2011). *The Kelly capital growth investment criterion.* Singapore: World Scientific. https://doi.org/10.1142/7598

McDonald, R., & Siegel, D. (1986). The value of waiting to invest. *The Quarterly Journal of Economics*, *101*(4), 707. https://doi.org/10.2307/1884175

Meder, D., Rabe, F., Morville, T., Madsen, K. H., Koudahl, M. T., Dolan, R. J., … Hulme, O. J. (2020). *Ergodicity-breaking reveals time optimal decision making in humans.* Retrieved from https://arxiv.org/abs/1906.04652

Newell, R., & Siikamäki, J. (2015). Individual time preferences and energy efficiency. *American Economic Review*, *105*(5), 196–200. https://doi.org/10.1257/aer.p20151010

Peters, O. (2011a). Optimal leverage from non-ergodicity. *Quantitative Finance*, *11*(11), 1593–1602. https://doi.org/10.1080/14697688.2010.513338

Peters, O. (2011b). The time resolution of the St. Petersburg paradox. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, *369*(1956), 4913–4931. https://doi.org/10.1098/rsta.2011.0065

Peters, O. (2019). The ergodicity problem in economics. *Nature Physics*, *15*(12), 1216–1221. https://doi.org/10.1038/s41567-019-0732-0

Peters, O., & Gell-Mann, M. (2016). Evaluating gambles using dynamics. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, *26*(2), 023103. https://doi.org/10.1063/1.4940236

Peters, O., & Klein, W. (2013). Ergodicity breaking in geometric Brownian motion. *Physical*

*Review Letters*, *110*(10). https://doi.org/10.1103/physrevlett.110.100603

Pirvu, T. A., & Žitković, G. (2009). Maximizing the growth rate under risk constraints. *Mathematical Finance*, *19*(3), 423–455. https://doi.org/10.1111/j.1467-9965.2009.00378.x

Sunding, D., & Zilberman, D. (2001). The agricultural innovation process: Research and technology adoption in a changing agricultural sector. In B. Gardner & G. Rausser (Eds.), *Handbook of agricultural economics* (Vol. 1, pp. 207–261). Amsterdam: Elsevier.

# Working Paper Series in Production and Energy

recent issues

**No. 55** Daniel Fett, Christoph Fraunholz, Dogan Keles: Diffusion and System Impact of Residential Battery Storage under Different Regulatory Settings

**No. 54** Joris Dehler-Holland, Marvin Okoh, Dogan Keles: The Legitimacy of Wind Power in Germany

**No. 53** Florian Diehlmann, Markus Lüttenberg, Lotte Verdonck, Marcus Wiens, Alexander Zienau, Frank Schultmann: Public-Private Collaborations in Emergency Logistics: A Framework based on Logistical and Game-Theoretical Concepts

**No. 52** Florian Diehlmann, Patrick S. Hiemsch, Marcus Wiens, Markus Lüttenberg, and Frank Schultmann: A Novel Approach to Include Social Costs in Humanitarian Objective Functions

**No. 51** Florian Diehlmann, Miriam Klein, Marcus Wiens, Markus Lüttenberg, and Frank Schultmann: On the Value of Accurate Demand Information in Public-Private Emergency Collaborations

**No. 50** Maximilian Schücking, Patrick Jochem: Two-Stage Stochastic Program Optimizing the Total Cost of Ownership of Electric Vehicles in Commercial Fleets

**No. 49** Max Kleinebrahm, Jacopo Torriti, Russell McKenna, Armin Ardone, Wolf Fichtner: Using neural networks to model long-term dependencies in occupancy behavior

**No. 48** Nico Lehmann, Jonathan Müller, Armin Ardone, Katharina Karner, Wolf Fichtner: Regionalität aus Sicht von Energieversorgungsunternehmen - Eine qualitative Inhaltsanalyse zu Regionalstrom in Deutschland

**No. 47** Marcus Wiens, Farnaz Mahdavian, Stephen Platt, Frank Schultmann: Optimal Evacuation-Decisions Facing the Trade-Off between Early-Warning Precision, Evacuation-Cost and Trust – the Warning Compliance Model (WCM)

**No. 46** Phuong Khuong, Fabian Scheller, Russell McKenna, Dogan Keles, Wolf Fichtner: Willingness to pay for residential PV: reconciling gaps between acceptance and adoption

**No. 45** Christoph Fraunholz, Emil Kraft, Dogan Keles, Wolf Fichtner: The Merge of Two Worlds: Integrating Artificial Neural Networks into Agent-Based Electricity Market Simulation

**No. 44** Fritz Braeuer, Rafael Finck, Russell McKenna: Comparing empirical and model-based approaches for calculating dynamic grid emission factors: An application to CO2-minimizing storage dispatch in Germany

**Impressum**

Karlsruher Institut für Technologie

Institut für Industriebetriebslehre und Industrielle Produktion (IIP)
Deutsch-Französisches Institut für Umweltforschung (DFIU)

Hertzstr. 16
D-76187 Karlsruhe

KIT – Universität des Landes Baden-Württemberg und
nationales Forschungszentrum in der Helmholtz-Gemeinschaft

Working Paper Series in Production and Energy
**No. 56**, March 2021

ISSN 2196-7296