

A New Method of Constructing Brand Switching Matrix

Akinori Okada and Hiroyuki Tsurumi

Abstract A new method of defining brand switching is introduced. Brand switching is defined by comparing two brands purchased in two consecutive periods. For durable consumer goods this does not cause any problem, because only one brand would be purchased in one period. It is difficult to define brand switching of non-durable consumer goods in the same manner, because two or more brands can be purchased by a consumer in one period. A practical method to cope with this difficulty is to compare the best selling brand in each period, but this ignores brands other than the best selling brand. The present method defines brand switching by the change of the rank of the purchase amount of the brand in two consecutive periods. The method is applied to derive the brand switching matrix among potato snack brands, and is compared with the method based on the best selling brand by asymmetric multidimensional scaling. The comparison shows that the present method represents the dominance relationships among brands more accurately than the method based on the best selling brand.

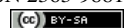
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1 Introduction

The brand switching would be defined by comparing two brands purchased by a consumer in two consecutive periods (e.g., periods 1 and 2). How to define brand switching varies depending on whether we deal with durable consumer goods or non-durable consumer goods. In the case of durable consumer goods like cars or refrigerators, only one car or one refrigerator would be purchased in a period by a consumer. In the case of durable consumer goods, how to define the brand switching is simple; it is enough to compare the brand purchased in period 1 and the brand purchased in period 2 for each consumer (e.g., Harshman et al. (1982)). In the case of non-durable consumer goods like yogurt or potato snack, each consumer would purchase not only once but several times in a period. And each consumer may purchase not only one brand but two or more brands in a period or even on one purchase occasion. How to define the brand switching is not easy, when two or more brands are purchased in a period. One practical method is to compare the brands which are the best selling brand purchased (in terms of the amount of money or quantity) in periods 1 and 2 for a consumer respectively (e.g., Okada and Tsurumi (2014)).

Brand switching from brands j to k for a consumer is represented by a brand switching matrix whose (j, k) element is unity and the other elements are null. The j -th diagonal element of a brand switching matrix is unity when brand j is the best selling brand in periods 1 and 2, and null otherwise. Brand switching in the case of non-durable consumer goods defined by the best selling brand relies on only one brand. Brand switching represents the dominance relationships among brands, and it is reasonable to define brand switching by not only one brand but by all brands. In the present study, the method to define brand switching which utilizes not only the best selling brand but all brands (Okada and Tsurumi (2018, 2019)), is described in detail. The present method has a concept in common with the 'share of wallet' or the 'share of customer' which is important in managing the customer relationship with brands (Peppers and Rogers (1993)), and which is used as an index of the relationship (Keiningham et al. (2011)). The purpose of the present study is twofold. The first one is to describe the method of defining brand switching based on all brands not only on the best selling brand. The second one is to compare two brand switching matrices. One constructed by the present method and the other constructed based on the best selling brand, by analyzing two brand switching matrices using asymmetric multidimensional scaling.

2 The method of constructing a brand switching matrix

The method of constructing a brand switching matrix based on all brands is described in this section. It utilizes the rank of the purchase amount (money) of brands for a consumer. Brand switching from brands j to k ($j \neq k$) for a consumer is defined as stated below. When the rank of the purchase amount of brand j at period 1 is overtaken by brand k at period 2, the (j, k) element of the brand switching matrix is unity, and null otherwise. This method makes it possible to take all brands into account in defining a brand switching matrix.

In the following we give small examples of constructing a brand switching matrix. Suppose there are five brands, namely A, B, C, D, and E, and the five \times five brand switching matrix, where the first row and column correspond to brand A, the second row and column correspond to brand B, ..., and the fifth row and column correspond to brand E, represents the brand switching. Let the rank vector of the purchase amount at period 1 be ABCDE, and that at period 2 BACDE. Brand A was overtaken by brand B at period 2. Then the $(1, 2)$ element of the brand switching matrix is unity, and the other elements are null. Let the rank of the purchase amount at period 1 be ABCDE, and that at period 2 CBADE. Brand A is overtaken by brands C and B, and brand B is overtaken by brand C at period 2. Then the $(1, 2)$, the $(1, 3)$, and the $(2, 3)$ elements of the brand switching matrix are unity, and the other elements are null.

The j -th diagonal element for a consumer is unity when the rank of the purchase amount of brand j at period 2 is higher or equal to that of period 1, and null otherwise.

The definition of the off diagonal element and that of the diagonal element of a brand switching matrix are slightly different. The (j, k) element ($j \neq k$) of a brand switching matrix shows whether brand j is switched to brand k (when (j, k) element=1) or not (when (j, k) element=0). On the other hand, The (j, j) element of a brand switching matrix shows whether the rank of brand j at period 1 is maintained at period 2 (when (j, j) element=1) or not (when (j, j) element=0). The diagonal element of a brand switching matrix shows how loyal a consumer is to the brand. When the rank of brand j at period 1 is maintained at period 2, it seems reasonable to think that the consumer is loyal to the brand.

The brand switching matrix for a group of consumers is derived by adding brand switching matrices of all consumers in the group. The (j, k) element

of the obtained brand switching matrix for a group represents the number of consumers who switched the brand from brands j to k in the group.

3 The data

The method was applied to the data on eight potato snack brands which were collected in 2009 by the Distribution Economics Institute of Japan. Of 47,633 customers who are members of the frequent shoppers' program of a supermarket chain, 8,431 customers purchased a potato snack from June 1 to August 31, 2009 at any of three stores which belong to the supermarket chain in the Tokyo metropolitan area. And 1,051 of the 8,431 customers purchased a potato snack more than once at both of period 1 (June 1 - July 16, 2009) and period 2 (July 17 - August 31, 2009). The eight potato snack brands are called A, B, ..., F, G, and O. Brand O represents brands other than brands A, ..., F, and G.

Table 1: Brand switching matrix among eight potato snack brands based on all brands.

Period 1	Period 2							
	A	B	C	D	E	F	G	O
Brand A	158	20	13	11	9	4	5	19
Brand B	37	182	38	33	48	30	38	50
Brand C	23	41	82	29	32	28	33	35
Brand D	12	37	26	78	21	19	19	31
Brand E	25	36	22	18	83	14	28	33
Brand F	2	8	6	4	6	25	4	8
Brand G	18	27	25	24	37	8	52	30
Brand O	13	25	26	19	18	10	13	29

Table 2: Brand switching matrix among eight potato snack brands based on the best selling brand.

Period 1	Period 2							
	A	B	C	D	E	F	G	O
Brand A	156	13	8	2	7	3	3	14
Brand B	29	145	19	12	28	18	14	35
Brand C	5	16	38	6	14	8	13	10
Brand D	3	8	13	40	6	3	4	8
Brand E	14	19	6	7	57	9	19	16
Brand F	2	1	1	0	1	14	0	1
Brand G	11	14	6	8	22	4	27	12
Brand O	8	10	15	5	10	4	7	20

Two brand switching matrices were derived from the data. One is the brand switching matrix shown in Table 1. Table 1 was derived by the method which was slightly adjusted the one based on all brands introduced in Section 2. In the adjusted method, even if the rank of brand j at period 1 is overtaken by brand k at period 2, or even if the rank of brand j at period 1 is maintained at period 2 for a consumer, the (j, k) element or the (j, j) element of a brand switching matrix for the consumer is 0 when the purchase amount of brand j is null at period 1. This can reduce the excessive influence of the long tail of markets on brand switching. The other is the brand switching matrix shown in Table 2

based on the best selling brand. The correlation coefficient between elements of two tables is 0.96 for all elements, and is 0.51 for off-diagonal elements. This two correlation coefficients tell that the frequency of the brand switching among eight potato snack brands are very similar for all elements, and fairly similar for off-diagonal elements.

4 Asymmetric multidimensional scaling

The (j, k) element of the brand switching matrix can be regarded as the similarity from brand j to k . Two brand switching matrices one derived by the present method based on all brands, and the other based on the best selling brand are analyzed respectively by the asymmetric multidimensional scaling which was used in Okada and Tsurumi (2014).

Let \mathbf{A} be the $n \times n$ matrix of the asymmetric similarity matrix among n brands. The (j, k) element of \mathbf{A} represents the similarity from brand j to k which can be not equal to the (k, j) element. By using singular value decomposition (Eckart and Young (1936)), \mathbf{A} is approximated by the product of three matrices;

$$\mathbf{A} \simeq \mathbf{X}_r \mathbf{D}_r \mathbf{Y}_r', \quad (1)$$

where \mathbf{D}_r is the $r \times r$ diagonal matrix having the r largest singular values in descending order at the diagonal elements, \mathbf{X}_r is the $n \times r$ matrix of the corresponding left singular vectors (the length is normalized to be unity), and \mathbf{Y}_r is the $n \times r$ matrix of the corresponding right singular vectors (the length is normalized to be unity).

The j -th element of the i -th column of \mathbf{X}_r (the i -th left singular vector), x_{ji} , represents the outward tendency of brand j along dimension i , which shows the weakness of brand j of being switched from brand j to the other brands. The k -th element of the i -th column of \mathbf{Y}_r (the i -th right singular vector), y_{ki} , represents the inward tendency of object k along dimension i , which shows the strength of brand k of being switched to brand k from the other brands.

5 Results

Each brand switching matrix (Tables 1 and 2) was analyzed by asymmetric multidimensional scaling. The five largest singular values are 282.7, 143.7, 94.9,

63.2, and 52.0 for the brand switching matrix based on all brands (Table 1), and 189.5, 134.7, 64.8, 41.9, and 30.5 for the brand switching matrix based on the best selling brand (Table 2). The two-dimensional result was chosen as the solution for both brand switching matrices, which makes it easy to compare the two results.

The two-dimensional result consists of two planar configurations of brands, one along dimension 1 and the other along dimension 2. Each configuration is represented in a plane. The abscissa of the plane along dimension i is the i -th left singular vector (the i -th column of \mathbf{X}_r) which represents the outward tendency. The ordinate of the plane along dimension i is the i -th right singular vector (the i -th column of \mathbf{Y}_r) which represents the inward tendency. Figure 1 shows the configuration of brands along dimension 1 based on all brands. Figure 2 shows the configuration of brands along dimension 1 based on the best selling brand.

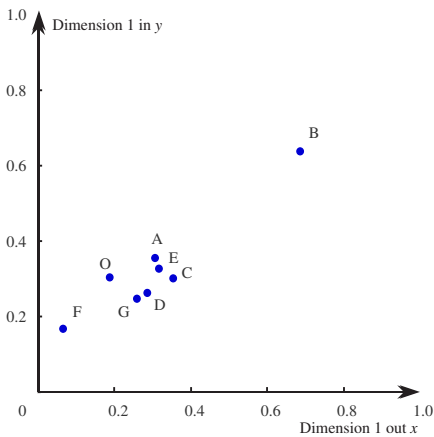


Figure 1: Configuration of brands along dimension 1 based on all brands.

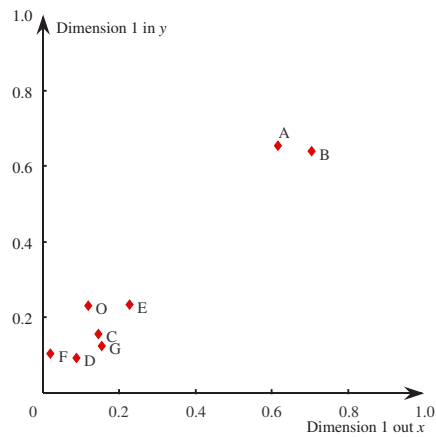


Figure 2: Configuration of brands along dimension 1 based on the best selling brand.

All brands are in the first quadrant in Figures 1 and 2. Before examining the configurations shown in Figures 1 and 2, it is useful to show how the asymmetric relationships of brand switching among brands are represented in the configuration. As shown by Equation (2), the frequency of the brand switching from brands j to k , a_{jk} , when $r = 2$, is approximated by the sum of two terms:

$$a_{jk} \simeq d_1 x_{j1} y_{k1} + d_2 x_{j2} y_{k2}, \quad (2)$$

where d_1 is the largest singular value, x_{j1} is the outward tendency of brand j along dimension 1, and y_{k1} is the inward tendency of brand k along dimension 1.

The first term of the right side of Equation (2) shows the frequency of the brand switching from brands j to k along dimension 1, which is the product of the outward tendency of brand j and the inward tendency of brand k along dimension 1 multiplied by the largest singular value. Similarly, the second term shows the frequency of the brand switching from brands j to k along dimension 2.

The frequency of the brand switching from brands F to B along dimension 1 is represented by $d_1 x_{F1} y_{B1}$ which means the area of a rectangle formed by two sides; x_{F1} and y_{B1} , multiplied by d_1 . The frequency of the brand switching from brands B to F along dimension 1 is represented by $d_1 x_{B1} y_{F1}$, which means the area of a rectangle formed by two sides; x_{B1} and y_{F1} , multiplied by d_1 . In Figures 1 and 2, $d_1 x_{F1} y_{B1} < d_1 x_{B1} y_{F1}$. This tells that the brand switching from brands B to F is larger than the brand switching from brands F to B along dimension 1, suggesting that brand F dominates brand B along dimension 1.

Figure 3 shows the configuration of brands along dimension 2 based on all brands. Figure 4 shows the configuration of brands along dimension 2 based on the best selling brand. As shown by the largest and the second largest singular values, dimension 1 based on all brands has an about two times larger effect than dimension 2 has in representing the relationships of the brand switching among brands, while dimension 2 based on the best selling brands has an about 1.4 times larger effect than dimension 2.

Figure 1 (all brands) represents substantial relationships of the brand switching among all brands along dimension 1. Figure 2 (the best selling brand) practically represents relationships of the brand switching between brands A and B along dimension 1, because points representing the other six brands are near to the origin.

In Figures 3 (all brands) and 4 (the best selling brand), brand A is in the third quadrant, and the other seven brands are in the first quadrant. The frequency of brand switching from brands A to j ($j \neq A$, and brand j is in the first quadrant) is represented by $d_2 x_{A2} y_{j2}$, and that from brand j to A is represented by $d_2 x_{j2} y_{A2}$. Both $d_2 x_{A2} y_{j2}$, and $d_2 x_{j2} y_{A2}$ are negative, because x_{A2} and y_{A2} are negative, whereas x_{j2} and y_{j2} are positive. This means that $d_2 x_{A2} y_{j2}$ and $d_2 x_{j2} y_{A2}$ along dimension 2, which corresponds to the second term of the right side of Equation (2), counterbalance the frequency of brand switching from

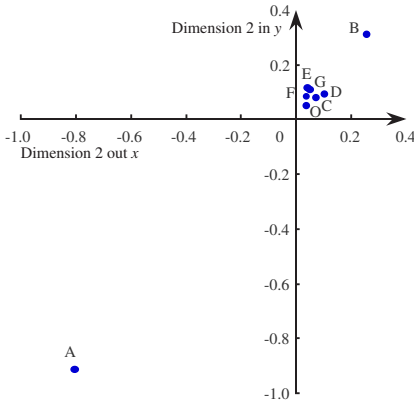


Figure 3: Configuration of brands along dimension 2 based on all brands.

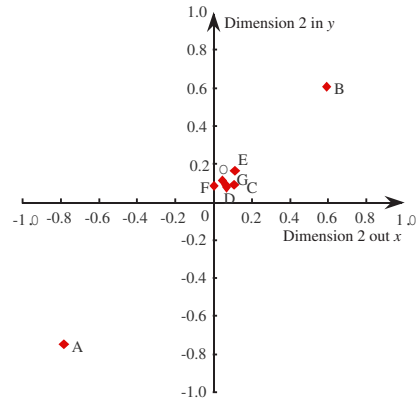


Figure 4: Configuration of brands along dimension 2 based on the best selling brand.

brands A to j and that from j to A along dimension 1, which corresponds to the first term of the right side of Equation (2).

Figure 3 shows that all brands but brand A are near to the origin, suggesting the relationships among the seven brands near to the origin are insignificant. Figure 3 essentially represents weak relationships of brand switching between brand A and the other seven brands. Figure 4 (the best selling brand) shows that six brands (brands other than A and B) are near to the origin, suggesting the relationships among the six brands are insignificant. Figure 4 mainly represents relationships between brands A and B, and weak relationships between brand A and the six brands, and those between brand B and the six brands.

Configurations shown in Figures 1 and 2 appear not so similar. The correlation coefficient between recovered elements of brand switching matrices along dimension 1 based on all brands and those based on the best selling brand is 0.68. Configurations shown in Figures 3 and 4 are somewhat similar, and the correlation coefficient between recovered elements of brand switching matrices along dimension 2 based on all brands and those based on the best selling brand is 0.87. These figures tell that the two configurations based on all brands and those based on the best selling brands along each dimension represent pretty or fairly similar relationships of brand switching. To compare the two dimensional configurations as a whole, the correlation coefficient between the

sum of recovered elements of brand switching matrices along dimensions 1 and 2 based on all brands and those based on the best selling brand was derived.

The correlation coefficient is 0.89 which is larger than the correlation coefficient between two sets of recovered elements of brand switching matrices along dimension 1 (0.68) as well as along dimension 2 (0.87). Two methods of defining brand switching resulted in brand switching matrices which show fairly similar relationships of brand switching represented in two-dimensional results as the analyses by asymmetric multidimensional scaling show. But the relationships among brands represented in configurations along each dimension are less similar than those represented in the two-dimensional results are. This suggests that two sorts of two-dimensional results, based on two different methods of defining brand switching, represent similar relationships among brands, and that each dimension discloses less similar aspects of the relationships represented in the two sorts of results. It should be pointed out that the recovered elements from/to brands A and B are very large (absolute) values compared to the other recovered elements. This can inflate the three correlation coefficients shown above.

6 Discussion

The asymmetry of brand switching or the dominance relationships among brands determines the increase or the decrease of the market share of the brand. Table 3 shows the market share of each brand at periods 1 and 2, and the increment/decrement of the market share from periods 1 to 2. The market share of brands A, E, F, and O increased from periods 1 to 2, conversely the market share of brands B, C, D, and G decreased.

Table 3: Market share (%) of eight potato snack brands based on the amount of money purchased.

Brand	Period 1	Period 2	Period 2 - Period 1
Brand A	22.6	32.9	10.3
Brand B	24.5	18.1	-6.5
Brand C	12.4	8.3	-4.1
Brand D	11.0	7.7	-3.3
Brand E	10.7	11.2	0.5
Brand F	1.2	3.7	2.6
Brand G	10.2	8.7	-1.4
Brand O	7.4	9.4	2.0

Let two points representing brands j and k respectively be in the same quadrant or in the neighboring quadrants of the configuration along dimension i . The frequency of brand switching from brands j to k along dimension i is represented by $d_i x_{ji} y_{ki}$ which means the signed area of a rectangle formed by two sides; x_{ji} and y_{ki} , multiplied by d_i . If brand k is ahead of the counterclockwise direction (azimuth at the origin) than brand j is in the configuration, $x_{ki} y_{ji} < x_{ji} y_{ki}$. This tells that brand k dominates brand j along dimension i .

It is worthwhile to examine the correspondence between the increment/decrement of the market share and the dominance relationships among brands represented in the configuration. The configuration based on all brands along dimension 1 (Figure 1) tells that the dominance relationships among brands are $F > O > E > A > G > D > B > C$, where $F > O$ represents that brand F dominates brand O (the brand switching from O to F is larger than from F to O). The configuration based on the best selling brand along dimension 1 (Figure 2) tells that the dominance relationships among brands are $F > O > A > C > D > E > B > G$. It seems that the dominance relationships along dimension 1 represented in Figure 1 more closely correspond to the increment/decrement of the market share than those based on the best selling brand do. The configuration based on all brands along dimension 2 (Figure 3) tells that the dominance relationships among brands are $E > F > G > O > B > C > D$, $D > A$, and $A > E > F > G > O > B > C$. The configuration based on the best selling brand along dimension 2 (Figure 4) tells that the dominance relationships among brands are $F > O > E > G > D > B > C$, $C > A$, and $A > F > O > E > G > D > B$. The dominance relationships of brand switching in Figures 3 and 4 only partially correspond to the increment/decrement of the market share. These tell that the present method of defining brand switching based on all brands corresponds a little more accurately to the dominance relationships of the brand switching among brands than the method based on the best selling brands does. The ratio of the second largest singular value to the largest singular value is 0.51 for Table 1 (all brands), and 0.71 for Table 2 (best selling brand). This indicates that dimension 2 is not significant, especially for all brands.

The present method of defining a brand switching matrix based on all brands was applied only to one set of data. And the obtained brand switching matrix was analyzed only by the asymmetric multidimensional scaling based on the singular value decomposition.

It is necessary to apply the method to other data sets, and to utilize other asymmetric multidimensional scaling methods to analyze the data in order to more comprehensively examine and evaluate the present method. The present method has an advantage in defining brand switching which is hardly influenced even if the sum total of the purchase amount of all brands shrank or expanded from periods 1 to 2, because the present method depends on the rank of the purchase amount not on the amount itself. The present method can be used to analyze the change of ranks. For example of universities, occupations, soccer or baseball teams, ...

In using the present method to analyze the change of ranks, we need ranks given by quite a few numbers of respondents which is sufficient to derive a substantive 'brand switching matrix' among universities, occupations, soccer or baseball teams and so on. This will make it possible to predict the brand switching matrix of the future (cf. Blattberg and Golanty (1978); Urban et al. (1984)). Suppose a customer gives the rank of brands he would like to purchase on the next purchase occasion. The obtained rank and the rank of the purchase amount of brands in the past, e.g., one month, can be used to derive the brand switching matrix from the last one month to the next purchase occasion. This will enable us to acquire dominance relationships among brands on the next purchase occasion.

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References

- Blattberg R, Golanty J (1978) Tracker: An Early Test Market Forecasting and Diagnostic Model for New Product Planning. *Journal of Marketing Research* 15(2):192–202. DOI: 10.2307/3151249.
- Eckart C, Young G (1936) The Approximation of One Matrix by Another of Lower Rank. *Psychometrika* 1(2):211–218. DOI: 10.1007/BF02288367.
- Harshman RA, Green PE, Wind Y, Lundy ME (1982) A Model for the Analysis of Asymmetric Data in Marketing Research. *Marketing Science* 1(2):205–242. DOI: 10.1287/mksc.1.2.205.
- Keiningham TL, Aksoy L, Buoye A, Cooil B (2011) Customer Loyalty Isn't Enough. Grow Your Share of Wallet. *Harvard Business Review* 89(10):29–31.
- Okada A, Tsurumi H (2014) Evaluating the Effect of New Brand by Asymmetric Multidimensional Scaling. In: Vicari D, Okada A, Ragozini G, Weihs C (eds.), *In Analysis and Modeling of Complex Data in Behavioral and Social Sciences*. Springer, Heidelberg, pp. 201–209. DOI: 10.1007/978-3-319-06692-9.
- Okada A, Tsurumi H (2018) Comparing Two Brand Switching Matrices by Asymmetric Multidimensional Scaling. *Book of Abstracts of the European Conference on Data Analysis ECDA2018*, p. 45, Hüllermeier E (ed.), University of Paderborn, Paderborn.
- Okada A, Tsurumi H (2019) A New Notion of Constructing Brand Switching Matrix With Diagonal Elements. *Book of Abstracts of the European Conference on Data Analysis ECDA2019*, pp. 28–29, Baier D (ed.), University of Bayreuth, Bayreuth.
- Peppers D, Rogers M (1993) *The One to One Future: Building Relationships One Customer at a Time*. Crown Business: New York. ISBN: 978-0-385425-28-5.
- Urban GL, Johnson PL, Hauser JR (1984) Testing Competitive Market Structures. *Marketing Science* 3(2):83–112, Sudhir K (ed.). DOI: 10.1287/mksc.3.2.83.