

# The analytical wind farm efficiency and wake model EFFWAKE

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EFFWAKE

Introduction

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Outlook

Analytical models are needed for rapid assessment of major features of planned wind farms

There are two types of analytical wind farm models:

- (1) bottom-up models: superposition of multiple wakes, suitable for small farms
- (2) top-down models: wind farm is seen as one homogeneous entity which contributes to momentum dissipation, suitable for large farms

# Short Characterization

EFFWAKE is a top-down analytical wind farm efficiency and wake model for an **indefinitely large** wind farm

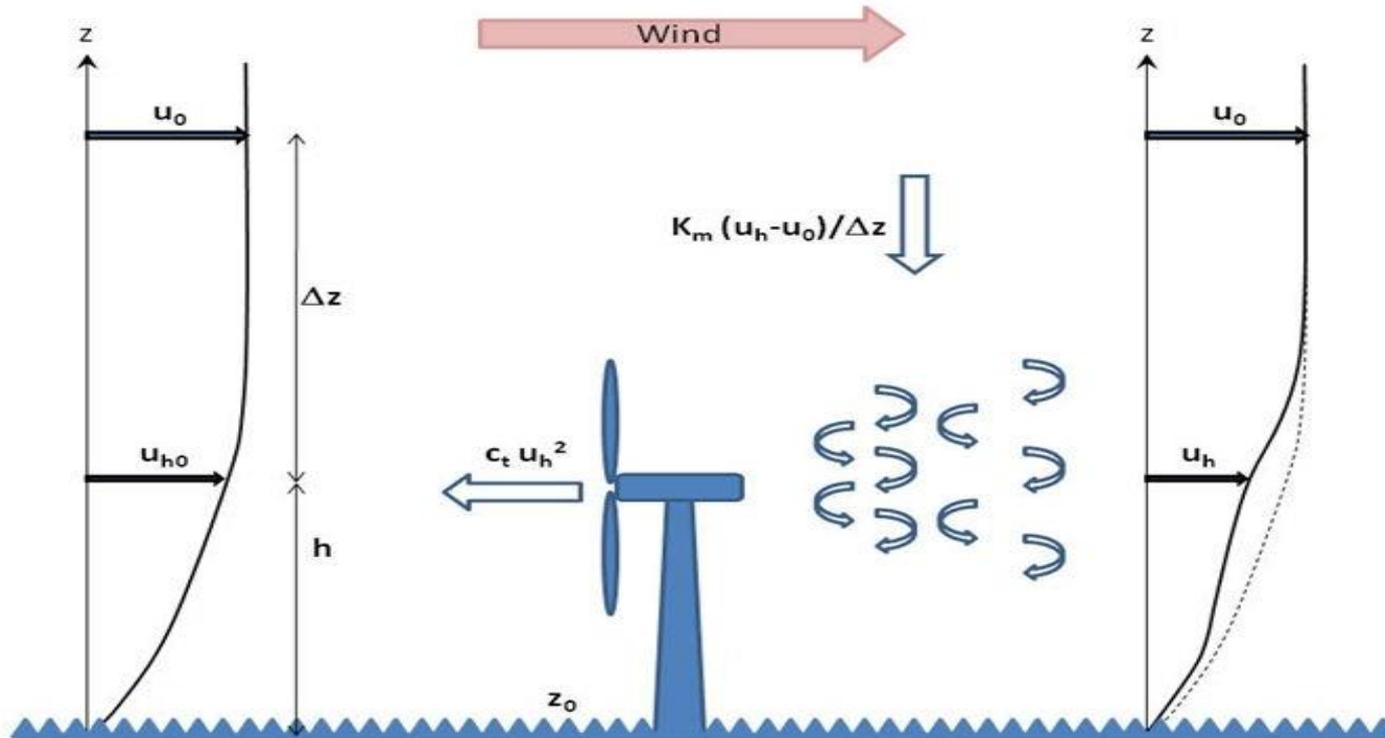
EFFWAKE is based on an **equilibrium** between momentum dissipation by WEA and re-supply by vertical turbulent fluxes from above

EFFWAKE is based on **Prandtl-layer** relations for the full WEA layer

EFFWAKE is **layout-independent** and wind direction-independent

EFFWAKE considers thermal stability of Prandtl layer and turbine-generated turbulence

# Short Characterization



# Short Characterization

farm efficiency (in terms of power)

$$R_p = R_t^3 = \frac{\left( \frac{\phi_m c_{s,h}}{\kappa^2} + f_z I_{ueff} \right)^3}{\left( \frac{\phi_m c_{teff}}{\kappa^2} + f_z I_{ueff} \right)^3}$$

recovery of wind speed in the wake

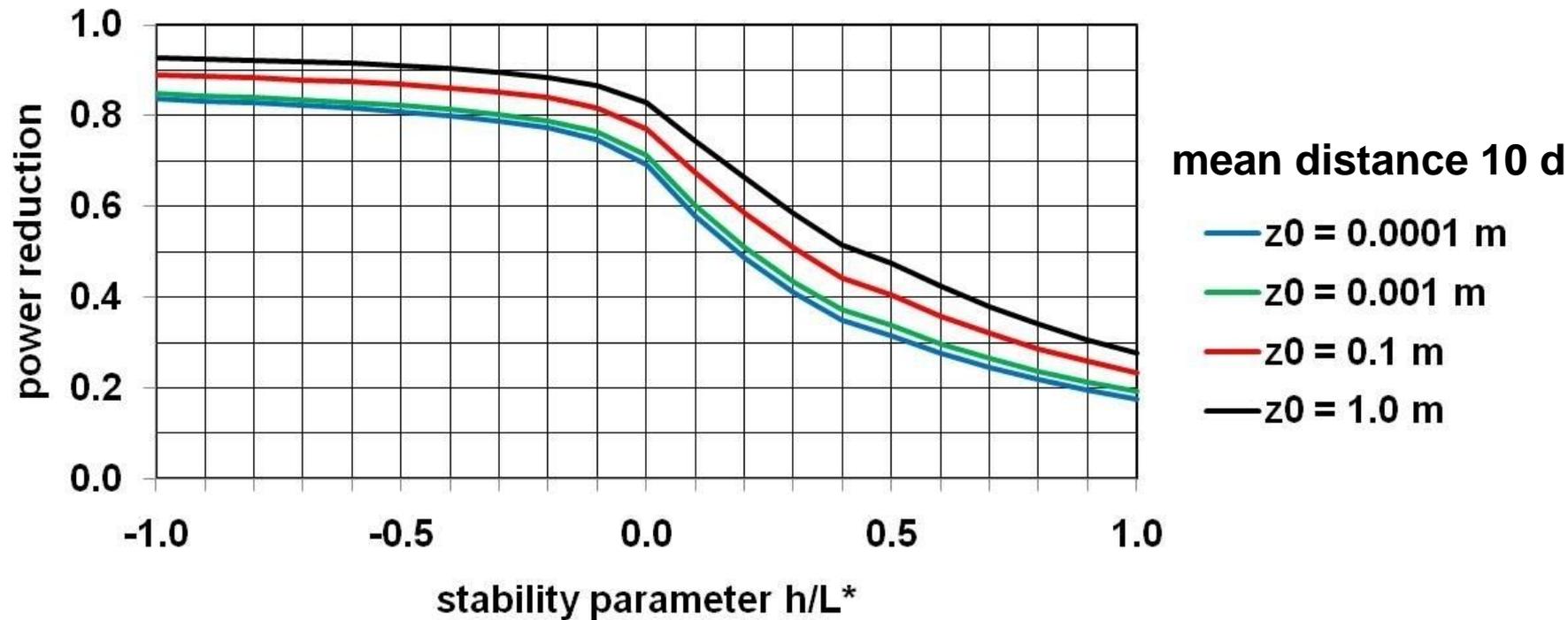
$$R_n(t) = \frac{u_{hn}(t)}{u_0} = 1 + \left( \frac{u_{uhn0}}{u_0} - 1 \right) \exp(-\alpha t)$$

(temporal → spatial domain)

$$R_{np} = R_n^3(x) = R_n^3(tu_{h0}) = 0.95$$

- **Newman (1977)**, wind speed at hub height as function of turbine density, wind farm considered as additional roughness
- **Bossanyi et al. (1980)**, extensive review on previous approaches, wind farm efficiency, wind speed-dependent turbine drag, assumptions on replenishment from above using a logarithmic profile, basis of all top-down models
- **Jensen (1983)**, redistribution of momentum in a single wake, basis of all bottom-up wake models
- **Frandsen (1992)**, first idea for this farm efficiency top-down model, two-layer model with geostrophic drag law in the outer layer used for the determination of the friction velocity

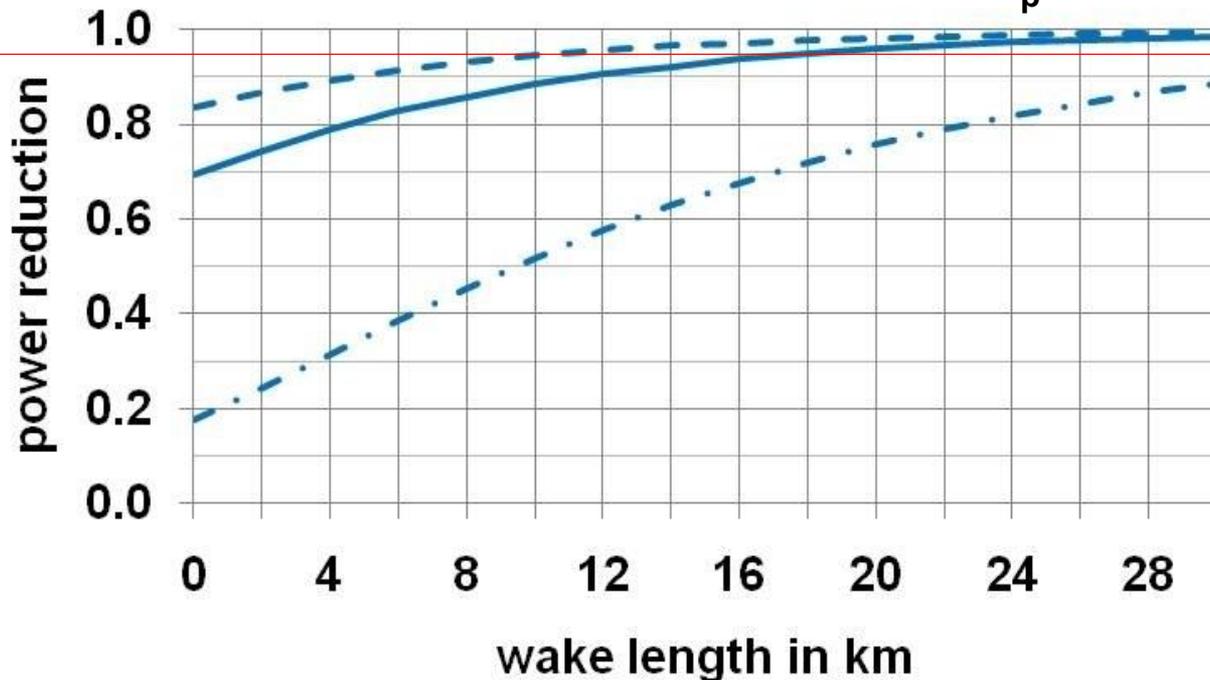
- **Emeis und Frandsen (1993)**, equations for farm efficiency for neutral stratification with no wind profile assumptions, ratio of mixing length to vertical distance to undisturbed flow is the only tuneable parameter (core idea of EFFWAKE)
- **Rooijmans (2004)**, enhancement of the E&F93 model for thermal stratification by making the mixing length stability-dependent using André (1978), comparison with MM5 shows very similar results
- **Emeis (2010)**, EFFWAKE, inspired by Rooijmans (2004) extension of the E&F93 model with surface drag and vertical exchange coefficient (instead of mixing length) **stability-dependent**, computes farm efficiency **and wake length**, without turbine-generated turbulence
- **Emeis (2018)**, EFFWAKE, second version with **turbine-generated turbulence** added



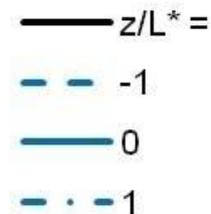
# Sample Results

## wake length

definition:  $R_p < 0.95$

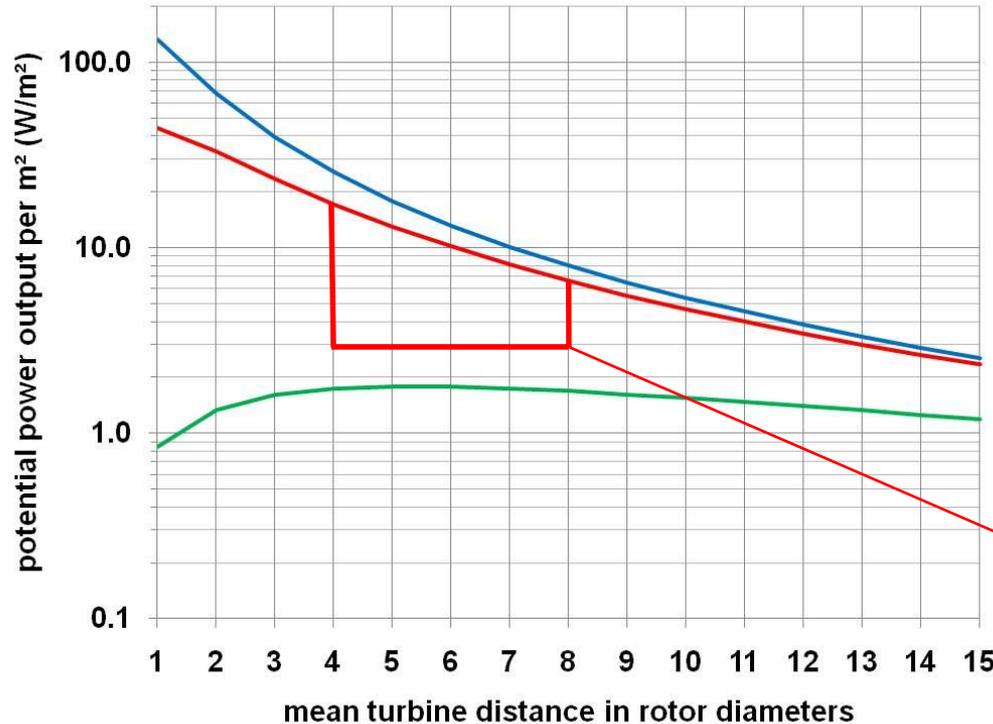


mean distance 10 d  
offshore ( $z_0 = 0.0001$  m)



# Sample Results

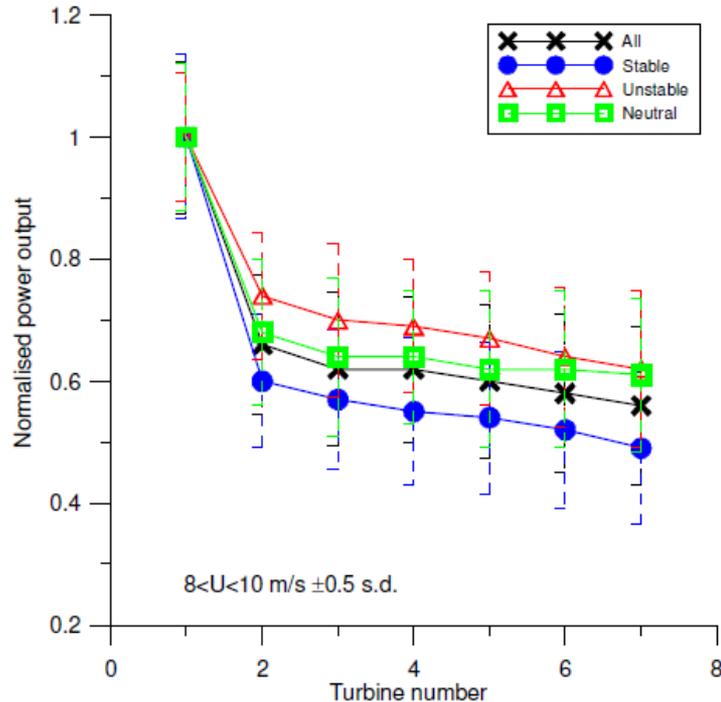
## optimum turbine density



offshore ( $z_0 = 0.0001$  m)

- ▶ halving the mean distance
- ➔ four times as much turbines
- ➔ yield increases by  $\sim 2.5$

# Validation



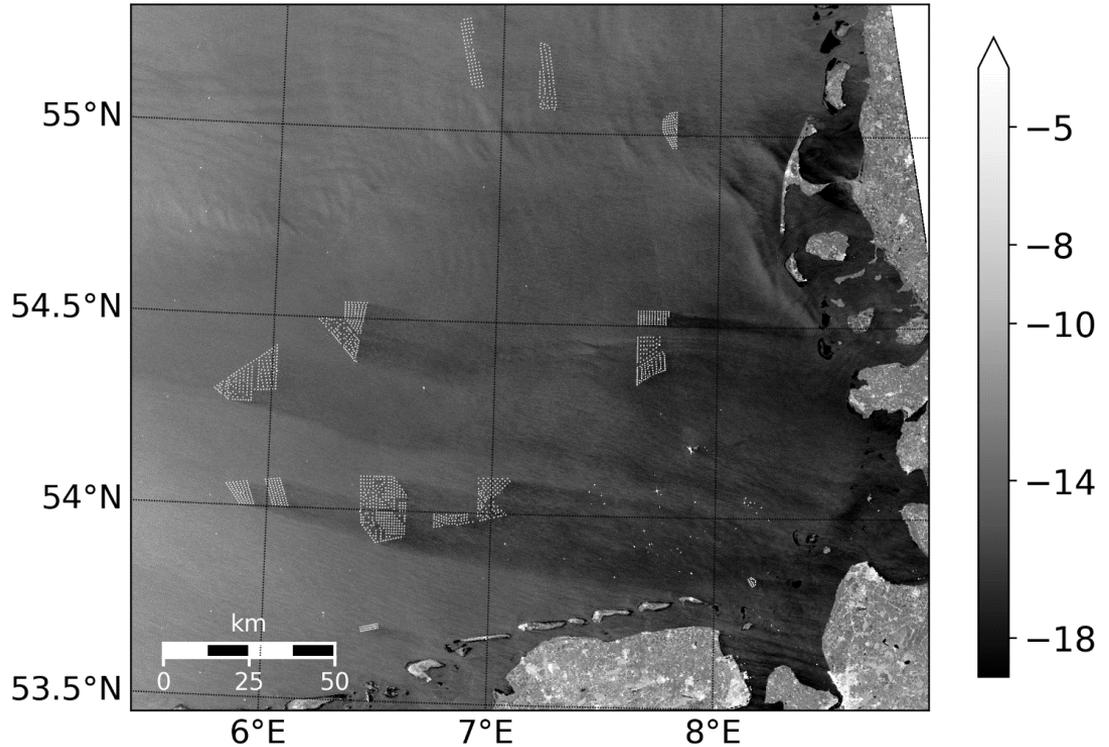
farm efficiency (in terms of power)

order of magnitude is correct even for rather small wind farms (Nysted: 7 by 7 turbines)

Barthelmie R, Frandsen ST, Rethore PE, Jensen L., 2007: Analysis of atmospheric impacts on the development of wind turbine wakes at the Nysted wind farm. Proceedings of the European Offshore Wind Conference, Berlin 4.-6.12.2007.

# Validation

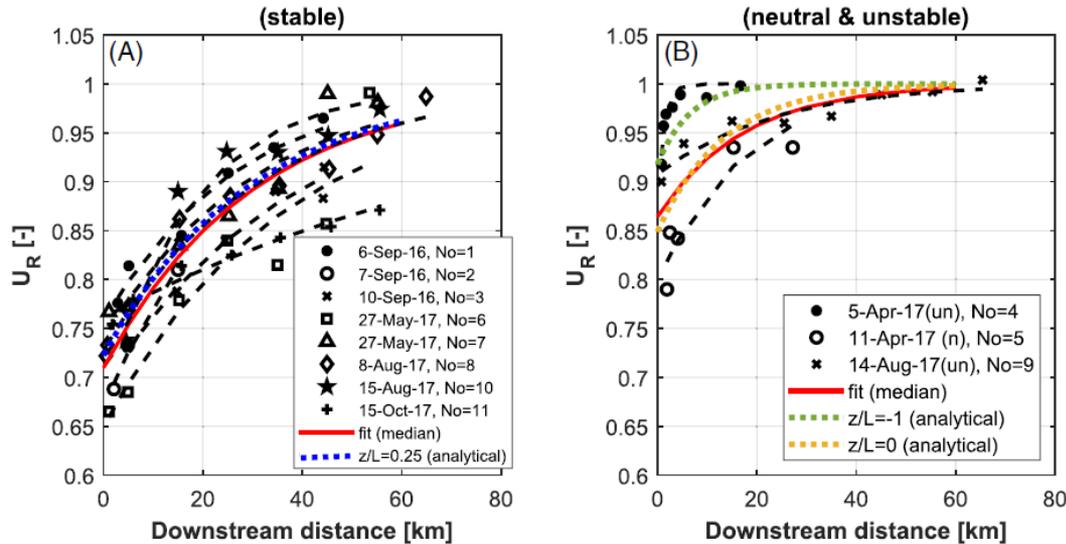
20200401 at 17:17 UTC (Copernicus 2020 data)



farm wakes in SAR satellite data

April 2020 ,  
produced by  
Bughsin Djath  
(Hereon) from  
Copernicus  
remote sensing  
data

(@) European  
Space Agency –  
ESA

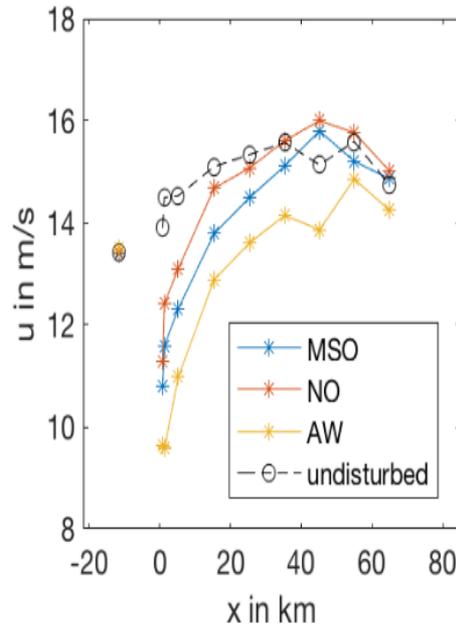


farm wakes from  
WIPAFF data (black  
symbols)

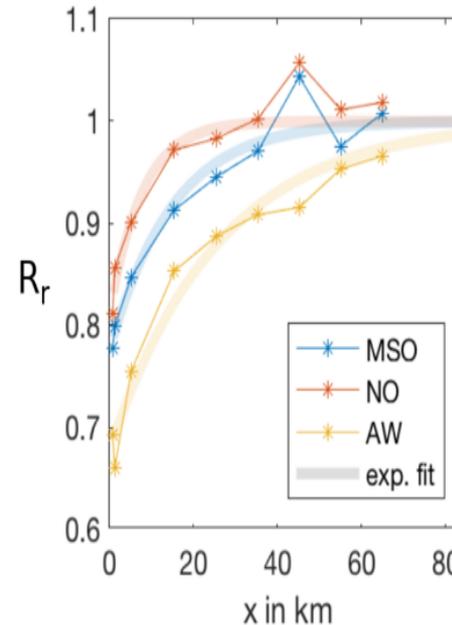
and EFFWAKE  
(dashed coloured  
curves)

**FIGURE 7** The wind speed recovery  $U_R(x)$  as a function of the downstream distance  $x$ . Each data point indicates the wind speed recovery  $U_R$  (Equation 5) at the minimum wake wind speed  $U_{\min}$  for each wake traversal at hub height ( $h \approx 100$  m) for stable A, and unstable/neutral B, stratification based on the stability criteria presented in Table 2. The black dashed curves are exponential fits to each case based on Equation (5), red curves correspond to the exponential function applied to the median values of the stable A, and unstable B, cases, and the dashed colored curves are calculated from the analytical model introduced in Section 2 for  $z/L = 0.25$  (A) and  $z/L = 0, -1$  (B) [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

**FIGURE 5** (A) Wind speed in the wake and undisturbed zone of Flight 31. The blue line indicates the wind speed in the wake downstream of the wind farm MSO, the red NO and the yellow one AW. The dashed black line indicates the wind speed of the undisturbed flow aside of the wake. (B) Relative wind speed deficit of Flight 31 [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]



(A)



(B)

farm wakes from  
WIPAFF data (lines  
and symbols)

and EFFWAKE  
(shaded curves)

EFFWAKE is a rapid assessment tool. Suitable chains of numerical models at different scales can yield higher precision (if sufficient input data is available).

(Financially) Optimum density of turbines in a large farm can be easily estimated with EFFWAKE when adding price tags to the turbines.

Turbines get larger which violates the assumption that wind turbines are fully immersed in the Prandtl layer.

Upstream blocking of larger farms is not (yet) considered.

Coastal effects (internal boundary layers) are not considered. Issue in the currently running project X-Wakes.

# Acknowledgements

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X-Wakes **FKZ 03EE3008**

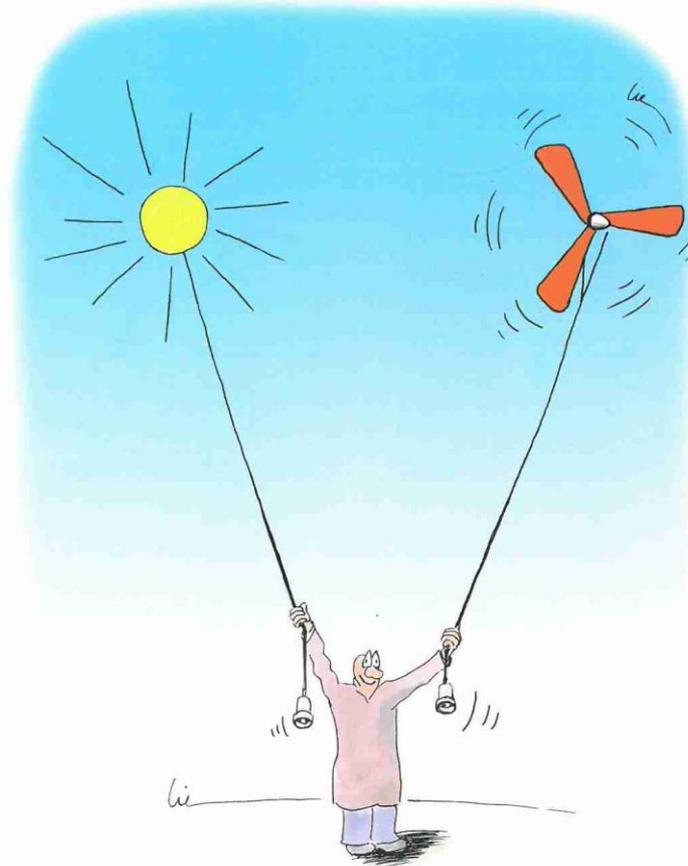
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and Energy

on the basis of a decision  
by the German Bundestag

**Thank you for  
your attention**



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Erik Liebermann