AUTOMATED CALIBRATION OF SMARTPHONE CAMERAS FOR 3D RECONSTRUCTION OF MECHANICAL PIPES

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Abstract

This paper outlines a new framework for the calibration of optical instruments, in particular smartphone cameras, using highly redundant circular black-and-white target fields. New methods were introduced for (i) matching targets between images; (ii) adjusting the systematic eccentricity error of target centres; and (iii) iteratively improving the calibration solution through a free-network self-calibrating bundle adjustment. The proposed method effectively matched circular targets in 270 smartphone images, taken within a calibration laboratory, with robustness to type II errors (false negatives). The proposed eccentricity adjustment, which requires only camera projective matrices from two views, behaved comparably to available closed-form solutions, which require additional a priori object-space target information. Finally, specifically for the case of mobile devices, the calibration parameters obtained using the framework were found to be superior compared to in situ *calibration for estimating the 3D reconstructed radius of a mechanical pipe (approximately 45% improvement on average).*

KEYWORDS: 3D reconstruction of pipes, circular target extraction and matching, ellipse eccentricity correction, smartphone camera calibration

INTRODUCTION: PHOTOGRAMMETRIC CALIBRATION

PHOTOGRAMMETRIC CALIBRATION OF OPTICAL instruments such as smartphone cameras is the process of modelling the effects of instrumental systematic errors in the acquired data. The procedure involves the estimation of the interior orientation parameters (IOPs) – and, in the case of self-calibration, the exterior orientation parameters (EOPs) – of the camera, given several point correspondences between two or more image views. Provided an adequate set of images and a well-distributed target field, camera calibration requires:

(1) an efficient procedure to define exact point correspondences between images; and
(2) an appropriate geometric camera model to describe the IOPs.

To find exact point correspondences, centres of circular targets, such as those shown in the calibration laboratory of Fig. 1, are almost exclusively utilised in high-precision close-range photogrammetry applications (Luhmann, 2014). Circular targets are geometrically approximated by ellipses in images. They offer several advantages such as a unique centre, invariance under rotation and translation, and low cost of production. As the number of targets, as well as images, increases, on the one hand the manual matching of corresponding targets becomes more tedious, time consuming and impractical. On the other hand, automated matching using only the geometric characteristics of ellipses, runs the risk of mismatches (type II errors, also termed false negatives or omission errors). Therefore, coded targets were recommended to reduce the effects of type II errors during target matching (Shortis and Seager, 2014). Coded targets, however, still require the design of a distinct unique identifier as well as manual labelling of targets individually, which may serve to be impractical in large target fields. A fully automated process to identify and match targets with minimal manual intervention is, hence, desirable for larger target fields with larger number of images.

In addition, even though the circular targets in object space are projected as ellipses in the image plane, the centres of the best-fitting ellipses in the images do not necessarily correspond to the actual centre of the circular target. This is commonly referred to as the eccentricity error, which is a consequence of the projective geometry. The systematic eccentricity error suggests that for two corresponding ellipses in two or more image views, the 3D reconstructed centres of the best-fitting ellipses, even in the absence of random or systematic errors, will not correspond to the centre of the original circular targets (Luhmann, 2014). Hence, the systematic eccentricity error in images must be corrected, especially in applications requiring high-precision metrology.

The other important consideration for camera calibration is the selection of the appropriate geometric camera model. Especially for new optical instruments, such as newly released smartphone cameras, the extent of the impact of the additional parameters, including the terms required to correct radial lens distortions, must be evaluated. In this study the goal is to model the IOPs and the requirements for metric calibration of 4K video recordings, acquired using
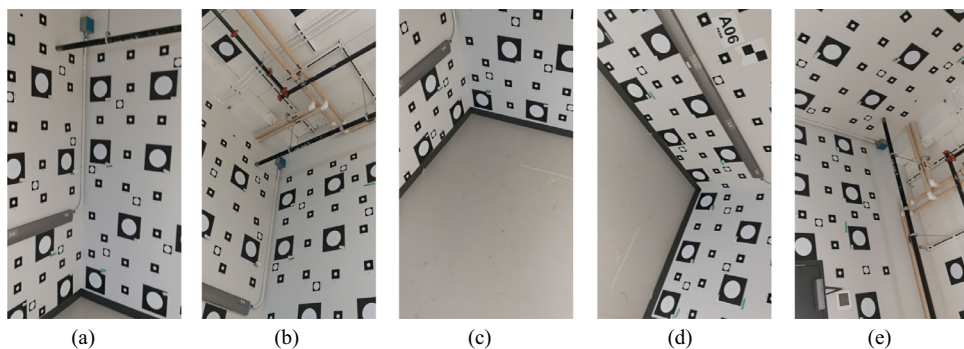

FIG. 1. Process to collect video data for calibration: (a) corner of the calibration laboratory; (b) bottom of the ladder facing the ceiling in portrait; (c) top of the ladder facing the floor in portrait; (d) top of the ladder facing the floor in landscape; and (e) bottom of the ladder facing the ceiling in landscape.

three of the latest smartphone cameras, namely, the iPhone 11, the Huawei P30 and the Samsung S10. Video recordings were utilised since the broader application of this study pertains to progress monitoring of mechanical pipes on construction projects, which is more practical using video recording (as opposed to single images). The real-world impact of the calibration parameters on estimating the geometric parameters of cylinders representing mechanical pipes, in support of as-built documentation of construction sites, will also be examined.

To this end, this study focuses on:

(i) developing an automated and robust method to detect and match circular targets between video images;
(ii) providing a simple approach to correct the eccentricity error;
(iii) identifying the geometric error modelling requirements of the mentioned smartphone cameras; and
(iv) determining the extent of the impact of the camera geometric modelling on the accuracy of a 3D reconstructed mechanical pipe.

LITERATURE REVIEW

The review of previous literature has been divided into two main categories: (i) matching conics between images; and (ii) geometric models for the calibration of smartphone cameras. These two are further explained in the following sections.

Matching Conics between Images

Given the camera matrices of two views, $\mathbf{P}$ and $\mathbf{P}'$, the necessary and sufficient conditions for matching conics is given by Δ as follows (Quan, 1996):

$$\begin{cases} \text{dev}(\mathbf{V}) = \det(A + \lambda \mathbf{B}) = I_1\lambda^4 + I_2\lambda^3 + I_3\lambda^2 + I_4\lambda + I_5 \\ \mathbf{A} = \mathbf{P}^T\mathbf{C}\mathbf{P} \\ \mathbf{B} = \mathbf{P}'^T\mathbf{C}'\mathbf{P}' \\ \Delta = I_3^2 - 4I_2I_4 = 0 \end{cases} \tag{1}$$

where $\mathbf{C}$ and $\mathbf{C}'$ are the conic's algebraic matrices corresponding to views $\mathbf{P}$ and $\mathbf{P}'$; $\mathbf{V}$ is the characteristic polynomial of matrices $\mathbf{A}$ and $\mathbf{B}$; $I_j : j = 1 \cdots 5$ are the coefficients of the determinant of $\mathbf{V}$, $\det(\mathbf{V})$; and $(.)^T$ denotes a matrix transpose. Equation (1) shows that, for two matching conics between two views, Δ is equal to zero (or very close to zero in the presence of measurement errors). The problem of automated matching of conics between two images, hence, requires the automated: (i) detection of conics; and (ii) determination of camera matrices. A comprehensive discussion of available and novel methods for detecting non-overlapping ellipses from images was given in Maalek and Lichti (2021b). The remainder of this section, therefore, focuses on the latter requirement, namely automated methods of recovering camera matrices, given only point correspondences.

Recovering Camera Matrices. Fundamental matrices are widely implemented in computer vision to provide an algebraic representation of epipolar geometry between two images. Given a sufficient set of matching points (at least seven), the fundamental matrix ($\mathbf{F}$) can be estimated (Hartley and Zisserman, 2004). An important property of fundamental matrices is that the estimated $\mathbf{F}$ between two views is invariant to projective transformation

of the object space. Therefore, the relative camera matrices can be obtained directly by the fundamental matrix up to a projective ambiguity (Luong and Viéville, 1996). If the camera model is assumed unchanged between two views, the reconstruction is possible up to an affine ambiguity. In case an initial estimate of the IOPs is available, the fundamental matrix can be decomposed into the essential matrix, $\mathbf{E} = \mathbf{K}^{\mathrm{T}}\mathbf{FK}$, where $\mathbf{K}$ represents the matrix of intrinsic camera parameters (IOPs). Given matrix $\mathbf{K}$, the relative orientation between two cameras can be retrieved through singular-value decomposition of the essential matrix (Hartley and Zisserman, 2004) with only five point correspondences (Stewénius et al., 2006). In such cases, it is possible to recover the reconstruction up to a similarity transformation (an arbitrary scale factor). The two-view process can also be extended to multiple image views. In fact, the camera matrices of m images can be recovered, given at least $m - 1$ pairwise fundamental matrices and epipoles, using the projective factorisation method of Sturm and Triggs (1996). The latter is an example of a global reconstruction framework. In practice, however, a sequential reconstruction and registration of new images typically produces more reliable results, due to the flexibilities and control inherent in the incremental improvement of the reconstruction solution (Schönberger, 2018).

Sequential Structure from Motion (SfM). Structure from motion is the process of retrieving camera IOPs and EOPs subject to rigid-body motion (namely rotation and translation (Ullman, 1979)). An overview of a typical sequential SfM comprises the following steps:

(1) Detect and match features between every pair of images and determine overlapping images. The point correspondences are typically obtained automatically using established computer vision feature extraction and matching methods such as the scale-invariant feature transform (SIFT: Lowe, 2004), speeded-up robust features (SURF: Bay et al., 2008), or their variants.
(2) Start with an initial pair of images (typically the two images obtaining the highest score for some geometric selection criterion; see Schönberger (2018).
(3) Estimate the relative orientation and camera matrices between the two images using corresponding feature points. Note that an initial estimate of the IOPs is typically required for this stage.
(4) Triangulate to determine 3D coordinates of the corresponding points.
(5) Perform a bundle adjustment to refine IOPs and EOPs.
(6) For the remaining images, perform the following:

    (a) Add a new overlapping image to the set of previous images.
    (b) Estimate the relative orientation parameters of the new image from the existing overlapping feature points.
    (c) Using step 1, determine the new matching feature points and triangulate them to estimate the object-space coordinates of the new feature points.
    (d) Perform a bundle adjustment to refine the IOPs and EOPs.
    (e) Perform steps 6(a) through (d) until all images are examined.

The output of SfM is a set of EOPs, IOPs and image feature points, as well as sparse object-space point clouds. Several approaches, as well as software packages, exist that perform different variants of SfM. In this study, COLMAP, an open-source software package comprised of many computational and scientific improvements to traditional SfM methods, as documented in Schönberger (2018), was utilised.

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Geometric Models for Smartphone Camera Calibration

Smartphone cameras can be considered as pinhole cameras for which the collinearity condition can be used to model the straight-line relationship between an observed image point (x, y), its homologous object point (X, Y, Z) and the perspective centre of the camera (X^c, Y^c, Z^c), as described in Luhmann et al. (2014). Random error departures from the hypothesised collinearity condition are modelled as additive, zero-mean noise terms $(\varepsilon_x, \varepsilon_y)$ while $(\Delta x, \Delta y)$ represent systematic error correction terms. The latter comprise the models for radial lens distortion and decentring distortion. Radial lens distortion, which is by far the larger of the two distortions, is most often modelled with three terms of the standard polynomial (such as Luhmann et al., 2016), though higher-order terms have been demonstrated to be required for wide-angle lenses (Lichti et al., 2020). Images collected with modern smartphone cameras are generally corrected for radial lens distortion. However, the extent of the correction and the metric impact, if any, on object-space reconstruction is not known and a subject of this investigation.

METHODOLOGY

The proposed method for metric calibration of smartphone cameras is formulated as follows:

(1) *Calibration data collection*: which involves the method for data collection from the calibration laboratory (Fig. 1).
(2) *Circular target centre matching*: which consists of the following stages:

   (a) Estimation of camera projective matrices (Fig. 2(a)).
   (b) Automated ellipse detection from images (Fig. 2(b)).
   (c) Automated ellipse matching between images (Fig. 2(c)).
   (d) Correction of ellipse eccentricity error (Fig. 2(d)).

(3) *Self-calibrating bundle adjustment*.

Each section is introduced in more detail in the following sections.

Calibration Data Collection

This study focuses on the calibration of smartphone cameras using video sequences. To improve the precision of the self-calibration and prevent projective compensation (coupling), the recordings must: (i) capture depth variation in the target field; (ii) be convergent; and (iii) rotate 90° about the camera's optical axis (thus both landscape and portrait images). One corner of a complete calibration laboratory was utilised (Fig. 1(a)), which consists of multiple targets attached to two right-angled intersecting walls. The two intersecting planar walls were utilised to create depth variation in the target field. A ladder was used to collect convergent images of the scene, starting from the bottom of the ladder facing the ceiling (Fig. 1(b)) and ending at the top of the ladder facing the floor (Fig. 1(c)). At the top, the camera was rotated 90° and the video was then recorded in the reverse order (Figs. 1(d) and (e)).

Circular Target Centre Matching

The problem of circular target matching for calibration requires: (i) estimation of camera projective matrices (Fig. 2(a)); (ii) automated detection of ellipses from each image
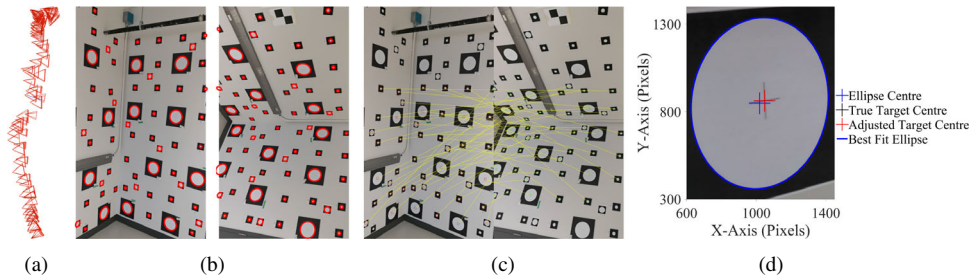
FIG. 2. Output of the proposed steps to acquire matching target centres between images: (a) estimated camera position and orientation (EOPs) using Algorithm 1; (b) detected ellipses using the method of Maalek and Lichti (2021a) for two images; (c) automatic matching of corresponding target centres using Algorithm 2; and (d) correction of the estimated target centre in image plane using Algorithm 3.

(Fig. 2(b)); (iii) correct matching of corresponding ellipses between different images (Fig. 2(c)); and (iv) correction of the eccentricity error of the ellipses' centres (Fig. 2(d)). These steps are discussed in more detail below.

Estimating the Camera Projective Matrices. Based on the discussions above, an initial estimate of the camera projective matrices (comprised of IOPs and EOPs) can be obtained using an SfM framework, such as COLMAP. Here, Algorithm 1 is proposed to further improve the estimated camera matrices, retrieved from the outputs of COLMAP's sparse reconstruction:

ALGORITHM 1*:* Refining Camera Matrices

(1) Determine the features used for sparse reconstruction in images from COLMAP.

(2) Between every two images with overlapping features, $i \in 1 \dots (n-1)$ and $j \in (i+1) \dots n$, where n is the total number of images, perform the robust least median of squares (LMedS; (Rousseeuw and Leroy, 1987)) fundamental matrix estimation, using the subsample bucketing strategy of Zhang (1998), to retrieve the inlier features and fundamental matrix.

(3) For the inlier matching feature points of step 2, perform the robust triangulation using LMedS with random subsampling. Identify the inlier feature points for every given sparsely reconstructed point.

(4) Perform the bundle adjustment only on the inlier features of steps 2 and 3 (thus weighting all outlying features as zero) to retrieve the refined EOPs and IOPs.

As a point of reference, Fig. 2(a) illustrates the estimated camera positions and orientations of a sequence of images collected with the proposed strategy using Algorithm 1. Fig. 3 shows the refinement of the matched features using Algorithm 1. As observed, two mismatched features, represented by red and green circles, were correctly removed. The red circle indicates a feature point that did not satisfy step 2 of Algorithm 1, whereas the
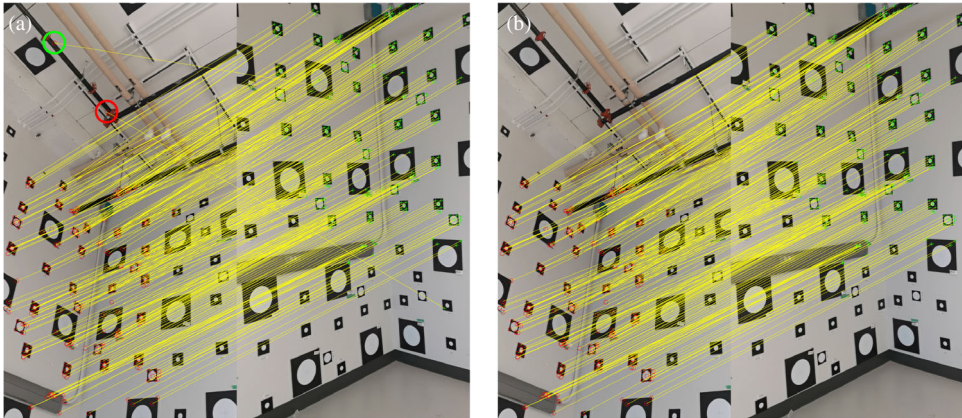
FIG. 3. Matched features between images. (a) Results from COLMAP. (b) Results of refinements using Algorithm 1. Note that the two mismatched features, represented by red and green circles in (a), were correctly removed in (b).

mismatched feature indicated by the green circle was removed using step 3. In this example, even though the refinement is marginal, two mismatches out of around 300 correct matches can still negatively affect the results of the estimated IOPs and EOPs, especially when most images contain mismatches.

Automated Ellipse Detection from Images. The robust ellipse detection method presented in Maalek and Lichti (2021b) is used to detect non-overlapping ellipses of the projected circular targets. The method was shown to provide ellipse detection with superior robustness to both type I errors (false positives; commission errors) and type II errors (false negatives; omission errors), compared to the established ellipse detection methods of Fornaciari et al. (2014) and Pătrăucean et al. (2017). Once the ellipses are detected, the best-fitting geometric parameter vector of each ellipse is estimated using the new confocal hyperbola ellipse fitting method, presented in Maalek and Lichti (2021a). The geometric parameters of the ellipse (centre, semi-major length, semi-minor length and rotation angle) are then converted to algebraic form, which are then transformed into matrix form to be used for conic matching through equation (1).

Automated Ellipse Matching between Images. Equation (1) is utilised here to verify possible matching conics between two images. A brute-force matching strategy suggests checking the correspondence condition, Δ of equation (1), for every detected ellipse of an image to all ellipses of all other images, which can become computationally expensive with a larger number of images with many targets. In addition, Δ calculated using equation (1) is not necessarily equal to zero in the presence of systematic and random measurement errors (Quan, 1996). Therefore, a threshold is required on $|\Delta|$ to model the matching uncertainties in the presence of measurement errors. An arbitrarily selected threshold for the correspondence condition Δ will, however, almost guarantee either mismatches (type II errors) or no matches (type I errors).

Here, instead of comparing all conics together in a brute-force fashion, first, only a select set of candidates are considered, whose centres in both images satisfy an adaptive

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closeness constraint on the corresponding epipolar distance. Amongst the available ellipses satisfying the epipolar constraint, that achieving the smallest $|\Delta|$ is chosen as the matching conic. This process is attractive for two reasons. First, since the robust fundamental matrix between two images is already computed automatically using the LMedS method in Algorithm 1 (step 2), the agreement of the inlier points to the estimated fundamental matrix can be used as a basis to compute the threshold for the epipolar constraint on the centres as well (no predefined subjective threshold is required). Second, the ellipse matching using the correspondence condition of equation (1) requires no threshold. To formulate the proposed process, Algorithm 2 is provided as follows:

Step 4(b) of Algorithm 2 performs the pairwise conic matching condition on only a selected set of inlier ellipse centres that lie near the epipolar line. Furthermore, instead of choosing an arbitrary threshold for Δ, the minimum of $|\Delta|$ is used. To further reduce the effects of type II errors (mismatching targets), the robust triangulation using LMedS is performed.

Correction of Ellipse Eccentricity Error. Modelling the eccentricity error for circular targets has been the subject of investigation, especially in high-precision metrology (Ahn et al., 1999; He et al., 2013; Luhmann, 2014). Closed formulations of the eccentricity error in both the image plane and object space exist (Dold, 1996; Ahn et al., 1999). The available correction formulations for the errors in the image plane, however, require the knowledge of the target's object-space parameters, such as the radius of the target, the object-space coordinates of the centre and the normal vector of the circular target's plane. This information, however, cannot be trivially retrieved. Even if a reliable external measurement of the object space exists, the additional constraint will be undesirable in free-network self-calibration practices, which is the focus of this study. In the following, a process is presented to retrieve the projection of the true centre of the circular target onto the image, given only the matching target parameters in two views.

Following the formulation of equation (1), Quan (1996) proposed a process to acquire the equation of the object-space plane where the conic lies, given two camera projection matrices and the conics' algebraic matrices in the image plane. The subsequent object-space conic's equation for the matching conics can then be retrieved by intersecting the plane with the cone's equation (see equation (1)). Ideally, the object-space conic should be represented by a circle in the case of circular targets, however, due to measurement errors and uncertainties in the estimation of the projection matrices, the object-space conic might be an ellipse. The object-space centre of the ellipse (or circle) can be directly extracted from the retrieved conic's equation. The corrected centre in the image can, hence, be extracted by back projecting the object-space centre of the retrieved ellipse onto the image planes. Since multiple views of a given target may be imaged, the final consideration is to determine the best view for a particular target. To this end, two criteria are used: (i) the convergence angle between the two views; and (ii) the uncertainty of the object-space centre estimation. First, for a given view, only the image views with an average convergence angle of more than 20° (Schönberger, 2018) are considered. Second, for a specific matched target, the uncertainty of the object-space centre estimation is characterised, here, as the covariance of the 3D reconstructed ellipse centres between a given view and another acceptable view. The covariance is estimated using the formulation provided by Beder and Steffen (2006). The two views achieving the minimum determinant of the covariance matrix (MCD) – representing the pair with the minimum uncertainty – are selected as the ideal candidate (Rousseeuw and Leroy, 1987). Intuitively, the latter selection criterion provides the pair of images whose object-space coordinates in the vicinity of the true centre are the least

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ALGORITHM 2: Automated Matching of Ellipses

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(1) Estimate the camera projective matrices using Algorithm 1.

(2) Detect the ellipses using the robust non-overlapping ellipse detection of Maalek and Lichti (2021a).

(3) For each detected ellipse, find the best-fitting ellipse using the method of Maalek and Lichti (2021b) and construct the equivalent algebraic conic matrix.

(4) For every pair of overlapping images, $i \in 1 \ldots (n-1)$ and $j \in (i+1) \ldots n$, where n is the total number of images, find the matching ellipses as follows:

    (a) From the LMedS algorithm of step 1:

        (i) Use the solutions obtained for the fundamental matrix, $\mathbf{F_{ij}}$.

        (ii) Calculate the standard deviation of the epipolar distance (Hartley and Zisserman, 2004), σ_{ij}, of the inlier matches.

    (b) For each ellipse centre in image i, c_{ik}, find the ellipse centres in image j, c_{jl}, that satisfies the following condition:

$$D\left(c_{ik}, c_{jl}\right) = \sqrt{d\left(c_{jl}, F_{ij}c_{ik}\right)^2 + d\left(c_{ik}, F_{ij}^T c_{jl}\right)^2} \leq \sigma_{ij}\sqrt{\chi^2_{0.975,2}} \qquad (2)$$

where $D\left(c_{ik}, c_{jl}\right)$ is the epipolar distance between image points $\left(c_{ik}, c_{jl}\right)$, and $\chi^2_{0.975,2}$ is the chi-squared cumulative probability distribution function with probability 97.5% and 2 degrees of freedom for 2D data.

    (c) For all ellipse candidates satisfying equation (2) between two images, perform the following:

        (i) Select all related matching ellipses between the two images. For instance, if ellipse labels 5 and 6 of image j satisfy equation (2) for the ellipse labelled 4 of image i, all other ellipses of image i that satisfy equation (2) for ellipses labelled 5 and 6 of image j should also be selected, and so on.

        (ii) Calculate Δ from equation (1) between the ellipse candidates (from the previous step) of images i and j.

        (iii) Two ellipses are considered to be matching if and only if both ellipses achieved the minimum $|\Delta|$ for each other. For instance, if the ellipse labelled 5 of image j achieved the smallest $|\Delta|$ for label 4 of image i, the two will only be matched if the ellipse labelled 4 of image i achieves the smallest $|\Delta|$ for label 5 of image j.

(5) Using the matched ellipses of step 4 in two views (image pairs), determine the matching ellipse between all other images.

(6) Using the multiview correspondences identified in step 5, for each ellipse perform the following steps to reduce the impact of type II errors (mismatches):

    (a) Use the camera projective matrices estimated in step 1.

    (b) Perform a robust multiview triangulation on the ellipse centres using LMedS (or any other preferred robust method).

    (c) Retain only the inlier set of corresponding ellipse centres.

(7) The inlier ellipse correspondences are the final set of matched ellipses.

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uncertain. The process is formulated using Algorithm 3 for each identified target with two or more image view correspondences, as follows:

The results of Algorithm 3 are fed to a free-network self-calibrating bundle adjustment that estimates the EOPs, IOPs and object-space coordinates of the targets. Once the EOPs and IOPs are determined, the centres can again be adjusted and recursively fed into the bundle adjustment to refine the results up to the required/satisfactory precision. Furthermore, Algorithm 3 can be utilised before the robust triangulation step of Algorithm 2 (after step 4) so that possible correct matches are not incorrectly rejected (improving type I errors in target matching). Fig. 4 illustrates the results of using Algorithm 3 within Algorithm 2. In this example, one additional target was correctly matched (amongst a total of 23 overlapping targets) when applying Algorithm 3 within Algorithm 2.

Self-calibrating Bundle Adjustment

The final step of the algorithm is the self-calibrating bundle adjustment, which is the accepted standard methodology for obtaining the highest accuracy (Luhmann et al., 2016). Provided that the aforementioned design measures have been incorporated into the imaging network, the IOPs will be successfully decorrelated from the EOPs and, thus, recovered accurately. Any outlier observations have been successfully removed by this stage of the algorithm, so a least-squares solution can be utilised for the parameter estimation. The IOPs (which are considered network-invariant – one set per camera), EOPs and object points are estimated. The singularity of the least-squares normal-equations matrix caused by the datum defect is removed by adding the inner constraints (free-network adjustment (Luhmann et al., 2014)) since this yields optimal object point precision.

Following the self-calibration adjustment, the solution quality can be examined with several computed quantities. Most important among these are the estimated IOPs and their precision estimates, together with derived correlation coefficient matrices that quantify the success of the parameter decorrelation. The residuals are crucial for graphically and statistically assessing the effectiveness of the lens distortion modelling. The presence of unmodelled radial lens distortion, for example, can be readily identified in a plot of the radial component of the image point residuals as a function of radial distance from the principal point. Moreover, reconstruction accuracy in object space can be quantified by comparing the photogrammetrically determined coordinates of (or derived distances between) targets with reference values from an independent measurement source.

Summary of Methods

The proposed camera calibration framework can be summarised as follows:

(1) Determine an initial estimate of the EOPs and IOPs using available SfM methods or software packages (here, COLMAP was used).
(2) Refine the estimated EOPs and IOPs to calculate the modified camera projective matrices for each image view using Algorithm 1.
(3) Determine the ellipses in each image using the robust non-overlapping ellipse detection of Maalek and Lichti (2021b).
(4) Match overlapping ellipses between all views using Algorithm 2.
(5) Adjust the eccentricity error of the estimated ellipse centres of all matched ellipses using Algorithm 3.

ALGORITHM 3: Correcting Eccentricity Error

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(1) Retain the object-space coordinate of the target's centre from Algorithm 2.

(2) Identify all images corresponding to the considered target, say images $1 \dots m$, where m is the number of views corresponding to a given target.

(3) For each image $i \in 1 \dots m$, find the best image pair in $j \in 1 \dots m, i \neq j$, (image pair with the least uncertainty in the estimated target's centre) as follows:
   (a) Find all images in $j \in 1 \dots m, i \neq j$ whose average convergence angle from image i is more than $20°$ (Schönberger, 2018).
   (b) Estimate the covariance matrix of the 3D reconstructed ellipse centres of the views obtained by the previous step using Beder and Steffen (2006).
   (c) For image i, find the corresponding pair whose determinant of the covariance matrix is minimum (MCD).
   (d) Repeat the steps 3(a) to 3(c) to find a best image pair for all images $1 \dots m$.

(4) For image k with the selected best image $\{k \neq l \mid k, l \in 1 \dots m\}$, perform the following steps to retrieve the corrected ellipse centre in image k:
   (a) Determine the object space plane parameters, $\mathbf{T}_{kl} = (t_1, t_2, t_3, t_4)^{\mathrm{T}}$, of the conic in space using the method of Quan (1996).
   (b) Intersect plane $\mathbf{T}_{kl}$ with the cone $\mathbf{A}_k$ (for image k) to find the conic equation as follows ($\mathbf{A}$ explained in equation (1)):
      (i) Parametrically derive the Z coordinate as a function of X and Y using the planes' equation as follows:
      $$Z = -\frac{t_1 X + t_2 Y + t_4}{t_3} \tag{3}$$
      (ii) Substitute Z into the cone's equation to derive the object-space conic matrix of the ellipse, $\mathbf{C}_k$, as follows:
      $$[X \ \ Y \ \ Z \ \ 1] \, \mathbf{A}_k \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \xrightarrow{\text{Substitute } Z} [X \ \ Y \ \ 1] \, \mathbf{C}_k \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = 0 \tag{4}$$
      (iii) Calculate the geometric centre (X_c, Y_c) of the ellipse corresponding to the conic matrix $\mathbf{C}_k$ as follows:
      $$X_c = \frac{\det \mathbf{C}_{\mathbf{k31}}}{\det \mathbf{C}_{\mathbf{k33}}} \quad , \quad Y_c = -\frac{\det \mathbf{C}_{\mathbf{k32}}}{\det \mathbf{C}_{\mathbf{k33}}} \tag{5}$$
      where $\mathbf{C}_{k_{ij}}$ is the 2×2 matrix constructed after removing row i and column j from $\mathbf{C}_k$.
      (iv) Substitute (X_c, Y_c) into equation (3) to calculate the Z component of the object-space 3D coordinates of the centre of the ellipse $\mathbf{X}_k = (X_c, Y_c, Z_c)$.
      (v) Project the centre of the conic back onto the image plane to find the corrected centre, $\mathbf{x}_k = \mathbf{P}_k \mathbf{X}_k$, where $\mathbf{P}_k$ is the camera projective matrix for image k.
      (vi) Convert the estimated centres into homogeneous coordinates for use in the bundle adjustment.

(5) Once the target's centre in all images has been corrected, perform triangulation to correct the object-space coordinates of the centre.
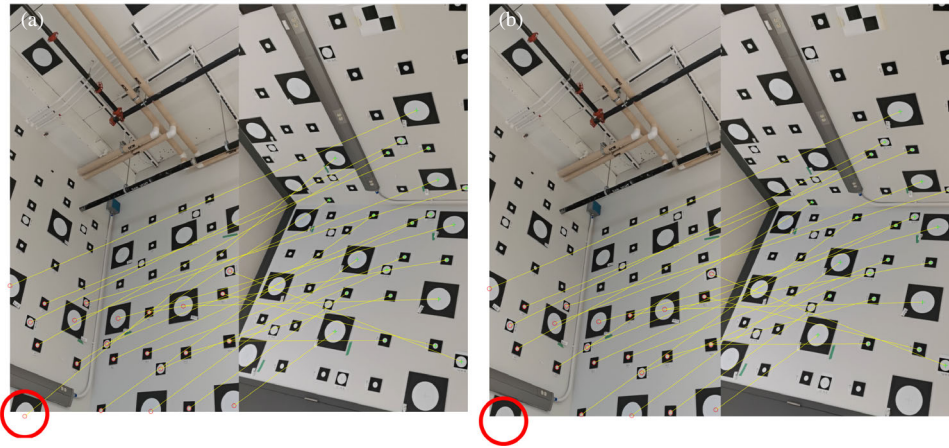

Fig. 4. Impact of centre correction on the results of the target matching between two sample images: (a) without centre correction; and (b) with centre correction.

(6) Perform the proposed free-network self-calibrating bundle adjustment to estimate the EOPs and IOPs.

(7) Perform steps 4 through 6 with the new EOPs and IOPs until the sets of matched ellipses between two consecutive iterations remain unchanged.

The final set of IOPs is the solution to the calibration.

Selection of Terms for Radial Lens Distortion

The accuracy of 3D reconstruction from a camera system affected by lens distortions can be significantly impacted by the choice of systematic error correction terms included in the augmented collinearity condition. The aim is to find a trade-off between goodness-of-fit and allowable bias. One must avoid an underparameterised model with an insufficient number of terms to describe the distortion profile that can lead to the propagation of bias into other model parameters and optimistic parameter precision. On the other hand, overparameterisation by adding more terms than necessary can introduce correlations among model variables that can inflate the condition number of the normal-equations matrix and, in turn, degrade reconstruction accuracy. To this end, a process similar to that described in Lichti et al. (2020) was employed. Using the final set of matched targets, an initial self-calibration solution was performed without any lens distortion parameters. The interior geometry of the camera in this adjustment was described only by the principal point and principal distance. Graphical analyses of the estimated residuals, supported with statistical testing and information criteria, were utilised to make an informed decision about the coefficients to be added. In particular, the radial component of the image point residuals, v_r, plotted as a function of radial distance from the principal point, r, was graphically assessed. A single parameter was then added to the model and the self-calibration was recomputed. The process was repeated until no systematic trends remain.

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Resulting Smartphone Calibration Parameters and Conditions

A total of 1.5 minutes of 4K videos at 30 frames per second (fps) was recorded as per the presented data collection method above using the Huawei P30, iPhone 11 and Samsung S10 smartphones. The portion of the calibration laboratory used in this study contained 130 targets. The recorded videos using each smartphone instrument were captured so as to fill the frame with the same 130 targets for consistency. The Open Camera app was utilised, where possible, to help access raw smartphone camera configurations, such as adjusting the focus to infinity, disabling autofocus and fixing the camera exposures. From the 1.5 minutes of video footage, 90 image frames at 1 fps were decomposed: these were used within the proposed frameworks of Algorithms 1 to 3 to calibrate the smartphone cameras. The convergence angles were 80°, 88° and 109° for the P30, iPhone 11 and S10 networks, respectively. The final set of IOPs, including the selected terms for correction of radial lens distortion, are shown in Table I.

Method of Validation of Results

The effectiveness of Algorithm 2 requires the quantification of the quality of ellipse matching between different images. Here, the four main metrics commonly used to determine the quality of object-extraction algorithms (precision, recall, accuracy and F-measure: Olson and Delen, 2008), were utilised:

$$
\begin{cases}
Precision = \dfrac{T_P}{T_P + F_P} \\[2ex]
Recall = \dfrac{T_P}{T_P + F_N} \\[2ex]
Accuracy = \dfrac{T_P + T_N}{T_P + T_N + F_P + F_N} \\[2ex]
F - measure = 2\dfrac{Precision \times Recall}{Precision + Recall}
\end{cases}
\tag{6}
$$

where T_P, T_N, F_P, F_N are the number of true positives, true negatives, false positives and false negatives of the object-extraction algorithm, respectively. To measure the accuracy of estimated parameters, such as centre adjustment in Algorithm 3, the Euclidian distance (or L2 norm) of the estimated parameters from the final ground-truth parameters were used. The ground truth in each experiment was determined manually.

TABLE I. Summary of the estimated IOPs for each smartphone using the proposed methodology.

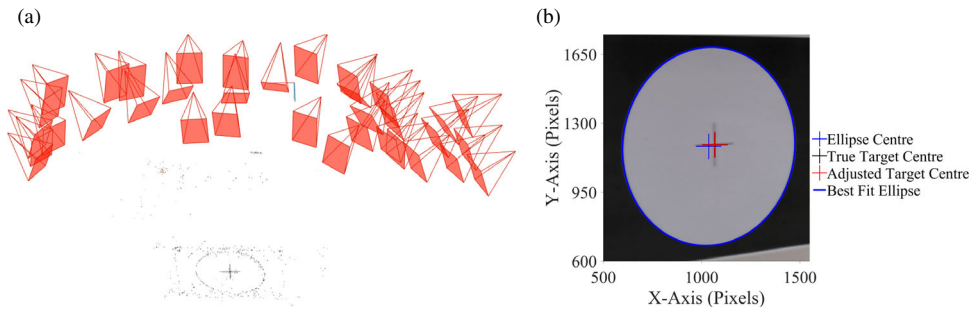| Device	Principal point (px)	Principal distance (px)	Radial distortion parameters				
			k1	k2	k3	k4	k5
Huawei P30	(1086.7, 1915.2)	3762.3	1.9E−08	−2.1E−14	1.1E−20	−2.5E−27	2.0E−34
$\hat{\sigma}$	(0.2, 0.3)	0.9	4.6E−10	5.7E−16	3.1E−22	7.4E−29	6.5E−36
iPhone 11	(1087.9, 1896.3)	3411.4	8.9E−09	−1.4E−15	4.4E−23	0	0
$\hat{\sigma}$	(0.1, 0.1)	0.6	5.3E−11	2.6E−17	3.9E−25	−	−
Samsung S10	(1136.5, 1953.8)	3002.6	2.0E−08	−7.5E−15	9.8E−22	−1.4E−25	0
$\hat{\sigma}$	(0.8, 0.8)	1.5	5.0E−10	2.5E−17	3.9E−24	6.1E−27	−

F<small>IG</small>. 5. Design of the evaluation of ellipse eccentricity adjustment experiment: (a) EOPs for the 28 image views; (b) ellipse centre, adjusted target centre and true target centre for a sample image and target.

<div align="center">E<small>XPERIMENT</small> D<small>ESIGN</small></div>

Four experiments were designed to assess the effectiveness of the proposed methods used in this study, namely: evaluation of ellipse eccentricity adjustment, incorporating two experiments (impact of centre adjustment and comparison of eccentricity error); quality assessment of ellipse matching; and evaluation of impact of calibration on pipe reconstruction. The four experiments are explained in more detail in the following.

Evaluation of Ellipse Eccentricity Adjustment

Algorithm 3 was developed to correct the eccentricity error of the estimated centre of the targets in images due to projective transformation. This experiment is designed to evaluate the effectiveness of the proposed method in practical settings. To this end, 28 image frames were taken from 4K video recording of a single target using a pre-calibrated Huawei P30. The EOPs of each image view were estimated using COLMAP and shown in Fig. 5(a). The ground-truth target centre in each image was manually determined from the pronounced black cross (Fig. 5(b)). The radius of the circular target was 100 mm and the images were taken from an average of 530 mm from the circular target. The scale of the 3D reconstruction was manually defined using the radius of the circular target. The precision of the 3D reconstructed centre using the ground-truth image centres and estimated EOPs was 0.13 mm. The ground-truth centre, the estimated best-fitting ellipse centre and the adjusted centre using Algorithm 3 for one sample image are shown in Fig. 5(b)). Two experiments were designed to: (i) quantify the impact of centre adjustment on the accuracy of the 3D reconstructed centre; and (ii) evaluate its performance compared to the closed formulation of eccentricity error by Luhmann (2014).

Impact of Centre Adjustment on the 3D Reconstruction. In this experiment, the impact of adjusting the eccentricity error compared to the estimated best-fitting ellipse centre (unadjusted) on the accuracy of the 3D reconstructed centre of the target was evaluated as the number of camera views increased from 2 to 28. Since different combinations of views will generate different reconstruction results, for a given number of (say k) image views, N different combinations of $k = 2...28$ images were selected. To this end, for k image views, the following steps were carried out:

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(1) Randomly select N different combinations of k images from the 28 images.
(2) For each set of k images, using the estimated best-fitting ellipse centre (no adjustment), adjusted centre using Algorithm 3 and camera projection matrices (Fig. 5(a)), perform triangulation (Hartley and Zisserman, 2004) and determine the object-space position of the target centres.
(3) Calculate the Euclidian distance between the object-space coordinates of the adjusted and unadjusted centres (separately) from the ground-truth centre.
(4) For the given number of image views, k, record the mean of the N distances obtained from step 3 for the adjusted and unadjusted centres.

Here, $N = 50$ combinations were selected.

Comparison of Eccentricity Error between Algorithm 3 and Luhmann's Formula. A closed formulation of the centre eccentricity error in the image plane was provided in Luhmann (2014), given the EOPs and IOPs of the view, 3D object-space target centre, object-space target radius and target plane's normal. To determine the eccentricity error using Luhmann's formula, the ground-truth 3D object-space centre, the radius, as well as the plane normal, were used. For each image, the eccentricity errors (in pixels) were calculated using both the authors' method (Algorithm 3) and Luhmann's formula. For each image, both eccentricity errors in each image were then compared to the ground-truth eccentricity error and reported.

Quality Assessment of Ellipse Matching

This experiment was designed to assess the quality of the ellipse matching results, obtained by Algorithm 2. The 270 images from the calibration laboratory, reported above in the section *Resulting Smartphone Calibration Parameters and Conditions*, were used. The ellipses of the 270 images were extracted using the method of Maalek and Lichti (2021b). The camera projective matrices as well as the detected ellipses for each image were then fed to Algorithm 2 to determine the matching ellipses between different images. The quality of matching was quantified for three settings: Algorithm 2 without robust triangulation; Algorithm 2 with robust triangulation; and Algorithm 2 with robust triangulation and corrected centres.

Evaluation of Impact of Calibration on Pipe Reconstruction

The broader objective of this study pertains to the application of smartphone cameras for 3D reconstruction of pipes. To this end, mechanical mock pipes were professionally installed at one corner of the calibration laboratory, shown in Fig. 6. This experiment was designed to assess the effectiveness of the proposed calibration process in estimating the radius of the pipe of interest after 3D reconstruction. Here, the accuracy of the estimated radius using the authors' pre-calibration process and lens distortion modelling was compared with that obtained using COLMAP's default SfM process. The latter involves an *in situ* automatic calibration, comprised of the first two terms of the radial lens distortion parameters, using the exchangeable image file (Exif) data as the initial IOP estimation. This latter process, from here on, is referred to as COLMAP-radial.

A 60-second 4K video was recorded around one pipe using each of the Huawei P30, iPhone 11 and Samsung S10 smartphones. The recording was divided into two 30-second videos (at 1 fps), one in portrait mode and the other in landscape mode. The
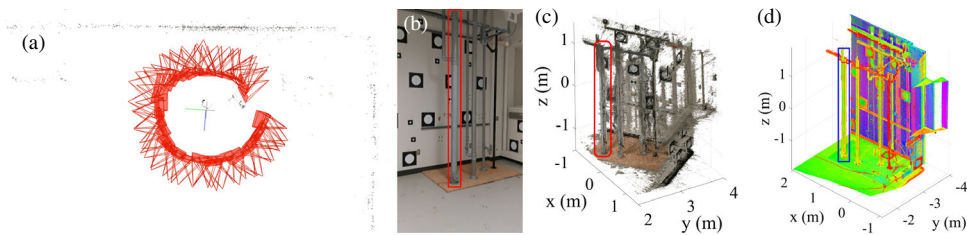
Fig. 6. Design of the "evaluation of impact of calibration on pipe reconstruction" experiment: (a) sample path around the pipe investigated; (b) sample image of the pipe with the targets on the background wall. 3D point cloud of the mock pipes using: (c) SfM 3D reconstruction; and (d) HDS6100 TLS.

camera was rotated 90° about its optical axis at a different height so as to provide COLMAP-radial a reasonable opportunity to calibrate the instruments without possible projective coupling. A dense 3D reconstruction was then carried out using COLMAP, once with the IOPs obtained by COLMAP-radial and then again with the authors' target-based calibration framework.

The final consideration for the 3D reconstruction was to define the scale. Since the accuracy of estimating the radius of the cylinder is being considered, it is important to define the scale of the 3D reconstruction independently from the cylinder's radius. The scale of the reconstruction is, hence, defined using the distance of two of the targets behind the mock pipes (Fig. 6(b)). The ground-truth distance between the targets was determined using a Leica Geosystems HDS6100 terrestrial laser scanner (TLS). The following process was then performed to determine the scale:

(1) Detect the ellipses in images using the ellipse detection method of Maalek and Lichti (2021b).
(2) Match the detected ellipses between images using Algorithm 2.
(3) Select two of the targets with the maximum number of image views.
(4) Adjust the centre eccentricity error of the two targets in each image view using Algorithm 3.
(5) Triangulate to determine the 3D coordinates of the two targets.
(6) Determine the ground-truth 3D coordinates of the centre of the two targets in the TLS point cloud using the method presented in Lichti et al. (2019b).
(7) The scale is defined by the ratio between the ground-truth distance (step 6) and the 3D reconstructed distance (step 5) of the two targets.

The radius of the cylinder of interest for the scaled 3D reconstruction (Fig. 6(c)) as well as the TLS point cloud (Fig. 6(d)) were then calculated using the robust cylinder fitting method of Maalek et al. (2019). The root mean square error (RMSE) of the best-fitting cylinder, representing the uncertainty, and the corresponding ground-truth pipe radius were 0.3 and 57.3 mm, respectively. The precision of the distance between the centres of the two targets, acquired from TLS, was approximately 0.1 mm, which is consistent with the sub-millimetre centre estimation precision reported in Lichti et al. (2019a). The accuracy of the estimated radius is reported for comparison.

A summary of the four experiments is provided in Table II.

TABLE II. Summary of the designed experiments.

Experiment description		Type of data	Purpose
Evaluation of ellipse eccentricity adjustment	Impact of centre adjustment on the 3D reconstruction	4K video images using calibrated Huawei P30	Comparing accuracy of 3D reconstructed centre of targets from the best fit ellipse centre and the adjusted centre of Algorithm 3
	Comparison of eccentricity error between Algorithm 3 and Luhmann's formula	4K video images using calibrated Huawei P30	Comparison of eccentricity error using Algorithm 3 and the Luhmann (2014) closed formula
Quality assessment of ellipse matching		4K video images using Samsung S10, iPhone 11 and Huawei P30	Quantifying the quality of matching ellipses between different images
Evaluation of impact of calibration on pipe reconstruction		4K video images using Samsung S10, iPhone 11 and Huawei P30 with both auto and fixed focus	Assessing the impact of the determined IOPs on the accuracy of estimating the radius of a cylindrical pipe

EXPERIMENTAL RESULTS

Evaluation of Ellipse Eccentricity Adjustment

Impact of Centre Adjustment on the 3D Reconstruction. The accuracy of the 3D object-space coordinates of the centre of the target (shown in Fig. 5), using the best-fitting ellipse centre and the adjusted centre using Algorithm 3, was determined as the number of image views increased. Fig. 7 shows the mean accuracy of the estimated centres (for the 50 selected combinations) with no adjustment (in blue) and with adjustment (in red). For both adjusted and unadjusted centres, it can be visually observed that the results of the mean accuracy of the 3D reconstructed centre remains almost constant as the number of image views increases from 5 to 28. The accuracy of the object-space 3D coordinates of the centre using the adjusted centre is, however, significantly better than that using the unadjusted ellipse centres. Using all 28 image views, the accuracy of the object-space coordinates was
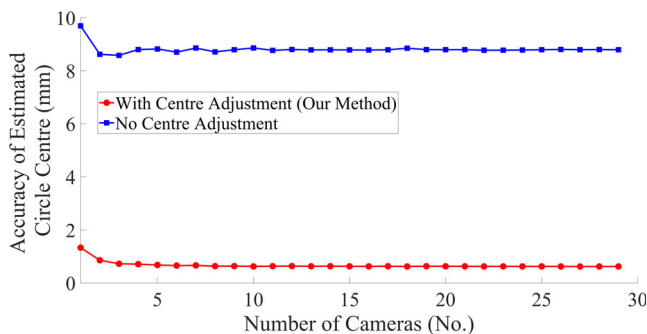

FIG. 7. Impact of adjusting the eccentricity error of elliptic targets in images on the accuracy of the 3D reconstructed centre.
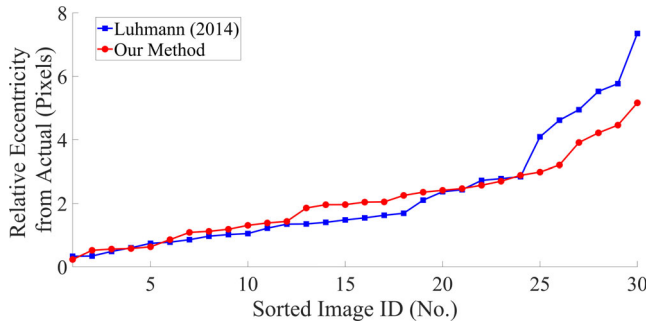
F*IG*. 8. Eccentricity errors from the ground truth, calculated using Algorithm 3 and the Luhmann (2014) method.

0.64 and 9.46 mm for the adjusted and unadjusted centres, respectively. On average, the results of the unadjusted centres were approximately 15 times that obtained using the adjusted centres. The result of this experiment demonstrates that reaching sub-millimetre accuracy for the 3D object-space coordinates of large target centres becomes possible using the method proposed in Algorithm 3, even for larger targets.

Comparison of Eccentricity Error between Algorithm 3 and Luhmann's Formula. The eccentricity error, which is the error between the estimated ellipse centre and the actual target centre in the image plane, was calculated for each image. This was done using both Algorithm 3 as well as Luhmann's closed formulation, given the object-space target information. The absolute deviation of the estimated eccentricity (here referred to as the "relative eccentricity") from the ground-truth eccentricity was calculated using both Algorithm 3 and Luhmann's formula. Fig. 8 shows the result of the difference between the ground-truth and estimated eccentricity errors for both the authors' and Luhmann's methods. As illustrated, the results are comparable, however, Luhmann's method achieved slightly better results for the best 22 images. The authors' method, on the other hand, achieved better results for the worst six images. The average of the relative eccentricity error for all images was 2.84 and 2.41 pixels using Luhmann's and the authors' methods, respectively. These results are comparable, but the authors' method provided around a 20% improvement compared to Luhmann's method. This result is attractive since the new formulation requires no a priori knowledge of the target's object-space information, which is a requirement for other available eccentricity formulations.

Quality Assessment of Ellipse Matching

The precision, recall, accuracy and F-measure for the ellipse matching were calculated for three settings: (i) Algorithm 2 without robust triangulation (before step 6); (ii) Algorithm 2 complete (with robust triangulation); and (iii) Algorithm 2 combined with Algorithm 3 to also adjust target centres. The quality of the ellipse matching method for the 270 images combined is presented in Table III. The ground-truth matches between every two images is manually identified. As observed, Algorithm 2 with robust triangulation achieved a considerably better recall, compared to Algorithm 2 without robust triangulation, which demonstrates its relative robustness to mismatching ellipses (no type II errors – false negatives). The robust triangulation, however, achieved a relatively lower precision compared to when no robust

TABLE III. Summary of precision, recall, accuracy and F-measure for the ellipse matching in three variations of Algorithm 2. Bold values show the best results.

Variations of Algorithm 2	Precision	Recall	Accuracy	F-measure
Without robust triangulation	**98.44%**	91.25%	93.28%	94.71%
With robust triangulation	93.71%	**100.00%**	96.15%	96.75%
With robust triangulation and adjusted centres	96.85%	**100.00%**	**98.08%**	**98.40%**

triangulation is performed. This shows that some correct matches are reduced, contributing to an increase in type I errors (false positives). When Algorithm 3 is combined with Algorithm 2 (before the robust triangulation step) to correct the target's eccentricity error, the robustness to type II was maintained (recall of 100%), and a portion of the correct matches that were eliminated were also recovered (an increase of about 3% in the precision). The F-measure, which provides a single value to explain both the contributions of type I and type II errors, shows that using Algorithm 2 in combination with Algorithm 3 provides the best result. The use of robust triangulation was also found to be necessary to eliminate type II errors and improve the F-measure, compared to Algorithm 2 without robust triangulation.

Evaluation of Impact of Calibration on Pipe Reconstruction

The impact of the pre-calibration using the model and IOPs extracted using the authors' method, compared to COLMAP-radial, for estimating the radius of a mechanical pipe was evaluated. Table IV shows the results obtained by COLMAP-radial as well as the pre-calibration for the three cameras using the authors' methodology. From the results presented in Table IV, the following four observations can be made:

(1) More inlier cylinder points were observed using the authors' pre-calibration, compared to COLMAP-radial, for all camera devices (about 1.3 times on average). This suggests that better feature matching was obtained from the same set of images when the correct radial lens distortion parameters and IOPs were used. This is attributed to the fact that the EOPs (particularly fundamental matrices) are impacted by radial lens distortion. In fact, given the same number of point correspondences, known radial lens distortion provides a better estimate of the fundamental matrix, even when the correct radial lens distortion model is considered (see Fig. 3 of Barreto and Daniilidis, 2005).

(2) The RMSE of the best-fitting cylinder was better using the pre-calibrated model compared to COLMAP-radial in all three devices, even though the number of inlier observations was higher in the pre-calibrated setting. The average difference was, however, only 0.1 mm, which may be considered negligible in many practical applications.

(3) The accuracy of the estimated radius was better for all devices using the authors' pre-calibration compared to COLMAP-radial (around 45% better accuracy on average). This demonstrates that the pre-calibration procedure outlined in this paper for each device provides a better cylinder reconstruction compared to the *in situ* calibration.

(4) The accuracy obtained using the iPhone 11 was better than that using the Huawei P30, which were both better than the Samsung S10. This is most likely to be attributed to the fact that the average number of detected features per image on the iPhone 11 was higher than that of the Huawei P30, both of which were higher than

that of the Samsung S10. The higher number of matched features are also observed from the number of inlier cylinder points reported using the iPhone, Huawei and Samsung, as shown in Table IV (regardless of the calibration procedure). In fact, more inlier points suggest the existence of more correct point correspondences between different images, which consequentially provides a better estimation of the EOPs, especially with the correct camera model (see explanations given in the literature review around the essential matrix). The higher number of matched features could be a consequence of factors such as calibration precision (see Table I), type of video compression (for example, H.265 and H.264 codec for the iPhone 11 and Huawei P30, respectively) and higher relative stability of calibration parameters, a common dilemma in smartphone camera calibration (Chikatsu and Takahashi, 2009; Elias et al., 2020). The determination of the exact reasons for the disparity between different instruments would be an interesting avenue for future exploration.

TABLE IV. Evaluation of the impact of the proposed calibration process on estimating the radius of mechanical pipe.

Device	Type	Accuracy of radius estimation (mm)	RMSE (mm)	Number of inlier cylinder points
Huawei P30	Pre-calibration	1.2	1.2	187 225
	COLMAP-radial	3.1	1.3	126 694
iPhone 11	Pre-calibration	0.5	1.0	219 297
	COLMAP-radial	1.4	1.1	166 238
Samsung S10	Pre-calibration	4.1	2.3	84 122
	COLMAP-radial	6.5	3.4	72 542

CONCLUSIONS

This paper provides a collection of new methods for the automatic calibration of optical instruments, in particular smartphone cameras. To this end, 4K videos, decomposed into images at 1 fps, were recorded in the calibration laboratory with a redundant set of black-and-white circular targets of different sizes. The method then utilises the potential of SfM for the sequential calibration and reconstruction of the scene to provide initial estimates of the EOPs and IOPs of each image. The ellipses, representing the boundaries of the circular black-and-white targets, were detected from each image. A new method was then proposed to match the ellipses between different camera views, given the initial camera projective matrices provided by SfM. The centre of each target, viewed by at least two images in the network, was then adjusted to correct for the eccentricity error using another newly developed method. Self-calibrating bundle adjustment was performed to re-estimate the EOPs and IOPs using the adjusted centres of each target. The EOPs and IOPs can be re-introduced into the previous steps for further iteration and refinement until the required precision for the adjusted centres is achieved (or until no more matches are found).

Four experiments were designed to assess the effectiveness of the proposed calibration methods using 4K video recordings captured via an iPhone 11, Huawei P30 and Samsung S10. The evaluation of the proposed centre adjustment showed that the adjusted centre provided around 15 times better 3D reconstructed centre estimation accuracy, compared to when no centre adjustment was performed. The results also revealed that, as long as the camera matrices are available with a superior precision, five camera views can be sufficient to provide a sub-millimetre accuracy for the 3D reconstructed centre of circular targets.

It was also shown that the proposed method for estimating the eccentricity error in each image plane was comparable (and in some cases outperformed) the closed formulation, provided by Luhmann (2014). The effectiveness of the proposed method in correcting the eccentricity error provides opportunities to utilise larger targets.

The third experiment assessed the quality of the ellipse matching algorithm. It was demonstrated that the proposed method performed best when using both robust triangulation to eliminate possible false matches (type II errors), and the adjusted centres to increase the correct matches (enhancing type I errors). The last experiment evaluated the accuracy of the estimated radius of professionally installed mechanical mock pipework using both the proposed calibration parameters and sequential SfM with *in situ* COLMAP-radial. It was observed that for all three smartphones in this research – iPhone 11, Huawei P30 and Samsung S10 – the calibration parameters, estimated using the proposed method, provided a better accuracy compared to COLMAP-radial (around 1.3 times better). The results of the experiments show that the proposed process for calibration is advantageous. Specifically, for the case of the smartphone cameras, the pre-calibration was found to be necessary to achieve better pipe-radius estimation results.

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Résumé

Cet article présente une nouvelle procédure pour l'étalonnage d'instruments optiques, en particulier les caméras de smartphones, utilisant des champs hautement redondants de cibles circulaires en noir et blanc. De nouvelles méthodes sont introduites pour (i) apparier les cibles entre les images; (ii) corriger l'erreur d'excentricité systématique sur les centres de cibles; et (iii) améliorer de manière itérative la solution d'étalonnage grâce à un ajustement de faisceaux d'auto-étalonnage en réseau libre. Cette procédure a apparié efficacement des cibles circulaires dans 270 images prises avec des smartphones dans un laboratoire d'étalonnage, avec une bonne robustesse aux erreurs de type II (faux négatifs). La méthode proposée pour la correction de l'erreur d'excentricité, qui nécessite seulement les matrices projectives des caméras pour deux

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prises de vue, s'est comportée de manière comparable aux solutions de forme fermée disponibles, qui nécessitent des informations a priori supplémentaires sur la cible dans l'espace objet. Enfin, dans le cas particulier des appareils mobiles, les paramètres d'étalonnage obtenus à l'aide de cette procédure se sont avérés de meilleure qualité que par l'étalonnage in situ pour estimer le rayon d'un tuyau mécanique reconstruit en 3D (amélioration d'environ 45% en moyenne).

Zusammenfassung

Dieses Manuskript bietet einen neuen Rahmen für die Kalibrierung optischer Instrumente, insbesondere von Smartphone-Kameras, unter Verwendung hochredundanter kreisförmiger Schwarz-Weiß-Zielfelder. Es wurden neue Methoden eingeführt für (i) den Abgleich von Zielen zwischen Bildern, (ii) die Anpassung des systematischen Exzentrizitätsfehlers der Zielzentren und (iii) die iterative Verbesserung der Kalibrierungslösung durch eine selbstkalibrierende Bündel Anpassung. Es wurde beobachtet, dass die vorgeschlagene Zielanpassung effektiv kreisförmige Ziele in 270 Smartphone-Bildern aus einem Kalibrierungslabor mit Robustheit gegenüber Typ-II-Fehlern abgleicht. Die vorgeschlagene Exzentrizitätsanpassung, die nur projektive Kameramatrizen aus zwei Ansichten benötigt, verhielt sich synonym zu verfügbaren Lösungen in geschlossener Form, die mehrere zusätzliche Objektraum-Zielinformationen a priori erfordern. Schließlich, speziell für den Fall von Smartphone-Geräten, die Kalibrierungsparameter mit unserem Rahmen erhalten wurde gefunden überlegen im Vergleich zu in-situ-Kalibrierung für die Schätzung der 3D-rekonstruierten Radius eines mechanischen Rohres (ca. 45% Verbesserung im Durchschnitt).

Resumen

Este manuscrito proporciona un marco conceptual nuevo para la calibración de instrumentos ópticos, en particular cámaras de teléfonos inteligentes, utilizando dianas circulares en blanco y negro altamente redundantes. Se introdujeron nuevos métodos para (i) la correspondencia de dianas entre imágenes; (ii) el ajuste del error de excentricidad sistemático de los centros de las dianas; y (iii) la mejora iterativa de la solución de calibración mediante un ajuste libre con autocalibración. El método propuesto realizó la correspondencia efectiva de dianas circulares en 270 imágenes de teléfono inteligente, tomadas en un laboratorio de calibración, y con robustez a los errores de tipo II (falsos negativos). El ajuste de excentricidad propuesto, que solo requiere de las matrices proyectivas de cámara desde dos orientaciones, se comportó de manera comparable a las soluciones de forma cerrada disponibles, que requieren información a priori adicional de las dianas en el espacio objeto. Finalmente, y específicamente para el caso de los dispositivos móviles, los parámetros de calibración obtenidos utilizando este marco conceptual, en comparación con la calibración in situ, proporcionaron mejor estimación del radio de una tubería mecánica en la reconstrucción 3D (aproximadamente un 45% mejor en promedio).

摘要

本文为光学仪器（尤其是智能手机相机）的检校提供了一种新方法，该方法使用了高度冗余的圆形黑白检校目标，引入了用于（i）在图像之间匹配目标的新方法；（ii）调整目标中心的系统性偏心误差；（iii）通过自由网自检校光束法平差来迭代地改进检校方案。所提出的方法有效地匹配了在检校实验室中拍摄的270个智能手机图像中的圆形目标，并具有对II型误差（假阴性）的鲁棒性。所提出的目标偏心调整仅需要来自两幅影像的相机投影矩阵，其性能与现有的封闭式解决方案相类似，后者需要附加的目标在物方空间的信息。最后，利用移动设备(相机)来计算估计机械管道三维重建的半径，使用本方法获得的校准参数要优于现场校准的结果(平均提高约45%)。