



# FlexKälte-Workshop: Flexibilisierung von Kälteversorgungssystemen

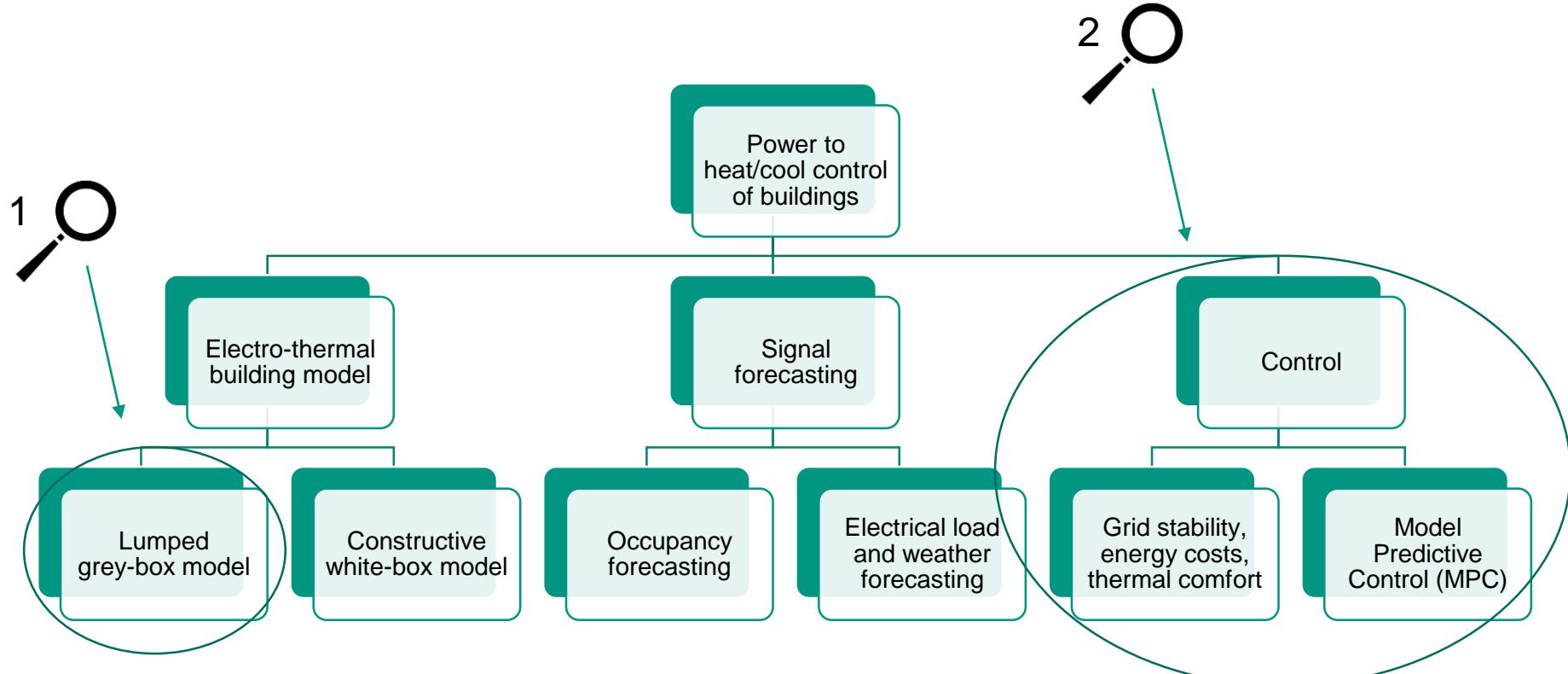
**Thema: Modellierung von Kälteversorgungssystemen**

**Autoren: Moritz Frahm, Philipp Zwickel, Felix Langner**

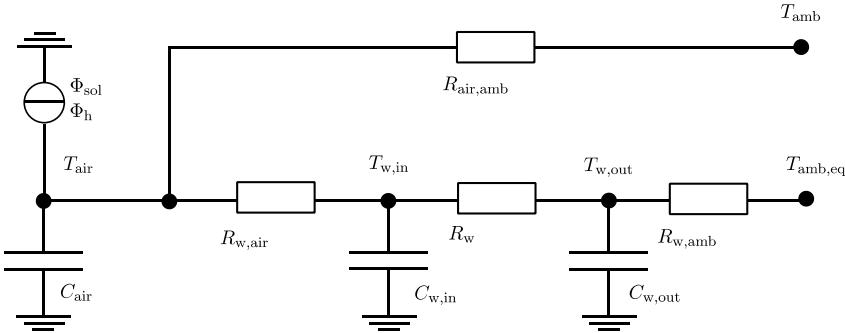
Institut für Automation und angewandte Informatik



# Introduction



# Lumped Electro-Thermal Building Model



Electro-thermal building model,  
inspired by Harb et al. [1] and modified

$$\frac{dT_{\text{air}}}{dt} = \frac{1}{C_{\text{air}}} \cdot \left( \frac{T_{w,\text{in}} - T_{\text{air}}}{R_{w,\text{air}}} + \frac{T_{\text{amb}} - T_{\text{air}}}{R_{\text{air},\text{amb}}} + \Phi_{\text{sol}} + \Phi_{h,\text{air}} \right)$$

$$\frac{dT_{w,\text{in}}}{dt} = \frac{1}{C_{w,\text{in}}} \cdot \left( \frac{T_{\text{air}} - T_{w,\text{in}}}{R_{w,\text{air}}} + \frac{T_{w,\text{out}} - T_{w,\text{in}}}{R_w} + \Phi_{h,\text{wall}} \right)$$

$$\frac{dT_{w,\text{out}}}{dt} = \frac{1}{C_{w,\text{out}}} \cdot \left( \frac{T_{w,\text{in}} - T_{w,\text{out}}}{R_w} + \frac{T_{\text{amb},\text{eq}} - T_{w,\text{out}}}{R_{w,\text{amb}}} \right)$$

Corresponding differential equations

$$\Phi_{h,\text{air}} = (1 - f_{\text{heat,rad}}) \Phi_h$$

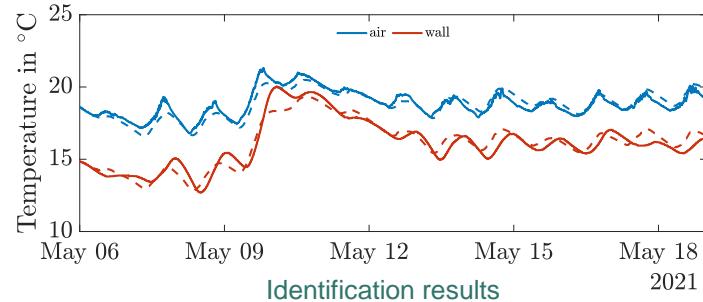
$$\Phi_{h,\text{wall}} = f_{\text{heat,rad}} \Phi_h$$

$$\Phi_{\text{sol}} = f_{\text{sol}} \phi_{\text{global}}$$

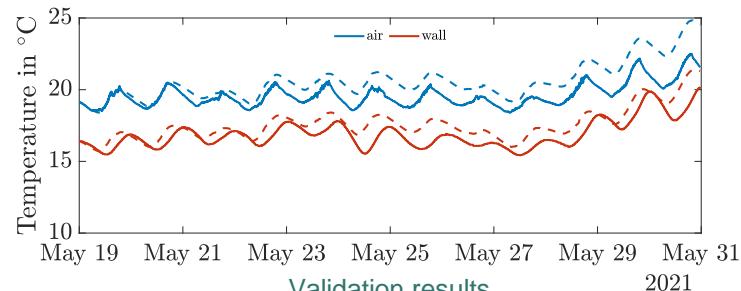
$$T_{\text{amb},\text{eq}} = T_{\text{amb}} + \phi_{\text{global}} \frac{a_f}{\alpha_A}$$

Model inputs

Comparison of the measured temperature (solid line) with the simulated temperature (dashed line)



Identification results



Validation results

# Economic Model Predictive Control

Optimization of cost function  $l(k, y, u)$

Time-discrete state-space representation of linear, time-invariant building model

$$\min_{u(\cdot|t)} \sum_{k=t}^{N-1} l(k, y(k|t), u(k|t))$$

subject to  $\forall k \in [0, N-1] :$

$$x(k+1|t) = A_d x(k|t) + B_d u(k|t) + B_{zd} z(k|t)$$

$$y(k|t) = C_d x(k|t)$$

$$x(0|t) = x(t), u(k|t) \in \mathcal{U}, y(k|t) \in \mathcal{Y}$$

$$l(k, y, u) = \underline{\lambda}(y - \tilde{y})^T(y - \tilde{y}) + (1 - \underline{\lambda})p(k)^T u,$$

Model-predictive optimal-control algorithm,  
based on Zwickel et al. [2]

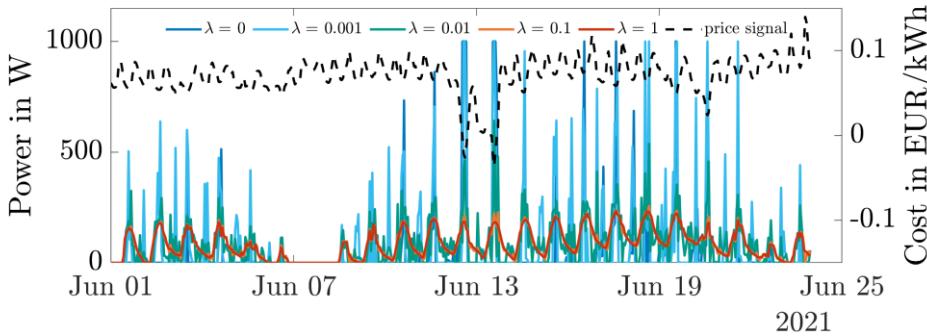
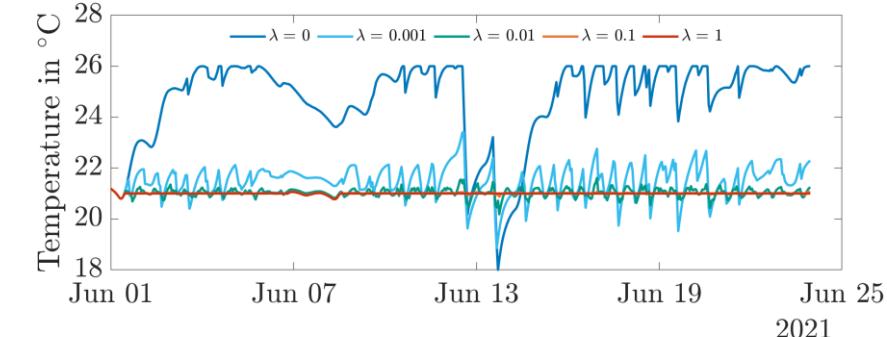
Boundaries

Temperature tracking  
 $\tilde{y} = 21^\circ\text{C}$

$$\begin{aligned} u_{\min} &= 0 \text{ kW} \\ u_{\max} &= 1 \text{ kW} \end{aligned}$$

$$\begin{aligned} y_{\min} &= 18^\circ\text{C} \\ y_{\max} &= 26^\circ\text{C} \end{aligned}$$

Energy cost



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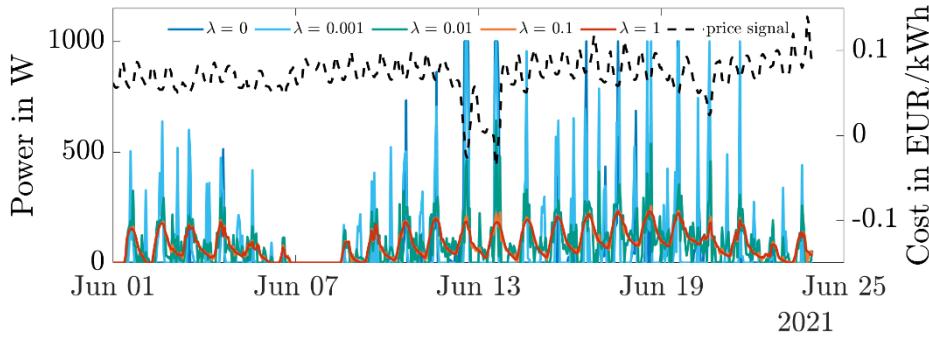
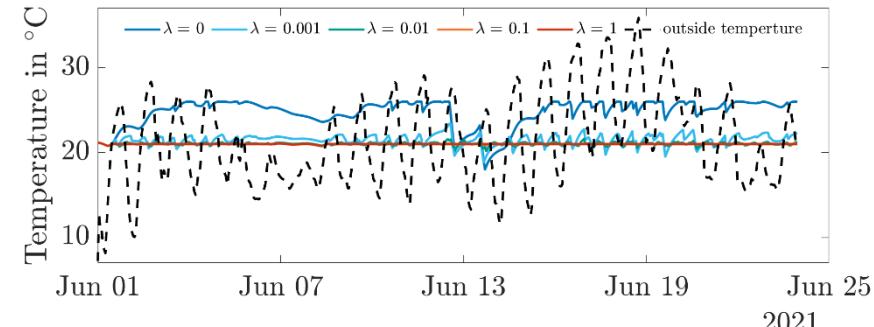
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# Discussion & Outlook

- Easily applicable tools for modeling and control
  - application on various buildings
  - sufficient accuracy of model for control (closed loop)



- Direct consideration of measurable and predictable signals
  - weather
  - energy price
  - occupancy

- Limitations
  - simulation-based results
  - linear-time-invariant model
  - simplified solar signal



- Future work
  - real-world application
  - release of open-source toolbox
  - model and control complexity

# Sources, Tools and Practical Application

## ■ Literature sources

- [1] Hassan Harb, Neven Boyanov, Luis Hernandez, Rita Streblow, and Dirk Müller. 2016. Development and validation of grey-box models for forecasting the thermal response of occupied buildings. *Energy and Buildings* 117 (2016), 199–207. <https://doi.org/10.1016/j.enbuild.2016.02.021>
- [2] Philipp Zwickel, Alexander Engelmann, Lutz Gröll, Veit Hagenmeyer, Dominique Sauer, and Timm Faulwasser. 2019. A Comparison of Economic MPC Formulations for Thermal Building Control. In 2019 IEEE PES Innovative Smart Grid Technologies Europe (ISGT-Europe). IEEE, 1–5. <https://doi.org/10.1109/ISGTEurope.2019.8905593>
- [3] Joel A E Andersson, Joris Gillis, Greg Horn, James B Rawlings, and Moritz Diehl. 2019. CasADi – A software framework for nonlinear optimization and optimal control. *Mathematical Programming Computation* 11, 1 (2019), 1–36. <https://doi.org/10.1007/s12532-018-0139-4>

## ■ Tools

- Matlab System Identification toolbox: <https://de.mathworks.com/help/ident/index.html>
- Matlab Symbolic Math toolbox: <https://de.mathworks.com/products/symbolic.html>
- CasADi efficient optimal control software: <https://web.casadi.org/>

## ■ Practical Application

- We are planning the release of the toolbox & paper ☺ <https://github.com/Building-Measurement-to-Control-Toolbox>