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IIB matrix model and regularized big bang

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The large-N master field of the Lorentzian IIB matrix model can, in principle, give rise to a particular degenerate metric relevant to a regularized big bang. The length parameter of this degenerate metric is then calculated in terms of the IIB-matrix-model length scale.

Subject Index B25

1. Introduction

Einstein's gravitational field equation [1] gives, in a cosmological context, the Friedmann–Lemaître–Robertson–Walker (FLRW) solution of a homogeneous and isotropic expanding universe with relativistic matter [2–8]. This solution has, however, a singularity with diverging energy density and curvature: the big bang singularity at cosmic-time coordinate $t = 0$.

Recently, we have suggested another solution [9], which has an additional length parameter b. This solution has maximum values of energy density and Kretschmann curvature scalar proportional to b^{-2} and b^{-4}, respectively. In a way, the length parameter b acts as a "regulator" of the big bang singularity, and the new solution has been called the regularized big bang solution. This new solution replaces the Friedmann big bang curvature singularity at $t = 0$ by a "spacetime defect" localized at $t = 0$. The spacetime defect is, in fact, described by a *degenerate* metric with a vanishing determinant at $t = 0$. The details of this new cosmological solution are discussed in Refs. [10–12], and further information on this particular type of spacetime defect appears in Refs. [13–15].

Up until now, the length parameter b of the degenerate metric is a mathematical artifact (regulator). But it is also possible that b is actually a *remnant* of a new physics phase that replaces Einstein gravity. In Appendix B of Ref. [10] and Appendix C of Ref. [12], we have explicitly mentioned loop quantum gravity [16] and string theory [17] as possible candidates for the physics of this new phase. Especially interesting may be the nonperturbative formulation of string theory, which may hold some surprises in store on the nature of the new phase [18].

A particular formulation of nonperturbative type-IIB superstring theory (M-theory) is given by the so-called IIB matrix model [19,20]. As this model involves only a finite number of matrices (traceless Hermitian matrices of size $N \times N$, where N is taken to infinity), spacetime and gravity must emerge dynamically. Numerical simulations [21,22] of the Lorentzian version of the IIB matrix model suggest, in fact, that a ten-dimensional classical spacetime emerges with three "large" spatial dimensions behaving differently from six "small" spatial dimensions. The previous literature [19–22] is, however, not entirely clear on from where precisely the spacetime points and metric come.

About a year ago, we suggested that, in the context of matrix models, the large-N master field [23] may play a crucial role for the emergence of a classical spacetime. This suggestion was detailed in Ref. [24] and several toy-model calculations were presented in two follow-up papers [25,26].

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We now pose the following question: does the master field of the Lorentzian IIB matrix model (assumed to be relevant for the physics of the Universe) give an emerging spacetime with a particular degenerate metric that corresponds to the regularized big bang solution of general relativity? At this moment, we cannot provide a definite answer, as we do not know the IIB-matrix-model master field. However, awaiting the final result on the master field, we can already investigate what properties the master field would need to have in order to be able to produce, if at all possible, an emerging metric resembling the metric of the regularized big bang solution. (It is far from obvious that the IIB-matrix-model expression for the emergent metric can give rise to such a type of metric.) The present paper is, therefore, solely exploratory in character.

2. Background material

2.1. *Regularized big bang solution*

In Sect. 1, we have already mentioned the main properties of the regularized big bang solution in general relativity. Here, we will briefly recall the relevant expressions of this metric.

The new line element is given by [5–9]

$$ds^2\,\Big|^{\text{(RWK)}} \equiv g_{\mu\nu}(x)\, dx^\mu\, dx^\nu\,\Big|^{\text{(RWK)}} = -\frac{t^2}{b^2+t^2}\, dt^2 + a^2(t)\;\delta_{ij}\, dx^i\, dx^j, \tag{1a}$$

$$b>0, \qquad a^2(t)>0, \tag{1b}$$

$$t \in (-\infty,\, \infty), \quad x^i \in (-\infty,\, \infty), \tag{1c}$$

where we have set $x^0 = c\,t$ and $c = 1$. The spacetime indices μ and ν run over $\{0,\, 1,\, 2,\, 3\}$, and the spatial indices i, j over $\{1,\, 2,\, 3\}$. Observe that the cosmic-time coordinate t covers the whole of the real line. The real function $a(t)$ corresponds to the cosmic scale factor.

The metric from Eq. (1) is degenerate, having a vanishing determinant at $t = 0$, and describes a spacetime defect with a characteristic length scale b. Further references on this type of spacetime defect have been given in Refs. [9,10]. The similarities and differences of the standard Robertson Walker (RW) metric and the degenerate metric (1) are discussed in a recent review [27].

Assuming a homogeneous perfect fluid for the matter, with energy density $\rho_M(t)$ and pressure $P_M(t)$, and inserting the metric (1) in the Einstein gravitational field equation (taking the appropriate limits [14] for $t = 0$) produces a modified Friedmann equation for $a(t)$, which has a bounce-type solution with a nonsingular behavior of the energy density and curvature at $t = 0$. The matter of the homogeneous perfect fluid is assumed to satisfy the standard energy conditions, for example the null energy condition $\rho_M + P_M \geq 0$.

Specifically, the modified spatially flat Friedmann equation, the energy-conservation equation of the matter, and the equation of state of the matter are [10]

$$\left[1+\frac{b^2}{t^2}\right]\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\,\rho_M, \tag{2a}$$

$$\dot\rho_M + 3\,\frac{\dot a}{a}\left[\rho_M + P_M\right] = 0, \tag{2b}$$

$$P_M = P_M\big(\rho_M\big), \tag{2c}$$

where the overdot stands for differentiation with respect to t and G is Newton's gravitational coupling constant. For relativistic matter ($P_M/\rho_M = 1/3$), the regular solution of Eq. (2) reads [2–9]

$$a(t)\,\Big|^{\text{(FLRWK)}}_{\text{(rel. mat.)}} \propto \left(t^2 + b^2\right)^{1/4}, \tag{3}$$

with $\rho_M(t) \propto 1/(t^2 + b^2)$. The new solution (3) appears, in slightly different notation, as Eq. (3.7) in Ref. [9]. The review [27] elaborates on how the degenerate metric (1) with cosmic scale factor $a(t)$ from Eq. (3) satisfies the Hawking–Penrose cosmological singularity theorems [28]. Further discussion of the resulting bouncing cosmology appears in Refs. [11,12], but here we highlight only one result.

Consider a perturbative Ansatz around the "bounce" at $t = 0$:

$$a(t) = 1 + a_2 \left(t/b\right)^2 + \cdots, \tag{4a}$$

$$\rho_M(t) = \rho_{M,0}\left(1 + r_2 \left(t/b\right)^2 + \cdots\right), \tag{4b}$$

with the energy-density scale $\rho_{M,0} > 0$ and real constants a_n and r_n, for $n \geq 2$. The modified Friedmann equation (2a) then gives the following parametric relation:

$$1/G \equiv 1/(l_{\text{Planck}})^2 \sim b^2\, \rho_{M,0}, \tag{5}$$

where we have assumed $a_2 \sim 1$ and have set $\hbar = 1$ and $c = 1$ in the definition of the Planck length $\left[\text{recall that, in general, we have } (l_{\text{Planck}})^2 \equiv \hbar\, G/c^3\,\right]$. From the measured value of G, we get $l_{\text{Planck}} \approx 1.62 \times 10^{-35}$ m; see Chapter 43 of Ref. [29] for further discussion. The relation (5) will be used in Sect. 4.

For comparison with later results, we give explicitly the degenerate metric (a rank-2 covariant tensor) corresponding to the line element (1),

$$g_{\mu\nu}\,\Big|^{\text{(RWK)}} = \begin{cases} -\dfrac{t^2}{t^2 + b^2}, & \text{for } \mu = \nu = 0, \\ a^2(t), & \text{for } \mu = \nu = m \in \{1,\,2,\,3\}, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

The inverse metric (a rank-2 contravariant tensor) is simply given by the matrix inverse (cf. p. 201 of Ref. [29])

$$g^{\mu\nu}\,\Big|^{\text{(RWK)}} = \begin{cases} -\dfrac{t^2 + b^2}{t^2}, & \text{for } \mu = \nu = 0, \\ a^{-2}(t), & \text{for } \mu = \nu = m \in \{1,\,2,\,3\}, \\ 0, & \text{otherwise,} \end{cases} \tag{7}$$

which, for $b^2 > 0$, has a divergent g^{00} component at $t = 0$.

2.2. *Emergent spacetime metric*

The IIB matrix model is extremely simple to formulate, having a finite number of matrices, but extremely hard to evaluate and interpret. More specifically, the model has a finite number of $N \times N$

traceless Hermitian matrices (N is taken to infinity later). Details of the IIB matrix model are given in the original papers [19,20] and have been briefly reviewed in Ref. [24]. Here, we only recall what is needed for the further discussion.

Adapting Eq. (4.16) of Ref. [20] to our master-field approach, we have obtained the following expression for the emergent inverse metric [24]:

$$g^{\mu\nu}(x) \sim \int_{\mathbb{R}^D} d^D y \, \langle\langle \, \rho(y) \, \rangle\rangle \; (y-x)^\mu \, (y-x)^\nu \, f(y-x) \; r(x,y), \tag{8}$$

with spacetime dimension $D = 10$ for the original matrix model and continuous spacetime coordinates x^μ. These spacetime coordinates x^μ have the dimension of length, which traces back to the IIB-matrix-model length scale ℓ that has been introduced in the path integral [24]. The average $\langle\langle \ldots \rangle\rangle$ in the integrand of Eq. (8) will be discussed shortly, after some other explanations have been given.

We refer to Refs. [24,25] for the details on how the discrete spacetime points $\widehat{x}_k^\mu$, with index $k \in \{1, \ldots, K\}$, are extracted from the bosonic master field $\underline{\widehat{A}}^{\,\mu}$. This bosonic master field corresponds to a set of ten $N \times N$ traceless Hermitian matrices for $N = Kn$, with positive integers K and n. The limit $K \to \infty$ carries along the limit $N \to \infty$, provided n stays constant or increases (the role of n will be explained below).

The quantities entering the integral (8) are the density function

$$\rho(x) \;\equiv\; \sum_{k=1}^{K} \delta^{(D)}\big(x - \widehat{x}_k\big) \tag{9}$$

for the emergent spacetime points $\widehat{x}_k^\mu$ as obtained in Refs. [24,25] and the dimensionless density correlation function $r(x, y)$ defined by

$$\langle\langle \, \rho(x) \, \rho(y) \, \rangle\rangle \;\equiv\; \langle\langle \, \rho(x) \, \rangle\rangle \, \langle\langle \, \rho(y) \, \rangle\rangle \; r(x,y). \tag{10}$$

In Eq. (8), there is also a localized symmetric real function $f(y - x)$, which appears in the effective action [20,24] of a low-energy scalar degree of freedom σ hopping over the discrete spacetime points $\widehat{x}_k^\mu$:

$$S_\text{eff}[\sigma] \sim \sum_{k,l} \frac{1}{2} f\big(\widehat{x}_k - \widehat{x}_l\big)\, \big(\sigma_k - \sigma_l\big)^2, \tag{11}$$

where σ_k is the field value at the point $\widehat{x}_k$ (the scalar degree of freedom σ arises from a perturbation of the master field $\underline{\widehat{A}}^{\,\mu}$ and σ has the dimension of length; see Appendix A of Ref. [24] for a toy-model calculation). As this function $f(x) = f\big(x^0, x^1, \ldots, x^{D-1}\big)$ has the dimension of $1/(\text{length})^2$, the inverse metric $g^{\mu\nu}(x)$ from Eq. (8) is manifestly dimensionless. The metric $g_{\mu\nu}$ is obtained as the matrix inverse of $g^{\mu\nu}$.

The extraction procedure [24,25] of the discrete spacetime points $\widehat{x}_k^\mu$ relies on $n \times n$ blocks positioned adjacently along the diagonal of the $N \times N$ matrices $\underline{\widehat{A}}^{\,\mu}$ of the bosonic master field (there are K blocks on each diagonal). Very briefly, the coordinates of the K discrete spacetime points $\widehat{x}_k^\mu$ are obtained as follows: the kth $n \times n$ block on the diagonal of the single $N \times N$ matrix $\underline{\widehat{A}}^{\,\sigma}$ (where σ is a fixed index) gives a single coordinate $\widehat{x}_k^\sigma$ as the average of the real eigenvalues of this particular $n \times n$ block. The average $\langle\langle \ldots \rangle\rangle$ entering Eqs. (8) and (10) then corresponds to averaging over different block sizes n and different block positions along the diagonal in the master field. The details of this averaging procedure still need to be clarified, but this does not affect the present discussion.

A few heuristic remarks may help to clarify expression (8) for the emergent inverse metric. In the standard continuum theory [i.e., a scalar field $\sigma(x)$ propagating over a given continuous spacetime manifold with metric $g_{\mu\nu}(x)$], two nearby points x' and x'' have approximately equal field values, $\sigma(x') \sim \sigma(x'')$, and two distant points x' and x''' generically have different field values, $\sigma(x') \neq \sigma(x''')$. The logic is inverted for our discussion. Two approximately equal field values, $\sigma_1 \sim \sigma_2$, still have a relatively small action (11) if $f(\widehat{x}_1 - \widehat{x}_2) \sim 1$, and inserting $f \sim 1$ in Eq. (8) gives a "large" value for the inverse metric $g^{\mu\nu}$ and, hence, a "small" value for the metric $g_{\mu\nu}$, meaning that the spacetime points $\widehat{x}_1$ and $\widehat{x}_2$ are close (in units of ℓ). Similarly, two very different field values σ_1 and σ_3 have a relatively small action (11) if $f(\widehat{x}_1 - \widehat{x}_3) \sim 0$, and inserting $f \sim 0$ in Eq. (8) gives a "small" value for the inverse metric $g^{\mu\nu}$ and, hence, a "large" value for the metric $g_{\mu\nu}$, meaning that the spacetime points $\widehat{x}_1$ and $\widehat{x}_3$ are separated by a large distance (in units of ℓ).

In the following, we will focus on the four "large" spacetime dimensions [21,22] and we have, for the emergent inverse metric,

$$g^{\mu\nu}(x) \sim \int_{\mathbb{R}^4} d^4y \; \rho_{\text{av}}(y) \; (y-x)^\mu \; (y-x)^\nu \; f(y-x) \; r(x,\, y), \tag{12a}$$

$$\rho_{\text{av}}(y) \equiv \langle\langle \, \rho(y) \, \rangle\rangle, \tag{12b}$$

with an effective spacetime dimension $D = 4$ and the abbreviated notation ρ_{av}. Perhaps it is not even necessary to do this additional averaging of ρ in the integrand of Eq. (12a), as that is already taken care of by the $N \to \infty$ limit [26].

In Ref. [26], we have rewritten the integral (12a) somewhat by using the integration variables $z^\mu \equiv y^\mu - x^\mu$ and introducing new functions h and $\bar{r}$. The resulting integral and the required definitions are

$$g^{\mu\nu}(x) \sim \int_{\mathbb{R}^4} d^4z \; \rho_{\text{av}}(z+x) \; z^\mu \, z^\nu \; h(z) \; \bar{r}(x,\, z+x), \tag{13a}$$

$$h(y-x) \equiv f(y-x) \; \widetilde{r}(y-x), \tag{13b}$$

$$r(x,\, y) \equiv \widetilde{r}(y-x) \; \bar{r}(x,\, y), \tag{13c}$$

where the new function $\bar{r}(x, y)$ has a more complicated dependence on x and y than the combination $x - y$, but the function is still symmetric, $\bar{r}(x, y) = \bar{r}(y, x)$. The advantage of using Eq. (13a) is that the x-dependence in the integrand has now been insolated in only two functions, ρ_{av} and $\bar{r}$.

For later use, we recall that the action of the ten-dimensional Lorentzian IIB matrix model [19–22] contains coupling constants $\widetilde{\eta}_{\mu\nu}$, for indices $\mu, \nu \in \{0, 1, \ldots, 9\}$. Reduced to $D = 4$ dimensions, these coupling constants are given by

$$\widetilde{\eta}_{\mu\nu} = \begin{cases} -1, & \text{for} \;\; \mu = \nu = 0, \\ +1, & \text{for} \;\; \mu = \nu = m \in \{1, 2, 3\}, \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

We emphasize that the above 16 numbers are only coupling constants and not yet a metric.

The purpose of the present paper is to investigate the integral (13a). It is not at all obvious that a Lorentzian inverse metric could appear with the required singular behavior. Indeed, we want to determine what would be required of the unknown functions ρ_{av}, h, and $\overline{r}$ (which trace back to the IIB-matrix-model master field), so that the integral (13a) gives the inverse metric (7), which has a divergent g^{00} component at $t = 0$.

3. Emergent degenerate metric

3.1. Basic idea

By choosing an appropriate length unit, we set the IIB-matrix-model length scale ℓ to unity, $\ell = 1$. In this way, the coordinates $\widehat{x}_k^{\mu}$ of the discrete emerging spacetime points are effectively dimensionless, and the same holds for the continuous spacetime coordinates x^{μ} used in Sect. 2.2. Moreover, we write, in a cosmological context, these continuous spacetime coordinates as follows:

$$x^{\mu} = \left(x^0, x^1, x^2, x^3\right), \tag{15a}$$

$$x^0 = \widetilde{c}\, t = t, \tag{15b}$$

where t is interpreted as the cosmic-time coordinate and $\widetilde{c}$ is set to unity by an appropriate choice of the time unit. The cosmic-time coordinate t is also effectively dimensionless.

In order to obtain an emergent inverse metric with a possibly divergent g^{00} component at $t = 0$, the convergence properties of the z^0 integral in Eq. (13a) need to be relaxed. Instead of the factor $\exp\left[-\left(z^0\right)^2\right]$ in $h(z)$ as used by Ref. [26] for the standard spatially flat RW inverse metric, we consider the following structure of the function $h(z)$ entering Eq. (13a):

$$h(z) \sim \frac{1}{\left(z^0\right)^2 + 1}\, \exp\left[-\left(z^1\right)^2 - \left(z^2\right)^2 - \left(z^3\right)^2\right]. \tag{16}$$

Focussing on the z^0 integral and neglecting other contributions, we then have

$$g^{00} \sim \int_{-z^0_{\text{cutoff}}}^{z^0_{\text{cutoff}}} dz^0\, \frac{\left(z^0\right)^2}{\left(z^0\right)^2 + 1}, \tag{17a}$$

$$g^{11} \sim \int_{-z^0_{\text{cutoff}}}^{z^0_{\text{cutoff}}} dz^0\, \frac{1}{\left(z^0\right)^2 + 1}, \tag{17b}$$

where the first integral diverges linearly as $z^0_{\text{cutoff}} \to \infty$ but the second does not.

Next, we must obtain $z^0_{\text{cutoff}} \sim 1/t^2$ and we use, for that, the following Ansatz:

$$\overline{r}(x, z + x) \sim \frac{p_1}{1 + \left(z^0\right)^2 \left(x^0\right)^4} + \frac{p_2}{1 + \left(z^0\right)^2 \left(z^0 + x^0\right)^4}, \tag{18}$$

where x^0 is identified with the cosmic-time coordinate t and where, later, we set $p_1 = p_2 = 1/\pi$. Note that the above function $\overline{r}(x, z + x)$, with equal p_1 and p_2, is symmetric in its arguments x and $z + x$, which explains the appearance of the second term proportional to p_2.

From Eq. (13) with $\rho_{\text{av}} = 1$ and the Ansatze (16) and (18), we find that the integrals with p_1 can be done analytically. The integrals with p_2 are more complicated but can be dealt with after a Taylor expansion with respect to $x^0 = t$. The following structure is obtained:

$$\left|g^{00}\right| \propto \frac{1}{t^2} + \mathrm{O}(1), \tag{19a}$$

$$\left|g^{11}\right| \sim \mathrm{O}(1). \tag{19b}$$

Further work is needed to get a t-independent term in $\left|g^{00}\right|$ exactly equal to unity and the Lorentzian signature. In a first reading, it is possible to skip the technical details and move forward to Sec. 4.

3.2. Core structure

With the basic idea of the previous subsection [namely, a mild cutoff on the z^0 integral of Eq. (13) at values of order $\pm t^{-2}$], we have not yet obtained the core structure of the desired inverse metric (7). For that, we need an extended Ansatz with additional freedom carried by four real parameters $\{\alpha, \beta, \gamma, \delta\}$. Remark that "core structure" refers to the inner structure of the spacetime defect [9], which, in this case, concerns the time coordinate and corresponds to a divergent g^{00} component.

Specifically, we take the following Ansatz functions:

$$\overline{r}(x, z+x) = \frac{p_1}{1+\alpha\left(z^0\right)^2\left(x^0\right)^4} + \frac{p_2}{1+\alpha\left(z^0\right)^2\left(z^0+x^0\right)^4}, \tag{20a}$$

$$h(z) = \xi\, \frac{\beta}{1+\gamma\left(z^0\right)^2}\, \exp\left[-\left(z^1\right)^2-\left(z^2\right)^2-\left(z^3\right)^2\right]\Big(\widetilde{\eta}_{00} + \widetilde{\eta}_{11}\,\delta\left[\zeta\left(z^1\right)^2-1\right] + \widetilde{\eta}_{22}\,\delta\left[\zeta\left(z^2\right)^2-1\right] + \widetilde{\eta}_{33}\,\delta\left[\zeta\left(z^3\right)^2-1\right]\Big), \tag{20b}$$

$$\rho_{\mathrm{av}}(z+x) = 1, \tag{20c}$$

$$\alpha > 0, \quad \gamma > 0, \tag{20d}$$

where we set, as before, $x^0 = t$ and $p_1 = p_2 = 1/\pi$. One of the constants ξ or β in Eq. (20b) is superfluous, but we keep them both in order to ease the comparison with the previous calculation of Ref. [26]. The $h(z)$ Ansatz involves, in addition, the coupling constants $\widetilde{\eta}_{\mu\nu}$ from the Lorentzian IIB matrix model reduced to $D = 4$ dimensions, as given by Eq. (14). For a different way of obtaining a Lorentzian signature in the emergent inverse metric, see Appendix B of Ref. [24] and Appendix D of Ref. [27].

Inserting the Ansatz functions (20) into the emergent-inverse-metric expression (13), we can perform all integrals analytically, except for the z^0 integral involving the p_2 term. For that integral, we make a Taylor expansion in $x^0 = t$ and then integrate analytically the resulting Taylor coefficients. As explained in Ref. [26], we set

$$\zeta = 2, \tag{21a}$$

$$\xi = 1/\pi^{3/2}, \tag{21b}$$

and obtain the following result ($m \in \{1, \ldots, 3\}$ is the spatial index):

$$g^{00} \sim (-1)\left[c_{-2}^{00}\, t^{-2} + c_0^{00} + \mathrm{O}(t^2)\right], \tag{22a}$$

$$g^{mm} \sim (+1)\left[c_0^{mm} + \mathrm{O}(t^2)\right], \tag{22b}$$

with all other components $g^{\mu\nu}$ vanishing by symmetry [the integrand of Eq. (13) then has a single factor z^1, z^2, or z^3]. The coefficients $c_n^{\mu\nu}$ in Eq. (22) are functions of the four real Ansatz parameters $\{\alpha, \beta, \gamma, \delta\}$.

In order to simplify the discussion, we immediately fix

$$\overline{\alpha} = 1, \quad \overline{\beta} = 10^2, \tag{23}$$

so that we only need to determine the appropriate values of the parameters γ and δ. In fact, the c_0^{00} coefficient now only depends on the parameter γ, as δ is absent and α and β have been fixed to the numerical values (23). From the requirement

$$\widetilde{c}_0^{\,00} = 1, \tag{24}$$

where the tilde indicates the use of Eq. (23), we obtain a seventh-order algebraic equation for $\sqrt{\gamma}$, which has two positive real roots. The analytic expressions for these two roots are rather cumbersome and we will just give their numerical values:

$$\gamma_1 \approx 10.1337, \tag{25a}$$

$$\gamma_2 \approx 34.5392. \tag{25b}$$

For definiteness, we take the first root from Eq. (25) and set

$$\overline{\gamma} = \gamma_1 \approx 10.1337. \tag{26}$$

Having found a suitable value for γ, we turn to the resulting coefficient c_0^{mm} of the inverse-metric component g^{mm}. From the requirement

$$\widetilde{c}_0^{\,mm} = 1, \tag{27}$$

where the tilde indicates the use of Eqs. (23) and (26), we obtain a linear equation for δ and find the following solution:

$$\overline{\delta} \approx 0.517689. \tag{28}$$

To summarize, we have, from the Ansatz functions (20) and the parameters

$$\left\{\overline{\alpha},\, \overline{\beta},\, \overline{\gamma},\, \overline{\delta}\right\} = \left\{1,\, 10^2,\, 10.1337,\, 0.517689\right\}, \tag{29}$$

the following result for the emergent inverse metric as given by the expression (13):

$$g^{\mu\nu}\Big|^{\text{(core-structure)}} \sim \begin{cases} (-1)\left[\left(\overline{\beta}/\overline{\gamma}\right) t^{-2} + 1 + \mathrm{O}(t^2)\right], & \text{for } \mu = \nu = 0, \\ (+1)\left[1 + \mathrm{O}(t^2)\right], & \text{for } \mu = \nu = m \in \{1, 2, 3\}, \\ 0, & \text{otherwise}, \end{cases} \tag{30}$$

where the numerical value of $\overline{\beta}/\overline{\gamma}$ is of the order of 10 (the actual numerical value will be given shortly).

Comparing to the general-relativity inverse metric (7), we interpret the first two nontrivial terms of g^{00} from Eq. (30) as follows:

$$g^{00}\Big|^{\text{(core-structure)}} \sim (-1)\left[\frac{b^2/\ell^2 + t^2/\ell^2}{t^2/\ell^2} + \ldots\right], \tag{31a}$$

$$b^2/\ell^2 = \overline{\beta}/\overline{\gamma}, \tag{31b}$$

where ℓ is the length scale of the IIB matrix model that we have previously set to unity. With the parameter values (29), we have

$$b^2/\ell^2 = \overline{\beta}/\overline{\gamma} \approx 9.8681, \tag{32}$$

but different numerical values are obtained if, for example, the β value is changed away from the value 10^2 or if the root γ_2 is chosen instead of γ_1. The general parametric behavior of the t^{-2} coefficient in g^{00} follows by adapting the elementary integral for g^{00} in Sect. 3.1 and gives

$$b^2 \sim \frac{\beta}{\sqrt{\alpha}\,\gamma}\,\ell^2, \tag{33}$$

for the particular Ansatz (20).

3.3. *First approximation*

In the previous subsection, we have shown that, in principle, the emergent inverse-metric expression (13) can give the core structure of the inverse metric (7), with an explicit numerical value of the classical-gravity length parameter b in units of the IIB-matrix-model length scale ℓ. See, in particular, the results of Eqs. (31), (32), and (33).

We now want to check that the higher-order terms in t of Eq. (31a) can be made to vanish. For that, we will use, instead of Eq. (20c), a nontrivial Ansatz of the ρ_{av} function. Specifically, we take

$$\rho_{\text{av}}(z+x) = 1 + \left(\sum_{k=0}^{2} r_{2k}\,\left(z^0+x^0\right)^{2k}\right)\,\exp\left[-\left(z^0+x^0\right)^2\right], \tag{34}$$

with real parameters r_n and an explicit exponential factor to guarantee the convergence of the z^0 integral (the r_n terms will, for this reason, not modify the coefficient of the t^{-2} term in g^{00}). Keeping the γ parameter equal to the numerical value $\overline{\gamma}$ from Eq. (26) but allowing for a change in the numerical value of δ, we find that the coefficients $c_n^{\mu\nu}$ of Eq. (22) have the following dependence:

$$\overline{c}_0^{\,00} = \overline{c}_0^{\,00}\big(r_0,\, r_2,\, r_4\big), \tag{35a}$$

$$\overline{c}_2^{\,00} = \overline{c}_2^{\,00}\big(r_0,\, r_2,\, r_4\big), \tag{35b}$$

$$\overline{c}_0^{\,11} = \overline{c}_0^{\,11}\big(r_0,\, r_2,\, r_4,\, \delta\big), \tag{35c}$$

where the overbar indicates the use of the numerical values (23) and (26).

Demanding

$$\overline{c}_0^{\,00} = 1, \quad \overline{c}_2^{\,00} = 0, \quad \overline{c}_0^{\,11} = 1 \tag{36}$$

gives three algebraic equations for the three parameters $\{r_0,\, r_2,\, \delta\}$ with the following solutions:

$$\overline{r}_0 \approx -0.297254 - 0.305679\, r_4, \tag{37a}$$

$$\overline{r}_2 \approx +0.491944 - 0.850548\, r_4, \tag{37b}$$

$$\overline{\delta} \approx \frac{12.013 - 3.77531\, r_4}{23.026 - 7.55061\, r_4}; \tag{37c}$$

which are still functions of the free parameter r_4. [Note that $r_0 = r_2 = r_4 = 0$ is not a solution of the the conditions (36).] The corresponding inverse metric reads

$$g^{\mu\nu}\Big|^{\text{(first approx.)}} \sim \begin{cases} (-1)\,\Big[\big(\overline{\beta}/\overline{\gamma}\big)\, t^{-2} + 1 + \mathrm{O}(t^4)\Big], & \text{for}\;\; \mu=\nu=0, \\ (+1)\,\Big[1 + \overline{c}_2\, t^2 + \mathrm{O}(t^4)\Big], & \text{for}\;\; \mu=\nu=m\in\{1,\,2,\,3\}, \\ 0, & \text{otherwise}, \end{cases} \tag{38a}$$

$$\overline{\beta}/\overline{\gamma} \approx 9.8681, \tag{38b}$$

$$\overline{c}_2 \approx \frac{4.17896 + 0.862081\, r_4}{23.026 - 7.55061\, r_4}, \tag{38c}$$

which is a significant improvement compared to the core-structure result (30). The result in Eq. (38) corresponds, in fact, to a first approximation of the desired inverse metric valid to order t^4.

We can invert the map (38c) and obtain the required input value r_4 for a desired value of c_2 ,

$$r_{4,\,\text{input}} \approx -\frac{4.84753 - 26.7098\, c_{2,\,\text{desired}}}{1 + 8.75859\, c_{2,\,\text{desired}}}. \tag{39}$$

In this way, we can get *any* Taylor coefficient c_2 in the g^{mm} component from Eq. (38) by choosing an appropriate value of the Ansatz parameter r_4.

From Eq. (38a), we obtain by matrix inversion the diagonal metric $g_{\mu\nu}$ which has the following 00 component:

$$g_{00}\Big|^{\text{(first approx.)}} \sim (-1)\,\frac{t^2}{\overline{\beta}/\overline{\gamma} + t^2 + \mathrm{O}(t^6)}. \tag{40}$$

It is already clear that this metric is degenerate, with a vanishing determinant at $t = 0$, but we postpone further discussion of this point to the next subsection.

3.4. *Conjectured final result*

As indicated on the left-hand side of Eq. (38a), we consider that result to be a first approximation of the inverse metric (7), as derived from the IIB-matrix-model master field under the assumptions stated. Better approximations, with more and more Taylor coefficients for g^{mm} and more and more t^n terms vanishing in g^{00}, will follow from higher orders in the Ansatz function ρ_{av} from Eq. (34) and possible further extensions of the Ansatz functions $\overline{r}$ and h. This procedure has been tested in Ref. [26] for the standard spatially flat RW inverse metric.

The final result for the emergent inverse metric is expected to have the following structure (in units with $\ell = 1$):

$$g^{\mu\nu}\Big|^{\text{(final-result)}} \stackrel{?}{\sim} \begin{cases} -\dfrac{t^2 + c_{-2}}{t^2}, & \text{for}\;\; \mu=\nu=0, \\ 1 + c_2\, t^2 + c_4\, t^4 + \dots, & \text{for}\;\; \mu=\nu=m\in\{1,\,2,\,3\}, \\ 0, & \text{otherwise}, \end{cases} \tag{41}$$

where the question mark indicates that, strictly speaking, this is a conjectured result. The real dimensionless coefficients c_n in g^{mm} of (41) result from the requirement that t^{2n} terms, for integer $n > 0$, vanish in g^{00}. The emergent metric is given by the matrix inverse of Eq. (41),

$$g_{\mu\nu}\Big|^{\text{(final-result)}} \overset{?}{\sim} \begin{cases} -\dfrac{t^2}{t^2 + c_{-2}}, & \text{for } \mu = \nu = 0, \\ \dfrac{1}{1 + c_2\, t^2 + c_4\, t^4 + \ldots}, & \text{for } \mu = \nu = m \in \{1, 2, 3\}, \\ 0, & \text{otherwise,} \end{cases} \tag{42}$$

which has, for $c_{-2} > 0$, a vanishing determinant at $t = 0$. In short, the emergent metric (42), obtained from the expression (13) with appropriate Ansatz functions and parameters, is degenerate.

The emergent metric (42) has indeed the structure of the metric (6), with the following effective parameters:

$$b_{\text{eff}}^2 \sim c_{-2}\, \ell^2, \tag{43a}$$

$$a_{\text{eff}}^2(t) \sim 1 - c_2\, \big(t/\ell\big)^2 + \mathrm{O}(t^4/\ell^4), \tag{43b}$$

where the IIB-matrix-model length scale ℓ has been restored and where we omit the question marks as we have explicit results for the coefficients shown. Indeed, the leading coefficients are given by $c_{-2} \approx \overline{\beta}/\overline{\gamma} > 0$ from Eq. (32) and $c_2 \approx \overline{c}_2(r_4)$ from Eq. (38c), for the particular Ansatz functions (20a), (20b), and (34) and Ansatz parameters (23) and (26). If the Ansatz parameter r_4 in Eq. (38c) is chosen appropriately, we get $c_2 \approx \overline{c}_2 < 0$ in the square of the cosmic scale factor (43b), so that the emergent classical spacetime corresponds to the spacetime of a nonsingular cosmic bounce at $t = 0$, as obtained in Refs. [9,10] from Einstein's gravitational field equation. The proper cosmological interpretation of the emergent classical spacetime will be discussed further in Sect. 4.

4. Conclusion

In the present article, we have started an exploratory investigation of how a new physics phase can give an emerging classical spacetime with an effective metric where the big bang singularity has been tamed [9].

In order to be explicit, we have used the IIB matrix model [19,20], which has been suggested as a nonperturbative definition of type-IIB superstring theory. If we interpret the numerical results [21,22] from the Lorentzian IIB matrix model as corresponding to an approximation of the genuine master field [23], then it appears that spacetime points emerge with three "large" spatial dimensions and six "small" spatial dimensions. But the numerical simulations are still far removed from providing results on the required density and correlation functions that build the inverse metric [20,24].

For the moment, we have adopted a leapfrogging strategy by jumping over the actual analytic or numeric evaluation of the IIB-matrix-model master field and by simply assuming certain types of behavior of the density and correlation functions that enter the inverse-metric expression (8). The explicit goal of the present article is to establish what type of functions are required in Eq. (8) to get, if at all possible, an inverse metric with the behavior shown in Eq. (7). [Note that, in principle, the origin of the expression (8) need not be the IIB matrix model but can be an entirely different theory, as long as the emerging inverse metric is given by a multiple integral with the same basic structure.]

For the integral (8), we have indeed been able to find suitable functions (these functions are, most likely, not unique), which give an emerging classical spacetime with an effective metric where the big bang singularity has been tamed. In fact, the big bang singularity is effectively regularized by a nonzero length parameter b_{eff} that is now calculated in terms of the IIB-matrix-model length scale ℓ ; see the last paragraph of Sect. 3.4. One important lesson, from the comparison with our previous calculation [26] of the Minkowski and RW metrics, appears to be that the relevant correlation functions must have long-range tails in the time direction, in order to get a divergent behavior of g^{00}, as explained in Sect. 3.1.

Note that we have not yet obtained the effective (Einstein?) gravitational field equation and the corresponding solution of the metric. Instead, we have used a general constructive expression for the inverse metric, as given by Eq. (13) after some redefinitions. The further consistency of the emerging field theories may then restrict the values of some of the parameters entering our explicit Ansatz functions (20a), (20b), and (34), fixing, for example, the values of α and β, or even demanding different functional forms of the functions $\rho_{\text{av}}(z+x)$, $h(z)$, and $\overline{r}(x, z+x)$.

Expanding on the previous paragraph, we observe that the IIB matrix model not only produces a classical spacetime but also its matter content [20]. Now, the IIB matrix model in the formulation of Ref. [24] has a single length scale ℓ, so that, for the cosmological quantities (4) near the bounce at $t = 0$, we expect an energy-density scale $\rho_{M,0} \sim 1/\ell^4$. If, moreover, general covariance [20] and the Einstein gravitational field equation are recovered, we have from the relation (5) with $b \sim \ell$ the following parametric relation:

$$l_{\text{Planck, eff}} \stackrel{?}{\sim} \ell, \tag{44}$$

where $l_{\text{Planck, eff}}$ corresponds to $\sqrt{G_{\text{eff}}}$ (using units to set $\hbar_{\text{eff}}$ and c_{eff} to unity) and where the question mark indicates that this is a conjectured result. If correct, the emergent Planck length would, not surprisingly, be of the same order as the IIB-matrix-model length scale ℓ. Reading Eq. (44) from right to left and inserting the experimental values for G, $\hbar$, and c on the left-hand side, we would also have an estimate for the actual value of the unknown IIB-matrix-model length scale ℓ,

$$\ell \stackrel{?}{\sim} 1.62 \times 10^{-35}\ \text{m}, \tag{45}$$

where the "experimental" numerical value for the Planck length was already given a few lines below Eq. (5).

The cosmological interpretation of the emergent classical spacetime is perhaps as follows. The new physics phase is assumed to be described by the IIB matrix model and the corresponding large-N master field gives rise to the points and the metric of a classical spacetime. If the master field has an appropriate structure, the emergent metric has a tamed big bang, with a metric similar the degenerate metric (6) of general relativity, but now having an effective length parameter b_{eff} proportional to the IIB-matrix-model length scale ℓ . In fact, one possible interpretation is that the new physics phase has produced a universe–antiuniverse pair [30], that is, a "universe" for $t > 0$ and an "antiuniverse" for $t < 0$.

As a final comment on our main result $b_{\text{eff}} \sim \ell$ from Eq. (43a) and the conjectured result (44), we recall that we have used a IIB-matrix-model length scale ℓ that was introduced directly into the path integral [24]. However, a more subtle origin of the length scale ℓ is certainly not excluded. One example of such an origin would be, in the emerging massless relativistic quantum field theory from the matrix model, the appearance of a length scale by the phenomenon of dimensional transmutation [31]. In any case, assuming the IIB matrix model to be relevant for physics, progress on fundamental questions such as the origin of the length scale or the birth of the Universe will only happen if more is known about the IIB-matrix-model master field.

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