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ON POSSIBILITY OF ALPHA-BUCKETS DETECTING AT THE KIT STORAGE RING KARA (KARLSRUHE RESEARCH ACCELERATOR)

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Abstract

Computer studies of longitudinal motion have been performed with the objective to estimate the possibility of detection of alpha-buckets at the KIT storage ring KARA (Karlsruhe Research Accelerator). The longitudinal equations of motion and the Hamiltonian were expanded to high order terms of the energy deviation of particles in a beam. Roots of third order equation for three leading terms of momentum compaction factor and free energy independent term were derived in a form suitable for analytical estimations. Averaged quadratic terms of closed orbit distortions caused by misalignment of magnetic elements in a ring lead to orbit lengthening independent of particle energy deviation. Particle transverse excursions were estimated and are taken into account. Simulations have been bench-marked on existing experiments at Metrology Light Source (MLS) in Berlin (Germany) and SOLEIL (France). A computer model of KARA was used to predict behavior and the dynamics of possible simultaneous beams in the ring.

INTRODUCTION

Special techniques are applied to reduce the bunch length in electron storage rings [1, 2]. In the so-called “squeezed” operation mode, the high degree of spatial compression of the optics with reduced momentum compaction factor (“low-α optics”) entails complex longitudinal dynamics of the electron bunches.

During experiments with low-α optics two and even three simultaneously rotating beams have been detected and observed at some light sources [3-7]. These beams are referred to as either ‘RF’ or ‘α–bucket’ beams. Operation with α–buckets is based on non-linear theory of longitudinal beam dynamics (LBD) with high order terms of momentum compaction factor [8-11].

COMPACTION FACTOR THEORY

The path length variation of beam orbit in a ring can be split into two parts [4, 10]; one independent of momentum deviation (χ) and the other dependent on high order terms of momentum offset δ

$$\Delta L/L_0 = \alpha(\delta)\cdot\delta + \chi \tag{1}$$

where momentum compaction factor itself depends on energy offset and is, up to the second order of energy deviation δ, defined as

$$\alpha(\delta) = \alpha_1 + \alpha_2\delta + \alpha_3\delta^2 \tag{2}$$

Linear and high order components of the momentum compaction factor depend on dispersion function terms

$$\alpha_1 = \frac{1}{L_0}\oint\left(\frac{D_0}{\rho}ds\right) \tag{3}$$

$$\alpha_2 = \frac{1}{L_0}\left[\oint\left(\frac{D_0^2}{2\rho^2}+\frac{D_1}{\rho}+\frac{D'^2_0}{2}\right)ds\right] \tag{4}$$

$$\alpha_3 = \frac{1}{L_0}\left[\oint\left(\frac{D_0D_1}{\rho^2}+\frac{D_2}{2\rho}+D'_0D'_1\right)ds\right] \tag{5}$$

The momentum independent term of relative orbit lengthening (χ) includes betatron oscillations and closed orbit distortions (COD) errors [10], and is now given as

$$\chi = \frac{1}{2L_0}\oint\left(x'^2_\beta + z'^2_\beta + x'^2_{cod} + z'^2_{cod} + \frac{x^2_\beta}{\rho^2} + \frac{x^2_{cod}}{\rho^2}\right)ds \tag{6}$$

The condition of fixed momentum offset $\frac{\partial H}{\partial\delta} = \frac{d(\Delta\varphi)}{dt} = 0$ of the Hamiltonian operator of longitudinal motion [4, 12] is realized for two phases $\varphi = \varphi_s$ and $\varphi = \pi - \varphi_s$. Condition of fixed phase $\frac{\partial H}{\partial(\Delta\varphi)} = -\frac{d\delta}{dt} = 0$ is fulfilled when the relative orbit lengthening is zeroed

$$\alpha_3\delta^3 + \alpha_2\delta^2 + \alpha_1\delta + \chi = 0 \tag{7}$$

Neglecting (at this moment only) the energy independent term ($\chi = 0$), one can simplify and split Eq. (7) into two parts. The first part has one root corresponding to on-momentum synchronous particles ($\delta_1 = 0$) and phase φ_s (RF bucket). Second part provides another two roots with momentum offset (α–buckets)

$$\delta_{2,3} = \frac{1}{2\alpha_3}\left[-\alpha_2 \pm \sqrt{\alpha_2^2 - 4\alpha_1\alpha_3}\right] \tag{8}$$

These roots are real if $|\alpha_2| > 2\sqrt{\alpha_1\alpha_3}$ or the first α_1, and third α_3 order terms are of different sign and $(\alpha_1\alpha_3) < 0$. In the general case when free term (χ) is not negligible, there is no solution of third order Cardano equation, see Eq. (7), with zero momentum offset ($\delta = 0$). All roots are shifted in momentum from the reference orbit if COD errors are not compensated for ($\chi \neq 0$) [13]. At some conditions, see, for example, [1, 4], three real roots of cubic equation might be found

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$$\delta_1 = -\frac{a_2}{3a_3} + \frac{2}{\sqrt[3]{2}(3a_3)}\left\{\sqrt[3]{\sqrt{\Delta_1^2+|-\Delta|}}\, cos\left(\frac{\varphi}{3}\right)\right\} \quad (9)$$

$$\delta_2 = -\frac{a_2}{3a_3} - \frac{1}{\sqrt[3]{2}(3a_3)}\left\{\sqrt[3]{\sqrt{\Delta_1^2+|-\Delta|}}\left[cos\left(\frac{\varphi}{3}\right)+\sqrt{3}sin\left(\frac{\varphi}{3}\right)\right]\right\}$$

$$\delta_3 = -\frac{a_2}{3a_3} - \frac{1}{\sqrt[3]{2}(3a_3)}\left\{\sqrt[3]{\sqrt{\Delta_1^2+|-\Delta|}}\left[cos\left(\frac{\varphi}{3}\right)-\sqrt{3}sin\left(\frac{\varphi}{3}\right)\right]\right\}$$

Here the second order determinant is $(-\Delta_0) = 3a_1a_3 - a_2^2$, third order term is $\Delta_1 = 2a_2^3 - 9a_1a_2a_3 + 27a_3^2\chi$ and general determinant $(-\Delta)$ is a composition of third and second order determinants $(-\Delta) = \Delta_1^2 + 4(-\Delta_0)^3$ [4, 13]. Tangent of angle φ is the ratio $tg(\varphi) = \sqrt{|-\Delta|}/(-\Delta_1)$. One should take care of the sign of the $sin(\varphi)$ function when angle φ<0. The synchrotron tune, F_s , depends on high order terms of the momentum compaction factor [4], where α can be defined [14, 15] as a derivative of the relative orbit lengthening with momentum offset $\alpha = \partial(\Delta L/L_0)/\partial\delta$

$$F_s(\delta) = F_0\sqrt{\frac{h_{rf}\, eU_{rf}(-cos\varphi_s)}{2\pi\beta_0^2 E_0}}\cdot\sqrt{(\alpha_1+2\alpha_2\delta+3\alpha_3\delta^2)} \quad (10)$$

CONDITIONS FOR α–BUCKETS AT KARA

We calculated parameters of both RF and α–buckets from SOLEIL [4] and MLS [6, 16] rings in order to benchmark our simulations and validity of Eq. (9). Based on Eq. (10), and converting the momentum offset to a variation of RF frequency

$$-\frac{\Delta F_{rf}}{F_{rf}} = \frac{\Delta L}{L_0} = (\alpha_3\delta^2+\alpha_2\delta+\alpha_1)\delta \quad (11)$$

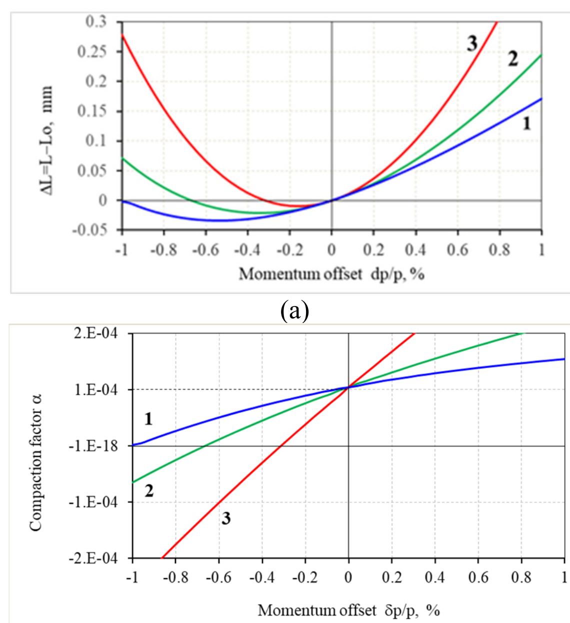
we precisely reproduce the synchrotron frequency as a function of RF frequency variation for SOLEIL and MLS with low–α optics [1]. We propose a consistent explanation of lifetime effects based on MLS tests at different low–α settings and octupoles current [16]. For a particular case of low–α optics when the first and third terms of momentum compaction factor are of different signs, and its product is negative ($\alpha_1\cdot\alpha_3$<0), one could estimate a momentum acceptance applying the condition of zero synchrotron tune ($F_s = 0$)

$$MA_{(Fs=0)} = \delta_{(Fs=0)} = \frac{1}{3\alpha_3}\left[-\alpha_2 \pm \sqrt{\alpha_2^2 - 3\alpha_1\alpha_3}\right] \quad (12)$$

Estimations of Touschek lifetime [1] based on Eq. (12) are in agreement with experiments at MLS [16]. We have applied computer model of KARA [1, 17] and calculated possible conditions to fill and store α–buckets at KARA taking into account limits imposed by the momentum acceptance of a ring at low–α operation. The energy deviations of stable fixed points (α–buckets) were estimated in three different ways : the first by calculating the roots, Eq. (8), of the second order equation, the second by calculating the roots, Eq. (9), of the third order equation, and the third by tracking the orbit for off-momentum particles [18]. For estimation purposes, the free momentum independent term was zeroed (χ=0). Thus, the focus point of RF buckets corresponds here to the reference energy with zero momentum offset (δ=0).

For positive values of the momentum compaction factor of KARA (α_1>0), the signs of the third and first terms are different, the curvature of compaction function as function of momentum offset is negative and three real roots exist. At high values of compaction factor α_1=+9·10^{-3}, the energy offsets of the stable fixed points, (±10%), well exceeds the momentum acceptance of the ring (±1%). Therefore, only RF buckets can be stored while in the 'user operation' regime of KARA.

Short bunch operation at 1.3 GeV and low positive compaction factor α_1=+1·10^{-4}, is another routine regime of the KARA storage ring. This mode is used for studies of beam bursting effects caused by coherent synchrotron radiation in the THz frequency range [19, 20]. In order to fill and realize simultaneous circulation of RF and α–buckets at a positive and negative low–α regime [21], one needs to adjust the energy gap between RF and α–buckets so as to fit both beams into the ring energy acceptance (Fig. 1). For this, we slightly decrease the strengths of SF and SD sextupole families, and reduce horizontal and vertical chromaticity. As a consequence, the value of the second term of momentum compaction factor



(a)

(b)

Figure 1: Reduction of the momentum offset of α–buckets with respect to the energy of the reference orbit at low–α mode with α_1=+1·10^{-4} by variation of second term of momentum compaction factor: (a) orbit lengthening as a function of momentum offset; (b) compaction factor vs energy deviation. Curve 1 (blue) corresponds to second term α_2=+7.13·10^{-3}, curve 2 (green) – α_2=+1.37·10^{-2} and curve 3 (red) – α_2=+3.26·10^{-2}.

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grows from α_2=+7·10^{-3} (curve 1 blue) up to α_2=+3.3·10^{-2} (curve 3 in red) and the slope of compaction factor curve α(δ) is increased.

This results in the orbit lengthening curve, ΔL(δ), as well as the momentum compaction factor curve, α(δ), crossing the zero axis (ΔL=0) and (α=0) at smaller values of energy offset (δ). The energy gap between RF and α–buckets, shown in Fig. 1, is reduced in absolute value from 1% down to 0.3%. One can capture both beams simultaneously providing the momentum offset between RF and α–buckets fits to the ring acceptance at low–α.

In Fig. 2 one can see the expected separation between two beams (RF and α–buckets) simultaneously stored in the KARA storage ring during low–α operation. Bunches at 0.5 GeV and 1.3 GeV are split in the vertical plane for clarity. Dispersion is negative D≈–1 m at the observation point It was assumed that the momentum offset between RF and α–buckets is 1%. The horizontal size of the low current 0.5 GeV beam is σ_x=0.7 mm (rms) and bunch width is increased to σ_x=1.2 mm (rms) at 1.3 GeV because the natural momentum spread grows from σ_E=+1.8·10^{-4} at 0.5 GeV to σ_E=+4.7·10^{-4} at 1.3 GeV.

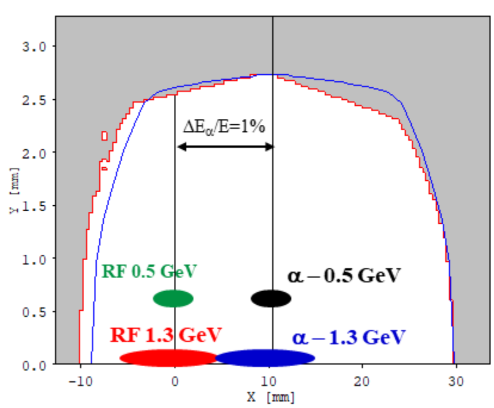

Figure 2: Expected images of RF and α–buckets to be simultaneously stored at KARA during low–α operation where the first term of momentum compaction factor is α_1=+1·10^{-4}. Bunches at 0.5 and 1.3 GeV are split in the vertical plane for clarity. Dispersion D≈–1m at observation point. It was assumed that momentum offset between RF and α–buckets is ~1%.

The blue contour line depicts the projection of the vacuum chamber at the observation point. For the dynamic aperture, shown as a red contour line, together with this momentum acceptance, is sufficient to store both beams. We simulated the energy offset of both RF and α–buckets at different values of orbit misalignments, associated with COD errors, by calculating roots, Eq. (9), of the third order equation Eq. (7) where the free term (χ) has been varied, see Fig. 3. The compaction factor was evaluated using the first term α_1=+1·10^{-4} and with the same starting conditions from Fig. 1. Initially on-momentum RF buckets ($\delta_1 = 0$) are progressively off-centered from the reference orbit because the momentum independent term of relative orbit lengthening, (χ), grows from a low level of orbit errors ($\chi < 10^{-9}$) to large errors of ($\chi > 1{\cdot}10^{-7}$); negative energy offset of α–buckets is reduced in absolute value at high levels of orbit errors.

For an initially small energy gap between RF and α–buckets (δ = 0.3%) it seems not to be possible to capture both beams simultaneously if the relative momentum independent term would exceed ($\chi > 5{\cdot}10^{-8}$). According to estimations, see Fig. 3, one should carefully center the beam and reduce orbit oscillations to less than 1 mm in order to limit relative free term to an acceptable level of ($\chi < 5{\cdot}10^{-8}$).

Precise adjustment of sextupole strength to limit the second term of momentum compaction factor within a small range between 7·10^{-3}< α_2<1·10^{-2} should allow to capture both beams simultaneously when the energy gap between RF and α–buckets is large enough to split both beams in space but still fit to a ring acceptance (0.6%<δ<1%). Momentum acceptance drops from 1% at second order term α_2≈0 down to 0.2% at α_2≈0.03. Similar estimations of possible α–buckets have been provided for negative low–α operation mode [1].

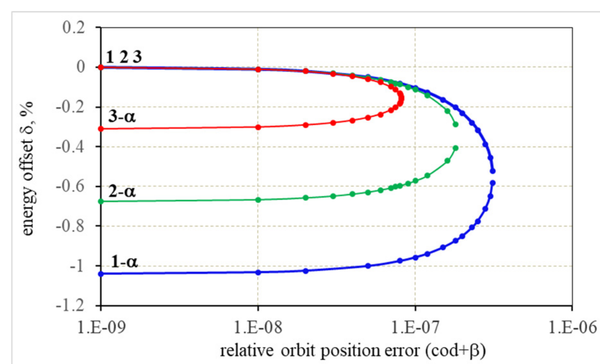

Figure 3: Energy offsets of α–buckets in the presence of momentum independent coherent orbit errors (χ). Positive low–α optics of the KARA storage ring with first term α_1=+1·10^{-4} was chosen as an example. Curves marked in blue represent RF (1) and α–buckets (1-α) with second term α_2=+7.13·10^{-3}, curves marked in green – RF (2) and α–buckets (2-α) with α_2=+1.37·10^{-2} and curves marked in red – RF (3) and α–buckets (3-α) with α_2=+3.26·10^{-2}.

CONCLUSION

Results of analytical studies based on second and third order equations were compared with high order computer tracking, and benchmarked against existing experiments at KARA, SOLEIL and MLS rings. We showed that for certain conditions, strong dependence of the synchrotron tune on the energy offset might limit the momentum acceptance and lifetime at low and negative–α operation.

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