DEVELOPMENT AND VALIDATION OF A PHASE-FIELD METHOD FOR AN ARBITRARY NUMBER OF IMMISCIBLE INCOMPRESSIBLE FLUIDS

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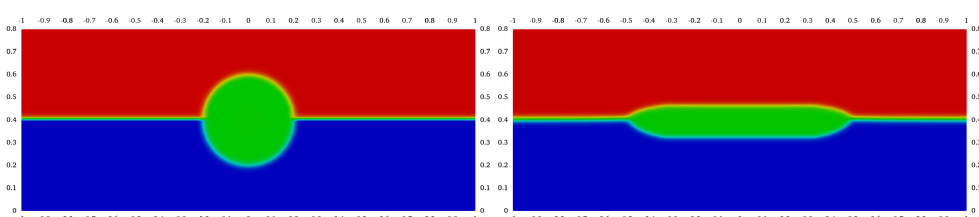
We present the most recent work concerning method development, implementation and validation of N ($N > 2$) immiscible, incompressible and isothermal fluids enhancing further our library for the diffuse interface phase-field interface capturing method in FOAM-extend 4.0/4.1. The phase-field method is an energic variational formulation based on the work of Cahn and Hilliard [1] where the interface is composed of a physical diffuse layer resembling realistic interfaces.

The extension from a two-phase ($N = 2$) to the generic multiphase ($N > 2$) flow framework requires obeying to consistency conditions. The model is fully reduction consistent [2, 3, 4], i.e. the N-phase model reduces to the traditional two-phase model upon existence of only two phases in the system. This principle is vital to eliminate generation of fictitious phases [4]. Furthermore, the model preserves phase volumes and ensures momentum conservation according to Huang et al. [4], which we follow in this study.

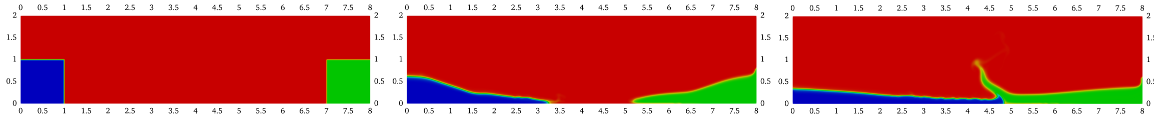
To validate the implemented model the floating liquid lens problem depicted in Figure 1 (a) is simulated. In this problem a liquid oil drop is initially floating on the air-water interface and will spread depending on the magnitude of the gravitational acceleration and surface tension coefficients. The final thickness of the drop will be compared to an equilibrium solution and an asymptotic analysis available in literature. Figure 1 (b) illustrates the final shape of the oil drop when $|g| = 9.8\, m^2/s$.



(a) Initial configuration (b) Final configuration $|g| = 9.8\, m^2/s$

Figure 1: Preliminary results for the floating liquid lens problem. Blue: Water, Red: Air, Green: Oil.

Moreover, the three-phase dam break problem depicted in Figure 2 is also considered as a validation case where both water and oil columns are initially held at rest by artificial walls. At $t = 0$ these walls are removed and both columns collapse. The results of this validation case will be compared with the experimental and numerical data available in the literature. Evolution of the phases at two different time instances are shown in Figure 2 (b) and (c), respectively.



(a) Initial configuration (b) $t = 0.15s$ (c) $t = 0.25s$

Figure 2: Preliminary results for the three phase dam break problem. Blue: Water, Red: Air, Green: Oil.

The validation of the liberary and solver will be subject to a comprehensive set of further validation covering both capillarity- and inertia-dominated cases.

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