COLLISION DETECTION FOR RIGID SUPERELLIPSOIDS USING THE NORMAL PARAMETERIZATION

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Summary The normal parameterization as an approach to describe geometries is introduced. The advantages of this description – as compared to other parameterizations or implicit functions – in the context of collision detection are: the possibility to explicitly calculate axis aligned bounding boxes for any convex geometry, an efficient iterative algorithm for collision detection between objects with arbitrary geometry that does not require any (analytical) derivatives. A system of several rigid superellipsoids is used to demonstrate the application and performance of the proposed approach in a multibody simulation.

MOTIVATION

Collision detection is an essential task in multibody models with unilateral contacts. While the geometry of complicated objects can be approximated using mesh-based approaches, good approximations require a high number of vertices, which in turn increases the computational effort and computation times. These approaches are widely used because they are very flexible and efficient collision detection algorithms are available for mesh-based geometry descriptions [1].

Alternative approaches use smooth representations based on implicit functions or some parameterization. This typically requires far fewer parameters than high-accuracy mesh approximations, however, it is more difficult to describe or approximate arbitrary geometries. A possible remedy to mitigate this drawback is to use unions of several objects. In this context, ellipsoids, superellisoids and superovoids are popular geometries for the approximation of objects such as particles in granular media [2], rigid foot models [3], robot fingers or human teeth [4], to name but a few examples.

Collision detection for objects with smooth geometry representations can be performed based on the common normal concept: the outer normal vector at the (potential) contact points on the boundaries of two objects are (anti)parallel. There are few explicit solutions for this problem, e. g. detecting collisions between two spheres. In general, efficient algorithms based on Newton iterations are proposed in the literature [4, 5]. For these collision detection problems, significant improvements in computation times compared to mesh-based approaches with high-accuracy are reported [4].

The necessity for iterative methods arises from the geometry representation by the commonly used parameterizations or by implicit functions. Any representation based on implicit functions requires an iterative solution to even determine points on a boundary surface; the calculation of two points on opposing boundaries with parallel normal vectors must therefore also be solved iteratively. Parameterizations, on the other hand, are based on a formula for the explicit calculation of all points on the boundary surface of an object, which also allows an explicit calculation of the corresponding normals. However, the inverse relationship, the calculation of a point on the boundary for a given normal direction, generally yields an implicit relationship which again must be solved iteratively.

With the aims to improve current approaches for collision detection problem between smooth objects and to contribute to the derivation of explicit solutions, we propose a new parameterization for convex objects which we call the *normal parameterization* which is based on [6]. A useful property is the possibility to calculate axis aligned bounding boxes explicitly for any geometry described in this way. To illustrate further properties and the possible use in collision detection applications, we demonstrate its use by means of a simulation with several rigid superellipsoids. Two collision detection algorithms, one based on a Newton iteration with analytic Jacobian and one based on a fixed-point iteration without the need of any derivatives, are presented. A transient time simulation with several rigid superellipsoids and multiple collisions is used to compare these algorithms to approaches from literature.

METHOD

Given a normal direction $\mathbf{n}$ in spherical coordinates $(\varphi, \theta) \in \mathbb{S}^2$, the normal parameterization gives the position $\mathbf{r}(\varphi, \theta) : \mathbb{S}^2 \to \mathbb{R}^3$ of the corresponding point P on the object's boundary relative to a body-fixed point B. In general, the normal parameterization of a strictly convex object follows from the *generating potential* $g(\varphi, \theta) : \mathbb{S}^2 \to \mathbb{R}$ as

$$\mathbf{r}(\varphi,\theta) = \begin{cases} \frac{\partial g(\varphi,\theta)}{\partial \varphi}\mathbf{e}_\varphi + (\sin\varphi)^{-1}\frac{\partial g(\varphi,\theta)}{\partial \theta}\mathbf{e}_\theta + g(\varphi,\theta)\mathbf{e}_\mathrm{n}\,, & \text{if } \varphi \in (0,\pi)\,, \\ \frac{\partial g(\varphi,\theta)}{\partial \varphi}\mathbf{e}_\varphi + (\cos\varphi)^{-1}\frac{\partial^2 g(\varphi,\theta)}{\partial \varphi \partial \theta}\mathbf{e}_\theta + g(\varphi,\theta)\mathbf{e}_\mathrm{n}\,, & \text{if } \varphi \in \{0,\pi\}\,, \end{cases}$$

where the normal direction $\mathbf{e}_\mathrm{n} = \mathbf{n}$ and the tangent directions $\mathbf{e}_\varphi, \mathbf{e}_\theta$ are rotated with respect to a body-fixed reference frame $\{\mathbf{b}_\mathrm{x}, \mathbf{b}_\mathrm{y}, \mathbf{b}_\mathrm{z}\}$ by

$$\begin{aligned} \mathbf{e}_\varphi &= \cos\varphi\cos\theta\,\mathbf{b}_\mathrm{x} + \cos\varphi\sin\theta\,\mathbf{b}_\mathrm{y} - \sin\varphi\,\mathbf{b}_\mathrm{z}\,, \\ \mathbf{e}_\theta &= -\sin\theta\,\mathbf{b}_\mathrm{x} + \cos\theta\,\mathbf{b}_\mathrm{y}\,, \\ \mathbf{e}_\mathrm{n} &= \sin\varphi\cos\theta\,\mathbf{b}_\mathrm{x} + \sin\varphi\sin\theta\,\mathbf{b}_\mathrm{y} + \cos\varphi\,\mathbf{b}_\mathrm{z}\,, \end{aligned}$$

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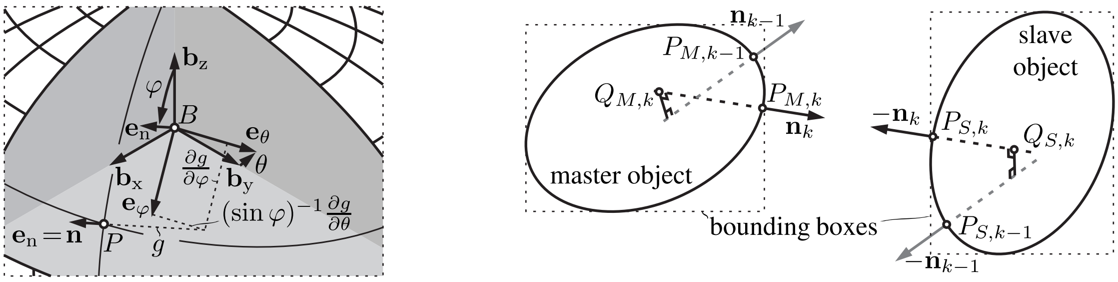

(a) (b)

Figure 1: (a) Reference frames for the definition of the normal parameterization; (b) collision detection between two objects.

cf. figure 1a. Superellipsoids follow from the generating potential

$$g(\varphi,\theta)=\Big(\Big(\big(a^2\sin^2\varphi\cos^2\theta\big)^{\frac{\delta}{2(\delta-1)}}+\big(b^2\sin^2\varphi\sin^2\theta\big)^{\frac{\delta}{2(\delta-1)}}\Big)^{\frac{\epsilon(\delta-1)}{\delta(\epsilon-1)}}\big(c^2\cos^2\varphi\big)^{\frac{\epsilon}{2(\epsilon-1)}}\Big)^{\frac{\epsilon-1}{\epsilon}}$$

with $a, b, c, \delta, \epsilon \in \mathbb{R}$, $a, b, c > 0$ and $\delta, \epsilon > 1$. With this, the parameterization fulfills the implicit equation

$$\Big(\big((x/a)^2\big)^{\delta/2}+\big((y/b)^2\big)^{\delta/2}\Big)^{\epsilon/\delta}+\big((z/c)^2\big)^{\epsilon/2}=1\,.$$

Collision detection is performed in two steps: first, axis aligned bounding boxes are used to quickly check whether two objects may overlap; if this test does not rule out a possible collision, one of two iterative procedures is employed for accurate collision detection. The calculation of an axis aligned bounding box for any object with arbitrary orientation with respect to an inertial frame $\{\mathbf{i}_\mathrm{x}, \mathbf{i}_\mathrm{y}, \mathbf{i}_\mathrm{z}\}$ is simple. To determine the distance of the bounding box from the body-fixed point B in any direction $\{\pm\mathbf{i}_\mathrm{x}, \pm\mathbf{i}_\mathrm{y}, \pm\mathbf{i}_\mathrm{z}\}$, calculate (φ, θ) from $\mathbf{e}_\mathrm{n}$ for the respective direction and evaluate $g(\varphi, \theta)$. If the bounding boxes of two objects do not overlap, there can be no collision.

If there is an overlap of the bounding boxes, two iterative algorithms for accurate collision detection based on the common normal concept between two objects with arbitrary position and orientation are considered. For both algorithms, chose one object as master object with index M and the other as slave with index S. Let $\mathbf{n}_k = x_k\mathbf{i}_\mathrm{x} + y_k\mathbf{i}_\mathrm{y} + z_k\mathbf{i}_\mathrm{z}$ with $x_k^2 + y_k^2 + z_k^2 = 1$ be the normal direction for the master object in the k-th iteration step. Determine the corresponding coordinates $(\varphi_{M,k}, \theta_{M,k})$ from $\mathbf{e}_{\mathrm{n},M,k} = \mathbf{n}_k$ and the coordinates $(\varphi_{S,k}, \theta_{S,k})$ from $\mathbf{e}_{\mathrm{n},S,k} = -\mathbf{n}_k$ due to the common normal concept. Evaluate the normal parameterization to determine the points $P_{M,k}$ and $P_{S,k}$ on the objects' boundaries, cf. figure 1b.

Algorithm I is based on a Newton iteration with analytic Jacobian and is adapted from [5]. Let $\mathbf{d}_{MS,k}$ be the vector from $P_{M,k}$ to $P_{S,k}$ and calculate a Newton step to zero the two tangential components $\mathbf{d}_{MS,k} \cdot \mathbf{e}_{\varphi,M,k}$ and $\mathbf{d}_{MS,k} \cdot \mathbf{e}_{\theta,M,k}$ and determine $\mathbf{n}_{k+1}$. Algorithm II is based on a fixed-point iteration for $\mathbf{n}$, which is designed such that no derivatives are required. Determine $Q_{M,k}$ as the point on the line given by $P_{M,k}$ and $\mathbf{e}_{\mathrm{n},M,k}$ which is closest to the same line from the previous iteration $k-1$ (cf. figure 1b). $Q_{M,k}$ is an approximation the center of curvature; $Q_{S,k}$ is determined analogously. The next iterate $\mathbf{n}_{k+1}$ is then taken as the unit vector pointing from $Q_{M,k}$ into the direction of $Q_{S,k}$.

RESULTS AND CONCLUSIONS

The collision detection approach is applied to a multibody system with several rigid superellipsoids. The investigated example systems show that the proposed approach works for rigid body simulations. Algorithm II which is based on a fixed-point iteration shows better robustness but slower convergence than algorithm I. The ability to explicitly calculate axis aligned bounding boxes is an advantage of the proposed normal parameterization, as is the possibility for an accurate and robust collision detection procedure which does not require any derivatives.

References

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