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Low cost estimation of Wöhler and Goodman–Haigh curves of Ti-6Al-4V samples by considering the stress ratio effect

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Abstract

The stress ratio effect on the fatigue life of materials is a topic which have been studied by two different approaches. On the one hand, several experiments, performed under different stress ratios are necessary to estimate the corresponding Wöhler curves. Afterwards, these curves are considered to estimate the fatigue life under a particular stress range. On the other hand, fatigue failure criteria for fluctuating stress like the Goodman–Haigh relationship, are applied to estimate the stress amplitude for a constant fatigue life. Based on the Stüssi function, this paper presents a low cost model to estimate Wöhler curves and constant fatigue Goodman–Haigh diagrams. This procedure requires a set of tests performed under a particular stress ratio from LCF to HCF, and data from minimum two additional stress ranges for each subsequent stress ratio. An application on data from Ti-6Al-4V samples manufactured by selective laser melting (SLM) is presented.

KEYWORDS

life estimation, mean stress, mean stress effects, stress ratio

1 | INTRODUCTION

The influence of stress ratio R on the fatigue life of materials is a relevant but also complex research topic.

Among the studies on the influence of the mean stress on the fatigue strength, the role of the maximum stress was first proposed in the work of Smith et al[1] together with the work of Walker.[2] Subsequently, many studies have been undertaken to propose relevant models but unfortunately, a deep study of this situation implies high costs and long time of experimentation.

Two different approaches are mainly applied in order to study how the stress ratio affects the fatigue lifetime of structures. On the one hand, based on experimental data, Wöhler or $S-N$ curves are defined for different stress ranges and stress ratios. Several deterministic and probabilistic models have been proposed in other studies[3–17] to model and depict the Wöhler curves, which afterwards can be used to perform estimations of the fatigue life, see Table 1.

Particularly, the linear model of Basquin, because its simple implementation, is the most used in the official

TABLE 1 Some models to represent the Wöhler curves

Model	Wöhler curves equation
Basquin (1910)	$\log N = A - B \log \Delta\sigma;\ \Delta\sigma \geq \Delta\sigma_\infty$
Stromeyer (1914)	$\log N = A - B \log(\Delta\sigma - \Delta\sigma_\infty)$
Palmgreen (1924)	$\Delta\sigma = b(N+B)^{-a} + \Delta\sigma_\infty$
Bastenaire (1972)	$N = \frac{A}{\Delta\sigma - E}\exp[-C(\Delta\sigma - E)] - B$
Ling and Pan (1997)	$F = \sum_{i=1}^{n}\left\{\ln\sigma(S_i) + \frac{[\log N_i - \mu(S_i)]^2}{2\sigma^2(S_i)}\right\}$
Kohout and Věchet (2001)	$\log\left(\frac{\Delta\sigma}{\Delta\sigma_\infty}\right) = \log\left(\frac{N+N_1}{N+N_2}\right)^b$
Castillo *et al.* (2009)	$p = 1 - \exp\left\{-\left[\frac{(\log N - B)(\log\Delta\sigma - C) - a}{b}\right]^c\right\}$

standards and guidelines such as Eurocode 3,[18] International Institute of welding (IIW),[19,20] ISO12107,[21] German railways,[22] Swiss standard,[23] and guidelines for design of wind turbines.[24] However, it has strong limitations, and it is based in several arbitrary assumptions, so that its reliability has been questioned, see Castillo and Fernández-Canteli[25] and Toasa Caiza.[26]

However, this methodology implies performing several experiments under different stress ranges and different stress ratios, which demands high costs of samples manufacturing and testing, and a long experimentation time. On the other hand, there are some failure criteria which have been proposed to estimate the fatigue strength for a particular fatigue life. These criteria, however, are empirical and require the knowledge of the fatigue limit at a stress ratio $R = -1$. Moreover, they have some limitations such as the assumption of linearity as in case of Goodman–Haigh or Soderberg criteria and the high costs of experimentation as well.

Besides these main approaches, some authors have studied the effect of stress ratio by considering other principles such the continuum mechanics,[27] the fatigue damage,[28] the eigenvalue optimization,[29] or the strain-energy-density approach.[30]

In order to overcome the limitations and constraints mentioned above, a method, which reduces the necessary costs to model both, the Wöhler curves, and the fatigue diagrams type Goodman–Haigh for different stress ratios, is proposed. To reach this goal, it is necessary to have a set of data corresponding to experiments performed under certain stress ratio and several stress ranges from low-cycle fatigue (LCF) to high-cycle fatigue (HCF) regimes. These data will be used to estimate the first Wöhler curve which will serve as reference for the subsequent estimations. Additionally, for every subsequent stress ratio, as a minimum two experimental data or their estimators are necessary. One data should come from a stress range in LCF regime and other from a stress range in HCF regime.

An application by considering experimental data of porous titanium (Ti-6Al-4V) samples made by selective laser melting (SLM), from Krijger et al[31] is presented in order to validate the proposed model. The obtained results are quite similar to those obtained by the common methods mentioned above.

2 | THE STRESS-BASED APPROACHES

In order to study the effect of stress ratio on the fatigue life, two stress-based approaches are usually applied. The first alternative is modeling the Wöhler curves from experimental data performed under different stress ranges and different stress ratios. The second one is based on the fatigue strength for a particular fatigue life, which is known as Goodman–Haigh or Soderberg criterion.

In this paper, the Stüssi function is applied to model the Wöhler curves and Goodman–Haigh diagrams.

2.1 | The Stüssi function

The modified Stüssi function proposed by Toasa Caiza and Ummenhofer in [32] will be considered in order to model the Wöhler curves. This function is given by

$$\Delta\sigma = \frac{R_m(1-R) + \alpha N^\beta \Delta\sigma_\infty}{1 + \alpha N^\beta} = \mathrm{S}(N,R), \qquad |R| \leq 1 \tag{1}$$

where

$\Delta\sigma$: stress range during the fatigue test
N: number of load cycles up to failure or up to end of the test
R_m: ultimate tensile strength
$\Delta\sigma_\infty$: fatigue or endurance limit corresponding to the stress ratio R.
α, β: geometrical parameters
R: stress ratio
S: Stüssi function.

The model given by Equation (1) depends on two geometrical parameters α and β which can be estimated by applying a linear regression and on two material parameters R_m and $\Delta\sigma_\infty$ which are supposed to be known. For more details about the estimation of the geometrical parameters see Toasa Caiza and Ummenhofer.[33] Considering that the upper bound of the curve depends on the ultimate tensile strength, R_m is an assumption to simplify the model. In fact, this phenomenon is described by the Bauschinger effect that results in a change in tensile or compression yield strength when the direction of loading is reversed after prior plastic deformation. However, since the fatigue strength estimation based on the stress is relevant in HCF and VHCF regimes, this assumption does not harm the goal of this work. If the goal is estimating in ULCF or LCF regimes, where the plastic behavior prevails, a better accuracy in the estimation can be achieved by considering the strain.[32] Assuming that the fatigue endurance $\Delta\sigma_\infty$ is known, may be controversial, but it can be estimated by applying the Weibull model proposed by Castillo and Fernandez-Canteli.[25] Nevertheless, the existence of the fatigue limit is still an open debate.[34–36]

The Stüssi function offers a good geometrical approach to depict the theoretical fatigue behavior of a material. Moreover, it describes clearly the asymptotic behavior regarding the ultimate tensile strength R_m and the fatigue limit $\Delta\sigma_\infty$ since

$$\lim_{N\to 0} \Delta\sigma = R_m,$$

$$\lim_{N\to\infty} \Delta\sigma = \Delta\sigma_\infty.$$

A typical Wöhler curve based on the Stüssi function given by Equation (1) is depicted in Figure 1.

The Stüssi function can be also applied to consider the strain as the variable of interest. In this case, together with the Ramberg–Osgood relationship allow to model curves of the type $\Delta\epsilon - N$, see.[32]

2.2 | The Goodman–Haigh relationship

In order to estimate the fatigue resistance of structures when subjected to a variation of both the mean stress σ_μ and the stress amplitude σ_a, some criteria have been proposed, see Table 2. Among these criteria, the modified Goodman–Haigh is the most used one, since its linearity allows to perform calculations and plots easily, see Budynas and Nisbett.[37]

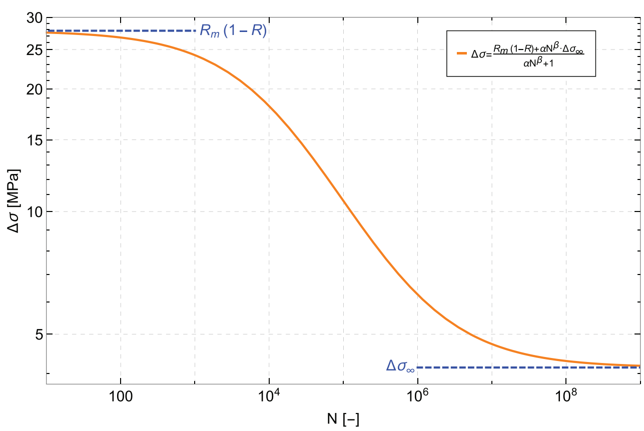


FIGURE 1 Wöhler curve based on the Stüssi function [Colour figure can be viewed at wileyonlinelibrary.com]

These criteria are depicted as fatigue diagrams with the mean stress σ_μ plotted along the abscissa and the stress amplitude σ_a on the ordinate. The yield stress R_{el} and the ultimate tensile strength R_m have to be plotted. If the fatigue strength S_e for a particular number of load cycles N and stress ratio $R = -1$ is known, it can be plotted as well, see Figure 2.

3 | PROCEDURE TO MODEL THE WÖHLER CURVES AND GOODMAN–HAIGH DIAGRAMS BASED ON THE STÜSSI FUNCTION FOR DIFFERENT STRESS RATIOS

In order to estimate the Wöhler curves and Goodman–Haigh diagrams based on the Stüssi function, fatigue experiments performed from LCF to HCF at the same stress ratio are necessary. Moreover, additional experiments only from one stress range at LCF and one stress

TABLE 2 Failure criteria models, where n is the design factor or factor of safety, R_m is the ultimate tensile strength, R_{el} is the yield stress and S_e is the fatigue strength at $R = -1$

Model	Equation
Goodman (1899) Haigh (1917)	$\frac{\sigma_a}{S_e} + \frac{\sigma_\mu}{R_m} = \frac{1}{n}$
Soderberg (1930)	$\frac{\sigma_a}{S_e} + \frac{\sigma_\mu}{R_{el}} = \frac{1}{n}$
Gerber	$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_\mu}{R_m}\right)^2 = 1$
ASME	$\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_\mu}{R_{el}}\right)^2 = 1$

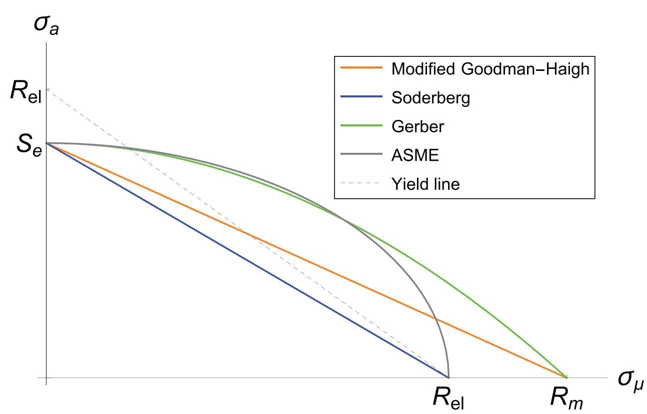


FIGURE 2 Fatigue diagram showing various failure criteria for a design factor $n = 1$. For each criterion, points on or above the respective curve indicate failure [Colour figure can be viewed at wileyonlinelibrary.com]

range at HCF for each subsequent stress ratio have to be available.

3.1 | Wöhler curves $\Delta\sigma - N$ and $\sigma_{max} - N$ based on the Stüssi function

In order to model the Wöhler curves for any stress ratio R, the linearization of the Stüssi function has to be considered.

By manipulating the Stüssi equation (1), this can be written as follows:

$$\alpha N^{\beta} = \frac{R_m(1-R)-\Delta\sigma}{\Delta\sigma-\Delta\sigma_{\infty}}. \tag{2}$$

Then, by applying logarithms in Equation (2), the Stüssi equation can be written in a linear form as follows:

$$\log(N) = \frac{1}{\beta}\log\left(\frac{R_m(1-R)-\Delta\sigma}{\Delta\sigma-\Delta\sigma_{\infty}}\right) - \frac{1}{\beta}\log(\alpha). \tag{3}$$

The Equation (3) is no more than an elementary linear equation of the type

$$Y = aX + b, \tag{4}$$

where

$Y = \log(N)$,
$X = \log\left(\frac{R_m(1-R)-\Delta\sigma}{\Delta\sigma-\Delta\sigma_{\infty}}\right)$,
$a = \frac{1}{\beta}$,
$b = -\frac{1}{\beta}\log(\alpha)$.

A graphical representation of Equation (4) is shown in Figure 3.

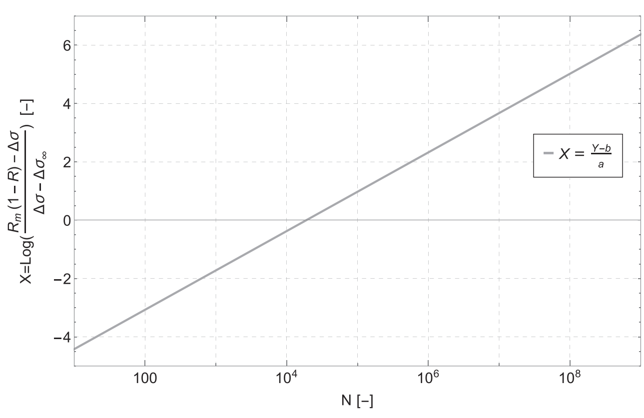


FIGURE 3 Linearization of the Stüssi function [Colour figure can be viewed at wileyonlinelibrary.com]

The parameters a and b of Equation (4) can be estimated by applying an elementary regression analysis on experimental data of fatigue failures. Afterwards, the geometrical parameters α and β of Equation (3) are given by

$$\beta = \frac{1}{a}, \tag{5}$$

$$\alpha = \exp(-b\cdot\beta). \tag{6}$$

Once the geometrical parameters α and β have been estimated, it is possible to depict the Wöhler curve $\Delta\sigma - N$ based on the Stüssi function, which was introduced by Equation (1).

Afterwards, according to the definition $\Delta\sigma = \sigma_{max}\cdot(1-R)$ and considering Equation (1), the Wöhler curve $\sigma_{max} - N$ based on the Stüssi function for a particular stress ratio R can be obtained by

$$\sigma_{max} = \frac{S(N,R)}{1-R}. \tag{7}$$

3.2 | Modeling the Wöhler curves $\Delta\sigma - N$ for a particular stress ratio R

In order to model the Wöhler curves for a particular stress ratio R, the linearization of the Stüssi function given by Equation (3) has to be considered.

From elementary geometry, it is known that a straight line can be uniquely defined by two points. In other words, it is possible to consider two given points ($\Delta\sigma_1$, N_1) and ($\Delta\sigma_2$, N_2) to define a straight line like Equation (3), which depends on the geometrical parameters α and β. These two points will correspond to two experiments performed under a particular stress ratio R.

It is important to keep in mind that in a deterministic model two points are enough to define a straight line. However, the fatigue of materials is far away of being a deterministic phenomenon. In fact its random nature demands additional considerations. For this reason, it is suggested to consider two series of experiments, each one performed under given stress ranges $\Delta\sigma_1$ and $\Delta\sigma_2$.

Both series can be denoted as

$$(\Delta\sigma_1, N_i), \quad i = 1,2,\ldots,n \tag{8}$$

and

$$(\Delta\sigma_2, N_j), \quad j = 1,2,...,m. \tag{9}$$

Afterwards, a suitable statistic method should be applied in order to estimate the fatigue lives N_i and N_j for each stress range. Since it has been proved, that an extreme value distribution describes properly the random variable corresponding to fatigue life, an analysis based on the Weibull distribution can be applied to perform the mentioned estimations, see Castillo and Fernández-Canteli[25] and Toasa Caiza.[26]

Also the physical characteristics of the fatigue should be taken into account. For this reason, in order to depict properly the transition from the plastic into the elastic behavior, it is suggested that the first stress range $\Delta\sigma_1$ belongs to LCF regime, and for the transition into the fatigue limit region, the second stress range $\Delta\sigma_2$ has to be obtained from HCF regime.

Denoting by $\widehat{N_1}$ and $\widehat{N_2}$ the estimations of the fatigue life corresponding to the stress ranges $\Delta\sigma_1$ and $\Delta\sigma_2$, the experimental points to be considered to define the straight line are the following.

$$Y_k = \log(\widehat{N_k}) \quad k = 1,2 \tag{10}$$

and

$$X_k = \log\left(\frac{R_m(1-R) - \Delta\sigma_k}{\Delta\sigma_k - \Delta\sigma_\infty}\right) \quad k = 1,2. \tag{11}$$

Then, according to the standard equation of a straight line, the parameters a and b from Equation (4) can be calculated as follows.

$$a = \frac{Y_2 - Y_1}{X_2 - X_1} \tag{12}$$

and

$$b = Y_1 - a \cdot X_1 = Y_2 - a \cdot X_2. \tag{13}$$

Consequently, the parameters α and β from Equation (3) can be defined by

$$\beta = \frac{1}{a} = \frac{X_2 - X_1}{Y_2 - Y_1}, \tag{14}$$

and

$$\alpha = \exp(X_1 - \beta \cdot Y_1) = \exp(X_2 - \beta \cdot Y_2). \tag{15}$$

Finally, by replacing the results given by Equations (14) and (15) in Equations (1) and (7), the Wöhler curves $\Delta\sigma - N$ and $\sigma_{max} - N$ based on the Stüssi function for a particular stress ratio R can be obtained.

3.3 | Constant fatigue life diagrams type Goodman–Haigh

Once the Wöhler curves $\Delta\sigma - N$ have been defined, it is possible to estimate and depict fatigue diagrams such as the Goodman–Haigh failure criteria for a constant fatigue life. These diagrams describe the fatigue strength for a constant fatigue life while the stress ratio is varying.

The first step is choosing a constant fatigue life given by a particular number of loading cycles N_i and a particular stress ratio R_j. Afterwards, by applying the Equation (1) for N_i, the corresponding mean stress $\sigma_{\mu,\,i,\,j}$ and stress amplitude $\sigma_{a,\,i,\,j}$ can be defined by

$$\sigma_{\mu,i,j} = \frac{S(N_i,R_j)}{2} \cdot \left(\frac{1+R_j}{1-R_j}\right) \tag{16}$$

and

$$\sigma_{a,i,j} = \frac{S(N_i,R_j)}{2}. \tag{17}$$

Then, by varying the stress ratio R_j and keeping constant the load cycles N_i, a set of points corresponding to a constant fatigue life are obtained. Moreover, in order to better visualize the values corresponding to the considered stress ratios R_j, the straight lines given by

$$\sigma_a = \sigma_\mu \cdot \left(\frac{1-R_j}{1+R_j}\right). \tag{18}$$

have to be plotted.

An example of a constant fatigue life diagram is shown in Figure 4.

4 | APPLICATION ON SAMPLES MADE OF POROUS TITANIUM TI-6AL-4V

In order to apply the theoretical results from the previous section, experimental data corresponding to compression fatigue data performed on samples made of porous titanium Ti-6Al-4 are considered. The samples were built on top of a solid titanium build plate from which they were subsequently removed using wire electrical discharge machining (EDM). For more details, see the article from de Krijger et al.[31] It is worth mentioning that in their work, the authors changed the notation of the variables

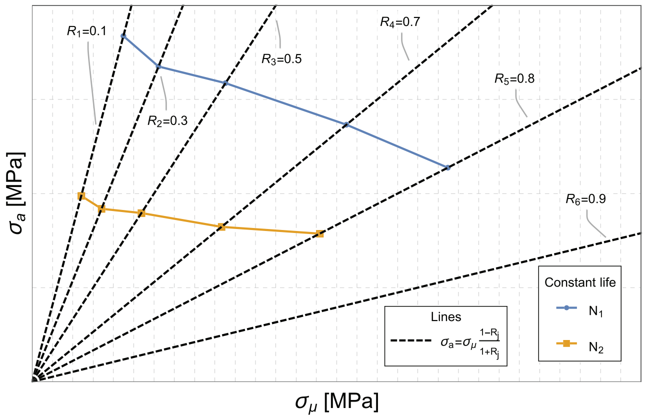


FIGURE 4 Constant fatigue life diagram based on the Stüssi function. Points on or above the respective curve indicate failure [Colour figure can be viewed at wileyonlinelibrary.com]

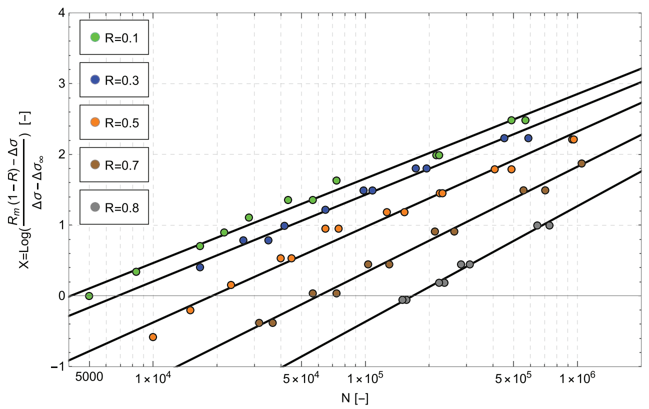


FIGURE 5 Linear regression of the Stüssi curves obtained by considering all of the experimental data [Colour figure can be viewed at wileyonlinelibrary.com]

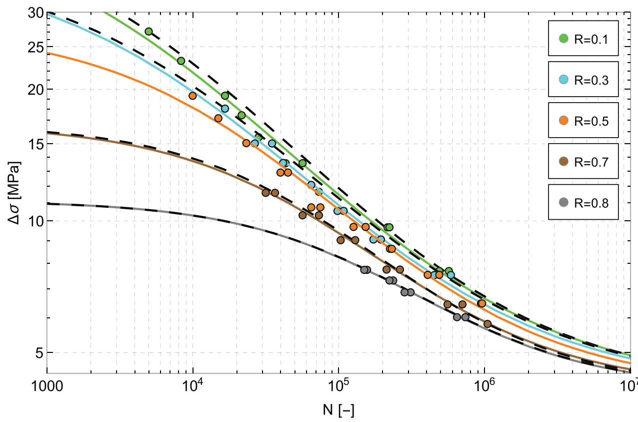


FIGURE 6 Wöhler curves by considering all of the experimental data (continuous lines) and by considering only the experimental data at $R = 0.5$ and only two experimental points (one with the lowest loading cycles and other with the highest loading cycles) at different stress ratios R (dashed black lines) [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 3 Mechanical properties of the porous structures made of Titanium Ti-6Al-4V used to manufacture the samples

Material properties of Titanium Ti-6Al-4V			
Ultimate Tensile strength	R_m	55.6	[MPa]
Yield strength	R_{el}	43.0	[MPa]
Fatigue limit	$\Delta\sigma_\infty$	4.14	[MPa]

TABLE 4 Geometrical parameters of the Stüssi function

Estimated geometrical parameters					
		Common method		Proposed method	
i	$\mathbf{R}_i$	α_i	β_i	$\hat{\alpha}_i$	$\hat{\beta}_i$
1	0.1	0.013	0.518	0.011	0.533
2	0.3	0.009	0.532	0.009	0.530
3	0.5	0.003	0.586	—	—
4	0.7	0.000	0.650	0.001	0.672
5	0.8	0.000	0.709	0.000	0.706

Note: Estimation based on all the experimental data and on the proposed method.

as follows. The σ_{max} is given by the absolute value of the maximum compression stress, while in the traditional notation the maximum compression will be considered as σ_{min}. This change of notation also affects the stress ratio. Thus, the stress ratio is given by $R =$ (minimum compression / maximum compression) $= \sigma_{max}/\sigma_{min}$, which in the traditional notation will be denoted as $1/R$. In this study, the changed of notation is kept as well.

The compression fatigue experiments were performed at five stress ratios $R = 0.1, 0.3, 0.5, 0.7, 0.8$.

The mechanical properties of the material are shown in Table 3.

The experimental values corresponding to the maximum stress σ_{max} and loading cycles N were obtained by applying the web plot digitizer[38] on the Fig. 5a of the article from Krijger et al.[31]

The estimation of the fatigue limit $\Delta\sigma_\infty$ for every stress ratio was performed by applying the method proposed by Castillo and Fernández-Canteli.[25] Theoretically, there is a different fatigue limit in the $\Delta\sigma - N$ field for every stress range R. However, from a conservative engineering point of view and for safety reasons, it is safe to choose the lowest estimation of the fatigue limit. Moreover, in case of compression, the Goodman–Haigh diagrams assume a constant fatigue limit as well.

4.1 | Wöhler curves $\Delta\sigma - N$ and $\sigma_{max} - N$

In order to validate the proposed model, the first step is considering all of the experimental results to find the equation of the Stüssi function for every stress ratio. As it was explained in Section 2.1, the Stüssi function depends on two geometrical parameters α and β which can be estimated based on the experimental data. Afterwards, the proposed estimation of these geometrical parameters is applied by using the Equations (14) and (15) which depend on data only from two stress ranges.

The results of the estimation of the geometrical parameters α and β obtained by evaluating all the experimental data for each stress ratio and obtained by applying the proposed method are shown in Table 4.

The linear regressions corresponding to the experimental data and to the geometrical parameters α_i and β_i of Table 4 are depicted in Figure 5.

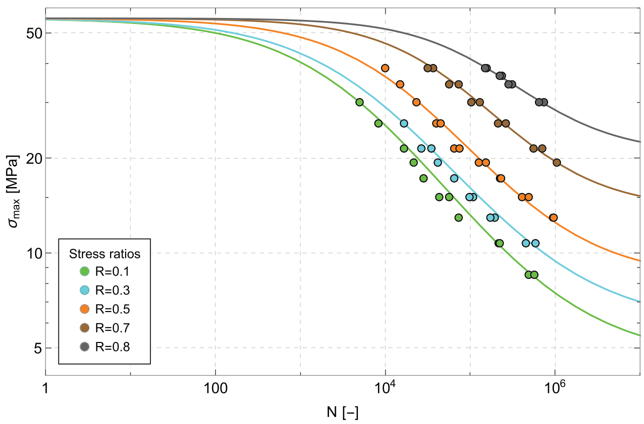


FIGURE 7 Wöhler curves $\sigma_{max} - N$ obtained by considering only the experimental data at $R = 0.5$ (in orange) [Colour figure can be viewed at wileyonlinelibrary.com]

The Wöhler curves obtained by applying the Stüssi function on each set of experimental data are shown in Figure 6.

Afterwards, only the experimental values corresponding to the stress ratio $R = 0.5$ and two experimental points of each stress range were considered in order to apply the proposed method to obtain the Wöhler curves for other stress ratios. The Wöhler curves obtained by applying the proposed method are shown in Figure 6 as well.

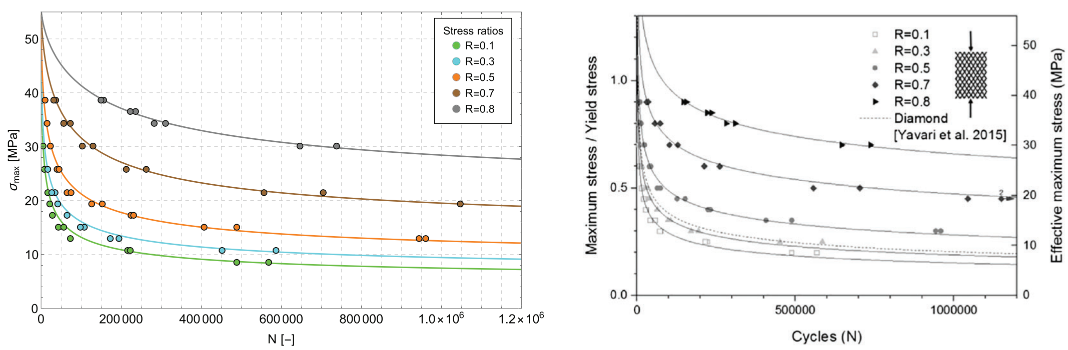
As it can been seen in Figure 6, the dashed black curves obtained by applying the proposed method are very similar to those obtained by considering all the experimental data for a particular stress ratio. This fact implies that it is possible to estimate the Wöhler curves for other stress ratios by considering minimum two experiments, ideally one from LCF and other from HCF. In other words, by applying the proposed method, the time and costs of the fatigue experiments can be markedly reduced.

As it was explained in Section 3.1, Wöhler curves $\sigma_{max} - N$ for a particular stress ratio can be defined by applying Equation (7) as well. The Wöhler curves obtained by applying the mentioned equation are shown in Figures 7 and 8A.

Particularly, the curves depicted in Figure 8A are quite similar to those obtained by Krijger et al. who applied an exponential fit. The curves obtained by Krijger et al. are depicted in Figure 8B which has been taken from de Krijger et al.[31]

4.2 | Constant fatigue life diagrams type Goodman–Haigh

In this section, the constant fatigue life diagrams type Goodman–Haigh will be estimated by applying the methodology presented in Section 3.3. In this



(A) Considering only the experimental data at R=0.5.

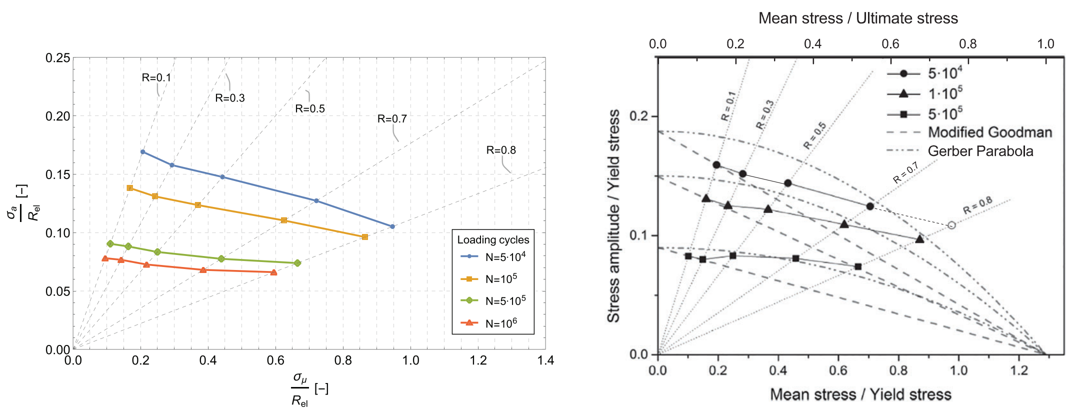
(B) See Figure 5.a in Krijger *et al.*[31]

FIGURE 8 (A,B) Cartesian Wöhler curves $\sigma_{max} - N$. Both are very similar [Colour figure can be viewed at wileyonlinelibrary.com] application, four constant fatigue lives were selected to depict their corresponding diagrams type Goodman–Haigh by applying the Equations (16) and (17). The estimations of the mean stress σ_μ and stress amplitude σ_a corresponding to the selected fatigue lives are shown in Table 5.

The diagrams corresponding to the obtained results are shown in Figure 9A. In this case, the values of the mean stress and stress amplitude were divided by the yield strength R_{el} in order to compare the obtained plot with that obtained by Krijger et al. shown in Figure 9B which has been taken from de Krijger et al.[31] In this case, the plots obtained by applying the proposed method are quite similar to those obtained by Krijger et al. as well. This fact show how suitable and reliable the proposed method is.

TABLE 5 Estimation of the mean stress and stress amplitude obtained by applying the proposed method

Mean stress and stress amplitude									
i	R_i	$N_1 = 5 \cdot 10^4$		$N_2 = 10^5$		$N_3 = 5 \cdot 10^5$		$N_4 = 10^6$	
		$\sigma_{\mu,i,1}$	$\sigma_{a,i,1}$	$\sigma_{\mu,i,2}$	$\sigma_{a,i,2}$	$\sigma_{\mu,i,3}$	$\sigma_{a,i,3}$	$\sigma_{\mu,i,4}$	$\sigma_{a,i,4}$
1	0.1	8.89	7.27	7.26	5.94	4.75	3.89	4.10	3.36
2	0.3	12.60	6.78	10.46	5.63	7.04	3.79	6.13	3.30
3	0.5	19.04	6.35	15.93	5.31	10.76	3.59	9.37	3.12
4	0.7	31.01	5.47	26.89	4.75	18.90	3.33	16.59	2.93
5	0.8	40.70	4.52	37.21	4.13	28.59	3.18	25.59	2.84



(A) Considering only the experimental data at R=0.5.

(B) See Figure 11 in Kriejger *et al.*[31]

FIGURE 9 Curves type Goodman–Haigh [Colour figure can be viewed at wileyonlinelibrary.com]

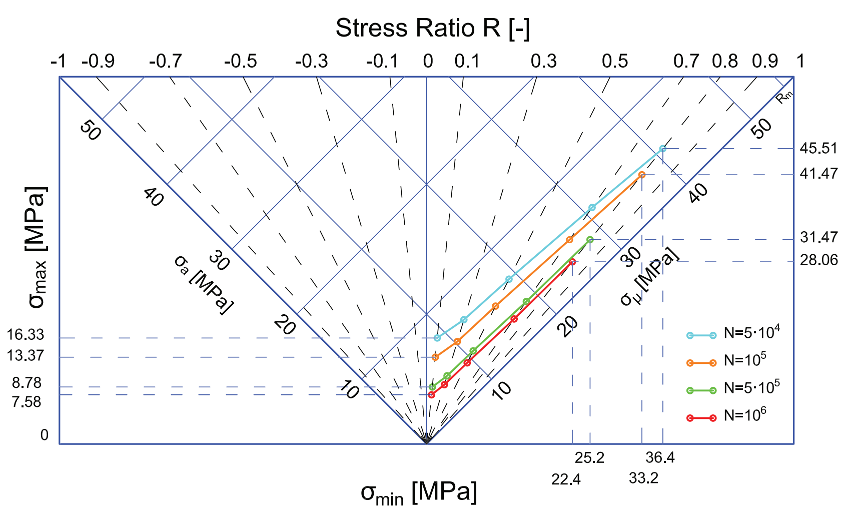


FIGURE 10 Master fatigue life diagram [Colour figure can be viewed at wileyonlinelibrary.com]

Finally, the master fatigue diagram corresponding to the obtained results is shown in Figure 10. This diagram displays four of the stress components as well as the stress ratios.

5 | CONCLUSIONS

As it has been in Section 4, the proposed method based on the modified Stüssi function allows to model Wöhler curves and diagrams type Goodman–Haigh with less costs than the traditional methods. To do this, it is necessary to have fatigue data performed under several stress ranges and a particular stress ratio like in a common experimental campaign. Additionally, minimum two additional data, ideally one from LCF and other from HCF for every subsequent stress ratio are required. Even though because of the random nature of fatigue, it is suggested considering statistical estimators of the fatigue life of one stress range in LCF and other stress range in HCF. Thus, the reliability of the method and the accuracy of the results will improve.

This fact, represents the main advantage of the proposed method, since the time and costs of the required experiments is markedly reduced.

The proposed method was applied on experimental data of samples made of porous titanium Ti-6Al-4V and provided similar results to those obtained by analyzing the data coming from extensive experimental campaigns.

Despite the encouraging results obtained during this investigation, it is necessary to perform additional experiments to evaluate the proposed method. These experiments, have to consider more different values of stress ratio and different materials or structures. In other words, the subsequent research about the suitability and reliability of the proposed model has just began.

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AUTHOR CONTRIBUTIONS

Mr. Toasa Caiza (TC) Developed the mathematical model proposed in this manuscript. Implementation of the algorithms and programming routines corresponding to the model. Evaluated the experimental data by applying the proposed model. Wrote the initial draft of this manuscript. Mr. Sire (S) Assisted TC to evaluate the suitability of the model and the reliability of the results obtained by the evaluation of the experimental data. Cooperate in the interpretation of the obtained results. Provided valuable suggestions and comments prior to the submission of this manuscript. Mr. Ummenhofer (Um) Assisted TC by mentioning important technical facts to be considered in the proposed model. Cooperate in the interpretation of obtained results. Provided valuable suggestions and comments prior to the submission of this manuscript. Mr. Uematsu (Ue) Assisted TC by proposing relevant technical considerations to be taken into account in the proposed model. Cooperate in the interpretation of obtained results. Provided valuable suggestions and comments prior of the submission to this manuscript. All authors agree to be accountable for all aspects on the work performed in this manuscript.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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