Complex-permittivity estimation of a polymer foam using microwave tomography for the application of microwave drying

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Abstract
A new kind of intelligent control for industrial microwave heating system for drying applications is under implementation in the TOMOCON [1] project network. The idea is to control the amount of total microwave power inside the applicator based on available volumetric moisture distribution inside the material. To achieve this goal a microwave tomography (MWT) imaging module is integrated with the industrial heating system. The MWT is applied to reconstruct the moisture content distribution (in terms of complex permittivity) in a polymer foam of large cross-section during its continuous processing. For the reconstruction algorithm, we use a statistical inversion framework. Generally, in this framework, the real and imaginary parts are treated as statistically independent quantities in the prior formulation. Thus, often it is found that the reconstructed real and imaginary parts are spatially imbalanced. As a result, when mapping the respective reconstructed parts of complex-permittivity to the available dielectric charaterization of the foam, conflicting and incorrect moisture distributions can be obtained. Therefore, we present a sample-based prior model in the statistical inversion framework to improve overall reconstruction accuracy and spatially balance the real and imaginary parts. The method is demonstrated with 2-D numerical MWT data at 8.2 GHz for different moisture scenarios.

1. Introduction
Controlled/localized heating in industrial microwave oven [2, 3] is paramount to address hot-spot formation and thermal runaway issues . Therefore, system efficiency and processed product quality may improve. Presently, we are working on a microwave oven technology called HEPHAISTOS. The system is characterized by hexagonal geometry for the cavity that supports a very high electromagnetic field homogeneity. Specifically, during drying of a porous polymer foam, thermal runaway and hot-spot formation occur [4]. Such situations lead to low-quality processing and may even damage the industrial unit. Therefore, automatic online control of power sources (magnetrons) to obtain a selective heating rate at each stage of the drying process is one option to eliminate these problems. To apply such precise control of power sources, non-invasive in-situ measurement of the unknown distribution of moisture, especially dominant wet-spots, inside the material is required. Thus, integration of microwave tomography (MWT) imaging modality operating in X-band range [5] (from 8 GHz to 12 GHz) with the drying system is proposed to estimate the moisture content distribution in a polymer foam. Based on the MWT tomographic output, an intelligent control strategy for power sources can be derived.

Microwave tomography (MWT) applications in the industry are mostly for inspection and monitoring purposes, as reported in [6]. In the X-band frequency range, performing quantitative MWT imaging is challenging especially when detecting the moisture levels in a porous material with a large cross-sectional dimension. Some prior information on the moisture levels and corresponding dielectric behavior of the foam is available. To integrate this prior information in the imaging algorithm, a statistical inversion approach based on the

Bayesian framework is applied in this work. In this framework, the real and imaginary parts are treated as statistically independent quantities in the prior formulation. Thus, often it is found that the reconstructed real and imaginary parts are spatially imbalanced. As a result, when mapping the respective reconstructed parts of complex-permittivity to the available dielectric charaterization of the foam, conflicting and incorrect moisture distributions can be obtained.

We present a sample-based prior model in the statistical inversion framework to improve overall reconstruction accuracy and spatially balance the real and imaginary parts. In our case, dielectric characterization of foam for wet-basis moisture level is available. Based on this information first we form a database of moisture distribution samples with different spatial variations. In these samples, dielectric values are chosen based on the dielectric characterization of the foam. In the second step, to get the prior covariance structure, we use the dataset to estimate the second-order statistics. The proposed sampled-based prior approach is tested with the 2-D numerical scattered field data for three different cases of moisture content distribution.

2. Microwave tomography setup and forward model

We consider a two-dimensional (2-D) foam domain $\Omega_{\text{foam}} = [-15, 15] \times [-1.5, 1.5]$ cm with an inhomogeneous moisture distrubution represented by relative permittivity $\epsilon_r = \epsilon_r' - j\epsilon_r''$ as shown in Fig. 1. The foam is surrounded by background domain Ω consisting of air with $\epsilon_b = 1 - j0$. For this 2-D numerical study, the antennas are modelled as $\hat{z}$-oriented electric line source; 6 such line sources are placed in a transceiver mode at 5 cm from the top and bottom surface of the foam, respectively. Thus, a total of $N = 12$ antennas is used for the measurements. In general, the scattered electric field E_{sct} under the illumination of time-harmonic transverse magnetic (TM) incident field is governed by the following scalar volume integral equation [7]

$$E_{\text{sct}}(r) = k^2 \int_{\Omega_{\text{foam}}} G(r, r')(\epsilon_r(r') - \epsilon_b)E(r')dr'. \quad (1)$$

The term is E the total electric field inside the foam. The wavenumber of the background medium is denoted by k. The term $G(r, r')$ is the free-space Green's function. The source and the observation points are denoted by the position vectors r and r', respectively. The total field inside the foam is given as

$$E(r) = E_{\text{inc}}(r) + k^2 \int_{\Omega_{\text{foam}}} G(r, r')(\epsilon_r(r') - \epsilon_b)E(r')dr'. \quad (2)$$

where E_{inc} is the incident field from the line source. As the total electric field depends on the dielectric constant of the foam, its mapping with the scattered electric field is non-linear.

In MWT, we seek to retrieve the dielectric distribution of the foam (in terms of $m \times n$ pixels) given the scattered electric field data. The nature of our inverse problem is ill-posed due to properties of the integral operator defined in (5) \cite{Colton98}. Also, part of this ill-posed nature comes from the fact that many parameters may result the same scattering data. Proper regularization terms in the quantitative inversion algorithms can alleviate this problem to some level. We have some prior information available on the dielectric behavior of the foam with respect to its moisture content level. Also, we expect and assume the moisture content distribution to have smooth distribution in the foam. To encode this prior information in a natural way in the regularization term we pursue a Bayesian inversion framework. With the Bayesian estimate, it can be quantified which parameters are more favorable/likely to generate this scattering data rather than fixed estimates which are given in classical, deterministic, inversion techniques.

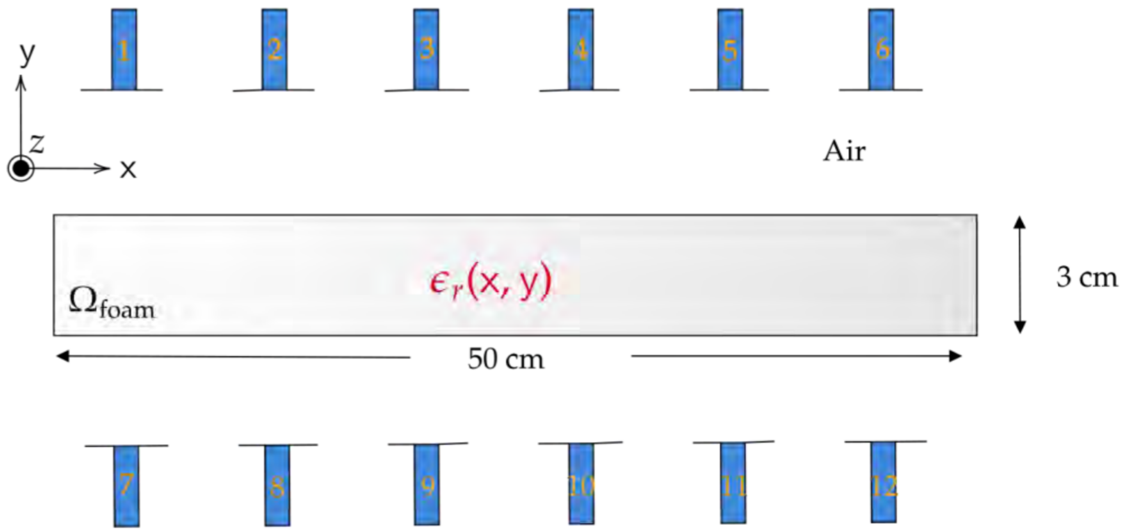

Fig. 1 : Schematic of the microwave tomography setup used in this study.

3. 2D reconstruction algorithm: Bayesian inversion framework

Consider an inverse problem of identifying an unknown complex parameter $\epsilon_r \in C^{m \times n}$ given noisy measurement scattering data $E_{\text{sct}} \in C^{N \times N}$ according to the observation model

$$\mathrm{E_{sct}} = \mathcal{F}(\epsilon_\mathrm{r}) + \mathrm{n}, \tag{3}$$

where n denotes the additive measurement noise component. The term $\mathcal{F}(\epsilon_r)$ denotes the integral operator defined in (1) and (2). The unknown parameter and noise terms are considered mutually independent. Note that the measurement data and unknown terms are complex quantities. In the present study, the real and imaginary parts are treated separately as real-valued random variables for the real-valued optimization problem.

In the Bayesian framework, the unknown parameters are treated as random variables, and information about them is expressed in terms of probability densities. The inverse problem is then expressed as given the measured scattering data; the task is to find the conditional probability density $\pi(\epsilon_r \mid E_{\text{sct}})$ for the unknown parameter ϵ_r. The conditional probability is constructed using Bayes' theorem as

$$\pi(\epsilon_r \mid E_{\text{sct}}) \propto \pi(E_{\text{sct}} \mid \epsilon_r)\pi(\epsilon_r), \tag{4}$$

where $\pi(\epsilon_r \mid E_{\text{sct}})$ is the posterior density, $\pi(E_{\text{sct}} \mid \epsilon_r)$ is the likelihood density which represents the distribution of the measured data if complex permittivity is known, and $\pi(\epsilon_r)$ is the prior density which contains the prior information available for the unknown ϵ_r. Here, the dielectric characterization data, often available in most of the application of MWT, becomes useful. The posterior density contains the complete solution of the inverse problem in the Bayesian framework. The solution can be expressed by point estimates. One of the most common point estimates in tomographic imaging problems is the *maximum a posteriori* (MAP) [8]. The MAP estimate can be computed from the posterior as

$$\widehat{\epsilon_{r_{\text{MAP}}}} = \arg\max_{\epsilon_r} \pi\,(\,\epsilon_r \mid E_{\text{sct}}\,)\,. \tag{5}$$

Under the assumption of Gaussian density for the likelihood term and prior term, this problem is equivalent of minimizing a logarithmic function which can be solved using non-linear least-square optimization problem.

3.1 Prior modelling

As prior information, it is first assumed that the moisture variation is smooth inside the foam. Such an assumption can be encoded using a Gaussian density [8] as

$$\pi(\epsilon_r) \propto \exp\left\{-\tfrac{1}{2}\left(\epsilon_r - \eta_{\epsilon_r}\right)^{\mathsf{T}}\Gamma_{\epsilon_r}^{-1}\left(\epsilon_r - \eta_{\epsilon_r}\right)\right\}. \tag{6}$$

Here, η_{ϵ_r} is the mean value of the prior density and Γ_{ϵ_r} is the covariance structure which defines the spatial smoothness inside the domain. The covariance structure in general is defined as

$$\Gamma_{\epsilon_r} = \begin{pmatrix} \Gamma_{\epsilon_{r'}} & \Gamma_{\epsilon_{r'}\epsilon_{r''}} \\ \Gamma_{\epsilon_{r''}\epsilon_{r'}} & \Gamma_{\epsilon_{r''}} \end{pmatrix}.$$

(7)

Since the permittivity is a complex number, the prior density is independently derived for the real part and imaginary part, respectively. Here, if we treat the real and imaginary parts to be uncorrelated then the cross-covariances $\Gamma_{\epsilon_{r'}\epsilon_{r''}} = \Gamma_{\epsilon_{r''}\epsilon_{r'}} = 0$. This type of prior model will be called as smoothness prior model. And its effect on the MAP estimates will be shown in the next section. However, to define the covariance structure in more accurate sense the correlation factor between the real and imaginary part should be well known. In this work to establish the correlation and to form the covariance structure we use sample-based prior densities.

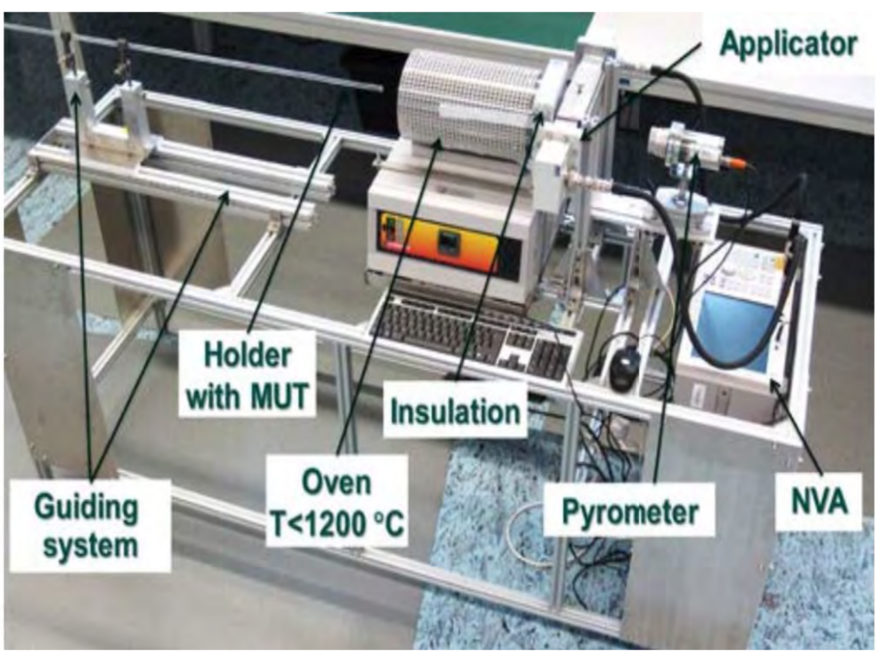

Fig. 2 : Dielectric charaterization of the polymer foam using cavity-perturbation method [9] at KIT laboratory, Germany.

In sample-based prior, we make use of a large set of previously obtained samples of the random variable in question. These datasets are known as samples. And then from this dataset, the mean and covariance structure can be straightforwardly calculated. The dielectric values used in the sample-based mean and covariance calculations are generated numerically, based on the dielectric characterization of the polymer foam in laboratory environment. In the characterization, a small cylindrical shape volume of the foam is characterized using a cavity perturbation technique at room temperature to obtain the complex permittivity value for different levels of wet-basis moisture content level. The developed dielectric measurement system is shown in Fig. 2. Extensive details on the moisture samples generation for generating the dataset and use of dielectric charaterization are given in [10][11].

4. Numerical study

In this section, we evaluate the performance of the MAP estimates with the smoothness prior and sample-based prior for smoothly distributed moisture scenario. To generate the synthetic measurement data from the MWT setup shown in Fig. 2 a 2-D finite element method (FEM) based COMSOL simulation tool is chosen. The scattered field data is generated at a frequency of 8.2 GHz and stored in terms of the scattering matrix of size 12 × 12.

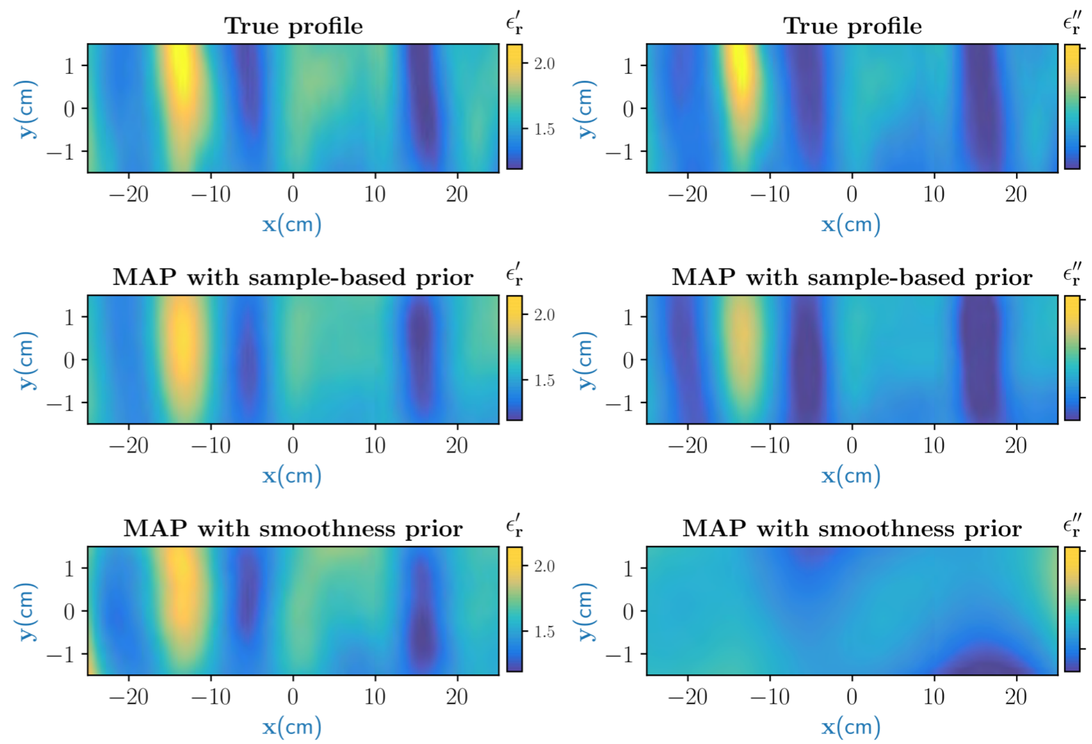

Fig. 3 : High moisture case : true distribution and MAP estimates with smoothness prior and sample-based prior with real (left) and imaginary part (right) of the complex-permittivity.

In the first set of experiments numerical scattered electric field measurement data for a high moisture scenario between 35% to 50% is considered. The moisture values correspond to permittivity values of 1.35 to 2.2 for the real part and 0.03 to 0.15 for the imaginary part. The MAP reconstruction with smoothness prior model and sample-based prior model is shown in Fig. 3. Though the real part is estimated well with both priors the imaginary part is much more accurate with the sample-based prior model. Notice that the certain moisture regions are much more clearly indicated in the estimated imaginary part with the sample-based prior model.

5. Conclusion

In this work, we used a microwave tomography to estimate the moisture distribution (in terms of complex permittivity) in a polymer foam using the Bayesian inversion framework. The imaging modality will be integrated with an industrial microwave drying system to derive approaches for the intelligent control. It is shown that when real and imaginary parts are treated uncorrelated in the smoothness-based prior model, reconstructed values are conflicting and incorrect. Thus, we proposed a sample-based prior model to preserve the spatial correlation in the reconstructed real and imaginary parts of the complex permittivity. The results presented show the efficacy of the sample-based prior model in spatially correlating the real and imaginary parts. A significant improvement in the reconstruction accuracy is achieved with sample-based prior in comparison to the smoothness prior model.

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