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On the influence of surface roughness on friction-induced oscillations

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The influence of surface roughness on friction-induced oscillations due to non-conservative coupling is investigated. The classical non-conservative coupling model is extended by a stochastic friction coefficient which is modeled as colored noise. Two coupled and parametrically excited stochastic differential equations are obtained. The almost sure stability is analyzed by means of the top Lyapunov exponent Λ_1 which indicates instability in the case of $\Lambda_1 > 0$ and asymptotic stability in the case of $\Lambda_1 < 0$. An influence of the stochastic friction coefficient on stability is found which manifests in the occurrence of parametrically excited fundamental and combination resonances.

1 Introduction

Possibly occurring but undesired friction-induced oscillations in engineering systems have already been extensively investigated and are mainly explained by two mechanisms in the literature. The first one relies on the negative slope of a velocity dependent friction force, for which oscillations occur in the case of a small structural damping. The second one can be described by a non-conservative coupling model as depicted in figure 1(a) and explains in particular friction-induced oscillations in the case of velocity independent friction forces [4]. Surprisingly, the vast majority of investigations in the literature rely on deterministic friction laws in contrast to experiments and detailed contact simulations which rather suggest a stochastic modeling. Consequently, the non-conservative coupling model is further extended with a stochastic friction coefficient.

2 Mechanical Model

The mechanical model depicted in figure 1 is extended by an additional time dependent stochastic friction coefficient motivated from experiments and simulations.

The friction coefficient is modeled as colored noise $\mu_\tau \sim \mathcal{N}\left(\overline{\mu}, \sigma^2/4D\Omega^3\right)$ by using a second order filter with Gaussian white noise χ_τ as input

$$\mu_\tau'' + 2D\Omega\mu_\tau' + \Omega^2\mu_\tau = \Omega^2\overline{\mu} + \sigma\chi_\tau, \qquad \mu_\tau(0) = \overline{\mu},\ \mu_\tau'(0) = 0, \tag{1}$$

in which Ω represents any system specific excitation frequency, e.g. the rpm of a brake disk or any asperity caused excitation frequency, which means that Ω is connected to the belt velocity v. D and σ represent additional filter coefficients which are used to fit any prescribed auto-correlation function or power spectral density and $\overline{\mu}$ represents the mean friction coefficient. Note that a harmonic excitation $\mu_\tau = \overline{\mu} + \Delta\mu\cos(\Omega\tau)$ is obtained with slightly changed initial conditions $\mu_\tau(0) = \overline{\mu} + \Delta\mu$ and $\sigma = D = 0$.

Two coupled stochastic differential equations for the horizontal and vertical degree of freedom ξ and ζ are obtained in dimensionless form

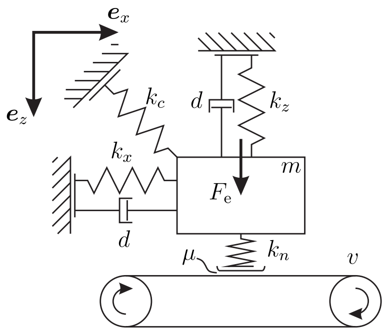
$$\begin{aligned} \xi_\tau'' + 2\delta\xi_\tau' + c_{11}\xi_\tau + \left[c_{12} + \mu_\tau \operatorname{sign}\left(\xi_\tau' - \nu\right)\right]\zeta_\tau &= 0 \\ \zeta_\tau'' + 2\delta\zeta_\tau' + c_{21}\xi_\tau + \left[1 + c_{22}\right]\zeta_\tau &= 1 \end{aligned} \tag{2}$$

in which the friction coefficient acts as a parametric excitation and δ as well as c_{ij} denote dimensionless damping and stiffness coefficients respectively.

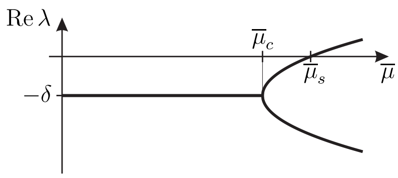
Following Arnold [1], the top Lyapunov exponent is calculated for the perturbed system (2) assuming $\xi' < \nu$

$$\Lambda_1 = \lim_{\tau\to\infty} \frac{1}{\tau} \ln \frac{A_\tau}{A_0} = \lim_{\tau\to\infty} \frac{1}{\tau} \int_0^\tau f_1\left(\varphi_s, \psi_s, \vartheta_s, \mu_s\right) \mathrm{d}s \tag{3}$$

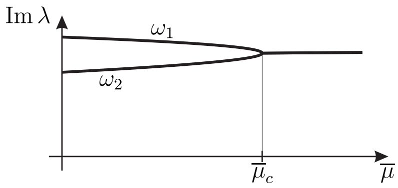
in which A_τ denotes the transient Euclidean norm of the disturbances. A transformation to new coordinates allows the reformulation as stochastic differential equation in



(a) Mechanical Model



(b) Real-parts of EV for $\mu_\tau = \overline{\mu}$



(c) Natural eigenfrequencies for $\mu_\tau = \overline{\mu}$

Fig. 1: Non-conservative coupling model

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the sense of Itô

$$\begin{array}{rcl} A_\tau & = & \sqrt{\Delta\xi_\tau^2 + \Delta\zeta_\tau^2 + \Delta\xi_\tau'^2 + \Delta\zeta_\tau'^2} \\ \tan\varphi_\tau & = & \Delta\xi_\tau'/\Delta\xi_\tau \\ \tan\psi_\tau & = & \Delta\zeta_\tau'/\Delta\zeta_\tau \\ \tan\theta_\tau & = & \sqrt{\Delta\zeta_\tau^2 + \Delta\zeta_\tau'^2}/\sqrt{\Delta\xi_\tau + \Delta\xi_\tau'^2} \end{array} \Rightarrow \begin{pmatrix} \mathrm{d}\ln A_\tau \\ \mathrm{d}\varphi_\tau \\ \mathrm{d}\psi_\tau \\ \mathrm{d}\vartheta_\tau \\ \mathrm{d}\mu_1 \\ \mathrm{d}\mu_2 \end{pmatrix} = \begin{pmatrix} f_1(\varphi_\tau,\psi_\tau,\vartheta_\tau,\mu_1) \\ f_2(\varphi_\tau,\psi_\tau,\vartheta_\tau,\mu_1) \\ f_3(\varphi_\tau,\psi_\tau,\vartheta_\tau,\mu_1) \\ f_4(\varphi_\tau,\psi_\tau,\vartheta_\tau,\mu_1) \\ \mu_2 \\ -2D\Omega\mu_2 - \Omega^2\mu_1 \end{pmatrix} \mathrm{d}\tau + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \sigma \end{pmatrix} \mathrm{d}W_\tau. \quad (4)$$

The final system of equations (4) is decoupled from the amplitude A_τ and consequently the top Lyapunov exponent from equation (3) only depends on the stationary angle processes. The numerical integration is performed with a stochastic Runge-Kutta integration scheme following a publication of Rössler [3] and using random initial conditions for the first four states. Each simulation lasted for 1e7 time steps and time increments of $\Delta\tau = \pi/1024$ are used. The evaluation of equations (3) was verified by comparison to Floquet multiplier λ as $\Lambda_1 = \Re[\ln(\lambda)\Omega/2\pi]$ in a pureley harmonic excitation case.

3 Simulation Results

Some results are shown for a constant, a harmonic and a stochastic friction coefficient respectively in order to distinguish between the different sources of instability.

In the case of a constant friction coefficient the stability can easily be evaluated by an eigenvalue analysis from which two conjugated complex pairs of eigenvalues are obtained. Their real parts are shown in figure 1(b) and coincide up to a critical value $\overline{\mu}_c$ where they split up and shortly after instability occurs due to one real part who crosses the zero axis at $\overline{\mu}_s$. Furthermore, at the same critical friction coefficient $\overline{\mu}_c$, the natural eigenfrequencies collide as depicted in figure 1(c).

In the case of a harmonic friction coefficient, the stability map is complemented by the occurrence of additional parametric resonances as shown in figure 2. Grey areas indicate a positive Lyapunov exponent and consequently instability. Fundamental and difference combination resonances according to $\Omega/\omega_1 = 2/n$, $\Omega/\omega_2 = 2/n$ and $\Omega/(\omega_1 - \omega_2) = 1/n$ for n $= 1, 2, \ldots$ are found which are typical for parametric excited systems. Summation combination resonances are not found which is in accordance with [2]. Furthermore, figure 3 shows the dependence of the unstable tongues on the excitation amplitude $\Delta\mu$.

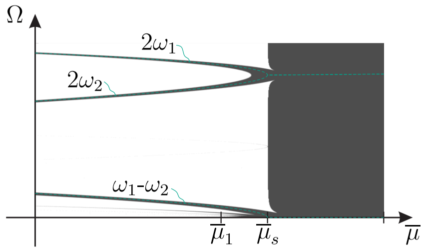

Fig. 2: Stability map for harmonic excitation and $\Delta\mu = 0.05\sqrt{2}$, $\delta = 0$

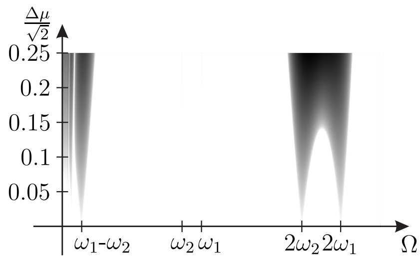

Fig. 3: Stability map for harmonic excitation and $\overline{\mu}_1 < \overline{\mu}_s$, $\delta = 0$

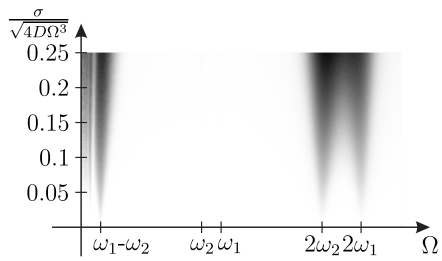

Fig. 4: Stability map for stochastic excitation and $\overline{\mu}_1 < \overline{\mu}_s$, $\delta = 0$, $D = 0.01$

In case of a stochastic friction coefficient $\mu_\tau \sim \mathcal{N}(\overline{\mu}, \sigma^2/4D\Omega^3)$, a qualitatively similar stability map is obtained as depicted in figure 4 despite the fact that the borders to unstable tongues seem blurred. This can be attributed to the fact that the spectral density of μ_τ still has a peak near Ω. It is noted that a stabilizing effect in the region $\overline{\mu} > \overline{\mu}_s$ could be found as well.

4 Conclusions

The extension of a classical non-conservative coupling model with a stochastic friction coefficient leads to stochastic differential equations with parametric excitation. A stability analysis by means of calculating the top Lyapunov exponent shows the occurrence of a combined self- and parametric excited dynamic system behavior. The consideration of noisy friction coefficients may therefore contribute to explain dynamic phenomena occurring in systems with friction.

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