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Digital Tools to Enable Collaborative Mathematical Modeling Online

Herramientas digitales para la modelización matemática colaborativa en línea

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Abstract

To enable collaborative modeling activities online digital tools are essential. In this paper we present a holistic and adaptable concept for the development and implementation of modeling activities – which could especially be fruitful in times of homeschooling and distance learning. The concept is based on two digital tools: Jupyter Notebooks and a communication platform with video conferences.We carried out this concept in the context of two types of modeling activities: guided modeling days, where the students work on previously prepared and didactically developed digital learning material, and modeling weeks, in which the students work on open problems from research and industry very freely. In this paper the usage of Jupyter Notebook in modeling activities is presented and illustrated with the example of the optimization of a solar power plant. On top, we share our experiences in online modeling activities with high-school students in Germany.

Para facilitar las actividades de modelización colaborativa en línea, las herramientas digitales son esenciales. En este trabajo presentamos un concepto holístico y adaptable para el desarrollo y la implementación de actividades de modelización – que podría ser especialmente provechoso en tiempos de educación a distancia. El concepto se basa en dos herramientas digitales: Jupyter Notebooks y una plataforma de comunicación con videoconferencia. Realizamos este concepto en el contexto de dos tipos de actividades de modelización matemática: días de modelización guiada, en los que los alumnos trabajan con material de aprendizaje digital previamente preparado y desarrollado didácticamente, y semanas de modelización, en las que los alumnos trabajan en problemas abiertos de la investigación o de la industria de forma libre. Se presenta el uso de Jupyter Notebook en las actividades de modelización y se ilustra con el ejemplo de la optimización de una planta solar. Además, compartimos nuestras experiencias en actividades de modelización en línea con estudiantes de secundaria en Alemania.

Keywords: Digital tools, mathematical modeling, collaborative learning online
Palabras clave: Herramientas digitales, modelización matemática, aprendizaje colaborativo en línea

1. Mathematical modeling activities

Mathematical modeling can be described as the process of translating the real world into mathematics and vice versa. It is one of the process-related competencies that students across Germany (and in many other countries) are expected to acquire during their education in mathematics (Blum & Leiß, 2007; Blum & Borromeo Ferri, 2009).
In short, the modeling process can be described as a cycle consisting of four steps: (1) structuring and simplifying the problem, (2) describing the simplified problem mathematically, (3) working within the mathematical model and (4) interpreting and validating the obtained mathematical results in relation to the real world. By additionally taking the solution progress into account, we can visualize the iterations over the cycle through a spiral (see fig. 1). More detailed descriptions of the modeling process and further models to describe this process can be found in Borromeo Ferri (2006) or Blum (2015).

Figure 1 – Solution Helix of Math of the initiative Computer Based Math (cf. computationalthinking.org/helix, accessed 29 January, 2021)

1.1. Teaching mathematical modeling

The importance of mathematical modeling in mathematics education in general and the problem selection, in particular, is emphasized by the German mathematics educationalist Heinrich Winter (1995). Among other important aspects, Winter says that mathematics education should allow students to perceive phenomena from the world that surrounds us in a specific way and to acquire problem-solving skills that go beyond mathematics when dealing with tasks (Winter, 1995).

Through a proper design of modeling activities and a suitable selection of modeling tasks, students should be enabled to gain the necessary competences to work through all steps of the modeling process independently (Blum et al., 2007; cf. Blum, 2015, p. 77). To support the way students perceive *the world around us*, the problems should be real and relevant (especially to the students). Additionally, fruitful problems should be authentic and (mathematically) rich to acquire profound *problem-solving skills going beyond mathematics*. By the term authentic we follow a definition of Niss (1992), that the extra-mathematical problem should, though moderately simplified, still be considered as relevant by experts in the field and recognized as a problem they would deal with in their work (Niss, 1992; cf. Vos, 2011; Kaiser & Schwarz, 2010; Maaß, 2010). The modeling activities presented in this paper were carried out based on problems that fulfill the mentioned criteria (to a varying extend). Some examples are described in sections 1.3 and 1.4.

Due to the fairly complex and diverse nature of the problems in our environment, working in teams typically plays an important role in problem-solving processes in the real world (e.g., in science, technology, economy). Therefore, *teamwork and thus collaborative mathematical modeling* is essential in the modeling activities presented here. Clearly, the proximity to real-world problem-solving processes is not the only advantage of collaborative or cooperative learning[1]. It might enhance students' self-esteem and motivates them to actively participate in the learning process (Panitz, 1999b, p. 59). Moreover, "collaborative learning has as its main feature a structure that allows for student talk: students are supposed to talk with each other...and it is in this talking that much of the learning occurs?" (Golub, 1988, p. 1). As we will point out in the following, communication between students is crucial in the modeling activities described in this paper. In Panitz (1999b) and Johnson & Johnson (1989) several further benefits of collaborative learning are shown.

1.2. Mathematical modeling with digital tools

Normally, most of the learning and teaching of mathematics takes place in school or, as in our case, in activities for high-school students organized by scientific staff (students lab). Especially in times of distance learning and homeschooling it has become crucial to allow the students to acquire the modeling competencies mentioned above by working on problems from home. Thus, we developed a holistic concept to enable collaborative mathematical modeling online. By definition, this can only be realized using *technology*.

In this paper we emphasize the outstanding role of digital tools, especially in section 2. However, the use of digital tools in mathematical modeling activities is by no means only useful when conducted online, but can also enrich the activities in a face-to-face context, (Siller & Greefrath, 2010). Various goals and benefits of using technologies for mathematical modeling activities with students, already highlighted by Greefrath and Siller (2010, 2018), include:

- Technologies make it easier or even possible to deal with real-world problems. These problems are often fairly complex and therefore require to work with a vast amount of data or to solve computationally intensive problems (Siller and Greefrath, 2010 p.2138; Greefrath & Siller, 2018 p. 5). Hence, technologies increase the variety of problems that can be examined with students. This is especially relevant in today's world, as a large number of problems and daily applications from students' everyday lives are based on huge amounts

[1]The terms collaborative and cooperative learning are often used interchangeably. However, Panitz (1999a) and Bruffee, K.A. (1995) distinguish between both terms. Following Bruffee, K.A. (1995) one difference is that collaborative learning is more student centered whereas cooperative learning is more teacher centered and thus closer controlled by the teacher. When using the terms collaborative and cooperative learning we refer to the definition by Bruffee.

of data (Big Data). Examples include music recognition apps, fitness trackers, automatic face recognition, or online streaming services. All of them are based on mathematical models that can be made accessible to students in a fundamental form (Wohak et al., 2021)

- The use of technology keeps the focus on the modeling process itself, because calculations are outsourced to the computer (Greefrath and Siller, 2018, p. 5).
- The active and creative use of technologies is an essential competence nowadays, as students are facing rapidly changing technologies and digital media in their everyday life.
- In science, industry and research, real - world problems are tackled and solved with the help of technologies. Therefore, we underline the integrity of the problem-solving process by exploiting digital tools.
- The use of technologies facilitates different types of visualizations (Siller & Greefrath, 2010, p. 2139).

The holistic approach for collaborative modeling activities online presented in this paper was developed within the Computational and Mathematical Modeling Program (CAMMP)[2]. Within CAMMP we aim to develop digital, computer-aided teaching and learning material. Our material covers problems satisfying the criteria mentioned above and has been used for numerous realizations of modeling activities with students so far – both online and face-to-face. We set up two different types of modeling activities: (a) guided modeling activities, so-called modeling days[3], and (b) completely open modeling activities, the so-called modeling weeks.
Besides the modeling days and weeks carried out within the program CAMMP several other modeling activities organized by universities exist in Germany. Two examples are the Hamburg modeling days described in Vorhölter and Freiwald (2022) and the modeling days and weeks realized in Kaiserlautern within the project KOMMS (Bock & Bracke, 2015).

In the following, the typical face-to-face course of our modeling days and weeks is described and essential methodological elements are emphasized. Subsequently the challenges and requirements of digital tools resulting from online implementations of these activities are highlighted.

1.3. Modeling days – guided modeling activities

In the context of a modeling day[4] the students get involved in solving real-world problems from their everyday life, technology, economics or science (Frank et al., 2018; Wohak, et al., 2021). Three of many problems for use in our modeling days are (a) How is it possible to obtain images of the internal structure of the body using X-rays? Within this workshop the students reconstruct the material composition of an examined object, which is used to create computer tomography images (Wohak & Frank, 2019). (b) How can we automatically classify faces or other objects on images? The students develop a classification model on the basis of a data set with two or three dimensional data. Afterwards they apply it to higher dimensional data in order to classify handwritten digits or faces of humans (Schmidt, 2019; Schönbrodt, 2019). (c) How can a solar thermal power plant be built to maximize energy gain (Schönbrodt & Frank,

[2]CAMMP is a program at the RWTH Aachen University and the Karlsruhe Institute of Technology (both in Germany) conducting modeling activities for students from grade 8 to 13 (corresponds to a range of age of approx. 13 – 19 years) since 2011, see www.cammp.online/english/index.php, accessed 29 January, 2021.

[3] A modeling day does not have to be carried out on one single day. Instead, it is possible to distribute the activity over several, separate school hours.

[4]Most modeling activities were supervised by scientific staff. A first implementation of a workshop by a high-school teacher was carried out in 2020.

2020)? This last problem is described in section 3 in more detail. All problems that we have worked out didactically can be found on `www.cammp.online/english/116.php` (accessed 29 January, 2021).

During a modeling day the problem-solving process is guided by digital learning material containing worksheets and various differentiation materials (Frank et al., 2018; Wohak et al., 2021; Gerhard et al.). In section 2.1 we describe the digital material in detail.

Main elements in the face-to-face implementation

When carrying out the modeling days in presence, the teacher and students are in the same room. In the beginning, the teacher gives an introduction to the problem using *presentation slides*. Students are engaged in the presentation through *discussion questions*. After the introduction the students start working on the problem *in groups* of two or three using a computer / laptop / tablet. During the group work there is usually a lively *communication* among the students.

Digital worksheets guide the students through the problem-solving process. Depending on the problem, *sketches* that students draw for (mutual) explanations play an important role in the modeling process. Communication but also joint *written documentation* are essential in the working phases.

In case the students get stuck they can use tip cards provided within the digital material. If they still cannot continue, they can *ask the teacher for help*. The students' difficulties are usually solvable through small hints, a short discussion and / or a sketch by the teacher. If the teacher notices that a problem occurs in several groups, he / she can briefly discuss it in a plenary session. By having everyone in the same room, the teacher can see the individual progress of each group at any time and get an overview of the students' work status and their motivation. If the teacher observes that a group seems to have major difficulties, he / she can offer support on his / her own initiative.

At various points during the modeling day, *intermediate plenary discussions* are held to compare and validate previous results or to discuss further questions and solutions. In the plenary discussions the teacher again uses presentation slides. It is also possible to use the *blackboard* so that students can visualize and share ideas with their classmates.

In order to address the heterogeneity of the learning group, faster groups of students can work on additional more advanced tasks or open subproblems.

A plenary discussion which is moderated by the teacher ends the modeling day. The full modeling process is summarized, the obtained results are interpreted and validated, and further model improvements are discussed.

1.4. Modeling weeks – open modeling activities

During a modeling week the students work on real problems from research, economy or everyday life in small *teams* (5–6 students). They are supervised by scientific staff (also referred to as supervisor). The problems are open which means that people from the practice area are still working on this problem. Thus, neither the students nor the supervisor have a solution at hand. Apart from a description of the problem and, if necessary, the prior preparation of a given data set, the students do not receive any learning materials. They structure and design the modeling process on their own. Some examples of problems that students have already worked on during our modeling weeks (biannually organized as part of CAMMP) are: (a) How should wind turbines in an offshore wind farm be positioned so that they do not overshadow each other, but at the same time use the available area in an optimal way? (b) How can the patient-to-room allocation in hospital stations be optimized? (c) Is the climate change man-made? How can we use real data to analyze the various influences on the Earth's energy budget and the effect of natural and non-natural causes on the global temperature? A possible course of a modeling week on the last topic is described in Hattebuhr and Frank (2022).

Main elements in the face-to-face implementation
An exemplary course of a modeling week is shown in figure 2.

Program

Sunday

bis 17:30	Arrival at the Youth Hostel
17:45	Welcoming & Getting To Know Each Other Game
18:30 - 19:30	Dinner
19:30	What is Mathematical Modelling?

Monday

08:00 - 09:00	Breakfast
09:00 - 10:00	Presentation of the Projects
10:30 - 12:30	Work in groups
12:30 - 14:00	Lunch
14:00 - 18:00	Work in groups
18:30 - 19:30	Dinner
19:30 Uhr	MATLAB - Crash course (optional)

Tuesday

08:00 - 09:00	Breakfast
09:00 - 12:30	Work in groups
12:30 - 14:00	Lunch
14:00 - 18:00	Work in groups
18:30 - 19:30	Dinner
19:30 Uhr	Advisor meeting

Wednesday

08:00 - 09:00	Breakfast
09:00 - 12:30	Work in groups
12:30 - 13:45	Lunch
14:00 - 14:45	Studying at RWTH Aachen or KIT
14:45 - 17:45	Sports Game
17:45	Barbecue
19:00	Tour of guests through the groups to discuss the intermediate results

Thursday

08:00 - 09:00	Breakfast
09:00 - 12:30	Work in groups
12:30 - 14:00	Lunch
14:00 - 15:00	Work in groups
15:00 - 18:00	Rehearsal Talks & Deadline for the group report
18:30 - 19:30 Uhr	Dinner
19:30 Uhr	Advisor meeting

Friday

08:00 - 09:00	Breakfast
09:00 - 09:30	Evaluation
11:15 - 16:30	Presentations of the students & Farewell

Figure 2 – Exemplary program of a modeling week

At the beginning of the week, the students receive a short *presentation* on the yet unsolved problems. Afterwards the working phase in the groups begins. For this purpose each group has its *own group room*[5] and several *laptops* are available per group. During the very intensive group work, the students visualize their ideas and structure upcoming tasks on a *flipchart*. Usually, several students sit together around one laptop and implement a program together.
During the working phases we often observed that the students split up into *smaller subgroups* in order to work on different subproblems. Repeatedly, there are periods of gathering or discussion in the whole group to review the current progress.

The supervisor of the group accompanies the entire working process and helps the group to structure their working progress (if necessary), to discuss results with the students and to keep an eye on the time. Since programming is not a prerequisite for participating in a modeling week, the scientific supervisor helps with hints regarding the development of a program when needed. Most often during *offline* modeling weeks the programming environment Matlab was used. In principle, however, any other language with a similarly "easy" syntax (e.g., Julia, Python or R) is feasible.

At the end of the modeling week, the *students present their results* to an interested audience (scientific staff, problem advisors, parents, teachers, friends, etc.) and hand in a report docu-

[5]The modeling weeks usually take place in a youth hostel or in seminar rooms of the university.

menting their modeling process.

As can be seen from the previous explanations, the students undergo different phases during the modeling week. In fact, the design corresponds to the typical seven components of the project method according to Frey (2012, pp. 58–159):

1. project initiative: a member of the learning group or an outsider suggests a project
2. examination of the project initiative: students find subtasks they want to pursue
3. joint development of the field of activity: students identify what they are going to work on
4. project implementation: students may split up into small groups to work on subtasks and afterwards put together what they have worked on
5. end of the project: as one option a presentation crowns the end oft the project
6. fixed points: they serve to stimulate exchange between students, to organize next steps, to visualize the work status in comparison to the project initiative etc.
7. meta-interaction: students reflect on the social course of the working process within the group

Based on Frey's method, similar concepts were further developed, e.g., from Ludwig (1997) who specifically focuses on projects in mathematics education.

In comparison, the modeling (see sec. 1.3) does not contain all components of Frey's project method. This is due to the stronger guided design of the modeling days: The processing of the modeling problem is pre-structured by the teacher. Smaller predefined sub-tasks guide the students through the modeling process. The two components included in the modeling days are (a) the end of the project realized as a plenary discussion where the students summarize the main steps of the modeling process and compare the final results with the real-world problem. And (b) the fixed points realized through several plenary discussions focusing on the students' intermediate results, on open questions and on how the current modeling step fits into the overall modeling process.

1.5. Challenges for the online implementation of modeling activities

As can be seen from the explanations of modeling days and weeks given above, teamwork and communication between students are essential in both types of modeling activities. In order to realize teamwork and interaction also during the implementation of modeling activities online[6], the following questions are central:

- How can the communication among students and with the supervisor be established online?
- How can online supervision be organized? How or when can the supervisor be contacted and provide support in case of problems?
- How can the teacher keep track of the students' progress?
- Which tools are suitable for realizing sketches and rich visualizations, which form an important part of the modeling activities?

[6] The initial reason for the development of a holistic concept for mathematical modeling activities online within CAMMP was the COVID-19 pandemic. Nevertheless, the learning material described in section 2.1 was already realized digitally for the use in our face-to-face workshops. This was due to the advantages of digital tools for solving real-world problems highlighted in section 2

- Plenary discussions take place frequently during modeling activities. How can they be realized virtually and, if necessary, spontaneously? How can plenary phases be designed so that the students are actively involved?
- Students already work with computer-aided, digital learning material in the face-to-face activities – a good basis for an online implementation. But how can the digital learning material be processed without the installation of software or programs on the students' devices (computer, laptop, tablet)?
- Especially in longer modeling activities, social and human interaction apart from the problem is crucial. How can this be integrated into online modeling weeks?

These questions can be linked to the research field of *Computer-Supported Collaborative Learning* (CSCL), where numerous publications point out the opportunities and challenges that arise when implementing collaborative learning in an online environment (Roberts, 2004; Stahl et al., 2006). Within this paper we try to answer the questions stated above by sharing our experience with the online modeling days and weeks we already carried out. In doing so we try to emphasize what was pointed out by Johnson & Johnson (2014): "Technology does not have to isolate and separate students. When used effectively, technology can bring students together in cooperative efforts and enhance student experiences."

Some additional questions are answered in this paper, which come up in the online implementation of guided modeling activities and are also essential in the face-to-face workshops:

- How can the learning material be structured so that students of different skill levels can work on the problems independently and with a sense of achievement?
- How should the learning material be structured so that the students can perform calculations and visualize models without assuming any programming knowledge from the students?

Within this paper we tackle all the mentioned questions – focusing on some of them in more detail than on others.
In the following, we present tools that were tested to answer the questions raised in this section and we explain how they can be implemented.

2. Digital tools for modeling activities

From above, it can be concluded that the digital tools must satisfy two indispensable requirements for the implementation of collaborative modeling activities online: On the one hand, they must allow to develop and edit digital learning material by the students. On the other hand, the digital tools have to allow communication between students and teachers in groups of varying sizes and constellations. Communication should be possible both by voice and chat. The latter requirement, if satisfied, already emphasizes one strong advantage of digital learning: efficient communication between any pair of students or teachers without classroom disturbances.

This section is structured as follows: First, we introduce the reader to two software tools to enable digital learning. The first one, *Jupyter Notebooks*, used both during guided modeling days as well as during open modeling weeks. Jupyter Notebooks is an authentic tool in the sense that they are actually used by experts. The second tool, *CoCalc*, is especially suitable for open modeling projects, where students work jointly but without much additional support. Note that within the tool CoCalc it is also possible to generate Jupyter Notebooks, but on top, the tool offers a variety of features for open modeling activities that plain Jupyter Notebooks do not include. We enhance the discussion about their usability and technical aspects by providing our own experience from past modeling days and weeks. Finally, we summarize requirements for communication platforms and explain our choice of tools.

2.1. Guided digital learning: Jupyter Notebook

One tool that can be used to develop interactive digital learning material are *Jupyter Notebooks*[7]. They are widely used in numerous branches of industry, research and economy. This means that not only the learning content but also the digital tool provides an authentic insight into current problem-solving strategies and technical implementations in applied mathematics or more generally in STEM fields. Put in simple terms, Jupyter Notebooks are digital documents which can be used to write texts, execute code (to compute mathematical terms) and can be edited interactively by the students. Given that students and teachers face different technical equipment in their school and have various operating systems on their private computers, a huge advantage is that Jupyter Notebooks are web-based and hence can be used without installing additional software (only a web browser is required[8]).

This section gives the reader a basic idea of how digital learning material can be realized based on Jupyter Notebooks. To get a better understanding of what Notebooks, designed for modeling days, could look like interested readers can simultaneously view the digital learning material developed by the authors. The access procedure is described in section 2.1.

Jupyter Notebooks offer great benefits compared to standard documents or code files: tasks (in form of narrative text), explanatory illustrations and small sections of code (which can be edited interactively) can be combined in a nicely formatted single document (see fig. 3 and fig. 4). On top, all elements (so-called cells) can be ordered flexibly. The basic cell types available in Jupyter Notebooks are text and code cells. Both are highlighted in figure 3.

- As the name implies, the main use of *text cells* is to provide narrative text, e.g., in form of longer explanations or small notes, which can be used to guide the students through the material. It can help to introduce a problem in a narrative way, show mathematical equations, link to other documents or worksheets that might be of value or explain concrete tasks to the students. Furthermore, tables, images and videos can be integrated easily and linked directly to the corresponding tasks.
- *Code cells* allow the user to write and execute code. The code can be written in one of several programming languages, e.g., *Julia*, *Python* or *R*. The worksheets (Jupyter Notebooks) we developed for our modeling days are based on the programming language Julia[9]. An important aspect why we chose Julia is its simple syntax, which is close to so-called *pseudocode*, i.e., reading and writing Julia is comparatively easy and similar to writing the task down in English.
  In the learning material, students edit code cells according to some task, which is usually described in a previous text cell. We follow a *"fill in the gaps"* approach, i.e., we define gaps in the code which for example have to be filled by formulas, numbers or equations[10]. Thereby we ensure that no advanced programming skills are required and keep the focus

[7]For further information see: `www.jupyter.org`, accessed 7 January, 2021.
[8]Note that we are discussing the case where the Jupyter software itself is installed and hosted on a server which is then accessed by students and teachers (see sec. 2.1).
[9]For further information see: `www.julialang.org`, accessed 7 January, 2021.
[10]In our learning material all gaps are marked by a particular placeholder, namely *NaN* (not a number).

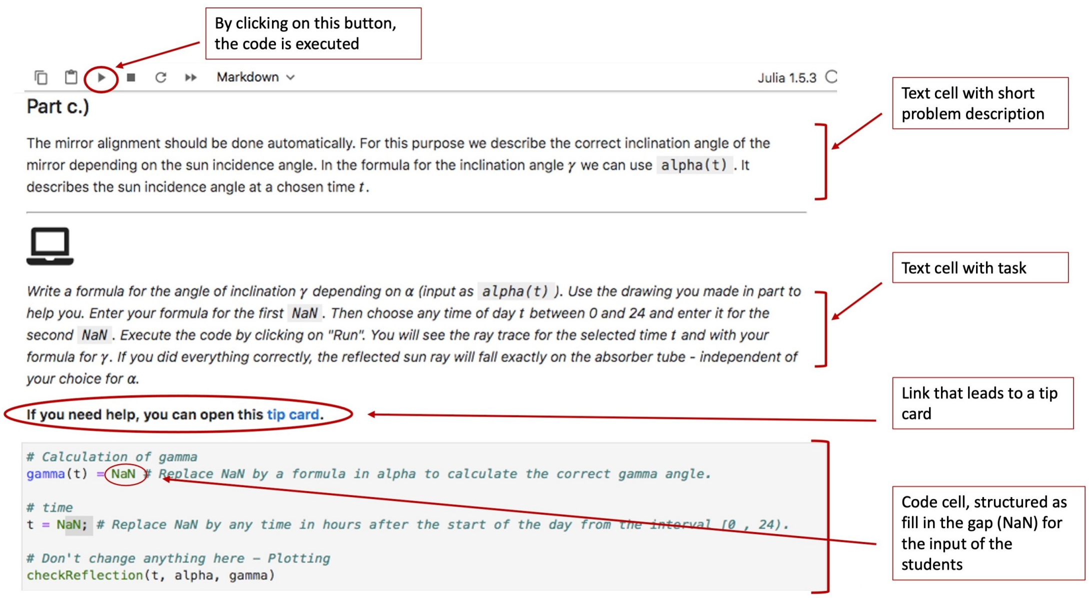

Figure 3 – Screenshot of a digital worksheet (Jupyter Notebook) from a workshop on the optimization of a solar power plant

on the actual (modeling) task.

After executing a code cell the students receive an output, e.g., images, tables, plots or simply results of mathematical computations. It is directly shown below the cell (see fig. 4). This enables students to quickly check their results or even try out different inputs to study the correlation between input and result.

From a didactic point of view, Jupyter Notebooks offer a huge step to promote self-directed learning in heterogeneous learning groups by providing optimal guidance for each student. Additional tips or explanatory information can be provided in the Notebooks. The students can access these by clicking on links embedded in the text cells (see fig. 3), ensuring that each student gets the amount of support he / she needs. The tips with different levels of assistance can contain hints, sketches, equations or examples and open in separate Jupyter Notebooks, (Hänze et al., 2010). In addition to tips, some learning materials include additional (optional) worksheets that can be accessed through links as well. These can be used as a further method of differentiation[11] and provide additional tasks or information (both mathematical and non-mathematical) for interested students.

On top, almost all of our workshops include further working material (mostly pdf files) to help students develop formulas or gather results. They mostly consist of but are not limited to illustrations (like diagrams), sketches which have to be edited by the students (drawing, taking measurements, etc.) or answer sheets. These can be filled out using a pdf editing program or a whiteboard tool to draw into given figures.

Depending on the amount of guidance or (visual) support the teacher wants to give, the text and code cells can contain more or less content. Two extreme forms of support can be stated as: (a) designing and structuring learning material with various explanations, support material and images (suitable for more guided modeling activities, like the modeling days described in section 1.3) or (b) open activities where students are encouraged to develop and design Jupyter Notebooks themselves (shown in section 2.2).

To go a step further into the direction of self-directed learning, the students' solutions can be

[11] Besides the tip cards and the optional worksheets containing further information many other differentiation methods could be used in heterogeneous classes. For a broad overview see Robi Kronberg et al.(1997).

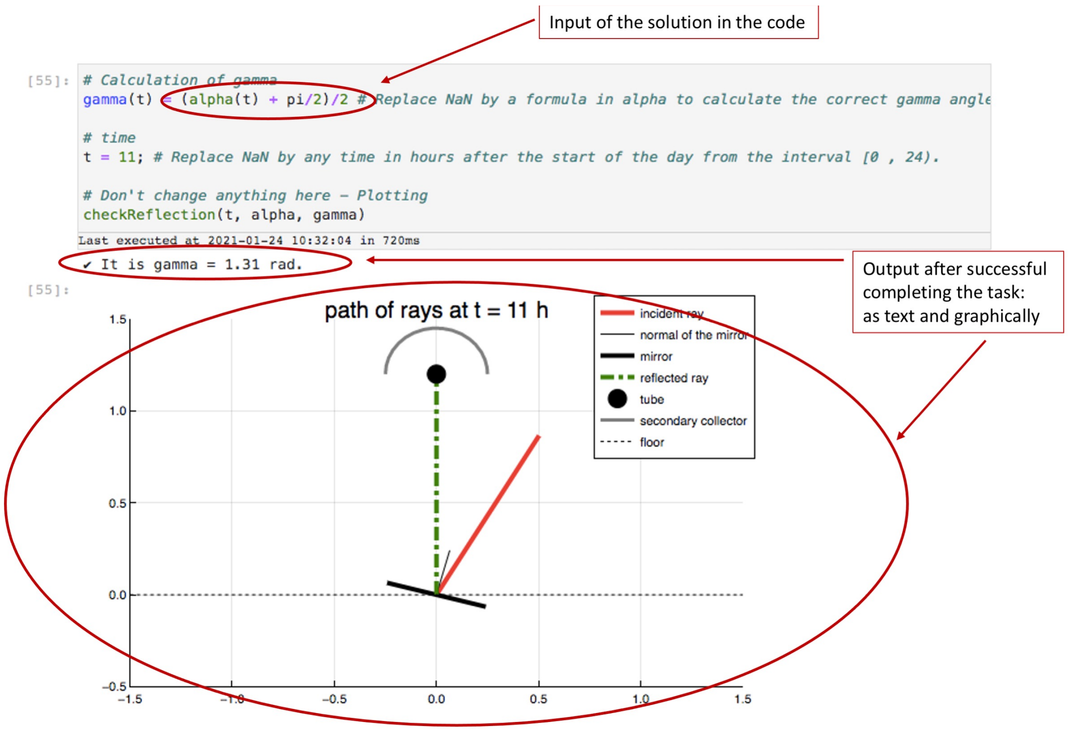

Figure 4 – Screenshot of input and output of a digital worksheet from the workshop on the optimization of a solar power plant. Here the correct solution of the task was already entered.

automatically checked and evaluated such that individual feedback and tips for correction are given. This will not only reduce the teacher's work, but also point towards mistakes early in the workflow. Note that these checking functions must be implemented by the teacher and are not available directly in the program.

Due to all the mentioned features, Jupyter Notebooks offer an excellent opportunity to develop learning material for mathematical modeling activities online.

Accessing our developed learning material

To give the reader a first impression of what digital learning material based on Jupyter Notebooks can look like, we want to finish the section by pointing out our own resources. The learning material we developed for modeling days are all located on an online accessible *workshop platform*[12] of the Karlsruhe Institute of Technology (KIT), Germany. Hence, the material can be used in any internet browser without installing any additional software. The only prerequisite for the realization of the learning material is a stable internet connection. The material can be accessed via `workshops.cammp.online`. Access data will be provided to interested readers at any time by mail to `cammp@scc.kit.edu`.

2.2. Collaborative calculation using Jupyter Notebooks on CoCalc

In contrast to the previous section, where we presented a way to develop guided, digital learning material, we now focus on the modeling weeks described in section 1.4, where students are supposed to work on the problems without a lot of guidance. An important requirement

[12] A server running the *JupyerHub* software allowing to manage multiple Jupyter Notebooks for several users working simultaneously. For more information see: `https://jupyter.org/hub`, accessed 25 January, 2021.

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http://polipapers.upv.es/index.php/MSEL

for these activities is that the students need to be able to work on the same code simultaneously. Therefore, a tool allowing for collaborative programming is required. One possible tool is the web-based computing platform for computational mathematics *CoCalc*[13]. CoCalc is open source, under active development and allows the creation of Jupyter Notebooks using different programming languages (Python, Julia, R, Octave).

The main difference between the use of the workshop platform[14] described in section 2.1 and CoCalc from a technical point of view is: CoCalc allows several students to work on the same Jupyter Notebook / code in real-time. Changes made by one student are immediately visible to all students in the same group. A screenshot of a notebook developed by a group of students during a modeling week using CoCalc is shown in figure 5. In contrast, on the workshop platform for the modeling days each student has his / her own version of the Jupyter Notebooks. Here, other students cannot make changes to these.

Furthermore, CoCalc has numerous additional functionalities that are not available on the workshop platform. These include, for example, a latex editor or a chat system. However, these additional functionalities increase the complexity and make it less intuitive to use. Experience has shown that CoCalc is better suited for activities that extend over longer periods. Whereas, due to its simplicity and the clarity the workshop platform is more suitable for shorter activities (such as the modeling days described in section 1.3).

Figure 5 – Jupyter Notebook developed by a group of students working on the problem: *How can initials and images on book pages be automatically recognized and distinguished from each other?* (The used illustrations are freely accessible via http://resolver.staatsbibliothek-berlin.de/SBB00006DFC00000000, accessed 1 June, 2021.)

[13] For further information see: https://cocalc.com/, accessed 8 January, 2021.
[14] By the term *workshop platform* we refer to the guided, digital learning material realized as Jupyter Notebooks running on JupyterHub.

2.3. Choosing a communication platform

As already mentioned, teamwork plays an essential role when dealing with (complex) modeling problems. In face-to-face classes, students can easily get together and discuss their problems and ideas, teachers answer questions and give encouragements. To replace this communication in a digital learning environment, a virtual communication platform is needed, which should fulfill the following requirements:

1. The tool should be open source so that there are no data protection problems when dealing with the students' personal data.
2. It should enable both oral and written exchange and discussion within a group of about 30 students.
3. Virtual breakout rooms are needed to allow students to exchange ideas in small teams.
4. An option to present slides and share screens is mandatory.
5. The platform should (ideally) provide a whiteboard where all members can exchange ideas through drawings.
6. The exchange of files should be possible.

All these functions should be clearly arranged, easy and intuitive to use and, like Jupyter Notebooks, operable in a web browser. Due to slow and unstable internet connections in many areas, the platform should be as robust as possible, i.e., allow for smooth reconnections.

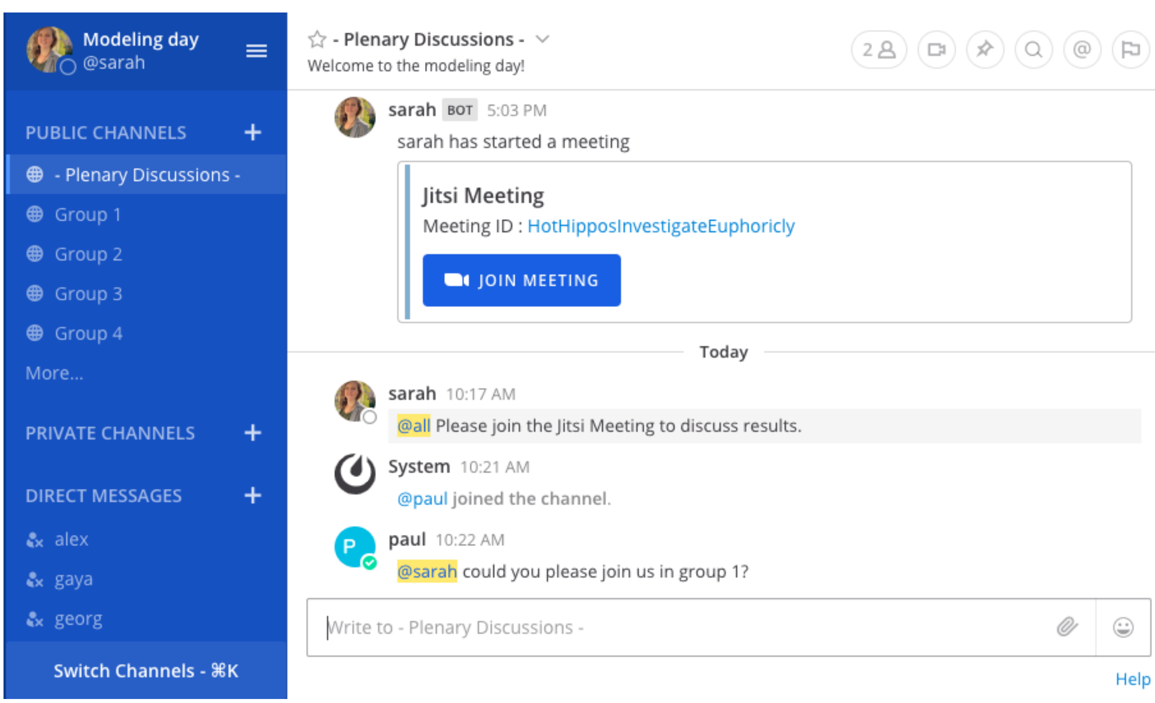

Figure 6 – The communication platform Mattermost

We tested several communication tools extensively during the past modeling days and weeks and decided to use *Mattermost*[15]. We now discuss to which extend Mattermost fulfills the mentioned requirements (based on our experience) and consequently explain our choice.
Mattermost is open source and can be used independently and flexibly by both teachers and students. One of the most important features leading to our decision is that within Mattermost all users (students and teachers) can create different channels if needed (see fig. 6). In these channels users can chat with one another and start video conferences using a Jitsi Plugin. Based on our experience it is most convenient to set up one channel for plenary phases and one channel for each group of students. Further key advantages of Mattermost when realizing online modeling activities are:

- Students can start Jitsi video conferences on their own and independently switch between different conferences. The teacher does not need to manually assign students to different subgroups (breakout rooms).
- The students can share their screen in the video conferences without having to obtain permission. This facilitates collaboration in a group.
- The chat history in the channels of each group of students is preserved. This is especially useful for multi-day activities when, e.g., older messages, links or screenshots need to be accessed again.
- The teacher can notify all students through a single message in the chat of the plenary channel (see fig. 6). This is useful, e.g., to give a hint on a task that concerns everyone, or to announce discussions in the plenary channel. Vice versa, students can contact the teacher at any time and ask him / her to join the video conference of their group.

3. Exemplary course of an online modeling day

The following section helps the reader to get a better understanding of our guided online modeling activities. Therefore, we describe an online modeling day on the problem of optimizing a solar power plant. We decided to put the focus on the technical and organizational aspects and will not describe the modeling of the problem in great detail. The digital learning material that is used during the modeling day can be accessed by the reader as described in section 2.1. Our modeling days typically last about five working hours. Depending on the learning group the different elements of the modeling days described below may vary in terms of time but also in their order and number. In some learning groups, for example, it may be reasonable to include more discussion phases and thus interrupt the individual working phases more frequently.

Technical check-in: If it is the first time that the students use Mattermost and Jitsi, we start with a quick technical check-in. The students join a Jitsi video conference in the plenary channel and are asked to unmute themselves (optionally to switch on the camera) to check if microphone and audio are working properly. The technical check-in takes approximately 5–10 minutes.

Introduction to the problem: The actual modeling day starts with a Jitsi video conference in the plenary channel with all students attending. The teacher gives a short introductory talk on the problem and on mathematical modeling. In the end the teacher presents the main question of the modeling day: *How should a solar thermal power plant be built and operated so that the energy gain is maximized?*

[15] https://mattermost.com/, accessed 23 January, 2021

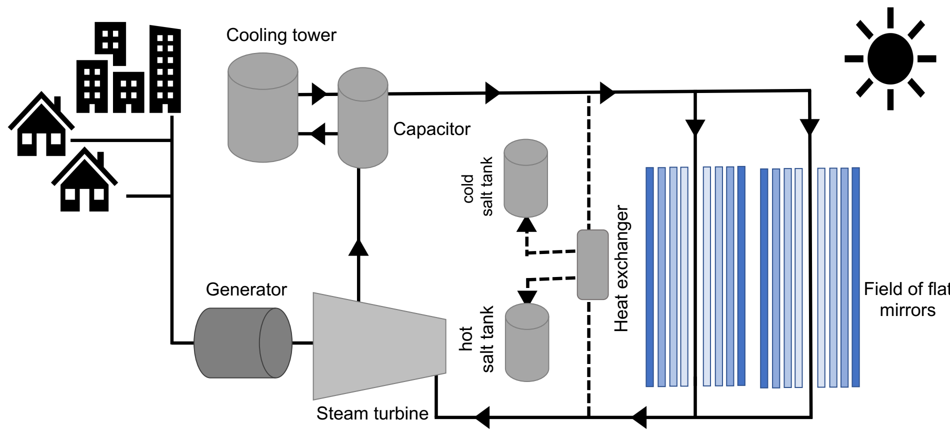

Figure 7 – Schematic structure of a Fresnel power plant

During the introduction, the teacher briefly describes the design of a so-called Fresnel power plant: flat mirrors focus sun rays onto an absorber tube (see fig. 7). The tube contains a heat transfer fluid, such as water, which is heated and, in the case of water, evaporated. Electrical energy is generated by a steam turbine.
The problem is explained using presentation slides shown via screen sharing. Short videos are also presented in between. The students are included in the discussion by sharing their prior knowledge on solar energy systems. As in face-to-face classes, students raise their (virtual) hands to speak up and ask questions.

Transition phase: After the introduction to the problem the teacher gives a short overview on how to use Mattermost during the group work. In addition, by sharing the screen he / she shortly shows how to access the digital learning material and explains the structure of the worksheets (Jupyter Notebooks) and how to use them. Afterwards the students continue by logging in to the workshop platform as described in section 2.1. The transition phase takes approximately 15 minutes.

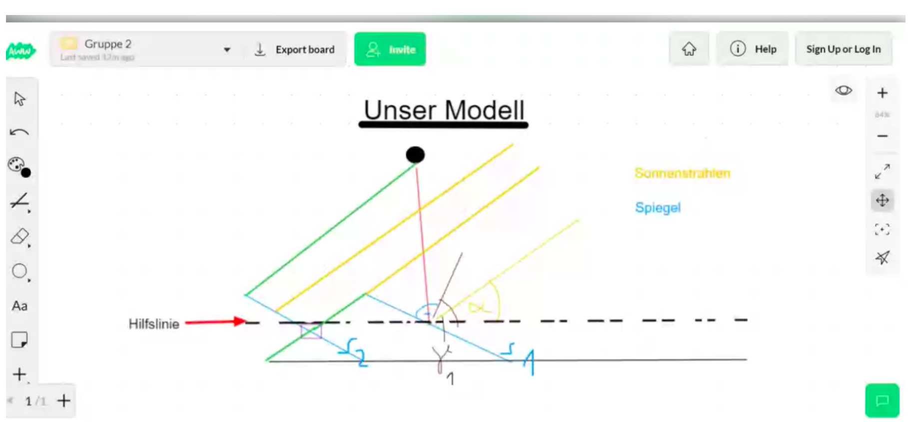

Figure 8 – Digital whiteboard (provided by awwapp) which was designed by a group of students during a modeling day

First working phase: The students start working on the digital learning material. For communication in small groups, two to three students join a channel in Mattermost. In this channel the students independently start their own video conference (via Jitsi). Furthermore, they can use the chat in their channel to send links, formulas and other material. While working on the worksheets the students discuss ideas or questions together – just as if they were in the same room.
In addition, to sketch ideas, students have the option to use a whiteboard tool.
Numerous free online whiteboard tools exist that can be used if there is no such tool integrated in the chosen communication platform. So far, we gained good experience with *awwapp*[16] (see fig. 8).
On the first worksheet of the solar power plant workshop, the students deal with the correct orientation of the mirrors. The inclination of the mirrors should be adjusted according to the movement of the sun so that the sun rays are reflected onto the absorber tube at all times of the day (see fig. 9)and with a Whiteboard-Plugin that can be used in Mattermost.

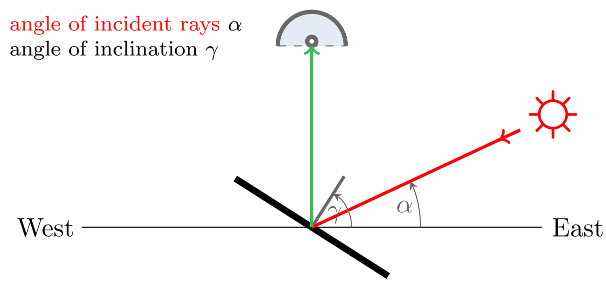

Figure 9 – The mirror is inclined in such a way that the incoming sun ray (red) gets reflected (green) onto the absorber tube

In case the students have further questions or difficulties they contact the teacher via a short message in the chat. During the working phases in groups the teacher is permanently available via the chat in the plenary channel. To keep an eye on the students' work and progress, the teacher can require the students to briefly write in the chat of the plenary channel whenever they complete a worksheet. He / she can also check in on each group's video conference at any time. If the teacher wants to grade the students' work, he / she can ask the students to hand in their documentations.

First intermediate discussion: The teacher uses the plenary channel to announce that the first plenary discussion is about to start. The focus lies on the students' results, but also leaves space for further open questions. Parts of the tasks can be drawn on a virtual whiteboard or can be presented by the students via screen sharing.
In the case of the solar power plant workshop the students briefly explain which mathematical / physical formulas and laws they used to obtain the correct equation describing the orientation of a mirror during the day.

Further working and discussion phases: Further working phases are structured in the same way as the first one described above. Students keep working on the learning material independently in small teams. At appropriate points in the course, the teacher gathers the students for plenary discussions again.
During the modeling day on solar power plants, plenary discussions are realized after the following modeling steps:

[16] For further information see: https://awwapp.com/, accessed 24 January, 2021.

- Developing a model for the orientation of the mirrors
- Developing a model for the power reaching the absorber tube
- Developing a model for the energy gain over a day
- Optimization of the positions of the mirrors

To ensure that faster groups are not running out of interesting problems, we provide additional material to continue, e.g., by developing a model to account for shading effects between mirrors or developing different algorithms to optimize the positions of the mirrors. Due to their fairly diverse nature, some of these questions leave plenty of freedom in abstraction levels and solution approaches to the students.

Final discussion: During the final discussion, the students reflect the whole modeling process. The results obtained are evaluated with respect to the real-world situation. The students are actively involved in the discussion by presenting their results orally, via screen sharing or on the whiteboard. Moreover, they discuss ideas to extend the modeling process. At the end of the modeling day, students are handed an online evaluation. In this questionnaire we ask for the experience and opinions of the students regarding the content and the organization of the online modeling day to improve our workshops.

4. Experiences and student feedback

In this section, we briefly describe our experiences with online modeling days and weeks. These are mainly based on observations of the authors during the modeling activities as well as on the students' feedback.

Experiences in guided modeling days using Jupyter Notebooks

The workshop platform described in section 2.1, where didactically prepared worksheets are available as Jupyter Notebooks, has already been used in numerous modeling activities *both online and in presence*. In total, more than 45 workshops with more than 650 students in grades 8–13 (this corresponds to an age range of approx. 13–19 years) from mainly German high schools have taken place. Few modeling events have already been held with groups of students from Mexico, France, China and the Netherlands.

The students' feedback after participating in a modeling day is mostly positive. Although many students get in touch with a programming language for the first time throughout the workshop, they are able to work through the learning material successfully. Mainly due to the simple syntax of Julia the students do not have any major difficulties. In addition, they take positive note of the material in general:

> "I really liked the interactive worksheets, I would like to have something like that in school" (participant of an online modeling day 2020).

Moreover, they positively emphasized that the material is compactly available in one place and that all the different documents and worksheets can be opened via links.

Experiences in Mattermost

Using the communication tool Mattermost with the Jitsi Plugin, we have already realized more than 15 guided workshops and two modeling weeks with over 220 students. The interaction with Mattermost during the workshops is very intuitive for the students and poses no problem as the following comments of students underline:

> "Everything worked very well within the given framework. Communication did not present any difficulties.", "We communicated through the available means of communication, as they were always reliable.", "The online communication was the best I have experienced so far" (participants of online modeling days and weeks 2020).

The students could easily switch between video conferences / chats with supervisors or with their own team. To reduce the amount of audio problems (which are definitely part of the online learning experience) and run the workshops as smoothly as possible, we usually offer a technical check-in period before the workshop starts as already mentioned in section 3. Due to (a) the successful online implementation of the learning material and (b) the continuous opportunity for interaction the students barely miss out on mathematical modeling experience:

> "I really enjoyed the CAMMP week. It was interesting and I especially liked the fact that I learned a lot and could exchange ideas with others" (participant of an online modeling week 2020).

This is promising and reinforces our efforts to continue in this direction.

Experiences in CoCalc

The tool CoCalc was used during two modeling weeks with over 45 high-school students. The feedback on the suitability and functionality of the CoCalc program was mainly positive. However, there is still some potential for improvement, because CoCalc refreshes itself from time to time. Nevertheless, the mix of Mattermost, Jitsi and CoCalc achieves exactly what we aimed for: students can both see what other students are implementing in CoCalc and talk about the code sections through Mattermost / Jitsi at the same time. As a result, the students were able to experience and learn what was intended, despite the online implementation:

> "I learned to work in a team, solve a problem independently, and present ideas to the group" (participant of an online modeling week 2020).

5. Summary and open challenges

Looking back at the questions we raised in section 1.5, we can summarize that the combination of Mattermost with CoCalc and Jupyter Notebooks respectively are well suited for our modeling activities. It enables many of the challenges posed: Students can communicate with each other, they can contact the teacher, the teacher can easily see how the students are coping and group discussions can spontaneously be carried out in plenary. On top, students can create sketches, exchange formulas, and receive outputs in different representation types without requiring any programming skills.

In case of the modeling days, students can self-regulate the amount of support they access and their solutions are checked automatically. Furthermore, during the modeling week, using CoCalc enabled collaborative and parallel programming.

One question that remains unaddressed is the integration of social human interaction since our focus, so far, has been on providing the technical implementation so that students can work together and communicate with one another. However, during the modeling activities that take place in presence (especially the modeling weeks[17]), the social exchange with peers is very important. That is why we always organize a social afternoon with games in the course of the weeks, especially to trigger the social component (see fig. 2). The following answers from

[17] Note that the participants of our modeling weeks normally attend different high-schools in Germany and do not know each other beforehand.

participants of a virtual modeling week 2020 to the question "What would you improve?" show how important this component is to students:

> "More time for social get together, which is hard to do online.", "More conversations away from work, which have come somewhat short due to the online format."

Consequently, we keep improving the social activities. Based on the feedback we have received we can conclude that we are on a promising way:

> "Overall, the CAMMP week was a lot of fun, despite the long distances between the participants, and I can definitely recommend it", "It was fun and I will miss my group" (participants of an online modeling week 2020).

These comments show that the students do get to know each other to a certain degree. Nevertheless, not everything can be realized to the same extent online:

> "I think you have implemented the online CAMMP week very well. Of course it would be better to meet in person, especially to get to know the people better, but I think you made the best out of it and I had a lot of fun too" (participant of an online modeling week 2020).

In the virtual format, the implementation of social events is more difficult as the students already pointed out. Many online games that are suitable for students getting to know each other already exist, but these, again, require the students to sit in front of the screen so that they get little movement or variety. This poses a challenge, for which we have not yet found a adequate solution.

In addition, the experiences described so far are all based on activities that were organized and carried out by scientific staff working at the program CAMMP. For the future, it would be interesting to see how in-service teachers cope with the digital tools and the learning material presented within this paper.

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