T violation in nonstandard neutrino oscillation scenarios

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We discuss time reversal (T) violation in neutrino oscillations in generic new physics scenarios. A general parametrization is adopted to describe flavor evolution, which captures a wide range of new physics effects, including nonstandard neutrino interactions, nonunitarity, and sterile neutrinos in a model-independent way. In this framework, we discuss general properties of time reversal in the context of long-baseline neutrino experiments. Special attention is given to fundamental versus environmental T violation in the presence of generic new physics. We point out that T violation in the disappearance channel requires new physics, which modifies flavor mixing at neutrino production and detection. We use time-dependent perturbation theory to study the effect of nonconstant matter density along the neutrino path and quantify the effects for the well-studied baselines of the DUNE, T2HK, and T2HKK projects. The material presented here provides the phenomenological background for the model-independent test of T violation proposed by us in [T. Schwetz and A. Segarra, Model-Independent Test of T Violation in Neutrino Oscillations, Phys. Rev. Lett. **128**, 091801 (2022).].

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I. INTRODUCTION

The search for the charge-parity (CP) symmetry violation is a central goal for current and upcoming long-baseline neutrino oscillation experiments. Fundamental CP violation is closely related to time reversal T violation thanks to the CPT theorem, which states that the product of these two transformations must be a symmetry. Early works on this topic include Refs. [1–3]. Fundamental CP or T violation are related to complex couplings in the Lagrangian. Indeed, in the standard three-flavor scenario [4], it is described by a single complex phase in the lepton mixing matrix [5–7], the so-called Dirac phase [8]. However, actual neutrino oscillation experiments involve neutrino passage through matter, and hence, observable transition probabilities are subject to matter effects [9]. Since a background of normal matter leads to violation of CPT in the neutrino flavor evolution [10], fundamental CP or T violation is not directly observable; see Refs. [11,12] for a recent discussion. In this respect, T violation has an advantage over CP violation for the following reason. Since the matter effect is different between neutrinos and antineutrinos, environmental CP violation is typically large and difficult to disentangle from fundamental CP violation. In contrast, it is well known that a symmetric matter density profile (symmetric between neutrino source and detector) does not introduce environmental T violation if the fundamental theory is T invariant; see, for instance, Refs. [13,14]. On the other hand, T violation itself is again difficult to observe experimentally since it formally corresponds to exchanging neutrino flavors of neutrino source and detector. There is extensive literature on T violation in neutrino oscillations; see Refs. [1,12–21] for an incomplete list.

The usual search for CP violation is highly model dependent. It relies on the standard unitary three-flavor paradigm, implying the absence of any new physics in neutrino interactions, mixing, and propagation. In this restricted framework, a parametric fit to the available data is performed in terms of the standard mass-squared differences Δm^2_{21}, Δm^2_{31}, mixing angles θ_{12}, θ_{23}, θ_{13}, and the complex phase δ. "Discovery of CP violation" is usually identified with the situation when the fit disfavors values of $\delta = 0$ and π at a certain confidence level. Indeed, within this approach, current results from the T2K [22] and NOvA [23] long-baseline experiments, combined with the global neutrino oscillation data, show already an indication for a preferred range of δ [24–26].

In Ref. [27], we have proposed a method, with the goal to address several limitations outlined above and to search for fundamental T violation in the neutrino sector in a more model-independent way. In order to achieve this goal, we have introduced two main ingredients:

(i) We prosed a rather general parametrization of neutrino evolution to describe the flavor system more model independently, and
(ii) We presented a potentially realistic way to search for fundamental T violation in long-baseline experiments.

Regarding (i), our general parametrization allows for effects of nonstandard interactions in neutrino source and detection, as well as arbitrary matter effect. Mixing can be nonunitary, and therefore, the presence of sterile neutrinos is allowed, as long as they do not introduce additional oscillation frequencies (i.e., we restrict to two independent oscillation frequencies). We will review this parametrization in detail in Sec. II below.

The main idea with respect to item (ii) is the following: We consider the oscillation probabilities within the general framework at different baselines but at the same energy. The reason for this assumption is that we want to be agnostic about the energy dependence of the new physics. Hence, we need to combine data from long-baseline experiments at different baselines L at the same neutrino energy. Then we check if the data requires T-odd (or equivalently L-odd) terms in the transition probability, based on the model-independent parametrization. We have shown in Ref. [27] that this test potentially can be performed already with data from three different long-baseline experiments (plus data from a near detector). This opens the possibility to apply the proposed test with actually planned and proposed experiments, such as DUNE ($L = 1300$ km) [28,29], T2HK ($L = 295$ km) [30], T2HK with a second detector in Korea, T2HKK ($L = 1100$ km) [31], and a long-baseline experiment at the European Spalation Source, ESSνSB ($L = 540$ km) [32,33]. The crucial requirement is the availability of measurements at the first and the second oscillation maxima (at the same energy), with sufficiently good energy reconstruction. Preliminary sensitivity estimates have been performed in Ref. [27].

The goals of the present paper are the following. We provide a more in-depth discussion of T violation, allowing for the general nonstandard physics described above to establish the theoretical basis for the test proposed in Ref. [27]. We show explicitly that several results known for the standard framework carry over to the new physics case considered here. For instance, we prove that any nonstandard matter effect does not introduce environmental T violation if the fundamental theory is T conserving, as long as the matter density profile is symmetric. Special care is given to nonstandard mixing effects in source and detector. We give a careful definition of the time reversal symmetry and discuss its effect in nonstandard mixing scenarios. Along the way, we establish the basic assumptions of the test in [27].

In Ref. [27], we have formulated the test by assuming a constant matter density along the neutrino path and that the matter density is the same for all experiments. These assumptions are only approximately valid for the experiments under consideration. Therefore, in the present article, we provide a quantitative estimate for corrections induced by realistic matter density profiles, based on the detailed investigations for the T2HKK and DUNE baselines from Refs. [34,35]. In general, our approach is based on a perturbation ansatz, based on the fact that both the new physics as well as the nonconstant density effects are small perturbations to the standard three-flavor and constant matter case. The methods developed below allow for a straightforward correction of the T violation test with respect to nonconstant density.

The outline of the remainder of the paper is as follows. In Sec. II, we review the model-independent parametrization of the neutrino flavor evolution and derive the transition amplitudes and probabilities, taking into account nonconstant matter densities as a small perturbation effect. In Sec. III, we consider the time reversal transformation within this model-independent setting and discuss which properties known for the standard oscillation case still hold in our model-independent framework and which do not. In Sec. IV, we provide some further discussion and comment on T violation in the disappearance channel within new physics scenarios. In Sec. V, we provide quantitative estimates of nonconstant matter density profiles for T2HK, T2HKK, and DUNE. We conclude in Sec. VI.

II. MODEL-INDEPENDENT DESCRIPTION OF FLAVOR EVOLUTION

We assume that propagation of the three Standard Model (SM) neutrino states is described by a hermitian Hamiltonian $H(E, x)$, which depends on neutrino energy E and, in general, on the matter density at the position x along the neutrino path. We follow the usual approximation in describing neutrino evolution by setting $x = t$ (in units where the speed of light is unity), i.e., localized neutrino wave packets propagating with the speed of light. Therefore, the space dependence of the Hamiltonian effectively becomes a time dependence. In the following, we will use x and t interchangeably. The evolution of the flavor state $|\psi\rangle$ is described by the equation,

$$i\partial_t|\psi\rangle = H(t)|\psi\rangle, \tag{1}$$

where, here and in the following, we suppress the energy dependence.

We consider neutrinos propagating from a source at position x_s at time t_s to a detector at position x_d, arriving at time t_d, with $x_d - x_s = t_d - t_s = L$. Let us define

$$H(t) = H_{\mathrm{vac}} + V_{\mathrm{tot}}(t) = H_0 + V(t), \tag{2}$$

$$V_{\mathrm{tot}}(t) = V_0 + V(t), \tag{3}$$

$$H_0 = H_{\mathrm{vac}} + V_0, \tag{4}$$

where H_0 contains contributions from the vacuum Hamiltonian, as well as the average matter potential V_0, and it is time and position independent, whereas $V(t)$ corresponds to the matter potential due to the varying matter density, with $\int_{x_s}^{x_d} dx V(x) = 0$. For our purposes (long-baseline experiments), we assume that $V(t)$ is a small perturbation, i.e., that the matter density is roughly constant along the neutrino path. This is a good approximation for experiments with baselines less than several 1000 km [34–37]. We will quantify this in Sec. V.

Let us diagonalize the position-independent part H_0 by $H_0 = W\lambda W^\dagger$, with W being a unitary matrix and $\lambda = (\lambda_i)$ a diagonal matrix of the real eigenvalues $\lambda_1, \lambda_2, \lambda_3$ of H. Both W and λ depend on the neutrino energy, and they will be different for neutrinos and antineutrinos due to the matter effect as well as fundamental CP violation. The energy eigenstates $|\nu_i\rangle$ fulfill

$$H_0|\nu_i\rangle = \lambda_i|\nu_i\rangle. \tag{5}$$

We allow for arbitrary (nonunitary) mixing of the energy eigenstates $|\nu_i\rangle$ with the flavor states $|\nu_\alpha\rangle$ relevant for detection and production,

$$|\nu_\alpha^{s,d}\rangle = \sum_{i=1}^{3} (N_{\alpha i}^{s,d})^* |\nu_i\rangle, \tag{6}$$

where * denotes complex conjugation. We make no specific assumption on the coefficients $N_{\alpha i}^s$ and $N_{\alpha i}^d$. They can include effects of heavy sterile neutrinos as well as nonstandard interactions. Note that generically, the unitary matrix W diagonalizing the Hamiltonian will contribute to these coefficients; in specific models, they may be related and the new physics entering in the coefficients $N_{\alpha i}^{s,d}$ and will also induce nonstandard contributions to the Hamiltonian $H(t)$. Some examples for specific models are nonunitary mixing [38,39], nonstandard neutrino interactions [40–43], or the presence of sterile neutrinos [44–47]. Our oscillation formalism has some similarities with the one developed in the context of nonunitary mixing; see, e.g., Refs. [48–50]. Reference [51] discusses the parametric relation between various nonstandard scenarios.

Here, we want to be more general and treat W and $N^{s,d}$ as independent. In particular, $N^{s,d}$ can be nonunitary, arbitrary functions of neutrino energy, and they can be different for processes relevant at the neutrino source and for detection (as indicated by the indices s and d). This implies that, in general, $|\nu_\alpha^s\rangle \neq |\nu_\alpha^d\rangle$, but we do assume that $N_{\alpha i}^{s,d}$ are the same for different experiments (at the same energy). Note, however, that while the mixing in Eq. (6) can be nonunitary, the induced matter potential $V_{\rm tot}$ as well as the total Hamiltonian will be still Hermitian, leading to a unitary evolution of the system via Eq. (1).[1]

As we will see in the following, complex phases of $N_{\alpha i}^{s,d}$ induce fundamental CP and T violation. In the standard scenario, there is only one relevant phase (the Dirac CP phase), whereas in nonstandard scenarios, there are several new sources for complex phases [38–47].

A. Transition probabilities

We are interested in the transition amplitude for a neutrino of flavor α at the source to a neutrino of flavor β at the detector, $\mathcal{A}(\nu_\alpha^s \to \nu_\beta^d) \equiv \mathcal{A}_{\alpha\beta}$, and the corresponding transition probability $P_{\alpha\beta} = |\mathcal{A}(\nu_\alpha^s \to \nu_\beta^d)|^2$. Consider first the unitary evolution operator S of the energy eigenstates, $|\nu_i(t^s)\rangle \to |\nu_j(t^d)\rangle$: $S_{ij}(t_d, t_s)$. Then using Eq. (6), we obtain

$$\mathcal{A}_{\alpha\beta} = \langle \nu_\beta^d | S(t_d, t_s) | \nu_\alpha^s \rangle = \sum_{ij} S_{ij}(t_d, t_s) N_{\alpha i}^{s*} N_{\beta j}^d. \tag{7}$$

Let us now consider $V(t)$ as a small perturbation and solve Eq. (1) using time-dependent perturbation theory; see, e.g., Ref. [52]. At zeroth order in $V(t)$, i.e., assuming only the constant Hamiltonian H_0, the evolution operator is just given by $S_{ij}^{(0)}(t_d, t_s) = e^{-i\lambda_i(t_d - t_s)}\delta_{ij}$ and

$$\mathcal{A}_{\alpha\beta}^{(0)} = \sum_i N_{\alpha i}^{s*} N_{\beta i}^d e^{-i\lambda_i(t_d - t_s)}. \tag{8}$$

At first order in $V(t)$, we find

$$S_{ij}^{(1)}(t_d, t_s) = -i e^{-i\lambda_j t_d + i\lambda_i t_s} \int_{t_s}^{t_d} dt V_{ij}(t) e^{i(\lambda_j - \lambda_i)t}, \tag{9}$$

$$\mathcal{A}_{\alpha\beta}^{(1)} = -i \sum_{ij} N_{\alpha i}^{s*} N_{\beta j}^d e^{-i\lambda_j t_d + i\lambda_i t_s} \int_{t_s}^{t_d} dt V_{ij}(t) e^{i(\lambda_j - \lambda_i)t}. \tag{10}$$

For the transition probability, we have

$$P_{\alpha\beta} \approx |\mathcal{A}_{\alpha\beta}^{(0)} + \mathcal{A}_{\alpha\beta}^{(1)}|^2 \approx P_{\alpha\beta}^{(0)} + P_{\alpha\beta}^{(1)},$$
$$P_{\alpha\beta}^{(0)} = |\mathcal{A}_{\alpha\beta}^{(0)}|^2, \qquad P_{\alpha\beta}^{(1)} = 2\,\mathrm{Re}[\mathcal{A}_{\alpha\beta}^{(0)*} \mathcal{A}_{\alpha\beta}^{(1)}]. \tag{11}$$

The parametrization discussed here allows one to cover a rather broad range of new physics scenarios, including nonstandard interactions in charged current and neutral current interactions, generic nonunitarity, as well as sterile neutrinos (as long as they do not introduce additional oscillation frequencies for the relevant energy and baselines). In principle, we can allow for an arbitrary energy dependence of $N_{\alpha i}^{s,d}$ and λ_i, although in the presence of finite energy resolution, we have to demand that the energy dependence is weak at the scale of the resolution. Following Ref. [27], due to the success of the standard three-flavor paradigm, we can assume that new physics effects are a small perturbation to the standard case. To leading order, our parametrization covers also nonstandard

[1]Per assumption, we do not allow for the decay of energy eigenstates, which would lead to a nonunitary evolution equation.

interactions within the most general effective field theory framework [53,54].[2]

III. PROPERTIES UNDER THE T TRANSFORMATION

With these generalized expressions for the transition amplitudes and probabilities at hand, we can now discuss their properties under the time reflection transformation.

A. Constant matter potential

Let us first assume that the matter potential is constant; i.e., we work at zeroth order in the time-dependent perturbation theory. In this case, we can define the T transformation by

$$T: t \to -t. \tag{12}$$

Then, we obtain from Eq. (8),

$$T\mathcal{A}^{(0)}_{\alpha\beta} = T\mathcal{A}^{(0)}(\nu^s_\alpha \to \nu^d_\beta) = [\mathcal{A}^{(0)}(\nu^d_\beta \to \nu^s_\alpha)]^*. \tag{13}$$

Note that if $N^s_{\alpha i} \neq N^d_{\alpha i}$, then $\mathcal{A}^{(0)}(\nu^d_\beta \to \nu^s_\alpha) \neq \mathcal{A}^{(0)}_{\beta\alpha} = \mathcal{A}^{(0)}(\nu^s_\beta \to \nu^d_\alpha)$. Therefore, the usual result $TP_{\alpha\beta} = P_{\beta\alpha}$ holds only if the mixing is the same for the processes relevant for neutrino production and detection. If there is new physics distinguishing between neutrino production and detection, the transformation (12) is *not* equivalent to exchanging only the neutrino *flavors* of source and detector, but also the type of interaction needs to be exchanged (formally $d \leftrightarrow s$).

For real $N^{s,d}_{\alpha i}$, it follows from Eq. (8) that $T\mathcal{A}^{(0)}_{\alpha\beta} = \mathcal{A}^{(0)*}_{\alpha\beta}$, and therefore, $TP_{\alpha\beta} = P_{\alpha\beta}$. As expected, the T transformation tests complex phases in the theory.

For the transition probabilities at zeroth order in $V(t)$, we obtain

$$P^{(0)}_{\alpha\beta} = \left|\sum\nolimits_i c_i e^{-i\lambda_i(t_d - t_s)}\right|^2 \tag{14}$$

$$= \sum_i |c_i|^2 + 2\sum_{j<i} \mathrm{Re}(c_i c_j^*)\cos(\omega_{ij}L) - 2\sum_{j<i} \mathrm{Im}(c_i c_j^*)\sin(\omega_{ij}L), \tag{15}$$

with the abbreviation $c_i \equiv N^{s*}_{\alpha i} N^d_{\beta i}$ and the frequencies $\omega_{ij} \equiv \lambda_j - \lambda_i$. As usual, we identify the baseline by $L = t_d - t_s$. Therefore, T is formally equivalent to $L \to -L$ [1]. We see that the first line of Eq. (15) is invariant under T, whereas the second line is T-odd. It is also apparent that T violation will be present only for nonzero $\mathrm{Im}(c_i c_j^*)$, i.e., nontrivial complex phases of $N^{s,d}_{\alpha i}$.

Hence, fundamental T violation can be established by proving the presence of the L-odd term in the probability. Or, put in other words, if data cannot be described by an L-even transition probability,

$$P^{\mathrm{even}}_{\alpha\beta}(L) = \sum_i c_i^2 + 2\sum_{j<i} c_i c_j \cos(\omega_{ij}L), \tag{16}$$

with c_i real, and fundamental T violation needs to be present in the theory. This is the test proposed in Ref. [27].

B. Nonconstant matter potential

Let us now consider the first-order correction in the case of a nonconstant matter potential $V(t)$. We recall that $V(t)$ is defined between the locations of the source x_s and the detector x_d. Therefore, we have to make sure that $V(t)$ is evaluated only for times in the interval $[t_s, t_d]$. This requirement has to be respected also when applying the T transformation in Eq. (12). One possible choice is to replace Eq. (12) by

$$T: t \to t_s + t_d - t. \tag{17}$$

This implies that T leads to $t_s \leftrightarrow t_d$ and that T is still equivalent to $L \to -L$. Applying this transformation to Eq. (10), we find

$$T\mathcal{A}^{(1)}_{\alpha\beta} = i\sum_{ij} N^{s*}_{\alpha j} N^d_{\beta i} e^{i\lambda_j t_d - i\lambda_i t_s} \int_{t_s}^{t_d} dt V^*_{ij}(t) e^{-i(\lambda_j - \lambda_i)t}. \tag{18}$$

Here, we have renamed the indices $i \leftrightarrow j$ and used the Hermiticity of the Hamiltonian, $V_{ij} = V^*_{ji}$. We observe that Eq. (13), which we have obtained for $\mathcal{A}^{(0)}_{\alpha\beta}$, and the comments thereafter hold also for the first-order correction $\mathcal{A}^{(1)}_{\alpha\beta}$.

As we have seen in Sec. III A for the case of constant matter density, the mixing coefficients $N^{s,d}_{\alpha i}$ are the only sources of complex phases relevant for the transition probabilities, and therefore, fundamental T violation or conservation can be characterized by the presence or absence of (nontrivial) complex phases in $N^{s,d}_{\alpha i}$. Note, however, that $N^{s,d}_{\alpha i}$ are defined with respect to the eigenbasis of the Hamiltonian H_0 for constant density; see Eqs. (5) and (6). Therefore, in general, the nonconstant perturbation $V(t)$ may contain nontrivial complex phases even if $N^{s,d}_{\alpha i}$ are real. Hence, we will define "fundamental T conservation" in the following as real $N^{s,d}_{\alpha i}$ *and* real $V_{ij}(t)$.[3]

Comparing Eqs. (10) and (18), we see that even for $N^{s,d}_{\alpha i}$ and $V_{ij}(t)$ real, we still have $T\mathcal{A}^{(1)}_{\alpha\beta} \neq \mathcal{A}^{(1)*}_{\alpha\beta}$ in general. Hence, we recover the result that a nonconstant matter density induces environmental T violation in neutrino

[2]We thank Martin Gonzalez-Alonso for illuminating discussions about this point.

[3]Note that $N^{s,d}_{\alpha i}$ includes possible phases from the constant matter potential. Therefore, having complex phases in $V(t)$ but not in $N^{s,d}_{\alpha i}$ would require rather special CP violating new physics, coupling only to density *variations*.

oscillations, even if the fundamental theory is T conserving. This is a well-known result in the standard oscillation scenario, e.g., [14].

We can also prove the result known for the standard case, namely that a symmetric matter profile does not induce T violation in the absence of fundamental T violation. Following Ref. [14], this can most easily seen by setting the time $t = 0$ at the baseline mid-point, such that the time interval $[t_s, t_d]$ becomes symmetric: $[-L/2, L/2]$. The transformation (17) still remains $L \to -L$. Rewriting Eq. (10), we obtain

$$\mathcal{A}^{(1)}_{\alpha\beta} = -i\sum_{ij} N^{s*}_{\alpha i} N^{d}_{\beta j} e^{-i(\lambda_j+\lambda_i)L/2} \int_{-L/2}^{L/2} dt V_{ij}(t) e^{i(\lambda_j-\lambda_i)t}, \tag{19}$$

$$T\mathcal{A}^{(1)}_{\alpha\beta} = i\sum_{ij} N^{s*}_{\alpha i} N^{d}_{\beta j} e^{i(\lambda_j+\lambda_i)L/2} \int_{-L/2}^{L/2} dt V_{ij}(-t) e^{-i(\lambda_j-\lambda_i)t}, \tag{20}$$

where, in the second line, we have performed the variable transformation $t \to -t$ in the integral. Comparing the two expressions, we see that if $N^{s,d}_{\alpha i}$ and V_{ij} are real (no fundamental T violation), and the matter potential is symmetric, $V_{ij}(t) = V_{ij}(-t)$, then $T\mathcal{A}^{(1)}_{\alpha\beta} = \mathcal{A}^{(1)*}_{\alpha\beta}$. Since the same holds for $\mathcal{A}^{(0)}_{\alpha\beta}$, we have $TP_{\alpha\beta} = P_{\alpha\beta}$ in this case. Hence, the above statement is proven. As in the standard scenario, in order to introduce environmental T violation in the absence of fundamental T violation, an asymmetric matter potential is needed.

Here, we derived this result at first order in perturbation theory. In Ref. [14], this statement was proven for the standard oscillation scenario for arbitrary matter profile. It is straightforward to generalize the proof given in [14] also to the nonstandard scenario considered here, which we briefly outline in the following. We depart from Eq. (7) and use general properties of the evolution operator:

$$S(t_d, t_s) S(t_s, t_d) = 1, \qquad S(t_d, t_s) S^\dagger(t_d, t_s) = 1. \tag{21}$$

The second relation follows from the unitarity of the evolution due to the Hermiticity of $H(t)$. Using now the T transformation $T: t_s \leftrightarrow t_d$ and combining the two properties above, we find

$$TS(t_d, t_s) = S(t_s, t_d) = S^\dagger(t_d, t_s). \tag{22}$$

Using again a symmetric time coordinate, we consider the evolution operator $S(t, -t)$. Its time evolution is given by [14]

$$i\frac{d}{dt} S(t, -t) = H(t) S(t, -t) + S(t, -t) H(-t). \tag{23}$$

Take now the transpose of this equation. If the Hamilton operator is real (no fundamental T violation), then it is symmetric. If, in addition, the density profile is symmetric, $H(t) = H(-t)$, we see that $S(t, -t)$ and $S^T(t, -t)$ follow the same evolution equation and are therefore equal; i.e., S is symmetric. Using this in Eq. (22), we obtain $TS(t_d, t_s) = S^*(t_d, t_s)$. With real $N^{s,d}_{\alpha i}$, we obtain then from Eq. (7) that $T\mathcal{A}_{\alpha\beta} = \mathcal{A}^*_{\alpha\beta}$ and therefore, $TP_{\alpha\beta} = P_{\alpha\beta}$.

IV. COMMENTS ON *T* ASYMMETRIES AND THE DISAPPEARANCE CHANNEL

Let us collect a few relations regarding time reversal asymmetries. We define the T asymmetry as

$$A_{\alpha\beta} = P_{\alpha\beta} - TP_{\alpha\beta}. \tag{24}$$

From Eq. (15), we find for the zeroth-order asymmetry,

$$A^{(0)}_{\alpha\beta} = 4\sum_{i<j} \mathrm{Im}(N^s_{\alpha i} N^{d*}_{\beta i} N^{s*}_{\alpha j} N^d_{\beta j}) \sin(\omega_{ij} L). \tag{25}$$

The first order correction to the probabilities from Eq. (11) can be written in the following way:

$$P^{(1)}_{\alpha\beta} = \sum_{ijk} \sin\left(\lambda^k_{ij}\frac{L}{2}\right) \mathrm{Re}G^k_{ij} + \sum_{ijk} \cos\left(\lambda^k_{ij}\frac{L}{2}\right) \mathrm{Im}G^k_{ij}, \tag{26}$$

where we have defined

$$\lambda^k_{ij} = 2\lambda_k - (\lambda_i + \lambda_j), \tag{27}$$

$$G^k_{ij} = 2N^s_{\alpha k} N^{d*}_{\beta k} N^{s*}_{\alpha i} N^d_{\beta j} I_{ij} \tag{28}$$

$$I_{ij} = \int_{-L/2}^{L/2} dt V_{ij}(t) e^{i(\lambda_j-\lambda_i)t}, \tag{29}$$

with $\lambda^k_{ij} = \lambda^k_{ji}$ and $I_{ji} = I^*_{ij}$. The T transformation corresponds to $L \to -L$, and it follows that $TI = -I$, $TG = -G$, and therefore, the first (second) line in Eq. (26) is T even (odd). Hence, we obtain for the first order asymmetry

$$A^{(1)}_{\alpha\beta} = 2\sum_{ijk} \cos\left(\lambda^k_{ij}\frac{L}{2}\right) \mathrm{Im}G^k_{ij}. \tag{30}$$

For a symmetric profile $V_{ij}(t) = V_{ij}(-t)$, we have

$$I^{\mathrm{sym}}_{ij} = \int_{-L/2}^{L/2} dt V_{ij}(t) \cos(\omega_{ij} t). \tag{31}$$

In the absence of fundamental T violation, with $N^{s,d}_{\alpha i}$ and V_{ij} real, $I^{\rm sym}_{ij}$ and G^k_{ij} are real as well, and $A^{(1)}_{\alpha\beta} = 0$, in agreement with our results in Sec. III B.

Consider now the disappearance probabilities $\beta = \alpha$. In the standard oscillation scenario, the T transformation becomes trivial since exchanging initial and final flavor has no effect. Let us reconsider this case in the extended new physics scenario. First we assume that mixing is identical at the source and detector $N^s_{\alpha i} = N^d_{\alpha i}$. Then we find that both $A^{(0)}_{\alpha\alpha} = 0$ and $A^{(1)}_{\alpha\alpha} = 0$. The first follows directly from Eq. (25). The second follows from Eq. (30) by noting that for $N^s_{\alpha i} = N^d_{\alpha i}$ and $\alpha = \beta$, we have $G^k_{ij} = G^{k*}_{ji}$. Hence, we conclude that for $N^s_{\alpha i} = N^d_{\alpha i}$ (which includes also standard mixing), no T asymmetry can be observed in the disappearance channel. This holds even in presence of complex phases as well as asymmetric matter density profiles.

On the contrary, if $N^s_{\alpha i} \neq N^d_{\alpha i}$, in the presence of nontrivial complex phases, we obtain $A^{(0)}_{\alpha\alpha} \neq 0$. If $N^s_{\alpha i} \neq N^d_{\alpha i}$ but both real, then $A^{(1)}_{\alpha\alpha} \neq 0$ for an asymmetric density profile.[4] We conclude that

(i) The observation of T violation in a disappearance channel would be a signal of new physics inducing different flavor mixing at source and detector;
(ii) If effects of asymmetric matter densities can be neglected, it requires fundamental T violation (in addition to $N^s_{\alpha i} \neq N^d_{\alpha i}$).

Let us briefly comment on the possible observability of such an effect, at least in principle. One can follow the approach of Ref. [27] and imagine measurements of $P_{\alpha\alpha}(L_b)$ at a number of baselines L_b at a fixed energy and, in this way, study the L dependence of the probability. Then one can check if this shape is consistent with an even function of L, or if data require the presence of L-odd terms. However, to map out the L dependence for the disappearance channel, one would need several data points, covering at least first and second oscillation maxima. Therefore, currently such an analysis seems not feasible with the proposed long-baseline experiments. The test studied in Ref. [27] is based on the interplay of disappearance and appearance channel and therefore, works already with four baselines (including the near detector). We leave for future studies whether the disappearance test could potentially be performed with atmospheric neutrinos.

V. ESTIMATION OF NONCONSTANT DENSITY CORRECTIONS

In this section, we are going to use this formalism to estimate the impact of a nonconstant density for our T violation test. We will address the following two points:

(1) When the average matter densities for different baselines are not exactly the same, and

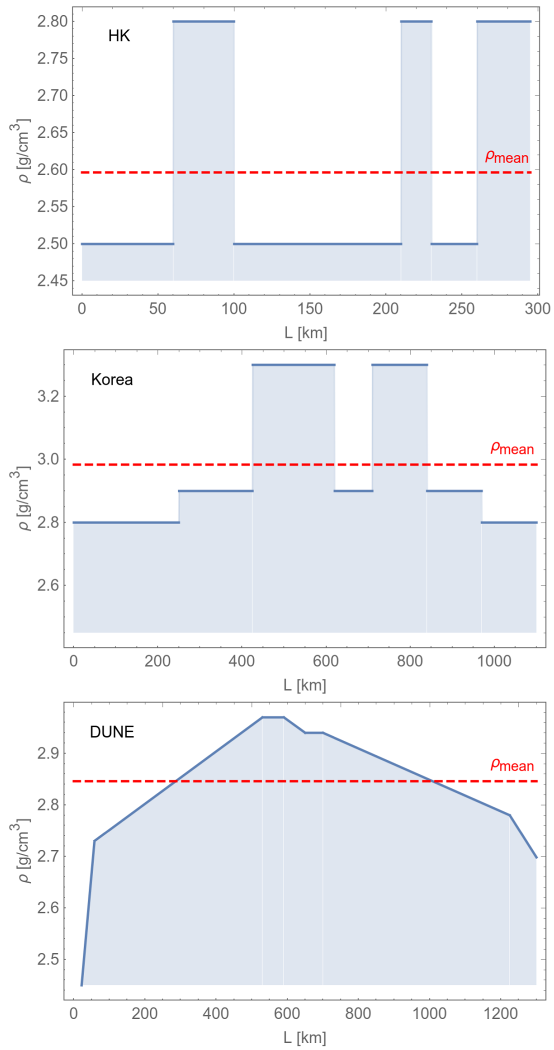

FIG. 1. Matter density along the baseline of the T2HK (top) T2HKK (middle) and DUNE (bottom) experiments. Data taken from Refs. [34,35].

(2) An asymmetric density profile at a given baseline.

We consider these two cases using existing density profile studies for the T2HK(K) [34] and DUNE [35] experiments.[5] The corresponding density profiles are shown in Fig. 1.

Since both nonconstant density effects and new physics are small corrections to the standard scenario with constant matter density, we work in leading order in small effects; we can neglect any new physics effect in estimating the size of the matter density corrections. In this way, the nonconstant density will introduce calculable corrections, which can be taken into account in the T violation test, as we describe below.

[4]Note, however, that this would be a second order effect, being suppressed by the small density variations *and* the new physics responsible for $N^s_{\alpha i} \neq N^d_{\alpha i}$.

[5]As shown in Ref. [27], the ESSνSB experiment contributes only very little to the sensitivity of the T violation test. Therefore, we focus here on the DUNE and T2HK(K) experiments.

A. Different average densities

Let us first assume that the matter density can be considered constant for each experiment, even if the average values are not the same. From the profiles shown in Fig. 1, we obtain

$$\bar{\rho}_{\rm HK} = 2.6\ {\rm g/cm^3},$$
$$\bar{\rho}_{\rm DUNE} = 2.85\ {\rm g/cm^3},$$
$$\bar{\rho}_{\rm HKK} = 3.0\ {\rm g/cm^3}. \tag{32}$$

The Hamiltonian of the system is thus reduced to the time-independent H_0 in Eq. (4). Assuming the standard neutrino model, it reads

$$H_b = \frac{1}{2E} U \begin{bmatrix} 0 & & \\ & \Delta m^2_{21} & \\ & & \Delta m^2_{31} \end{bmatrix} U^\dagger + \begin{bmatrix} \bar{v}_b & & \\ & 0 & \\ & & 0 \end{bmatrix} \tag{33}$$

in the flavor basis, where E is the neutrino energy, U is the standard PMNS mixing matrix, $\Delta m^2_{ij} \equiv m^2_i - m^2_j$ are the neutrino mass-squared differences, and

$$v(\rho) = \sqrt{2} G_F n_e(\rho) \approx 3.78 \times 10^{-14}\ {\rm eV} \left[\frac{\rho}{\rm g/cm^3}\right]. \tag{34}$$

$\bar{v}_b$ denotes the potential corresponding to the average density $\bar{\rho}_b$, and the index b labels the different baselines, which emphasizes that each baseline may have a different mean density, and thus, a different Hamiltonian. We study the time evolution of the system by diagonalizing this Hamiltonian numerically, which leads to a set of effective masses $m^2_b = 2E\lambda_b$ and mixings $N^s_b = N^d_b = W_b$ for each experiment.

In Table I, we give the values of disappearance and appearance oscillation probabilities for the three baselines, assuming different values for the mean densities for a few choices of the CP phase δ. The relative size of the effect for the appearance probabilities is shown in the upper panel of Fig. 2. For the table and the figure, we have chosen the neutrino energy $E = 0.75$ GeV, which has been found to provide the best sensitivity in Ref. [27]. For the figure, we assume the mean density $\bar{\rho} = 2.85\ {\rm g/cm^3}$, corresponding to the DUNE experiment, and show the relative error induced for the T2HK and T2HKK baselines. As mentioned above, we can use the standard oscillation scenario to estimate this effect, considering only leading order terms in density variations and new physics. We show the size of the correction as a function of the CP phase δ; all other oscillation parameters are set to their best fit values [24]. We see the effect is below 1% for all values of the CP phase δ. For the disappearance probabilities, the effect is even smaller; compare Table I.

TABLE I. Disappearance (top) and appearance (bottom) oscillation probabilities at the T2HK, T2HKK, and DUNE baselines, assuming different mean densities, for $\delta = 0, 90°, 180°$ and $E = 0.75$ GeV. The bold values correspond to the correct $\bar{\rho}$ for each experiment.

$P_{\mu\mu}(\%)$		$\bar{\rho}({\rm g/cm^3})$		
	L (km)	2.6	2.85	3.0
$\delta = 0$	295	**11.69**	11.68	11.68
	1100	2.19	2.19	**2.19**
	1300	41.77	**41.78**	41.78
$\delta = 90°$	295	**12.05**	12.05	12.05
	1100	2.82	2.82	**2.82**
	1300	38.94	**38.91**	38.89
$\delta = 180°$	295	**12.41**	12.41	12.41
	1100	3.53	3.53	**3.53**
	1300	36.22	**36.15**	36.12

$P_{\mu e}(\%)$		$\bar{\rho}({\rm g/cm^3})$		
	L (km)	2.6	2.85	3.0
$\delta = 0$	295	**4.77**	4.80	4.81
	1100	5.77	5.72	**5.69**
	1300	2.65	**2.76**	2.83
$\delta = 90°$	295	**3.53**	3.55	3.56
	1100	1.63	1.62	**1.60**
	1300	2.06	**2.17**	2.23
$\delta = 180°$	295	**4.11**	4.13	4.15
	1100	4.69	4.65	**4.63**
	1300	7.93	**8.12**	8.24

Regarding the T violation test [27] based on Eq. (16), the main effect of the different mean densities is that the fit parameters, the amplitudes c_i, and frequencies ω_{ij} are no longer the same for all baselines. Since the density dependence is small, we can include it in the fit, assuming

$$c^b_i = \bar{c}_i + \delta c^b_i, \qquad \omega^b_{ij} = \bar{\omega}_{ij} + \delta\omega^b_{ij}, \tag{35}$$

with the reference parameters $\bar{c}_i$ and $\bar{\omega}_{ij}$ corresponding to standard oscillations and a common density $\bar{\rho}$ taken the same for all baselines. Expanding up to first order in these L-dependent perturbations, the L-even probability in Eq. (16) becomes

$$\begin{aligned} P^{{\rm even},b}_{\alpha\beta} = {} & \sum_i \bar{c}^2_i + 2\sum_{j<i} \bar{c}_i \bar{c}_j \cos(\bar{\omega}_{ij} L) \\ & + 2\sum_i \bar{c}_i \delta c^b_i + 2\sum_{j<i} (\bar{c}_i \delta c^b_j + \bar{c}_j \delta c^b_i) \cos(\bar{\omega}_{ij} L) \\ & - 2\sum_{j<i} \bar{c}_i \bar{c}_j \delta\omega^b_{ij} L \sin(\bar{\omega}_{ij} L). \end{aligned} \tag{36}$$

Thus, the crucial effect of different (mean) densities is the appearance of new terms in the (previously) L-even

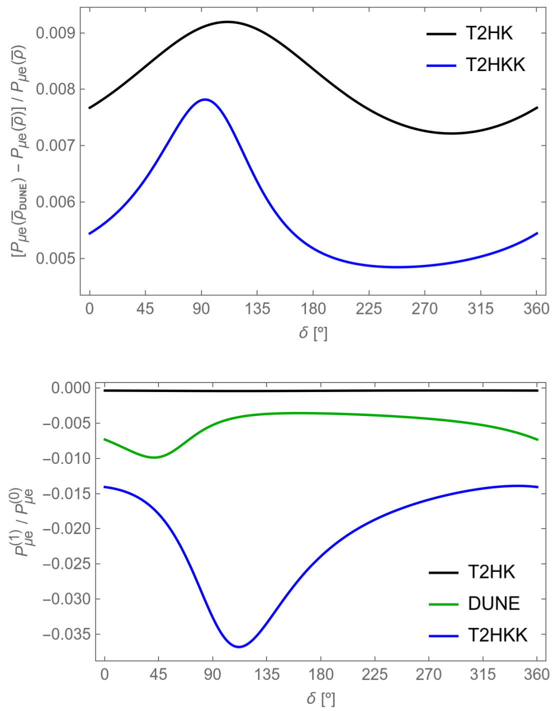

FIG. 2. Relative correction of the standard appearance probabilities as a function of the CP phase δ for $E = 0.75$ GeV. The upper panel shows the relative error for T2HK and T2HKK if the constant mean density of 2.85 g/cm^3 is assumed instead of the correct ones according to Eq. (32). The lower panel shows the relative size of the correction due to the nonconstant matter potential $V(t)$.

oscillation probability. Note the corrections shown in the second and third lines of Eq. (36) are known and fixed and can be just included in the test described in Ref. [27] as constant correction terms for each baseline. The fit itself can be performed with an expression equivalent to the firsts line in Eq. (36), consistent with the leading order perturbation approach mentioned above. From Fig. 2, we see that at $\delta = 0$ and π relevant for the test, the corrections are $\lesssim 0.5\%$ on the appearance probabilities, which themselves are only few %. Therefore, with realistic statistical uncertainties, these corrections are negligible.

B. Nonconstant density

Let us now discuss the effect of a nonconstant and nonsymmetric matter profile at a given baseline. We use the formalism developed Sec. III B to calculate how much this affects the probabilities to be probed in T2HK(K) and DUNE, assuming the standard neutrino model (using the same perturbative argument as above).

Within our perturbation theory in $V(t)$, the zeroth-order result corresponds to the diagonalization of the Hamiltonian with the constant mean density of the previous subsection. Therefore, the procedure described above yields the eigenvalues λ_b and mixing matrices $N^s_b = N^d_b = W_b$ for each experiment. As above, we obtain the mean probabilities from Eq. (8) as $P^{(0)}_{\alpha\beta} = |\mathcal{A}^{(0)}_{\alpha\beta}|^2$. The first order correction to the oscillation amplitudes is then given by Eq. (19) in terms of these parameters, and the matrix elements of the perturbation in the H_0 eigenbasis are $V_{ij}(t) = W_{ei} W^*_{ej} [v(t) - \bar{v}]$. With this, we can calculate the first order correction to the probabilities given in Eq. (11). The relative size of this correction is shown in the lower panel of Fig. 2 as a function of δ. We observe that for T2HK, these corrections are negligible and not visible on the scale of the plot. For DUNE, the effect is subpercent for all values of δ. For T2HKK, it can become as large as 3.5% for $\delta \simeq 110°$; for $\delta = 0$ and π, it is around 1.5%. Similar as in Sec. V A, these are calculable and fixed corrections to the probabilities, which can be taken into account in the test proposed in Ref. [27]. However, considering that these are percent-level correction on probabilities that themselves are only a few %, this effect is again negligibly small, given realistic statistical errors.

Notice, however, that the size of such corrections does not directly determine the amount of environmental T violation induced by the matter profile. Even for the case of a symmetric profile with real mixings, where no extra T violation is introduced, the oscillation probabilities themselves get a nonvanishing correction. In order to get a feeling for the size of environmental T violation at the DUNE and T2HKK baselines, we calculate the asymmetries (24) for the standard oscillation case considered above:

$$\begin{aligned} \text{T2HKK:}\ & A^{(1)}_{\mu e} \approx 3.5(5.3) \times 10^{-4}, \\ \text{DUNE:}\ & A^{(1)}_{\mu e} \approx -2.9(-2.0) \times 10^{-4}, \end{aligned} \tag{37}$$

for $\delta = 0(180°)$. These can be compared to the case of $\delta = 90°$, where we find

$$\begin{aligned} \text{T2HKK:}\ & A^{(0)}_{\mu e} \approx 7.1 \times 10^{-2}, \qquad A^{(1)}_{\mu e} \approx -9.1 \times 10^{-4}, \\ \text{DUNE:}\ & A^{(0)}_{\mu e} \approx 6.5 \times 10^{-2}, \qquad A^{(1)}_{\mu e} \approx 3.8 \times 10^{-4}. \end{aligned} \tag{38}$$

We conclude that for these realistic density profiles, environmental T violation is typically a % level effect compared to generic fundamental T violation [34,35].

VI. CONCLUSIONS

In this paper, we have studied some aspects of the time reversal transformation in a generic nonstandard neutrino oscillation framework. The motivation for our study is the model-independent T violation test proposed recently in Ref. [27]. This test can potentially be performed with three long-baseline experiments, such as T2HK, DUNE, and the proposed T2HKK. Here, we provide a theoretical discussion of the formalism for the model-independent new physics parametrization proposed in Ref. [27]. We derive the relevant flavor transition amplitudes and probabilities and study their behavior under the T transformation.

The proposed parametrization covers a wide range of new physics scenarios in a model-independent way, including nonstandard neutrino interactions with arbitrary Lorentz structures in the charged and neutral-current interaction, generic nonunitarity as well as sterile neutrinos. We provide a discussion of fundamental versus environmentally induced T violation, where the former is related to complex phases in the theory while the latter is due to (standard or nonstandard) matter effects along the neutrino path. We show that a result well known for standard oscillations holds also in our extended scenario: In the absence of fundamental T violation, environmental T violation can only be induced by an asymmetric matter density profile.

We show that in general new physics scenarios, the disappearance channel can be sensitive to T violating effects. This requires new physics generating different flavor mixing at neutrino source and detector. Although difficult to realize in practice, such an observation offers, in principle, a clear signal of new physics since in the standard oscillation scenario, no T violation is expected in the disappearance channel.

Focusing on long-baseline accelerator neutrino experiments, we have treated density variations along the neutrino path as a small perturbation. Using detailed matter density profile studies for the DUNE and T2HK (K) baselines from the literature, we have provided some quantitative estimates on the corrections induced by a nonconstant matter density. Typically, they are of order few percent or smaller. Considering that appearance probabilities are themselves typically only few percent, these corrections are much smaller than realistic experimental uncertainties and hence, do not affect the test proposed in Ref. [27].

To conclude, the material presented here provides background information to the model-independent T violation test from Ref. [27]. This lies out the basis for the possibility to test one of the fundamental symmetries of nature, the time reversal symmetry, in a model-independent way using actually planned neutrino oscillation experiments.

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[1] N. Cabibbo, Time reversal violation in neutrino oscillation, Phys. Lett. **72B**, 333 (1978).
[2] S. M. Bilenky, J. Hosek, and S. T. Petcov, On oscillations of neutrinos with dirac and majorana masses, Phys. Lett. **94B**, 495 (1980).
[3] V. D. Barger, K. Whisnant, and R. J. N. Phillips, *CP* Violation in Three Neutrino Oscillations, Phys. Rev. Lett. **45**, 2084 (1980).
[4] P. Zyla *et al.* (Particle Data Group), Review of particle physics, Prog. Theor. Exp. Phys. (**2020**), 083C01.
[5] B. Pontecorvo, Neutrino experiments and the problem of conservation of leptonic charge, Zh. Eksp. Teor. Fiz. **53**, 1717 (1967) [Sov. Phys. JETP **26**, 984 (1968)].
[6] V. N. Gribov and B. Pontecorvo, Neutrino astronomy and lepton charge, Phys. Lett. **28B**, 493 (1969).
[7] Z. Maki, M. Nakagawa, and S. Sakata, Remarks on the unified model of elementary particles, Prog. Theor. Phys. **28**, 870 (1962).
[8] M. Kobayashi and T. Maskawa, *CP* violation in the renormalizable theory of weak interaction, Prog. Theor. Phys. **49**, 652 (1973).
[9] L. Wolfenstein, Neutrino oscillations in matter, Phys. Rev. D **17**, 2369 (1978).
[10] P. Langacker, S. T. Petcov, G. Steigman, and S. Toshev, On the Mikheev-Smirnov-Wolfenstein (MSW) mechanism of amplification of neutrino oscillations in matter, Nucl. Phys. **B282**, 589 (1987).
[11] J. Bernabéu and A. Segarra, Signatures of the genuine and matter-induced components of the *CP* violation asymmetry in neutrino oscillations, J. High Energy Phys. 11 (2018) 063.
[12] J. Bernabéu and A. Segarra, Do T asymmetries for neutrino oscillations in uniform matter have a *CP*-even component? J. High Energy Phys. 03 (2019) 103.
[13] P. Krastev and S. Petcov, Resonance amplification and *T* violation effects in three neutrino oscillations in the earth, Phys. Lett. B **205**, 84 (1988).
[14] E. K. Akhmedov, P. Huber, M. Lindner, and T. Ohlsson, *T* violation in neutrino oscillations in matter, Nucl. Phys. **B608**, 394 (2001).
[15] T.-K. Kuo and J. T. Pantaleone, *T* nonconservation in three neutrino oscillations, Phys. Lett. B **198**, 406 (1987).
[16] S. Toshev, Maximal *T* violation in matter, Phys. Lett. B **226**, 335 (1989).
[17] S. Toshev, On *T* violation in matter neutrino oscillations, Mod. Phys. Lett. A **06**, 455 (1991).
[18] J. Arafune and J. Sato, *CP* and *T* violation test in neutrino oscillation, Phys. Rev. D **55**, 1653 (1997).
[19] S. J. Parke and T. J. Weiler, Optimizing T violating effects for neutrino oscillations in matter, Phys. Lett. B **501**, 106 (2001).
[20] Z.-z. Xing, Leptonic commutators and clean *T* violation in neutrino oscillations, Phys. Rev. D **88**, 017301 (2013).
[21] S. T. Petcov and Y.-L. Zhou, On neutrino mixing in matter and *CP* and *T* violation effects in neutrino oscillations, Phys. Lett. B **785**, 95 (2018).

[22] K. Abe *et al.* (T2K Collaboration), Constraint on the matter–antimatter symmetry-violating phase in neutrino oscillations, Nature (London) **580**, 339 (2020); Erratum, Nature (London) **583**, E16 (2020).
[23] M. Acero *et al.* (NOvA Collaboration), First Measurement of Neutrino Oscillation Parameters Using Neutrinos and Antineutrinos by Nova, Phys. Rev. Lett. **123**, 151803 (2019).
[24] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, and A. Zhou, The fate of hints: Updated global analysis of three-flavor neutrino oscillations, J. High Energy Phys. 09 (2020) 178.
[25] P. de Salas, D. Forero, S. Gariazzo, P. Martínez-Miravé, O. Mena, C. Ternes, M. Tórtola, and J. Valle, 2020 global reassessment of the neutrino oscillation picture, J. High Energy Phys. 02 (2021) 071.
[26] F. Capozzi, E. Di Valentino, E. Lisi, A. Marrone, A. Melchiorri, and A. Palazzo, Unfinished fabric of the three neutrino paradigm, Phys. Rev. D **104**, 083031 (2021).
[27] T. Schwetz and A. Segarra, companion Letter, Model-Independent Test of T Violation in Neutrino Oscillations, Phys. Rev. Lett. **128**, 091801 (2022).
[28] B. Abi *et al.* (DUNE Collaboration), Deep underground neutrino experiment (DUNE), far detector technical design report, volume I introduction to DUNE, J. Instrum. **15**, T08008 (2020).
[29] B. Abi *et al.* (DUNE Collaboration), Deep underground neutrino experiment (DUNE), far detector technical design report, volume II: Dune physics, arXiv:2002.03005.
[30] K. Abe *et al.* (Hyper-Kamiokande Collaboration), Hyper-Kamiokande design report, arXiv:1805.04163.
[31] K. Abe *et al.* (Hyper-Kamiokande Collaboration), Physics potentials with the second Hyper-Kamiokande detector in Korea, Prog. Theor. Exp. Phys. (**2018**), 063C01.
[32] E. Baussan *et al.* (ESSnuSB Collaboration), A very intense neutrino super beam experiment for leptonic CP violation discovery based on the European spallation source linac, Nucl. Phys. **B885**, 127 (2014).
[33] M. Blennow, E. Fernandez-Martinez, T. Ota, and S. Rosauro-Alcaraz, Physics potential of the ESSνSB, Eur. Phys. J. C **80**, 190 (2020).
[34] K. Hagiwara, N. Okamura, and K.-i. Senda, The earth matter effects in neutrino oscillation experiments from Tokai to Kamioka and Korea, J. High Energy Phys. 09 (2011) 082.
[35] B. Roe, Matter density versus distance for the neutrino beam from Fermilab to Lead, South Dakota, and comparison of oscillations with variable and constant density, Phys. Rev. D **95**, 113004 (2017).
[36] T. Miura, T. Shindou, E. Takasugi, and M. Yoshimura, The matter fluctuation effect to T violation at a neutrino factory, Phys. Rev. D **64**, 073017 (2001).
[37] H. Yokomakura, K. Kimura, and A. Takamura, Overall feature of CP dependence for neutrino oscillation probability in arbitrary matter profile, Phys. Lett. B **544**, 286 (2002).
[38] E. Fernandez-Martínez, M. B. Gavela, J. Lopez-Pavon, and O. Yasuda, Cp-violation from nonunitary leptonic mixing, Phys. Lett. B **649**, 427 (2007).
[39] F. J. Escrihuela, D. V. Forero, O. G. Miranda, M. Tórtola, and J. W. F. Valle, Probing CP violation with nonunitary mixing in long-baseline neutrino oscillation experiments: DUNE as a case study, New J. Phys. **19**, 093005 (2017).
[40] S.-F. Ge and A. Y. Smirnov, nonstandard interactions and the CP phase measurements in neutrino oscillations at low energies, J. High Energy Phys. 10 (2016) 138.
[41] A. de Gouvêa and K. J. Kelly, nonstandard neutrino interactions at DUNE, Nucl. Phys. **B908**, 318 (2016).
[42] P. Coloma, I. Esteban, M. C. Gonzalez-Garcia, and M. Maltoni, Improved global fit to nonstandard neutrino interactions using coherent energy and timing data, J. High Energy Phys. 02 (2020) 023; J. High Energy Phys.12 (2020) 071(A).
[43] P. B. Denton, J. Gehrlein, and R. Pestes, CP-Violating Neutrino Nonstandard Interactions in Long-Baseline-Accelerator Data, Phys. Rev. Lett. **126**, 051801 (2021).
[44] J. Kopp, P. A. N. Machado, M. Maltoni, and T. Schwetz, Sterile neutrino oscillations: The global picture, J. High Energy Phys. 05 (2013) 050.
[45] R. Gandhi, B. Kayser, M. Masud, and S. Prakash, The impact of sterile neutrinos on CP measurements at long baselines, J. High Energy Phys. 11 (2015) 039.
[46] A. Palazzo, 3-flavor and 4-flavor implications of the latest T2K and NOνA electron (anti-)neutrino appearance results, Phys. Lett. B **757**, 142 (2016).
[47] J. M. Berryman, A. de Gouvêa, K. J. Kelly, and A. Kobach, Sterile neutrino at the deep underground neutrino experiment, Phys. Rev. D **92**, 073012 (2015).
[48] S. Antusch, C. Biggio, E. Fernandez-Martínez, M. B. Gavela, and J. Lopez-Pavon, Unitarity of the leptonic mixing matrix, J. High Energy Phys. 10 (2006) 084.
[49] F. J. Escrihuela, D. V. Forero, O. G. Miranda, M. Tortola, and J. W. F. Valle, On the description of nonunitary neutrino mixing, Phys. Rev. D **92**, 053009 (2015); Erratum, Phys. Rev. D **93**, 119905 (2016).
[50] C. S. Fong, H. Minakata, and H. Nunokawa, nonunitary evolution of neutrinos in matter and the leptonic unitarity test, J. High Energy Phys. 02 (2019) 015.
[51] M. Blennow, P. Coloma, E. Fernandez-Martínez, J. Hernandez-Garcia, and J. Lopez-Pavon, nonunitarity, sterile neutrinos, and nonstandard neutrino interactions, J. High Energy Phys. 04 (2017) 153.
[52] J. Sakurai and J. Napolitano, *Modern Quantum Mechanics* (Cambridge University Press, Cambridge, England, 2017).
[53] A. Falkowski, M. González-Alonso, and Z. Tabrizi, Consistent QFT description of nonstandard neutrino interactions, J. High Energy Phys. 11 (2020) 048.
[54] I. Bischer and W. Rodejohann, General neutrino interactions from an effective field theory perspective, Nucl. Phys. **B947**, 114746 (2019).