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Passage through resonance of two coupled exciters

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A minimal model of two coupled exciters is considered to investigate passage through resonance of self-synchronizing systems. An averaging method for partially strongly damped systems [1] is used for asymptotic analysis. Stationary solutions of the system is derived and compared with previous work [2]. The dependency of such solutions on damping and excitation power is shown.

1 Introduction

Application of several unbalanced exciters instead of one is a widely used design for vibratory machines [2]. It can be used to distribute the excitation along the machine or decrease the load of the exciter's bearing, if several low power exciters are used instead of one powerful exciter. It also offers the possibility of coordinating rotors dynamics without kinematic connections between them (self-synchronization), which can be utilized for generating required excitation forces with specific directions in vibratory machines.

Prior research about self-synchronization [2] investigated only different types of synchronous solutions, their stability and existence, while neglecting damping. Goal of this work is to analyze transient behavior of synchronizing systems with damping analytically with the help of suitable averaging methods.

2 Investigated model

The simplest model of two coupled exciters, where self-synchronization can occur, is investigated, see Figure 1. It consists of two unbalanced rotors of mass m_i, moment of inertia J_i and eccentricity e_i, where $i = 1, 2$ is the index describing the number of rotors. They are driven in the same direction by induction engines of limited power with a linearized torque characteristic given as $T_{\text{mot}} = U_i(\omega_i^* - \dot{\varphi}_i)$. The variable φ_i describes the motion of the rotors, parameter U_i the slope of motor characteristic and ω_i^* the nominal rotation speeds. Both rotors are mounted on a carrier of mass M, which is elastically suspended with a spring-damper element of stiffness c and damping d in horizontal direction. The variable x describes the motion of the carrier.

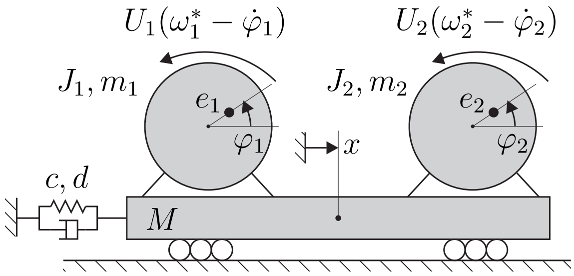

Fig. 1: Investigated model

With the non-dimensional parameters

$$\mu_i = \frac{m_i}{M^*}, \quad \nu_i = \frac{e_i}{e^*}, \quad \varepsilon s_i = \frac{1}{1 + J_i/m_i e_i^2}, \quad e^* = (e_1 + e_2)/2, \quad u_i = \frac{U_i s_i}{k m_i e_i^2}, \quad \lambda_i = \frac{\omega_i^*}{k},$$
$$\xi = \frac{x}{e^*}, \quad k^2 = \frac{c}{M^*}, \quad 2\sigma = \frac{d}{kM^*}, \quad M^* = M + m_1 + m_2, \quad \tau = kt,$$

the equations of motion of the system read

$$\xi'' + 2\sigma\xi' + \xi = \sum_{i=1}^{2} \mu_i \nu_i (\varphi_i'' \sin\varphi_i + \varphi_i'^2 \cos\varphi_i),$$
$$\varphi_i'' = \varepsilon \left(\frac{s_i}{\nu_i} \xi'' \sin\varphi_i + u_i(\lambda_i - \varphi_i') \right) = \varepsilon f_{\varphi_i}, \qquad i = 1, 2.$$

The parameter ε is assumed to be small and the damping parameter σ is not small. The investigation is performed for a system with two identical rotors with different nominal speeds, which means that parameters of both rotors are identical except $\lambda_1 \neq \lambda_2$.

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3 Asymptotic analysis

The averaging method, introduced in [1, 3], is used for the asymptotic analysis of partially strongly damped system. Using an analogous procedure shown in [3], the motion of the carrier can be replaced by its forced solution and the differential equation for ξ can be neglected in further analysis. Defining the new variables $\omega_i = \varphi_i'$, $\psi = (\varphi_1 + \varphi_2)/2$, $\delta = \varphi_2 - \varphi_1$, $p = \omega_1 + \omega_2$, $v = (\omega_2 - \omega_1)/\sqrt{\varepsilon}$, the equations in standard form for a second order approximation with $\sqrt{\varepsilon}$ as the small parameter are derived:

$$\frac{\mathrm{d}\delta}{\mathrm{d}\psi} = \frac{2\sqrt{\varepsilon}v}{p}, \qquad \frac{\mathrm{d}v}{\mathrm{d}\psi} = \frac{2\sqrt{\varepsilon}(f_{\varphi_2} - f_{\varphi_1})}{p}, \qquad \frac{\mathrm{d}p}{\mathrm{d}\psi} = \frac{2\varepsilon(f_{\varphi_2} + f_{\varphi_1})}{p}.$$

Averaging these equations, the sixth-order system can be reduced to a third-order system for the averaged variables $\bar{\delta}$, $\bar{v}$ and $\bar{p}$.

4 Results

System is analyzed for $\lambda_1 = 1.5$ and $\lambda_2 = 1.7$, which has the synchronization frequency of $\lambda_S = 1.6$ according to [2]. Examining the differential equation of δ, it can be seen that all stationary solutions of the system are synchronous solutions ($\bar{v} = 0$, i.e. $\bar{\omega}_1 = \bar{\omega}_2 = \bar{\omega}$), including capture into resonance of both rotors. These solutions can be seen in Figure 2 for different values of damping parameter σ. It depicts the stationary solutions of the averaged system as a function of parameter u ($u_1 = u_2 = u$) corresponding to the slope of motor characteristic. Both, capture into resonance at the vicinity of resonance frequency of the carrier ($\bar{\omega} = 1$) and synchronous solution at λ_S can be seen. Also shown in the same figure is the existence condition $u^* \approx 4.2$, which is derived with the Poincaré method without factoring in damping, see [2]. It states that synchronous solution at λ_S is only possible, if the condition $u < u^*$ holds. It can be seen that with smaller values of σ the top peak of the curve at $\lambda \approx \lambda_S$ is closer to the result of Poincaré method, which is the expected result.

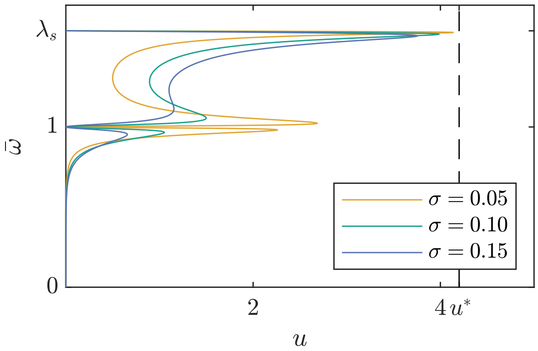

Fig. 2: Stationary solutions of the averaged system for different values of damping

Lastly, two stationary solutions of the full and averaged systems are compared. Figure 3 shows capture into resonance for $u = 0.2$ and the synchronous solution at λ_S for $u = 2$. Amplitude of the carrier and rotor speeds are depicted. In both cases, the results of the full and averaged systems are in reasonable agreement.

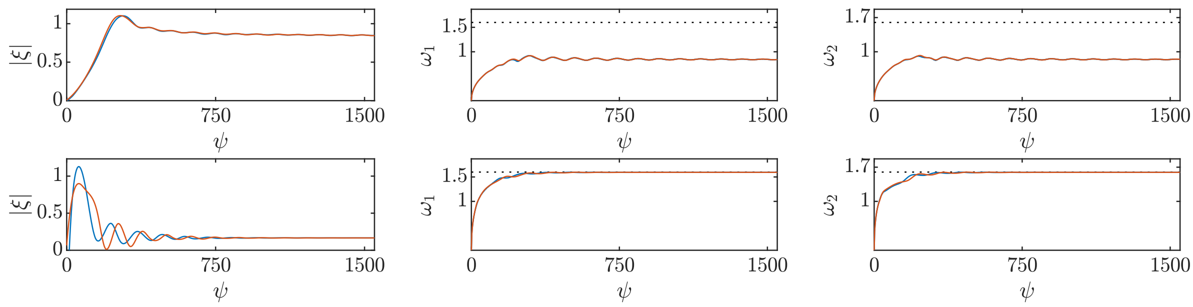

Fig. 3: Comparison of the full (blue) and averaged (orange) systems for the parameter $u = 0.2$ (above) and $u = 2$ (below)

5 Conclusion

A model of two coupled exciters is investigated using an averaging method for partially strongly damped systems. Averaged equations of second order approximation are analyzed. Furthermore, the results are compared to previous work and an improved existence condition of synchronous solutions is derived, which considers damping in the system. Next, phase space should be investigated to determine attraction domain of different stable solutions.

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