# Investigation of a centrifugal exciter with two coaxial unbalances on a carrier performing planar motion 

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Passing through and capturing into the resonance of a centrifugal exciter is investigated. The current work is the continuation of [1], which considered a minimal model of a centrifugal exciter consisting of a carrier with one degree of freedom. The analysis is conducted using an averaging method for partially strongly damped systems. Additional stationary and periodic solutions in vicinity of the resonances of the carrier system have been found, which don't exist in the simpler system investigated previously.
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## 1 Introduction

Passing through and capturing into the resonance of rotating machines is an important topic for scientific research, which is also relevant for industrial use. To avoid the capture into resonance, also known as Sommerfeld effect, which may keep the system from reaching the operating point, the dynamics of such systems should be examined thoroughly.

This work expands the model of the centrifugal exciter considered in [1] and allows the carrier of the rotor to perform planar motion. The system is strongly damped, which is a valid assumption, if physical dimensions of real technical systems are taken into consideration. Different kinds of capturing into resonance are investigated using asymptotic analysis and numerical simulations.

## 2 Investigated system

The simple model of the centrifugal exciter, shown in Fig. 1, is considered. It consists of an unbalanced rotor of mass $M$ and moment of inertia $J_{r}$ with eccentricity $e$. It is driven by an induction engine of limited power, whose torque characteristic is linearized and given as $T_{m o t}=U\left(\omega^{*}-\dot{\varphi}\right)$. The parameter $U$ describes the slope of motor characteristic and $\omega^{*}$ the nominal rotation speed of motor. The rotor is elastically suspended with spring-damper elements of stiffness $c_{x}$ and $c_{y}$ and damping $\beta_{x}$ and $\beta_{y}$ in horizontal and vertical directions, respectively. A pendulum of mass $m$, moment of inertia $J_{p}$ and length $r$ is mounted on the rotor. The rotational damping $\beta_{\varphi}$ is acting between the rotor and the pendulum. The resonance frequencies of the horizontal and vertical motion are chosen different but of the same order of magnitude as the nominal rotation speed of the motor.

With the non-dimensional parameters


Fig. 1: Model of the investigated system.

$$
\begin{aligned}
& M^{*}=M+m ; \quad \tau=k t ; \quad k^{2}=\frac{c_{x}}{M^{*}} ; \quad \kappa_{y}^{2}=\frac{c_{y}}{c_{x}} ; \quad 2 \sigma_{x}=\frac{\beta_{x}}{k M^{*}} ; \quad 2 \sigma_{y}=\frac{\beta_{y}}{k M^{*}} ; \\
& \xi=\frac{M *}{m r} x ; \quad \eta=\frac{M *}{m r} y ; \quad \varepsilon=\frac{1}{1+J_{r} / M e^{2}} ; \quad p=\frac{1}{1+J_{p} / m r^{2}} ; \quad b_{r}=\frac{\beta_{\varphi} \varepsilon}{k M e^{2}} ; \quad b_{p}=\frac{\beta_{\varphi} p}{k m r^{2}} ; \\
& \lambda=\frac{\omega^{*}}{k} ; \quad u=\frac{U}{k M e^{2}} ; \quad \mu=p \frac{m}{M^{*}} ; \quad s=\frac{M e}{m r} ; \quad w=\frac{m r}{M^{*}} ;
\end{aligned}
$$

the equations of motion of this system can be written as follows:

$$
\begin{aligned}
& \xi^{\prime \prime}+2 \sigma_{x} \xi^{\prime}+\xi=s\left(\varphi^{2} \cos \varphi+\varphi^{\prime \prime} \sin \varphi\right)+\left(\varphi^{\prime}+\psi^{\prime}\right)^{2} \cos (\varphi+\psi)+\left(\varphi^{\prime \prime}+\psi^{\prime \prime}\right) \sin (\varphi+\psi) \\
& \eta^{\prime \prime}+2 \sigma_{y} \eta^{\prime}+\kappa_{y}^{2} \eta=s\left(\varphi^{\prime 2} \sin \varphi-\varphi^{\prime \prime} \cos \varphi\right)+\left(\varphi^{\prime}+\psi^{\prime}\right)^{2} \sin (\varphi+\psi)-\left(\varphi^{\prime \prime}+\psi^{\prime \prime}\right) \cos (\varphi+\psi) \\
& \varphi^{\prime \prime}-b_{r} \psi^{\prime}=\varepsilon w\left(\xi^{\prime \prime} \sin \varphi-\eta^{\prime \prime} \cos \varphi\right)+\varepsilon u\left(\lambda-\varphi^{\prime}\right)=\varepsilon f_{\varphi} \\
& \psi^{\prime \prime}+\left(b_{r}+b_{p}\right) \psi^{\prime}=\mu\left(\xi^{\prime \prime} \sin (\varphi+\psi)-\eta^{\prime \prime} \cos (\varphi+\psi)\right)-\varepsilon w\left(\xi^{\prime \prime} \sin \varphi-\eta^{\prime \prime} \cos \varphi\right)-\varepsilon u\left(\lambda-\varphi^{\prime}\right)=\varepsilon f_{\psi}
\end{aligned}
$$

The parameters $\varepsilon$ and $\mu$ are assumed to be small and the damping parameters $\sigma_{x}, \sigma_{y}, b_{r}$ and $b_{p}$ are not small.

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## 3 Asymptotic analysis

For the asymptotic analysis of this system an averaging method for partially strongly damped systems, introduced in [1,2], is used. The procedure in this work is analogous to [1].

In the first order approximation the strongly damped variables can be ignored. Choosing the angle $\varphi$ as the new independent variable the equations in standard form for the first order approximation are derived:

$$
\frac{\mathrm{d} \omega}{\mathrm{~d} \varphi}=\frac{\varepsilon f_{\psi} b_{r}}{\omega\left(b_{r}+b_{p}\right)}+\frac{\varepsilon f_{\varphi}}{\omega} ; \quad \frac{\mathrm{d} \gamma}{\mathrm{~d} \varphi}=\frac{\varepsilon f_{\psi}}{\omega} .
$$

These equations can be averaged analytically. Thus the full eight-order system is reduced to an averaged second-order system, which describe the averaged rotation speed $\bar{\omega}$ of the rotor and the averaged phase shift $\bar{\gamma}$.


Fig. 2: Stationary solutions of the averaged system

## 4 Results

The stationary and the periodic solutions of a system with the nondimensional resonance frequencies 1 and 1.6 in horizontal and vertical directions, respectively, and nominal rotation speed $\lambda=1.8$ are analyzed. Fig. 2 depicts the stationary solutions of the averaged system as a function of the parameter $u$ corresponding to the slope of the motor characteristic. The solutions at $\bar{\omega} \approx 1.8$ correspond to passing through the resonance whereas all other solutions correspond to capturing into resonance. Compared to the system in [1], the considered system features additional stationary solutions corresponding to capturing into resonance (blue lines in Fig. 2). Performing a planar motion, the system has more possibilities to not pass through the resonances of the carrier system.

Two types of periodic solutions orresponding to nonstationary capturing into the resonans were also found. Fig. 3 shows the evolution of periodic solutions for $u=9.2$ and $u=20$. The amplitudes $\xi_{\max }$ and $\eta_{\max }$ of the horizontal and vertical motions respectively and the rotational speed $\omega$ modulate in both cases with a very low frequency and the phase shift $\psi$ increases constantly over time. The solutions of the full and the averaged system are in a reasonable agreement. These two periodic solutions result from different bifurcations. The system in [1] posses only one kind of periodic solution corresponding to the nonstationary capture into the resonance.

## 5 Conclusions

A centrifugal exciter with two coaxial unbalances performing planar motion is investigated using an averaging method for partially strongly damped systems to obtain approximated asymptotic solutions. Different kinds of stationary and periodic solutions are found, which correspond to either capturing into or passing through the resonance. Stability and bifurcation analysis is the topic of further research.

## References

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