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## Comparison of modeling methods to determine cutting tool profile for conventional and synchronized whirling

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### Abstract

The determination of cutting tool profiles for machining operations with coupled rotational kinematics like gear and screw generation can be a complex task which is executed by either numerical or analytical methods. The cutting tool profile for whirling is derived from process parameters and desired workpiece geometry by both a numerical dextral-based model and an analytical model based on the condition of tangential motion. The models are adapted to a process variant of whirling with synchronized rotation of tool and workpiece and compared regarding accuracy, computation time and geometrical flexibility.

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### 1. Introduction

Whirling is a variant of milling with a circular tool holder that encompasses the workpiece and the cutting tools rotating internally, see Fig 1. The axis of the workpiece is inclined to the tool holder axis by the tilt angle. The workpiece is positioned at an offset in the ring.

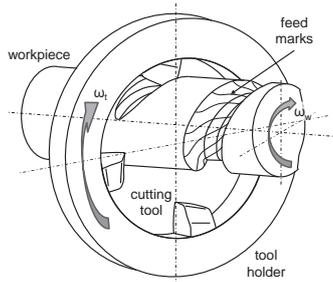


Fig. 1. Machining of a screw geometry by whirling [1]

Whirling is used primarily for the production of thread geometries like screws and worm gears. The resulting workpiece geometry is defined by the cutting tool profile and process kinematics. In processes with two non-parallel axes like whirling, the workpiece is formed in a generation motion. The resulting workpiece geometry complies with the envelope of the generation motion of the cutting tool profile. Thus the cutting tool profile cannot only be derived from the targeted workpiece geometry but has to be determined by taking the process kinematics into consideration. In some process settings the generation motion leads to undercutting of the workpiece. If undercutting is detected early on in the design of the process, it can be compensated by changing the cutting tool profile or process parameters. The cutting tool profile can be determined by different methods both numerical and analytical. The numerical models are based on spatial and temporal discretization of the targeted workpiece geometry and the machining motion. One method is to project the intersection of the workpiece geometry and the cutting tool plane pointwise onto the cutting tool plane for a finite number of steps of the relative motion.

The cutting tool profile can be constructed by deriving the envelope of all projected points of the workpiece geometry. The envelope method is often used for analysis of gear generation [2] and was successfully applied to whirling [3]. Although the determination of non-convex envelopes is a numerical problem without singular solution [4, 5]. Another numerical method is determining the cutting tool profile by pointwise trimming of a blank cutting tool in each step by detecting intersections with the surface of the workpiece. The intersections are found by elementwise comparison of the point positions. The trimming of intersecting geometries in machining processes can be simulated efficiently by dextral modeling [6, 7]. Analytical models are based on the equation of meshing and solved by methods of differential geometry. This method is used for determining the cutting tool profile for gear grinding processes and milling of screw geometries like pump rotors [2, 8]. While analytical problems can be solved in form of an explicit formula this is not possible in general [9]. When this is the case the set of analytic nonlinear formulae describing the problem can be solved numerically [10].

#### Nomenclature

$\omega_t$	rotational speed of the tool
$\omega_w$	rotational speed of the workpiece
$h_{max}$	cutting thickness
$v_t$	tangential speed of the tool (cutting speed of conventional whirling)
$v_w$	tangential speed of the workpiece
$v_{rel}$	relative speed between tool and workpiece (cutting speed of synchronized whirling)
$N_w$	surface normal of the workpiece

## 2. Whirling process variants

The performance of thread whirling exceeds other machining processes at meeting high geometric and surface quality requirements under difficult to machine conditions. In contrast to turning the intermittent cut leads to fragmented chips and allows the cutting tool to cool down. At the same time the concave tool motion encompasses the workpiece diameter resulting in steadier cutting conditions and smaller feed marks compared to thread milling. Screws for medical applications are made of biocompatible materials like titanium most of which are considered difficult to machine. Nonetheless surface quality and geometric accuracy has to be high as no consecutive grinding process is applied. These requirements and a high productivity are best met by whirling although the efficiency of the process is limited by the material removal between the major diameter and the diameter of the feedstock necessary for the head of the screw [11]. This material can either be removed by turning before the whirling operation or during whirling with the whirling tool. The first increases the main time of the process, the second increases tool wear. Both may render the process uneconomic.

Synchronized whirling a variant of the whirling process was developed for the generation of multi-start threads in a single pass [12]. Multiple thread starts are cut by increasing the rotational speed of the workpiece so that it revolves around its

axis in between the engagement of two consecutive cutting tools. Thus each cutting tool meets a different thread. The rotational speeds of tool holder and workpiece are synchronized in a whole-numbered ratio according to the number of thread starts and cutting tools in the tool holder in order to not remove the crest. Furthermore the tilt angle between the axes of the workpiece and the tool holder is to be adapted. The synchronized whirling process allows the integration of turning operations due to the higher rotation speed of the workpiece. The parallelization of turning and whirling was shown to reduce main time in the production of bone screws and to increase productivity significantly [13].

### 2.1. Modeling of whirling processes

The kinematics of the conventional whirling process can be modeled by transformations between the tool holder and workpiece coordinate systems in homogenous coordinates [3]. The tool holder axis is moved out of the workpiece axis by the eccentricity and rotated around the eccentricity by the tilt angle. The tilt angle corresponds with the lead angle of the workpiece. The cutting tool rotates around the tool holder axis. The workpiece shifts along its axis by the feed rate and rotates accordingly as the zone of engagement moves along the thread. The synchronized whirling process differs from conventional whirling by an additional rotation of the workpiece. The process variant sets different constraints to the process parameters like a fixed ratio of rotational speeds between tool and workpiece and a different tilt angle. The adaptation of the tilt angle is necessary to align the resulting vector of relative motion between cutting tool and workpiece with the lead of the thread, as illustrated in figure 2.

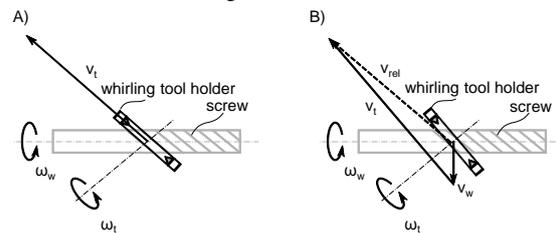


Fig. 2. Kinematics of (A) conventional whirling and (B) synchronized whirling with the combination of speed vectors

The synchronized whirling process with two rotating axes can be modeled by attaching the observing coordinate system to the workpiece. Thus the additional rotary motion of the workpiece is projected to the tool holder. Effects of the modified process parameters on the cutting conditions can be derived easily by this modeling approach, as illustrated in figure 3. For example the positive impact of the additional rotation on cutting thickness. The projection of the workpiece rotation to the tool holder shifts the axis during cutting tool engagement and the motion encompasses the workpiece closer. The additional motion stretches the chip geometry and decreases cutting thickness while feed per tooth and chip volume stay constant.

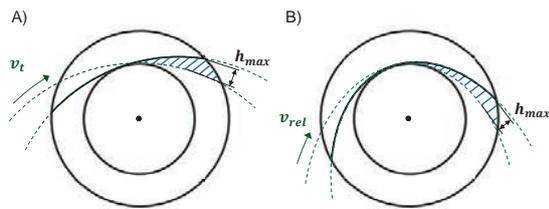


Fig. 3. Relative tool motion encompassing the workpiece cylinder and the machined volume between two revolutions of the cutting tool for (A) conventional whirling and (B) synchronized whirling with a smaller cutting thickness for the tighter encompassment

### 3. Determination of cutting tool profile

The cutting tool profile for conventional and synchronized whirling is determined by both numerical and analytical modeling. The numerical method presented is based on a dixel model as it is efficient, easily scalable and adaptable to different geometries and operations. The alternative analytical method is based on the meshing equation best known for its applications in gearing theory but universal for mechanisms of solid bodies.

#### 3.1. Numerical method

The numerical simulation trims a dixel blank in the plane equivalent to the face of the cutting tool as illustrated for conventional whirling in figure 4. The geometry of the targeted thread is modeled as a triangulated surface. The dixel blank representing the cutting tool is moved through the thread. Instead of the tool cutting the workpiece the dexels are trimmed by the workpiece geometry to render the cutting tool profile as illustrated in figure 4 steps I to IV. The model can easily be adapted to synchronized whirling by additionally rotating the workpiece.

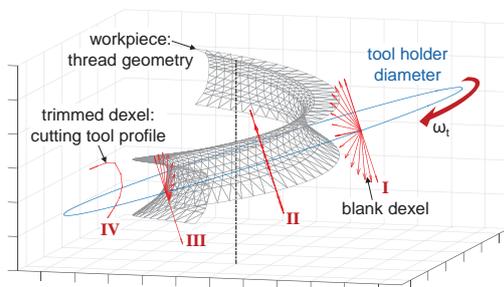


Fig. 4. Numerical determination of cutting tool profile for synchronized whirling by trimming dexels

The accuracy of this method is limited by the discretization of the workpiece, the motion and the number of dexels in the cutting tool plane. The consequential error is reduced by increasing the number of points on cutting tool and workpiece as well as steps of the motion. The problem of the dixel approach is the relation between discretization and calculation time. The computational effort increases proportional to the amount of steps simulated. Penetration of the two surfaces is

determined in each step by elementwise calculation of intersections between each plane of the triangulated surface and each dixel ray on the cutting tool blank. The computational effort increases proportional to the amount of discrete points on each surface and along each spatial degree of freedom or direction in space. Thus the computational effort of the model presented in figure 4 increases cubically for a reduction of the discretization error along the cutting tool profile and in both directions of the workpiece surface. The computational effort can be decreased significantly by projecting a cut through the workpiece surface onto the cutting tool plane as a two dimensional profile and eliminating one spatial dimension. Also advanced algorithms like clustering can be used to reduce the amount of the elementwise comparisons.

#### 3.2. Analytical method

The analytical method is based on the general equation of meshing together with a fundamental condition of contact. These equations state that two surfaces in contact share a common surface normal in the point of contact (1) and relative movement between the two can only be tangential to the surfaces, so perpendicular to the common surface normal (2).

$$\vec{N}_t = \vec{N}_w \quad (1)$$

$$\vec{N} \cdot \vec{v}_{rel} = 0 \quad (2)$$

The equation of meshing is applied to the machining process by assuming a finished workpiece surface. The interaction between cutting tool and workpiece is reduced to a non-engaging contact necessarily satisfying the equation of meshing. The points of contact between the two idealized meshing partners are obviously identical to the points on the cutting tool defining the workpiece profile in the machining process. Based on this assumption the condition can be applied for determining the cutting tool profile. The projection of points of contact of the idealized workpiece and cutting tool onto the cutting tool plane will render the cutting tool profile. The remaining problem is determining the contact points. Four degrees of freedom exist for every contact point, three spatial coordinates and a temporal one for the moment of contact during the engaging motion. For every such point of contact three conditions must be met:

- it lies in the known workpiece surface
- it complies with the equation of meshing
- it lies in the instantaneous cutting tool plane

Each condition eliminates a spatial degree of freedom. The set of contact points is left with the temporal degree of freedom forming a line of contact points emerging throughout the cutting tool engagement with the workpiece. The three conditions for the line of contact points can be described as formulae forming a set of equations. The definition of the workpiece surface is a critical element within the set of equations. Most of the screw geometries in industrial

applications are a concatenation of simple, continuous functions like lines and circles in the axial section of the thread. The set of equations is to be adapted to every section of the screw geometry. A simplification of the set of equations is neither trivial nor general as it depends on the surface geometry and therefore appears unpractical. The set of equations is evaluated over a discretized space constituting the remaining temporal degree of freedom.

The contact points on the straight flank and the circular fillet at the minor diameter of a thread for conventional whirling are shown in figure 5. A finite number of contact points within the limits of cutting tool engagement with the work blank or major diameter was determined for discrete points in time. The set of contact points forms continuous lines on each geometric section. The kinematics of conventional whirling are treated as a particular, simpler case of synchronized whirling. The workpiece rotation due to the feed is virtually nonexistent for industrially relevant ratios of feed rate and cutting speed. Thus the relative motion of the tool coordinate system resulting from the workpiece rotation can be neglected for conventional whirling. The vector field of relative motion between workpiece and tool holder stays invariant over time and so does the equation of meshing.

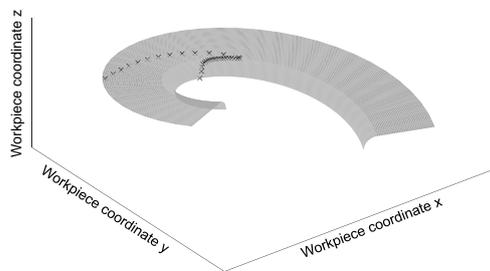


Fig. 5. Contact points between cutting tool and workpiece on a straight sided flank and adjacent fillet for conventional whirling.

For synchronized whirling, the tool rotation needs to be considered as it is significantly faster. The contact points on the same detail of the thread are shown for synchronized whirling in figure 6. The line of contact points stretches over a longer section of the flank. While along the fillet, the differences to conventional whirling are less distinct.

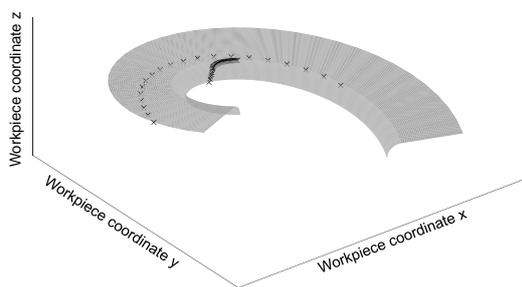


Fig. 6. Contact points between cutting tool and workpiece on a straight sided flank and adjacent fillet for synchronized whirling.

The tighter encompassment of the workpiece complies with the descriptions of chapter 2. The accuracy of the proposed analytical method depends only on one dimension of discretization. It can be increased by reducing the temporal increment. Thus the computational effort scales proportionately with the discretization error.

### 3.3. Evaluation and comparison of determined cutting tool profiles

The described numerical and analytical method to determine cutting tool profiles are compared by applying them to two different screw geometries listed in table 1. The first geometry is a metric ISO M6 screw, the second is derived from an asymmetric bone screw. The analysis of the profiles is simplified to only two geometric sections of the workpiece profile a straight flank and a circular fillet at the minor diameter. Though arbitrary screw profiles could be composed by further combining straight lines and circles.

Table 1. Parameters of screw geometries used for comparison

	Geometry 1 M6	Geometry 2 Bone screw
Threads	1	2
Pitch (mm)	1	1.75
Major diameter (mm)	6	5.93
Minor diameter (mm)	4.773	3.93
Flank angle (°)	30	3
Fillet at minor diameter (mm)	0.1443	0.3
Inner cutting tool diameter	12	10
Number of cutting tools in the tool holder	1	2
Ratio of rotation speeds in synchronized whirling	-1	-1
Pitch angle and tilt angle in conventional whirling (°)	3.3819	12.7339
Tilt angle in synchronized whirling (°)	4.8254	18.4033

The results of the numerical simulation of conventional and synchronized whirling of the metric screw are shown in figure 7.

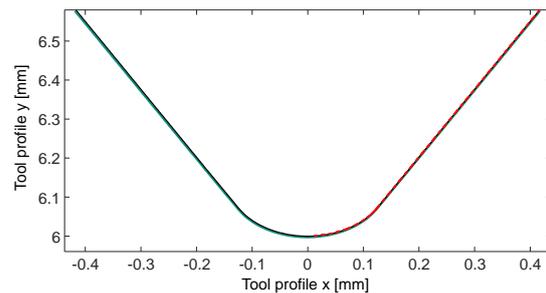


Fig. 7. Comparison of cutting tool profiles for a metric ISO M6 screw determined numerically for conventional (green) and synchronized whirling (black) and analytically for synchronized whirling only (dashed, red)

The difference of the cutting tool profiles for this parameter set is smaller than the discretization error. The analytical result for synchronized whirling is also shown. It matches perfectly except for a submicron discretization error. The comparison of results of the numerical simulation of conventional and synchronized whirling of the bone screw is shown in figure 8. The cutting tool profile of synchronized whirling is thinner. The profiles diverge especially on the steep left flank. Both profiles show a concave contour on the steep flank. On the cutting tool plane the relative generation motion between cutting tool and workpiece replicates the straight sided flank as a curve. This effect can be observed in many manufacturing processes with two rotating axes and is often associated with undercutting. The effect increases with the generation aspect of the kinematics and has a stronger impact on steep flanks. Therefore it is more relevant for synchronized whirling because of its coupled rotations and is more visible on geometry 2. The profile determined for synchronized whirling with the analytical simulation is also shown on the right. The profiles determined numerically and analytically match except for deviations at the top right and the transition between straight flank and fillet. The deviation at the top right of the profile is a fillet that was not modeled in the analytical simulation. The deviation at the lower transition between the geometric sections amounts to  $3\ \mu\text{m}$  as depicted in the detail and is of more peculiar origin. In the analytical simulation contact points from the straight sided flank and the fillet are projected onto the same diameter of the cutting tool plane. The analytically determined cutting tool profile shows a bifurcation that cannot be replicated by the numerical method.

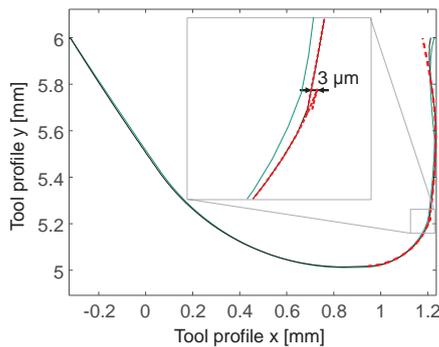


Fig. 8. Comparison of cutting tool profiles for the generic bone screw determined numerically for conventional (green) and synchronized whirling (black) and analytically for synchronized whirling only (dashed, red) and detail of bifurcation of the analytically determined cutting tool profile

The dexels in the numerical simulation are trimmed to the innermost point. Therefore the bifurcation shown in figure 8 is neglected. To model the result of the bifurcation on the workpiece profile in machining, the numerical simulation is reversed. The flexibility of discretized numerical methods like the dixel model are revealed in the process. The cutting tool profile is projected into a workpiece plane and cuts the workpiece profile in a generation motion resulting in the envelope of the cutting tool profiles of each step. The result of the reverse simulation is presented in figure 9.

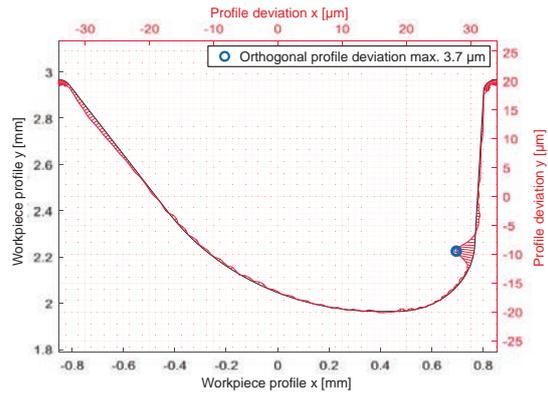


Fig. 9. Deviation (red) of the workpiece profile (black) after machining with the numerically determined cutting tool profile resulting in undercutting

The deviations between the workpiece created by the numerically determined cutting tool profile and the ideal workpiece geometry are shown as orthonormal projections on the profile. Apart from discretization errors in the top corners the error generated by the numerically determined cutting tool profile on the lower right flank is clearly visible. The same point on the tool cuts two points on the workpiece one on the fillet and one on the flank. As the numerically determined cutting tool is trimmed to cut the flank an error is generated on the fillet.

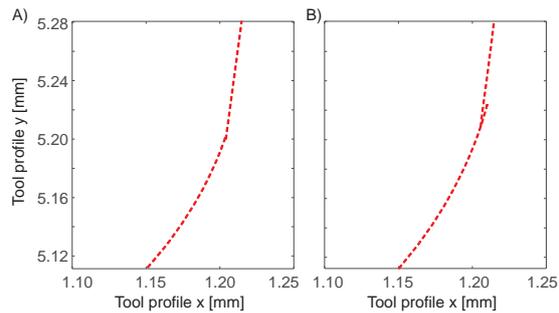


Fig. 10. Cutting tool profile with (A) and without (B) correction of tilt angle

The error of a bifurcating analytical cutting tool profile cannot be eliminated for this workpiece geometry and process parameters. It can be partially compensated distributing the error on the workpiece profile by selecting points in between the bifurcation. To eliminate the bifurcation the process kinematics have to be corrected as shown in figure 10. The tilt angle was corrected by  $-0.5^\circ$  to reduce the bifurcation to the submicron level. Analytical modeling allows the detection of bifurcations in the cutting tool profile in a single step while numerical methods need a second simulation modeling a simplified intersection with the workpiece. Thus the optimization of the process parameters and kinematics regarding the cutting tool profile and subsequent geometrical errors on the workpiece is simpler and faster with an analytical model.

#### 4. Validation of cutting tool profile

The workpiece geometry of a metric ISO M6 screw generated by the numerically determined cutting tool profile is tested in whirling experiments. The cutting tool used is a carbide insert manufactured by Paul Horn GmbH and grinded to fit the simulated cutting tool profile. The whirling process was performed on a Traub TNL18 sliding headstock lathe. The tool holder rotated at 2020 and the workpiece at 2000 rpm. The difference in rotation speed is necessary to compensate for the feed. The process parameters were set to a cutting speed of 113 m/min and a feed per tooth of 0.02 mm so that one pitch of the thread is cut 100 times. The screw geometry was manufactured from a brass blank to demonstrate geometric accuracy under easy to machine conditions. This way surface deviations resulting from unfavorable cutting conditions were ruled out. The manufactured screw profile was measured on a Nanofocus  $\mu$ -surf confocal microscope.

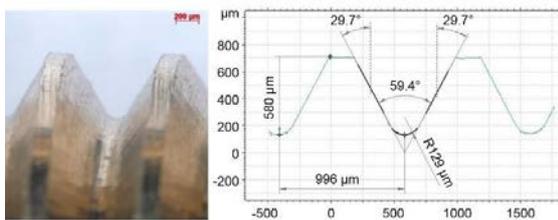


Fig. 11. Result of synchronized whirling with numerically simulated cutting tool profile.

The result for the M6 screw is shown in figure 11. The pitch and flank angle show errors of about 1 %. The determined cutting tool profile is correct and the accuracy is sufficient for creating screw geometries for technical and medical applications.

#### 5. Conclusion

The determination of cutting tool profiles for conventional and synchronized whirling by numerical and analytical methods is presented and compared. Based on the state of the art on the kinematics of whirling the kinematics of the synchronized whirling process was modeled by homogenous coordinate transformation. The kinematic condition is applied in a numerical and analytical model to determine the cutting tool profile. The numerical method uses the calculation of intersections of a workpiece surface and a cutting tool blank modeled by dexels which are trimmed on intersection. The analysis of the numerical model shows a high flexibility. The problem of the numerical model is the cubically scaling

computation time for small discretization errors. The analytical model is derived from the state of the art on other processes with similar kinematics and uses the equation of meshing. The workpiece surface is modeled by basic geometries. Thus the model is less flexible when adopted to different workpieces. The set of analytical equations is solved numerically. The advantage of the analytical model is the proportional scaling of computation time and discretization error. Both models rendered the same cutting tool geometries except the analytical model showed bifurcations that result in undercutting while machining the workpiece. It was shown that the bifurcations could be minimized by adapting the process parameters. Basically the decision between numerical and analytical modeling is a tradeoff between flexibility and calculation time. But the analytical model is a convenient tool to identify undercutting and optimize the process accordingly.

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