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Assessing Rigid and Non-Rigid Spatial Thinking

K. Maheshwary [1,✉], B. Eisenbart [2], S. Zorn [3], T. Nelius [4], K. Gericke [3], S. Matthiesen [4] and L. Blessing [1]

[1] Singapore University of Technology and Design, Singapore, [2] Swinburne University of Technology, Australia,
[3] University of Rostock, Germany, [4] Karlsruhe Institute of Technology, Germany

✉ Khushbu_maheshwary@mymail.sutd.edu.sg

Abstract

Spatial Thinking (ST) is an important part of reasoning. In contrast to Rigid ST (R-ST), Non-Rigid ST (NR-ST) has hardly been researched and tests do not exist, even though NR-ST is crucial for professions that deal with space and time such as engineering and design. Our study at 4 different universities confirms that R-ST does not predict performance in NR-ST; that performance in Non-Rigid ST is significantly lower than in R-ST; and that performance of students in later semesters is not better than in the first semesters. These results highlight the need for further research into NR-ST.

Keywords: spatial thinking, design education, design competences, cognitive capabilities

1. Introduction

A designer developing a 3D model using a software tool needs to first visualize the object and the best way to model that object. This ability to visualize an object in the mind involves Spatial thinking (ST) (Bennett et al. 1947; Atit et al. 2013; Zorn and Gericke 2020). Spatial thinking is

> *"the mental process of representing, analysing, and drawing inferences from spatial relations between objects or within objects" (Uttal et al. 2013) "to help remember, understand, reason, and communicate about the properties of and relations between objects" (National Research Council 2006)*

ST has been identified as a promising identifier to gauge student progress, particularly in STEM (Science, Technology, Engineering and Mathematics): children and young adults with higher ST skills tended to choose STEM majors and performed better in STEM (Shea et al. 2001; Wai et al. 2009). Many ST tests exist (see Section 3.1). They tended to be based on *Rigid* transformations, i.e., spatial changes where the distance between two points on the object under transformation remains unchanged (McCarthy 1990), like rotation and translation (see Figure 1). Research increasingly highlights the importance of *Non-Rigid* transformations in many disciplines, not the least engineering, design, and architecture (Shipley et al. 2012; Atit et al. 2013; Harris et al. 2013; Spröte and Fleming 2016). Such transformations involve surface deformations, e.g., twisting, turning, and bending, in which the distance between two points on an object is not preserved (see Figure 2) and which usually change over time, for example due to loading (see Section 2.2 for examples). Unfortunately, the vast majority of the many ST-tests that are in use to date still involve rigid transformations only.
In this paper we report on the results of an online quiz we developed to compare Rigid ST (R-ST) and Non-Rigid ST (NR-ST) abilities. Based on literature (Atit et al. 2013), we hypothesise that spatial thinking abilities differ across the two types of spatial intelligence: rigid and non-rigid. I.e., a person's performance in R-ST is not a predictor for performance in NR-ST. If this hypothesis holds, separate

tests and training material will have to be developed for those disciplines for which NR-ST is essential. Previous studies did not find a correlation between R-ST and NR-ST (Atit et al. 2013), but these studies involved psychology students and only compared rotation and bending. We further examine whether ST ability is correlated to educational system (country), field of study, hands-on activities, gender, and study progress.

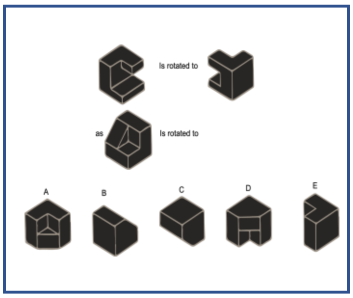

Figure 1. Rigid Spatial Thinking (R-ST) (left) and Non-Rigid Spatial Thinking (NR-ST) (right)

This is the first study of a project in which we aim to understand the process, dimensions and determinants of spatial thinking, in particular NR-ST; to develop NR-ST ability tests for tertiary education; and to develop an NR-ST methodology and toolkit for educators.

2. Literature Review

2.1. Importance of Spatial Thinking education

ST is an important part of reasoning that has been found to be crucial in visualizing events, data and objects across various professions: not only in engineering (Duesbury 1996), architecture and design (Ilić and Đukić 2017), and geology (Orion *et al.* 1997), but also in medicine (Crum et al. 2004), artificial intelligence (Bhatt et al. 2011), physics (Mac Raighne et al. 2015), and chemistry (Wu and Shah 2004). A meta-analysis showed that education and experience can improve ST ability (Uttal et al. 2013), e.g. through interaction with physical objects, virtual or augmented reality, video games, sketching and drawing (Sorby 2009a; Zorn and Gericke 2020). Interestingly, integrating spatial content into formal and informal instruction was found to not only improve spatial functioning in general, but also reduce differences related to gender and socioeconomic status that may impede full participation in a technological society (Newcombe and Frick 2010). The use of multimedia and workbooks to improve 3-D spatial skills proved useful for first-year engineering students, especially for women, and non-engineering middle and high school students (Sorby 2009b; Sorby 2009a). Relevant in this context is Buckley's framework for spatial cognition which contains unique spatial factors to support translation of theory into educational practices (Buckley et al. 2019).

2.2. Importance of Non-Rigid Spatial Thinking abilities

While ST has been well researched and shown to play a role in our reasoning and understanding of STEM, the vast majority of studies and tests focus on few R transformations, e.g. rotation. NR-ST remains largely unexplored and tests are lacking. The human brain can perceive complex NR transformations (Spelke et al. 1982; Gibson and Walker 1984), but this requires more complex reasoning using both ST and domain knowledge. This may e.g. explain why ST ability was a predictor for the performance of novice geologists but not of experts (Hambrick et al. 2012). The following examples highlight the importance of NR-ST for a variety of tasks that involve continuously varying transformations, indicating the need for ST abilities that involve changes in both time and space, i.e., NR-ST.

Ideation, creativity and innovation: Design and creativity require divergent thinking. ST abilities have shown to correlate with divergent thinking (Kell et al. 2013). According to (Bhatt et al. 2011)

> *"spatial ability not only plays a unique role in assimilating and utilizing pre-existing knowledge, but also plays a unique role in developing new knowledge. Without spatial*

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ability, the psychological architecture supporting creative thought and innovative production is incomplete—and many applied and theoretical activities in the psychological sciences are destined to be suboptimal."

Architects, engineers and designers need NR-ST when using and switching between different projections, perspectives and transformations while imagining, sketching and drawing creations that involve continuously varying or abrupt changes due to moving parts or to deformations caused by e.g. loading, heating or damage. However, a study of ST tests used in architectural education found that these do not cover the dynamic spatial abilities that are required (Ilić and Đukić 2017), such as bending, twisting or other transformations over time.

Data interpretation and visualization: ST ability maximizes the usefulness of interaction with visual computer interfaces (Norman 1994). Meteorologists need 'meta-representational competence' to select and use external representations to predict the likelihood of, e.g. heatwaves and thunderstorms from the development and interaction of pressure systems, surface topologies and humidity levels (Hegarty 2010). Such tasks involve rotations, imagining different perspectives, and mental construction of patterns from elementary shapes. Domain knowledge and ST ability improved forecasting performance based on interpretation of images (Trafton and Hoffman 2007). Because of the nature of the task, we assume that NR-ST will further improve this performance giving the dynamics of weather phenomena.

Image recognition: While operating on patients, surgeons need NR-ST to register and recall images of organs in different forms and states. NR-ST is also required for computer vision applications such as robot navigation, image-guided surgery, motion tracking, and face recognition (Myronenko et al. 2007)

Geological inferences: Geologists, when analysing the causes for folds and bends in the layers of the earth's crust, need NR-ST to extract features to deduce the involved transformations and determine the material properties, temperatures, magnitude and direction of the pressure that caused these rock formations (Wakabayashi and Ishikawa 2011). In a test based on deformed words (which requires NR-ST), geologists outperformed other participants (Shipley et al. 2012).

Concerns exist over the limited research into NR-ST, particularly on the simulation of more naturalistic NR transformations with continuously varying changes (Shipley *et al.* 2012 and Atit *et al.* 2013). Shipley, Atit and colleagues measured mental brittle transformation (MBT), which combines R and NR transformations, and observed that R-ST and NR-ST require different skills (Shipley et al. 2012). When they changed domain and introduced MBT of English words by fragmenting letters, they found that MBT experience in one domain helps MBT performance in other domains (Atit et al. 2013).

2.3. Assessment of spatial abilities

Assessment of ST abilities has become a standard part of aptitude and attitude tests when hiring employees. Whether the tests actually measure the relevant abilities is increasingly being questioned: many focus on spatial visualization and orientation, refer to small-scale spaces, hardly assess spatial relations and are usually based on R transformations (Atit et al. 2013). These tests, and any education derived thereof, therefore do not reflect the ST needs of practice. Bednarz and Lee (Bednarz and Lee 2019) developed strategies to improve student's ST abilities in geography and earth sciences, but did not find any test suitability to evaluate these abilities. In response, they developed STAT, a Spatial Thinking Ability Test that includes R and NR transformations. STAT is not suitable for other domains, but even for geospatial thinking several relevant R and NR concepts are not covered, such as scale, frames of reference, regionalization and spatial diffusion (Verma 2014).

It is evident that NR-ST is crucial for professions that deal with changes in space and over time, yet tests to assess or teach NR-ST abilities in different domains do not exist. Rigorous investigation of NR-ST processes, dimensions, and determinants, for specific tasks, domains and educational levels is required to develop valid tests and evidence-based interventions to assess and improve NR-ST abilities.

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3. Methodology

3.1. Test Development

We developed an online multiple-choice test for university students comprising 10 R-ST and 10 NR-ST questions (see Figures 2 and 3) of different complexity, and 4 questions on gender, field of study, study progress (semester), and hands-on experience. Question complexity scores (1-6) were based on number and types of required spatial skills, shapes and transformations. The 10 R-ST questions were chosen from 7 established ST tests to cover a wide range of transformations**:** Differential Aptitude Test (Bennett *et al.* 1947), Purdue Spatial Visualization Test (Guay 1976), the Mental Cutting Test (College Entrance Examination Board 1939), Form Board Test and Hidden Figures test (Ekstrom and Harman 1976), Paper Folding test (Guilford and Lacey 1947), Snake In Cube test (Górska and Juscakova 2003).
For the 10 NR-ST questions we introduced anamorphosis, a perspective technique that gives a distorted image which appears normal when viewed from a particular angle or with a suitable mirror or lens (Lee *et al.* 2009; Čučaković and Paunović 2015) (see Figure 2).

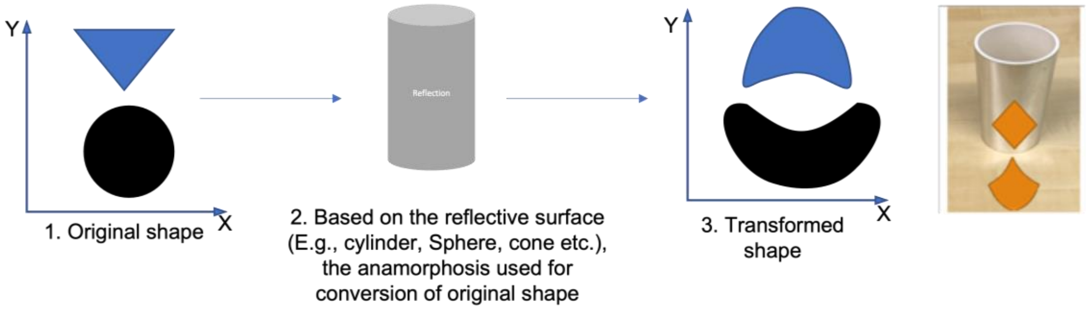

Figure 2. Example anamorphic transformation used to develop NR-ST questions

This technique first appeared in Leonardo da Vinci's notebooks and is still applied as experimental technique in art and architecture, and more recently in encryption and counterterrorism (Di Lazzaro *et al.* 2019). We choose mirror anamorphosis, using a cylindrical reflecting surface. Anamorphosis has never been used for ST ability assessment tests, but its ability to cover a wide range of NR transformations by varying - e.g., the shapes to transform, the shape of the reflecting surface, and the class of spatial transformations (object-based or observer-based) - allows the creation of NR-ST tests for different educational levels, domains, and complexity.

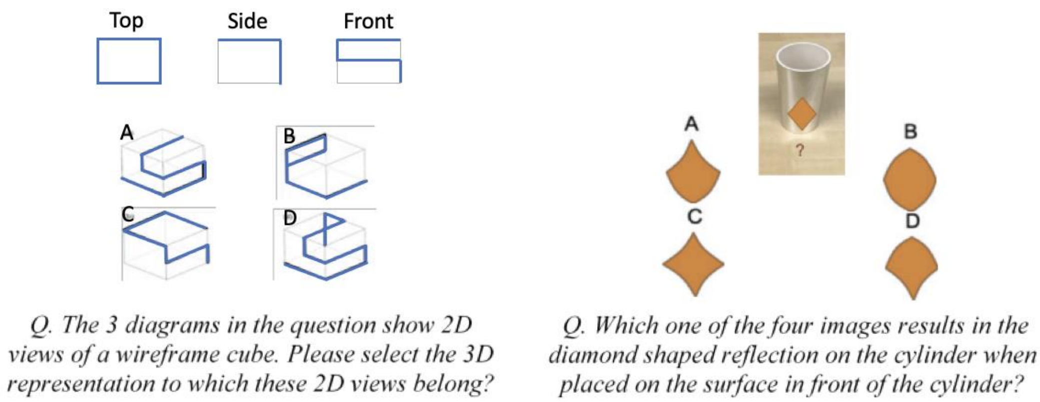

Figure 3. Sample test questions for R (left) and NR transformations (right)

3.2. Participants

The 413 participants in our study were mainly engineering students from 4 universities: Singapore University of Technology and Design (SUTD), Singapore; Karlsruhe Institute of Technology (KIT) and Rostock University (RU), Germany; and Swinburne University of Technology (SUT), Australia.

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Table 1. Universities, participant numbers, fields of study, and study progress (semester)

University	#	Fields of study	Semesters
SUTD	255	General Engg (Fresh more), Engg Product Development (EPD), Architecture and Sustainable Design (ASD), Information Systems, Technology and Design (ISTD), Engineering Systems and Design (ESD)	3, 5, 7
RU	40	Mechanical Engg, Mechatronics Engg, Biomedical Engg, Business Administration and Engg	1,3,5
KIT	22	Mechanical Engg, Mechatronics and IT Engg, Electrical Engg	1-3,5,7, 9-11
SUT	96	General Engg, Mechatronics Engg, Mechanical Engg, Product Design Engg, Biomedical Engg, Electrical Engg, Architectural Engg	1,3,5,7, 8

3.3. Procedure

Students were sent an invitation email with a short explanation and the URL of the quiz. Participants were informed and asked for consent for the data to be collected anonymously and used for research. The participants had 3 minutes for each of the 20 multiple-choice questions.

At SUTD, the invitation email was sent to all, ca.1200 SUTD undergraduate students, at RU to 240 students after a brief explanation in their course "Fundamentals of engineering design", at KIT to the ca. 70 students of the course "Mechatronic Systems and Products", and at SUT to two cohorts: those enrolled in core Bachelor of Engineering courses in year 3/4 and those enrolled in the common first year, which all engineering specializations share. At SUTD vouchers were awarded to the 50 best performing students based on correct answers and time, at SUT to the best 40 students. At RU and KIT no incentives were offered for participation.

4. Results and Discussions

4.1. Rigid versus non-rigid spatial thinking

To verify our hypothesis that spatial thinking abilities differ across the two types of spatial intelligence in engineering, design and architecture, we compared the performance for R-ST and NR-ST questions of all participants and per university. We used non-parametric tests as Skewness and Kurtosis tests indicated that data was not normally distributed.

Overall, and at each university, the number of correct answers (the score) was considerably lower for NR-ST than for R-ST (see figure 4). A statistically significant correlation, albeit weak, was found between the individual performances of all students for R-ST and NR-ST questions. The correlations at SUTD and SUT were weak to moderate. No correlations were found at KIT and RU. Although NR-ST scores were found to correlate to R-ST scores for SUTD and SUT (Table 2), i.e. students who are better in R-ST also score better in NR-ST, this is only relative. Figure 4 clearly shows the differences in scores between R-ST and NR-ST. This confirms our hypothesis: high performance in R-ST does not imply high performance in NR-ST. NR-ST requires additional spatial abilities.

Table 2. R-ST and NR-ST performance (scores) and correlation between individual R-ST and NR-ST scores. Significance: * = weak, **= weak to moderate

University	n	R-ST Score (%)	NR-ST Score (%)	Spearman's Rho correlation
All	413	79.01	43.66	r_s = -0.297, p (2-tailed) = 0.01*
SUTD	255	78.71	43.25	r_s = 0.310, p (2-tailed) < 0.001**
RU	40	79.25	43.00	r_s = -0.078, p (2-tailed) = 0.63
KIT	22	83.64	49.55	r_s = 0.14689, p (2-tailed) = 0.51
SUT	96	76.77	43.65	r_s = 0.41594, p (2-tailed) = 3E-05 **

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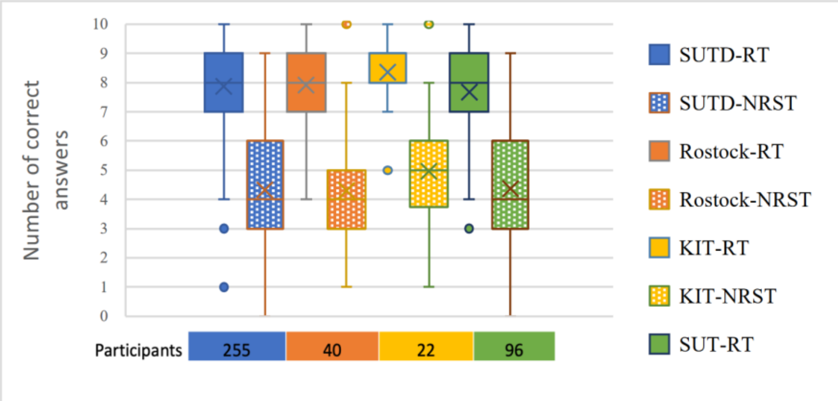

Figure 4. Number and variability of correct answers for R-ST and NR-ST questions per university

Complexity seems to affect the correctness of the answers, but not consistently (Figure 5). More than 3/4 of all students correctly answered the R-ST questions of complexity levels 2, 3 and 4 (average score 88.4%). The 2 R-ST questions with complexity levels 5 and 6 scored with an average of 42.3% correct answers in the same range as the NR-ST questions (average 43.7% correct answers). For the NR-ST questions of complexity 4, the scores varied strongly. The same is true for NR-ST complexity level 6. The results indicate that the complexity level definition requires further attention.

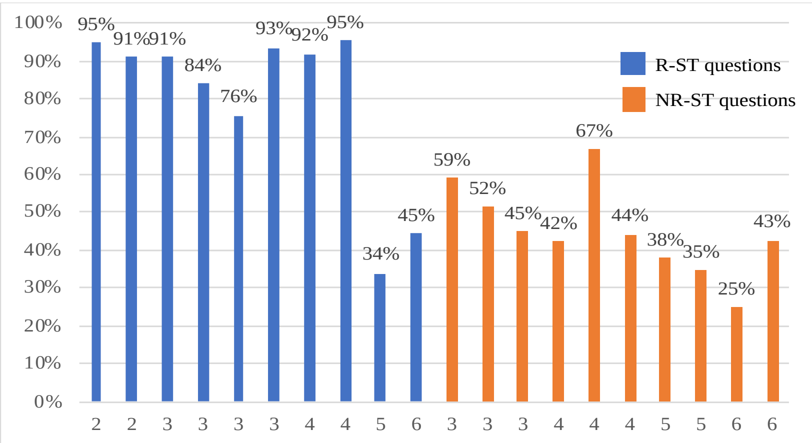

Figure 5. Percentage of correct answers for each R-ST question (left) and NR-ST question (right) ranked according to complexity (1-6), N= 413

4.2. Spatial Thinking and educational system

Educational systems differ in the material taught, the internship requirements, the opportunities for hands-on activities, etc. To investigate the impact on ST, we compared the four universities. To ensure a fair comparison, we compared course material and included only participants from engineering courses with a similar curriculum (e.g. Mechanical Engineering, Engineering Product Development, Mechatronics, etc.). This also included SUTD and SUT students from their respective common 1st year. For both R-ST and NR-ST engineering students at KIT performed, on average, considerably better than engineering students at the other universities, whose average scores were very similar (Table 3). This difference, however, is not statistically significant at $p<0.5$ (One-way Anova, fR-ST = 0.27, fNR-ST = 0.90).

4.3. Spatial Thinking and Discipline

By comparing participants from different fields of study, we can identify the influence of the discipline on ST abilities. Possible explanations are the number or depth of the courses and assignments involving ST, or the inclination of the students studying such disciplines. Further investigation is required to establish the most likely explanation.

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Table 3. Performance of participants from engineering courses with a similar curriculum

University	# of students	R-ST score (%)	NR-ST score (%)
SUTD	132	79.09	42.80
RU	38	79.47	40.79
KIT	20	85.50	50.00
SUT	82	76.83	42.93

For this analysis, we only used the data from SUTD. The SUTD participants covered each of its four programmes in similar numbers (Table 4). At the other universities the participants were mainly from engineering due to the method of distribution (compare Tables 2 and 3).

Table 4. Performance of participants of different disciplines at SUTD

SUTD – Disciplines	#	Highest rank	R-rank	NR-rank	R-ST Score	NR-ST Score
Common 1st year	101	4	3	1	79.50%	43.96%
Engineering Product Design (EPD)	31	3	2	1	77.74%	39.03%
Engineering Systems and Design (ESD)	43	1	0	1	74.88%	41.86%
Architecture and Sustainable Design (ASD)	29	8	3	5	80.69%	47.93%
Information Systems, Technology and Design (ISTD)	51	4	2	2	79.80%	42.94%

As shown, the overall performance of ASD students was higher than for the other disciplines. ASD ranked highest for 8 of the 20 questions (3 R-ST and 5 NR-ST), i.e., for 8 questions, the percentage of ASD students with correct answers was higher than the other disciplines. ASD students indeed have more courses and assignments requiring spatial thinking. However, the differences between the disciplines was not significant at $p < .05$ for both R-ST and NR-ST (Kruskal Wallis $H_{(R\text{-}ST)} = 1.6744$ (4, N = 50) with p= 0.80, and $H_{(NR\text{-}ST)} = 2.1242$ (4, N = 50) with p= 0.71). One reason may be that at SUTD, design activities take place in each of the programmes, as reflected in the name.

4.4. Spatial Thinking and experience

We also analysed whether increased exposure to tasks and assignments involving ST could increase ST ability, i.e., if students in higher semesters perform better than those who just started. As shown in Figure 6, the average scores for R-ST and NR-ST performance do not increase with study progress.

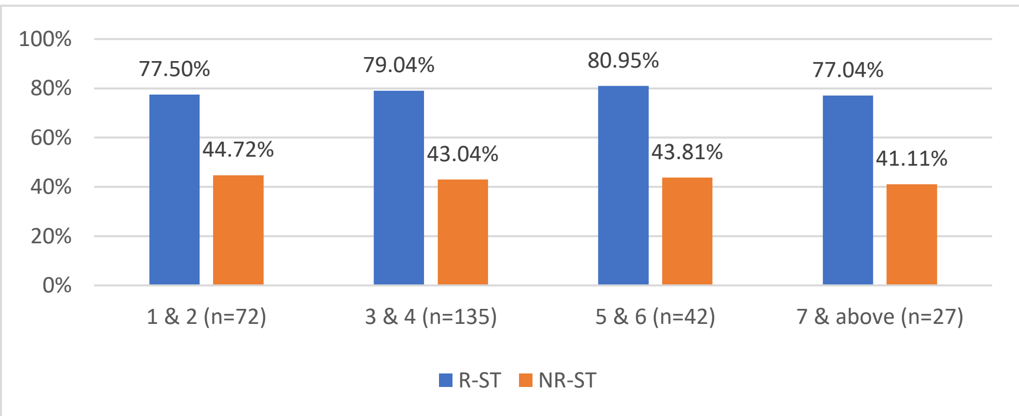

Figure 6. Average percentage of correct answers according to semester, for R-ST and NR-ST

Given the high level of R-ST performance, room for R-ST improvement is limited. For NR-ST, however, considerable room for improvement does exist as the scores were rather low. It has to be noted

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that not at each university did we have participants from each semester (see Table 1), and the number of students in some semesters was very low.

4.5. Spatial Thinking and Hands-on Activities

Literature suggests that hands-on activities improve spatial thinking abilities. We asked students at RU, KIT and SUT whether they did any hands-on activities, mentioning origami, 3d printing and woodworking as examples. (This question was not part of the original quiz at SUTD). To have similar numbers of participants we combined both German universities (RU and KIT). A total of 80.3% of the participants had experience with hands-on activities. As shown in Table 5, KIT & RU students without experience in hands-on activities perform slightly worse than those with experience. At SUT we see the opposite: participants without experience perform better. The results are not statistically significant at $p<0.05$ (Mann-Whitney U test).

Table 5. Average performance (R-ST and NR-ST) and experience in hands-on activities

University	R-ST score	NR-ST score	# Participants
RU & KIT- Experience with hands-on activities	81.49%	47.23%	47
RU & KIT - No experience with hands-on activities	78.67%	39.33%	15
SUT- Experienced with hands-on activities	76.00%	42.00%	80
SUT- No experience with hands-on activities	80.63%	51.88%	16

4.6. Analysis based on Gender

Although some literature suggest that ST ability is linked to gender (e.g. (Newcombe et al. 1983)) we did not find any significant differences in R-ST (p-value = 0.89, $p<0.5$) and NR-ST (p -value= 0.93, $p<0.5$) scores of our 251 male and 160 female participants (Mann-Whitney U test).

5. Conclusion, Limitations, and future work

For this initial study into R-ST and NR-ST abilities we developed a quiz comprising 10 R-ST and 10 NR-ST questions at different levels of complexity for students from four different universities. It has provided us with the following insights into R-ST and NR-ST abilities and factors that play a role.

- High performance in R-ST (number of correct answers) does not imply high performance in NR-ST, even though those who performed better in R-ST tended to perform better in NR-ST questions. NR-ST clearly requires additional spatial abilities.
- Complexity seems to affect the correctness of the answers, but not consistently. The complexity level definition requires further attention.
- For both R-ST and NR-ST, engineering students at KIT performed, on average, considerably better than engineering students at the other universities, whose average scores were very similar. However, the numbers of students at KIT and another of the four universities were rather small. No statistically significant differences in performance were found.
- The differences between the disciplines were not significant, although the overall performance of ASD (architecture) students was higher than for the other disciplines at SUTD. One reason may be that at SUTD, extensive design activities take place in each of the programmes.
- R-ST and NR-ST performance did not increase with study progress, but the low number of students in some semesters may have had an effect.
- The impact of having experience with hands-on activities was not found to improve performance. Interestingly, at KIT and RU there was a slight positive impact on the performance, whereas at SUT there was a slight negative impact. This merits further investigation.
- Contrary to literature we did not find an effect of gender on the students' performance.

Given the high level of R-ST performance, room for improvement is limited. For NR-ST, however, considerable room for improvement does exist and is necessary given its importance for a variety of professions, particularly in STEM. Further research is required. This includes not only a larger number

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of students and further disciplines, but also the validity of our novel NR-ST test and the complexity metrics. We did not have sufficient data from each university to do some of the more detailed analyses, e.g., only at SUTD were sufficient students of different domains involved. Furthermore, the lack of NR-ST understanding and tests did not allow us to verify our test material and the suitability of anamorphosis for NR transformations in the test, nor did it allow us to verify the measure of complexity. We need to address questions such as: do we measure what we think we measure (NR-ST ability as well as complexity)? Can the two types of abilities be compared? How do students' reason, and does this change from question to question?

Through our research, we aim to kindle research into NR-ST as a crucial ability for our discipline and for many others. We plan to extend and deepen our research to further increase our understanding of Spatial Thinking, to develop suitable tests and a toolkit to teach ST, with emphasis on NR-ST. Furthermore, we hope that our results will contribute to a much-needed update in standard aptitude and attitude tests to ensure they test the spatial thinking abilities required in the 21st century.

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