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Addendum: Towards next-to-next-to-leading-log accuracy for the width difference in the $B_s - \bar{B}_s$ system: fermionic contributions to order $(m_c/m_b)^0$ and $(m_c/m_b)^1$

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Abstract: We calculate the three-loop master integrals of ref. [1] in analytic form. This allows us to present the fermionic contributions to the $\Delta B = 2$ Wilson coefficients of the B–$\bar{B}$ decay matrix in next-to-next-to-leading order of QCD with full analytic dependence on the mass of the charm quark in the fermionic loops.

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1 Introduction

The off-diagonal element Γ_{12} of the 2×2 decay matrix of the B–$\bar{B}$ mixing problem, where B represents B_s or B_d, must be calculated to predict the width difference $\Delta\Gamma$ between the B meson mass eigenstates and the CP asymmetry in flavour-specific B decays. The contributions with fermionic loops of the next-to-next-to-leading order (NNLO) prediction calculated in ref. [1] are functions of $z = m_c^2/m_b^2$, where m_c and m_b are the masses of charm and bottom quark, respectively. The three-loop master integrals have been derived as an expansion in z and all but two integrals are given in analytic form, while the remaining two integrals involve numerically calculated coefficients. In this Addendum, we present analytic results for these as well and arrive at concise analytic results for the contributions of current-current operators to the $\Delta B = 2$ Wilson coefficients entering Γ_{12}. The corresponding contributions with penguin operators are also known in analytic form [2].

2 Updated master integrals

Our updated analytic results concern the imaginary parts originating from four-particle-cuts of the master integrals in eqs. (B.2) and (B.6) of ref. [1]. The expansion in $z = m_c^2/m_b^2$ up to $\mathcal{O}(z^4)$ in the $\overline{\text{MS}}$ scheme reads:

$$
\begin{aligned}
&Im^{(4)} \int [dk] \frac{1}{(k_2^2 - m_b^2)\, k_3^2\, ((k_1 - p_b)^2 - m_c^2)\, ((k_1 - k_2)^2 - m_c^2)\, (k_2 - k_3)^2} \\
&= \frac{(\mu/m_b)^{6\epsilon - 2}}{8192\pi^5} \left[-\frac{7}{2} + \frac{\pi^2}{3} - z\left(4 + \frac{2\pi^2}{3} + 4\log(z)\right) + z^2\left(\frac{27}{2} - 7\log(z) + \log^2(z)\right) \right. \\
&\qquad + z^3\left(-\frac{11}{3} + 2\log(z)\right) + z^4\left(\frac{7}{24} + \frac{1}{2}\log(z)\right) \\
&\quad + \epsilon\left(-\frac{175}{4} + \frac{3\pi^2}{2} + 22\zeta(3) + z\left(-56 + \frac{2\pi^2}{3} - 44\zeta(3) - 32\log(z) + 2\log^2(z)\right) + \frac{32\pi^2}{3} z^{3/2} \right. \\
&\qquad + z^2\left(-\frac{187}{4} + \frac{20\pi^2}{3} + 12\zeta(3) + \left(\frac{3}{2} - \frac{4\pi^2}{3}\right)\log(z) + \frac{9}{2}\log^2(z) - \log^3(z)\right) - \frac{32\pi^2}{15} z^{5/2}
\end{aligned}
$$

$$+z^3\left(\frac{16}{3}-2\pi^2-\frac{4}{3}\log(z)\right)-\frac{32\pi^2}{105}z^{7/2}+z^4\left(-\frac{6691}{1800}-\frac{\pi^2}{2}+\frac{43}{10}\log(z)\right)\Bigg)\Bigg]$$
$$+\mathcal{O}\left(z^5,\epsilon^2\right), \tag{2.1}$$

$$Im^{(4)}\int[dk]\frac{1}{k_1^2(k_1-p_b)^2\left((k_2-p_b)^2-m_b^2\right)(k_1-k_2)^2\left(k_3^2-m_c^2\right)\left((k_2-k_3)^2-m_c^2\right)}$$
$$=\frac{(\mu/m_b)^{6\epsilon}}{8192\pi^5}\Bigg[-4+\frac{\pi^2}{3}+z\left(2-\frac{2\pi^2}{3}+12\zeta(3)-\left(2-\frac{2\pi^2}{3}\right)\log(z)+\log^2(z)-\frac{1}{3}\log^3(z)\right)$$
$$+z^2\left(-\frac{19}{2}+3\log(z)\right)+z^3\left(-\frac{1}{6}+\frac{1}{2}\log(z)\right)+z^4\left(\frac{67}{216}+\frac{1}{6}\log(z)\right)\Bigg]$$
$$+\mathcal{O}\left(z^5,\epsilon^1\right), \tag{2.2}$$

with the loop measure defined as

$$\int[dk]=\int\frac{dk_1^d}{(2\pi)^d}\int\frac{dk_2^d}{(2\pi)^d}\int\frac{dk_3^d}{(2\pi)^d}. \tag{2.3}$$

The eq. (2.1) was obtained with the differential-equation method, while for eq. (2.2) we have used the formulas for the calculation of four-particle phase space integrals derived in [3].

3 Analytic results for $\Delta B = 2$ coefficients at order $\alpha_s^2 N_f$

The results above entail the following updated analytic functions $F_{ij}^{(2),N_v}(z)$ and $F_{S,ij}^{(2),N_v}(z)$ (determining the NNLO charm-loop contribution to the $\Delta B = 2$ Wilson coefficients), superseding eqs. (4.8)–(4.13) of ref. [1]:

$$F_{11}^{(2),N_V}(z)=-\frac{386}{9}\log\left(\frac{\mu_1}{m_b}\right)+\frac{176}{9}\log\left(\frac{\mu_2}{m_b}\right)-\frac{40}{3}\log\left(\frac{\mu_1}{m_b}\right)\log\left(\frac{\mu_2}{m_b}\right)+\frac{20}{3}\log^2\left(\frac{\mu_2}{m_b}\right)$$
$$-\frac{2689}{54}+\frac{5\pi^2}{9}+32\zeta(3)-4\pi^2\sqrt{z}+z(37-24\log(z))-4\pi^2z^{3/2}$$
$$+z^2\left(\frac{2219}{9}+\frac{8\pi^2}{9}-192\zeta(3)-\frac{572}{9}\log(z)+2\log^2(z)\right)$$
$$+z^3\left(\frac{46714}{2025}+\frac{128\pi^2}{27}+\frac{4846}{135}\log(z)-\frac{128}{9}\log^2(z)\right)+\mathcal{O}(z^4), \tag{3.1}$$

$$F_{12}^{(2),N_V}(z)=\frac{554}{27}\log\left(\frac{\mu_1}{m_b}\right)+\frac{352}{27}\log\left(\frac{\mu_2}{m_b}\right)-\frac{80}{9}\log\left(\frac{\mu_1}{m_b}\right)\log\left(\frac{\mu_2}{m_b}\right)+\frac{68}{3}\log^2\left(\frac{\mu_1}{m_b}\right)$$
$$+\frac{40}{9}\log^2\left(\frac{\mu_2}{m_b}\right)+\frac{3473}{324}+\frac{10\pi^2}{27}+\frac{64\zeta(3)}{3}-\frac{8\pi^2}{3}\sqrt{z}-z\left(\frac{334}{3}+16\log(z)\right)$$
$$-\frac{8\pi^2}{3}z^{3/2}+z^2\left(\frac{7345}{27}-\frac{86\pi^2}{27}-128\zeta(3)-\frac{3184}{27}\log(z)+\frac{55}{3}\log^2(z)\right)$$
$$+z^3\left(\frac{189512}{6075}+\frac{256\pi^2}{81}+\frac{8468}{405}\log(z)-\frac{256}{27}\log^2(z)\right)+\mathcal{O}(z^4), \tag{3.2}$$

$$
\begin{aligned}
F_{22}^{(2),N_V}(z) =& \left(\frac{236}{27}+\frac{4\pi^2}{9}\right)\log\left(\frac{\mu_1}{m_b}\right)+\frac{58}{27}\log\left(\frac{\mu_2}{m_b}\right)-\frac{32}{9}\log\left(\frac{\mu_1}{m_b}\right)\log\left(\frac{\mu_2}{m_b}\right)+\frac{20}{3}\log^2\left(\frac{\mu_1}{m_b}\right)\\
&+\frac{16}{9}\log^2\left(\frac{\mu_2}{m_b}\right)+\frac{3911}{324}+\frac{13\pi^2}{54}+\frac{16\zeta(3)}{3}-\frac{16\pi^2}{3}\sqrt{z}-z\left(\frac{94}{3}-\frac{4\pi^2}{3}+32\log(z)\right)\\
&+\frac{32\pi^2}{9}z^{3/2}+z^2\left(\frac{70}{3}+\frac{40\pi^2}{27}-20\zeta(3)-\frac{899}{54}\log(z)-\frac{17}{6}\log^2(z)\right)\\
&+z^3\left(-\frac{28369}{6075}+\frac{40\pi^2}{81}+\frac{3764}{405}\log(z)-\frac{40}{27}\log^2(z)\right)+\mathcal{O}(z^4), \qquad (3.3)
\end{aligned}
$$

$$
\begin{aligned}
F_{S,11}^{(2),N_V}(z) =& -\frac{80}{9}\log\left(\frac{\mu_1}{m_b}\right)+\frac{320}{9}\log\left(\frac{\mu_2}{m_b}\right)+\frac{128}{3}\log\left(\frac{\mu_1}{m_b}\right)\log\left(\frac{\mu_2}{m_b}\right)-\frac{64}{3}\log^2\left(\frac{\mu_2}{m_b}\right)\\
&-\frac{470}{27}+\frac{56\pi^2}{9}+32\zeta(3)-16\pi^2\sqrt{z}+136z-16\pi^2 z^{3/2}\\
&+z^2\left(\frac{1964}{9}+\frac{80\pi^2}{9}-192\zeta(3)-\frac{680}{9}\log(z)+8\log^2(z)\right)\\
&+z^3\left(\frac{43744}{2025}+\frac{128\pi^2}{27}+\frac{5296}{135}\log(z)-\frac{128}{9}\log^2(z)\right)+\mathcal{O}(z^4), \qquad (3.4)
\end{aligned}
$$

$$
\begin{aligned}
F_{S,12}^{(2),N_V}(z) =& \frac{464}{27}\log\left(\frac{\mu_1}{m_b}\right)+\frac{640}{27}\log\left(\frac{\mu_2}{m_b}\right)+\frac{256}{9}\log\left(\frac{\mu_1}{m_b}\right)\log\left(\frac{\mu_2}{m_b}\right)+\frac{32}{3}\log^2\left(\frac{\mu_1}{m_b}\right)\\
&-\frac{128}{9}\log^2\left(\frac{\mu_2}{m_b}\right)+\frac{734}{81}+\frac{112\pi^2}{27}+\frac{64\zeta(3)}{3}-\frac{32\pi^2}{3}\sqrt{z}+\frac{80}{3}z-\frac{32\pi^2}{3}z^{3/2}\\
&+z^2\left(\frac{5296}{27}+\frac{112\pi^2}{27}-128\zeta(3)-\frac{2320}{27}\log(z)+\frac{40}{3}\log^2(z)\right)\\
&+z^3\left(\frac{132704}{6075}+\frac{256\pi^2}{81}+\frac{10016}{405}\log(z)-\frac{256}{27}\log^2(z)\right)+\mathcal{O}(z^4), \qquad (3.5)
\end{aligned}
$$

$$
\begin{aligned}
F_{S,22}^{(2),N_V}(z) =& \left(\frac{704}{27}-\frac{32\pi^2}{9}\right)\log\left(\frac{\mu_1}{m_b}\right)-\frac{320}{27}\log\left(\frac{\mu_2}{m_b}\right)-\frac{128}{9}\log\left(\frac{\mu_1}{m_b}\right)\log\left(\frac{\mu_2}{m_b}\right)\\
&+\frac{32}{3}\log^2\left(\frac{\mu_1}{m_b}\right)+\frac{64}{9}\log^2\left(\frac{\mu_2}{m_b}\right)\\
&+\frac{2018}{81}-\frac{148\pi^2}{27}-\frac{32\zeta(3)}{3}+\frac{32\pi^2}{3}\sqrt{z}-z\left(\frac{208}{3}+\frac{32\pi^2}{3}\right)+\frac{608\pi^2}{9}z^{3/2}\\
&+z^2\left(-\frac{5552}{9}+\frac{352\pi^2}{27}-32\zeta(3)+\frac{6380}{27}\log(z)-\frac{172}{3}\log^2(z)\right)\\
&+z^3\left(-\frac{243592}{6075}+\frac{64\pi^2}{81}+\frac{11552}{405}\log(z)-\frac{64}{27}\log^2(z)\right)+\mathcal{O}(z^4). \qquad (3.6)
\end{aligned}
$$

The contribution of any light quark u, d, s can be obtained by setting $z=0$ in eqs. (3.1)–(3.6).

The new analytic results presented in this addendum are in excellent agreement with the previous numerical results presented in ref. [1].

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