Validation of a New Fast-Time Scale Code for Advanced Simulations of Gyrotron Cavities

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***Abstract* — Gyrotrons for fusion applications are microwave vacuum tubes that are capable to produce an output power in the megawatt range at long pulses up to continuous wave (CW) and at frequencies above 100 GHz. That is possible due to the working principle of gyrotrons which allows using cavities with a very large electrical size (in the order of several cm) compared to the operating wavelength (in the order of a few mm). This mandatory requirement for high output power is a challenge in simulating the interaction between the electromagnetic (EM) field and the electron beam in a gyrotron resonator. Due to this, the simulation of the electron interaction in gyrotrons are typically carried out by using computer codes which make use of the very specific properties of the EM problem to simplify the calculations. At KIT, a new code names "SimpleRick" is under development. A fast-time scale Particle-in-Cell (PIC) method is implemented to complement the classical models used for gyrotron simulation. The PIC code introduces significantly fewer assumptions than the classical model and may therefore represent more physical details. For example, in contrast to the classical models, the new model can represent non-symmetric electron beams. In this work, the numerical implementation and the performance of this PIC model are verified and a new method for the calculation of the eigenvalues of coaxial gyrotron resonators is shown in more detail.**

***Keywords* — electron tubes, gyrotrons, coaxial resonators, computer simulation, numerical models.**

I. Introduction

The currently used models for the simulation of the beam-wave interaction in gyrotron resonators involve inherent linearization and assumptions. They are permissible for simulating gyrotrons at the designed operation points, but could lead to inaccuracies in the simulation of unwanted, parasitic oscillations.

During the last year, a new Paricle-in-Cell (PIC) model was developed and introduced at KIT-IHM. Details can be found in [1]. The model was initially developed for the simulation of gyro-TWTs. In order to describe these, assumptions made in previous interaction models, such as [2], [3], are dropped. The adiabatic description of the particles [2], as used in previously common gyrotron interaction models, are abandoned and the motion of the particles are described by a Particle-in-Cell approach. Likewise, the amplitude of the eigenmode of the EM field is not assumed to be slowly varying. The EM field, as well as the interaction, is described on a fast timescale. This offers a broad simulation bandwidth and allows effects such as the mixing of the EM modes and harmonics at the nonlinear relativistic electron beam to be taken into account.

Such a generalized model can also be beneficial for simulating gyrotrons. For example, distortions of the EM field at the nonlinear electron beam can be correctly represented in the new model. The validation of the new model for gyrotron simulations is demonstrated in this work. For this purpose, simulation results of a gyrotron interacting on the second harmonic are compared with the results of the classical approach in the following. Additionally, the broadband capability of the new model is demonstrated.

II. Implemented Particle-in-Cell Model

A. Relativistic Motions of Electrons

The new interaction model was developed and described in [1]. In contrast to the classical modeling of the particles [2], [4], [5] as beamlets, the motions of electrons in the presented method are directly calculated from the Lorentz force

$$m_0 \frac{\mathrm{d}\,\boldsymbol{u}}{\mathrm{d}\,t} = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)\,, \tag{1}$$

where $\boldsymbol{u} = \gamma\,\boldsymbol{v}$, γ is the Lorentz factor and m_0 is the rest mass of an electron. This equation is numerically solved by the explicit Boris method [6] at every time step.

Compared to the classical method, the presented model of electrons allows the simulation of arbitrary electron distributions, e.g. non-symmetric beams. Since there is no assumption in modeling the static background magnetic field, non-symmetric and even non-adiabatic magnetic field can be additionally considered in the simulations. These features can be used to simulate the effects of non-ideal electron beams or misaligned magnetic fields. In the future, this could enable the design of even more robust cavities. In addition, it is possible to simulated devices with a second electron beam such as proposed in [7], [8] for the suppression of competing modes.

B. Wave Envelopes in a Weakly Varied Waveguide

The new model assumes, like the classical model, the EM field in the gyrotron cavity as cylindrical eigenmodes of a weakly irregular waveguide [9]. The electric field of a TE mode is assumed to be an amplitude modulation of a time dependent envelope function $a(z,t)$ on a carrier frequency ω_0:

$$\boldsymbol{E} = \boldsymbol{E}_\perp = a(z,t)\,e^{j\,\omega_0\,t}\,\mathfrak{e}_\perp \tag{2}$$

where $\mathfrak{e}_\perp$ is the normalized eigenvector of the TE mode at a given z position. It is assumed for a weakly irregular waveguide that $\left|\partial^2 \mathfrak{e}_\perp/\partial z^2\right| \ll \left|k_z^2\, \mathfrak{e}_\perp\right|$. Assuming $|\partial \mathfrak{e}_\perp/\partial z|/|\mathfrak{e}_\perp| \ll |(\partial a/\partial z)/a|$, one can obtain the following equation for the envelope function from the Helmholtz equation:

$$\frac{\partial^2 a}{\partial z^2} + k_{z0}^2\, a - j\frac{2\omega_0}{c_0^2}\frac{\partial a}{\partial t} - \frac{1}{c_0^2}\frac{\partial^2 a}{\partial t^2} = R \tag{3}$$

where k_{z0} is the axial wave number at frequency ω_0 and

$$R = \mu_0 \iint_{S_\perp} \mathfrak{e}_\perp^* \cdot \frac{\partial}{\partial t}\boldsymbol{J}\, \mathrm{e}^{-j\,\omega_0\, t}\, \mathrm{d}S_\perp \tag{4}$$

is the excitation. The cavity cross-section is denoted by $S_\perp$ and the current density by $\boldsymbol{J}$.

In the classical method [2], [5] the second temporal derivative of a, the gray term in eq. (3), is neglected; whereas in the presented model, this term can be optionally considered. There are two advantages for considering this term. First, by considering the second time derivative, the dispersion of the wave is considered correctly, even distant from the introduced carrier angular frequency ω_0. This is shown in more detail in section III-B. Second, due to non-linear distortions at the relativistic electron beam, mixing products can be produced. If the second time derivative is neglected, these mixed products can no longer be represented correctly. Consequently, with a proper broadband boundary condition [10], the influence of such broadband distortions on the interaction can be studied with the new model.

In the classical models, the source term R was modelled on the assumption, that the amplitude of the field varies much slower than the electron transit time τ_e. However, in the present model, the individual motions of all calculated electrons are aggregated at each time step. Therefore, the excitation bandwidth is not limited, as $1/\tau_\mathrm{e}$. It can be derived from the velocity and the assignment of each particle to a finite number of spatial discretization points. Therefore, it can be used for arbitrary electromagnetic fields and arbitrary electron beams. Furthermore, no assumption about the interaction on harmonics has to be made and is therefore valid for all harmonics. This is not the case with previous models. By using Graf's additions theorem [11] in the derivation of the source term, it is limited to the fundamental or exactly one harmonic [3].

An important application for the new simulation model will be the study of so-called After-Cavity-Interactions (ACI) [12]. These can occur at frequencies that are significantly away from the main operation frequency and are therefore difficult to model using classical methods.

III. Validation of the Model for the Use in Gyrotron Interaction Simulation

A. Solving the Characteristic Equation for Coaxial Cavities

In order to reduce ohmic losses, highly overmoded cavities are used for high-power gyrotrons, where the mode spectrum is dense. In such oversized cavities, competing modes can be suppressed by a coaxial insert [13]. To simulate a coaxial cavity, the eigenvalues χ_{mp} of the coaxial waveguide eigenmodes must be found [14]. Programs to calculate the eigenvalues χ_{mp} of a specific resonator geometry exist. For example, the code CHImp as part of the EURIDICE program package [5]. However, in order to fully integrate the eigenvalue calculation into the program flow of "SimpleRick", a new method for the calculation of the eigenvalues is designed.

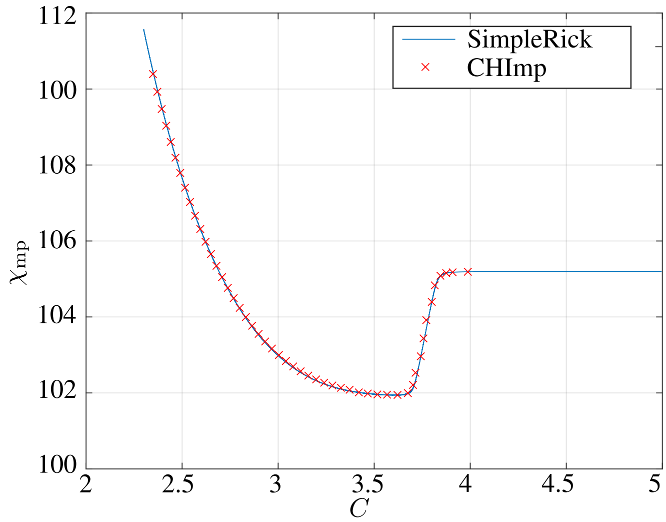

Fig. 1. Comparison of eigenvalue solutions in CHImp and in the newly implemented eigenvalue solver in SimpleRick for the coaxial $\mathrm{TE}_{34,19}$ mode, where the coaxial insert has longitudinal corrugations with azimuthal periodicity of 0.5 and depth of $0.15\,\lambda_c$.

For an optimal suppression of competing modes by a coaxial cavity, the coaxial insert can be corrugated [13]. The eigenvalue of a coaxial waveguide is a function of the outer to inner radius ratio $C = r_\mathrm{o}/r_\mathrm{i}$, the corrugation depth and periodicity. Due to the tapering of outer and inner conductor, C changes along the cavity axis, the eigenvalues have to be solved for every mesh segment of the cavity. One specific scenario is shown in fig. 1 for the verification of the newly implemented method. If the radius of the inner conductor is small compared to the radius of the outer cavity wall, the coaxial eigenvalue approaches the eigenvalue of the corresponding hollow cavity eigenmode ($C > 4$ in fig. 1).

The new method is based on a secant method, while the standard Newton method is unsuitable [15] because the first derivative near χ_{mp} becomes small due to the oscillating behavior of the first and second order Bessel functions. In addition, the required number of calculations is optimized for a given mesh grid.

B. Systematic Verification of the Interaction Code

To perform advanced interaction simulations using the new tool, the implementation of the model had to be reviewed and tested. Several test cases of varying complexities were defined for this purpose. Two of them are listed below.

First, to verify the numerical solution of eq. (3) taking the second temporal derivative into account, the wave dispersion in a hollow $\mathrm{TE}_{0,3}$ waveguide is checked. The simulated group and phase velocities of monochromatic waves are compared with its analytical solution, in absent of particles. The carrier frequency is chosen with $f_0 = 150\,\mathrm{GHz}$. As can be seen in

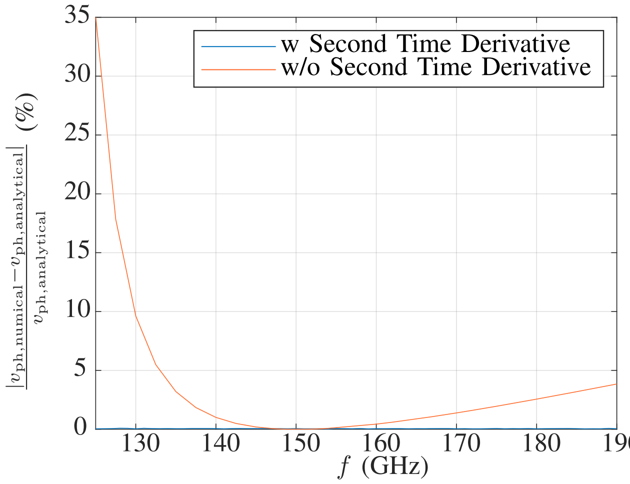

Fig. 2. Simulated dispersion error of $TE_{0,3}$ mode in a hollow cylindrical waveguide with $r = 4.015\,\text{mm}$ and $f_0 = 150\,\text{GHz}$

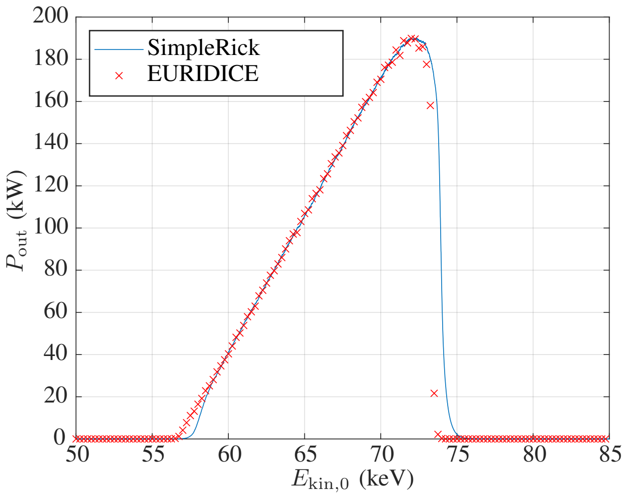

Fig. 3. Start-up scenario of the gyrotron in [16] for the second harmonic $TE_{7,4}$ operating mode

fig. 2, the dispersion without consideration of the second time derivative is valid only near the carrier frequency. In contrast, the new model is valid over a very broad frequency bandwidth.

Second, the simulation results of the new code were checked against the EURIDICE classical model using specific test cases. In order to compare the models, ideal electron beams are injected. Also, a weak relativistic operating point is chosen. The EURIDICE results can be considered correct in this case. For this purpose, the simulation of a startup scenario of the $TE_{7,4}$ mode in the gyrotron from [16] is shown in fig. 3. The chosen $TE_{7,4}$ operating mode interacts with the electron beam at the second harmonic of the electron cyclotron frequency. The results of the different models show only slight deviations in output power. The new model can now also be used for more advanced gyrotron simulations.

IV. Conclusion

The validity of a new fast-time scale interaction simulation tool was proven by a systematic verification of the model and its implementation. For this purpose, multiple test cases were designed. The investigated model does require fewer assumptions than the classical ones. First, the adiabatic modeling of the particles can be omitted. Further, since no slow-varying approximation is chosen, the second time derivative of the Helmholtz equation is taken into account. Therefore, the dispersion of the EM mode is correctly represented even for frequencies distant from the selected carrier frequency. In combination with the novel formulation of the source term, this allows a broadband gyrotron simulation. As a result, parasitic oscillations at higher harmonics can also be correctly represented. The validity of the simulation results was demonstrated using well known scenarios. In the future, the new tool will be used to investigate issues such as ACI.

Acknowledgment

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