

# BEAM DYNAMICS OBSERVATIONS AT NEGATIVE MOMENTUM COMPACTION FACTORS AT KARA

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## Abstract

For the development of future synchrotron light sources new operation modes often have to be considered. One such mode is the operation with a negative momentum compaction factor to provide the possibility of increased dynamic aperture. For successful application in future light sources, the influence of this mode has to be investigated. At the KIT storage ring KARA (Karlsruhe Research Accelerator), operation with negative momentum compaction has been implemented and the dynamics can now be investigated. Using a variety of high-performance beam diagnostics devices it is possible to observe the beam dynamics under negative momentum compaction conditions. This contribution presents different aspects of the results of these investigations in the longitudinal and transversal plane.

## INTRODUCTION

At the accelerator test facility KARA (Karlsruhe Research Accelerator) a new optics with negative momentum compaction factors  $\alpha_c$  has been implemented in recent years. The aim is to investigate the effects of the negative sign of  $\alpha_c$  on beam dynamics as well as to confirm the feasibility of using negative  $\alpha_c$  optics in order to allow reduced sextupoles without incurring instabilities such as the head-tail instability. Previous publications showed effects of the switch in sign of  $\alpha_c$  such as a shorter bunch length at negative  $\alpha_c$  [1]. In this contribution the effect of changes to the sextupole magnet currents, and therefore the chromaticity, on the first and second order of  $\alpha_c$  for the currently implemented optics at positive and negative  $\alpha_c$  are explored. Furthermore, the transverse stability in regard to head-tail effects at negative  $\alpha_c$  is investigated.

## SECOND ORDER OF $\alpha_c$

The negative  $\alpha_c$  optics are operated with negative chromaticities and therefore reduced sextupoles. This reduction in sextupole magnets strengths affects higher order terms of the momentum compaction factor. When including higher orders  $\alpha_c$  is a function of momentum

$$\alpha(\delta) = \alpha_0 + \alpha_1\delta + \alpha_2\delta^2 + \dots \quad (1)$$

As an analysis of this the second order (first non-linear order) has been investigated by means of measurements of the synchrotron frequency  $f_s$  as function of accelerating frequency  $f_{RF}$  used as tuning knob for the energy offset. The

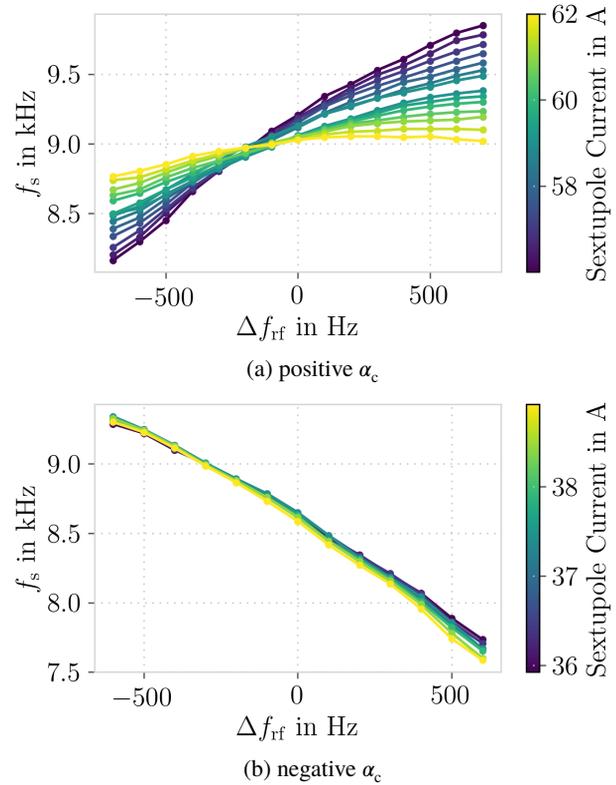


Figure 1: Synchrotron frequency as function of the frequency of the RF system at positive and negative  $\alpha_c$  for various sextupole magnet strengths.

synchrotron frequency in this case considering the first two orders of  $\alpha_c$  is given as [2]

$$f_s = f_{rev} \sqrt{\frac{heV_{RF} \cos \psi_s}{2\pi\beta_0^2 E}} \cdot \sqrt{\frac{\alpha_0}{2} + \sqrt{\frac{\alpha_0^2}{4} - \alpha_1 \frac{\Delta f_{RF}}{f_{RF}}}}, \quad (2)$$

where  $f_{rev}$  is the revolution frequency,  $h$  the harmonic number,  $V_{RF}$  the accelerating voltage,  $\psi_s$  the synchronous phase,  $\beta_0 = \frac{v}{c}$  and  $E$  is the particle energy. From this equation the first and second order of  $\alpha_c$  ( $\alpha_0$  and  $\alpha_1$ ) can be identified by fitting the equation to the mentioned measurements.

The synchrotron frequency has been measured at positive and negative  $\alpha_c$  in order to allow comparison. In both cases the beam energy was set to 1.3 GeV. The measurements were done by using the RF system to vary  $f_{RF}$  and by using the BBB feedback system [3, 4] which calculates the beam spectrum from BPM data via a Fourier transformation. In this data the synchrotron frequency is then given as a

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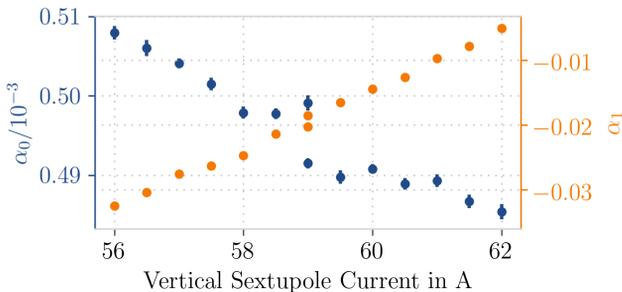


Figure 2: First two orders of  $\alpha_c$  extracted as fit of Eq. (2) on the data shown in Fig. 1 for positive  $\alpha_c$  as a function of the sextupole current.

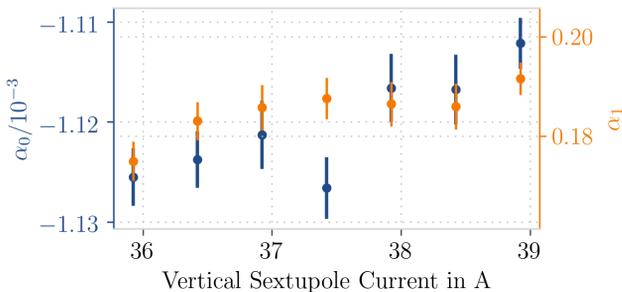


Figure 3: First two orders of  $\alpha_c$  extracted as fit of Eq. (2) on the data shown in Fig. 1 for negative  $\alpha_c$  as a function of the sextupole current.

peak. These measurements have been repeated for different currents powering the sextupole magnets to explore the dependency. In Fig. 1 the measurements are displayed. By fitting Eq. (2) to this data the mentioned two orders of  $\alpha_c$  can be extracted. The results are shown in Fig. 2 and Fig. 3.

The measurements at negative  $\alpha_c$  were taken at relatively high absolute values of  $\alpha_c$  due to the reduction in lifetime associated with a drastic reduction of  $|\alpha_c|$ . Furthermore, the necessary reduction in  $f_{RF}$  leads to beam loss at lower  $|\alpha_c|$  for negative  $\alpha_c$ . For both signs of  $\alpha_c$  the first order  $\alpha_0$  reduces in absolute value with increasing sextupole magnet strengths (indicated in the plots via the sextupole current). However, as the sign at negative  $\alpha_c$  is negative, this means the sextupoles have a different effect on the dispersion. At positive  $\alpha_c$ , where the value of  $\alpha_0$  decreases, it can be concluded that the dispersion gets more stretched, while at negative  $\alpha_c$ , where the values of  $\alpha_0$  increases, it can be concluded that the dispersion gets relaxed. In both cases the change seems linearly with sextupole current. The change in first order  $\alpha_c$  at positive  $\alpha_c$  is about 4% for a change with sextupole magnet current of about 10%. The equivalent for negative  $\alpha_c$  is a change of 1.25% in  $\alpha_0$  for a sextupole magnet current change of about 8%. Therefore, it seems the first order of  $\alpha_c$  is less sensitive to changes of the vertical sextupole current at negative  $\alpha_c$  than at positive  $\alpha_c$  for the currently implemented optics at KARA.

The second order  $\alpha_1$  seems linearly dependent on the sextupole magnet current for positive  $\alpha_c$ . At negative  $\alpha_c$ , a dependency is visible as well, albeit not as clearly linear as

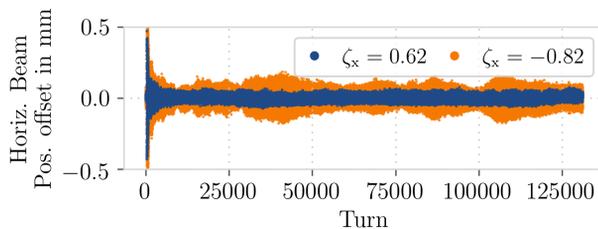


Figure 4: Beam position data after a horizontal kick on a turn-by-turn basis at positive  $\alpha_c$  for a positive and a negative horizontal chromaticity.

for positive  $\alpha_c$ . For this order the dependency is the same for both signs of  $\alpha_c$ , both increase with increasing sextupole magnet currents. Furthermore, the sign of the second order  $\alpha_1$  flips when flipping the sign of the first order  $\alpha_0$ .

These observations show the effect of sextupole magnets on the higher orders of  $\alpha_c$  and that these effects have to be considered when implementing optics with different chromaticity values. Influence of different orders of higher orders of  $\alpha_c$  on the longitudinal beam dynamics have been studied e.g. in [5]. In general the third order  $\alpha_2$  is of interest as well, for example in regard to alpha-buckets [2], and should be determined in the future.

## TRANSVERSE STABILITY

A reduction of sextupole magnet strengths would be ideal for multi-bend achromat lattices as this could increase the dynamic aperture. However, at positive  $\alpha_c$  a reduction too far, resulting in negative chromaticities, brings the risk of losing head-tail damping effects and even incurring the head-tail instability. Therefore, one solution could be the use of negative  $\alpha_c$  optics which would in theory allow negative chromaticities without the aforementioned downsides.

In order to investigate this with the negative  $\alpha_c$  optics at KARA a combination of kicks and BPM measurements was used. By firing one of the horizontal injection kickers every second and measuring the beam position on a turn-by-turn basis, the stability of the beam was tested at positive as well as negative  $\alpha_c$ . In theory the head-tail damping time  $\tau$  for mode  $n$  is given by [6]

$$\beta_n = \frac{1}{\tau} = \frac{NS}{\pi^2 c \gamma m_e} \frac{\zeta}{\alpha_c} \frac{\sigma_z}{4n^2 - 1}, \quad (3)$$

where  $\tau$  is the corresponding damping time,  $N$  is the number of particles in a bunch,  $S$  is the strength of the wakefield and  $\sigma_z$  is the bunch length. The electron mass is denoted with  $m_e$ , the chromaticity with  $\zeta$  and  $\gamma$  is the relativistic Lorentz factor. From this a larger damping time is expected at lower energies which is why the measurements presented here were performed at injection energy (0.5 GeV).

For positive alpha with the usual (positive) chromaticity, an initial damping of the large position offset is visible in Fig. 4 in blue. Afterwards over the remaining turns oscillations of a fairly constant amplitude are present. These oscillations conform to the synchrotron and betatron frequencies

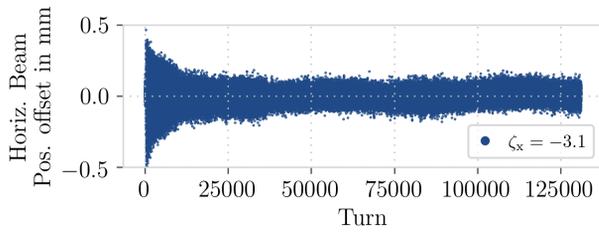


Figure 5: Beam position data after a horizontal kick on a turn-by-turn basis at negative  $\alpha_c$  for a negative horizontal chromaticity.

and are expected. For the kick at a negative chromaticity for positive  $\alpha_c$  the situation is different. This is shown in Fig. 4 in orange. After a short initial damping, large non-constant in amplitude oscillations are present. These oscillations are not linked to synchrotron or betatron oscillation frequencies and are not expected for a stable beam. From this it can already be guessed that head-tail damping effects are present at the measurement with a positive chromaticity and absent in the measurement for a negative chromaticity.

To test whether this effect can be reversed at negative chromaticities when using negative  $\alpha_c$  such measurements were also performed in these settings. Figure 5 shows such a measurement. Again an initial damping is visible followed by fairly large but constant in amplitude oscillations. These residual oscillations are in accordance to synchrotron and betatron frequencies. The larger amplitude can be explained by the largely increased dispersion necessary for the negative momentum compaction factor. For comparison, at positive  $\alpha_c$  the dispersion at the used BPM is about 0.17 m while at negative  $\alpha_c$  it is about 1.25 m. Through this increased dispersion the usual energy oscillations from the synchrotron oscillation manifest in enlarged horizontal position offsets. Therefore, combined with the rather constant amplitude of the oscillations, these measurements hint at the presence of the head-tail damping effect at negative  $\alpha_c$  with negative chromaticity.

From the BPM data a damping time can be extracted, at least in the damped cases of positive  $\alpha_c$  with positive chromaticity and negative  $\alpha_c$  with negative chromaticity. While the absolute damping time is a composition of at least radiation damping and the head-tail effect, the presence of the head-tail effect can be assessed by analyzing the bunch current dependency of the extracted damping time. In order to do this, the previously shown measurements were repeated at multiple bunch currents and the damping time extracted. The current dependency for positive  $\alpha_c$  with positive chromaticities can be seen from the results in Fig. 6. A clear increase of  $\frac{1}{\tau}$  with current is visible. The same measurements were conducted at negative  $\alpha_c$  with negative chromaticities which are shown in Fig. 7. Again a clear increase of  $\frac{1}{\tau}$  with current is visible. In both cases the relative sign between  $\alpha_c$  and  $\zeta_x$  is positive. Therefore, the observed current dependency of the damping time is in accordance to the theory in Eq. (3). Thus, it can be concluded that indeed head-tail damping is present

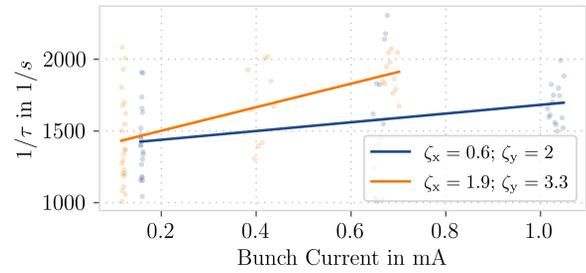


Figure 6: Inverse damping time  $\frac{1}{\tau}$  after an initial kick as function of bunch current for positive  $\alpha_c \approx 8.1 \cdot 10^{-3}$  at two different positive chromaticities.

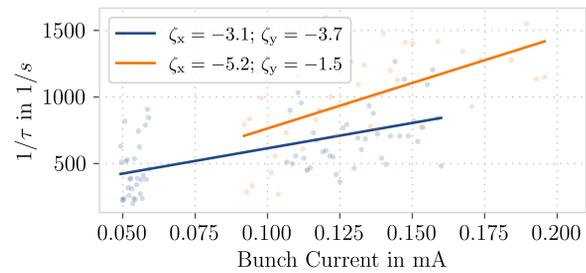


Figure 7: Inverse damping time  $\frac{1}{\tau}$  after an initial kick as function of bunch current for negative  $\alpha_c \approx -1.5 \cdot 10^{-3}$  at two different negative chromaticities.

in both cases. This means the negative  $\alpha_c$  operation mode at KARA successfully circumvented the head-tail instability when using negative chromaticities.

## SUMMARY

The negative momentum compaction regime is a possible solution to circumvent the need for high sextupole currents in multi-bend achromat structures. This contribution described the effects of possible changes in sextupole current on the first and second order of  $\alpha_c$  seen at KARA in both, positive and negative  $\alpha_c$  operation.

Furthermore, the validity of this mode has been investigated by studying the transverse beam stability. Kick measurements show a loss of head-tail damping at positive  $\alpha_c$  with negative chromaticity and a regaining of head-tail damping at negative  $\alpha_c$  with negative chromaticity.

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