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Full and partial compression fatigue tests on welded specimens of steel St 52-3. Effects of the stress ratio on the probabilistic fatigue life estimation

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ABSTRACT

The fatigue strength of structures subjected to cyclic loading depends strongly on the stress ratio. Particularly, in case of welded steel structures this fact is not considered in the corresponding standards nor in the guidelines. Experimentally, two approaches are used to study the effect of stress ratio on the fatigue life. On the one hand, based on the S-N curves obtained from tests performed at different stress ratios, the fatigue life under a particular stress range is estimated. On the other hand, the stress amplitude corresponding to a constant fatigue life is estimated by applying the failure criteria for fluctuating stress like the Goodman–Haigh relationship. This paper presents a general probabilistic model, which estimates the S-N and Goodman–Haigh curves for any stress ratio. Afterwards, this model is applied on data obtained from full and partial cyclic compression loading tests performed on welded specimens made of steel St 52-3. The tested details correspond to the permissible notch condition limit occurred in highly stressed structures used to build ships.

1. Introduction

In the field of steel structures some methods have been suggested in order to evaluate the effect of the stress ratio R on the fatigue strength estimation. These models are given by a function of the type

$$\Delta\sigma_D(R) = \widehat{\Delta\sigma_D} \cdot f(R), \quad (1)$$

where $f(R)$ is a predefined function, known as enhancement factor and $\widehat{\Delta\sigma_D}$ is a predefined fatigue strength. Note that the different standards use the same kind of function, however, for different stress ratios (IIW, $R = 0.5$ Hobbacher, 2016; FKM, $R = -1$ fkm, 2003) and compensate the discrepancy by different enhancement factors (Schork et al., 2020; Kaffenberger, 2012).

In case of welded joints, Hobbacher suggests multiplying the detail fatigue class by an enhancement factor $f(R)$, which depends on the level and direction of the residual stresses, see Hobbacher (2016). An additional proposal of the enhancement factor $f(R)$ is made by Yuen in Yuen et al. (2013) in case of welded joints with severe stress concentrations typical of ship details under constant and variable amplitude fatigue loading.

When the riveted structures are under study, on the one hand Taras and Greiner proposed in Greiner et al. (2007) and Taras and Greiner (2010) an enhancement factor $f(R)$, which depends on the material (wrought iron or steel) and on the period of construction (before or after 1900) of the structure. Thus, in this model a detail category $\Delta\sigma_C = 80$ MPa for riveted specimens is suggested. The proposed factor has not been validated by experimental tests, but it seems that they are based on the results of a technical report from the Utrecht University (Unterweger and Taras, 2010; Unterweger et al., 2013), and on some considerations made on the standard of the German railway (DB Netz AG, 2002) and of the Austrian standard (CEN-CENELEC, 2006).

On the other hand, Heydarinouri et al. proposed in Heydarinouri et al. (2019) a different factor, which is obtained by considering the Constant Life Diagram (CLD) approach and a modified Johnson criterion, see Budynas and Nisbett (2019) and Ghafoori et al. (2015). However, in case of remaining fatigue life estimation under variable amplitude loading, the authors themselves suggest not to apply their proposed method for the determination of the finite fatigue life since CLD is a local approach, but an estimation method based on the S-N curves.

Besides the empirical characteristics of the proposed factors $f(R)$, it is important to keep in mind that in every case it depends on a given fatigue strength $\widehat{\Delta\sigma_D}$ or detail category. However, the fatigue strength

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cannot be a predefined value but it depends on the experimental data. In other words, the fatigue strength is not a deterministic value but a random variable, so that, a suitable statistical analysis should be taken into account in order to obtain reliable estimations. Some remarks about this issue were already done in Sire and Ragueneau (2019).

Unfortunately, most of the classic and new methods assume that the fatigue strength can be described by a linear model linked with a normal distribution. Some of these new proposals are, the six parameters model of Leonetti et al. which is based on a uncertainty analysis (Leonetti et al., 2017, 2018), the model of Hensel, which proposes a mean stress correction based on the sensitivity to mean stress (Hensel, 2020), the four 4R method, which aims to determine the local stress and is applied by Ahola et al. in Ahola et al. (2020) and Ahola et al. (2021), and the analysis of the influence of the residual stress made by Baumgartner in Baumgartner and Bruder (2013). Nevertheless, these assumptions are not exclusive of the fatigue studies on structural steel, these are also considered in research of aeroengine disks (Lu et al., 0000), preloaded threaded fasteners (Li et al., 2020), fiber reinforced laminates (Meng et al., 2018) or lightweight composite structures (Böhm and Głowacka, 2020).

As a matter of fact, it is already known that this phenomenon actually is governed by a nonlinear model and a Generalized Extreme Value (GEV) distribution.

In order to overcome the limitations of the models mentioned above, a model based on the Stüssi function was proposed by Toasa and Ummenhofer in Toasa Caiza and Ummenhofer (2018) for $R = 0$, in Toasa Caiza and Ummenhofer (2020) for full reversed data and then enhanced and applied in Toasa Caiza et al. (2021). However, this proposals consider only the case when the experimental data come from full tension, tension dominated or full reversed fatigue tests, in other words when $|R| \leq 1$. Moreover, the enhanced model proposed in Toasa Caiza et al. (2021) provides only a deterministic estimation of the fatigue life.

For these reasons, a general model has to be built by considering any stress ratio R and the probability of fatigue failure. In order to generalize the mentioned model in Toasa Caiza et al. (2021) for any stress ratio R, it is necessary to consider the geometry of the loading wave and the elasto-plastic behavior of the material.

As it is known, the geometry of the loading wave depends on the relationship between the maximum and minimum stresses given by the stress ratio. Under these circumstances, the behavior of the loading wave can be classified according to four regions or quadrants shown in Fig. 1.

Based on Fig. 1, it can be assumed that the limits of the stress range $\Delta\sigma$ may depend on the ultimate tensile strength R_m. Thus, the mentioned limit behavior can be described by the function

$$T(R_m, R) = \begin{cases} R_m(1-R) & \text{if } R \in [0,1], \\ R_m(1-\frac{1}{R}) & \text{if } R \in [-\infty,-1[\, \cup\,]1,\infty], \end{cases} \tag{2}$$

which is shown in Fig. 2.

The definition given by Eq. (2) assumes that after plastic deformation no changes in the yield strength R_{el} or ultimate tensile strength R_m are caused. This fact becomes relevant, when the absolute value of tension or compression loads is bigger than R_{el}. In other words, when the fatigue test corresponds to the Ultra Low Cycle Fatigue (ULCF) or Low Cycle Fatigue (LCF) regimes.

Thus, it is important to keep in mind, that this assumption ignores that the stress reversal after yielding may cause a kinematic hardening of the material, which leads to a shift of the yield surface. This phenomenon is described by the Bauschinger[1] effect that describes a change in tensile or compression yield strength when the direction of loading is reversed after prior plastic deformation (Vlado A, 2001; Gothivarekara et al., 2021).

For these reasons, the fatigue strength estimation where the plastic behavior prevails is usually strain based, and where the elastic behavior prevails, the stress is taken into account.

As might be expected, if the ultimate tensile strength R_m is replaced by the yield strength R_{el} in Eq. (2), the effect of Bauschinger may be disregarded. This fact, however would imply that the estimation in VLCF and LCF regimes may not be performed.

The following sections on this paper are organized as follows. Section 2 presents the deterministic approach to model the Wöhler and Goodman–Haigh curves for any stress ratio R. Section 3 presents a general probabilistic proposal to model the curves mentioned in previous section. Section 4 presents an application of the methods proposed in Sections 2 and 3, which considers the fatigue data of welded specimens of steel St 52-3. These specimens are related with high stress structures used to build ships. Finally, Section 5 presents the conclusions of this study and propose the subsequent research to be performed.

2. Deterministic Stüssi model for any stress ratio R

Taking into account the assumption of no Bauschinger effect and Eq. (2) defined in the previous section, a general Stüssi equation to model the S-N curves for any stress ratio R can be written as

$$\Delta\sigma = \frac{T(R_m,R) + \alpha N^{\beta}\Delta\sigma_{\infty}}{1+\alpha N^{\beta}} = \mathrm{S}(N,R), \tag{3}$$

where

- $\Delta\sigma$: stress range during the fatigue test
- N : number of load cycles up to failure or up to end of the test
- R_m : ultimate tensile strength
- $\Delta\sigma_{\infty}$: fatigue limit[2]
- α, β : geometrical parameters
- R : stress ratio
- S: Stüssi function.

The model given by Eq. (3) depends on two geometrical parameters α and β which can be estimated by applying a linear regression and on two material parameters R_m and $\Delta\sigma_{\infty}$, which are supposed to be known.

2.1. Parameter estimation

The estimation of the geometrical parameters α and β from Eq. (3) can be performed as follows. By manipulating the Stüssi equation, (3), this can be written as

$$\alpha N^{\beta} = \frac{T(R_m,R) - \Delta\sigma}{\Delta\sigma - \Delta\sigma_{\infty}}. \tag{4}$$

Then, by applying logarithms in Eq. (4), the Stüssi equation can be written in a linear form as follows

$$\log(N) = \frac{1}{\beta}\log\left(\frac{T(R_m,R)-\Delta\sigma}{\Delta\sigma-\Delta\sigma_{\infty}}\right) - \frac{1}{\beta}\log(\alpha). \tag{5}$$

The Eq. (5) is no more than an elementary linear equation of the type

$$Y = AX + B, \tag{6}$$

[1] Johann Bauschinger (Nürnberg, 11.06.1834–25.11.1893). German mathematician and professor of Engineering Mechanics at Munich Polytechnic.

[2] The existence of the fatigue limit is still an open debate, see for example Bathias (1999), Miller and O'donnell (1999), Pyttel et al. (2011) and Fernández-Canteli et al. (2020).

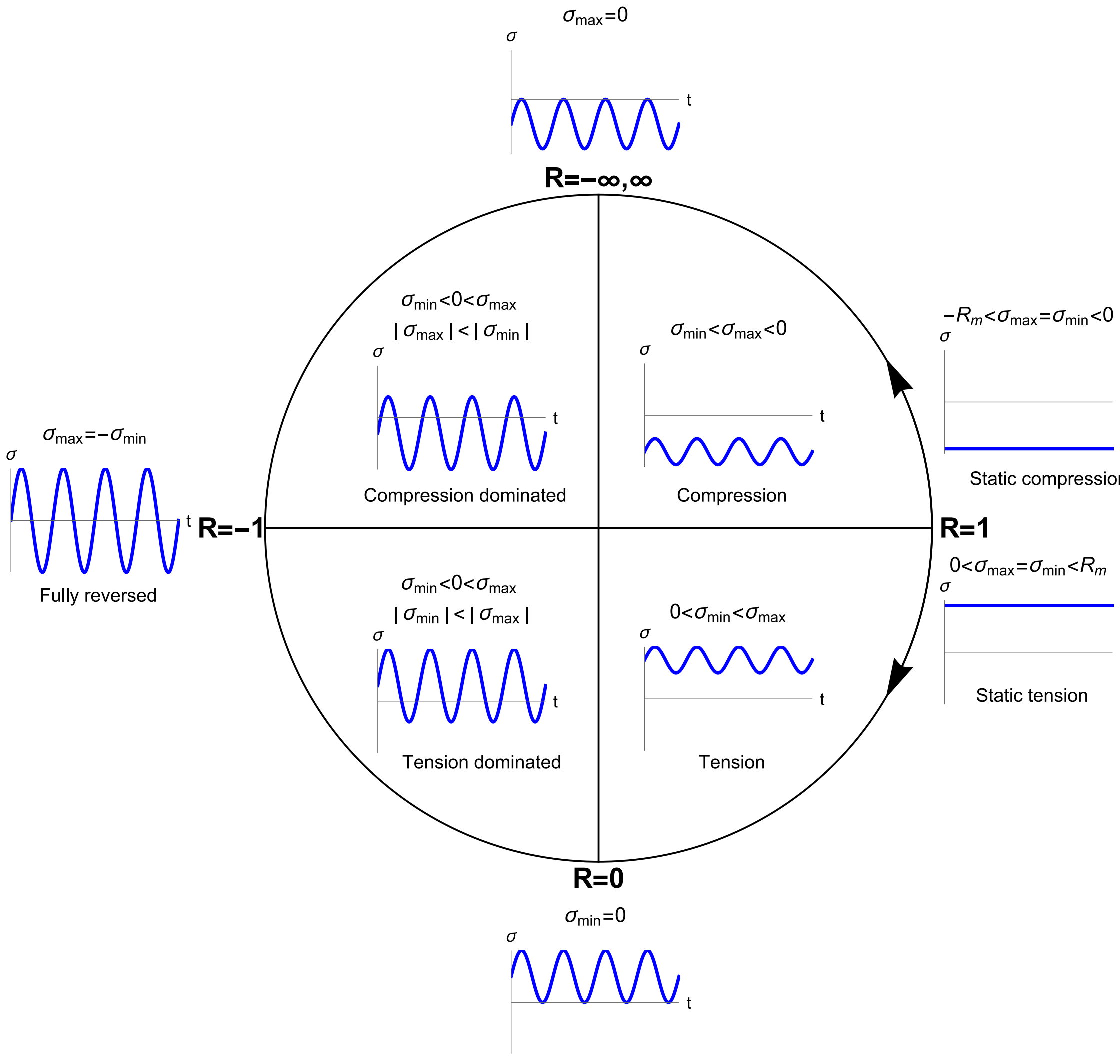


Fig. 1. Effects of the stress ratio on the loading wave geometry. $|\sigma_{max}| < R_m$, $|\sigma_{min}| < R_m$.

where

$Y = \log(N)$,

$X = \log\left(\dfrac{T(R_m, R) - \Delta\sigma}{\Delta\sigma - \Delta\sigma_\infty}\right)$,

$A = \dfrac{1}{\beta}$,

$B = -\dfrac{1}{\beta}\log(\alpha)$.

The parameters A and B of Eq. (6) can be determined from the experimental data of fatigue failures by applying a linear regression model. Then, the geometrical parameters α and β are given by

$$\beta = \frac{1}{A} \tag{7}$$

and

$$\alpha = e^{\frac{-B}{A}}. \tag{8}$$

A graphical representation of Eq. (6) for different stress ratios R is shown in Fig. 3.

2.2. Graphical representation

The Stüssi function offers a good geometrical approach to depict the theoretical fatigue behavior of a material. Moreover, it describes clearly

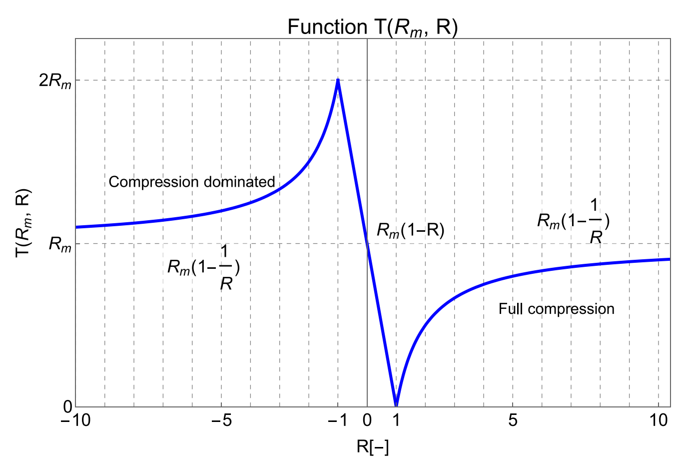


Fig. 2. Function $T(R_m, R)$.

the asymptotic behavior regarding the ultimate tensile strength R_m and the fatigue limit $\Delta\sigma_\infty$, since for Eq. (3)

$$\lim_{N\to 0} S(N, R) = T(R_m, R), \tag{9}$$

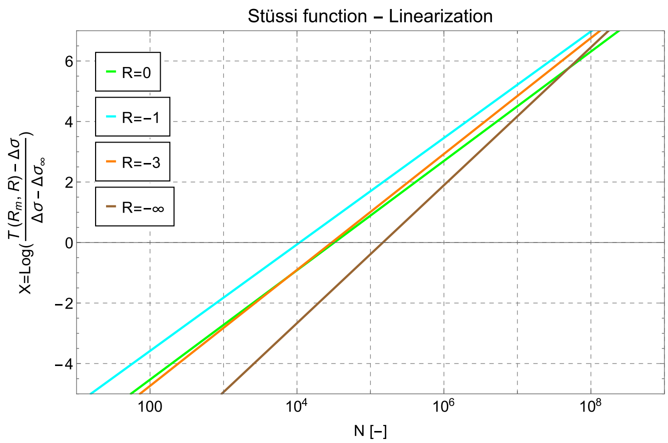


Fig. 3. Linear regressions of the Stüssi function $S(N,R)$ for different stress ratios.

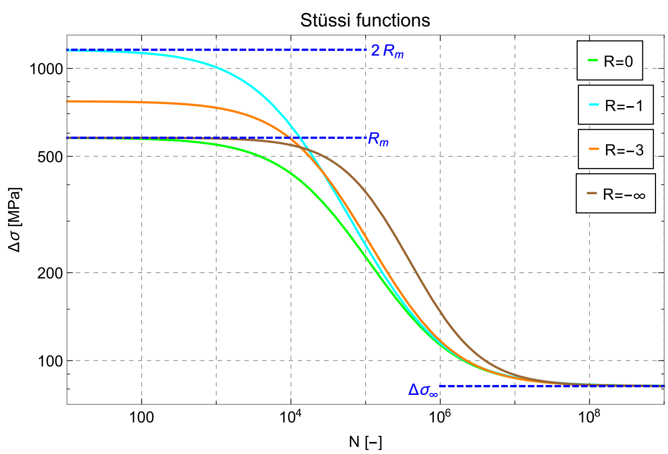


Fig. 4. S-N or Wöhler curves based on the Stüssi function for different stress ratios.

and

$$\lim_{N\to\infty} S(N,R) = \Delta\sigma_{\infty}. \tag{10}$$

A graphical representation of S-N curves given by Eq. (3) for different values of stress ratio R is shown in Fig. 4.

2.3. Goodman–Haigh diagrams

Once the S-N curves have been defined, the Goodman–Haigh diagrams can be plotted. These diagrams describe the fatigue strength for a constant fatigue life while the stress ratio is varying.

By applying Eq. (3), the mean stress $\sigma_{\mu,i,j}$ and stress amplitude $\sigma_{a,i,j}$ as function of a fatigue life N_i and stress ratio R_j are given by

$$\sigma_{\mu,i,j} = \frac{S(N_i,R_j)}{2}\cdot\left(\frac{1+R_j}{1-R_j}\right) \tag{11}$$

and

$$\sigma_{a,i,j} = \frac{S(N_i,R_j)}{2}. \tag{12}$$

Then, by varying the stress ratio R_j and keeping constant the loading cycles N_i, a set of points corresponding to a constant fatigue life are obtained.

Finally, for a better visualization, the straight lines given by

$$\sigma_a = \sigma_\mu\cdot\left(\frac{1-R_j}{1+R_j}\right) \tag{13}$$

have to be plotted.

Table 1
Experimental data of the fatigue tests under constant amplitude loading and the estimations obtained from them.

(a) Experimental data		(b) Estimations	
Experimental data		Estimations	
$\Delta\sigma_i$	$N_{i,j}$	Weibull parameters	Fatigue life $\widehat{N}_i$
$\Delta\sigma_1$	$N_{1,1}, N_{1,2}, \ldots, N_{1,j_1}$	$\hat{a}_1, \hat{b}_1, \hat{c}_1$	$\widehat{N}_1(P=p1)$
$\Delta\sigma_2$	$N_{2,1}, N_{2,2}, \ldots, N_{2,j_2}$	$\hat{a}_2, \hat{b}_2, \hat{c}_2$	$\widehat{N}_2(P=p1)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\Delta\sigma_n$	$N_{n,1}, N_{n,2}, \ldots, N_{n,j_n}$	$\hat{a}_n, \hat{b}_n, \hat{c}_n$	$\widehat{N}_n(P=p1)$

3. Probabilistic Stüssi model for any stress ratio R

It is well known that the fatigue life time or fatigue strength is a random variable. However, for decades, it was assumed that its random behavior can be described by a linear model linked to a Gaussian distribution. This fact is even today considered in the official standards and guidelines such as Eurocode 3 (CEN-CENELEC, 2010), International Institute of welding (IIW) (Hobbacher, 2009, 2010), ISO12107 (ISO/TC 164/SC 4, 2012), German railways (DB Netz AG, 2002), Swiss standard (SIA, 2011), guidelines for design of wind turbines (Det Norske Veritas, 2002), ship structures (Yuen et al., 2013) and much more.

However, it has been shown that this phenomenon actually is properly described by nonlinear model together with a Generalized Extreme Value (GEV) distribution, see Castillo et al. (1985), Castillo and Galambos (1987), Castillo (1988) and Castillo and Hadi (1994). Among the GEV distributions, the Weibull[3] distribution is the most common used in case of life time is the random variable under study, see Kotz and Nadarajah (2000) and Rinne (2008). The applicability of the Weibull distribution to model the fatigue strength is well presented in the book of Castillo and Fernández-Cantelli, see Castillo and Fernández-Canteli (2009).

According to the remarks presented above, the loading cycles N will follow a three parameter Weibull Distribution $W(a,b,c)$, whose cumulative distribution function (CDF) is given by

$$F_N(N_i) = P(N \le N_i) = 1 - \exp\left[-\left(\frac{N_i - a}{b}\right)^c\right], \quad x \ge a, \tag{14}$$

where

N : Random loading cycles
N_i : Fixed experimental loading cycles
$a \in \mathbb{R}$: Location or translation parameter, also known as threshold,
$b > 0$: Scale or statistical dispersion parameter,
$c > 0$: Shape parameter,
P : Probability of failure.

In other words, Eq. (14) gives the probability of reaching a fatigue life of at least N_i loading cycles.

Thus, the first goal is estimating the Weibull parameters corresponding to the fatigue life N for a given stress range $\Delta\sigma_i$.

3.1. Parameter estimation

Before making estimations about the fatigue life under a particular stress range, it is necessary to estimate the parameters of the Weibull distribution corresponding to this stress range.

By considering the random loading cycles given in the second column of Table 1a, the parameters of the three parameter Weibull Distribution for every stress range $\Delta\sigma_i$ can be estimated, see first column of Table 1b.

[3] Ernst Hjalmar Waloddi Weibull. Vittskövle (Sweden), 18.06.1887 - Annecy (France), 12.10.1979. Swedish engineer and mathematician.

Table 2
Chemical composition of the steel ST 52-3 used to manufacture the specimens.

Chemical composition of the specimens [%]			
C = 0,20	P = 0.017	Nb = 0.004	N = 0,006
M = 1.49	S = 0.016	V = 0.005	
Si = 0.41	Al = 0.030	Ti = 0.0003	

There are several methods to estimate these parameters, see Dupuis (1999), Gourdin et al. (1994), Gupta and Panchang (1989), Hirose (1996), Hosking et al. (1985), Marković et al. (2009), Offinger (1996), Rinne (2008) and Zanaquis and Kyparisis (1986). Within this paper the general probability weighted moments (PWM) method proposed in Toasa Caiza and Ummenhofer (2011) will be applied.

Once the Weibull parameters have been estimated, the quantile of the fatigue life $\hat{N}$ for a specific probability p can be estimated, see second column of Table 1a. Thus, by manipulating Eq. (14) and considering the estimated Weibull parameters, the fatigue life is estimated by

$$\hat{N}(P=p) = \hat{a} + \hat{b}\left[-\log(1-p)\right]^{\frac{1}{\hat{c}}}. \quad (15)$$

Finally, the pairs $(\Delta\sigma_i, \widehat{N_i})$ are considered to estimate the probabilistic Stüssi function by applying the linear regression method proposed in Section 2.1.

3.2. Goodman–Haigh diagrams

Similar to the method explained in Section 2.3, probabilistic Goodman–Hiagh diagrams can be plotted as well. In this case, the mean stresses and stress amplitudes have to be calculated by considering the fatigue life estimations $\widehat{N_i}$ in Eqs. (11) and (12).

4. Application on welded specimens

In order to apply and evaluate the suitability of the method proposed in the previous sections, the experimental data corresponding to welded details with a double side longitudinal stiffener with non-load carrying fillet were considered, see Rörup and Petershagen (2000) and Rörup (2005). The geometry of these details represent the permissible limit of notch condition in highly stressed ship structures.

4.1. Specimens

The test specimens were manufactured of steel St 52-3, whose chemical composition and mechanical properties are described in Tables 2 and 3. Since the application of the Stüssi model requires to know the fatigue limit $\Delta\sigma_\infty$, it is worth mentioning that in this application, the fatigue limit is assumed to be the equal the constant amplitude fatigue limit (CAFL), according to the definition given in the Eurocode EN-1993-1-9. In other words, the fatigue limit corresponds to the estimation of the CAFL with 95% confidence at $5 \cdot 10^6$ loading cycles.

According to the standards, the minimum yield strength of the steel St 52-3 has to be 355 MPa. The fillet joint was manually arc welded in the horizontal position by using an electrode of the type E 43 22R(C)3 according to DIN 1913. The throat thickness of the joint was 3,5 mm.

For the fatigue tests, details with a longitudinal stiffener with non-load carrying fillet welds were selected, see Fig. 5. These details represent the permissible notch condition limit occurred in highly stressed structures used to build ships. Furthermore, high residual tensile stresses could be expected in the fatigue critical area in small-scale specimens such as those considered in this work.

Table 3
Mechanical properties of the steel used to manufacture the specimens.

Mechanical properties of the steel St 52-3	
Yield strength R_{el}	406 MPa
Tensile strength R_m	579 MPa
Rupture elongation A_5	27%
Fatigue limit $\Delta\sigma_\infty$	58.57 MPa
Charpy V impact values at T = 20 °C	
1. Test	110 J
2. Test	100 J
3. Test	114 J

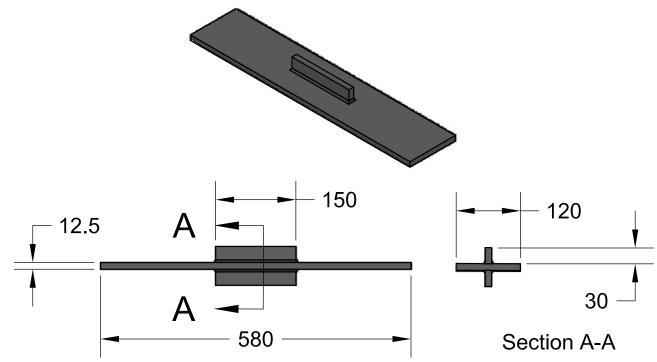


Fig. 5. Geometry of the specimen.

Table 4
Results of the fatigue tests under constant amplitude loading.

$\Delta\sigma$ [MPa]	$R=0$	$R=-1$	$R=-3$	$R=-\infty$
220	94 700	113 600	136 400	348 000
	108 600	115 000	138 000	350 000
	124 070	122 100	142 700	357 000
		123 300	154 700	370 000
		146 400	163 000	395 000
		151 400	166 500	398 000
		168 800	172 500	436 000
			185 500	457 000
140	306 300	381 500	409 000	755 400
	325 000	421 500	430 000	974 000
	375 000	457 900	501 000	1 021 000
	428 000	464 300	525 000	1 210 000
	539 100	480 600	526 000	1 311 000
	654 900	502 300	583 800	1 350 000
		520 000	588 500	1 520 000
			645 500	

(Loading cycles up to failure)

4.2. Fatigue experiments

The specimens were tested under controlled axial loading. Most of experiments was executed on a servo hydraulic machine at an average frequency of 6 Hz. While, some experiments were carried out on a resonance-testing machine at 33 Hz. The higher frequency did not have a noticeable influence on the fatigue life of the specimens. The applied stresses were nominal stresses corresponding to the cross section of the basic plate.

According to the data documentation given in Rörup and Petershagen (2000) and Rörup (2005), no information about strain gauges is presented, so that, it is assumed that misalignment of the specimen during the test is negligible.

The rupture of the specimen was used as failure criterion, which is considered as equivalent to a visible crack in a real ship structure. In the tests with stress ratio $R = -\infty$ the small or almost null maximum stress σ_{max} resulted in a minimum surface of ductile collapse. This fact leaded to a longer fatigue life, see Table 4.

Table 5
Geometrical parameters of the deterministic Stüssi model.

Geometrical parameters of the Stüssi model				
	$R = 0$	$R = -1$	$R = -3$	$R = -\infty$
α	0.001 16	0.003 93	0.000 97	0.000 056
β	0.6518	0.6185	0.6822	0.8223

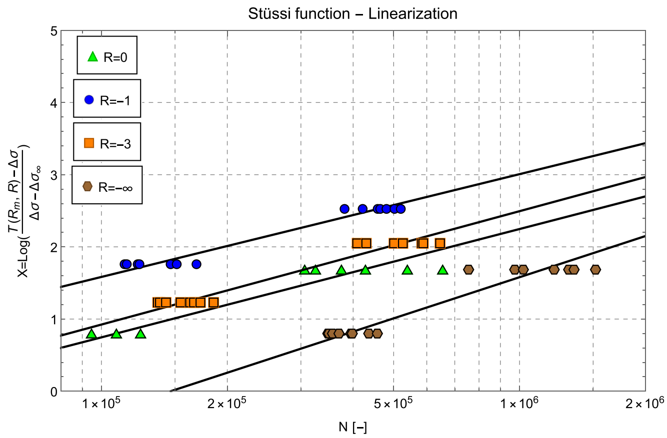


Fig. 6. Linear regression of the Stüssi function $S(N,R)$.

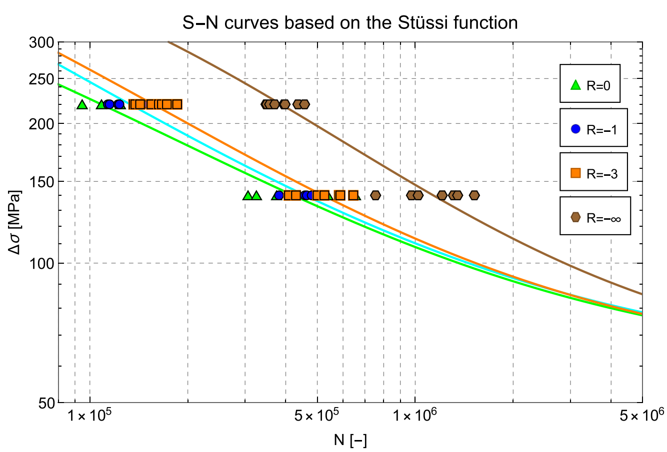


Fig. 7. S-N curves for different stress ratios.

4.3. Deterministic fatigue life estimation

As it was mentioned in Section 2.1, by performing a regression analysis the parameters A and B of Eq. (6) can be calculated. Then, by applying the Eqs. (7) and (8) the geometrical parameters are obtained. The Table 5 shows the obtained parameters.

The Fig. 6 shows the linear regressions corresponding to the experimental data.

Afterwards, the deterministic S-N curves based on the Stüssi function can be plotted, see Fig. 7.

The obtained S-N curves fit properly the experimental data. Nevertheless, the expected big variance of the data at 140 MPa at $R = 0$ and $R = \infty$ suggest that a statistical evaluation is necessary in order to obtain reliable estimations, see Section 4.4.

Besides the S-N curves, the proposed method allows to obtain Goodman–Haigh diagrams as it was explained in Section 2.3. Thus, the mean stresses and stress amplitudes corresponding to four constant lives can be calculated, see Table 6. The Goodman–Haigh diagram shows the expected fatigue behavior for every stress ratio.

Based on the values presented in Table 6, the Goodman–Haigh diagram can be plotted, see Fig. 8.

Table 6
Mean stresses and stress amplitudes for constant fatigue lives based on the deterministic Stüssi model.

Mean stresses and stress amplitudes for constant fatigue lives								
N	$R = 0$		$R = -1$		$R = -3$		$R = -\infty$	
	σ_μ	σ_a	σ_μ	σ_a	σ_μ	σ_a	σ_μ	σ_a
10^5	113	113	0	122.9	−65.3	130.7	−179.6	179.6
$5{\cdot}10^5$	66.3	66.3	0	68.1	−35.5	71	−98.8	98.8
$2{\cdot}10^6$	45.7	45.7	0	46.4	−23.4	46.7	−56.4	56.4
$5{\cdot}10^6$	38.6	38.6	0	39.1	−19.4	38.8	−42.8	42.8

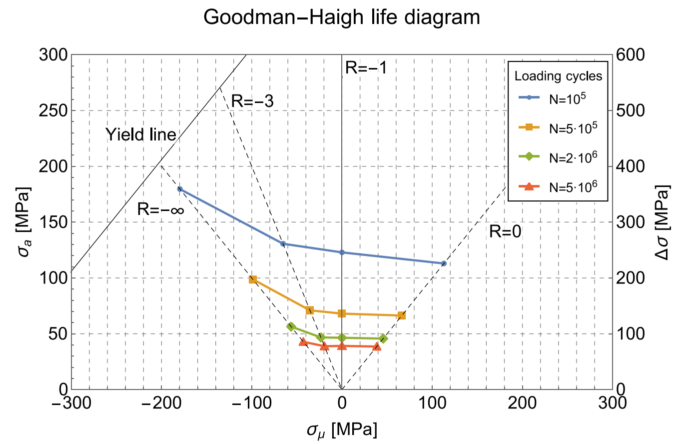


Fig. 8. Goodman–Haigh or constant fatigue life diagram.

Table 7
Weibull parameters of the probabilistic Stüssi model.

Weibull parameters of the Stüssi model				
	$R = 0$	$R = -1$	$R = -3$	$R = -\infty$
$\Delta\sigma = 220$ [MPa]				
$\hat{a}$	67 059.8	105 386	116 380	331 918
$\hat{b}$	47 348.7	30 985.4	46 314.6	61 376.8
$\hat{c}$	2.6163	1.227 02	2.308 93	1.270 73
$\Delta\sigma = 140$ [MPa]				
$\hat{a}$	264 779	−5 213 350	174 021	−545 095
$\hat{b}$	179 289	5 698 150	384 782	1 824 680
$\hat{c}$	1.094 79	137.363	4.711 44	7.101 48

4.4. Probabilistic fatigue life estimation

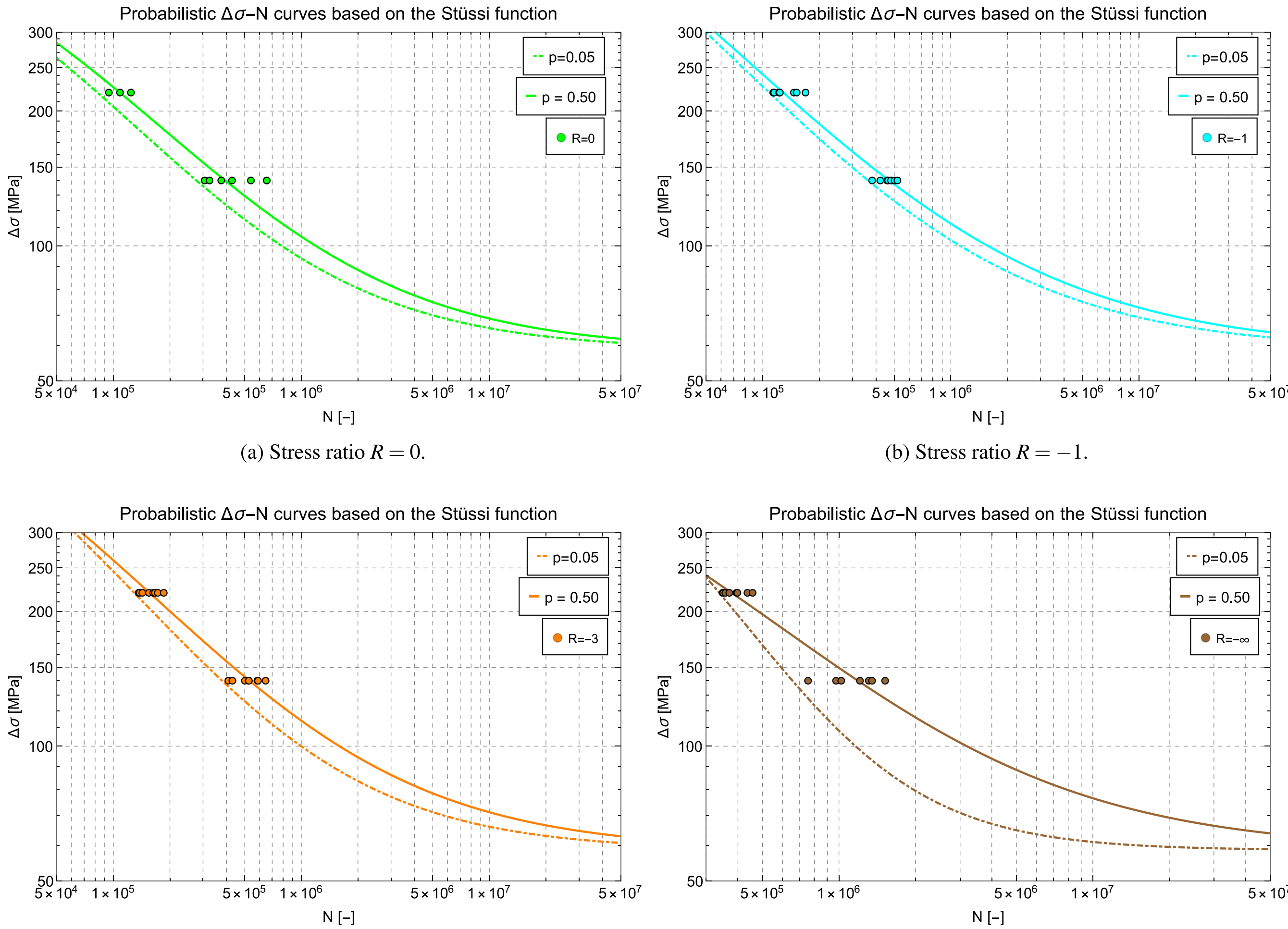
As it was mentioned in Section 3.1, probabilistic S-N curves can be plotted by considering the proposed method. To do that, the Weibull parameters corresponding to a particular stress range $\Delta\sigma_i$ have to be estimated, in this paper the PWM method is applied. Afterwards, the fatigue life for a given probability p can be estimated. The estimator of the Weibull parameters are shown in Table 7

After the estimation of the Weibull parameters, the quantile of the fatigue life N for probabilities of failure $p = 0.05$ and $p = 0.5$ were estimated according to Eq. (15). These fatigue lives are shown in Table 8.

Then, the geometrical parameters of the Stüssi model can be estimated, see Table 9

Afterwards, the probabilistic S-N curves based on the Stüssi function can be plotted, see Fig. 9.

By applying the proposed method, the geometry of the S-N curves are properly described. However, in case of $R = -\infty$ the big variance of the data corresponding to 140 MPa causes that the confidence intervals of the fatigue life increases notably under this stress range. This issue may be prevented by performing additional experiments at stress ranges below 100 MPa.



(a) Stress ratio $R = 0$.

(b) Stress ratio $R = -1$.

(c) Stress ratio $R = -3$.

(d) Stress ratio $R = -\infty$.

Fig. 9. Probabilistic S-N curves for different stress ratios $R = 0, -1, -3, -\infty$ and for a probability $p = 0.5, 0.05$.

Table 8
Fatigue life estimations for $p = 0.05$ and $p = 0.5$.

Fatigue life estimations				
$\widehat{N_i}$	$R = 0$	$R = -1$	$R = -3$	$R = -\infty$
$\Delta\sigma_1 = 220$ [MPa]				
$\widehat{N_1}(P = 0.05)$	82 274.7	108 140	129 175	337 846
$\widehat{N_1}(P = 0.5)$	108 219	128 371	155 896	377 917
$\Delta\sigma_2 = 140$ [MPa]				
$\widehat{N_2}(P = 0.05)$	276 673	362 917	378 866	655 908
$\widehat{N_2}(P = 0.5)$	393 060	469 621	530 004	1 187 810

Table 9
Geometrical parameters corresponding to the probabilistic Stüssi model.

Geometrical parameters of the Stüssi model				
	$R = 0$	$R = -1$	$R = -3$	$R = -\infty$
Probability p = 0.05				
α	0.000 573 098	0.003 790 23	0.000 436 961	0
β	0.730 15	0.632 809	0.761 764	1.334 73
Probability p = 0.5				
α	0.000 777 762	0.005 578 53	0.001 136 65	0.000 108 261
β	0.686 542	0.590 725	0.669 832	0.773 236

Finally, in order to plot the probabilistic Goodman–Haigh diagrams the estimations of mean stresses and stress amplitudes for a given probability can be calculated, see Table 10.

Table 10
Mean stresses and stress amplitudes for constant fatigue lives based on the probabilistic Stüssi model.

Mean stresses and stress amplitudes for constant fatigue lives								
N	$R = 0$		$R = -1$		$R = -3$		$R = -\infty$	
	σ_μ	σ_a	σ_μ	σ_a	σ_μ	σ_a	σ_μ	σ_a
Probability p = 0.05								
10^5	102.3	102.3	0	113.5	−61.4	122.8	−210.2	210.2
$5{\cdot}10^5$	57.3	57.3	0	63	−31.5	63	−84	84
$2{\cdot}10^6$	40.2	40.2	0	43.8	−20.9	41.8	−39.7	39.7
$5{\cdot}10^6$	35	35	0	37.5	−17.8	35.6	−32.5	32.5
Probability p = 0.5								
10^5	113	113	0	120.7	−65	130.1	−174.2	174.2
$5{\cdot}10^5$	64.6	64.6	0	68.6	−35.7	71.4	−98.5	98.5
$2{\cdot}10^6$	44.2	44.2	0	47.4	−23.6	47.2	−58	58
$5{\cdot}10^6$	37.4	37.4	0	40	−19.6	39.2	−44.3	44.3

The big variance of the data corresponding to $R = -\infty$ and $\Delta\sigma = 140$ MPa can be also observed in the Goodman–Haigh diagram, see the red lines in Fig. 10. As it was mentioned before, additional experiments at tress ranges below 100 MPa could prevent this issue and reduce the size of the corresponding confidence intervals.

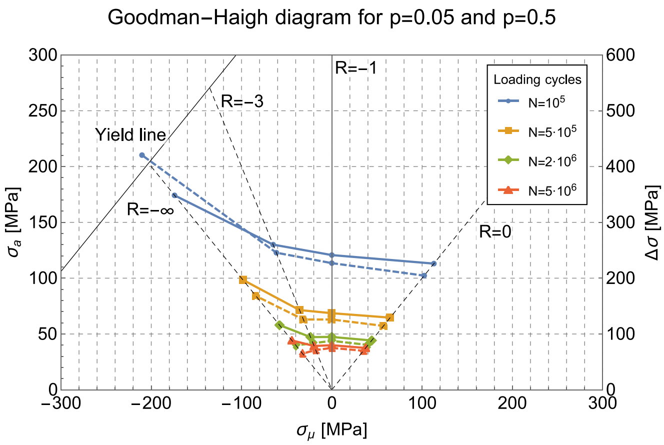


Fig. 10. Goodman–Haigh diagram for probabilities $p = 0.05$ and $p = 0.5$.

5. Conclusions and subsequent research

Analyzing and evaluating the influence of the stress ratio on the fatigue strength of steel structures represents a relevant design factor. It has been seen that most of the attempts to model the effect of the stress ratio are empirical and based on questionable assumptions, such as the linearity of the relationship stress-loading cycles, the normal distribution of the fatigue life and the inclusion of a enhancement factor. Under these circumstances, only a limited judgment of the experimental results can be performed, so that, the obtained estimations lack of reliability.

As an attempt to overcome the limitations of common models and trying to include the appropriate statistical concept, the authors of this work have proposed a probabilistic model based on the Stüssi function and on a Weibull distribution. The proposed method allows to depict the S-N curves and Goodman–Haigh diagrams for a given probability and any stress ratio. From the technical point of view, this model offers a reliable alternative to evaluate the effect of the stress ratio on the estimation of the fatigue life. However, in order to cover a wide fatigue range, it is mandatory to have data from a wide experimental frame. In other words, the fatigue experiments should be performed from the LCF to VHCF regimes. Thus, this method may help the engineers to improve the design criteria and motivate the experts to consider or at least to discuss the actual statistical results in the committees responsible for the standards updates.

Particularly, in this work the proposed method has been applied to evaluate the fatigue data from welded specimens made of steel St 52-3, which are typical used in ship structures subjected to high stresses. These data correspond only to finite fatigue life experiments performed under four different stress ratios. Although this fact limits the application of the method in a wider fatigue range, the results offer reliable estimations in the experimental frame.

Despite the promising results, it is necessary to perform further fatigue tests in LCF and HCF in order to provide additional arguments to support the feasibility of the proposed model. Moreover, subsequent research is necessary to study the influence of the Bauschinger effect and the role of the strain in the fatigue life in ULCF regime. Finally, in order to obtain more reliable estimations of the fatigue limit, additional experiments in HCF or VHCF regimes with different stress ratios have to be performed as well.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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