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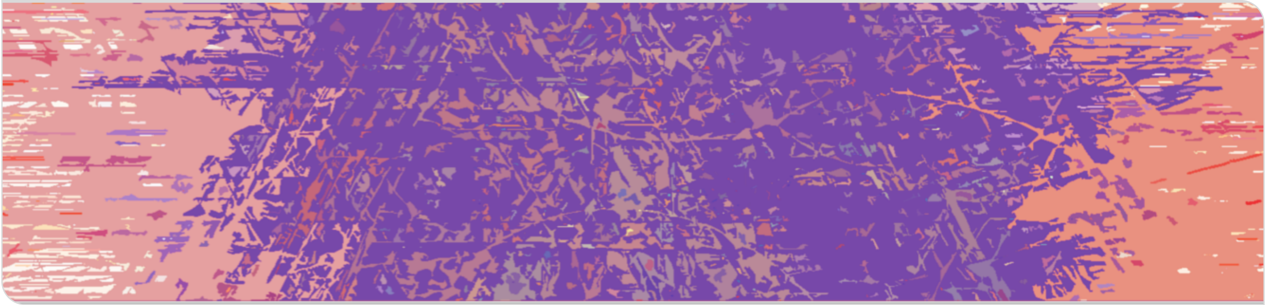
A Comprehensive Study of k-Portfolios of Recent SAT Solvers

SAT 2022 | Haifa, Israel

Jakob Bach, Markus Iser, and Klemens Böhm | August 2, 2022



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  - ... together with *lstech_maple* (Place 13 in Main Track)

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  - Model-based: Prediction model selects solver based on instance features

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$$\min_{x,y} \quad \frac{1}{|I|} \cdot \sum_{i \in I} \sum_{s \in S} c(i, s) \cdot x_{i,s}$$

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\forall s \in S: \quad & \sum_{i \in I} x_{i,s} \leq |I| \cdot y_s & \text{(only use solvers from portfolio)} \\
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Experimental Design

- Two datasets (from Main Tracks of recent SAT Competitions):
  1) *SC2020* (316 instances, 48 solvers) [4]
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- Four solution approaches:
  - *Optimal solution* via integer programming [19]
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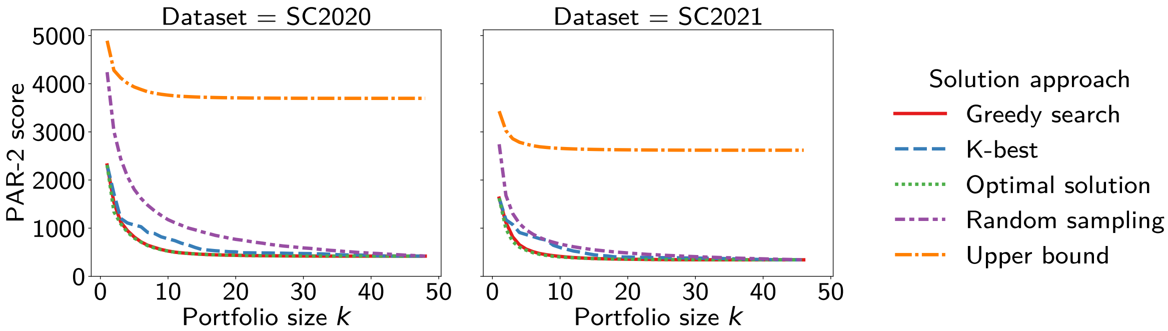
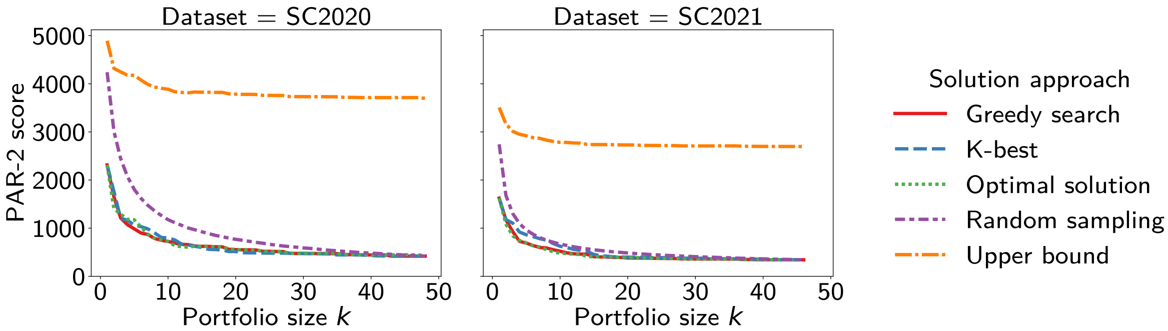
- Two multi-class prediction models: Random forests [6, 18] and XGBoost [7] with 100 trees each

Results – Portfolio Search (VBS on Training Set)



Training-set VBS performance for different datasets, values of k, and portfolio-search approaches.

Results – Portfolio Search (VBS on Test Set)



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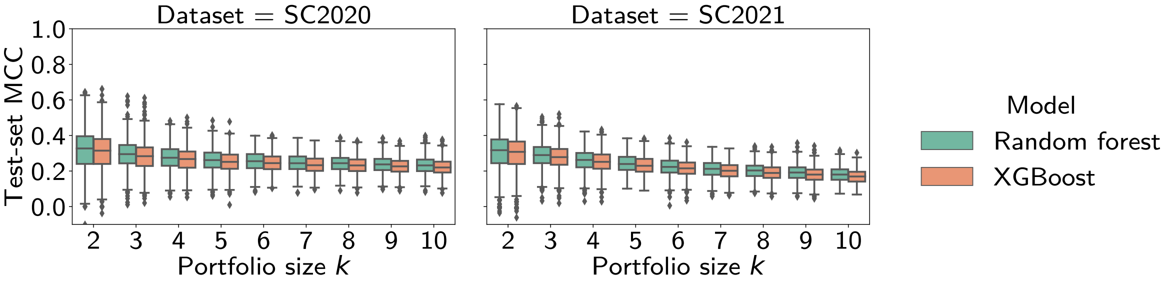
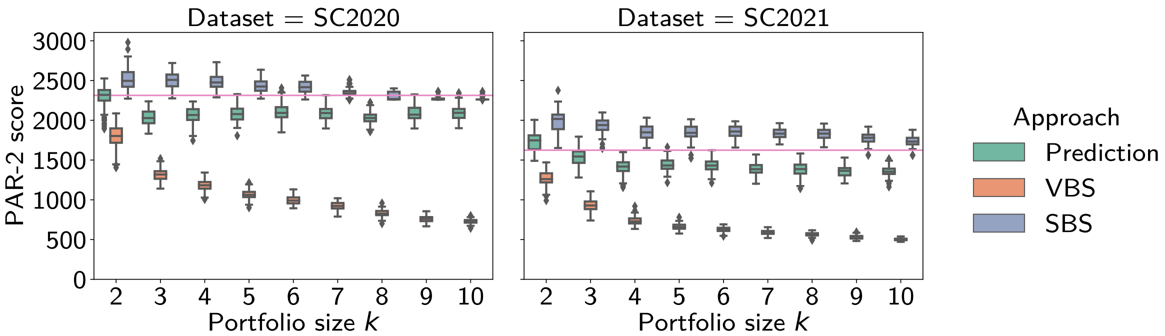
Results – Recommending Solvers (MCC)



Test-set prediction performance in terms of Matthews correlation coefficient (MCC) [15, 11] for different datasets, values of k, and prediction models. Randomly sampled portfolios.

Results – Recommending Solvers (PAR-2 Score)



Test-set solver performance for different datasets, values of k, and solver-recommendation approaches. Global SBS pictured as horizontal line. Portfolios from *beam search* with $w = 100$. Random forests for predictions.

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  - Compare to sophisticated portfolio approaches like SATzilla [23, 24]

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