DOI: 10.1002/pamm.202100221

Deformations of a piezo-actuated bearing ring in journal bearings

Alexander Bitner^{1,*} and Wolfgang Seemann¹

¹ Karlsruhe Institute of Technology, Institute of Engineering Mechanics, Karlsruhe, Germany

An elastically deformable bearing ring is investigated. The piezo-actuated shaping of the elastic bearing ring as well as the journal motion are coupled and determine the height of the lubrication gap which takes a crucial part in the theory of hydrodynamic lubrication. The influence of the active-dynamic shaping of the bearing ring on the rotor dynamic behaviour has been investigated in [1], especially for a two-lobe journal bearing. The present investigations are aiming at the activestatic shaping of a bearing ring, i.e. a finite number of piezo actuators is used to cause deformations of the bearing ring. Moreover, the realisation of different shapes by the introduced piezo actuators is examined, whereas the desired shapes are not necessarily limited to two-lobe geometries.

© 2021 The Authors. Proceedings in Applied Mathematics & Mechanics published by Wiley-VCH GmbH.

Introduction 1

Nowadays, fluid film bearings are used frequently in high-speed turbomachinery. However, such bearings are subject to various difficulties, e.g. self-excited vibrations and instabilities. To ensure the safe operation of the rotordynamic system, the influence of passively/actively varying bearing geometries has been investigated [1]. One way to change the shape of a bearing ring is the use of piezoelectric actuators. These piezoelectric elements can also be used as sensors in order to measure displacements at distinct points. With that measured displacements it is possible to estimate the shape of the bearing ring. Hence, with piezoelectric sensors and actuators smart, controllable bearings can be designed for the condition monitoring and control of rotordynamic systems.

Model and analytical approach 2

In the following a model of the bearing ring deformed by piezo actuators will be introduced. A scheme of such a bearing is depicted in figure 1 for instance with three evenly distributed actuators. The realisation of a desired shape requires the solution of an inverse problem in order to determine the voltages U_i to be applied.

The bearing ring is modelled as a weakly curved Euler-Bernoulli beam. The radial displacement $w(\varphi)$ and tangential displacement $u(\varphi)$ are assumed to be small. Due to the inextensibility assumption the radial displacement $w(\varphi)$ is described by an ordinary differential equation, whereas the tangential displacement $u(\varphi)$ is coupled by a kinematic constraint equation, namely

$$\frac{EI}{r^2} \left(\frac{\mathrm{d}^2 w}{\mathrm{d}\varphi^2} + w \right) = -M(\varphi) \quad \text{and} \quad \frac{\mathrm{d}u}{\mathrm{d}\varphi} + w = 0 \,. \tag{1}$$

The piezoelectric actuators acting at distinct angles φ_i are taken into account by radial and tangential forces

$$F_{i,r} = k_1 w(\varphi_i) + k_2 U_i$$
 and $F_{i,t} = k_3 u(\varphi_i)$

for i = 1, ..., n where the finite number of actuators is given by n.

The ring equations (1) were integrated piecewise in the regions $0 \le \varphi < \varphi_1, \varphi_i \le \varphi < \varphi_{i+1}, \dots, \varphi_n \le \varphi < \varphi_{n+1} = 2\pi$. Both the integration constants $C_{i,1}$, $C_{i,2}$, $C_{i,3}$, $C_{n+1,1}$, $C_{n+1,2}$, $C_{n+1,3}$ as well as the unknown stress resultants N_0 , Q_0 , M_0

distribution and reproduction in any medium, provided the original work is properly cited.

PAMM · Proc. Appl. Math. Mech. 2021;21:1 e202100221. https://doi.org/10.1002/pamm.202100221



© 2021 The Authors, Proceedings in Applied Mathematics & Mechanics published by Wiley-VCH GmbH.





Fig. 1: piezo-actuated bearing ring model

^{*} Corresponding author: e-mail alexander.bitner@kit.edu

 $[\]odot$ This is an open access article under the terms of the Creative Commons Attribution License, which permits use,

can be determined by the following boundary conditions

$$\varphi = \varphi_i: \quad w_i(\varphi_i) = w_{i+1}(\varphi_i), \qquad \qquad w'_i(\varphi_i) = w'_{i+1}(\varphi_i), \qquad \qquad u_i(\varphi_i) = u_{i+1}(\varphi_i), \\ \varphi = 2\pi: \quad w_{n+1}(2\pi) = w_1(0), \qquad \qquad w'_{n+1}(2\pi) = w'_1(0), \qquad \qquad u_{n+1}(2\pi) = u_1(0)$$

and the static equilibrium equations

2 of 3

$$\sum_{i=1}^{n} F_{i,r} \cos \varphi_i - F_{i,t} \sin \varphi_i = 0, \quad \sum_{i=1}^{n} F_{i,r} \sin \varphi_i + F_{i,t} \cos \varphi_i = 0, \quad \sum_{i=1}^{n} F_{i,t} = 0$$

As a result the radial displacement w is given as a piecewise function with respect to the coordinate φ and appears linearly dependent on the applied voltages U_i . Hence, it is possible to formulate a linear least-squares problem in order to minimize the total sum of residuals between the analytic radial displacement w and desired radial displacement \bar{w} at distinct angles φ_i

 $j = 1, \dots, m:$ $r_j = w(\varphi_j, U_1, U_2, \dots, U_n) - \bar{w}(\varphi_j) \quad \iff \mathbf{r} = \mathbf{A}\mathbf{U} - \bar{\mathbf{w}}$

where $\mathbf{r}, \mathbf{\bar{w}} \in \mathbb{R}^m, \mathbf{U} \in \mathbb{R}^n$ and $\mathbf{A} \in \mathbb{R}^{m \times n}$. This inverse problem has a unique solution given by $\mathbf{U} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{\bar{w}}$.

3 Results

Results are shown for a bearing ring with 8 attached piezo actuators. The desired radial displacement is given by

$$\bar{w}(\varphi) = \frac{w_0}{3} \sum_{k=1}^{3} \sin(\lambda_k(\varphi - \varphi_0))$$

with $w_0 = 20 \,\mu\text{m}$, $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 3$ and $\varphi_0 \in [0, 2\pi)$. Analytic displacements and desired displacements in fig. 2 are in very good agreement for all values of φ_0 , i.e. for any tilted position of the bearing ring. The maximum error between analytic and desired displacements is about 1%. The voltages that have to be applied are plotted in fig. 3 with respect to the parameter φ_0 .



Fig. 2: displacements of the bearing ring for $\varphi_0 = 0$



The other parameters used in the calculations are given by n = 8, m = 360, r = 0.05 m, h = 0.0025 m, d = 0.01 m, E = 205 GPa, $I = \frac{h^3 d}{12}$, $n_{disc} = 25$, b = 0.001 m, $d_{33} = 400 \cdot 10^{-12} \frac{\text{C}}{\text{N}}$, $S_{33}^E = 27.7 \cdot 10^{-12} \frac{\text{m}^2}{\text{N}}$, $k_1 = -\frac{\pi d^2}{n_{disc} b S_{33}^E}$, $k_2 = \frac{n_{disc} \pi d_{33}}{b S_{33}^E}$, $k_3 = 10^{-9} k_1$.

4 Conclusions

In order to achieve a desired shape specific voltages U_i must be applied. These voltages can uniquely be determined by a linear least-squares problem. Changing the number of piezo actuators reveals that the shape ability is limited by a cutoff frequency $\lambda_c < \frac{n}{2}$ so that not all shapes can be realized in any tilted position. By increasing the number of actuators the cutoff frequency can be increased and the errors between the analytic and desired displacements can be decreased. Determining the right number of actuators also depends on deformations that are caused by the pressure and shear forces present in fluid film bearings. This aspect will be considered in future investigations.

Acknowledgements Open access funding enabled and organized by Projekt DEAL.

References

[1] BECKER, K. (2019). Dynamisches Verhalten hydrodynamisch gelagerter Rotoren unter Berücksichtigung veränderlicher Lagergeometrien (Dissertation, Karlsruher Institut für Technologie (KIT)). DOI: 10.5445/IR/1000117516.