$B^0 - \bar{B}^0$ entanglement for an ideal experiment for the direct CP violation ϕ_3/γ phase

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B^0–$\bar{B}^0$ entanglement offers a conceptual alternative to the single charged B-decay asymmetry for the measurement of the direct CP-violating γ/ϕ_3 phase. With $f = J/\Psi K_L, J/\Psi K_S$ and $g = (\pi\pi)^0, (\rho_L\rho_L)^0$, the 16 time-ordered double-decay rate intensities to (f, g) depend on the relative phase between the f- and g-decay amplitudes given by γ at tree level. Several constraining consistencies appear. An intrinsic accuracy of the method at the level of $\pm 1°$ could be achievable at Belle-II with an improved determination of the penguin amplitude to g channels from existing facilities.

I. INTRODUCTION

There is considerable interest in improving the precision for the direct CP violation ϕ_3/γ phase $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$ in the $b - d$ unitarity triangle of the Cabibbo-Kobayashi-Maskawa flavor mixing matrix of quarks [1,2]. This angle connects the sides for decay amplitudes of the B system dominated by tree diagrams, so that its measurement is a bona fide determination of the Standard Model (SM) parameters. This is important in order to search for new physics in the loop contributions of penguin—rare decays—and box—mixing—diagrams.

The most precise result from a single analysis at the LHCb experiment is [3] $\gamma = (65.4^{+3.8}_{-4.2})°$. It uses the Giri-Grossman-Soffer-Zupan method [4] for $B^\pm \to DK^\pm$ with the choice of $D \to K_S\pi^+\pi^-$, $D \to K_SK^+K^-$ three-body decays. In charged B decays, the observation of CP violation (CPV) needs [5] the interference of two amplitudes with different weak phases—changing sign from particles to antiparticles—and strong phases—invariant under CP. This mismatch is what originates a CP-violating asymmetry in the corresponding decay rates for B^+ and B^-. In this case D represents a D^0 or $\bar{D}^0$ meson reconstructed from a final state that is common to both, D^0 and $\bar{D}^0$ being produced, respectively, by $b \to c\bar{u}s$ and $b \to u\bar{c}s$ tree-level diagrams. The parameters of their mixing have been simultaneously determined in the analysis in Ref. [3].

The amplitude of the decay $B^- \to DK^-$, $D \to K_S h^+ h^-$ can be written as a sum of $B^- \to D^0 K^-$ and $B^- \to \bar{D}^0 K^-$ contributions as

$$A_B(m_-^2, m_+^2) = A_D(m_-^2, m_+^2) + r_B e^{i(\delta_B - \gamma)} \bar{A}_D(m_-^2, m_+^2), \quad (1)$$

where $m_\pm^2$ are the squared invariant masses of the $K_S h^\pm$ particle combinations that define the position of the decay in the Dalitz plot. The parameter r_B is the ratio of the magnitudes of the $B^- \to \bar{D}^0 K^-$ and $B^- \to D^0 K^-$ amplitudes, δ_B being their strong relative phase. Neglecting CP violation in charm decays, the charge-conjugated amplitudes for B^+ decay satisfy $\bar{A}_D = A_D$. The sensitivity to γ is obtained by comparing the distributions in the Dalitz plots of D decays from B^+ and B^- mesons. As a consequence, the variation of the strong phase within the Dalitz plot is needed. Complementary information from measurements performed by CLEO [6] and BESIII [7–9] is available. Alternative methods [10–12] correspond to different choices for the decay channels of the D's.

In this paper, we discuss an ideal conceptual experiment for γ by exploiting the $B^0 - \bar{B}^0$ Einstein-Podolsky-Rosen (EPR) entanglement [13]. The use of the EPR correlation was proposed in Refs. [14–17] for several decay channels in the B factories. Entanglement has been instrumental in the past for the observation of time-reversal violation by the *BABAR* Collaboration [18] using the concept and method given in Refs. [19–21] and with far-reaching information [22–24]. The method for γ consists in the observation of the coherent double decay to flavor-nonspecific products. In it, the extraction of the γ phase is free from the essential strong phases contamination needed in charged B decays. The necessary interference between amplitudes containing the

$V_{cd}V_{cb}^*$ and $V_{ud}V_{ub}^*$ sides of the unitarity triangle is automatic from the two terms of the entangled $B^0 - \bar{B}^0$ system. The double rate intensity to the (f, g) and (g, f) pairs of CP-eigenstate decay products, with $f = J/\psi K_S, J/\psi K_L$ and $g = \pi^+\pi^-, \pi^0\pi^0, \rho_L^+\rho_L^-, \rho_L^0\rho_L^0$, will do the job from CP-conserving and CP-violating transitions, as we demonstrate below. The measurement of the time-ordered intensities for these $16 = 2(f) \times 4(g) \times 2$(time ordering) combined processes is rich in physics and consistencies, leading to the relative phase γ responsible of direct CP violation. The Belle-II experiment at the upgraded KEK facility would have the opportunity to perform the analysis presented here if enough integrated luminosity is accumulated in the coming years.

II. TIME-ORDERED DOUBLE-DECAY INTENSITIES

The choice of the $\rho_L\rho_L$ channels together with $\pi\pi$ channels is motivated by their common CP properties, as seen in the change of basis [25] from the two-particle states with definite helicity to $L - S$ coupling

$$\langle JM; LS|JM; \lambda_1\lambda_2 \rangle = \sqrt{\frac{2L+1}{2J+1}} C(LSJ|0\lambda)$$
$$\times C(s_1 s_2 S|\lambda_1 - \lambda_2). \quad (2)$$

In particular, for helicities $\lambda_1 = \lambda_2 = 0$, the $\rho\rho$ system from B decay is in states with $L = S = 0, 2$, so it has definite symmetry properties under $C = +$, $P = +$, and $CP = +$. Therefore, we may use a unified theoretical framework for the discussion of the time-ordered intensities associated to the double decays (f, g) and (g, f) with decay times t_1 and t_2 such that $\Delta t \equiv t_2 - t_1 > 0$. For any g decay products, the choice of $f = J/\psi K_L$ defines for the Δt living partner a CP-forbidden transition, whereas $f = J/\psi K_S$ corresponds to a CP-allowed transition.

Taking the transition amplitude from the $C = -$entangled $B^0 - \bar{B}^0$ state to the time-ordered decay products f and g, its square and integration over the initial decay time at fixed $\Delta t \equiv t$ leads to the double-decay intensity [24]

$$I(f, g; t) = \frac{e^{-\Gamma|t|}}{16\Gamma|pq|^2} |e^{i\Delta Mt/2}A_H^f A_L^g - e^{-i\Delta Mt/2}A_L^f A_H^g|^2, \quad (3)$$

with Γ the common decay width of the eigenstates $B_H = pB^0 + q\bar{B}^0$ and $B_L = pB^0 - q\bar{B}^0$ with definite time evolution, $\Delta M = M_H - M_L$ their mass difference, and $A_{H,L}^{f,g} = \langle f, g|T|B_{H,L}\rangle$ their decay amplitudes. In the absence of CP violation in the mixing for this system, $|p/q| = 1$. As anticipated, this intensity presents interference terms between f and g, either direct or through mixing. With $\Delta\Gamma = 0$ for B_d decays, there are time-independent and oscillatory terms in t with different physics. Because of the definite (anti)symmetry of the $C = -$entangled state,

Eq. (3) satisfies the following expected symmetry property: The combined transformation $t \to -t$ and $f \leftrightarrow g$ is the identity, hence the interest in the separate measurements of $I(f, g; t) + I(g, f; t)$ and $I(f, g; t) - I(g, f; t)$ in order to separate even and odd terms in t. As a consequence, we find it convenient to express Eq. (3) in the basis $\{\cos^2(\Delta Mt/2), \sin^2(\Delta Mt/2), \sin(\Delta Mt)\}$ of time dependencies as

$$\hat{I}(f, g; t) \equiv \frac{\Gamma}{\langle\Gamma_f\rangle\langle\Gamma_g\rangle} e^{\Gamma|t|} I(f, g; t)$$
$$= \mathcal{I}_d^{fg}\cos^2\frac{\Delta Mt}{2} + \mathcal{I}_m^{fg}\sin^2\frac{\Delta Mt}{2}$$
$$+ \mathcal{I}_{od}^{fg}\sin(\Delta Mt), \quad (4)$$

where $\langle\Gamma_f\rangle$ is the average decay probability to f from B^0 and $\bar{B}^0$. $\hat{I}(f, g; t)$ is a reduced intensity, with $\mathcal{I}_d^{fg} = \mathcal{I}_d^{gf}$, $\mathcal{I}_m^{fg} = \mathcal{I}_m^{gf}$, and $\mathcal{I}_{od}^{fg} = -\mathcal{I}_{od}^{gf}$ the "intensity parameters" for each decay pair. The $\mathcal{I}_d$ parameter shows up since the $t = 0$ separation between the two decays for each (f, g) pair, so it is the signal for a direct correlation between the decay amplitudes.

We introduce the usual mixing $\times$ decay interference $\lambda \equiv \frac{q}{p}\frac{\bar{A}}{A}$ from B^0 and $\bar{B}^0$ relevant at any time, for each decay amplitude—either λ_f or λ_g. In terms of the complex λ, we have the three combinations

$$C = \frac{1 - |\lambda^2|}{1 + |\lambda^2|}, \qquad R = \frac{2\text{Re}(\lambda)}{1 + |\lambda^2|}, \qquad S = \frac{2\text{Im}(\lambda)}{1 + |\lambda^2|}, \quad (5)$$

with the constraint $C^2 + R^2 + S^2 = 1$. The calculation of the intensity parameters in Eq. (4) for each double-decay rate (f, g) is then obtained from the combinations (5) as

$$\mathcal{I}_d^{fg} = \frac{1}{2}(1 - R_f R_g - S_f S_g - C_f C_g)$$
$$\mathcal{I}_m^{fg} = \frac{1}{2}(1 - R_f R_g + S_f S_g + C_f C_g)$$
$$\mathcal{I}_{od}^{fg} = \frac{1}{2}(S_f C_g - C_f S_g). \quad (6)$$

Whereas $\mathcal{I}_d$ and $\mathcal{I}_m$ contain real number terms and then select the real part of the time evolution, $\mathcal{I}_{od}$ contains imaginary terms selecting the imaginary part of the time evolution. In addition, we observe that the time-even parameters are symmetric under the $f \leftrightarrow g$ exchange, and the odd parameter is antisymmetric, as anticipated.

III. DECAY AMPLITUDES—ISOSPIN ANALYSIS

The decay channels $f = J/\psi K_L, J/\psi K_S$ are known to be well described by their tree-level amplitudes, which satisfy $|\lambda_f| = 1$, i.e., $C_f = 0$. As an important consequence of Eq. (6), a nonvanishing $\mathcal{I}_{od}$ intensity parameter is trapping penguin amplitudes through their modulus

contribution $C_g \neq 0$ ($|\lambda_g| \neq 1$) for any decay channel g. The phase of λ_f is to a great accuracy the mixing phase $q/p = e^{-2i\phi_M}$, with $\phi_M = \beta$ in the SM. Assuming also $\Delta F = \Delta Q$, that is, no wrong sign decays, we have

$$\lambda_S = -\lambda_L = -e^{-2i\phi_M} \tag{7}$$

imposed by the two opposite CP eigenvalues for the decay products $J/\psi K_L$ and $J/\psi K_S$. The surviving terms in Eq. (6) are linear in λ_f, implying *consistency relations* for the absolute and relative normalizations of the intensity parameters:

$$\mathcal{I}_d^{Lg} + \mathcal{I}_d^{Sg} = 1, \quad \mathcal{I}_m^{Lg} + \mathcal{I}_m^{Sg} = 1, \quad \mathcal{I}_{od}^{Lg} + \mathcal{I}_{od}^{Sg} = 0, \ \forall \, g, \tag{8}$$

leading, in turn, to consistencies for the double-decay time-dependent reduced intensities

$$\hat{I}(L, g; t) + \hat{I}(S, g; t) = 1, \quad \forall \, g, \quad \forall \, t. \tag{9}$$

Using the exchange symmetry properties, Eqs. (8) and (9) are also valid for the time-ordered $(g; L, S)$ decays. They provide a controlled connection between the CP-forbidden and CP-allowed time-dependent transitions for any of the four decay products g.

The λ_g amplitudes ($g = \pi\pi, \rho_L\rho_L$) can be parametrized as

$$\lambda_g = \rho_g e^{-i2(\phi_M + \phi_g)}, \tag{10}$$

where ϕ_g is a weak phase in the decay $b \to u\bar{u}d$ and both $\rho_g \neq 1$ and $\phi_g \neq \gamma$ are due to the penguin contributions. At tree level, all g states considered here would have $\phi_g = \gamma$. Notice that the $(R_f R_g + S_f S_g)$ combination appearing in the $\mathcal{I}_d^{fg}$ intensity parameter is blind to the phase of q/p and it directly probes

$$\lambda_{L \atop S} \lambda_g^* = \pm \rho_g e^{i2\phi_g}, \tag{11}$$

where the $\pm$ corresponds to $f = L, S$, respectively. As anticipated, no mediation of the mixing is present in the $\mathcal{I}_d$ parameter of the intensity. Thus, the determination of this direct correlation between the two decay products in Eq. (4) for these processes becomes

$$\mathcal{I}_d^{Sg \atop L} = \frac{1}{2}\left[1 \mp \frac{2\rho_g}{1 + \rho_g^2}\cos(2\phi_g)\right], \tag{12}$$

where clearly the mixing is not present.

If penguin contributions were not relevant, we would have at tree level

$$\mathcal{I}_d^{Lg} = \sin^2\gamma \quad \text{for all } CP\text{-forbidden transitions},$$

$$\mathcal{I}_d^{Sg} = \cos^2\gamma \quad \text{for all } CP\text{-allowed transitions}. \tag{13}$$

With the expected g-dependent penguin contributions through both ρ_g and $\gamma - \phi_g \equiv \epsilon_g$, to be discussed below, Eq. (12) provides a powerful consistency from the four g's and the two f's for the extraction of the CPV γ phase.

Let us focus now on the different information to be accessed by the measurement of the other intensity parameters. In the case of $\mathcal{I}_m$, the combination in Eq. (6) involves the $\lambda_f \lambda_g$ product, which connects the f, g decay amplitudes through the mixing. The use of Eqs. (7) and (10) leads to

$$\mathcal{I}_m^{Sg \atop L} = \frac{1}{2}\left[1 \mp \frac{2\rho_g}{1 + \rho_g^2}\cos(4\phi_M + 2\phi_g)\right]. \tag{14}$$

The result (14) depends on the phase $2\phi_M + \phi_g$, indicating explicitly that the $\mathcal{I}_m$ parameter denotes a correlation between the two f and g decay channels induced through the mixing. As already advertised, $\mathcal{I}_m^{Lg} + \mathcal{I}_m^{Sg} = 1 \ \forall \, g$, as for the other term even in time.

The two (f, g) time-even intensity parameters combine in the observable sum of intensities for the time-ordered exchange of decay products $f \leftrightarrow g$. We obtain the result

$$\begin{aligned}
\hat{I}(f, g; t) &+ \hat{I}(g, f; t) \\
&= 2[\mathcal{I}_d^{fg}\cos^2(\Delta M t/2) + \mathcal{I}_m^{fg}\sin^2(\Delta M t/2)] \\
&= 1 \mp \frac{2\rho_g}{1 + \rho_g^2}\{\cos(2\phi_g)\cos^2(\Delta M t/2) \\
&\quad + \cos(4\phi_M + 2\phi_g)\sin^2(\Delta M t/2)\},
\end{aligned} \tag{15}$$

for $f = L, S$ correspondingly. As seen, the contributions of the direct CPV phase ϕ_g and the mixing-induced CPV phase $2\phi_M + \phi_g$ separate in two different time-dependent behaviors, the second naturally needing a time slice to become apparent. For any of the two f channels and the four g channels, these two terms are separately apparent when

$$\frac{\Delta M t}{2} = n\pi, \qquad \frac{\Delta M t}{2} = (2n + 1)\frac{\pi}{2}, \tag{16}$$

with $n = 0, 1, 2\ldots$.

The third intensity parameter $\mathcal{I}_{od}$ can be separated out from the difference of the two time-ordered intensities:

$$\begin{aligned}
I(f, g; t) - I(g, f; t) &= 2\mathcal{I}_{od}^{Lg \atop S}\sin(\Delta M t) \\
&= \mp \left[\frac{1 - \rho_g^2}{1 + \rho_g^2}\sin(2\phi_M)\right]\sin(\Delta M t),
\end{aligned} \tag{17}$$

where $\mathcal{I}_{od}^{Lg} + \mathcal{I}_{od}^{Sg} = 0 \ \forall \, g$ in this case. It is worth remarking that this intensity parameter would vanish iff the penguin contribution were absent in the g decay channels. As the CPV mixing $\sin 2\phi_M$ ($\sin 2\beta$ in the

SM) is the best measured parameter in this field, Eq. (17) can be used to measure the deviation of ρ_g in each of the four g channels from 1, induced by the penguin amplitude, and check its prediction from the isospin analysis given below. Consistently, the measurement of observable (17) for both $f = L$ and $f = S$ has to reproduce a change of sign, providing, in particular, the relative normalization of events in these two decay channels.

Besides the factor depending on ρ_g, the observables are also affected by the penguin amplitudes in a departure of the phase ϕ_g from a common γ/ϕ_3 through

$$\epsilon_g = \gamma - \phi_g, \qquad (18)$$

to be extracted from a dedicated isospin analysis. The procedure follows the original ideas of Gronau and London along the path described in Refs. [26,27]. The neutral and charged B-meson decays differ in the presence versus absence, respectively, of the penguin contribution to the amplitudes for each final $h = \pi, \rho_L$ system. The charged decay amplitudes $A^{+0} = A(B^+ \to h^+ h^0)$ and $\bar{A}^{+0} = A(B^- \to h^- h^0)$ have a final $(h^\pm h^0)$ isospin-2 state and, therefore, only the $\Delta I = 3/2$ tree-level amplitude contributes with the weak phase γ. It is convenient to define, with the same notation for both neutral decay channels $\pi\pi$ and $\rho_L \rho_L$ and using $g = \pm$ or 00 for the corresponding decay charges:

$$a_g = \frac{A_g}{A_{+0}}; \qquad \bar{a}_g = \frac{\bar{A}_g}{\bar{A}_{+0}}, \qquad (19)$$

in such a way that the double ratio gives

$$\rho_g e^{2i\epsilon_g} = \frac{\bar{a}_g}{a_g}. \qquad (20)$$

The isospin triangular relations with these complex ratios are

$$\frac{1}{\sqrt{2}} a_{+-} = 1 - a_{00}; \qquad \frac{1}{\sqrt{2}} \bar{a}_{+-} = 1 - \bar{a}_{00}. \qquad (21)$$

Equations (21) allow one to get a_g and $\bar{a}_g$ by using all the branching ratios of the processes $B^\pm \to h^\pm h^0$; B^0, $\bar{B}^0 \to h^+ h^-$, $h^0 h^0$. In Table I, we give the summary of our isospin analysis with the present Particle Data Group data

[28]. Taking into account that the $\rho_L^+ \rho_L^-$ channel is the one with larger branching ratio, we must conclude that the error in $\epsilon_{\rho_L^+ \rho_L^-}$, $\delta\epsilon_{\rho_L^+ \rho_L^-} = 0.091 = 5.2°$, gives us an estimate of the uncertainty due to the present knowledge of the penguin pollution in the determination of γ/ϕ_3. An important improvement in the branching ratios entering in the isospin analysis is expected as an outcome of Belle-II and LHC experiments that will reduce this error significantly.

IV. RESULTS FOR THE EXTRACTION OF γ

The intrinsic accuracy of the method proposed in this paper is controlled by our ability to extract ϕ_g. In order to estimate the expected uncertainty in that extraction, we proceed as follows (further details are provided in the Appendix). First, we fix input values of ϕ_M and γ. For each decay channel $g = \rho_L^+ \rho_L^-$, $\rho_L^0 \rho_L^0$, $\pi^+ \pi^-$, $\pi^0 \pi^0$, we also fix input values of ρ_g and ϵ_g, which fix $\phi_g = \gamma - \epsilon_g$, following the three different benchmark cases in Table II. Next, considering the decay channels $f = L, S$, we compute the six coefficients $\mathcal{I}^{Sg}_{d,m,od}$, $\mathcal{I}^{Lg}_{d,m,od}$, which control the four time-dependent decay channels (f, g), (g, f), for each g. Then, for each g, a given number of events is generated according to the four double-decay intensities. The procedure is repeated in order to produce our simulated data, from which $\mathcal{I}^{Sg}_{d,m,od}$ are extracted including uncertainties and $\mathcal{I}^{Lg}_{d,m,od}$ are given by Eq. (8). Finally ρ_g, ϕ_g, and ϕ_M are obtained with a simple fit. Notice that the intensity parameters $\mathcal{I}_{d,m,od}$ depend, respectively, on ϕ_g, $\phi_M + \phi_g$, and ϕ_M phases. Therefore, the inclusion of the $\mathcal{I}_m$ term together with $\mathcal{I}_d$ in the fit allows one to avoid the discrete degeneracy $\phi_g \to \phi_g + \pi$, with the information of the quadrant for ϕ_M.

We show the results of our analysis in two scenarios A and B taking into account the Belle-II projected luminosity [29–31] and the corresponding branching ratios: Scenario A assumes 1000 $\rho_L^+ \rho_L^-$ events of type $B_{\rho\rho}$, 50 $\rho_L^0 \rho_L^0$ events of type $B_{\rho\rho}$, 200 $\pi^+ \pi^-$ events of type $B_{\pi\pi}^-$, and 50 $\pi^0 \pi^0$ events of type $B_{\pi\pi}^-$. Scenario B assumes 500 $\rho_L^+ \rho_L^-$ events of type $B_{\rho\rho}$ and 100 $\pi^+ \pi^-$ events of type $B_{\pi\pi}^+$. The results of the fit to the generated $\mathcal{I}^{fg}_{d,m,od}$ in both scenarios are given in Table III. From the results in scenario A, we conclude that, since $\gamma = \phi_g + \epsilon_g$, the error $\delta\phi_{\rho_L^+ \rho_L^-} = 0.020 = 1.1°$ gives an idea of the intrinsic statistical limiting error we would expect in the determination of γ for the assumed number of events. Combining $\phi_{\rho_L^+ \rho_L^-}$ with $\epsilon_{\rho_L^+ \rho_L^-} = 0.008 \pm 0.091$ would bring the error in γ to the present error in $\epsilon_{\rho_L^+ \rho_L^-}$, hence the importance of its improvement, as already mentioned.

TABLE I.   Summary of isospin analysis results.

g	ρ_g	ϵ_g
$\rho_L^+ \rho_L^-$	1.007 ± 0.076	0.008 ± 0.091
$\rho_L^0 \rho_L^0$	0.972 ± 0.241	0.007 ± 0.345
$\pi^+ \pi^-$	1.392 ± 0.062	$\pm(0.307 \pm 0.170)$
$\pi^0 \pi^0$	1.306 ± 0.206	$\pm(0.427 \pm 0.172)$

TABLE II.   Benchmark cases used in the numerical simulations.

Benchmark	ρ_g	ϵ_g	g
$B_{\rho\rho}$	1	0	$\rho_L^+ \rho_L^-, \rho_L^0 \rho_L^0$
$B_{\pi\pi}^+$	1.35	$+0.35$	$\pi^+ \pi^-, \pi^0 \pi^0$
$B_{\pi\pi}^-$	1.35	-0.35	$\pi^+ \pi^-, \pi^0 \pi^0$

TABLE III.   Results of the fit.

g	Scenario A		Scenario B	
	ϕ_g	ρ_g	ϕ_g	ρ_g
$\rho_L^+ \rho_L^-$	1.222(020)	1.00(06)	1.222(31)	1.00(08)
$\rho_L^0 \rho_L^0$	1.22(09)	1.00(24)		
$\pi^+ \pi^-$	1.57(12)	1.35(12)	0.87(07)	1.36(35)
$\pi^0 \pi^0$	1.57(18)	1.35(24)		
	$\phi_M = 0.384(31)$		$\phi_M = 0.384(40)$	

Even before these improvements, we can do better and fit the three $\mathcal{I}_{\mathrm{d,m,od}}^{fg}$ for all channels in terms of γ, ϵ_g, and ρ_g including all the information of the isospin analysis. In this case, the result is $\gamma = 1.222 \pm 0.080 = (70.0 \pm 4.6)°$. Note that the error on γ is smaller than the error in $\epsilon_{\rho_L^+ \rho_L^-}$ due to a unique γ in all channels, which presents a quantitative conclusion: The present proposal could provide a measurement of γ below the 1° error if the errors in the isospin analysis can be reduced to the level of $\delta\phi_{\rho_L^+ \rho_L^-} \simeq 1°$. For the more conservative scenario B, we get an intrinsic error $\delta\phi_{\rho_L^+ \rho_L^-} = 1.8°$. Again, using all the information used in the isospin analysis and ϕ_M, we estimate $\gamma = 1.221 \pm 0.085 = (70.0 \pm 4.9)°$, which reinforces the idea that, with this method, it could be statistically possible to go below 1° of precision in the determination of γ, thanks to the expected improvements in the data entering in the isospin analysis.

From the theoretical side, the inclusion of penguin pollution in the reference channels $J/\psi K_{S,L}$ is done with the phase substitutions $\phi_M \to \phi'_M = \phi_M - \Delta\phi_d/2$ and $\phi_g \to \phi'_g = \phi_g + \Delta\phi_d/2$, where $|\Delta\phi_d/2| \leq 0.34°$ has been estimated in Ref. [32] and comfortably below the statistical 1°. The corresponding moduli corrections to the right-hand sides in Eqs. (8) are below 10^{-2}. For the most important channel $\rho_L \rho_L$, these equations remain valid up to corrections of the order of 10^{-3} due to additional suppression factors like $(1 - \rho_g^2)/(1 + \rho_g^2)$ and $\sin(2(\phi_g + \phi_M))$. Effects from the corrections to the leading $(q/p)_{B,K}$ values are below the considered penguin pollution.

V. OUTLOOK

In conclusion, with B^0–$\bar{B}^0$ entanglement, we consider the double-decay rate intensity to flavor-nonspecific channels governed by the c and u quarks. It offers a conceptual alternative to the decay of single $B^{\pm}$ mesons for the extraction of the direct CPV γ/ϕ_3 phase. The needed interference between two decay amplitudes is provided by the exchanged terms of the entangled state, and no strong phases appear as essential ingredients. The eight time-symmetric (f, g) intensities with $f = J/\Psi K_L, J/\Psi K_S$, $g = (\pi\pi)^0$, $(\rho_L \rho_L)^0$ have a tree-level common γ phase, $g = \rho_L^+ \rho_L^-$ being the benchmark channel. Several constraining consistencies among the different intensities appear. We find that an intrinsic accuracy of the order of 1° could be achievable for the relative phase of the f and g amplitudes. The present limitation of $\pm 5°$, to be improved by the existing experimental facilities, comes from the phase of the penguin contribution in the g amplitude, extracted from an isospin analysis to neutral and charged B decays.

APPENDIX: SIMULATION

As discussed in the main text, the intrinsic limitation of the method is controlled by our ability to extract ϕ_g. The procedure to estimate the expected uncertainty in that extraction is the following.

(1) We fix input values of $\phi_M = \beta = 0.384$ and $\gamma = 1.222$, and, for each decay channel $g = \rho_L^+ \rho_L^-$, $\rho_L^0 \rho_L^0$, $\pi^+ \pi^-$, $\pi^0 \pi^0$, we also fix input values of ρ_g and ϵ_g, which fix $\phi_g = \gamma - \epsilon_g$. We consider the three different benchmark cases in Table II.

(2) Considering the decay channels $f = L, S$, the six coefficients $\mathcal{I}_{\mathrm{d,m,od}}^{Sg}$, $\mathcal{I}_{\mathrm{d,m,od}}^{Lg}$ are computed: They control the four time-dependent combinations (f, g), (g, f), for each g.

(3) For each g, we generate values of t, the events, distributed according to the four double-decay intensities. In order to incorporate the effect of experimental time resolution, each t is randomly displaced following a normal distribution with zero mean and $\sigma = 1$ ps. Additional experimental effects such as efficiencies are not included. Generation proceeds until a chosen number of events N_g with $|t| \leq 5\tau_{B^0}$ has been obtained with the four (f, g), (g, f) combinations altogether. These N_g events are binned.

(4) The procedure is repeated in order to obtain mean values and standard deviations in each bin: These constitute our simulated data, as illustrated in Fig. 1, which corresponds to $g = \rho_L^+ \rho_L^-$ (benchmark $B_{\rho\rho}$ in Table II), $N_g = 1000$ events and 20 bins in $[0; 5\tau_{B^0}]$. The black dots with bars are the mean values and uncertainties, the red curves are the extracted double-decay intensities, and the blue curves correspond to the $\mathcal{I}_d^{fg}$ term in each intensity. There are no significant differences if one considers, for example, 15 or ten bins.
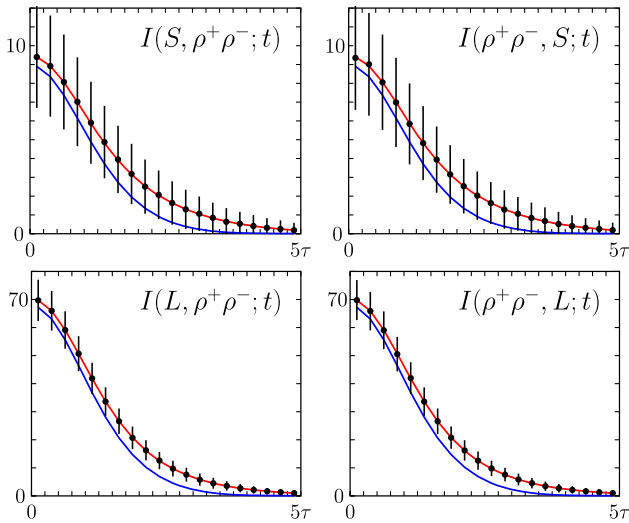
FIG. 1. Simulated data, 1000 events, benchmark $B_{\rho\rho}$. Black dots with bars indicate mean values and associated uncertainties; the red curves are the extracted double-decay intensities, while the blue curves correspond to the $\mathcal{I}_{\mathrm{d}}^{fg}$ term in each intensity.

(5) From the simulated data, one can obtain $\mathcal{I}_{\mathrm{d}}^{S\rho_L^+\rho_L^-}=0.1170\pm0.0138$, $\mathcal{I}_{\mathrm{m}}^{S\rho_L^+\rho_L^-}=0.1658\pm0.0456$, and $\mathcal{I}_{\mathrm{od}}^{S\rho_L^+\rho_L^-}=0.000\pm0.0198$, with $\mathcal{I}_{\mathrm{d,m,od}}^{L\rho_L^+\rho_L^-}$ given by Eq. (8), and similarly for decay channels $\rho_L^0\rho_L^0$, $\pi^+\pi^-$, $\pi^0\pi^0$ according to the different benchmarks $B_{\rho\rho}$ and $B_{\pi\pi}^\pm$ in Table II.

(6) Finally, we extract ρ_g, ϕ_g, and ϕ_M, with a simple fit to the $\mathcal{I}_{\mathrm{d,m,od}}^{Sg}$.

Concerning the number of events, with the Belle-II design luminosity [29] and the branching ratios BR(g) and BR(f), we assume that it would be possible to collect 1000 events for $g=\rho_L^+\rho_L^-$, 200 events for $g=\pi^+\pi^-$, and 50 events for both $g=\rho_L^0\rho_L^0$ and $g=\pi^0\pi^0$ channels. We show the results of our analyses for two scenarios.

(i) Scenario A assumes 1000 $\rho_L^+\rho_L^-$ events of type $B_{\rho\rho}$, 50 $\rho_L^0\rho_L^0$ events of type $B_{\rho\rho}$, 200 $\pi^+\pi^-$ events of type $B_{\pi\pi}^-$, and 50 $\pi^0\pi^0$ events of type $B_{\pi\pi}^-$.

(ii) In scenario B, we assume 500 $\rho_L^+\rho_L^-$ events of type $B_{\rho\rho}$ and 100 $\pi^+\pi^-$ events of type $B_{\pi\pi}^+$.

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